Higgs Pair @ Hadron Colliders

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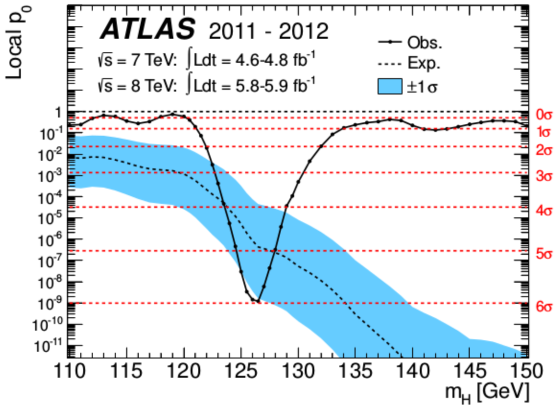
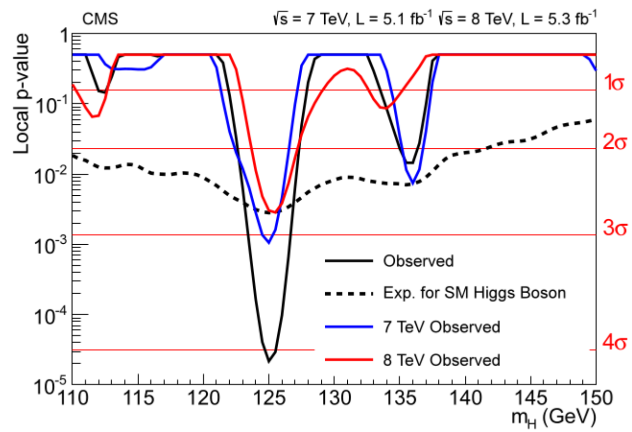
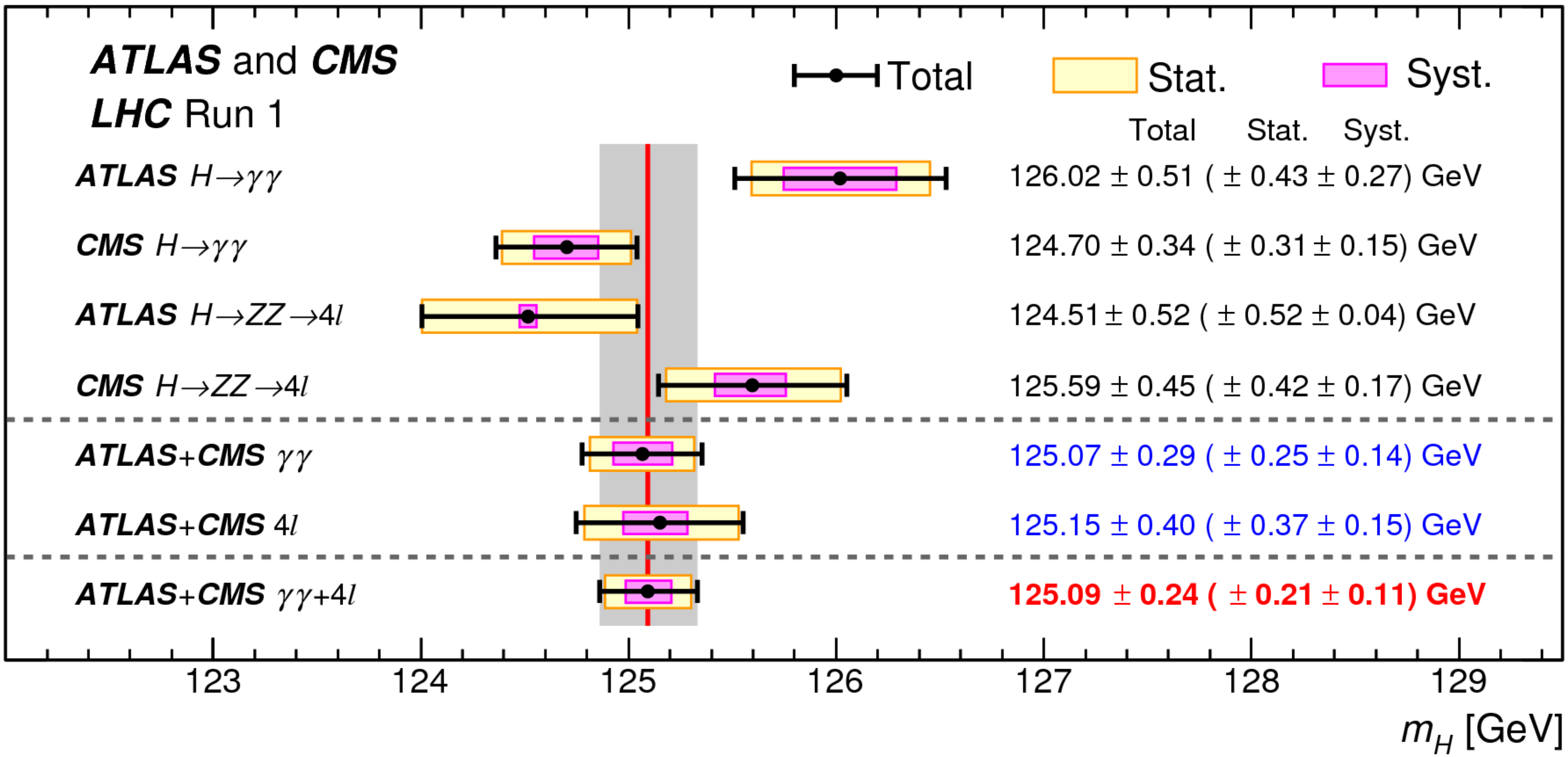
2012 July 4th Higgs announced





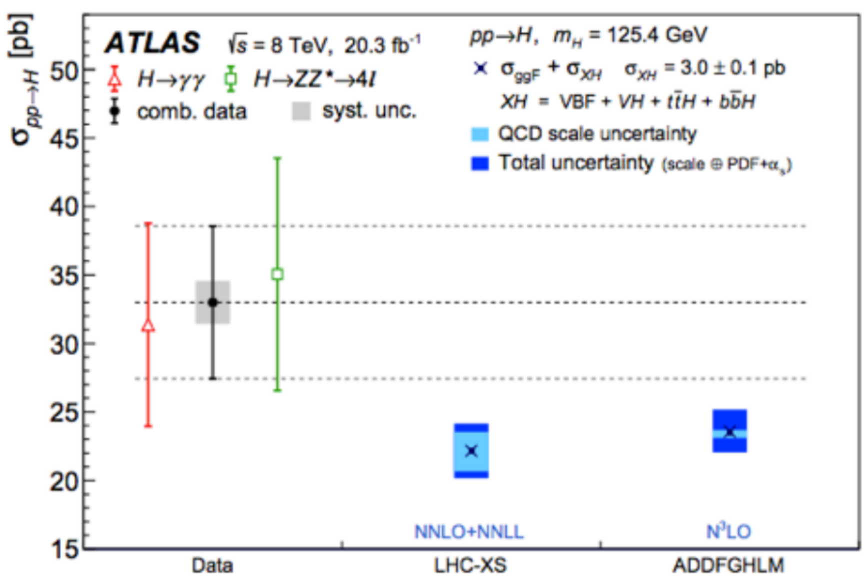
Higgs Of all the collisions over three years of LHC operation, only one in 5 billion produced the elusive particle.	**1.2 million**
Collisions The LHC produced roughly 90 billion particle collisions a day over the period of operation.	**5.8 quadrillion**
Physicists Collaborators from more than 40 countries worked on two of LHC's detectors, ATLAS and CMS.	**6,600**
Dollars The cost of running the LHC for three years came to about $62 per Higgs.	**74 million**
Power (MW) The electric power required to run the LHC and its experiments was enough to power about 50,000 homes.	**50**
Laps per second Traveling near the speed of light, protons in the LHC travel nearly 300,000 kilometers per second.	**11,245**

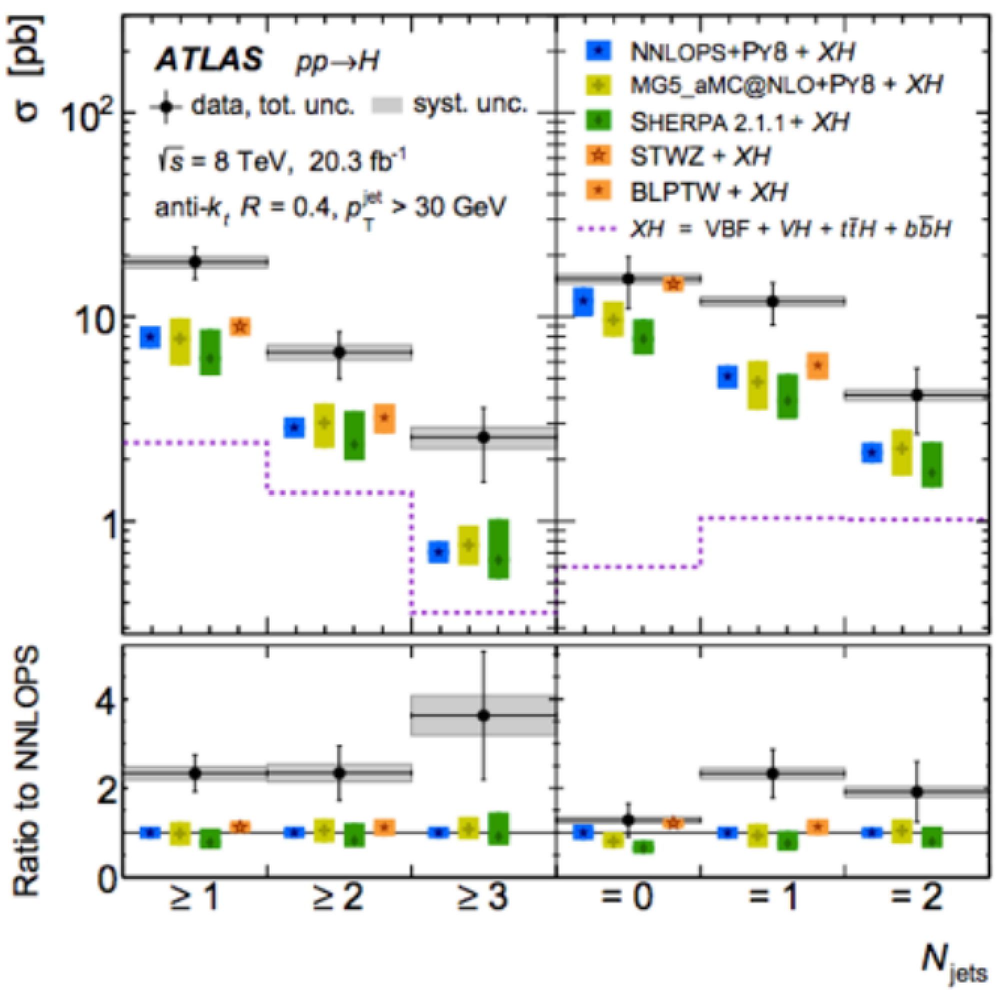
Properties of Higgs



100% satisfied?

Higgs Xsection reconstruction



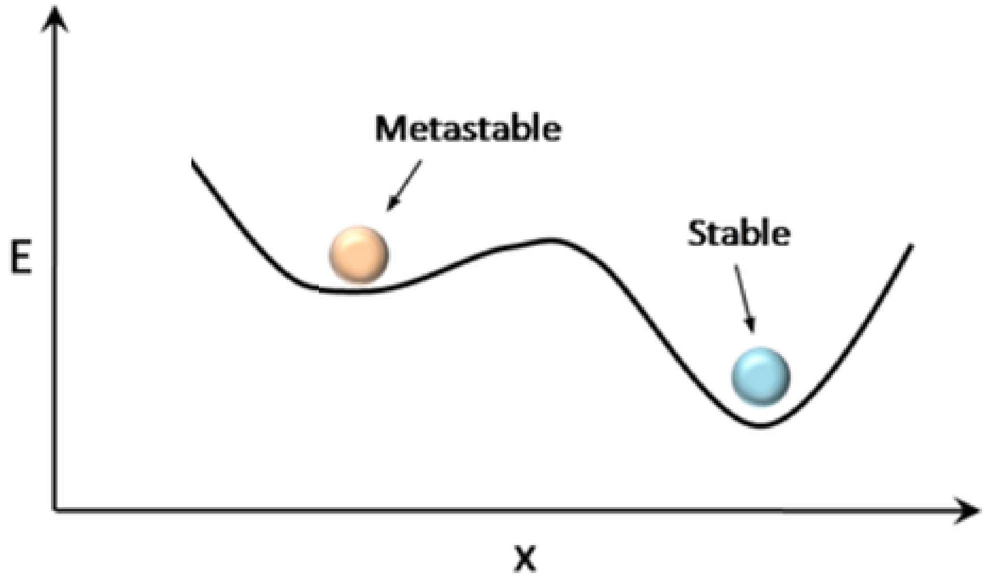


Les Houches wishlist

Process	known	desired	motivation
H	d\sigma @ NNLO QCD d\sigma @ NLO EW finite quark mass effects @ NLO	d\sigma @ NNNLO QCD + NLO EW MC@NNLO finite quark mass effects @ NNLO	H branching ratios and couplings
H+j	d\sigma @ NNLO QCD (g only) d\sigma @ NLO EW	d\sigma @ NNLO QCD + NLO EW finite quark mass effects @ NLO	H p_T
H+2j	\sigma_tot(VBF) @ NNLO(DIS) QCD d\sigma(gg) @ NLO QCD d\sigma(VBF) @ NLO EW	d\sigma @ NNLO QCD + NLO EW	H couplings
H+V	d\sigma(V decays) @ NNLO QCD d\sigma @ NLO EW	with H→bb @ same accuracy	H couplings
t\bar tH	d\sigma(stable tops) @ NLO QCD	d\sigma(NWA top decays) @ NLO QCD + NLO EW	top Yukawa coupling
HH	d\sigma @ LO QCD finite quark mass effects d\sigma @ NLO QCD large m_t limit	d\sigma @ NLO QCD finite quark mass effects d\sigma @ NNLO QCD	Higgs self coupling

Higgs Pair Production

HH	d\sigma @ LO QCD finite quark mass effects d\sigma @ NLO QCD large m_t limit	d\sigma @ NLO QCD finite quark mass effects d\sigma @ NNLO QCD	Higgs self coupling



Higgs Self Coupling & New Physics?

SM

$$\lambda_{HHH} = 3M_H^2/M_Z^2$$

Higgs Self Coupling & New Physics?

SM

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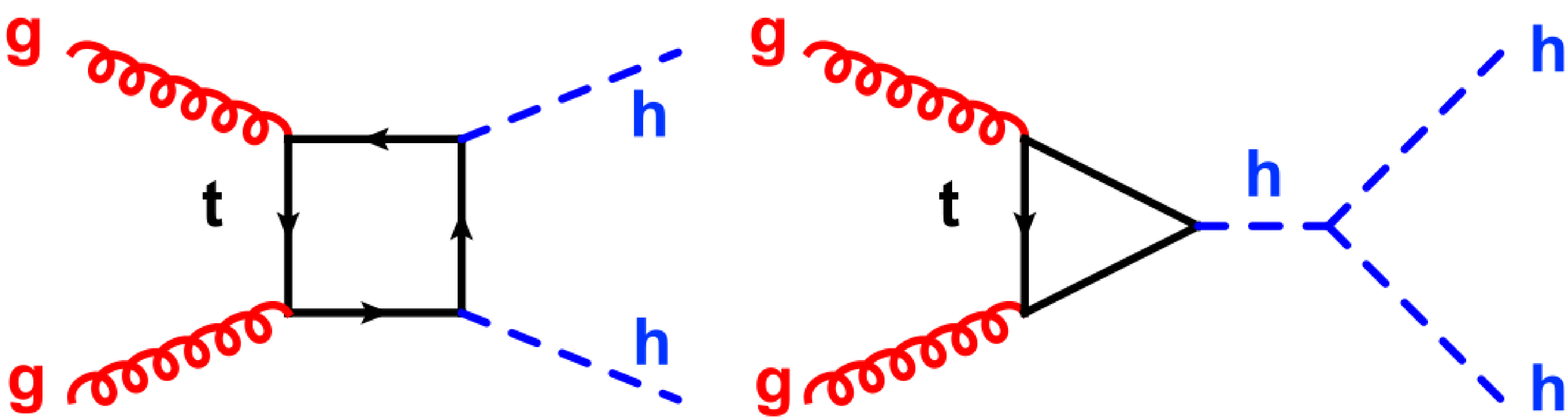
MSSM (2HDM)

$$\lambda_{hhh} \quad = \quad 3\cos 2\alpha \sin(\beta + \alpha) + 3\frac{\epsilon}{M_Z^2}\frac{\cos\alpha}{\sin\beta}\cos^2\alpha$$

$$\epsilon \;=\; 3G_F M_t^4/(\sqrt{2}\pi^2 \sin^2\beta)$$

gg→HH @ LO

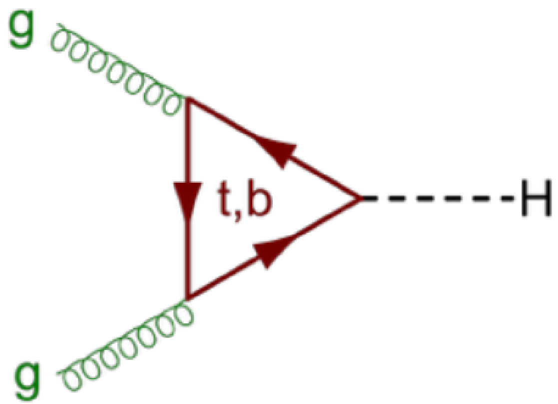
- NPB309 (1988) 282, E.W.N.Glover, J.J. van der Bij (aimed for SSC & LHC, Mt>40GeV)
- NPB479 (1996) 46, T.Plehn, M.Spira, P.M.Zerwas (corrected Mt & MSSM)

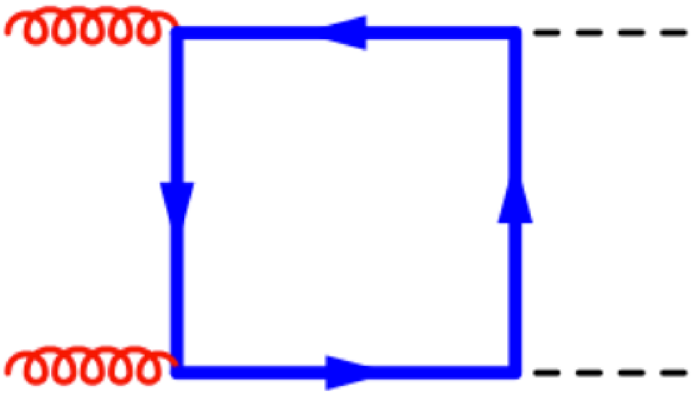


gg→HH @ NLO and beyond

- **PRD58(1998)115012, S.Dawson, S.Dittmaier, M.Spira.**
  (heavy Mt limit, QCD correction K~2)
- **JHEP1411(2014)079, F.Maltoni, E.Vryonidou, M.Zaro;**
  NPB875(2013)1, J.Grigo, J.Hoff, K.Melnikov, M.Steinhauser.
  (uncertainty from finite Mt may be +-10%)
- **PRL111(2013)201801, D.de Florian, J.Mazzitelli.**
  (NNLO QCD correction, another ~20%)
- **NPB900(2015)412, J.Grigo, J.Hoff, M.Steinhauser.**
  (1/Mt expansion estimates 10% for NLO & 5% for NNLO)
- **JHEP1509(2015)053, D.de Florian, J.Mazzitelli.**
  (NNLL correction ~7%)

Mt → Infinity



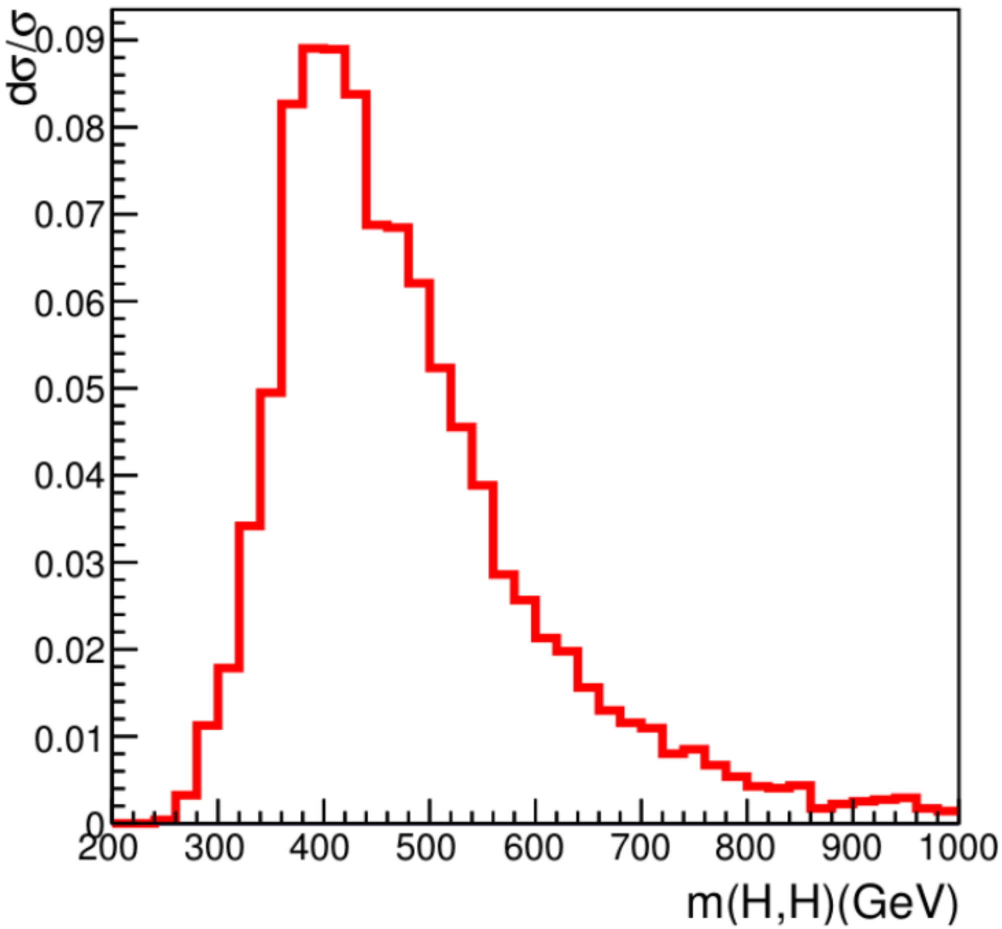


Large Mt Approx.

$$m_t^2 \gg s, t, m_H^2$$

Failed approximation

m(H,H)



Large Mt Approx.

$$m_t^2 \gg s, t, m_H^2$$

Large Mt approx. must be removed for Higgs investigations

Avaiable Algorithms

- **Analytical approach:**

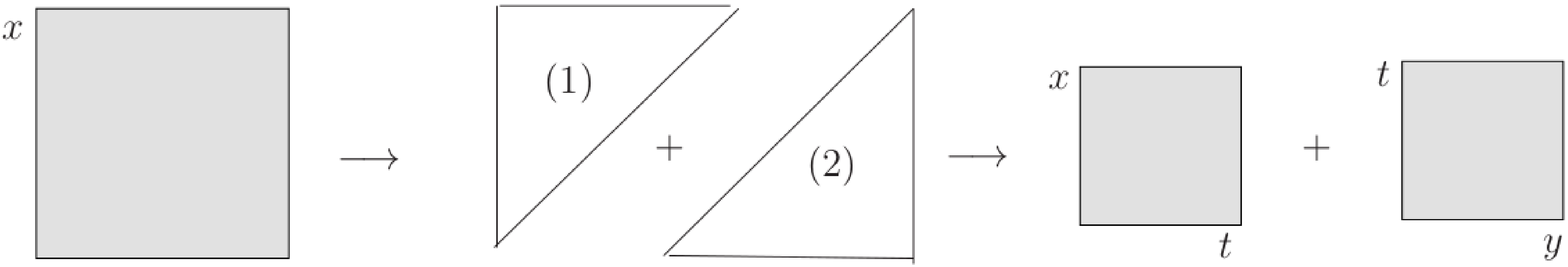
  Simple processes only, i.e. not too many scales. No breakthrough after more than ten years struggle.

- **Numerical approach:**

  Conventionally too slow. Popular algorithms are “Sector Decomposition” and “Mellin-Barnes”

Sector Decomposition

$$I = \int_0^1 dx \int_0^1 dy \; x^{-1-\epsilon} y^{-\epsilon} (x + (1-x)y)^{-1}$$



$$I = \int_0^1 dx \; x^{-1-\epsilon} \int_0^1 dt \;\; t^{-\epsilon} (1 + (1-x)t)^{-1}$$

$$+ \int_0^1 dy \;\; y^{-1-2\epsilon} \int_0^1 dt \;\; t^{-1-\epsilon} (1 + (1-y)t)^{-1}$$

Sector Decomposition

$$G^{\mu_1\cdots\mu_R}_{l_1\cdots l_R} = \int \prod_{l=1}^{L} d^D\kappa_l \, \frac{k_{l_1}^{\mu_1}\cdots k_{l_R}^{\mu_R}}{\prod_{j=1}^{N} P_j^{\nu_j}\left(\{k\},\{p\},m_j^2\right)} \,,$$

$$d^D\kappa_l = \frac{\mu^{4-D}}{i\pi^{\frac{D}{2}}}\, d^D k_l \,, \qquad P_j\left(\{k\},\{p\},m_j^2\right) = \left(q_j^2 - m_j^2 + i\delta\right) ,$$

Feynman parameterization

$$\frac{1}{\prod_{j=1}^{N} P_j^{\nu_j}} = \frac{\Gamma(N_\nu)}{\prod_{j=1}^{N}\Gamma(\nu_j)} \int_0^\infty \prod_{j=1}^{N} dx_j\, x_j^{\nu_j-1}\delta\left(1-\sum_{i=1}^{N} x_i\right) \frac{1}{\left[\sum_{j=1}^{N} x_j P_j\right]^{N_\nu}} \,,$$

where $N_\nu = \sum_{j=1}^{N} \nu_j$, leads to

$$G^{\mu_1\cdots\mu_R}_{l_1\cdots l_R} = \frac{\Gamma(N_\nu)}{\prod_{j=1}^{N}\Gamma(\nu_j)} \int_0^\infty \prod_{j=1}^{N} dx_j\, x_j^{\nu_j-1}\delta\left(1-\sum_{i=1}^{N} x_i\right) \int d^D\kappa_1\cdots d^D\kappa_L$$

$$\times\, k_{l_1}^{\mu_1}\cdots k_{l_R}^{\mu_R}\left[\sum_{i,j=1}^{L} k_i^{\mathrm{T}} M_{ij} k_j - 2\sum_{j=1}^{L} k_j^{\mathrm{T}}\cdot Q_j + J + i\delta\right]^{-N\nu} ,$$

Sector Decomposition

Integrate out loop momenta

$$G^{\mu_1\cdots\mu_R}_{l_1\cdots l_R} = (-1)^{N_\nu}\frac{1}{\prod_{j=1}^{N}\Gamma(\nu_j)}\int_0^\infty \prod_{j=1}^{N} dx_j\, x_j^{\nu_j-1}\delta\left(1-\sum_{l=1}^{N}x_l\right)$$

$$\times\sum_{m=0}^{[R/2]}\left(-\frac{1}{2}\right)^m \Gamma(N_\nu - m - LD/2)[(\tilde{M}^{-1}\otimes g)^{(m)}\tilde{l}^{(R-2m)}]^{\Gamma_1,\ldots,\Gamma_R}$$

$$\times\frac{\mathcal{U}^{N_\nu-(L+1)D/2-R}}{\mathcal{F}^{N_\nu-LD/2-m}}, \tag{7}$$

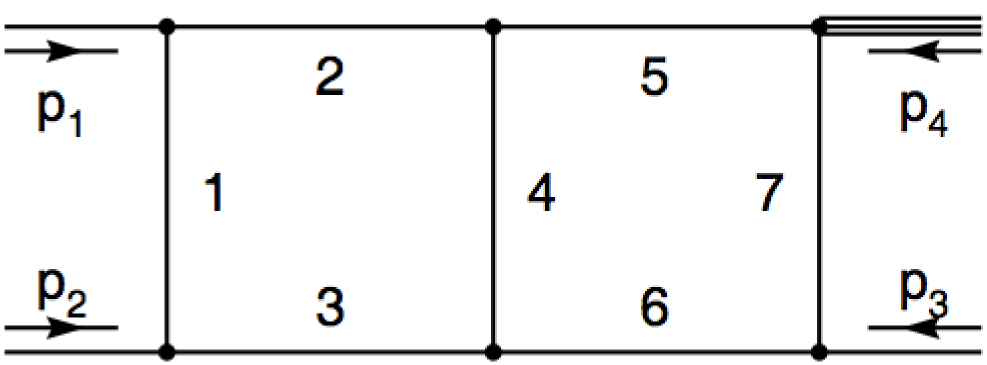
where

$$\mathcal{F}(\mathbf{x}) = \det(M)\left[\sum_{j,l=1}^{L} Q_j M^{-1}_{jl} Q_l - J - i\delta\right], \tag{8}$$

$$\mathcal{U}(\mathbf{x}) = \det(M), \quad \tilde{M}^{-1} = \mathcal{U}M^{-1}, \quad \tilde{l} = \mathcal{U}v$$

Sector Decomposition

U and F can
be determined
geometrically



$$\mathcal{U}(\mathbf{x}) = \sum_{T \in \mathcal{T}_1} \Bigg[\prod_{j \in \mathcal{C}(T)} x_j \Bigg] \, ,$$

$$\mathcal{F}_0(\mathbf{x}) = \sum_{\hat{T} \in \mathcal{T}_2} \Bigg[\prod_{j \in \mathcal{C}(\hat{T})} x_j \Bigg] (-s_{\hat{T}}) \, ,$$

$$\mathcal{F}(\mathbf{x}) = \mathcal{F}_0(\mathbf{x}) + \mathcal{U}(\mathbf{x}) \sum_{j=1}^{N} x_j m_j^2 \, .$$

$$\mathcal{U} = x_{123} x_{567} + x_4 x_{123567} \, ,$$

$$\mathcal{F} = (-s_{12})(x_2 x_3 x_{4567} + x_5 x_6 x_{1234} + x_2 x_4 x_6 + x_3 x_4 x_5) + (-s_{23}) x_1 x_4 x_7 + (-p_4^2) x_7 (x_2 x_4 + x_5 x_{1234}) \, ,$$

where $x_{ijk\ldots} = x_i + x_j + x_k + \cdots$ and $s_{ij} = (p_i + p_j)^2$.

Sector Decomposition

First generate primary sectors to eliminate Delta function

$$\int_0^\infty d^N x = \sum_{l=1}^{N} \int_0^\infty d^N x \prod_{\substack{j=1 \\ j\neq l}}^{N} \theta(x_l \geq x_j)\,.$$

$$x_j = \begin{cases} x_l t_j & \text{for } j < l\,, \\ x_l & \text{for } j = l\,, \\ x_l t_{j-1} & \text{for } j > l \end{cases}$$

$$G_l = \int_0^1 \prod_{j=1}^{N-1} dt_j \, \frac{\mathcal{U}_l^{N_\nu-(L+1)D/2}(\mathbf{t})}{\mathcal{F}_l^{N_\nu-LD/2}(\mathbf{t})}\,, \quad l = 1, \ldots, N\,.$$

Sector Decomposition

Determine a sub-set of parameters ti

$$\mathcal{S} = \{t_{\alpha_1}, \ldots, t_{\alpha_r}\}$$

Then divide into r sub-sectors

$$\prod_{j=1}^{r} \theta(1 \geq t_{\alpha_j} \geq 0) = \sum_{k=1}^{r} \prod_{\substack{j=1 \\ j \neq k}}^{r} \theta(t_{\alpha_k} \geq t_{\alpha_j} \geq 0) \, .$$

$$t_{\alpha_j} \to \begin{cases} t_{\alpha_k} t_{\alpha_j} & \text{for } j \neq k \, , \\ t_{\alpha_k} & \text{for } j = k \, . \end{cases}$$

Sector Decomposition

$$G_{lk} = \int_0^1 \left(\prod_{j=1}^{N-1} dt_j \, t_j^{a_j - b_j \epsilon} \right) \frac{\mathcal{U}_{lk}^{N_\nu - (L+1)D/2}}{\mathcal{F}_{lk}^{N_\nu - LD/2}}, \quad k = 1, \ldots, r\,.$$

$$\mathcal{U}_{lk_1k_2\cdots} = 1 + u(\mathbf{t})\,, \quad \mathcal{F}_{lk_1k_2\cdots} = -s_0 + \sum_\beta (-s_\beta) f_\beta(\mathbf{t})\,,$$

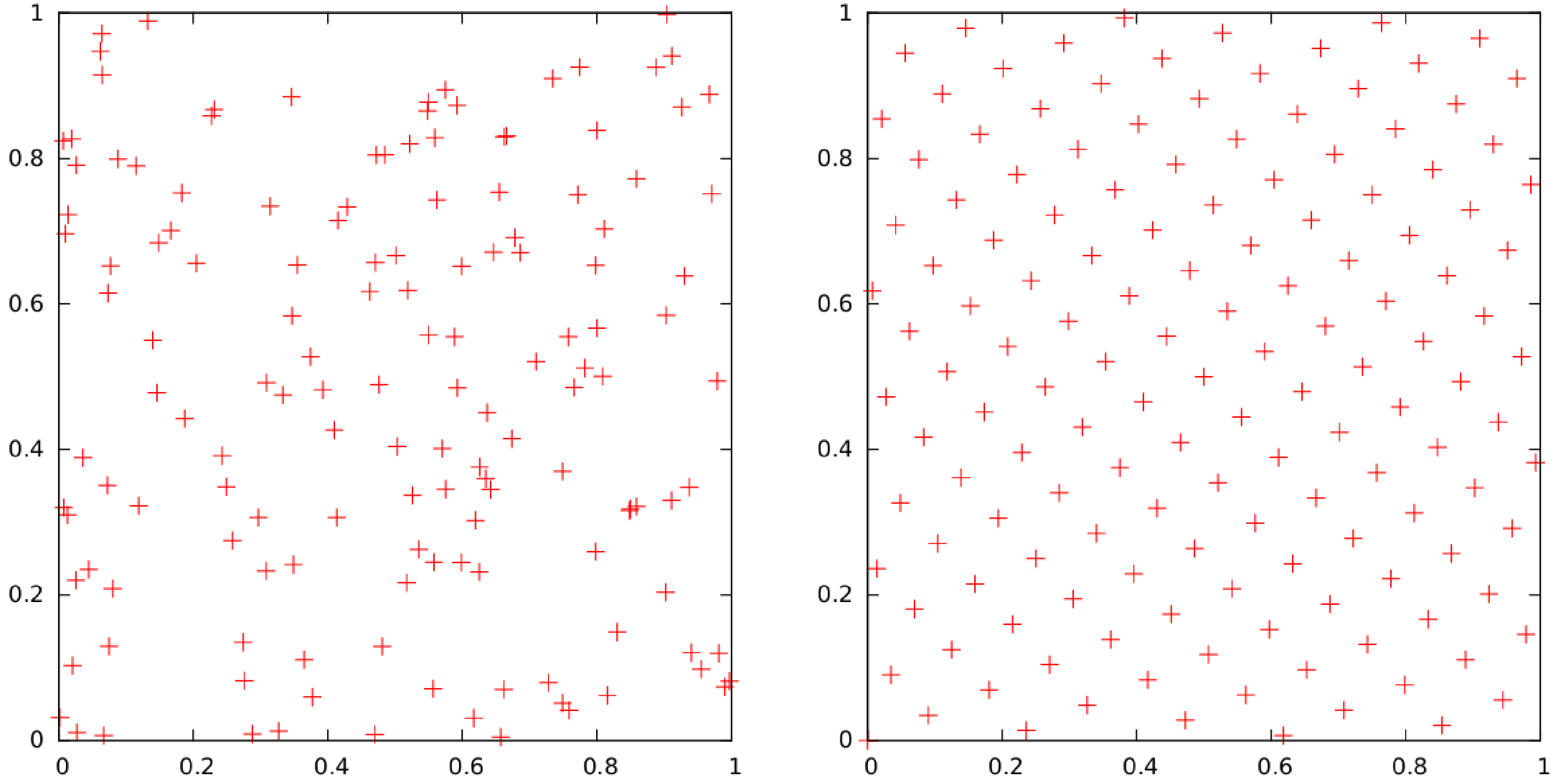
All the coefficients of divergences are finite (complicated).

Sector Decomposition

Diagram	A	B	C	S	X	H	This method	Exponential S.D.
Bubble	2	2	2	2*	2		2	2
Triangle	3	3	3	3*	3		3	3
Box	12	12	12	12	12		12	8
Tbubble	58	48	48	48*	48		48	36
Double box, $p_i^2 = 0$	775	586	586	362	293	282	266	106
Double box, $p_4^2 \neq 0$	543*	245*	245*	230*	192*	197	186	100
Double box, $p_i^2 = 0$ nonplanar	1138	698	698	441*	395		360	120
D420	8898	564	564	180	F		168	100
3 loop vertex (A8)	4617*	1196*	1196*	871*	750*	684	684	240
Triple box	M	114256	114256	22657	10155		6568	856

Quasi-Monte-Carlo

$$I(f) = \int_0^1 d^s x f(\vec{x}) \qquad I_{estimate}(f) = \sum_{i=0}^{n-1} f(\vec{x}_i)$$



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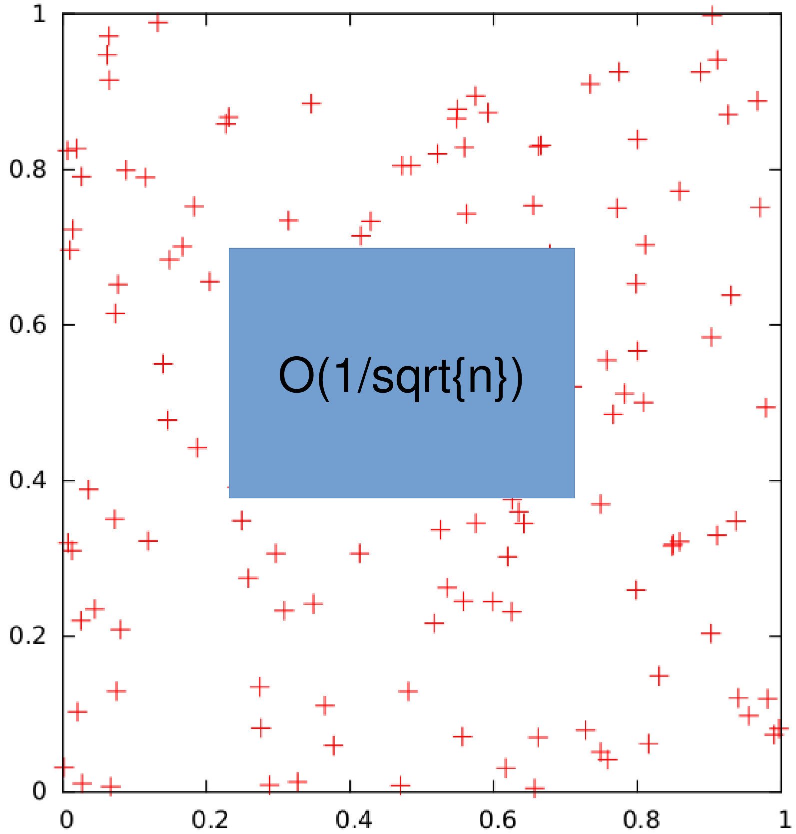
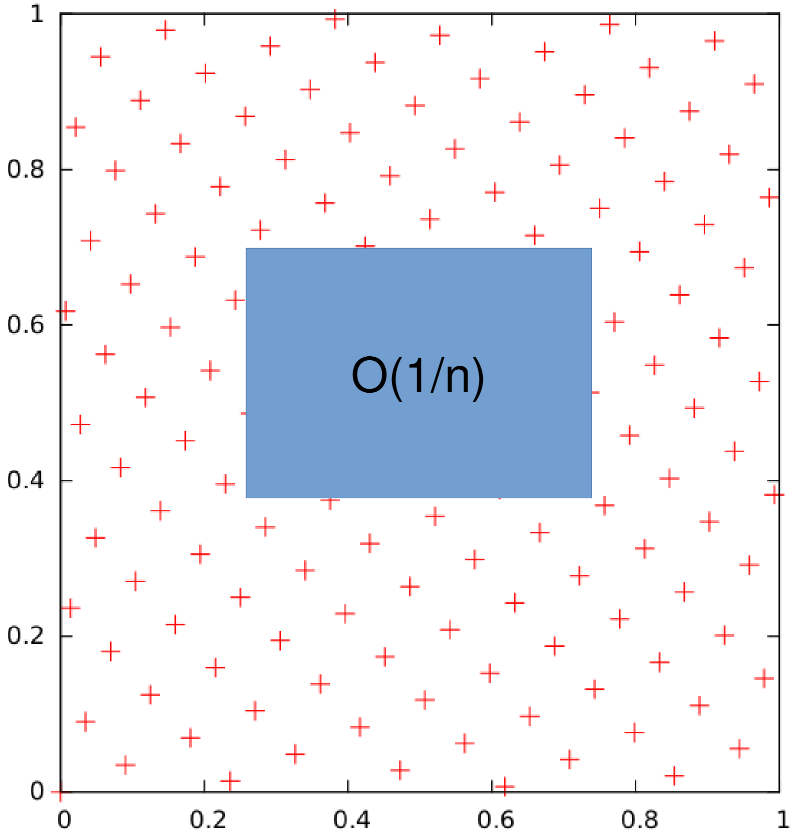


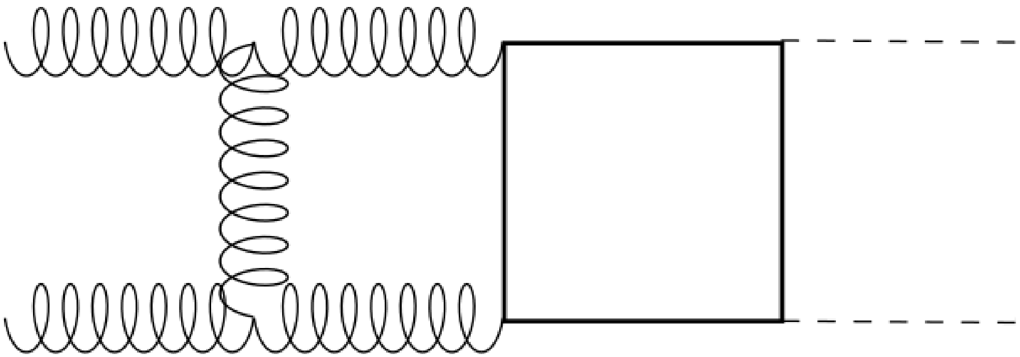


Improve Sector Decomposition

- **Quasi-Monte-Carlo algorithm**

- **Parallel (GPU) technique**

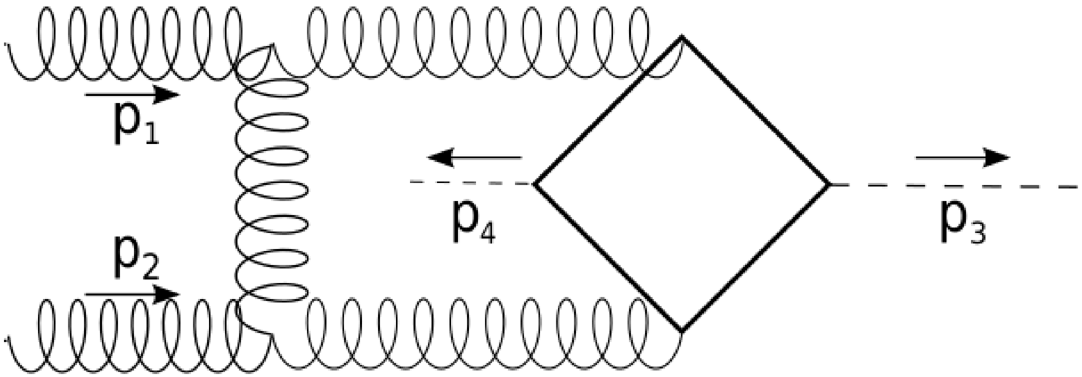
Planar Two-Loop Master Integral



$$I_C = e^{-2\epsilon\gamma_E} s^{-3-2\epsilon} \sum_{i=0}^{i=2} \frac{P_i}{\epsilon^i}.$$

	Vegas/CPU	QMC/GPU
P_2	$-7.959 \pm 0.009 - 10.586i \pm 0.009i$	$-7.949 \pm 0.003 - 10.585i \pm 0.005i$
P_1	$3.9 \pm 0.1 - 28.1i \pm 0.1i$	$3.831 \pm 0.005 - 28.022i \pm 0.005i$
P_0	$-3.9 \pm 0.8 + 92.3i \pm 0.8i$	$-4.63 \pm 0.07 + 92.13i \pm 0.07i$
Integration Time	45540s	19s

Non-Planar Two-Loop Master Integral



$$I_D = e^{-2\epsilon\gamma_E} s^{-3-2\epsilon} \sum_{i=0}^{i=2} \frac{P_i}{\epsilon^i}.$$

	Vegas/CPU	QMC/GPU
P_2	$-3.848 \pm 0.004 + 0.0005i \pm 0.003i$	$-3.8482 \pm 0.0007 + 0.0004i \pm 0.0003i$
P_1	$3.81 \pm 0.03 - 6.41i \pm 0.03i$	$3.83 \pm 0.02 - 6.40i \pm 0.02i$
P_0	$77.2 \pm 0.2 + 20.1i \pm 0.2i$	$77.2 \pm 0.1 + 19.9i \pm 0.1i$
Integration Time	54290s	20s

gg→HH @NLO with finite top mass

- 2-loop virtual contribution totally has 122 Feynman diagrams, 126 color ordered amplitudes, 8 integral families/topologies.
- IBP reduction is difficult due to 4 scales.
- Implementation of QMC on phase space integration can also improve the efficiency.
- SecDec group are also working on the same mission, but different technique approaches.

Conclusion & Prospect

- **Large Mt approximation is not suitable for Higgs associated production investigation.**
- **For the first time, we present the results of two-loop master integrals for gg→HH with finite top quark mass.**
- **Our improved numerical algorithm can obtain results with reasonable accuracy in acceptible time for complete gg->HH@NLO (two-loop) with finite top mass.**

Conclusion & Prospect

- **H+j @ NLO and beyond with finite top mass**
- **HH @ NNLO with finite top mass**
- **e+e-=>HZ @ CEPC EW+QCD correction**
- **X750 production via GF && its decay**
- **Single top @ NNLO**
- **At HL-LHC & SPPC, ttH @ NNLO**
- **…**

Thank You