Selected topics on QCD evolution

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arXiv:1603.0742; ZJ

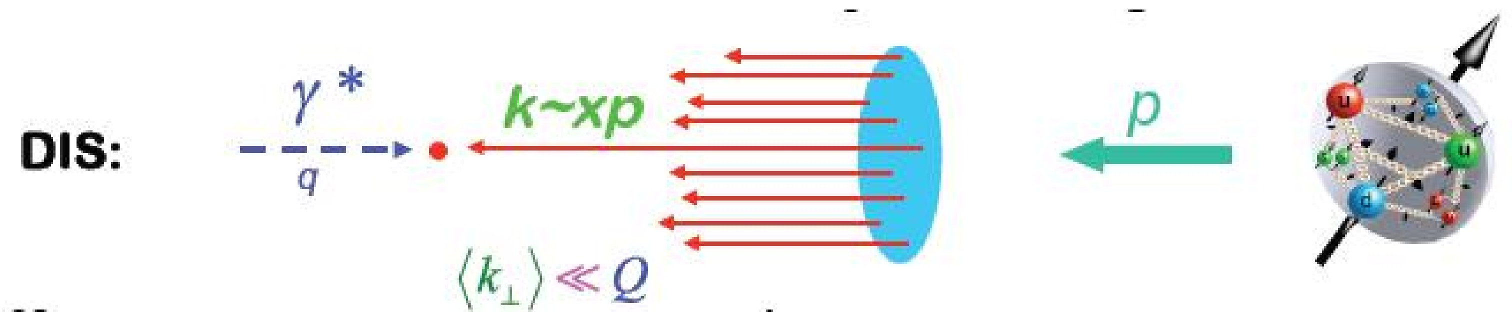
QCD study group, 2016, SJTU

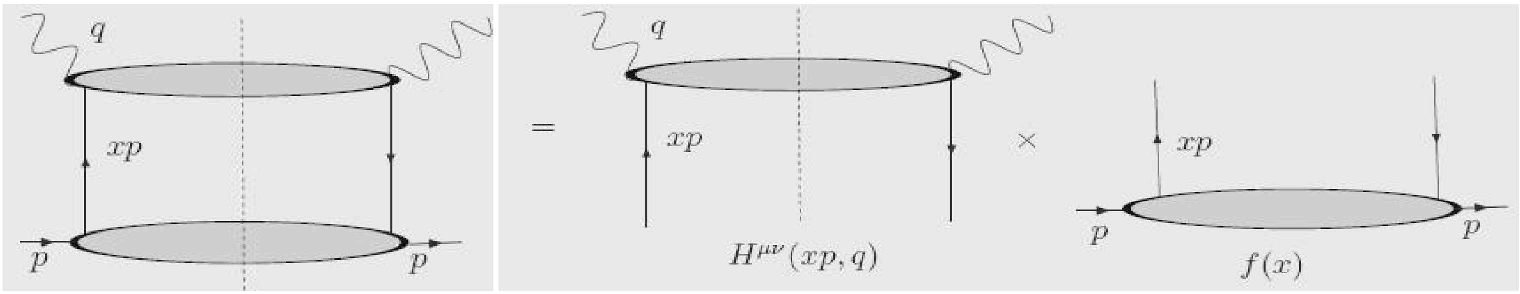
Outline:

- The scale dependence of the Burkardt sum rule
- The evolution of the small x gluon TMD
- Summary

Parton distribution function

DIS process:





$$W^{\mu\nu} \propto H^{\mu\nu}(xp, q) \otimes f(x)$$

Operator definiation:

$$f(x,\mu^2) = \int \frac{dy}{4\pi} e^{-ixp^+y^-} \langle P | \overline{\psi}(y^-) \not{n} L(y^-;0^-) \psi(0^-) | P \rangle$$

Sum rule

Longitudinal momentum conservation:

$$\sum_{q+\overline{q}+g} \int xf(x,\mu^2)dx = 1$$

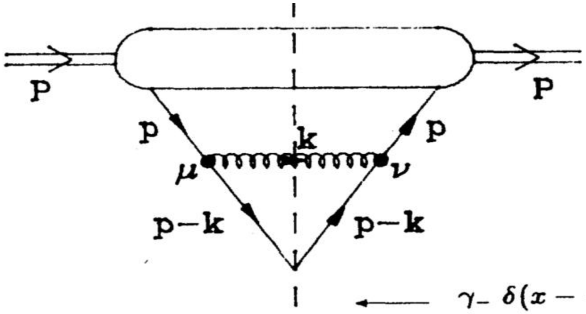
The scale dependence?

$$\frac{\partial \sum_{q+\overline{q}+g} \int xf(x,\mu^2)dx}{\partial \ln \mu^2} = ?$$

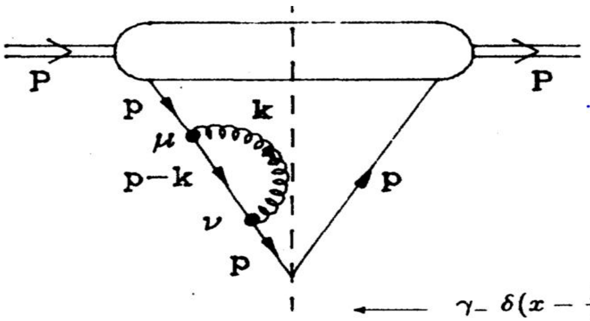
DGLAP type evolution

Starting point: $f(x,\mu^2)=\int \frac{dy}{4\pi} e^{-ixp^+y^-}\langle P|\overline{\psi}(y^-)\not{n}L(y^-;0^-)\psi(0^-)|P\rangle$

In the light cone gauge calculation: quark---->quark channel:



$\longleftarrow \quad \gamma_-\ \delta\left(x-\frac{(p-k)\cdot n}{P\cdot n}\right)\frac{1}{2P\cdot n}$



$\longleftarrow \quad \gamma_-\ \delta\left(x-\frac{p\cdot n}{P\cdot n}\right)\frac{1}{2P\cdot n}$

1978, Mueller

$$\frac{\partial f(x,\mu^2)}{\partial \ln \mu^2}=\frac{\alpha_s}{2\pi}\int_x^1\frac{dx_1}{x_1}\left[\frac{1+z^2}{(1-z)_+}-\frac{3}{2}\delta(1-z)\right]f(x_1,\mu^2)$$

$$z=x_1/x$$

Computing all channels:

quark ----> quark
quark ----> gluon
gluon ----> quark, antiquark
antiquark ----> antiquark
antiquark ----> gluon

One finds:

$$\frac{\partial \sum_{q+\bar{q}+g} \int xf(x,\mu^2)dx}{\partial \ln \mu^2} = 0$$

- The sum rule is stable under QCD scale evolution.

Parton transverse motion inside a nucleon

$$\Phi(x, k_T)$$

1982-1983，Collins and Soper

$$\Phi(x,k_T) = \int \frac{dy}{4\pi} \frac{d^2 y_T}{(2\pi)^2} e^{-ixp^+ y^- + ik_T y_T} \langle P | \overline{\psi}(y^-, y_T) \not{n} L L^+ \psi(0^-, 0_T) | P \rangle$$

For an unpolarized target,

$$\int d^2 k_T \Phi(x, k_T) \vec{k}_T = 0$$

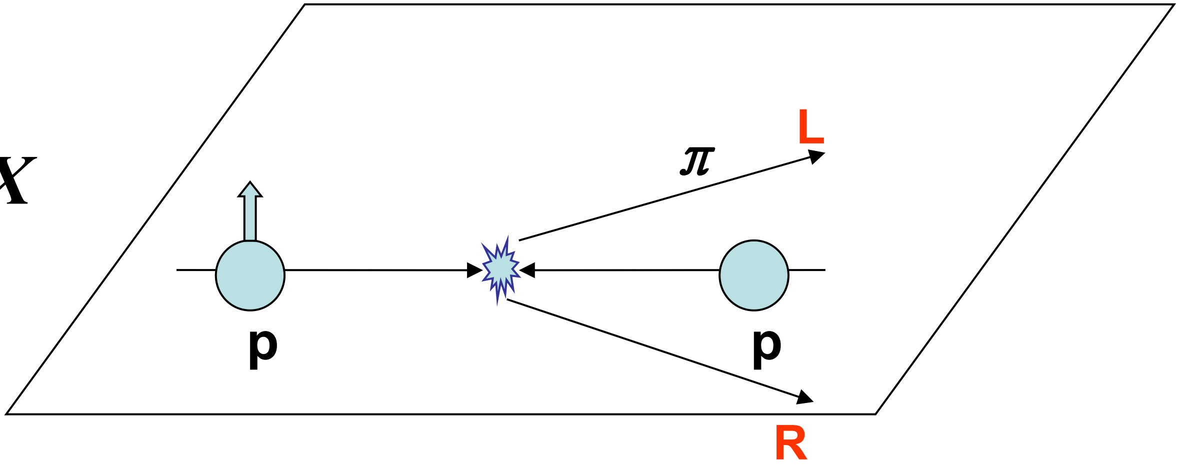
Parton distribution inside a transversely polarized target

parameterized as,

$$\Phi(x, k_T, S_T) = f(x, k_T^2) - \frac{\vec{k}_T \times \vec{S}_T}{M} f_{1T}^{\perp}(x, k_T^2)$$

1990, D. Sivers

$$p(\uparrow) + p \rightarrow \pi + X$$



Average transverse momentum:

$$\int dx d^2 k_T \Phi(x, k_T, S_T) \vec{k}_T^{\alpha} \propto \hat{S}_T^{\alpha} \int dx d^2 k_T f_{1T}^{\perp}(x, k_T^2) k_T^2 \neq 0$$

Transverse momentum conservation

Sum over all flavors

$$\sum_{q+\bar{q}+g} \int_0^1 dx \int d^2 k_T f_{1T}^{\perp}(x, k_T^2) k_T^2 = 0$$

The Burkardt sum rule, **2004, M. Burkardt**

The relations between the Sivers functions and twist-3 correlations,

2003, Boer, Mulders and Pijlman

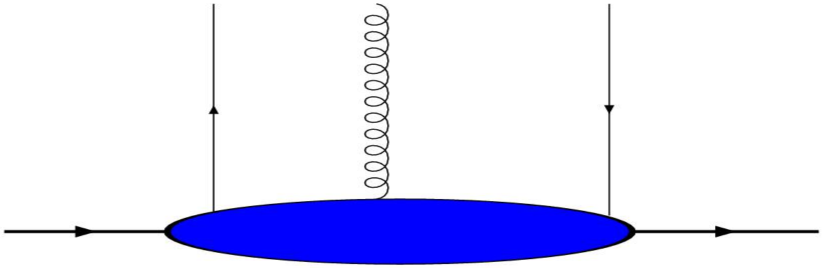
$$\int d^2 k_T f_{1T}^{\perp,q}(x, k_T^2) k_T^2 \propto T_F^q(x,x) \qquad \int d^2 k_T f_{1T}^{\perp,g}(x, k_T^2) k_T^2 \propto T_G^{(+)}(x,x)$$

The sum rule is re-expressed as,

$$\int_0^1 dx\, T_G^{(+)}(x,x) + \int_0^1 dx\, T_F^{q+\bar{q}}(x,x) = 0$$

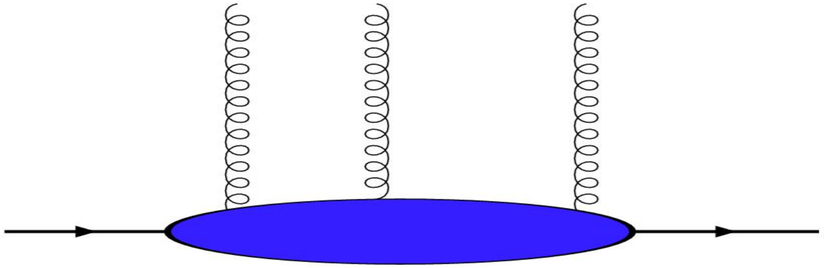
Operator definition of the twist-3 correlation functions

$$T_F(x,x,\mu^2) = \varepsilon_T^{S_T\sigma} \int \frac{dy_1^-}{2\pi} \int dy_2^- e^{ixP^+ y_1^-} \langle P | \overline{\psi}(0) \frac{\not{n}}{2} F_\sigma^+(y_2^-) \psi(y_1^-) | P \rangle$$



Qiu-Sterman function

1991, Qiu and Sterman



Tri-gluon function

1992, X. D. Ji; 2010, Beppu, Koike, Tanaka and Yoshida

For more discussions on high twist functions, see Liang's talk

The scale dependence of the Burkardt sum rule

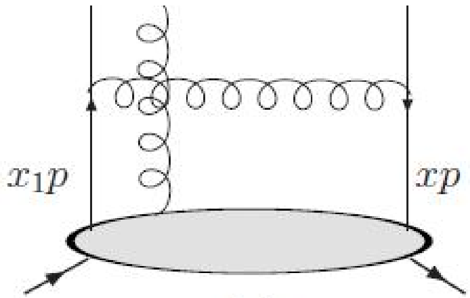
$$\sum_{q+\bar{q}+g} \langle \vec{k}_T \rangle (\mu^2) = \int_0^1 dx\, T_G^{(+)}(x,x,\mu^2) + \int_0^1 dx\, T_F^{q+\bar{q}}(x,x,\mu^2) = 0$$

Is the sum rule stable under QCD corrections?

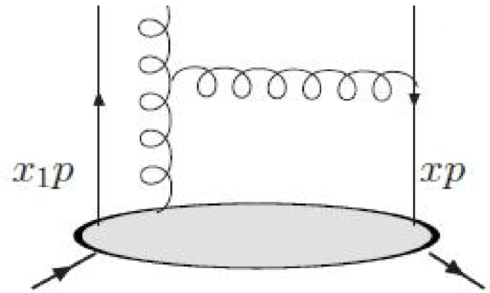
$$\frac{\partial \sum_{q+\bar{q}+g} \langle \vec{k}_T \rangle (\mu^2)}{\partial \ln \mu^2} = ?$$

DGLAP evolution of the twist-3 functions

Diagams contributing to the evolution kernel of the Qiu-Sterman function:



(a)



(b)

$$
\begin{aligned}
\left.\frac{\partial T_F^q(\xi,\xi,\mu^2)}{\partial \ln\mu^2}\right|_{q,\bar{q}\to q} = \frac{\alpha_s}{2\pi}\int_\xi \frac{dx}{x}\Bigg[& C_F\left\{\frac{1+z^2}{(1-z)_+}+\frac{3}{2}\delta(1-z)\right\}T_F^q(x,x)\\
&+\frac{C_A}{2}\left\{\frac{1+z}{1-z}T_F^q(\xi,x)-\frac{1+z^2}{1-z}T_F^q(x,x)-2\delta(1-z)T_F^q(x,x)\right\}\\
&-\frac{N_c}{2}\tilde{T}_F^q(\xi,x)+\frac{1}{2N_c}(1-2z)T_F^q(\xi,\xi-x)-\frac{1}{2N_c}\tilde{T}_F^q(\xi,\xi-x)\Bigg]
\end{aligned}
$$

2009, Kang and Qiu
2009, JZ, Yuan and Liang
2009, Vogelsang and Yuan
2011, Ma and Sang

2009, Braun, Manashov and Pirnay

2012, Schafer and JZ
2012, Ma and Wang
2012, Kang and Qiu
2013, Sun and Yuan

Computing all channels

quark+antiquark--->gluon

$$\left.\frac{\partial T_G^{(+)}(\xi,\xi,\mu^2)}{\partial \ln \mu^2}\right|_{q,\bar{q}\to g} = \frac{\alpha_s}{2\pi}\sum_{q,\bar{q}}\int_\xi \frac{dx}{x}\frac{C_A}{2}\left\{\frac{1+(1-z)^2}{z}[T_F^q(x,x)+T_F^{\bar{q}}(x,x)] - \frac{2-z}{z}[T_F^q(x,x-\xi)+T_F^{\bar{q}}(x,x-\xi)]+[\tilde{T}_F^q(x-\xi,x)+\tilde{T}_F^{\bar{q}}(x-\xi,x)]\right\}$$

2009, Braun, Manashov and Pirnay
2013, Schafer and JZ

gluon--->quark(or antiquark)

$$\left.\frac{\partial T_F^q(\xi,\xi,\mu^2)}{\partial \ln \mu_F^2}\right|_{g\to q} = \frac{\alpha_s}{2\pi}\int_\xi \frac{dx}{x}\frac{1}{2}[z^2+(1-z)^2]T_G^{(+)}(x,x)$$

2009, Kang and Qiu
2012, Ma and Wang

gluon--->gluon

$$\left.\frac{\partial\left[\frac{N(\xi,\xi)-N(\xi,0)}{\xi}\right]}{\partial \ln \mu_F^2}\right|_{g\to g} = \frac{\alpha_s}{2\pi}C_A\int_\xi^1 \frac{dx}{x^2}\left\{\frac{(z^2-z+1)^2}{z(1-z)_+}[N(x,x)-N(x,0)]+\frac{1+z^2}{2z(1-z)_+}N(\xi,x)-\frac{1+(1-z)^2}{2z(1-z)_+}N(x,x-\xi) - \frac{z^2+(1-z)^2}{2z(1-z)_+}N(\xi,\xi-x)-\delta(1-z)[N(x,x)-N(x,0)]\right\}+\frac{\alpha_s}{2\pi}\left(C_A\frac{11}{6}-\frac{n_f}{3}\right)[N(\xi,\xi)-N(\xi,0)]$$

$$\frac{x}{2\pi}T_G^{(+)}(x,x) = -4M_N[N(x,x)-N(x,0)]$$

2009, Kang and Qiu
2009, Braun, Manashov and Pirnay
2013, Schafer and JZ

We agree on all other evolution equations except for the tir-gluon one.

2009, Kang and Qiu
2009, Braun, Manashov and Pirnay
2013, Schafer and JZ

- Here we use the one derived by the third group.

$$\frac{\partial\left[\frac{N(\xi,\xi)-N(\xi,0)}{\xi}\right]}{\partial \ln \mu_F^2}\bigg|_{g\to g} = \frac{\alpha_s}{2\pi} C_A \int_\xi^1 \frac{dx}{x^2}\left\{\frac{(z^2-z+1)^2}{z(1-z)_+}[N(x,x)-N(x,0)] + \frac{1+z^2}{2z(1-z)_+}N(\xi,x) - \frac{1+(1-z)^2}{2z(1-z)_+}N(x,x-\xi)\right.$$
$$\left. - \frac{z^2+(1-z)^2}{2z(1-z)_+}N(\xi,\xi-x) - \delta(1-z)[N(x,x)-N(x,0)]\right\} + \frac{\alpha_s}{2\pi}\left(C_A\frac{11}{6}-\frac{n_f}{3}\right)[N(\xi,\xi)-N(\xi,0)]$$

Collecting all results:

$$\frac{\partial \sum\limits_{q+\bar{q}+g} \langle \vec{k}_T \rangle (\mu^2)}{\partial \ln \mu^2} = -\frac{\alpha_s}{2\pi} C_A \sum_{q+\bar{q}+g} \langle \vec{k}_T \rangle (\mu^2)$$

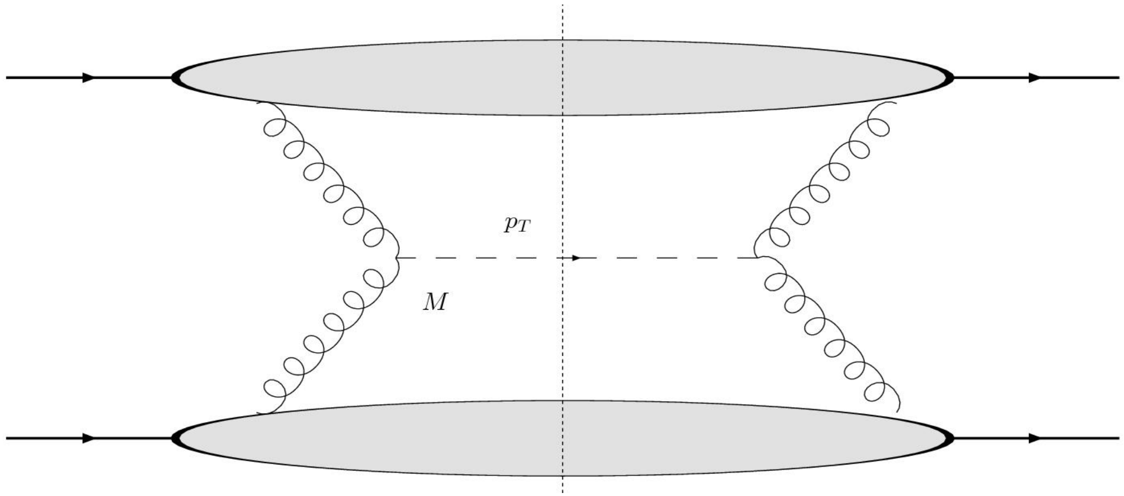
2015, ZJ

Solution:

$$\sum_{q+\bar{q}+g} \langle \vec{k}_T \rangle (Q^2) = e^{-\frac{C_A}{2\pi}\int_{Q_0^2}^{Q^2} \frac{d\mu^2}{\mu^2} \alpha_s(\mu^2)} \sum_{q+\bar{q}+g} \langle \vec{k}_T \rangle (Q_0^2)$$

The evolution of the small x gluon TMD

Gluon initiated Drell-Yan process



Inclusive process: Collinear factorization;

large logarithm $\ln\frac{M^2}{\mu^2}$ resummed by DGLAP equation

Semi-inclusive process:

- $M^2 >> p_T^2$, TMD factorization, $\ln\frac{M^2}{p_T^2}$ resummed by Collins-Soper equation

  1982-1983, Collins, Soper

- $S >> M^2$, Kt factorization, $\ln\frac{S}{M^2}$ resummed by BFKL equation

  1991, Catani, Ciafaloni and Hautmann

  1991, Collins and R. K. Ellis

The overlap region

$$S >> M^2 >> p_T^2$$

Both the TMD and Kt factorization apply.

1999, Li and Lim

An explicit NLO cross section calculation shows that both the large logarithm appear.

$$\ln\frac{S}{M^2} \qquad \ln\frac{M^2}{p_T^2}$$

2013, Mueller, Xiao, Yuan

Such joint resummation has been also disscussed in other literatures.

2015, Kovchegov and Sievert; 2015, Balitsky and Tarasov; 2015 Marzani

Our starting point

$$xG(x,l_\perp,x\zeta)=\int\frac{dy^-d^2y_\perp}{(2\pi)^3P^+}e^{-ixP^+y^-+il_\perp\cdot y_\perp}\langle P|F_\mu^+(y^-,y_\perp)\mathcal{L}_{\tilde{n}}^\dagger(y^-,y_\perp)\mathcal{L}_{\tilde{n}}(0,0_\perp)F^{\mu+}(0)|P\rangle$$

Gloun---->gloun splitting kernel;

$$\mathcal{P}_{gg}(z)=2C_A\frac{(z^2-z+1)^2}{z(1-z)}$$

When z(or x) --->0, large logarithm $\ln\frac{1}{x}$ summed by BFKL

When z--->1, light cone divergence, introduce ζ to regularize

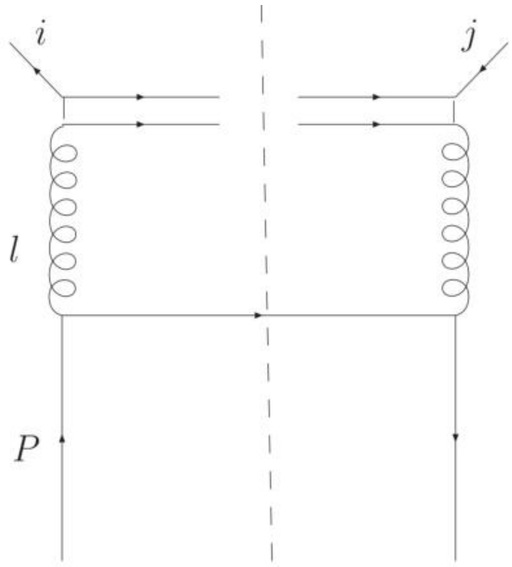
large logarithm $\ln\frac{x^2\varsigma^2}{k_T^2}$ summed by CS

See Li's talk

$$\ln\frac{1}{x}\Longrightarrow\ln\frac{S}{M^2}\qquad\qquad\ln\frac{x^2\varsigma^2}{k_T^2}\Longrightarrow\ln\frac{M^2}{p_T^2}$$

The main strategy

In a simple quark model, at tree level:

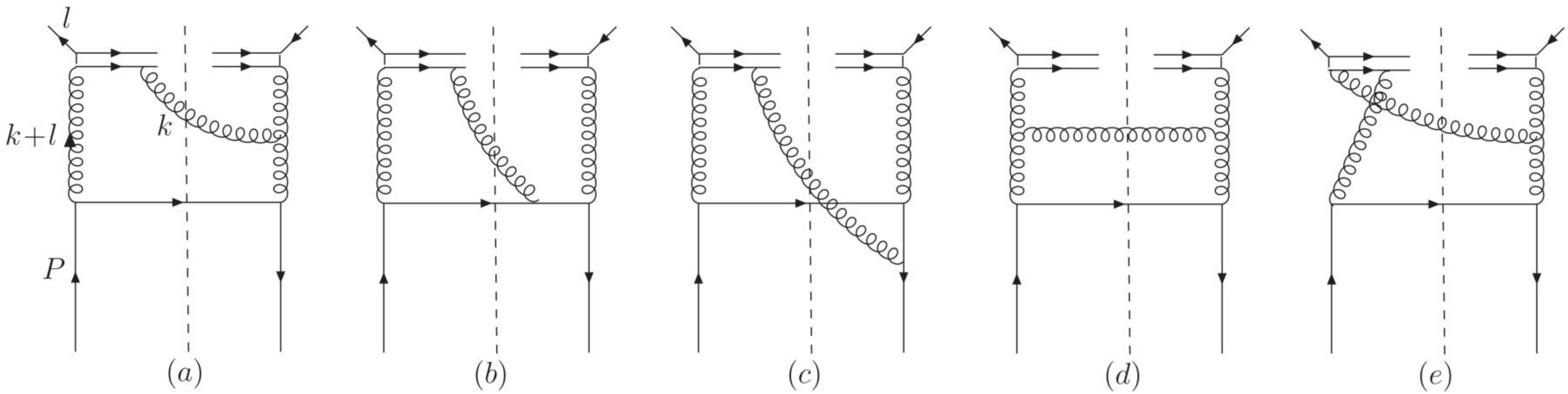


$$xG(x, l_\perp, x\zeta)_{LO}\big|_{x\to 0} = \frac{\alpha_s C_F}{\pi^2}\frac{1}{l_\perp^2}$$

- both large logarithms are absent at LO;
  how is it dressed by quantum corrections at NLO?

Real graphs

Sample diagrams:



- Fig.a is the only diagram contributing to both the CS and the BFKL evolution kernel

- Calculation formulated in the Ji-Ma-Yuan scheme.

Virtual graphs

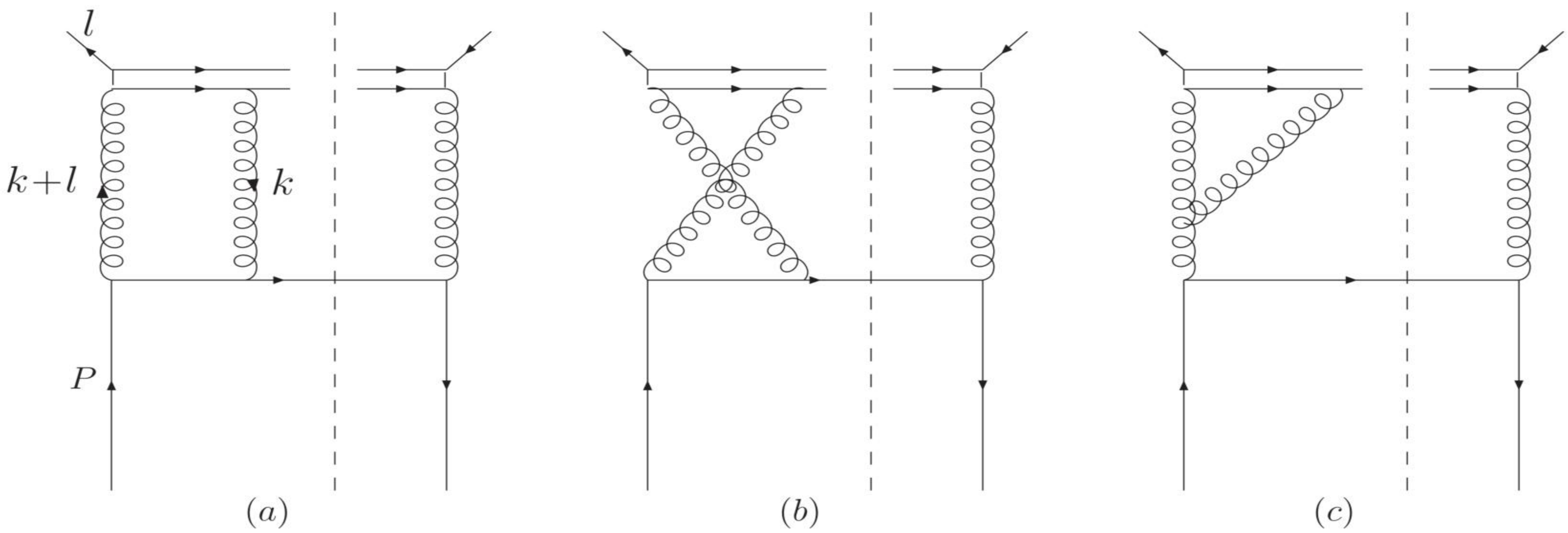


Fig.a & Fig.b ⟹ BFKL kernel

Fig.c ⟹ CS kernel

Collecting all results

In the leading logarithm approximations,

Small x gluon TMD at NLO reads,

$$
\begin{aligned}
xG(x,l_\perp,x\zeta)_{NLO} &= xG(x,l_\perp,x\zeta)_{LO} \\
&+ \frac{\alpha_s N_c}{\pi^2}\ln\frac{1}{x}\int\frac{d^2k_\perp}{k_\perp^2}\left[xG_{LO}(x,k_\perp+l_\perp,x\zeta)-\frac{l_\perp^2}{2(l_\perp+k_\perp)^2}xG_{LO}(x,l_\perp,x\zeta)\right] \\
&+ \frac{\alpha_s N_c}{2\pi}\left[\ln\frac{x^2\zeta^2}{l_\perp^2}-\frac{1}{2}\ln\frac{x^2\zeta^2}{\mu^2}-\left(\ln\frac{x^2\zeta^2}{l_\perp^2}\right)^2\right]xG_{LO}(x,l_\perp,x\zeta) \\
&+ \frac{\alpha_s N_c}{\pi^2}\int\frac{d^2k_\perp}{2\left[k_\perp^2+l_\perp^4/x^2\zeta^2\right]}\ln\frac{k_\perp^2(k_\perp^2+x^2\zeta^2)}{(k_\perp^2+l_\perp^2)^2}xG_{LO}(x,k_\perp+l_\perp,x\zeta)
\end{aligned}
$$

2016, ZJ

The resulting gluon TMD indeed simultaneously satisfies the both

BFKL equation:

$$\frac{\partial\left[xG(x,l_\perp,x\zeta)\right]}{\partial\ln(1/x)} = \frac{\alpha_s N_c}{\pi^2}\int\frac{d^2k_\perp}{k_\perp^2}\left\{xG(x,k_\perp+l_\perp,x\zeta)-\frac{l_\perp^2}{2(l_\perp+k_\perp)^2}xG(x,l_\perp,x\zeta)\right\}$$

CS equation:

$$\frac{\partial\left[G(x,b_\perp,x\zeta)\right]}{\partial\ln\zeta} = -\frac{\alpha_s N_c}{\pi}\ln\left[\frac{x^2\zeta^2 b_\perp^2}{4}e^{2\gamma_E-\frac{1}{2}}\right]G(x,b_\perp,x\zeta)$$

slightly different from Ji-Ma-Yuan's result

Summary

- Transverse momentum conservation is preserved under QCD scale evolution.
- TMD factorization and kt factorization should be jointly employed when three scales are well separated.

Thank you for your attention!