An Introduction to Chiral Magnetic Effect

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A game of collective rotation

Magnetic field

Rotation of many charged particles

Hydro-dynamics,

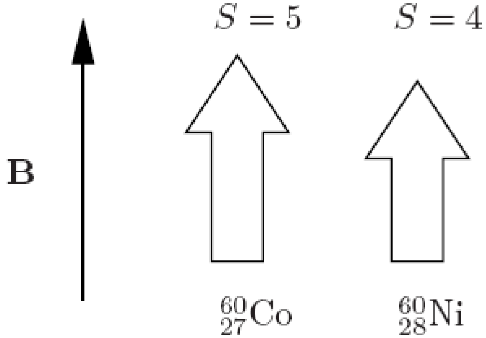
Kinetic Theory

Orbital angular momentum

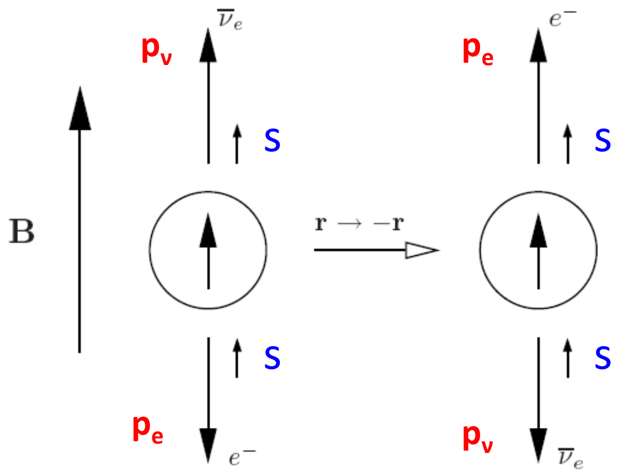
Outline

- A little history
- Topological configurations of gauge fields and instanton solution
- Chiral Magnetic Effect: an introduction
- Approaches to CME/CVE: Field theory, Hydrodynamics, Quantum kinetic theory
- Other related and new phenomena

P-violation in beta-decay



$$^{60}_{27}\mathrm{Co} \rightarrow {}^{60}_{28}\mathrm{Ni} + e^- + \overline{\nu}_e$$



C. S. Wu, et al,
Phys.Rev. 105(1957)1413-1414

C.S. Wu, 1957

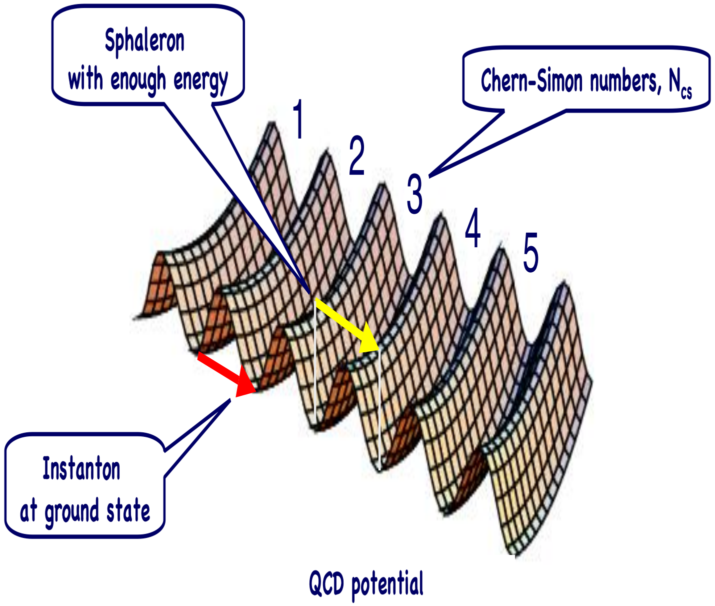

T.D. Lee, C.N. Yang
Nobel prize 1957

Experimental Result: p_e || (-B), implying
(1) Neutrinos are left-handed, p || (-s)
(2) Anti-neutrinos are right-handed, p || s

Milestone discovery in history of weak interaction
➔ Unification of electromagnetic and weak interaction

Vacuum of non-Abelian gauge field

- Vacuum of gauge field has topological structure which can be characterized by Chern-Simons (CS) number
- Transition between vacua is quantum tunneling effect → quantum anomaly → P and CP violatiion
- Two transitions: instanton (zero T) and sphaleron (hight T)
- In electroweak theory, quantum anomaly → lepton and baryon non-conservation → leptogenesis and baryogenesis in the early universe
- Quantum anomaly in QCD → axial charge non-conservation



QCD vacuum has topological structure

High T + strong B → local P/CP violation of strong interaction in HIC?

A little history

- Existence topologically non-trivial configurations of gauge fields [Belavin, Polyakov, Shvarts, Tyupkin, Phys. Lett. B 59 (1975) 85]
- Tunneling between topologically different configurations ['t Hooft, Phys. Rev. Lett. 37 (1976) 8; 't Hooft, Phys. Rev. D 14 (1976) 3432; 't Hooft, Phys. Rev. D 18 (1978) 2199, Erratum]
- Transition between different configuration state $\rightarrow$ changes in the topological charge $\rightarrow$ anomalous processes (processes forbidden by the classical process are allowed in quantum theory) [Adler, Phys. Rev. 177 (1969) 2426; Bell, Jackiw, Nuovo Cimento A 60 (1969) 47]

A little history

- Quantum tunneling between topologcally different configurations as non-perturbative phenomena is suppressed by $e^{-2\pi/\alpha}$ where α is the interaction strength of the gauge theory. High temperature $\rightarrow$ disappearance of exponential suppression (there is sufficient energy to pass classically over the barrier)
  [Manton, Phys. Rev. D 28 (1983) 2019; Klinkhamer, Manton, Phys. Rev. D 30 (1984) 2212]

A little history

- The rate is sufficiently large: important for electroweak baryogenesis [Kuzmin, Rubakov, Shaposhnikov, Phys. Lett. B155(1985)36; Shaposhnikov, Nucl. Phys. B287(1987)757; Arnold, McLerran, Phys. Rev. D36(1987)581; Arnold, McLerran, Phys. Rev. D37(1988)1020]
- Electroweak baryon number violation $\leftrightarrow$ Helicity non-conservation in QCD [McLerran, E. Mottola, M.E. Shaposhnikov, Phys. Rev. D 43 (1991) 2027]

A little history

- Topological charge changing process involves P-odd and CP-odd field configurations $\rightarrow$ Existence of axion [Peccei, Quinn, Phys. Rev. D16 (1977) 1791; Weinberg, Phys. Rev. Lett. 40 (1978) 223; Wilczek, Phys. Rev. Lett. 40 (1978) 279; Kim, Phys. Rev. Lett. 43 (1979) 103; Shifman, Vainshtein, Zakharov, Nucl. Phys. B 166 (1980) 493]
- It is thus possible that an excited vacuum domain which may be produced in heavy ion collisions can break P and CP spontaneously [T.D. Lee, Phys. Rev. D 8 (1973) 1226]

A little history

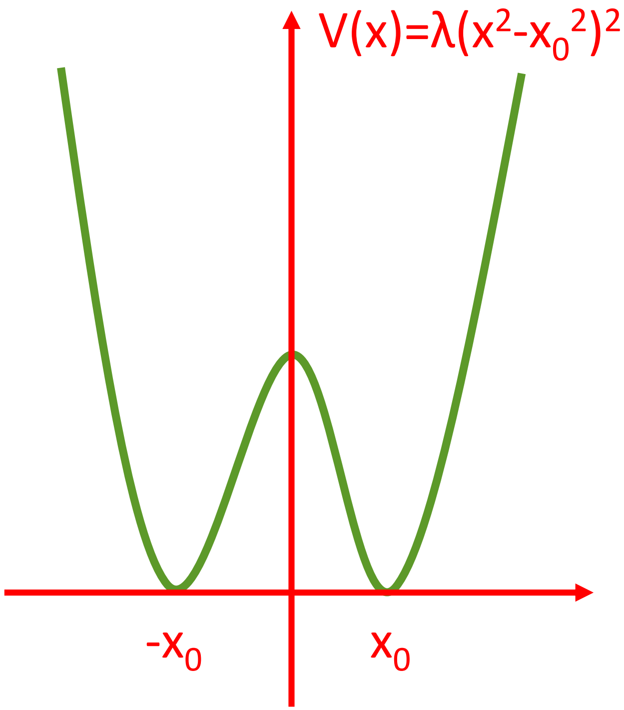
- In deconfinement phase transition, QCD vacuum can possess metastable domains or P-odd bubbles (space–time domains with non-trivial winding number).
  [Kharzeev, Pisarski, M.H.G. Tytgat, Phys. Rev. Lett. 81 (1998) 512]
- P-odd bubbles does not contradict the Vafa–Witten theorem (P and CP cannot be broken in the true ground state of QCD for $\theta = 0$), which does not apply to QCD dense and hot matter where Lorentz-non-invariant P-odd operators have non-zero expectation values. [Vafa, Witten, Phys. Rev. Lett. 53 (1984) 535; Vafa, Witten, Nucl. Phys. B 234 (1984) 173; Son, Stephanov, Phys. Rev. Lett. 86 (2001) 592]

A little history

- Examples of P-odd or C-odd ground states in hot or dense matter: P-odd pion-condensation phase, P-odd color superconducting phase, P-even but C-odd metastable states in hot gauge theory. [Migdal, Rev. Mod. Phys. 50 (1978) 107; Pisarski, Rischke, Phys. Rev. Lett. 83 (1999) 37; Bronoff, Korthals Altes, Phys. Lett. B 448 (1999) 85]
- Non-zero angular momentum (or equivalently of magnetic field) in heavy ion collisions → P-odd and CP-odd domains induce charge separation in the produced particles, because they carry net chirality and break symmetry between right-hand and left-hand fermions. [Voloshin, Phys. Rev. C 70 (2004) 057901; Kharzeev, Phys. Lett. B 633 (2006) 260]

Double well in quantum mechanics

- Is mirror symmetry x→-x broken in the ground state?



$$
\begin{aligned}
E_0 &= \frac{1}{2}\omega\left[1-\sqrt{\frac{2\omega^3}{\pi\lambda}}\underline{\exp\left(-\frac{\omega^3}{12\lambda}\right)}\right] \\
&\Longrightarrow \psi_0=\psi_+ + \psi_- \Rightarrow \langle x\rangle = 0 \\
E_1 &= \frac{1}{2}\omega\left[1+\sqrt{\frac{2\omega^3}{\pi\lambda}}\underline{\exp\left(-\frac{\omega^3}{12\lambda}\right)}\right] \\
&\Longrightarrow \psi_1=\psi_+ - \psi_-
\end{aligned}
$$

Non-perturbative correction unaccesible in perturbation

Topological configurations of gauge fields

- We consider SU(2) pure gauge field theory in Euclidean space.
- Field and strength tensor (dual tensor)

$$A_\mu = \frac{1}{2}\sigma_a A_\mu^a, \;\; F_{\mu\nu} = \frac{1}{2}\sigma_a F_{\mu\nu}^a$$

$$\begin{aligned} F_{\mu\nu}^a &= \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g\varepsilon_{abc}A_\mu^b A_\nu^c \\ F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu] \end{aligned}$$

$$\tilde{F}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}F_{\alpha\beta}$$

Topological configurations of gauge fields

- Gauge transformation

$$\begin{aligned} A'_\mu &= U A_\mu U^{-1} + \frac{i}{g} U \partial_\mu U^{-1} \\ F'_{\mu\nu} &= U F_{\mu\nu} U^{-1} \end{aligned}$$

- Define Chern-Simons current

$$\begin{aligned} K_\mu &= \frac{1}{4}\epsilon_{\mu\nu\alpha\beta}\left(A_\nu^a \partial_\alpha A_\beta^a + \frac{g}{3}\varepsilon_{abc} A_\nu^a A_\alpha^b A_\beta^c\right) \\ &= \epsilon_{\mu\nu\alpha\beta}\mathrm{Tr}\left(\frac{1}{2}A_\nu \partial_\alpha A_\beta - i\frac{g}{3} A_\nu A_\alpha A_\beta\right) \end{aligned}$$

- Satisfying

$$\boxed{\partial_\mu K_\mu = \frac{1}{4}\mathrm{Tr} F_{\mu\nu}\tilde{F}_{\mu\nu} = \frac{1}{8} F_{\mu\nu}^a \tilde{F}_{\mu\nu}^a}$$

Topological configurations of gauge fields

- Proof of CS current equation

$$
\begin{aligned}
\partial_\mu K_\mu &= \frac{1}{4}\epsilon_{\mu\nu\alpha\beta}\left[(\partial_\mu A_\nu^a)(\partial_\alpha A_\beta^a) + g\varepsilon_{abc}(\partial_\mu A_\nu^a)A_\alpha^b A_\beta^c\right] \\
\frac{1}{8}F_{\mu\nu}^a \tilde{F}_{\mu\nu}^a &= \frac{1}{16}\epsilon_{\mu\nu\alpha\beta}(\partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g\varepsilon_{abc}A_\mu^b A_\nu^c) \\
&\quad \times(\partial_\alpha A_\beta^a - \partial_\beta A_\alpha^a + g\varepsilon_{abc}A_\alpha^b A_\beta^c) \\
&= \frac{1}{4}\epsilon_{\mu\nu\alpha\beta}(\partial_\mu A_\nu^a)(\partial_\alpha A_\beta^a) + \frac{1}{8}\boxed{g\epsilon_{\mu\nu\alpha\beta}\varepsilon_{abc}(\partial_\mu A_\nu^a)A_\alpha^b A_\beta^c} \\
&\quad + \frac{1}{8}\boxed{g\epsilon_{\mu\nu\alpha\beta}\varepsilon_{abc}(\partial_\alpha A_\beta^a)A_\mu^b A_\nu^c} + \cancel{\frac{1}{8}g^2\epsilon_{\mu\nu\alpha\beta}A_\mu^b A_\nu^c A_\alpha^b A_\beta^c}
\end{aligned}
$$

equal

equal

Topological configurations of gauge fields

- Topological charge

$$
\begin{aligned}
q &= \frac{g^2}{16\pi^2}\int d^4x \mathrm{Tr} F_{\mu\nu}\tilde{F}_{\mu\nu} \\
&= \frac{g^2}{4\pi^2}\int d^4x \partial_\mu K_\mu = \frac{g^2}{4\pi^2}\int d^3\sigma_\mu K_\mu
\end{aligned}
$$

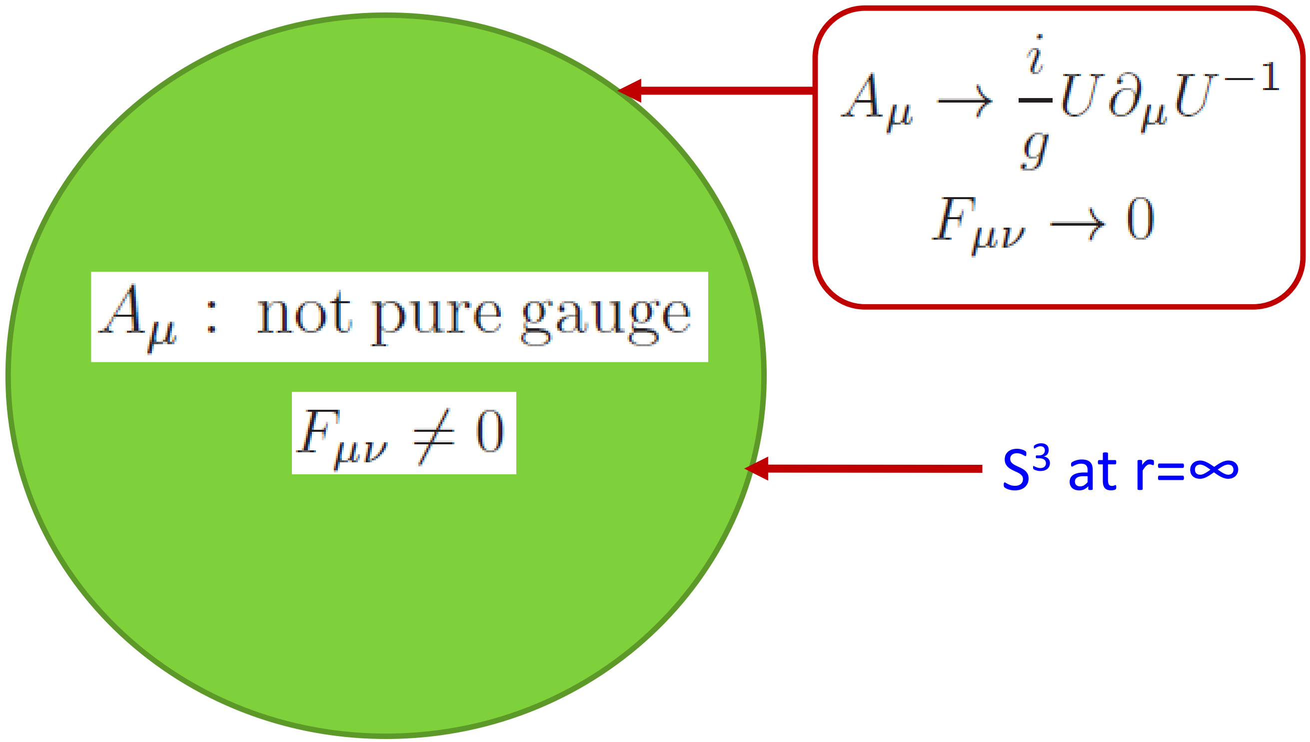
3-dim hyper-surface at infinity

- Impose condition for finite energy

$$
\begin{aligned}
F_{\mu\nu} &\underset{x\to\infty}{\Longrightarrow} O(x^{-3}) \qquad x^2 = \mathbf{x}^2 + x_4^2 \\
A_\mu &\underset{x\to\infty}{\Longrightarrow} \frac{i}{g}U\partial_\mu U^{-1} + O(x^{-2})
\end{aligned}
$$

Topological configurations of gauge fields

- At r → ∞, we have A_μ→ pure gauge, $F_{\mu\nu}$ →0,



Topological configurations of gauge fields

- Use $\bar{A}_\mu = U\partial_\mu U^{-1}, \;\; A_\mu = \frac{i}{g}\bar{A}_\mu$
- The CS current becomes

$$\begin{aligned} K_\mu &= \epsilon_{\mu\nu\alpha\beta}\mathrm{Tr}\left(\frac{1}{2}A_\nu\partial_\alpha A_\beta - i\frac{g}{3}A_\nu A_\alpha A_\beta\right) \\ &= \frac{1}{6g^2}\epsilon_{\mu\nu\alpha\beta}\mathrm{Tr}\left(\bar{A}_\nu\bar{A}_\alpha\bar{A}_\beta\right) \end{aligned}$$

- Topological charge depends on U(x)

$$q(U) = \frac{1}{24\pi^2}\int_{S^3} d^3\sigma_\mu \epsilon_{\mu\nu\alpha\beta}\mathrm{Tr}\left(\bar{A}_\nu\bar{A}_\alpha\bar{A}_\beta\right)$$

Topological configurations of gauge fields

- Since U(x) is a SU(2) matrix, we have a map

$$U(x \in S^3) \in SU(2) = S^3$$

$$\mathrm{map}: \ S^3 \Longrightarrow S^3$$

- which fall into homotopic classes labeled by winding number

Topological configurations of gauge fields

- Calculate topological charge

$$\begin{aligned} q(U) &= \frac{1}{24\pi^2}\int_{S^3} d^3\sigma n_\mu \epsilon_{\mu\nu\alpha\beta}\mathrm{Tr}\left[(U\partial_\nu U^{-1})(U\partial_\alpha U^{-1})(U\partial_\beta U^{-1})\right] \\ &= \frac{1}{24\pi^2}\int d^3\sigma \frac{\partial(g)}{\partial(\sigma)} = \frac{1}{24\pi^2}\int_G d^3 g = \text{integer} \end{aligned}$$

- From q=1 to q=n map

$$\begin{aligned} U_1(x) &= \frac{1}{x}(x_4 + i\mathbf{x}\cdot\boldsymbol{\sigma}) \\ &= \cos\theta + i(\hat{\mathbf{x}}\cdot\boldsymbol{\sigma})\sin\theta = e^{-i\theta\hat{\mathbf{x}}\cdot\boldsymbol{\sigma}} \\ \Rightarrow & \quad U_n = U_1^n(x) \end{aligned}$$

(q=1)

(q=n)

$$x^2 = \mathbf{x}^2 + x_4^2$$

$$|\hat{\mathbf{x}}|^2 = 1$$

Topological configurations of gauge fields

- Let's check q(U) for U_1

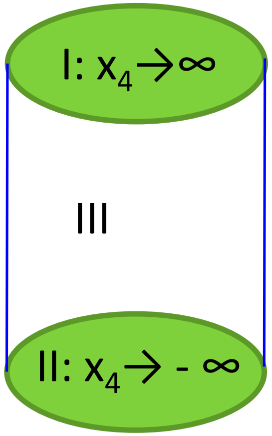
$$\boxed{\sigma_\mu = (i\boldsymbol{\sigma}, 1),\ \sigma_\mu^+ = (-i\boldsymbol{\sigma}, 1)}$$

$$\begin{aligned}
\bar{A}_\mu &= U_1 \partial_\mu U_1^{-1} = \frac{1}{x}(x_4 + i\mathbf{x}\cdot\boldsymbol{\sigma})\partial_\mu \frac{1}{x}(x_4 - i\mathbf{x}\cdot\boldsymbol{\sigma}) \\
&= \frac{1}{x^2}[(x\cdot\sigma)\sigma_\mu^+ - x_\mu] \\
K_\mu &= \frac{1}{6g^2}\epsilon_{\mu\nu\alpha\beta}\mathrm{Tr}\left(\bar{A}_\nu \bar{A}_\alpha \bar{A}_\beta\right) = \frac{2x_\mu}{g^2 x^4} \\
q(U_1) &= \frac{g^2}{4\pi^2}\int_{S^3} d^3\sigma_\mu K_\mu = \frac{1}{2\pi^2}\int_{S^3} d^3\sigma_\mu \frac{x_\mu}{x^4} = 1
\end{aligned}$$

Topological configurations of gauge fields

- BPST instanton solution

$$
\begin{aligned}
A_\mu &= \frac{i}{g}\frac{x^2}{x^2+\rho^2}U_1\partial_\mu U_1^{-1} \\
&\Longrightarrow \frac{i}{g}U_1\partial_\mu U_1^{-1} \quad (x\to\infty,\ \text{pure gauge})
\end{aligned}
$$



- Calculate q(U) in a different way

$$
\begin{aligned}
q(U) &= \frac{1}{24\pi^2}\int_{S^3} d^3\sigma_\mu \epsilon_{\mu\nu\alpha\beta}\mathrm{Tr}\left(\bar{A}_\nu \bar{A}_\alpha \bar{A}_\beta\right) \\
&= \frac{1}{24\pi^2}\left(\int_I - \int_{II}\right) d^3\sigma\epsilon_{4ijk}\mathrm{Tr}\left(\bar{A}_i \bar{A}_j \bar{A}_k\right) + \cancel{\frac{1}{24\pi^2}\int_{III} d^3\sigma_i\epsilon_{i\nu\alpha\beta}\mathrm{Tr}\left(\bar{A}_\nu \bar{A}_\alpha \bar{A}_\beta\right)} \\
&\Longrightarrow \frac{1}{24\pi^2}\left(\int_I - \int_{II}\right) d^3\sigma\epsilon_{4ijk}\mathrm{Tr}\left(\bar{A}_i \bar{A}_j \bar{A}_k\right)
\end{aligned}
$$

make A_4=0 by gauge-trans
one of $\nu\alpha\beta$ must be 4

Topological configurations of gauge fields

- By gauge transformation to make A_4=0, we obtain

$$\begin{aligned} A_i &\rightarrow iU_{n-1}\partial_i U_{n-1}^{-1}, \ \ x_4 \rightarrow -\infty \\ A_i &\rightarrow iU_n \partial_i U_n^{-1}, \ \ x_4 \rightarrow \infty \end{aligned}$$

- where $U_n = U_1^n, \ \ U_1 = \exp\left[i\pi \frac{\mathbf{x}\cdot\boldsymbol{\sigma}}{(x^2+\rho^2)^{1/2}} \right]$
- The instanton describes a solution of gauge field equation in which a vacuum with homotopy class n-1 at $x_4=-\infty$ to a vacuum with homotopy class n at $x_4=\infty$

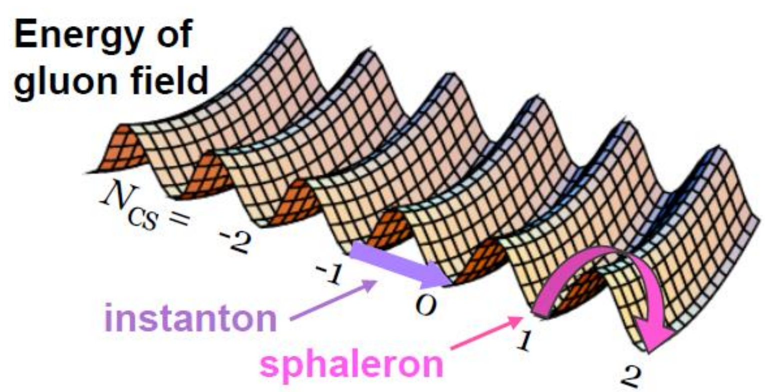
$$\boxed{q(U) = n - (n-1) = 1}$$

Topological configurations of gauge fields

- Self-dual property of instanton solution $F_{\mu\nu} = \tilde{F}_{\mu\nu}$
- The action in Euclidean space-time

Vacuum transtion probability: highly suppressed

$$
\begin{aligned}
S &= -\frac{1}{2}\int d^4x \mathrm{Tr} F_{\mu\nu}F_{\mu\nu} \\
&= -\frac{1}{2}\int d^4x \mathrm{Tr} F_{\mu\nu}\tilde{F}_{\mu\nu} = -\frac{8\pi^2}{g^2}, \quad \text{for } q(U)=1 \\
\Longrightarrow & \\
\boxed{P = \exp\left(-\frac{8\pi^2}{g^2}\right)}
\end{aligned}
$$



- high T, Sphaleron transition greatly enhanced

θ vacuum of QCD

- Geniune vacuum state of QCD

$$\psi_\theta = \sum_n e^{-in\theta}\psi_n \quad \Rightarrow \quad L_\theta = -\frac{1}{4}F^a_{\mu\nu}\tilde{F}^a_{\mu\nu} + \theta\frac{g^2}{32\pi}F^a_{\mu\nu}\tilde{F}^a_{\mu\nu}$$

- P and CP is broken by θ-term, θ≈0 (very small) [Neutron EDM not observed]
- Strong CP problem in QCD or the naturalness of QCD: Why is θ is so small?
- Axion conjecture. [Peccei-Quinn mechanism, 1977]

Chirality and Helicity

- Chiraltiy $\psi_L = \frac{1}{2}(1-\gamma^5)\psi, \quad \psi_R = \frac{1}{2}(1+\gamma^5)\psi$

- Helicity $h = \boldsymbol{\sigma} \cdot \frac{\mathbf{p}}{|\mathbf{p}|}$

- In the chiral limit (massless quark) with $m_f = 0$

Helicity	RH chirality	LH chirality
Particle	+1	-1
Anti-particle	-1	+1

Axial Anomaly and Winding number

- All gauge field configurations are classified by the topological winding numbers

$$
\begin{aligned}
Q_W &= \frac{g^2}{16\pi^2}\int d^4x \mathrm{Tr} F_{\mu\nu}\tilde{F}_{\mu\nu} \\
&= n_{CS}(t=\infty) - n_{CS}(t=-\infty)
\end{aligned}
$$

- Axial anomaly

$$j_\mu^5 = \sum_f \langle \bar{\psi}_f \gamma_\mu \gamma_5 \psi_f \rangle_A$$

Average over gluon field configuration

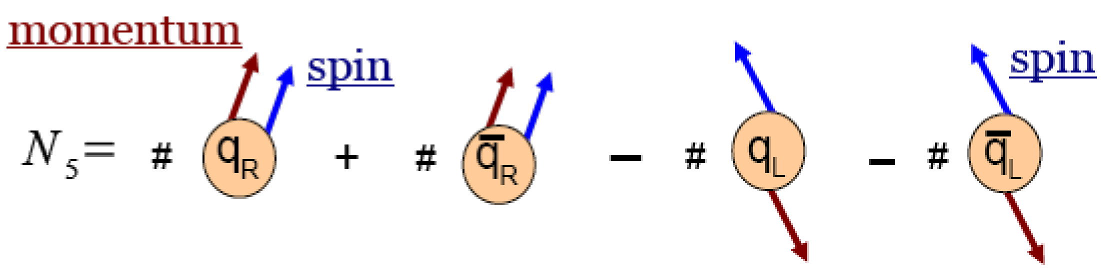
$$\partial^\mu j_\mu^5 = 2\sum_f m_f \langle \bar{\psi}_f i\gamma_5 \psi_f \rangle_A - \frac{N_f g^2}{16\pi^2} F_{\mu\nu}^a \tilde{F}_a^{\mu\nu}$$

Axial Anomaly

- Chiral charge number at chiral limit:

$$N_5 = N_R - N_L = (n_R - \bar{n}_R) - (n_L - \bar{n}_L)$$
$$= (n_R + \bar{n}_L) - (n_L + \bar{n}_R)$$
$$= n(h = +1) - n(h = -1)$$

R,L denote chirality

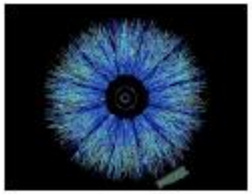


R,L denote helicity

- Assuming $N_R(t = 0) = N_L(t = 0)$, then we have

$$N_5(t = \infty) = -2N_f Q_w = -2N_f \Delta N_{CS}$$

Magnetic fields in Nature

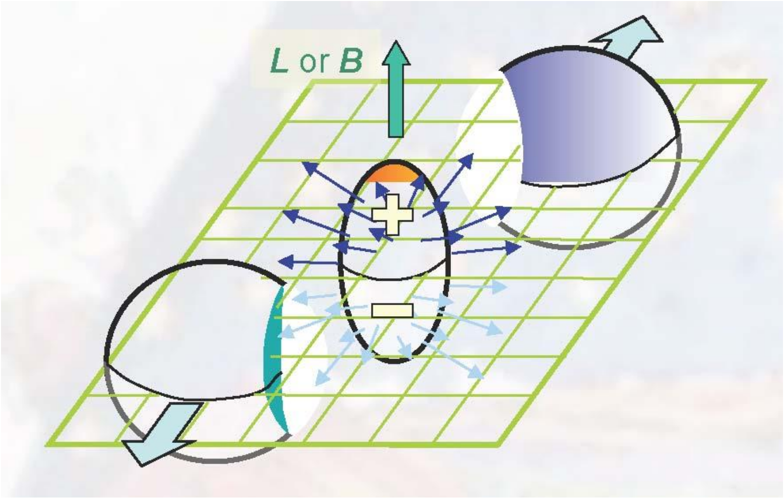


- Earth: 0.6 Gs → compass
- Hand-held magnet in daily life: 100 Gs
- Strongest steady fields in lab: 4.5×10^5 Gs = 45 T
- Strongest magnetic pulse in lab (not in HIC): 10^7 Gs
- Surface fields of pulsars: 10^{13} Gs
- Surface fields of magnetar: 10^{15} Gs
- High energy HIC: Au+Au at 200 GeV, $eB \approx 10^{18} - 10^{19}$ Gs

Magnetic fields in heavy ion collision

- **High energy HIC**

$$v = \sqrt{(s-m_n^2)/s} \sim 1 - \frac{m_n^2}{2s}$$
$$\gamma = 1/\sqrt{1-v^2/c^2} \sim \frac{\sqrt{s}}{m_n}$$



- **Electric field in cms frame of nucleus,**

$$\mathrm{E} = \frac{Ze}{R^2}\hat{\mathrm{r}}$$

- **Boost to Lab frame (v_z= 0.99995 c for 200GeV),**

Scale of strong interaction

$$\mathrm{B} = -\gamma \mathbf{v}_z \times \mathrm{E} \to eB \to 2\gamma v_z \frac{Ze^2}{R^2} \sim \boxed{1.3 m_\pi^2} \sim 2.6 \times 10^{18}\ \mathrm{Gs}$$

Kharzeev, McLerran, Warringa (2008), Skokov (2009), Deng & Huang (2012), Bloczynski,Huang, Zhang, Liao (2012); many others ……

About unit for magnetic field: from cgs to natural unit

$$
\begin{aligned}
1\,c &= 3\times 10^{10}\,\mathrm{cm}\cdot\mathrm{s}^{-1}\\
1\,\hbar &= 1.05\times 10^{-27}\,\mathrm{g}\cdot\mathrm{cm}^2\cdot\mathrm{s}^{-1}\\
1\,\mathrm{eV} &= 1.6\times 10^{-12}\,\mathrm{g}\cdot\mathrm{cm}^2\cdot\mathrm{s}^{-2}
\end{aligned}
$$

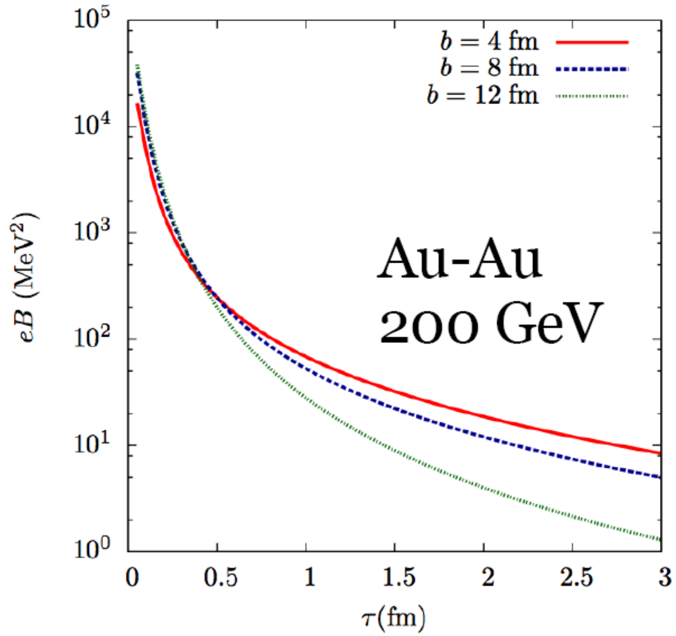
$$
\begin{aligned}
1\,\mathrm{s} &= 1.52\times 10^{15}\,\hbar\cdot\mathrm{eV}^{-1}\\
1\,\mathrm{cm} &= 5.06\times 10^{4}\,\hbar\cdot\mathrm{eV}^{-1}\cdot c\\
1\,\mathrm{g} &= 5.6\times 10^{32}\,\mathrm{eV}\cdot c^{-2}
\end{aligned}
$$

$$E = \frac{1}{8\pi}\int d^3x B^2$$

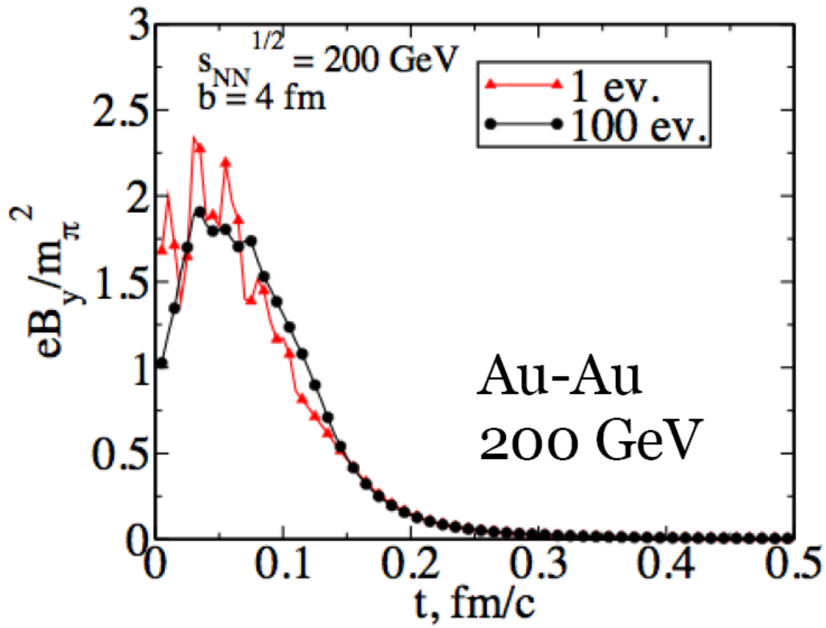
$$
\begin{aligned}
1\,\mathrm{Gauss} &= 10^{-4}\,\mathrm{T}\\
&= 1\,\mathrm{g}^{1/2}\mathrm{cm}^{-1/2}\mathrm{s}^{-1}\\
&= 6.92\times 10^{-2}\,(\hbar c)^{-3/2}\cdot\mathrm{eV}^2
\end{aligned}
$$

$$
\begin{aligned}
1\,\mathrm{MeV}^2 &= 1.44\times 10^{13}\,\mathrm{Gauss}\\
m_\pi^2 &\sim 2.8\times 10^{17}\,\mathrm{Gauss}
\end{aligned}
$$

Ultra-high Magnetic field



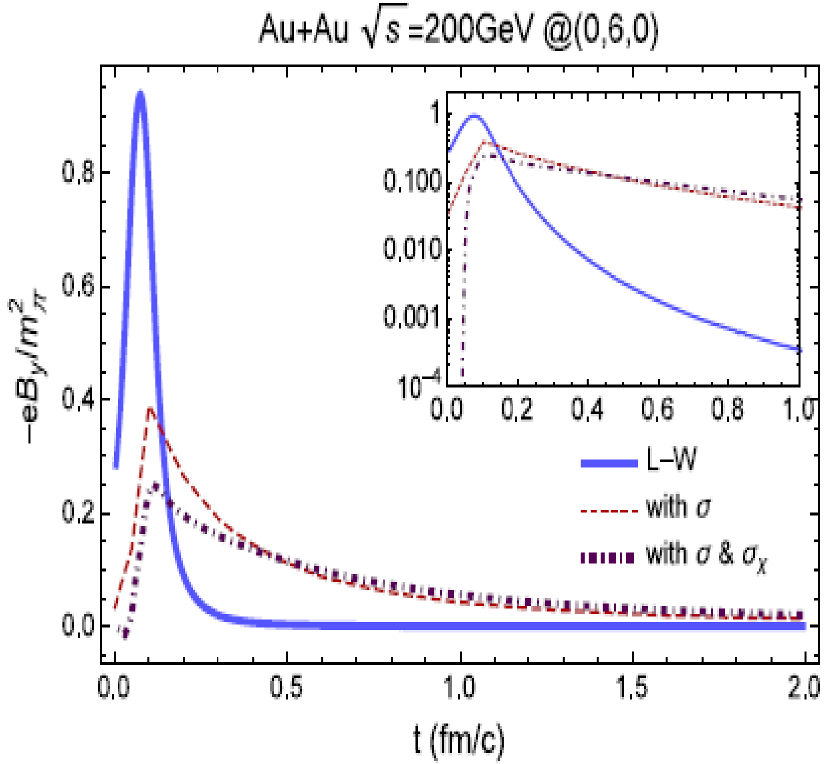
Pancake approximation
Kharzeev, McLerran & HJW ('08)
See also Minakata and Müller ('96)

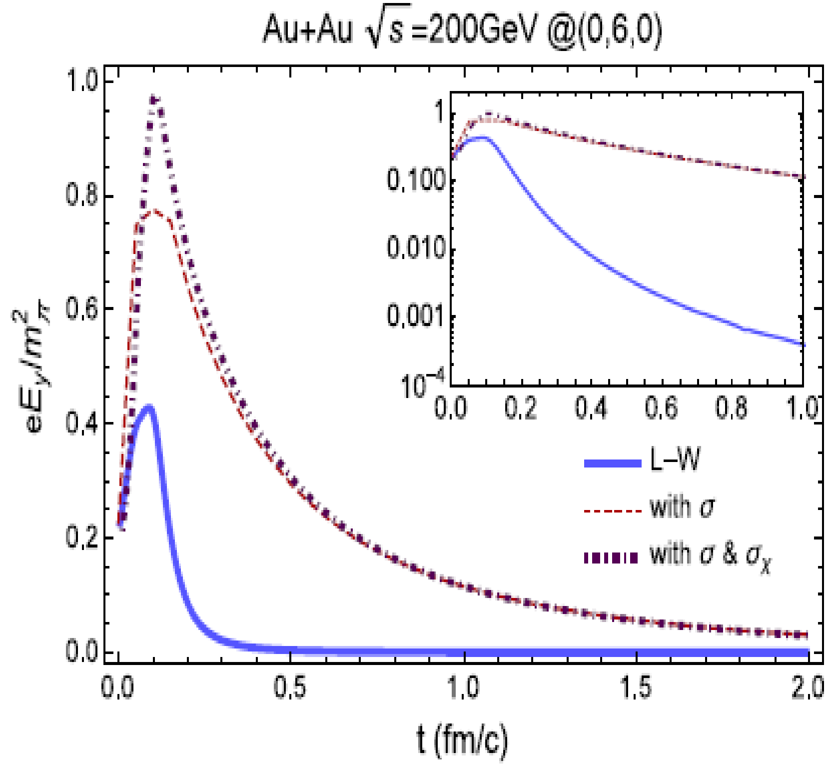


URQMD calculation
Skokov, Illarionov, Toneev ('09)

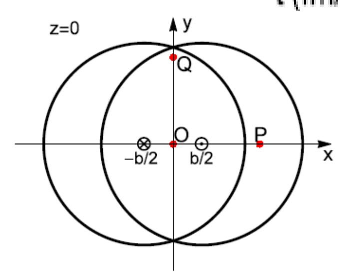
$$eB(\tau = 0.2 \text{ fm/c}) \approx 10^3 \sim 10^4 \, \text{MeV}^2 \approx 10^{18} \text{G}$$

EM Fields in nuclear collisions with electric and chiral conductivity

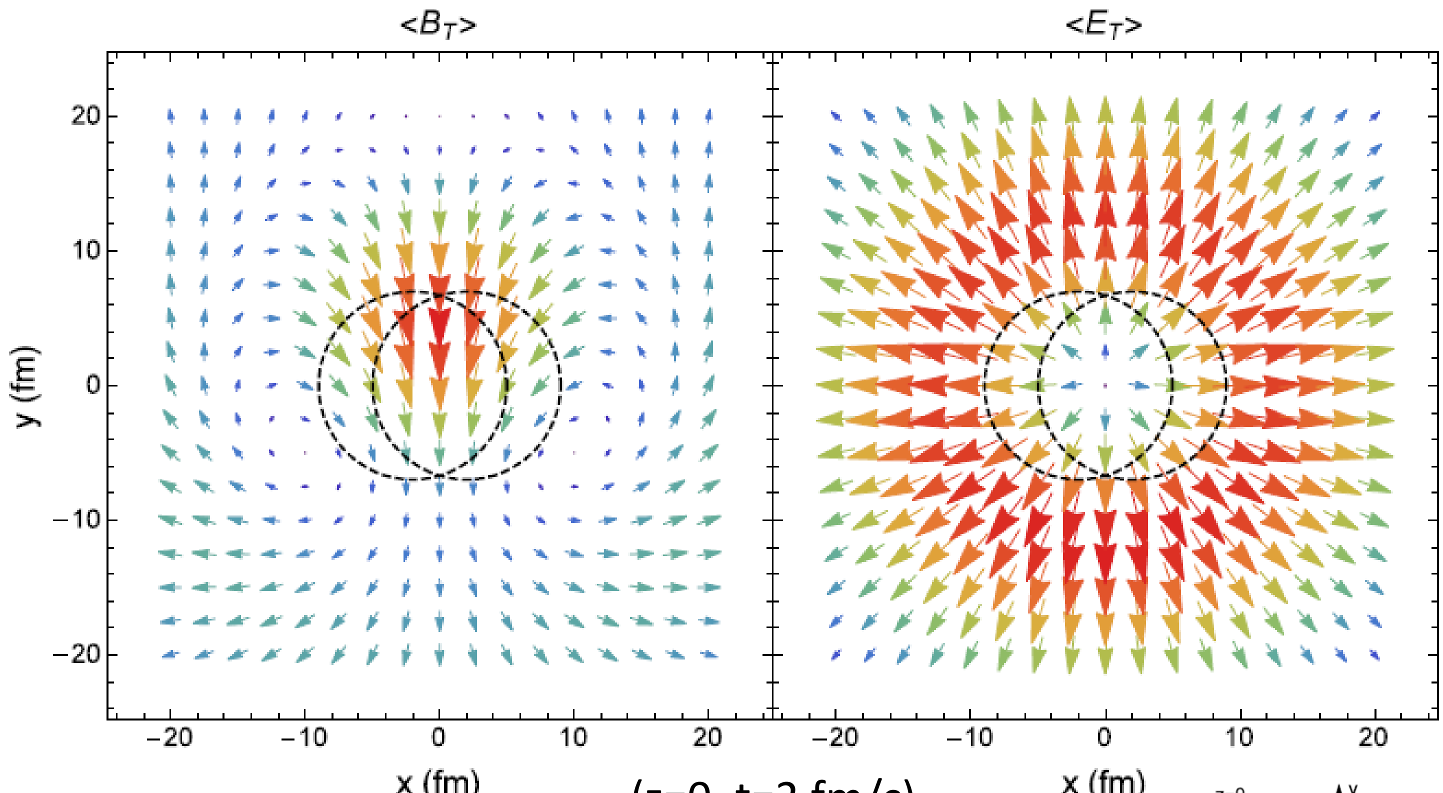




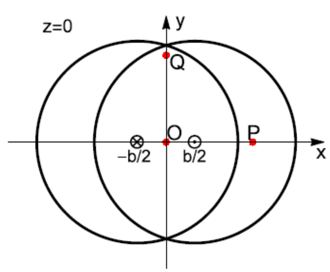
H.Li, X.-l Sheng, Q.Wang, arXiv: 1602.02223



EM Fields in nuclear collisions with electric and chiral conductivity



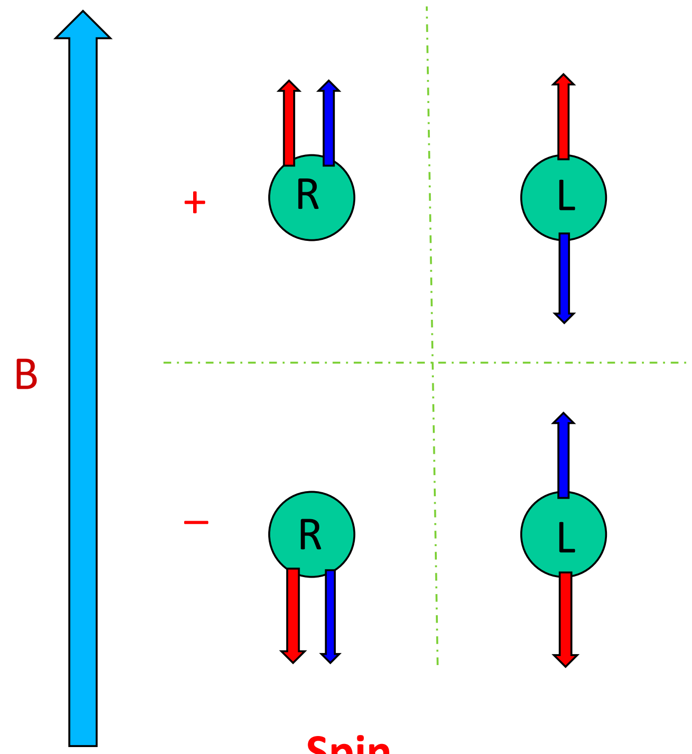
(z=0, t=2 fm/c)



H.Li, X.-l Sheng, Q.Wang, arXiv: 1602.02223

Charge separation and chiral magnetic effect

- Strong magnetic fields polarize quarks: positive charged quarks || B, negative charged quarks || (-B)
- RH quarks: s || p, LH quarks: s || (-p)
- QCD anomaly → non-conservation of axial charge → asymmetry in the number of RH and LH quarks.
- In one event, if there is an excess of number RH quarks, positive charges go up and negative charges go down, and vice versa.
- Charge separation effect (chiral magnetic effect)



Spin
Momentum

D. Kharzeev, L. McLerran, H. Warringa, NPA803, 227 (2008).
K. Fukushima, J.F. Liao, D.T. Son, M. Stephanov, H. Yee, I. Zahed, ……

Induced current by magnetic field

- Chiral chemical potential $\mu_5 = \frac{1}{2}(\mu_R - \mu_L)$
- Induced current

$$j = \frac{N_c \sum q_f{}^2}{2\pi^2}\mu_5 B$$

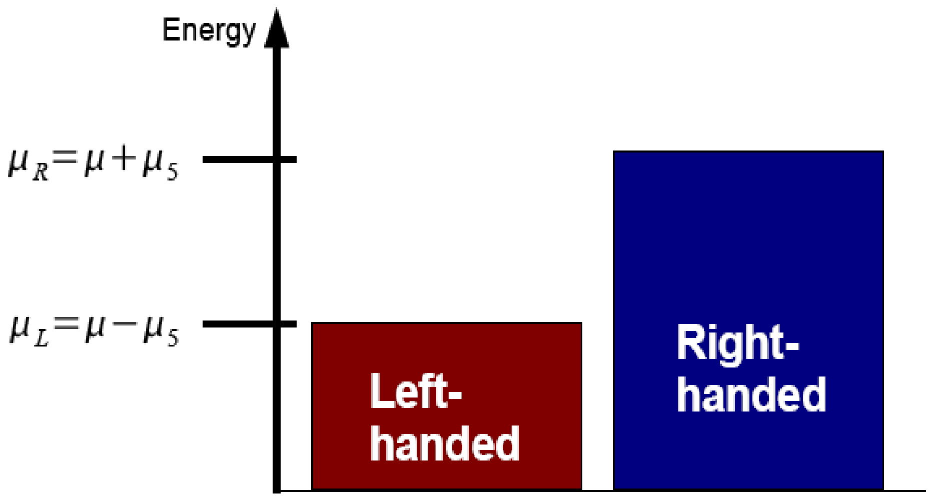
- Derivation:

Thermodynamic potential,

Linear response theory,

Propagator in magnetic field,

Kubo-Formula, Chern-Simons term



CME conductivity: discrete symmetries

$$\mathbf{j} = Q^2 \frac{\mu_5}{2\pi^2} \mathbf{B}$$

P-odd

T-odd

P-even

T-odd

P-odd effect

T-even effect
Non-dissipative current
Topologically protected

T: Time reversal
P: Parity

Ohm conductivity

$$\mathbf{j} = \sigma \mathbf{E}$$

T-odd
Dissipative

Anomalous conductivities

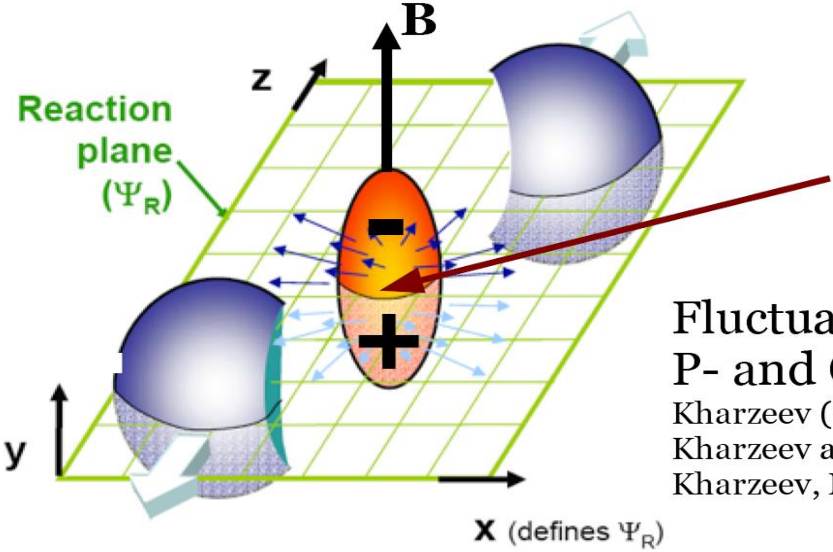
	Magnetic field	Vorticity
Vector current	$Q\frac{\mu_5}{2\pi^2}$	$\frac{\mu\mu_5}{\pi^2}$
Axial current	$Q\frac{\mu}{2\pi^2}$	$\frac{\mu^2+\mu_5^2}{2\pi^2}+\frac{T^2}{6}$

Chiral Magnetic Effect and Charge Separation

- Average over charge and charge squared
  $\langle Q_e \rangle = 0, \langle {Q_e}^2 \rangle \neq 0$

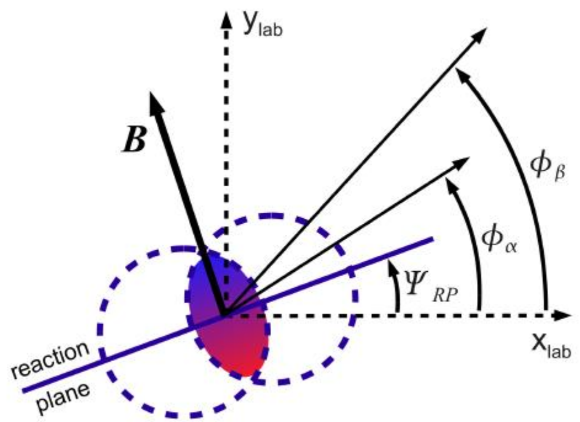
$\langle Q \rangle = 0$

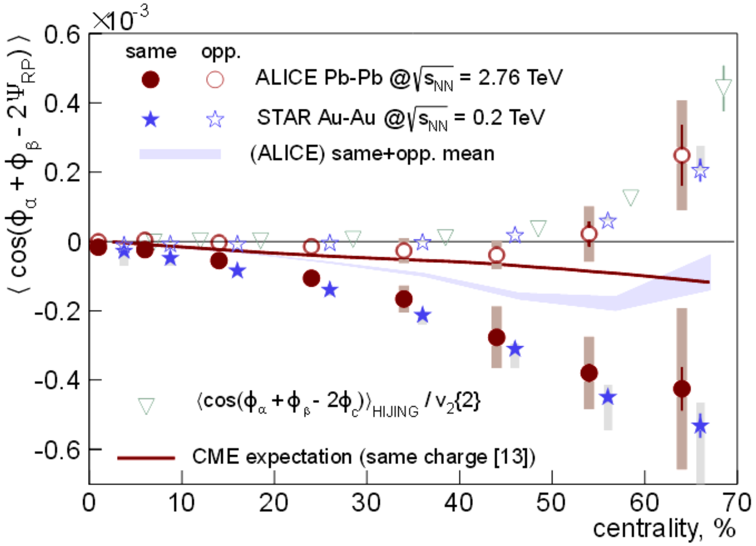
$\langle Q^2 \rangle \neq 0$



Fluctuating EDM of QGP
P- and CP-odd effect
Kharzeev ('06),
Kharzeev and Zhitnitsky ('07)
Kharzeev, Mclerran and Warringa ('08)

Charge Separation Effect in HIC





$$\langle \cos(\phi_\alpha + \phi_\beta - 2\Psi_{RP}) \rangle = \langle \cos\Delta\phi_\alpha \cos\Delta\phi_\beta \rangle - \underline{\langle \sin\Delta\phi_\alpha \sin\Delta\phi_\beta \rangle}$$

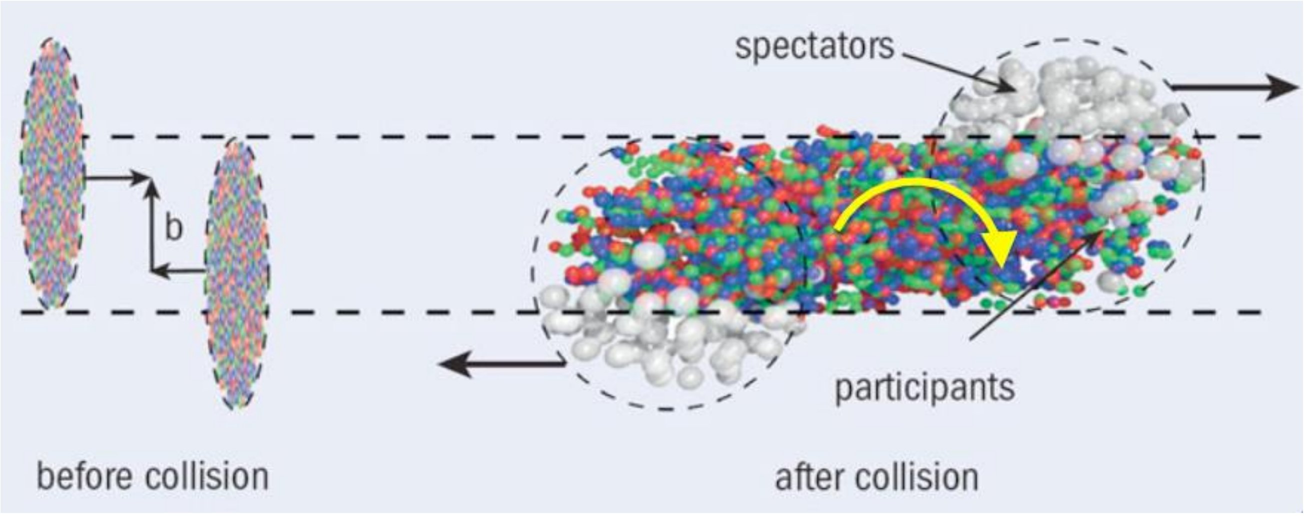
P-violation term

STAR Collab., PRL 103, 251601 (2009); PRC 81, 054908(2010)
ALICE Collab., PRL 110, 012301 (2013).

The interpretation of STAR and ALICE data is under debate. The mechanism behind the Charge Separation Effect is still inconclusive.

Global polarization in heavy ion collision

- Peripheral collisions generate $L \approx 10^5 h$



- Concequences: local polarization, alignment of spin to total angular momentum

Hyperon polarization

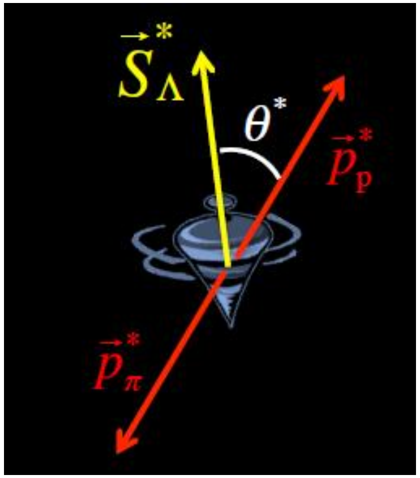
- Polarization of Λ and anti-Λ
- For an esemble of Λ with polarizarion P

$$\frac{dW}{d\Omega^*} = \frac{1}{4\pi}(1 + \alpha \mathbf{P} \cdot \hat{\mathbf{p}}^*) = \frac{1}{4\pi}(1 + \alpha P \cos\theta)$$

$$\mathbf{P} = \frac{3}{\alpha} \langle \hat{\mathbf{p}}^* \rangle \qquad 0 < P < 1$$

- Vortical or QCD spin-obital couplings

Z.T. Liang and X.N. Wang, PRL94 102301 (2005)
Z.T. Liang and X.N. Wang, PLB629 20 (2005)
B. Betz, M. Gyulassy, G. Torrieri PRC76 044901 (2007)
F. Beattini, F. Piccinini, J. Rizzo, Phys. Rev. C77, 024906 (2008)



Local polarization effect

Consider 3-flavor quark matter (u,d,s), the axial baryonic current

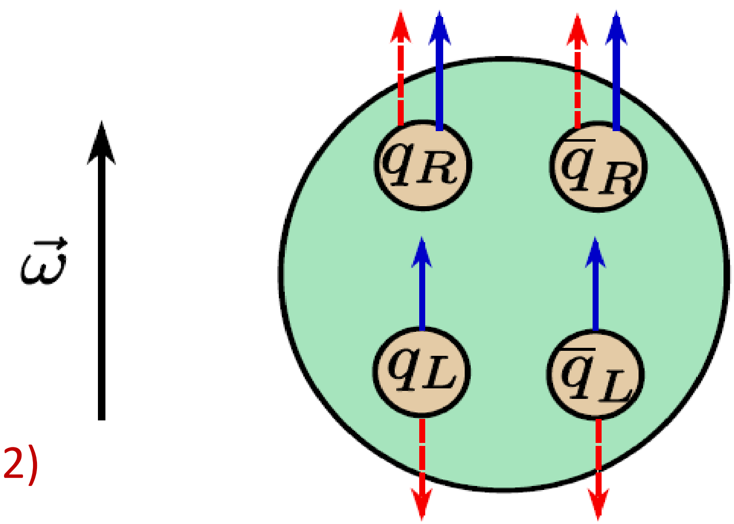
$$j_5^\sigma = n_5 u^\sigma + {\color{red} + \xi_5 \omega^\sigma + \xi_{5B} B^\sigma}$$

$$\xi_5 = N_c \left[\frac{1}{6} T^2 + \frac{1}{2\pi^2} (\mu^2 + \mu_5^2) \right],$$

$$\xi_{B5} = \frac{N_c}{6\pi^2} \mu \sum_f Q_f = 0.$$

Quadratic in temperature, chemical potential, chiral chemical potential
→
No cancellation!

Leading to Local Polarization Effects!
(either for high or low energy HIC)
by the hadron (e.g. hyperon) polarization along the vorticity direction once it is fixed in the event.



J.H. Gao, Z.T. Liang, et al. PRL 109, 232301 (2012)

Chiral magnetic effect in materials

- Oberservation of CME in $ZrTe_5$

nature physics

LETTERS

PUBLISHED ONLINE: 8 FEBRUARY 2016 | DOI: 10.1038/NPHYS3648

Chiral magnetic effect in $ZrTe_5$

Qiang Li[1*], Dmitri E. Kharzeev[2,3*], Cheng Zhang[1], Yuan Huang[4], I. Pletikosić[1,5], A. V. Fedorov[6], R. D. Zhong[1], J. A. Schneeloch[1], G. D. Gu[1] and T. Valla[1*]

BNL-Stony Brook-Princeton-Berkeley collaboration

CME in materials

- Put the crystal in parallel fields E||B, the anomaly induces chiral charge

$$\frac{d\rho_5}{dt} = \frac{e^2}{4\pi^2\hbar^2 c}\mathbf{E}\cdot\mathbf{B} - \frac{\rho_5}{\tau_V}$$

τ_V is chirality changing time

- Chiral charge and chemical potential at $t \gg \tau_V$

$$\rho_5 = \frac{e^2}{4\pi^2\hbar^2 c}\mathbf{E}\cdot\mathbf{B}\tau_V \Longrightarrow \mu_5 = \frac{3}{4}\frac{v^3}{\pi^2}\frac{e^2}{\hbar^2 c}\frac{\mathbf{E}\cdot\mathbf{B}}{T^2 + \frac{\mu^2}{\pi^2}}\tau_V$$

CME in materials

- CME current and conductivity

$$\sigma_{\rm CME}^{zz} = \frac{e^2}{\pi\hbar}\frac{3}{8}\frac{e^2}{\hbar c}\frac{v^3}{\pi^3}\frac{\tau_{\rm V}}{T^2+\frac{\mu^2}{\pi^2}}B^2$$

$$\mathbf{J}_{\rm CME} = \frac{e^2}{2\pi^2}\mu_5\mathbf{B}$$

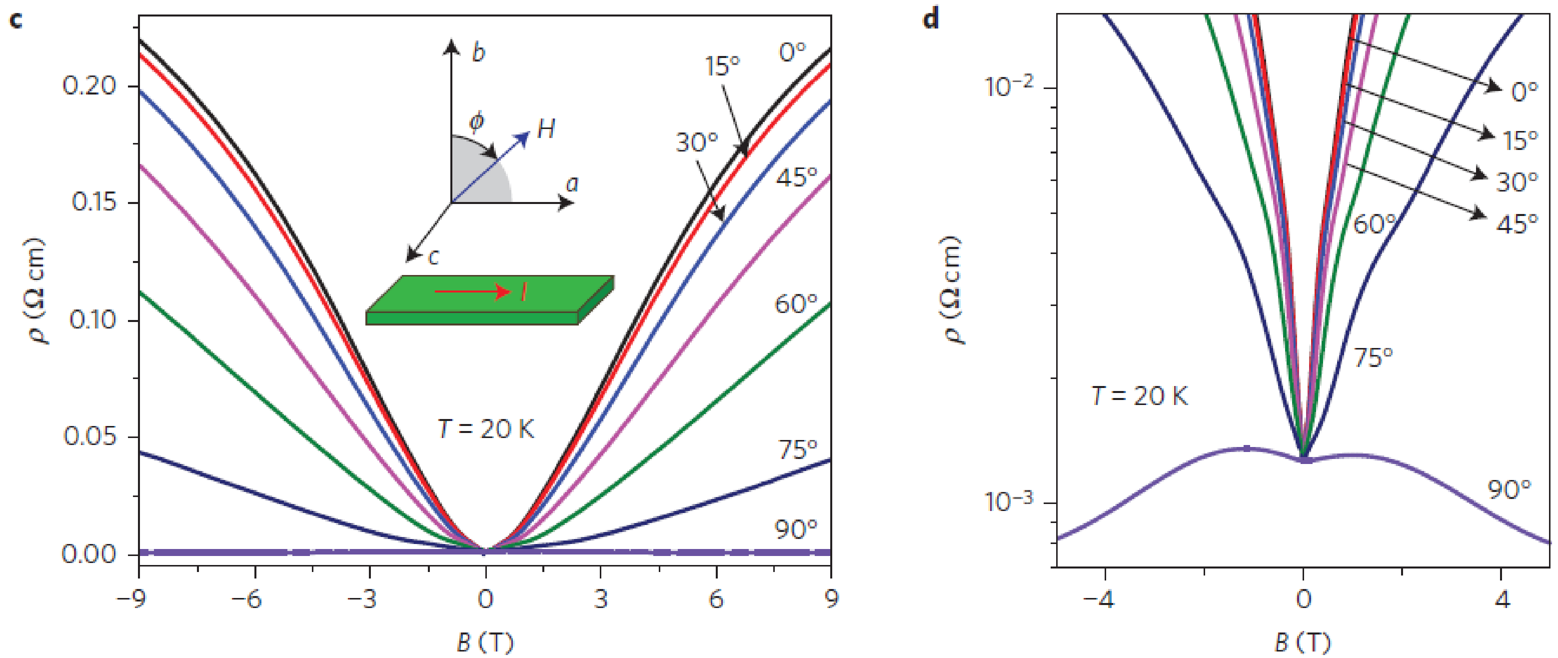
$$J_{\rm CME}^i = \frac{e^2}{\pi\hbar}\frac{3}{8}\frac{e^2}{\hbar c}\frac{v^3}{\pi^3}\frac{\tau_{\rm V}}{T^2+\frac{\mu^2}{\pi^2}}B^iB^kE^k \equiv \sigma_{\rm CME}^{ik}E^k$$

$$J = J_{\rm Ohm} + J_{\rm CME} = (\sigma_{\rm Ohm} + \underline{\sigma_{\rm CME}})E$$

Negative magnetoresistence

CME in materials

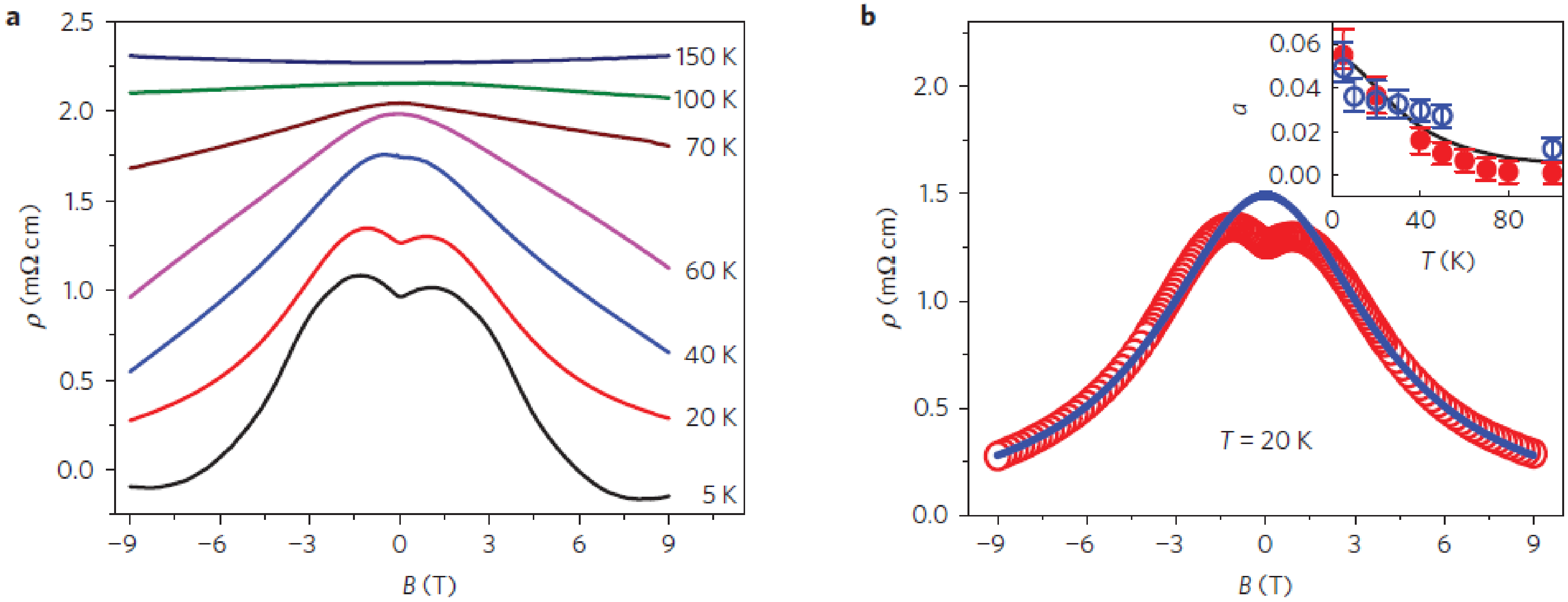
- Magnetoresistance



- (c) Magnetoresistance at 20 K for several angles of the applied field with respect to the current, as depicted in the inset. (d) Extremely large positive magnetoresistance for the magnetic field perpendicular to the current (B||b) and the negative magnetoresistance for the magnetic field parallel to the current (B||a).

CME in materials

- Magnetoresistance in magnetic fields parallel to the current (B||a) in ZrTe5.

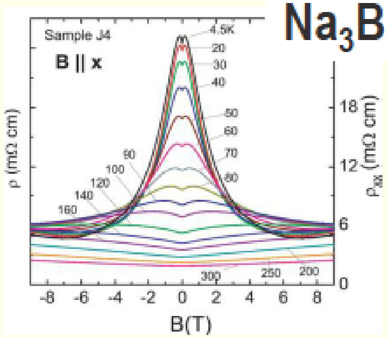


- A negative magnetoresistance is observed for T<100 K, increasing in magnitude as temperature decreases, consistent to $\sigma_{CME} = \sigma_0 + a(T)B^2$

CME in Dirac/Weyl semi-metals

Dirac semimetals:

Na_3Bi



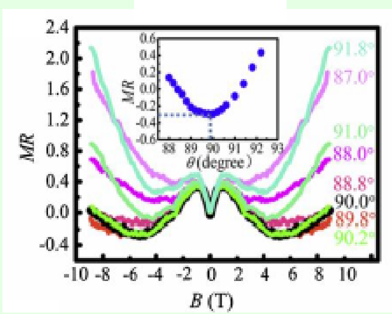
$ZrTe_5$ - Q. Li, D. Kharzeev, et al (BNL and Stony Brook Univ.) arXiv:**1412.6543**; doi:10.1038/NPHYS3648

Na_3Bi - J. Xiong, N. P. Ong et al (Princeton Univ.) arxiv:**1503.08179**; Science 350:413,2015

Cd_3As_2- C. Li et al (Peking Univ. China) arxiv:**1504.07398**; Nature Commun. 6, 10137 (2015).

Weyl semimetals

TaAs



TaAs - X. Huang et al (IOP, China) arxiv:**1503.01304**; Phys. Rev. X 5, 031023

NbAs - X. Yang et al (Zhejiang Univ. China) arxiv:**1506.02283**

NbP - Z. Wang et al (Zhejiang Univ. China) arxiv:**1504.07398**

TaP - Shekhar, C. Felser, B. Yang et al (MPI-Dresden) arxiv:**1506.06577**

Status and Remarks

- Anomalous transport of charge: a gateway to the geometry of gauge fields that defines the properties of QCD. It has broad implications across many fields (condensed matter, astrophysics, cosmology, ...)
- Experiment: established observations of charge-dependent hadron correlations from STAR and ALICE. The theory community should work with experimentalists to reliably interpret the data and extract the physics from backgrounds!
- Theory: goal is to use CME, CVE, CMW to extract the information about topological contents of QGP. We need reliable theoretical tools (Chiral Magneto-Hydro-Dynamics, CMHD, Chiral Kinetic Theory, …) and fully quantitative studies.

Backups

- The end, thank you!

Approaches to CME/CVE

- Field theory
- Hydrodynamics
- Quantum kinetic theory

Quantum Kinetic Approach in Wigner function

- To describe dynamics of chiral fermions, we have to explicitly know their helicity (equivalently p), therefore we need to know information of (t,x,p), that's why we use kinetic approach
- Classical kinetic approach: f(t,x,p)
- Quantum kinetic approach: W(t,x,p)

4D Wigner Function

Gauge invariant Wigner operator/function

Vasak, Gyulassy, Elze, Annals Phys. 173 (1987) 462-492

$$W(x,p) = <:\hat{W}(x,p):>$$

$$\hat{W}_{\alpha\beta}(x,p) = \int \frac{d^4y}{(2\pi)^4} e^{-ip\cdot y} \bar{\psi}_\beta\left(x+\frac{1}{2}y\right) \mathcal{P}U\left(A, x+\frac{1}{2}y, x-\frac{1}{2}y\right) \psi_\alpha\left(x-\frac{1}{2}y\right)$$

Gauge link $\mathcal{P}U\left(A, x+\frac{1}{2}y, x-\frac{1}{2}y\right) \equiv \mathcal{P}\mathrm{Exp}\left(-iey^\mu \int_0^1 ds A_\mu\left(x-\frac{1}{2}y+sy\right)\right)$

Dirac equation in electromagnetic field

$$[i\gamma^\mu D_\mu(x) - m]\,\psi(x) = 0, \qquad \bar{\psi}(x)\left[i\gamma^\mu D_\mu^\dagger(x) + m\right] = 0$$

Quantum Kinetic Equation for Wigner function for massless fermion in homogeneous electromagnetic field

$$\gamma_\mu\left(p^\mu + \frac{1}{2} i\nabla^\mu\right) W(x,p) = 0$$

phase space derivative

$$\nabla^\mu \equiv \partial_x^\mu - QF^{\mu\nu}\partial_\nu^p$$

4D Wigner Function

- **For massless and collisionless fermions in constant EM field**

$$\gamma_\mu\left(p^\mu+\frac{1}{2}i\nabla^\mu\right)W(x,p)=0.$$

- **Wigner function decomposition in 16 generators of Clifford algebra**

$$W=\frac{1}{4}\left[\mathscr{F}+i\gamma^5\mathscr{P}+\gamma^\mu\mathscr{V}_\mu+\gamma^5\gamma^\mu\mathscr{A}_\mu+\frac{1}{2}\sigma^{\mu\nu}\mathscr{S}_{\mu\nu}\right]$$

scalar p-scalar vector axial-vector tensor

$$j^\mu=\int d^4p\mathscr{V}^\mu,\qquad j_5^\mu=\int d^4p\mathscr{A}^\mu,\qquad T^{\mu\nu}=\int d^4pp^\mu\mathscr{V}^\nu$$

Vasak, Gyulassy and Elze, Annals Phys. 173, 462 (1987);
Elze, Gyulassy and Vasak, Nucl. Phys. B 276, 706(1986).

Solution of Wigner function

- **The solutions to $(F_{\mu\nu})^1$ and $(\partial_x)^1$ encodes a lot of information !!**

$$\mathscr{J}^{\rho}_{s}(x,p) = \mathscr{V}^{\rho} + s\mathscr{A}^{\rho}$$

$$\boxed{\begin{aligned} \mathscr{J}^{\rho}_{(0)s}(x,p) &= p^{\rho} f_s \delta(p^2) \\ \mathscr{J}^{\rho}_{(1)s}(x,p) &= -\frac{s}{2}\tilde{\Omega}^{\rho\beta} p_{\beta} \frac{df_s}{dp_0}\delta(p^2) - \frac{s}{p^2} Q\tilde{F}^{\rho\lambda} p_{\lambda} f_s \delta(p^2) \end{aligned}}$$

- **where**

Fermi-Dirac Distr.

$$f_s(x,p) = \frac{2}{(2\pi)^3}\left[\Theta(p_0)\underline{f_F(p_0-\mu_s)} + \Theta(-p_0)\underline{f_F(-p_0+\mu_s)}\right]$$

$$\tilde{F}^{\rho\lambda} = \frac{1}{2}\epsilon^{\rho\lambda\mu\nu}F_{\mu\nu}$$

$$\mu_s = \mu + s\mu_5$$

$$\tilde{\Omega}^{\xi\eta} = \frac{1}{2}\epsilon^{\xi\eta\nu\sigma}\Omega_{\nu\sigma} \qquad \Omega_{\nu\sigma} = \frac{1}{2}(\partial_{\nu}u_{\sigma} - \partial_{\sigma}u_{\nu})$$

Vector and axial current

$$
\begin{aligned}
j^{\mu} &= \int d^4p \mathscr{V}^{\mu} = n u^{\mu} + \xi \omega^{\mu} + \xi_B B^{\mu}, \\
j_5^{\mu} &= \int d^4p \mathscr{A}^{\mu} = n_5 u^{\mu} + \xi_5 \omega^{\mu} + \xi_{B5} B^{\mu},
\end{aligned}
$$

$$
\begin{aligned}
T^{\mu\nu} &= \frac{1}{2} \int d^4p (p^{\mu} \mathscr{V}^{\nu} + p^{\nu} \mathscr{V}^{\mu}) \\
&= (\epsilon + P) u^{\mu} u^{\nu} - P g^{\mu\nu} + n_5 (u^{\mu} \omega^{\nu} + u^{\nu} \omega^{\mu}) \\
&\quad + \frac{1}{2} Q \xi (u^{\mu} B^{\nu} + u^{\nu} B^{\mu})
\end{aligned}
$$

Coefficients and conservation law

Transport coefficients

Absent in hydro-derivation

$$\xi = \frac{1}{\pi^2}\mu\mu_5,$$

$$\xi_B = \frac{Q}{2\pi^2}\mu_5,$$

$$\xi_5 = \boxed{\frac{1}{6}T^2} + \frac{1}{2\pi^2}\left(\mu^2 + \boxed{\mu_5^2}\right),$$

$$\xi_{B5} = \frac{Q}{2\pi^2}\mu.$$

$$n = \frac{1}{3\pi^2}\mu\left(\pi^2T^2 + \mu^2 + 3\mu_5^2\right)$$

$$n_5 = \frac{1}{3\pi^2}\mu_5\left(\pi^2T^2 + \mu_5^2 + 3\mu^2\right)$$

$$\epsilon = \frac{1}{2\pi^2}\left[\frac{7\pi^4}{30}T^4 + \pi^2T^2(\mu^2 + \mu_5^2) + \frac{1}{2}(\mu^4 + \mu_5^4) + 3\mu^2\mu_5^2\right]$$

comes out naturally

All the conservation laws and anomaly can be derived naturally

$$\partial_\sigma j_5^\sigma = -\boxed{\frac{Q^2}{2\pi^2}}E\cdot B \qquad \partial_\sigma j^\sigma = 0 \qquad \partial_\sigma T^{\sigma\nu} = QF^{\nu\rho}j_\rho$$

An Independent derivation of chiral anomaly from quantum kinetic theory !

Local polarization effect

Consider 3-flavor quark matter (u,d,s), the axial baryonic current

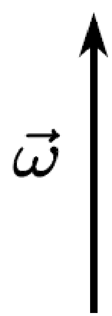
$$j_5^\sigma = n_5 u^\sigma + +\xi_5 \omega^\sigma + \xi_{5B} B^\sigma$$

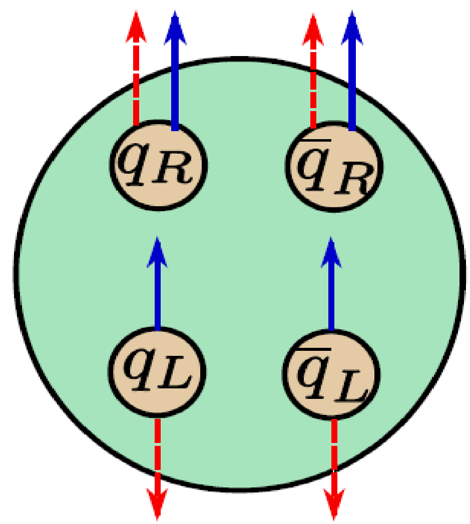
$$\xi_5 = N_c \left[\frac{1}{6} T^2 + \frac{1}{2\pi^2} (\mu^2 + \mu_5^2) \right],$$

$$\xi_{B5} = \frac{N_c}{6\pi^2} \mu \sum_f Q_f = 0.$$

Quadratic in temperature, chemical potential, chiral chemical potential
→
No cancellation!

Leading to Local Polarization Effects!
(either for high or low energy HIC)
By the hadron (e.g. hyperon) polarization along the vorticity direction once it is fixed in the event.





J.H. Gao, Z.T. Liang, et al. PRL 109, 232301 (2012)

Decoding the solution:
(2) CKE from 4D to 3D and Berry Phase

- **Covariant Chiral Kinetic Equation in 4D (CCKE)**

$$\nabla_\mu \mathscr{J}_s^\mu = 0 \quad \Rightarrow \quad \delta\left(p^2\right)\left(\frac{dx^\mu}{d\tau}\partial_\mu^x f_s + \frac{dp^\mu}{d\tau}\partial_\mu^p f_s\right) = 0$$

$$\frac{dx^\mu}{d\tau} \equiv p^\mu + s\epsilon^{\mu\nu\alpha\beta} b_\nu F_{\alpha\beta}, \quad \frac{dp^\mu}{d\tau} \equiv F^{\mu\nu} p_\nu - s\left(E\cdot B\right) b^\mu$$

Berry Curvature In 4D $\quad b^\mu \equiv -\frac{p^\mu}{p^2}$

- **Chiral Kinetic Equation in 3D**

$$\int dp_0 \nabla_\mu \mathscr{J}_s^\mu = 0 \quad \Rightarrow \quad \partial_t f_s + \frac{d\mathbf{x}}{dt}\cdot\nabla_{\mathbf{x}} f_s + \frac{d\mathbf{p}}{dt}\cdot\nabla_{\mathbf{p}} f_s = 0$$

$$\frac{d\mathbf{x}}{dt} \equiv \frac{\hat{\mathbf{p}} + s\left[(\hat{\mathbf{p}}\cdot\mathbf{\Omega})\mathbf{B} + \mathbf{E}\times\mathbf{\Omega}\right]}{1 + s\mathbf{\Omega}\cdot\mathbf{B}}, \quad \frac{d\mathbf{p}}{dt} \equiv \frac{\mathbf{E} + \hat{\mathbf{p}}\times\mathbf{B} + s(\mathbf{E}\cdot\mathbf{B})\mathbf{\Omega}}{1 + s\mathbf{\Omega}\cdot\mathbf{B}}$$

J.W. Chen, S.Pu, QW, X.N. Wang, PRL 110 (2013) 262301
D.T. Son, N. Yamamoto, PRL 109 (2012) 181602
M.A. Stephanov,Y. Yin, PRL 109 (2012) 162001

Berry Curvature In 3D $\quad \mathbf{\Omega} = \frac{\hat{\mathbf{p}}}{2\mathbf{p}^2}$

Decoding the solution:
(3) Energy shift and magnetic Moment

- **Particle and energy density with** $\mu_s(x,\mathbf{p}) \quad (\mu_s \equiv \mu + s\mu_5)$

$$
\begin{aligned}
n_s &= \int d^3\mathbf{p} f_s + \int d^3\mathbf{p}\frac{s}{2E_p^2}(\mathbf{v}\cdot\mathbf{B})f_s - \int d^3\mathbf{p}\frac{s}{2E_p}(\mathbf{v}\cdot\mathbf{B})\frac{d}{dE_p}f_s \\
&\approx \int \frac{d^3\mathbf{p}}{(2\pi)^3}\sqrt{\gamma_s}\left[f_F\left(E_p' - \mu_s(x,\mathbf{p})\right) - f_F\left(E_p' + \mu_s(x,\mathbf{p})\right)\right],
\end{aligned}
$$

$$
\begin{aligned}
\epsilon_s &= \int d^3\mathbf{p} E_p f_s + \int d^3\mathbf{p}\frac{s}{2E_p}(\mathbf{v}\cdot\mathbf{B})f_s - \int d^3\mathbf{p}\frac{s}{2}(\mathbf{v}\cdot\mathbf{B})\frac{d}{dE_p}f_s \\
&\approx \int \frac{d^3\mathbf{p}}{(2\pi)^3}\sqrt{\gamma_s}E_p'\left[f_F\left(E_p' - \mu_s(x,\mathbf{p})\right) + f_F\left(E_p' + \mu_s(x,\mathbf{p})\right)\right],
\end{aligned}
$$

- **Phase-space measure:** $\sqrt{\gamma_s} \equiv (1 + s\Omega\cdot\mathrm{B})$
- **Berry curvature:** $\Omega = \dfrac{\hat{\mathrm{p}}}{2\mathrm{p}^2}$

J.H.Gao, QW, PLB749 (2015) 542.

Decoding the solution:
(3) Energy shift and magnetic moment

- **Effective Energy of Chiral Fermion:**

$$E'_p = |\mathbf{p}| + \Delta E_B$$

- **Energy Shift:**

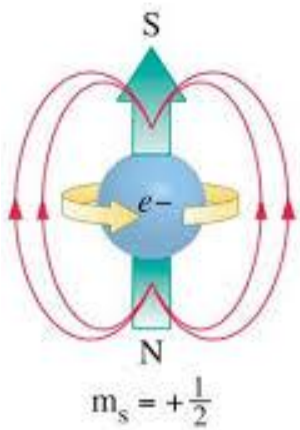
$$\Delta E_B \;=\; -\boldsymbol{\mu}_m \cdot \mathbf{B}$$

- **Magnetic moment of massless fermion:**

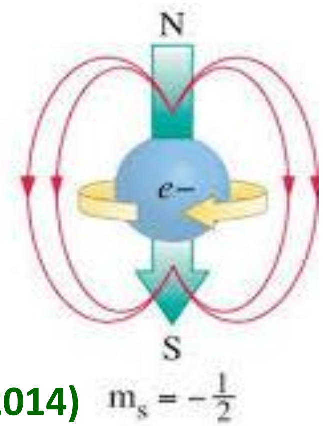
$$\boldsymbol{\mu}_m \;=\; \frac{Q}{|\mathbf{p}|}\mathbf{S}$$

$s = \pm 1$

- **Spin:** $S \;=\; \frac{s}{2}\hat{\mathbf{p}}$



$$\boldsymbol{\mu}_m \;=\; \frac{Q}{\mathrm{m}}\mathbf{S}$$



Son, Yamamoto (2013); Manual, Torres-Rincon (2014); Satow, Yee (2014)

Is it the energy for quasi-particle?

- **Effective Energy of Chiral Fermion with magnetic energy shift:**

$$
\begin{aligned}
E'_p &= |\mathrm{p}| - \frac{sQ}{2|\mathrm{p}|^2}(\mathrm{p}\cdot\mathrm{B}) \\
\mathrm{v}'_p &= \nabla_{\mathrm{p}} E'_p = \hat{\mathrm{p}} + \frac{sQ}{2|\mathrm{p}|^2}(2\hat{\mathrm{p}}\hat{\mathrm{p}} - 1)\cdot\mathrm{B} \\
\mathrm{v}'_p\cdot\hat{\mathrm{p}} &= \underbrace{1 + \frac{sQ}{2|\mathrm{p}|^2}(\hat{\mathrm{p}}\cdot\mathrm{B}) > 1}_{\text{Superluminal}} \qquad \text{for} \quad sQ(\hat{\mathrm{p}}\cdot\mathrm{B}) > 0
\end{aligned}
$$

$\boxed{sQ = \pm 1}$

- **Can E'_p be regarded as quasi-particle dispersion relation? Is this a problem?**
- **In our formalism, we don't have such superluminal problem. It appears only when re-writing the number/energy density in a 'quasi-particle' way in weak field approximation**

Decoding the solution:
(4) Energy Shift from Spin-Vorticity Coupling

- **Particle and energy density with** $\mu_s(x,\mathbf{p})$

$$\boxed{\begin{aligned} n_s &= \int d^3\mathbf{p} f_s - \int d^3\mathbf{p}\frac{s}{2}(\boldsymbol{\omega}\cdot\mathbf{v})\frac{d}{dE_p}f_s \\ &\approx \int \frac{d^3\mathbf{p}}{(2\pi)^3}\left[f_F\left(E'_p-\mu_s(x,\mathbf{p})\right)-f_F\left(E'_p+\mu_s(x,\mathbf{p})\right)\right], \end{aligned}}$$

$$\boxed{\begin{aligned} \epsilon_s &= \int d^3\mathbf{p} E_p f_s - \int d^3\mathbf{p}\frac{s}{2}(\mathbf{v}\cdot\boldsymbol{\omega})E_p\frac{d}{dE_p}f_s \\ &\approx \int \frac{d^3\mathbf{p}}{(2\pi)^3}E'_p\left[f_F\left(E'_p-\mu_s(x,\mathbf{p})\right)+f_F\left(E'_p+\mu_s(x,\mathbf{p})\right)\right], \end{aligned}}$$

- **No phase-space measure, no Berry curvature**
- **Effective energy:**

$$\Delta E_\omega = -\boldsymbol{\omega}\cdot\mathbf{S}$$

$$E'_p = E_p + \Delta E_\omega$$

J.H.Gao, QW, PLB749 (2015) 542.

Another solution:
Linear Response Theory for Wigner Functions

- **Expansion in $(F_{\mu\nu})^n$ [not in $(\partial_x)^n$!] , the first order equation:**

$$
\begin{aligned}
p^\mu \mathscr{J}_{s\mu}^{(1)} + \delta\Pi^\mu \mathscr{J}_{s\mu}^{(0)} &= 0, \quad \partial_x^\mu \mathscr{J}_{s\mu}^{(1)} + \delta G^\mu \mathscr{J}_{s\mu}^{(0)} = 0, \\
\epsilon_{\mu\nu\rho\sigma}\left[\partial_x^\rho \mathscr{J}_s^{(1)\sigma} + \delta G^\rho \mathscr{J}_s^{(0)\sigma}\right] &= -2s\left(p_\mu \mathscr{J}_{s\nu}^{(1)} - p_\nu \mathscr{J}_{s\mu}^{(1)}\right) - 2s\left[\delta\Pi_\mu \mathscr{J}_{s\nu}^{(0)} - \delta\Pi_\nu \mathscr{J}_{s\mu}^{(0)}\right]
\end{aligned}
$$

- **Formal solution:** $\Delta \equiv \partial^p \cdot \partial_x$

$$
\begin{aligned}
\mathscr{J}_{s\mu}^{(1)} &= -\frac{s}{2p\cdot\partial_x}\epsilon_{\mu\nu\rho\sigma}\partial_x^\nu\left[j_0\left(\frac{\Delta}{2}\right)F^{\rho\lambda}\partial_\lambda^p \mathscr{J}_s^{(0)\sigma}\right] + \frac{1}{p\cdot\partial_x}p_\mu j_0\left(\frac{\Delta}{2}\right)F^{\nu\lambda}\partial_\lambda^p \mathscr{J}_{s\nu}^{(0)} \\
&\quad -\frac{1}{2p\cdot\partial_x}\partial_x^\nu\left[j_1\left(\frac{\Delta}{2}\right)\left(F_{\mu\lambda}\partial_p^\lambda \mathscr{J}_{s\nu}^{(0)} - F_{\nu\lambda}\partial_p^\lambda \mathscr{J}_{s\mu}^{(0)}\right)\right]
\end{aligned}
$$

- **Parity-odd part of the Wigner function in momentum space:**

$$
\mathscr{J}_{s\mu}^{(1)}(k,p) = -i\frac{s}{2p\cdot k}\epsilon_{\mu\nu\rho\sigma}k^\nu p^\sigma A^\rho(k) j_0(\Delta)(k\cdot\partial_p)[f_s\delta(p^2)]
$$

J.H.Gao, QW, PLB 749 (2015) 542

Another solution:
Chiral Magnetic Conductivity

- **Chiral Magnetic Conductivity:**

J.H.Gao, QW, PLB 749 (2015) 542

$$\vec{j}(\omega,\mathbf{k}) = \int d^4p \left(\vec{\mathscr{J}}_+^{(1)} + \vec{\mathscr{J}}_-^{(1)}\right) = \sigma_\chi(\omega,\mathbf{k})\vec{B}$$

$$\sigma_\chi = \frac{\mathbf{k}^2-\omega^2}{16\pi^2|\mathbf{k}|^3}\int d|\mathbf{p}| f(|\mathbf{p}|) \sum_{t=\pm1}(2|\mathbf{p}|+t\omega)\ln\left[\frac{(\omega+i\epsilon+t|\mathbf{p}|-(\omega+|\mathbf{p}|)^2}{(\omega+i\epsilon+t|\mathbf{p}|-(\omega-|\mathbf{p}|)^2}\right]$$

- **HTL/HDL results from Wigner function:** $\omega, |\mathbf{k}| \ll |\mathbf{p}|$

$$\sigma_\chi(\omega,\mathbf{k}) = \sigma_\chi^{(0)}\left(1-\frac{\omega^2}{|\mathbf{k}|^2}\right)\left[1-\frac{\omega}{2|\mathbf{k}|}\ln\frac{\omega+|\mathbf{k}|}{\omega-|\mathbf{k}|}\right]$$

$$\boxed{\sigma_\chi^{(0)} = \frac{1}{2\pi^2}\mu_5}$$

M.Laine, JHEP 0510 (2005) 056;
D.Kharzeev, H.Warringa, PRD80 (2009) 034028;

Status and Remarks

- **Anomalous transport of charge: a gateway to the geometry of gauge fields that defines the properties of QCD. It has broad implications across many fields (condensed matter, astrophysics, cosmology, ...)**
- **Experiment: established observations of charge-dependent hadron correlations from STAR and ALICE. The theory community should work with experimentalists to reliably interpret the data and extract the physics from backgrounds!**
- **Theory: goal is to use CME, CVE, CMW to extract the information about topological contents of QGP. We need reliable theoretical tools (Chiral Magneto-Hydro-Dynamics, CMHD, Chiral Kinetic Theory, …) and fully quantitative studies.**