Three-body B decays

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Outlines

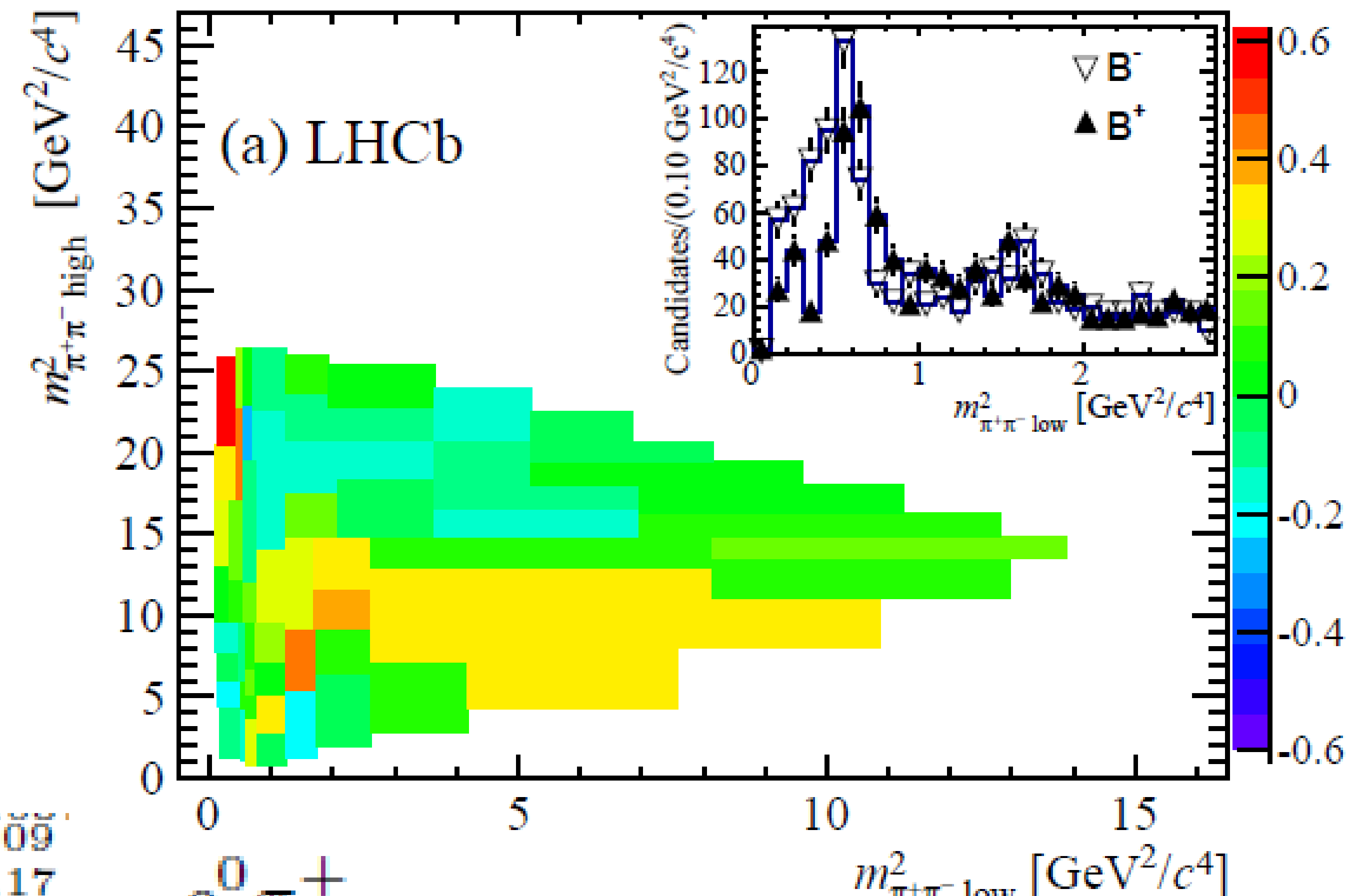
- Introduction
- Two-hadron distribution amplitudes
- Resonant contributions
- Summary

Introduction

See also Xin-Qiang and Deshan‘s talks

Dalitz plot

- LHCb has measured CP asymmetries in whole Dalitz plot



$0.18^{+0.09}_{-0.17}$

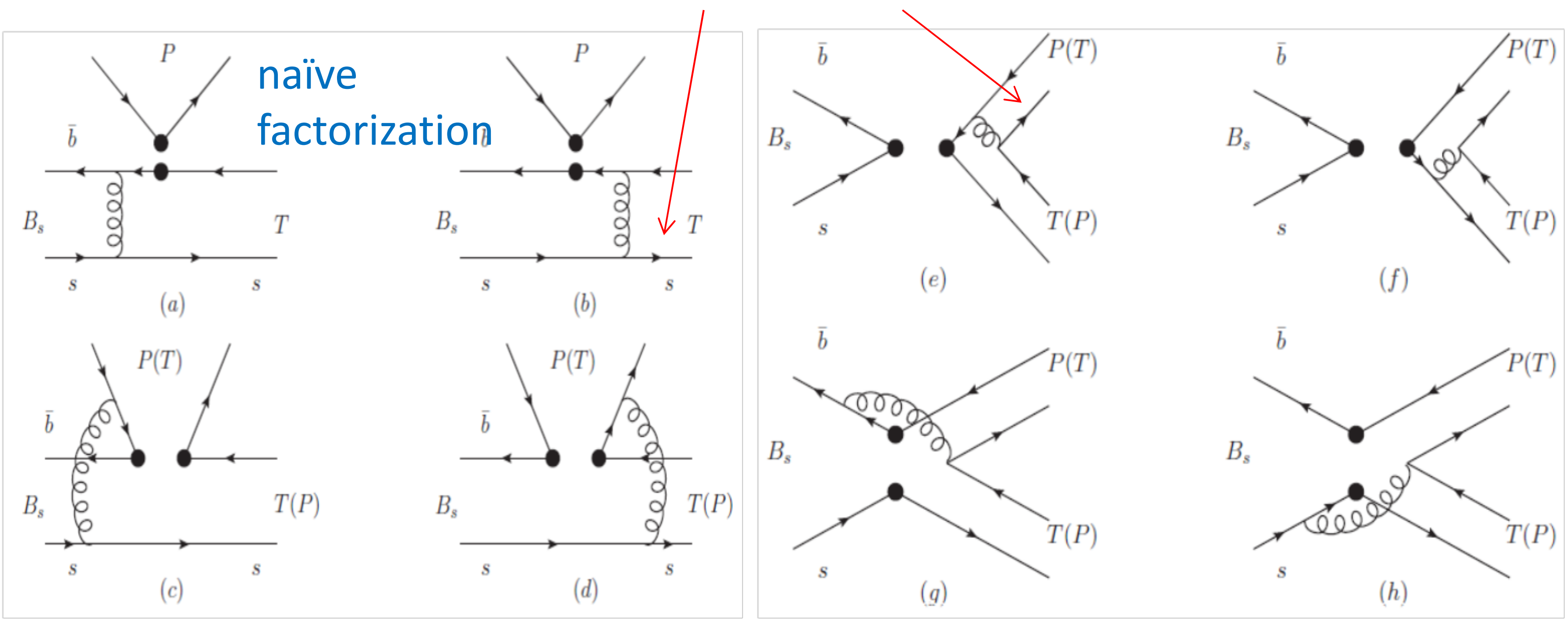
$\rho^0\pi^+$

Goals

- Develop a theoretical approach to 3-body hadronic B decays
- Understand data of direct CP asymmetries in localized regions, focusing on 3pi, Kpipi
- Predict direct CP asymmetries in other 3-body decay modes
- Predict direct CP asymmetries in whole phase space (resonant + nonresonant) . Very challenging

PQCD for 2-body B decays

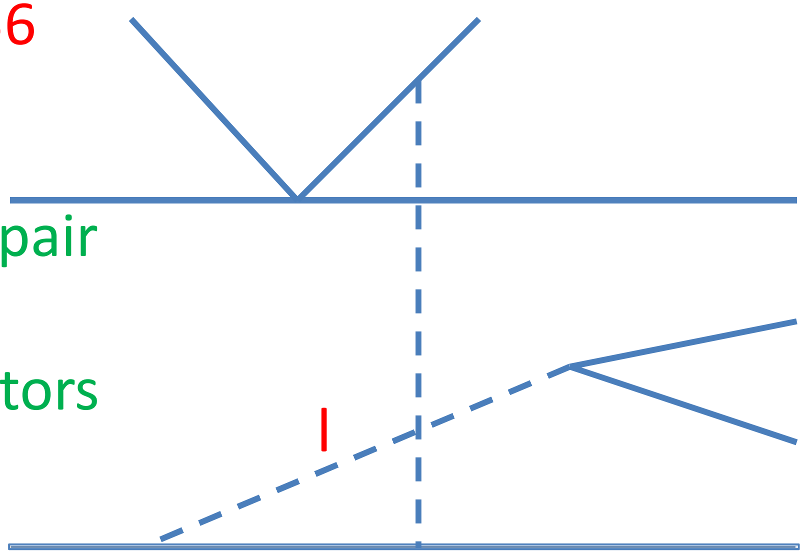
- PQCD approach to 2-body B decays based on kT factorization: b quark decay kernel convoluted with TMD hadron wave functions
- Parton kT smears end-point singularity



Typical diagram for 3-body decay

partial counting
8 X 2 X 8 X2= 256

attachment of l
location of pion pair
LO diagrams
4-fermion operators



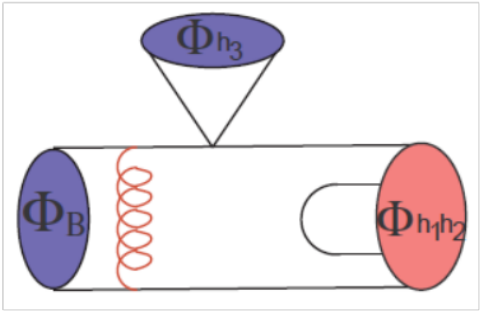
$$l^2 \sim w^2$$

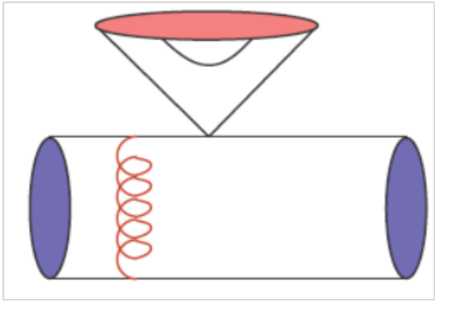
$w^2 \sim m_B^2$ power suppressed compared to

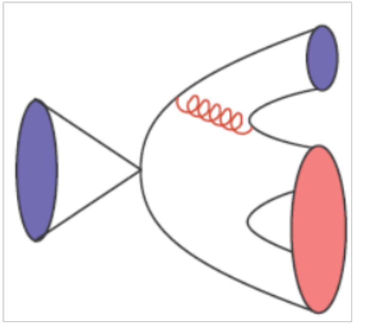
$$w^2 \sim \Lambda m_B, \Lambda^2$$

Approaches in literature

- Based on parameterizations of
  current-induced process
  transition process
  But, annihilation process?
- Nonfactorizable contribution?
- Resonant via Breit-Wigner then double counting of nonresonant?
- Rescattering strong phases?



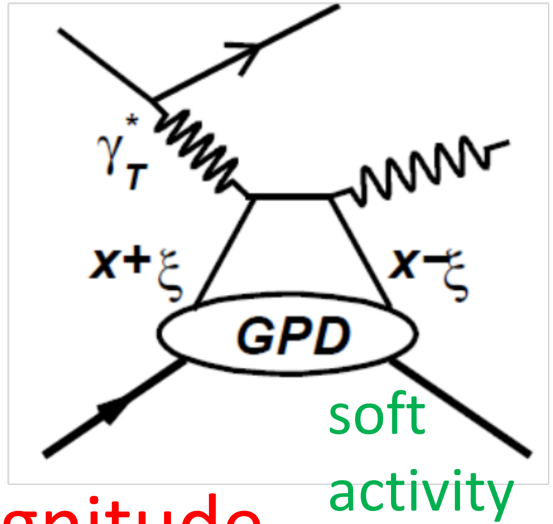


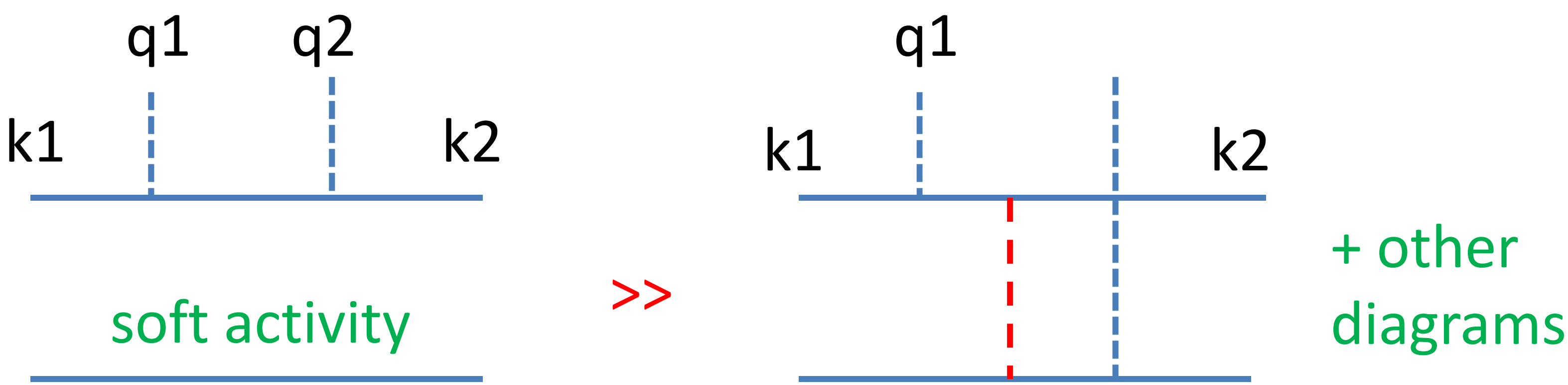


Two-hadron distribution amplitudes

Our proposal in 2002 (Chen, Li)

- Inspired by generalized parton distribution (GPD) based on dominance of hand-bag diagram in forward scattering
- Non-forward, same order of magnitude



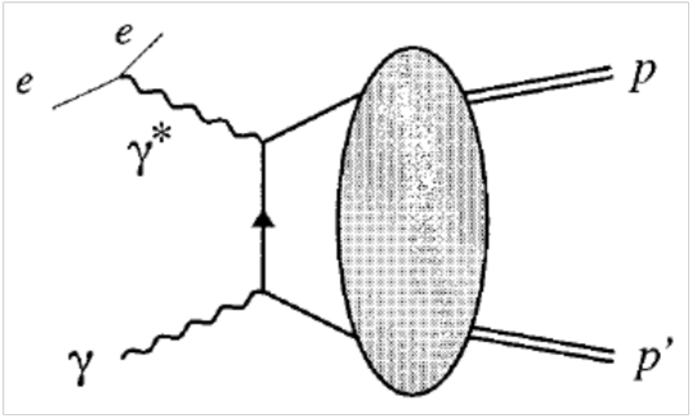


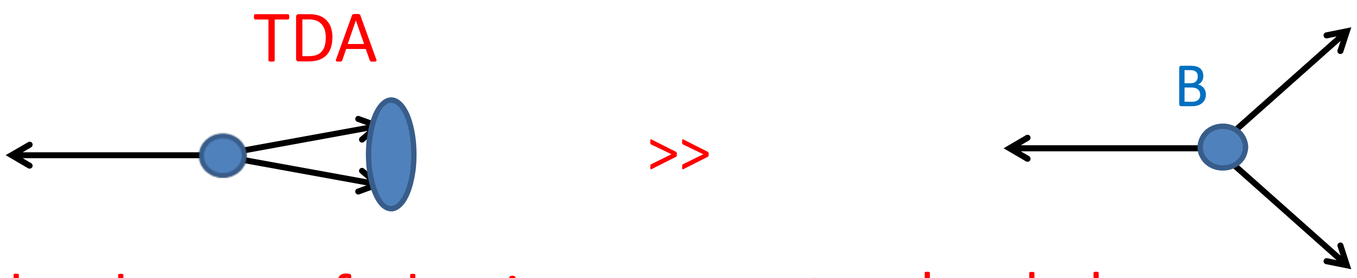
k1 // k2
no need of hard gluon

k1+q1 = k2? k2 off-shell
need hard gluon

Two-hadron DA

- Introduce two-hadron distribution amplitude (TDA, crossing of GPD) for dominant region in 3-body B decays





one hard, one soft dominant as two hadrons collimate

two hard-gluon power suppressed

Definitions of TDAs

- TDAs for vector, scalar, tensor currents (from Fierz transformation for factorizing quark flow)

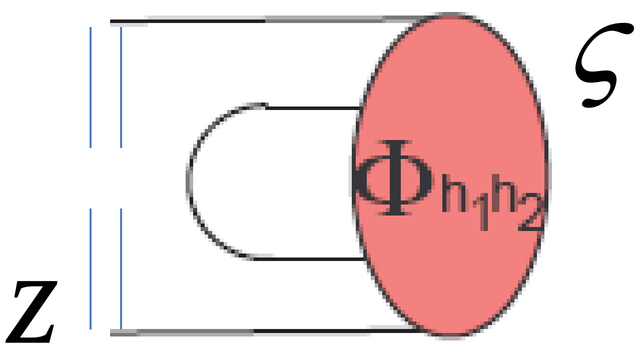
$\sigma^3/2$

$$\Phi_v(z,\zeta,w^2) = \frac{1}{2\sqrt{2N_c}} \int \frac{dy^-}{2\pi} e^{-izP^+y^-} \langle \pi^+(P_1)\pi^-(P_2)|\bar{\psi}(y^-)\; \not{n}_- T\psi(0)|0\rangle \,,$$

$$\Phi_s(z,\zeta,w^2) = \frac{1}{2\sqrt{2N_c}} \frac{P^+}{w} \int \frac{dy^-}{2\pi} e^{-izP^+y^-} \langle \pi^+(P_1)\pi^-(P_2)|\bar{\psi}(y^-) T\psi(0)|0\rangle \,,$$

$1/2$

$$\Phi_t(z,\zeta,w^2) = \frac{1}{2\sqrt{2N_c}} \frac{f_{2\pi}^{\perp}}{w^2} \int \frac{dy^-}{2\pi} e^{-izP^+y^-} \langle \pi^+(P_1)\pi^-(P_2)|\bar{\psi}(y^-) i\sigma_{\mu\nu} n_-^{\mu} P^{\nu} T\psi(0)|0\rangle$$



I = 1 with vector, tensor (C-parity odd)

I = 0 with scalar (C-parity even)

Kinematics

- Meson momenta in light-cone coordinates

$$p_B = \frac{m_B}{\sqrt{2}}(1, 1, 0_{\rm T}), \quad p = \frac{m_B}{\sqrt{2}}(1, \eta, 0_{\rm T}), \quad p_3 = \frac{m_B}{\sqrt{2}}(0, 1-\eta, 0_{\rm T})$$

- Two-hadron invariant mass

$$\omega^2 = p^2 \qquad p = p_1 + p_2 \qquad \eta = \frac{\omega^2}{m_B^2}$$

pi+ pi-

- pi+ momentum fraction

$$p_1^+ = \zeta \frac{m_B}{\sqrt{2}}, \quad p_1^- = (1-\zeta)\eta \frac{m_B}{\sqrt{2}}, \quad p_2^+ = (1-\zeta)\frac{m_B}{\sqrt{2}}, \quad p_2^- = \zeta\eta \frac{m_B}{\sqrt{2}}$$

Parameterization of TDAs

- Normalization

$$\propto (p_1 - p_2)^\mu F_\pi$$

$$\int_0^1 dz \Phi_{\parallel}^{I=1}(z,\zeta,w^2) = (2\zeta - 1)F_\pi(w^2)$$

$$\int_0^1 dz \Phi_{\perp}^{I=1}(z,\zeta,w^2) = (2\zeta - 1)F_t(w^2)$$

- Up to leading partial wave expansion

$$\Phi_{v,t}(z,\zeta,w^2) = \frac{3F_{\pi,t}(w^2)}{\sqrt{2N_c}} z(1-z)(2\zeta - 1)$$

complex time-like

$$\Phi_s(z,\zeta,w^2) = \frac{3F_s(w^2)}{\sqrt{2N_c}} z(1-z)$$

correspond to l = 1
P wave…

correspond to l = 0
S wave…

form factors Fs, Ft, twist-3,
suppressed by a power in PQCD

Direct CP asymmetry

- Factorization formula for decay amplitude

$$\mathcal{M} = \Phi_B \otimes H \otimes \Phi_{h_1 h_2} \otimes \Phi_{h_3}$$

- B meson and hadron DAs have been widely adopted in analysis of 2-body decay
- Calculate B+ and B- decays

$$A_{CP}^{reg}(B^{\pm} \to \pi^{\pm}\pi^{+}\pi^{-}) = 0.52^{+0.12}_{-0.22}(\omega_B)^{+0.11}_{-0.09}(a_2^{\pi})^{+0.03}_{-0.03}(m_0^{\pi})$$

+-0.05 +-0.15 +-0.1

- Data $A_{CP}^{\rm region}(\pi^{+}\pi^{-}\pi^{-}) = 0.584 \pm 0.082 \pm 0.027 \pm 0.007$
- Short-distance phase is important
- P wave phase doubled, Acp increases up to 0.7

Resonant contributions

Pire's communication

- After posting our preprint, B. Pire wrote to me about his works
- 0202231[hep-ph] contains an expression

$$\Phi^{I=0}(z,\zeta,m_{2\pi}) = 10z(1-z)(2z-1)\,R_\pi$$

$$\left[-\frac{3-\beta^2}{2}\,e^{i\delta_0(m_{2\pi})}\,|BW_{f_0}(m_{2\pi}^2)| + \beta^2\,e^{i\delta_2(m_{2\pi})}\,|BW_{f_2}(m_{2\pi}^2)|\,P_2(\cos\theta)\right]$$

$$BW_{f_{0/2}}(m_{2\pi}^2) = \frac{m_{f_{0/2}}^2}{m_{f_{0/2}}^2 - m_{2\pi}^2 - i\,m_{f_{0/2}}\Gamma_{f_{0/2}}}$$

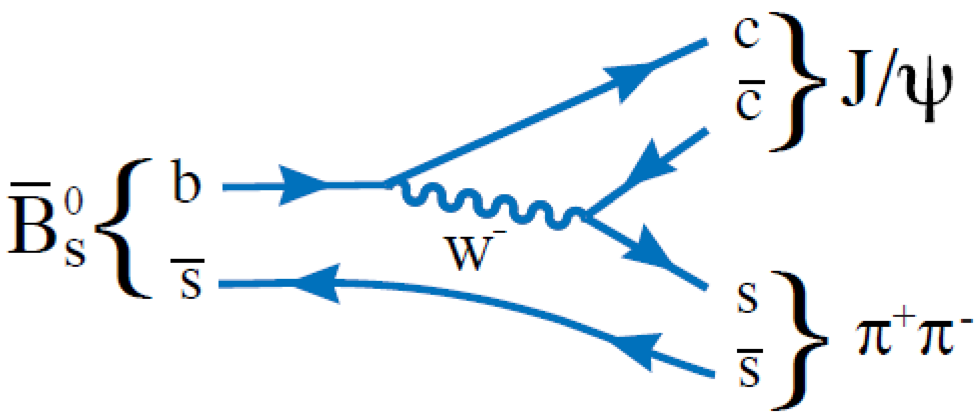
- Nonresonant ~ 1/w^2 asymptotically
- Resonant ~ 1/w^4, faster than nonresonant

LHCb measurement

CERN-PH-EP-2013-024
LHCb-PAPER-2013-069
February 26, 2014

1402.6248

Measurement of resonant and *CP* components in $\overline{B}^0_s \to J/\psi \pi^+\pi^-$ decays



I=0, S wave

Fit fractions

Fit fractions (%) of contributing components for both solutions

Component	Solution I	Solution II
$f_0(980)$	$70.3 \pm 1.5^{+0.4}_{-5.1}$	$92.4 \pm 2.0^{+\ 0.8}_{-16.0}$
$f_0(1500)$	$10.1 \pm 0.8^{+1.1}_{-0.3}$	$9.1 \pm 0.9 \pm 0.3$
$f_0(1790)$	$2.4 \pm 0.4^{+5.0}_{-0.2}$	$0.9 \pm 0.3^{+2.5}_{-0.1}$
$f_2(1270)_0$	$0.36 \pm 0.07 \pm 0.03$	$0.42 \pm 0.07 \pm 0.04$
$f_2(1270)_\parallel$	$0.52 \pm 0.15^{+0.05}_{-0.02}$	$0.42 \pm 0.13^{+0.11}_{-0.02}$
$f_2(1270)_\perp$	$0.63 \pm 0.34^{+0.16}_{-0.08}$	$0.60 \pm 0.36^{+0.12}_{-0.09}$
$f_2'(1525)_0$	$0.51 \pm 0.09^{+0.05}_{-0.04}$	$0.52 \pm 0.09^{+0.05}_{-0.04}$
$f_2'(1525)_\parallel$	$0.06^{+0.13}_{-0.04} \pm 0.01$	$0.11^{+0.16+0.03}_{-0.07-0.04}$
$f_2'(1525)_\perp$	$0.26 \pm 0.18^{+0.06}_{-0.04}$	$0.26 \pm 0.22^{+0.06}_{-0.05}$
NR	-	$5.9 \pm 1.4^{+0.7}_{-4.6}$

Flatte ad BW models

$$F_s^{s\bar{s}}(\omega^2) = \frac{c_1 m_{f_0(980)}^2 e^{i\theta_1}}{m_{f_0(980)}^2 - \omega^2 - i m_{f_0(980)}(g_{\pi\pi}\rho_{\pi\pi} + g_{KK}\rho_{KK})} + \frac{c_2 m_{f_0(1500)}^2 e^{i\theta_2}}{m_{f_0(1500)}^2 - \omega^2 - i m_{f_0(1500)}\Gamma_{f_0(1500)}(\omega^2)} + \frac{c_3 m_{f_0(1790)}^2 e^{i\theta_3}}{m_{f_0(1790)}^2 - \omega^2 - i m_{f_0(1790)}\Gamma_{f_0(1790)}(\omega^2)},$$

Flatte PLB, 1976

$$\rho_{\pi\pi} = \frac{2}{3}\sqrt{1 - \frac{4m_{\pi^\pm}^2}{\omega^2}} + \frac{1}{3}\sqrt{1 - \frac{4m_{\pi^0}^2}{\omega^2}} \qquad g_{\pi\pi} = 0.167\ \mathrm{GeV}$$

$$\rho_{KK} = \frac{1}{2}\sqrt{1 - \frac{4m_{K^\pm}^2}{\omega^2}} + \frac{1}{2}\sqrt{1 - \frac{4m_{K^0}^2}{\omega^2}} \qquad g_{KK} = 3.47 g_{\pi\pi}$$

PQCD results

$$c_1 = 1.17, \quad c_2 = 0.12, \quad c_3 = 0.06,$$
$$\theta_1 = -\frac{\pi}{2}, \quad \theta_2 = \frac{\pi}{4}, \quad \theta_3 = 0.$$

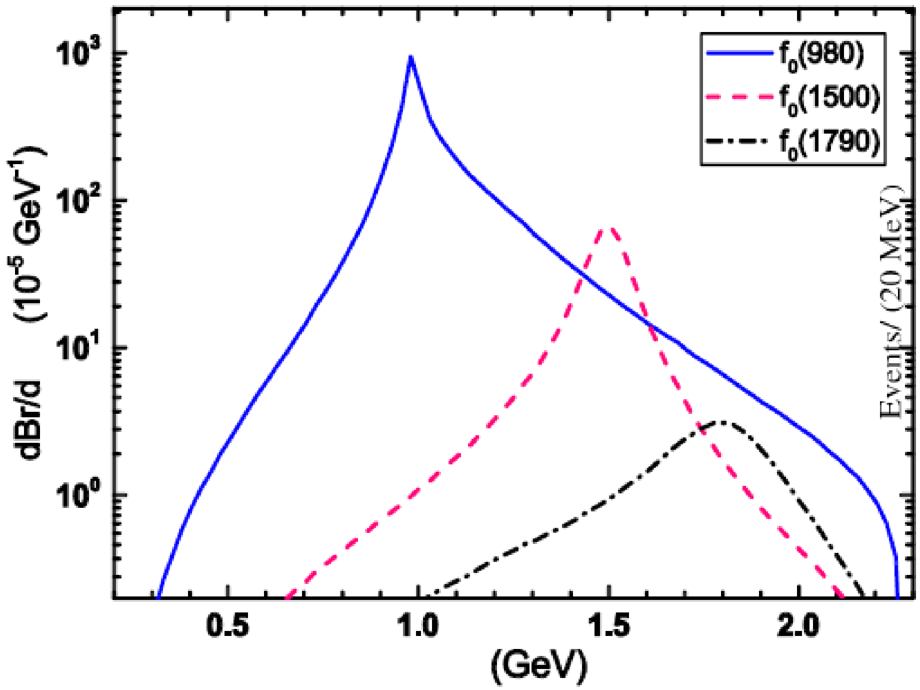
$$\mathrm{Br}(B_s^0 \to J/\psi f_0(980)[f_0(980) \to \pi^+\pi^-])$$
$$\mathrm{Br}(B_s^0 \to J/\psi f_0(1500)[f_0(1500) \to \pi^+\pi^-])$$
$$\mathrm{Br}(B_s^0 \to J/\psi f_0(1790)[f_0(1790) \to \pi^+\pi^-])$$

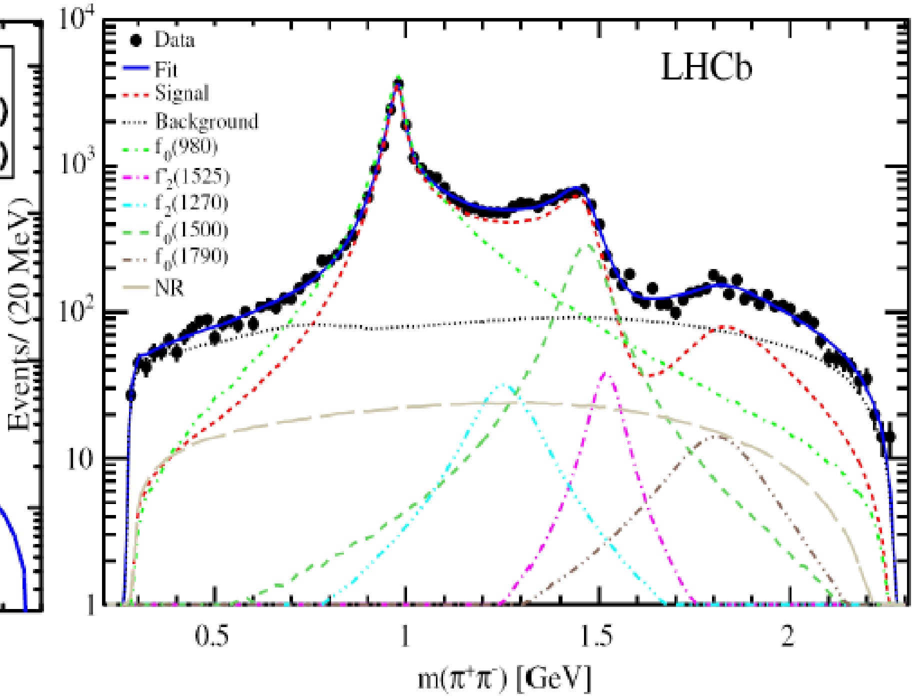
$$\left(1.33^{+0.51}_{-0.36}(\omega_{B_s})^{+0.19}_{-0.16}(a_2^{I=0})^{+0.03}_{-0.02}(m_c)\right) \times 10^{-4}\ , \qquad 75.1\%$$
$$\left(1.77^{+0.53}_{-0.39}(\omega_{B_s})^{+0.30}_{-0.25}(a_2^{I=0}) \pm 0.02(m_c)\right) \times 10^{-5} \qquad 10.0\%$$
$$\left(2.15^{+0.58}_{-0.49}(\omega_{B_s})^{+0.34}_{-0.32}(a_2^{I=0}) \pm 0.03(m_c)\right) \times 10^{-6} \qquad 1.2\%$$

closer to Solution I of LHCb data

Comparison with data

$$B_s^0 \to J/\Psi\pi^+\pi^-$$





PQCD(NLO)

LHCb **(Sol 1)**

Summary

- Systematic approach to 3-body B decays with TDA has been established
- Short-distance and rescattering P-wave phases are equally important for predicting Acp
- Can include both resonant and nonresonant contributions at the same time
- Can explain and predict direct CP asymmetries of 3-body B decays in various localized regions of phase space
- This approach is getting mature

Back-up slides

3-body reduced to 2-body

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Three-body nonleptonic *B* decays in perturbative QCD

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Abstract

We develop perturbative QCD formalism for three-body nonleptonic *B* meson decays. Leading contributions are identified by defining power counting rules for various topologies of amplitudes. The analysis is simplified into the one for two-body decays by introducing two-meson distribution amplitudes. This formalism predicts both nonresonant and resonant contributions, and can be generalized to baryonic decays.

Motivation

- Recent LHCb data of direct CP asymmetries in localized regions of phase space

$$A_{CP}^{\text{region}}(K^+K^-K^-) = -0.226 \pm 0.020 \pm 0.004 \pm 0.007,$$

for $m^2_{K^+K^-\text{high}} < 15\ \text{GeV}^2$ and $1.2 < m^2_{K^+K^-\text{low}} < 2.0\ \text{GeV}^2$

$$A_{CP}^{\text{region}}(K^-\pi^+\pi^-) = 0.678 \pm 0.078 \pm 0.032 \pm 0.007$$

for $m^2_{K^-\pi^+\text{high}} < 15\ \text{GeV}^2$ and $0.08 < m^2_{\pi^+\pi^-\text{low}} < 0.66\ \text{GeV}^2$

$$A_{CP}^{\text{region}}(K^+K^-\pi^-) = -0.648 \pm 0.070 \pm 0.013 \pm 0.007$$

for $m^2_{K^+K^-} < 1.5\ \text{GeV}^2$

rho resonance

$$A_{CP}^{\text{region}}(\pi^+\pi^-\pi^-) = 0.584 \pm 0.082 \pm 0.027 \pm 0.007$$

for $m^2_{\pi^+\pi^-\text{high}} > 15\ \text{GeV}^2$ and $m^2_{\pi^+\pi^-\text{low}} < 0.4\ \text{GeV}^2$

C-parity

- C-parity (charge parity) for mesonic state is equivalent to parity

$$\mathcal{C}\,|\pi^+\,\pi^-\rangle = (-1)^L\,|\pi^+\,\pi^-\rangle$$

- C-parity for quark fields (spinors)

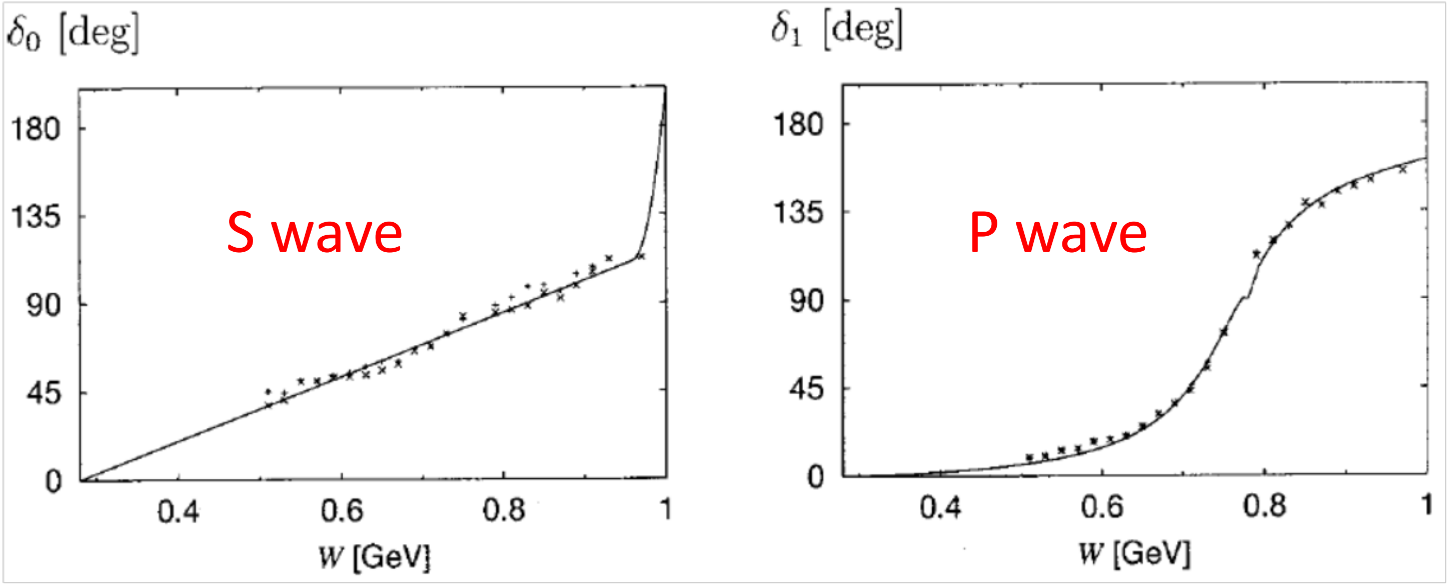
$$\psi^{(c)} = C\psi^\star \qquad C = i\gamma^2$$

$$C^\dagger\gamma^\mu C = -(\gamma^\mu)^\star$$

- C-parity is odd for vector and tensor currents, and even for scalar current

Rescattering phases

- LHCb data of CP asymmetries in localized regions (nonresonant only) offered a chance to confront our theory
- Data for rescattering phases in localized region ($m_{\pi\pi}^2 < 0.4\,GeV^2$) are available



Complex time-like form factors

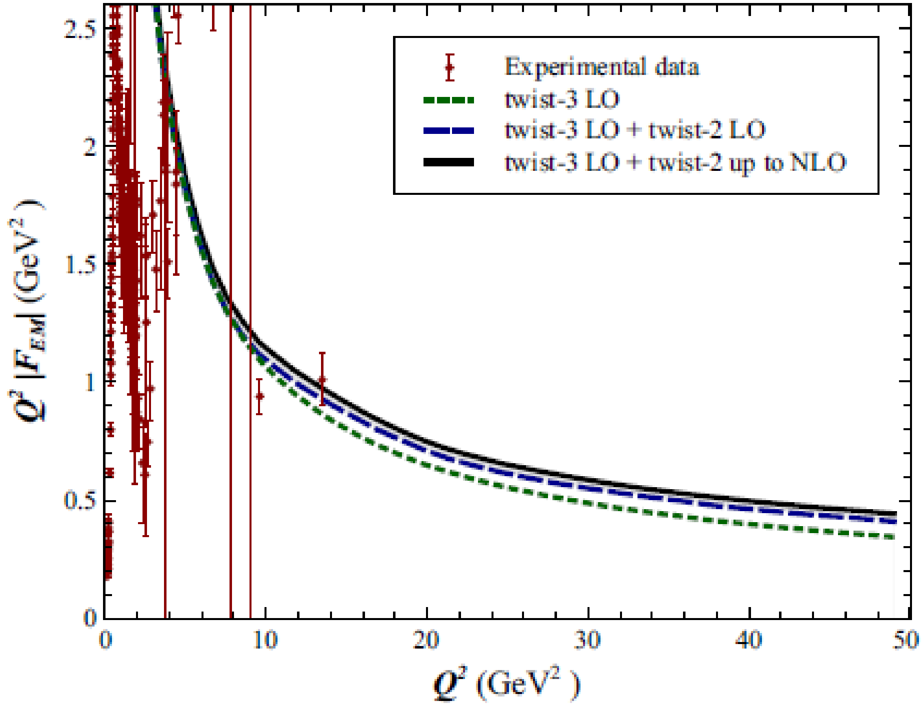
- P and S waves

$$F_\pi(w^2) = \frac{m^2 \exp[i\delta_1^1(w)]}{w^2 + m^2}$$

m=1 GeV

from data

$$m_{J/\psi}^2 |F_\pi(m_{J/\psi}^2)|^2 \sim 0.9 \text{ GeV}^2$$



$$F_s(w^2) = \frac{m_0^\pi m^2 \exp[i\delta_0^0(w)]}{w^3 + m_0^\pi m^2} \qquad F_t(w^2) = \frac{m_0^\pi m^2 \exp[i\delta_1^1(w)]}{w^3 + m_0^\pi m^2}$$

Watson theorem

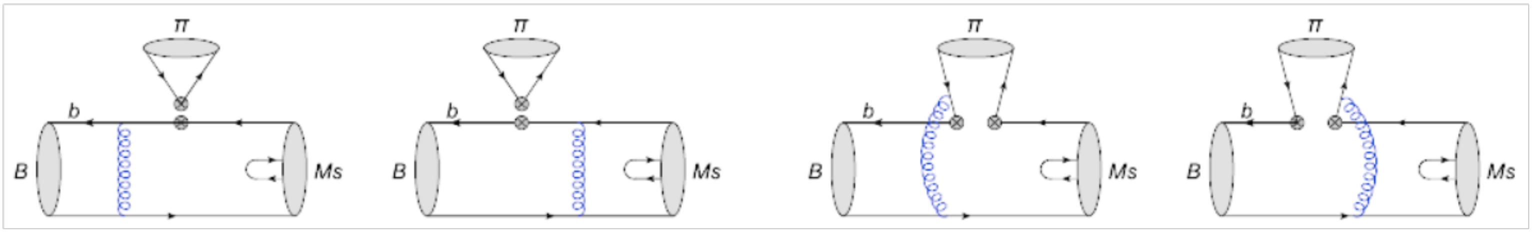
$$F_{s,t}(w^2)/F_\pi(w^2) \sim m_0^\pi / w$$

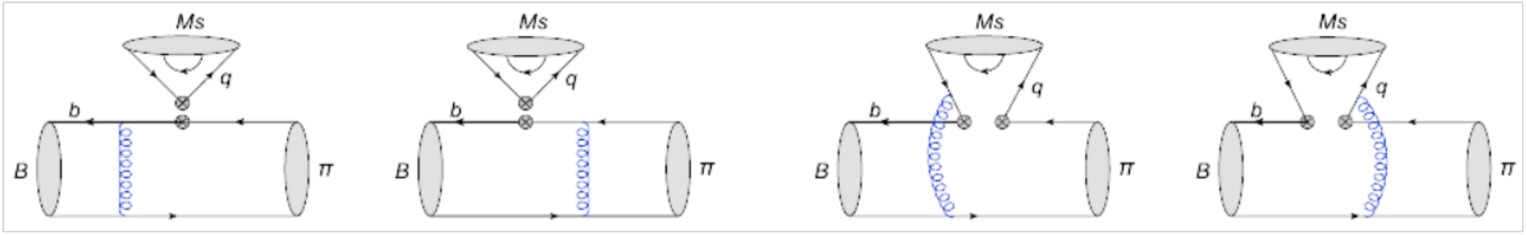
$$\frac{M_\pi^2}{m_u + m_d} = 1.4 \text{ GeV}$$

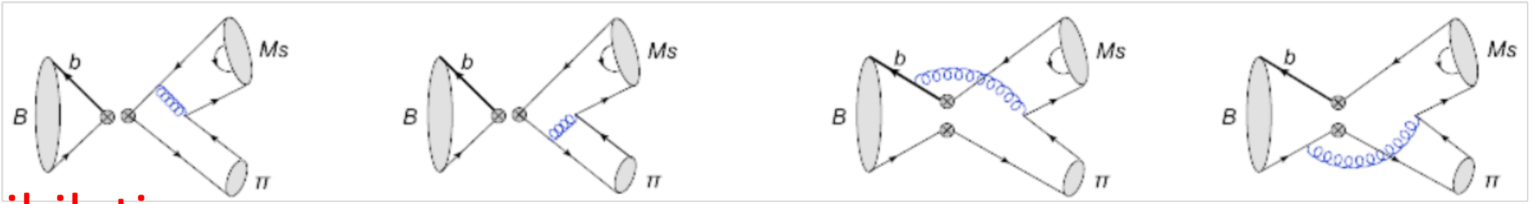
Feynman diagrams

- All inputs are ready, go ahead to calculate 16 diagrams (load 10 times lower)

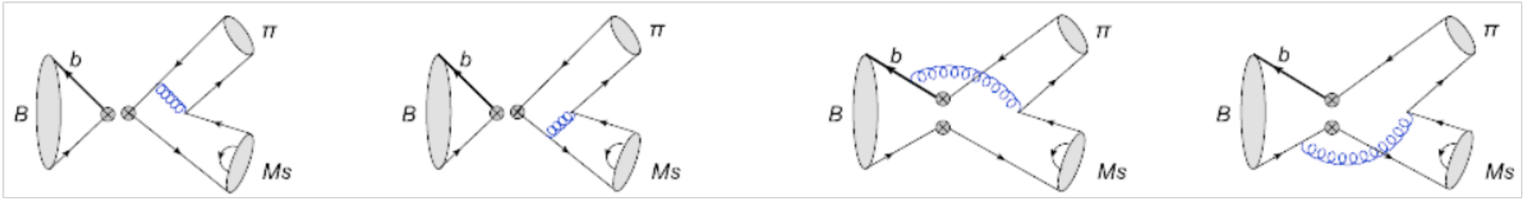
nonfactorizable





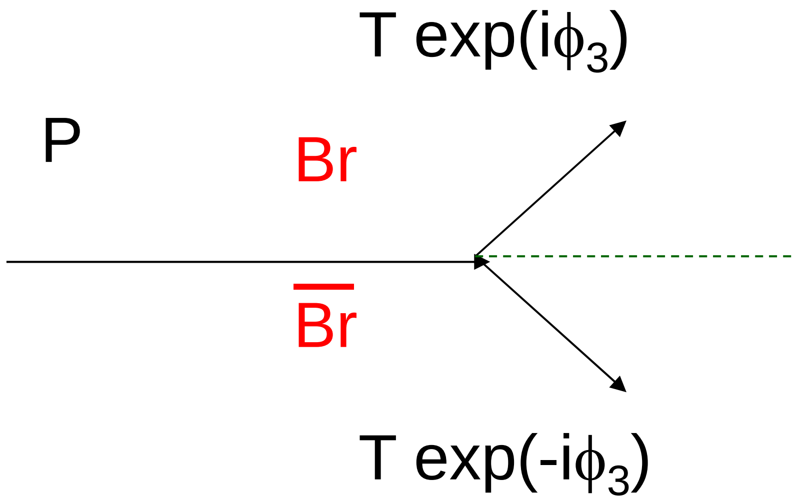


annihilation



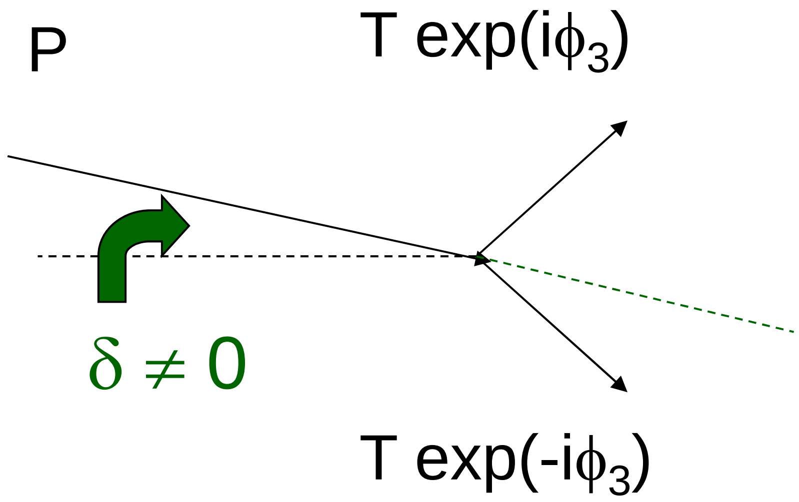
Direct CP asymmetry

- Require tree (T) and penguin (P) contributions, weak and strong phases
- Penguin annihilation provides (short-distance) strong phase in 2-body decays



If strong phase δ=0

$\overline{Br}$ = Br, no direct $\not{CP}$

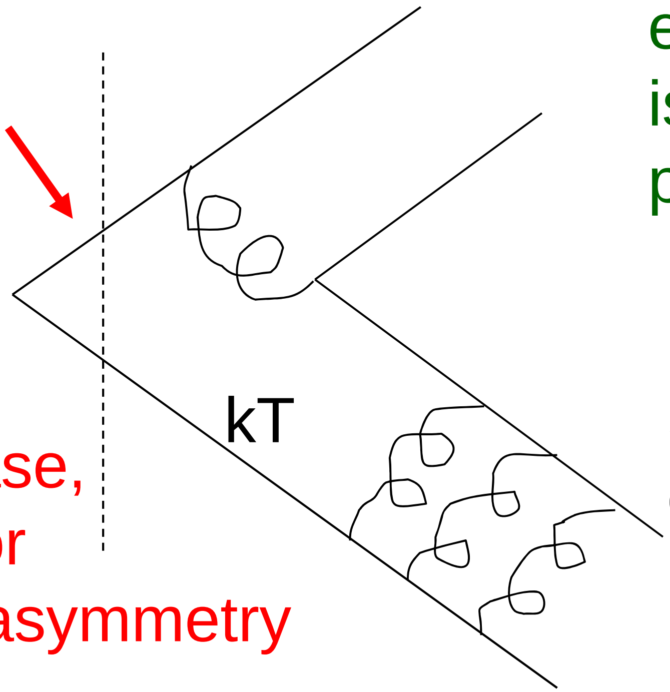


Br $\neq$ $\overline{Br}$ direct $\not{CP}$

Short-distance phase in PQCD

internal line goes on-shell

strong phase, required for direct CP asymmetry



end-point singularity is regularized by parton kT

Sudakov gluons guarantee finite kT

$$\frac{1}{xm_B^2 - k_T^2 + i\epsilon} = \frac{P}{xm_B^2 - k_T^2} - i\pi\delta(xm_B^2 - k_T^2).$$

⇨ kT also leads to complex annihilation in PQCD

$$A_{CP}(B^{\pm} \to K^{\pm}\pi^{+}\pi^{-})$$

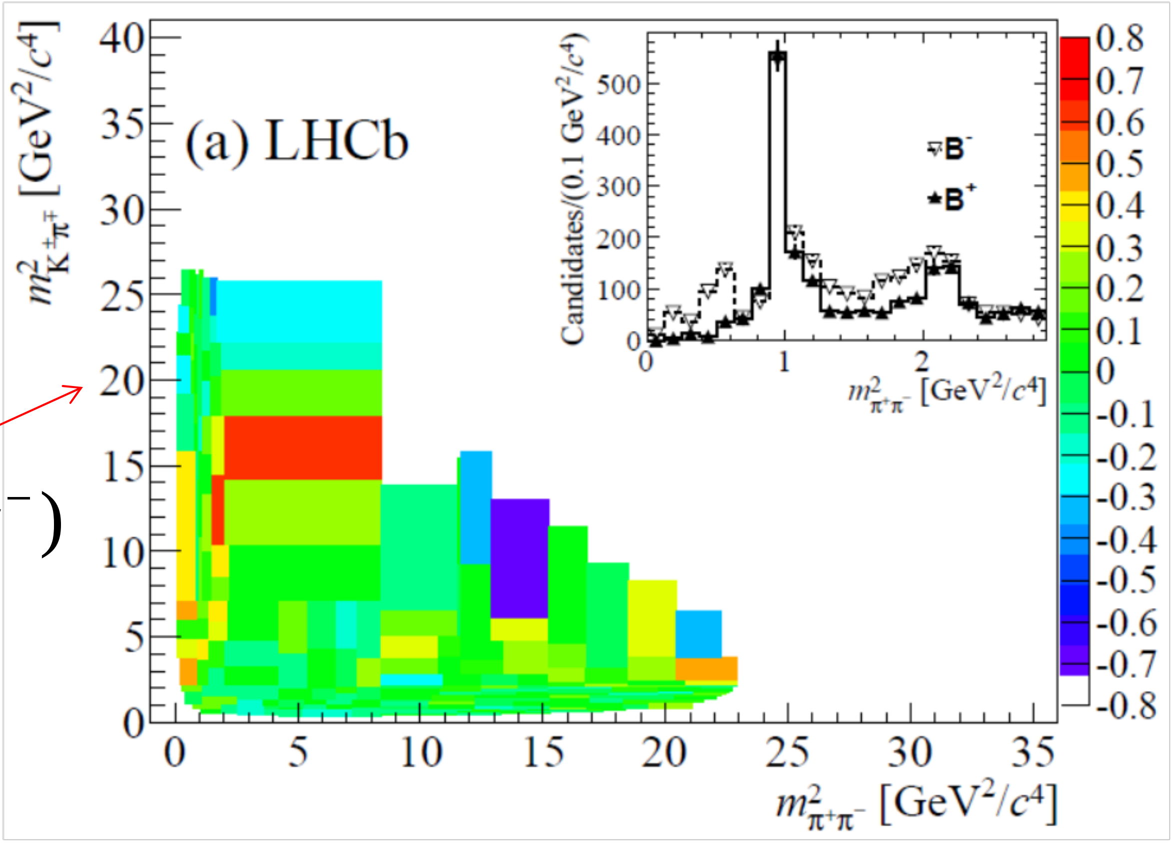
- PQCD (Mishima, Li): $A_{CP}(B^{\pm} \to K^{\pm}\rho^{0}) = 0.71^{+0.25}_{-0.35}$

$$A_{CP}^{\text{region}}(K^{-}\pi^{+}\pi^{-}) = 0.678 \pm 0.078 \pm 0.032 \pm 0.007$$

for $m^2_{K^-\pi^+\text{high}} < 15\ \text{GeV}^2$ and $0.08 < m^2_{\pi^+\pi^-\text{low}} < 0.66\ \text{GeV}^2$

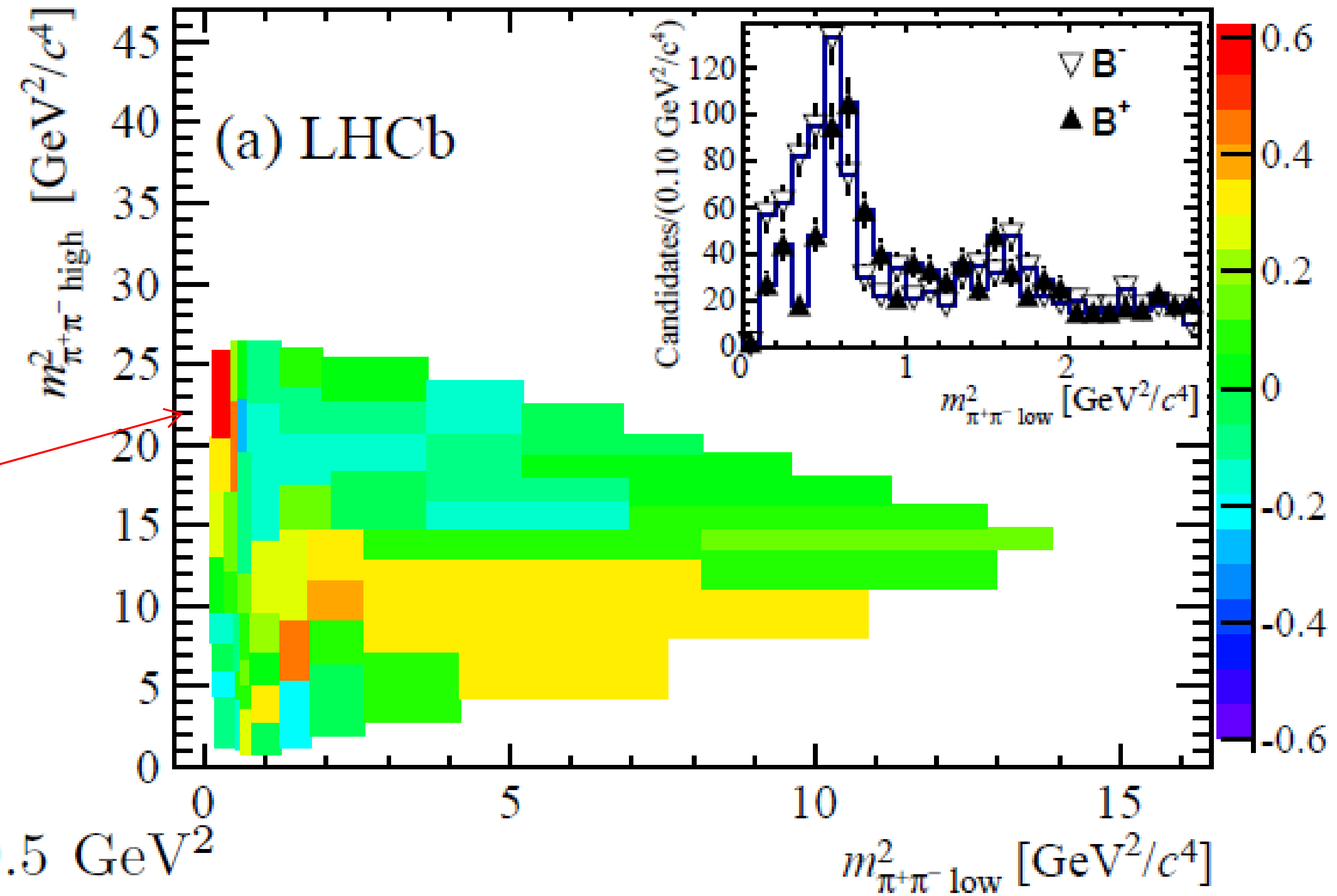
- In the same low pipi invariant mass

$A_{CP}^{reg}(B^{\pm} \to K^{\pm}\pi^{+}\pi^{-}) = -0.02$



$$A_{CP}(B^{\pm} \to \pi^{\pm}\pi^{+}\pi^{-})$$

- In higher pipi invariant mass



$$A_{CP}^{reg}(\pi^{\pm}\pi^{+}\pi^{-}) = 0.63$$

$m^2_{\pi^+\pi^-\mathrm{high}} > 20.5\ \mathrm{GeV}^2$

$m^2_{\pi^+\pi^-\mathrm{low}} < 0.4\ \mathrm{GeV}^2$

S-wave 2-pion DAs

$$\Phi_{\pi\pi}^{S-wave} = \frac{1}{\sqrt{2N_c}} \left[\not{p} \Phi_{v\nu=-}^{I=0}(z,\zeta,w^2) + \omega \Phi_s^{I=0}(z,\zeta,w^2) \right.$$
$$\left. + \omega(\not{n}_+ \not{n}_- - 1) \Phi_{t\nu=+}^{I=0}(z,\zeta,w^2) \right]$$

$$\phi_0 = \frac{9F_s(w^2)}{\sqrt{2N_c}} a_2^{I=0} z(1-z)(1-2z)$$

$$\phi_s = \frac{F_s(w^2)}{2\sqrt{2N_c}}, \quad \phi_\sigma = \frac{F_s(w^2)}{2\sqrt{2N_c}}(1-2z)$$