Functional evolution equations in QCD

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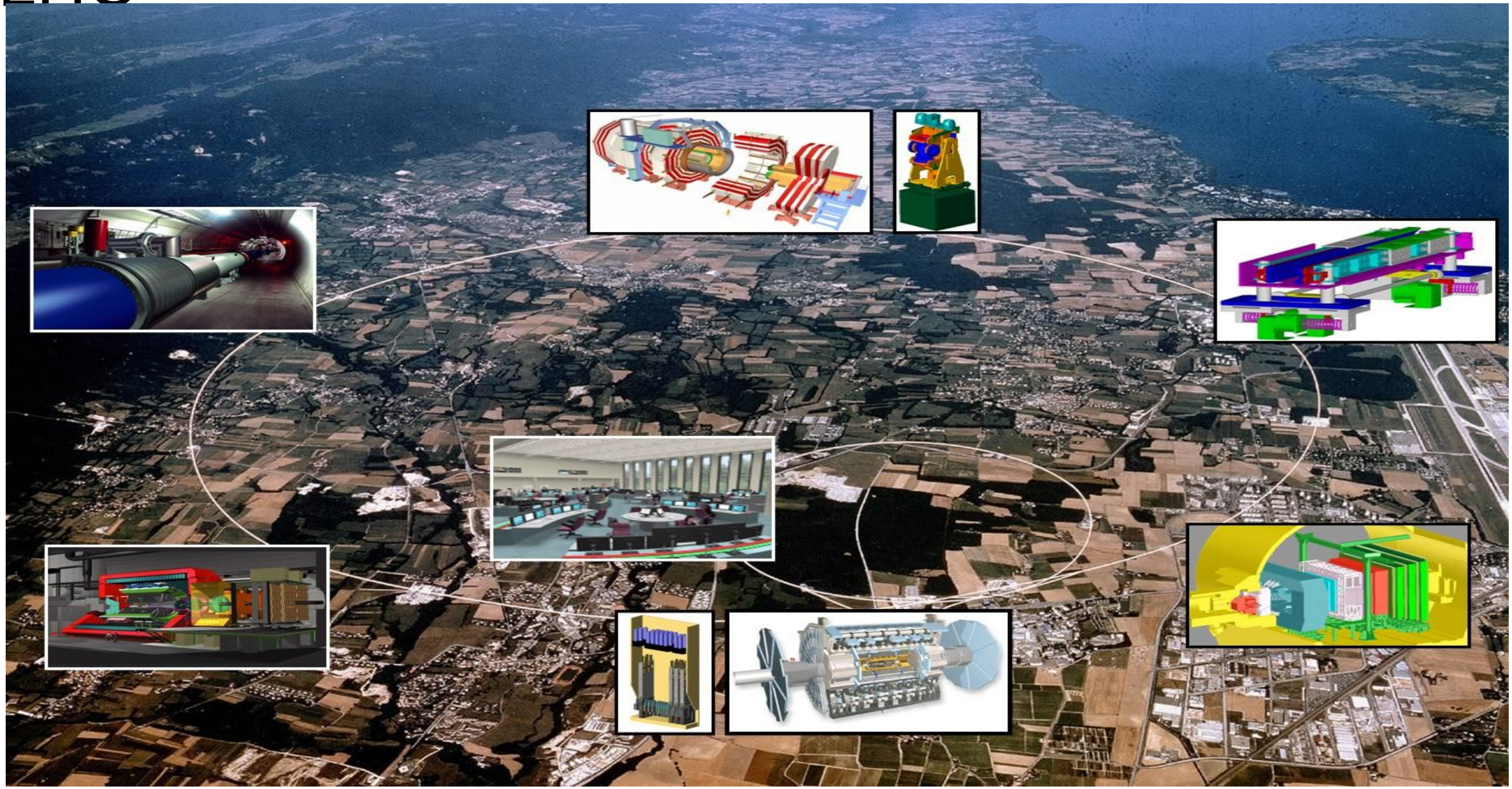
Outline

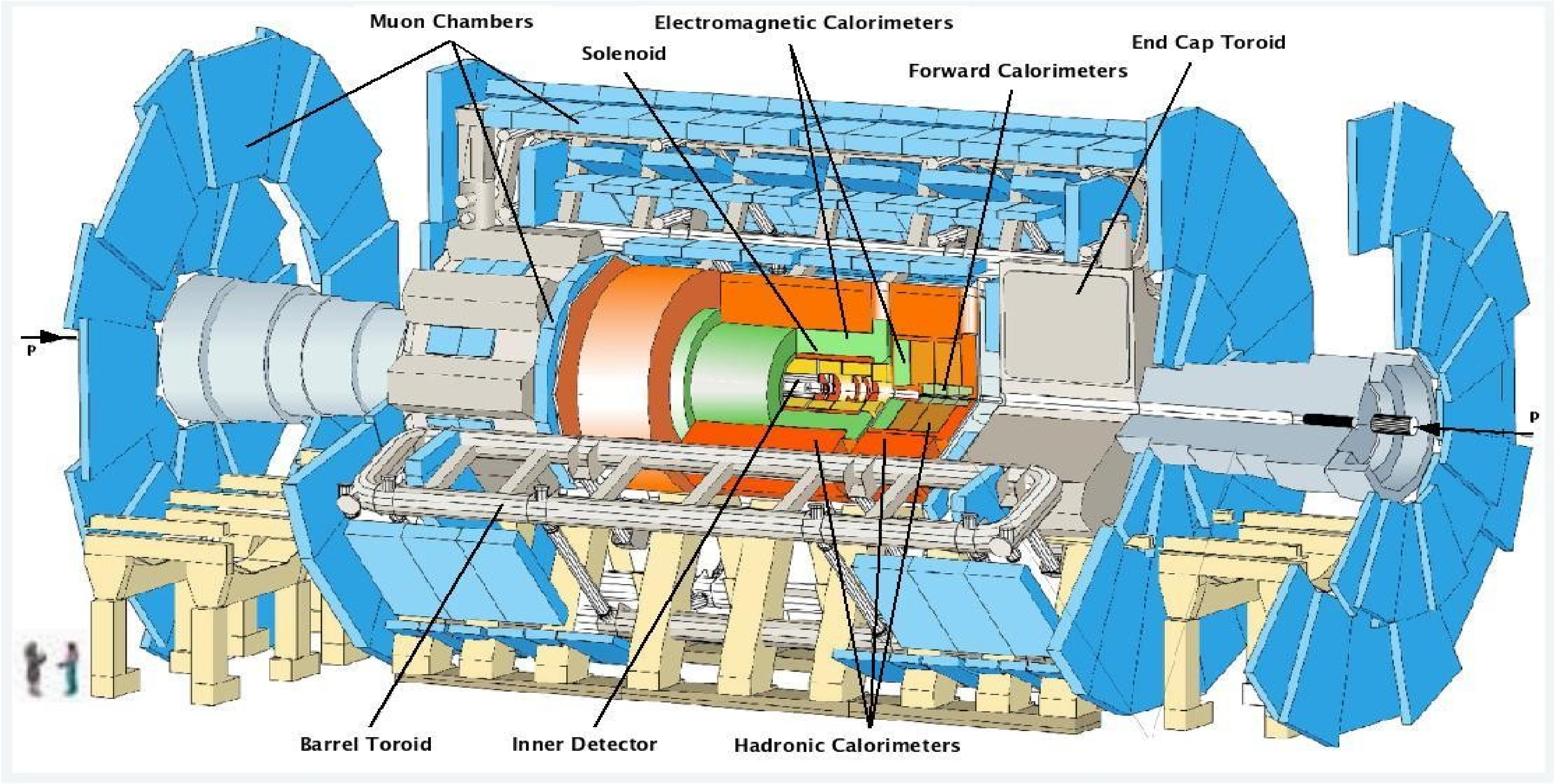
- Geometry of particle collisions
- Quantum Field Theory and phenomenology
- Analytic structure of perturbation theory
- Attempt of axiomatization

Collider experiments in particle physics

- LHC



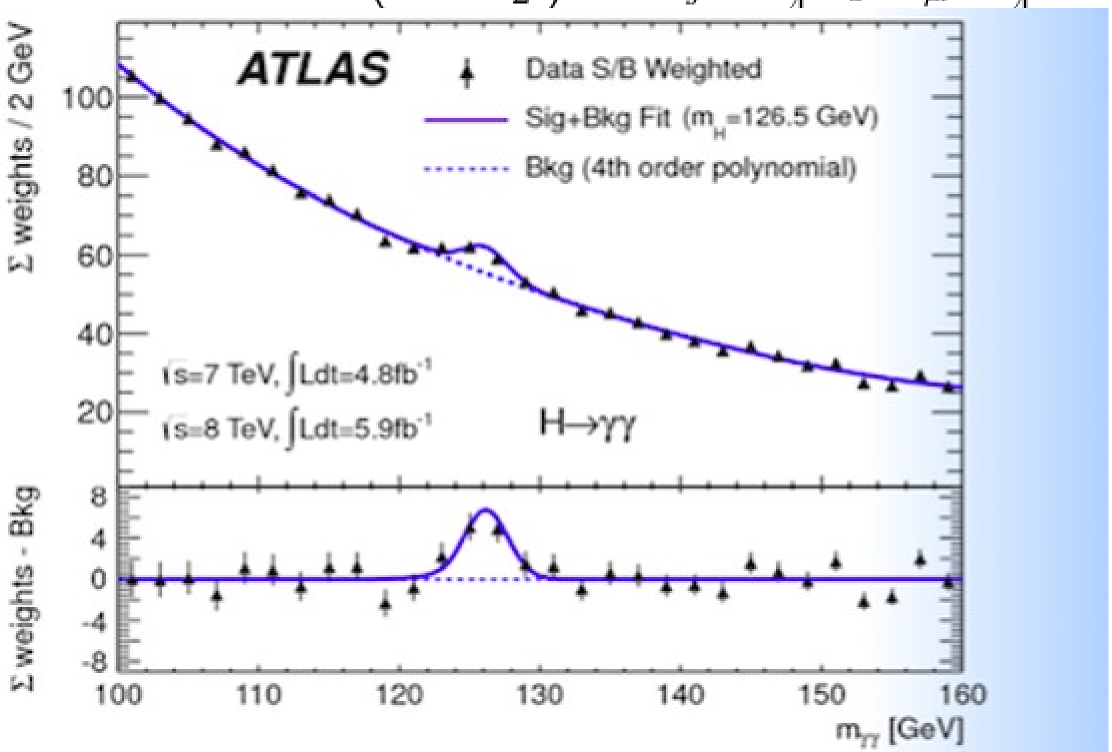
Particle detectors: ATLAS

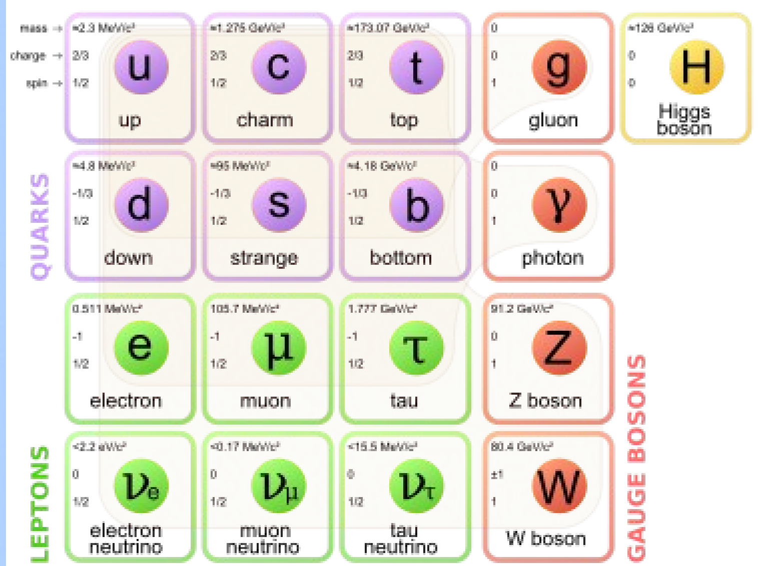


Particles as field quanta

- Elementary particles as quanta of fields
- Classical fields: $L_{QED}(x) = \bar{\psi}(x)(i\hat{\partial} - m)\psi(x) + F^2_{\mu\nu}(x) + e\bar{\psi}(x)\hat{A}(x)\psi(x)$
- The Standard Model $L_{SM} = L_{EW} + L_{Higgs} + L_{QCD} + L_{leptons} + L_{quarks}$

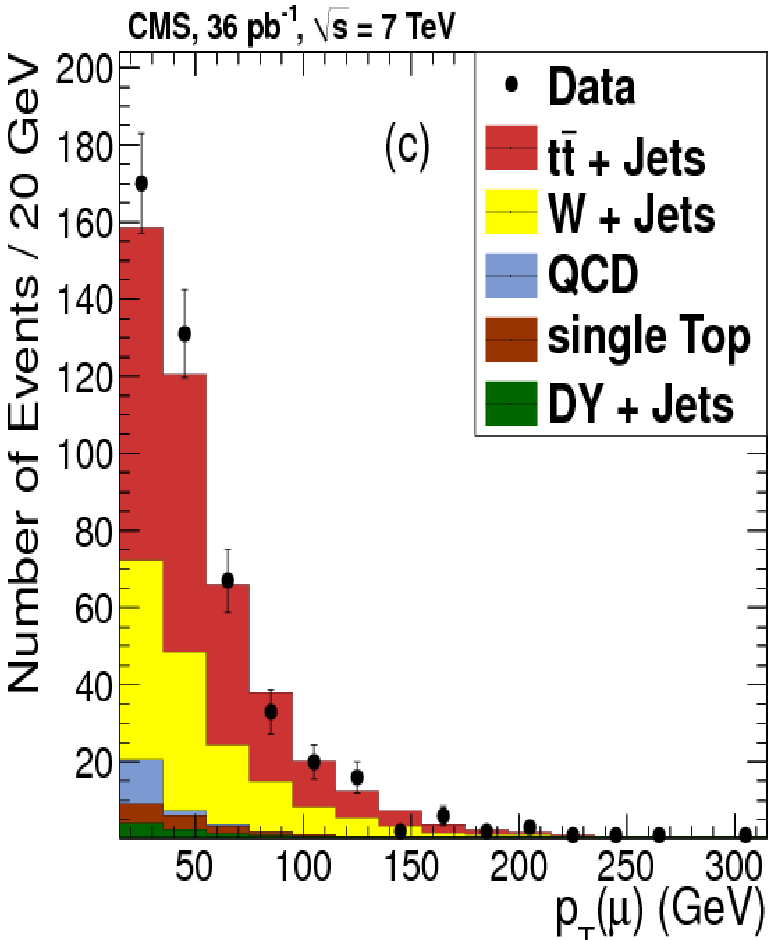
$$\mathcal{L}_h = |D_\mu h|^2 - \lambda\left(|h|^2 - \frac{v^2}{2}\right)^2 \qquad \mathcal{L}_g = -\frac{1}{4}W_a^{\mu\nu}W^a_{\mu\nu} - \frac{1}{4}B^{\mu\nu}B_{\mu\nu}$$

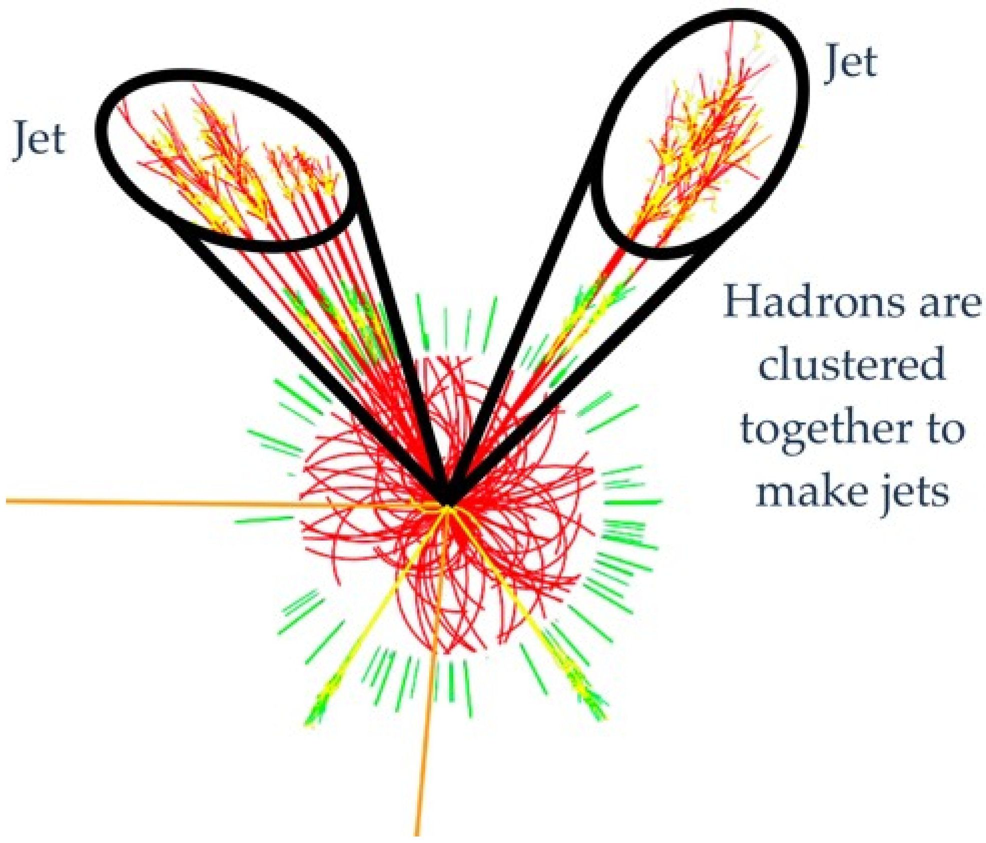




Events

- We study geometry and topology of interaction region. Effective geometry, nonlocality.





Plane wave solutions an quantization

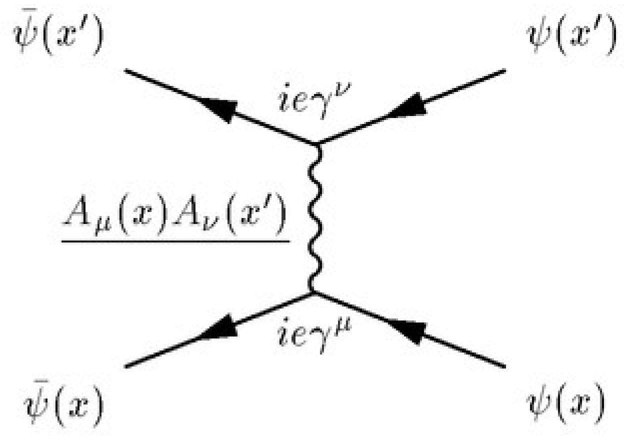
$$(i\hat{partial} - m)\psi = 0 \Rightarrow \psi(x) = \int \frac{d^3p}{(2\pi)^3/2\omega_p}(a_s^+(\bar{p})u_s(\bar{p})e^{-ipx} + b_s(\bar{p})v_s(\bar{p})e^{ipx})$$

- Momentum operator and normalization $E = \int d^3p\omega_p(a_p^+a_p + b_p^+b_p)$
- Hyperboloid, negative curvature, “creation” of new spaces
- The universal configuration space in p.T.
- Algebra of operators, commutation relations $\{a_p, a_q^+\} = \delta(\bar{p} - \bar{q})$
- Electromagnetic coupling constant $\alpha^{-1} = 137.035\,999\,139(31).$
- Hamiltonian $H(t) = e\int d^3x\bar{\psi}(x)\gamma_\mu\psi(x)A^\mu(x)$
- Self energy problem, normal ordering

S-matrix

- Heisenberg representation
- Schroedinger eqn $\partial_t F(t) = H_I(t)F(t)$
- T exponent $S = Texp(i\int H_I(t)dt)$
- Normal ordering and Wick's theorem
- Final states as free particles $|\bar{p}_1, ..., \bar{p}_n >= a^+_{p1} ... a^+_{p_n}|0>$

Feynman Diagrams



Enumerate p. T terms by graphs

- Their use at collider phenomenol
- Final state, creation of particles
- LO diagrams, rad corr
- Analogy for non linear PDEs. Greens functions on domains
- Chern Simons: metric independence
- Causality and backward forward mixing

Propagators

- Wick theorem
- Vacuum expectations of time ordered products

$$\mathcal{C}(x_1, x_2) = \langle 0|\mathcal{T}\phi_i(x_1)\phi_i(x_2)|0\rangle = \overline{\phi_i(x_1)\phi_i(x_2)} = i\Delta_F(x_1 - x_2) = i\int \frac{d^4k}{(2\pi)^4}\frac{e^{-ik(x_1-x_2)}}{(k^2-m^2)+i\epsilon}.$$

- Propagator in x-space $i\Delta_F^{ij}(x-y) \simeq \frac{-\delta^{ij}}{4\pi^2}\frac{1}{(x-y)^2 - i\epsilon}$

Amplitudes

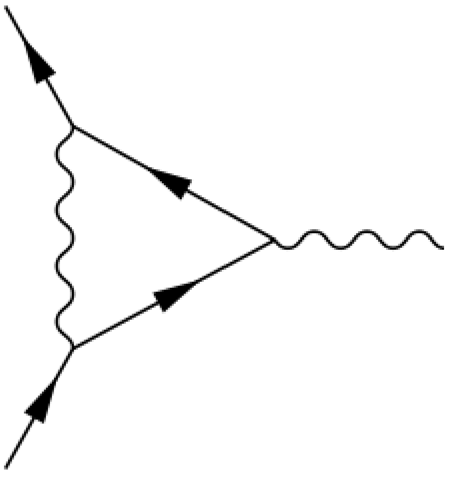
- The integral expression $G^D(p_1, ..., p_n) = \int dq_1 ... dq_n \prod_\alpha \frac{1}{(\sum p_i + \sum q_a)^2 - m_\alpha^2 + i\epsilon)}$
- Analytic continuation, the contour
- Divergences. $\int d^4q \frac{1}{q^2(q+p)^2}$
- Cut off $\int_{-\infty}^{\infty} d^4q \to \int^L d^4q$
- Dimensional regularization $\int \frac{(\hat{l}_i + m)}{l_i^2 - m^2 + i\epsilon} dq \to \int_\zeta d^nq \prod P_i^{\kappa_i}(q,p)$

$$G_r^D eg(p_1, ..., p_n) = \int d^{4-\eta}q_1 ... d^{4-\eta}q_n \prod_\alpha \frac{1}{(\sum p_i + \sum q_a)^2 - m_\alpha^2 + i\epsilon)}$$

Divergences and renormalization

- Single regularization parameter, cut off (or complex dimension)
- Series in ln(L)
- Observables are independent of the cutoff, calculation -does depend $(\partial/\partial log(\mu^2) + \beta(\alpha)\partial/\partial\alpha + \gamma(\alpha))G(s,\alpha,\mu) = 0$
- Beta function. It is defined on pro-finite riemann surface. Number theory (Connes). Foliations, Teichmuller theory. Can we get 1/137.03599 out of some number theoretic constructs? Noncommutative geometry. Coupling itself is a dynamic object. Algebra with infinity of operations.
- Need for geometry of particle configuration spaces.

Magnetic moment



- The triangle diagram

$$e^3 \int \frac{dq}{(2\pi)^4} \frac{1}{q^2 + i\epsilon} \gamma_\mu \frac{\hat{p} + \hat{q} + m}{(p+q)^2 - m^2 + i\epsilon} \gamma_\lambda \frac{\hat{p}' + \hat{q} + m}{(p'+q)^2 - m^2 + i\epsilon} \gamma_\mu$$

- UV and IR divergent $F(q^2)\gamma_\mu + \sigma_{\mu\nu} G(q^2) q_\nu$
- The limits: $p^2 = m^2, p'^2 = m^2, q^2 \to 0$

$$\frac{\alpha}{2\pi}$$

- Off shell- dilogarithm

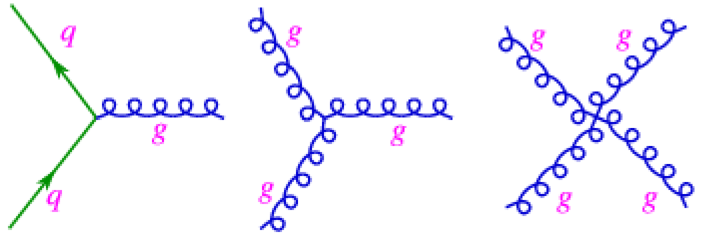
QCD

- Lagrangian

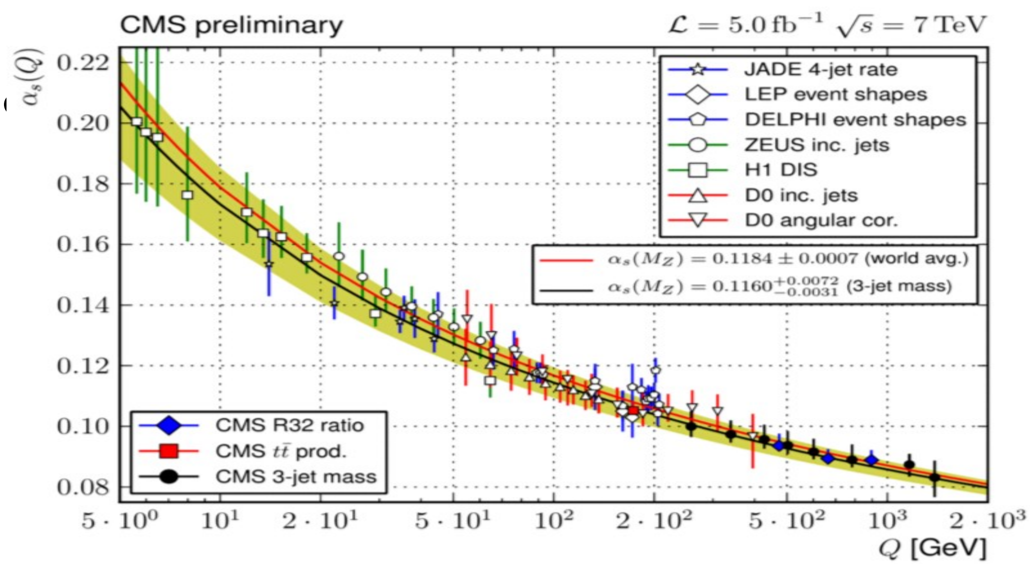
$$\mathcal{L}_{\mathrm{QCD}} = \bar{\psi}_i \left(i(\gamma^\mu D_\mu)_{ij} - m\,\delta_{ij} \right) \psi_j - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a$$

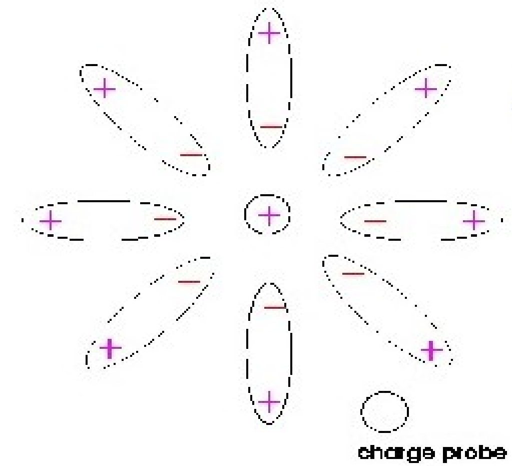
$$G^a_{\mu\nu} = \partial_\mu \mathcal{A}^a_\nu - \partial_\nu \mathcal{A}^a_\mu + g f^{abc} \mathcal{A}^b_\mu \mathcal{A}^c_\nu \,,$$

- Vertices

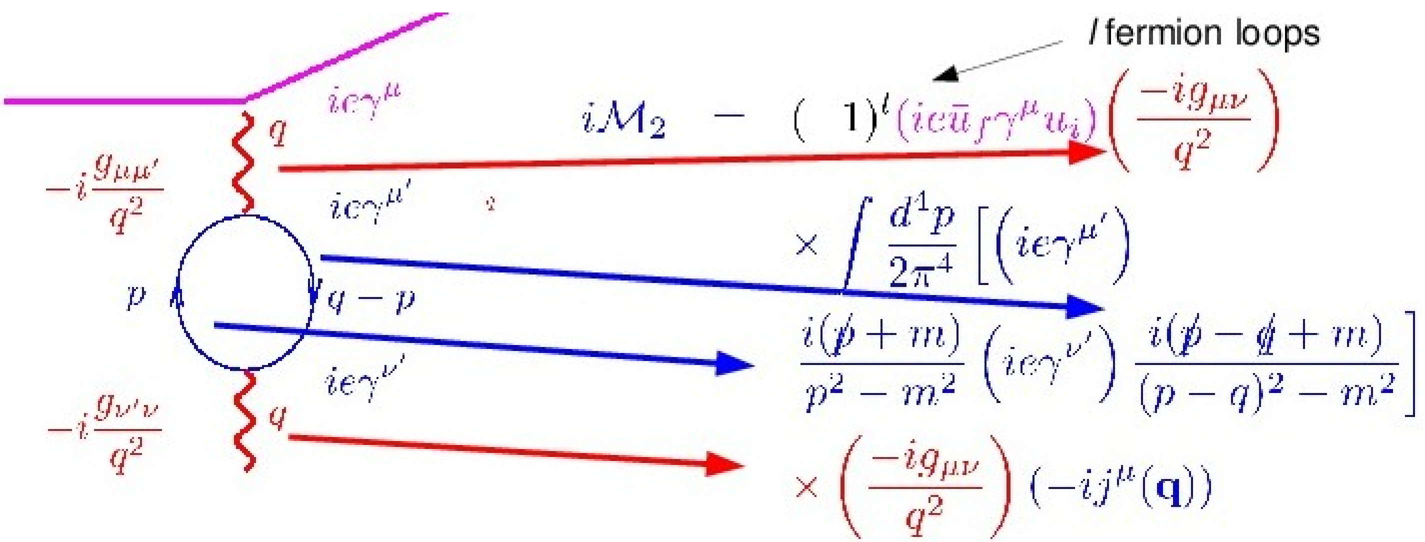


QCD:asymptotic freedom

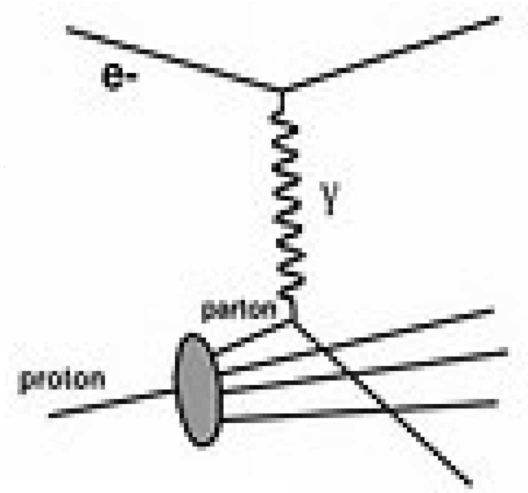




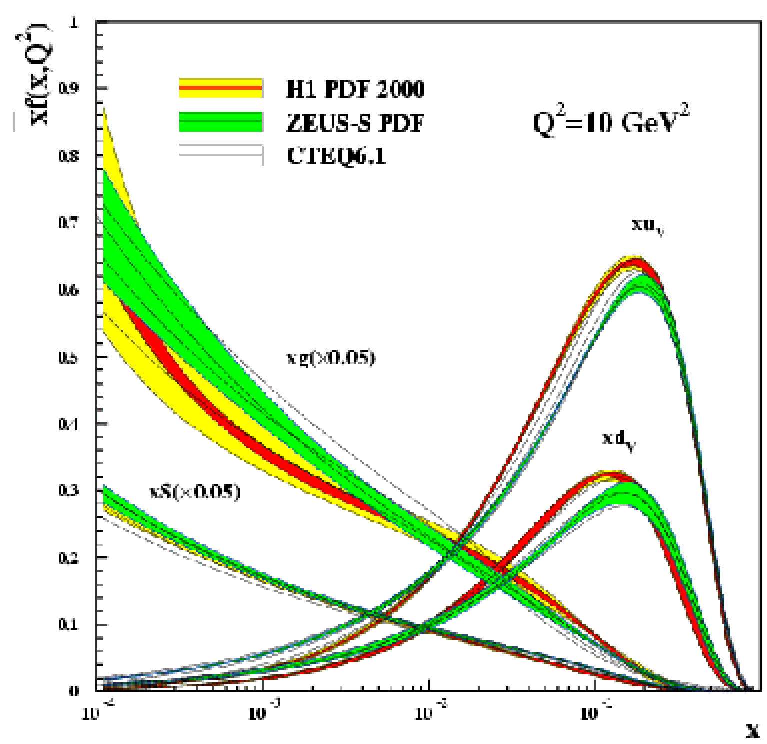
$$\alpha_S(\mu) = \frac{\alpha_S(\mu_0^2)}{1 + \alpha_S(\mu_0^2) b_0 log(\frac{\mu^2}{\mu_0^2})}, b_0 = \frac{33 - 2n_f}{12\pi}$$



QCD and the parton model



- Deep inelastic scattering, picture, photon, Lorenz contraction,
- f(x,Q2) as matrix element
- f_proton, different q
- Anomalous dimens



$$d\sigma = \frac{1}{2s}\frac{4\pi e^4}{Q^4}L^{\mu\nu}W_{\mu\nu}dl$$

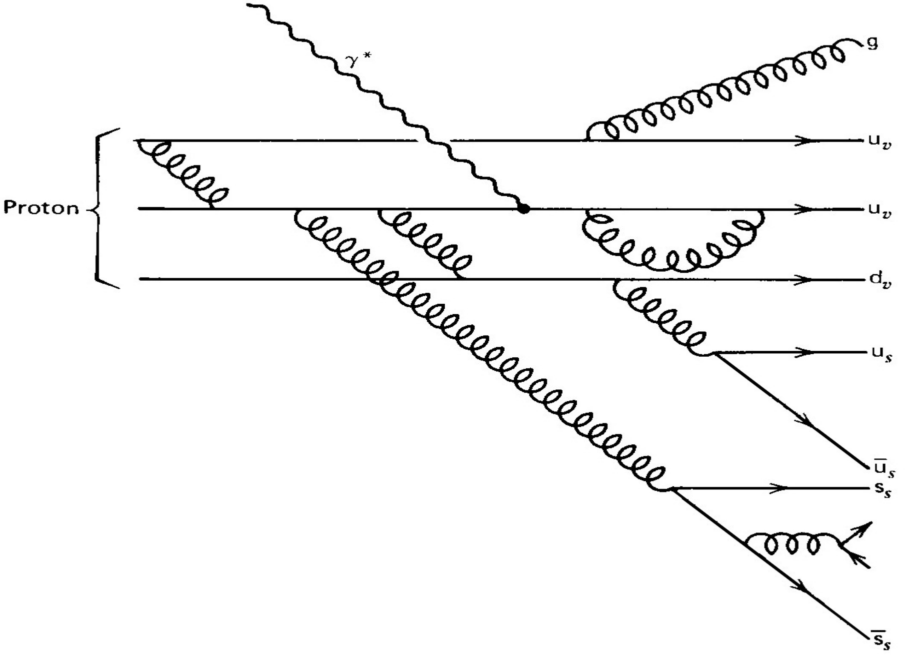
$$W_{\mu\nu} = \frac{1}{4\pi}\int e^{-iqx} < P|j^{\mu}(x)|X >< X|j^{\mu}(0)|P >$$

$$W_{\mu\nu} = W_1(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}) + \frac{W_2}{M^2}(p_\mu - q_\mu\frac{pq}{q^2})(p_\nu - q_\nu\frac{pq}{q^2})$$

$$2mW_1 = Q^2/2m\nu\delta(\nu - \frac{Q^2}{2m\nu}), \nu W_2 = \delta(\nu - \frac{Q^2}{2m\nu})$$

Infrared safeness

- Collinear divergences
- Soft-collinear effective theory
- Summation of large logarithms. What about other function forms?
- Quark-gluon "sea"



BFKL and DGLAP evolution

- Scaling violation: Q^2 dependence
- Collinear divergences
- Evolution equations, solutions
- Hera data, singularity at small x,BK

DGLAP

Integro(x)–differential(Q^2) eq^n for *integrated* gluon dist., g:

$$\frac{dg(x,Q^2)}{d\ln Q^2} = \int \frac{dz}{z} P_{gg}(z) g\left(\frac{x}{z}, Q^2\right)$$

BFKL

Integro(k)–differential(x) eq^n for *unintegrated* gluon dist., G:

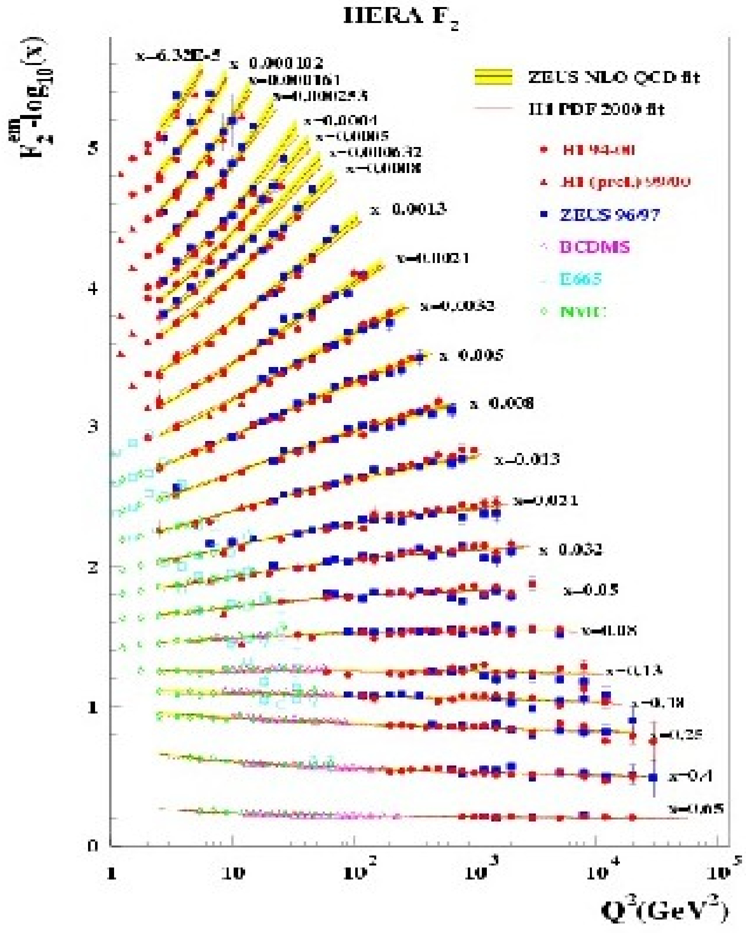
$$\frac{dG(x,k^2)}{d\ln 1/x} = \int \frac{dk'^2}{k'^2} K(k/k') G(x,k'^2)$$

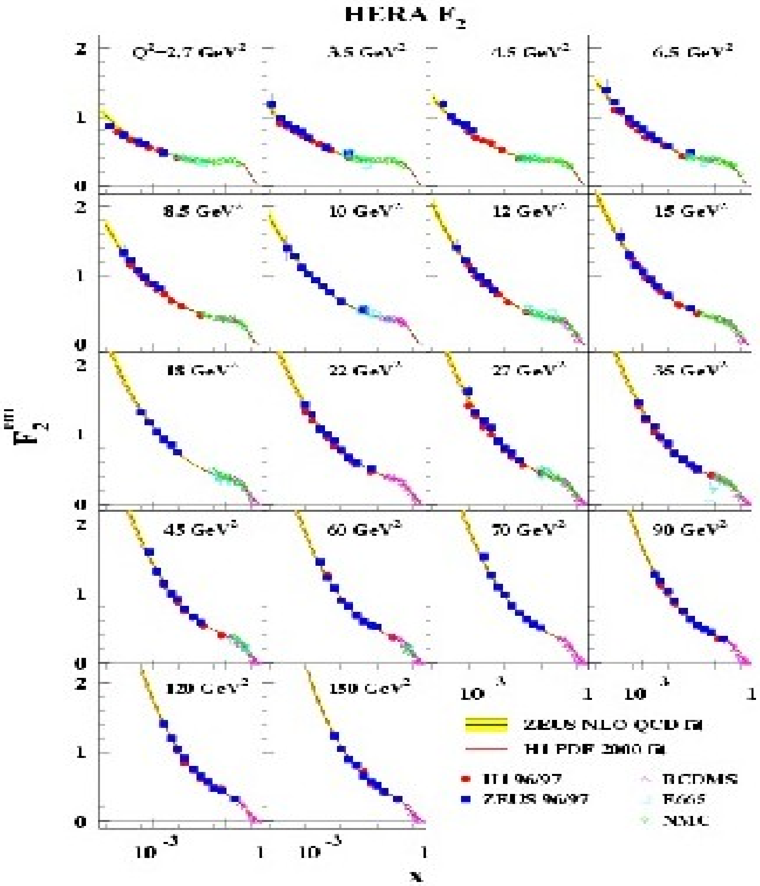
k, Q are transverse scales; x is longitudinal mom. fraction

$$xg(x,Q^2) = \int^Q d^2k\, G(x,k^2)$$

G.Salam

Evolution of parton densities





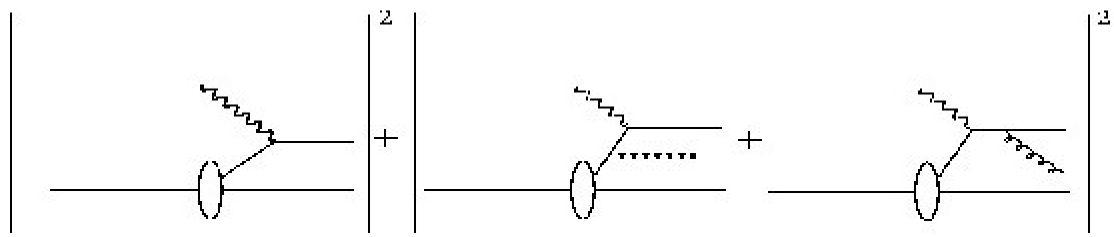
DGLAP- Q^2 evolution

- Collinear splitting of on-shell partons

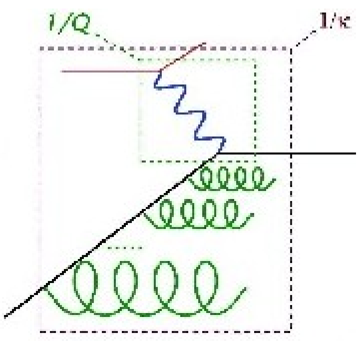
$$\partial/\partial log(Q^2) \begin{pmatrix} f_q(x,Q^2) \\ f_g(x,Q^2) \end{pmatrix} = \frac{\alpha_S(Q^2)}{2\pi} \int_x^1 d\xi \begin{pmatrix} P_{qq}(x/\xi,Q^2) & P_{qg}(x/\xi,Q^2) \\ P_{gq}(x/\xi,Q^2) & P_{gg}(x/\xi,Q^2) \end{pmatrix} \begin{pmatrix} f_q(\xi,Q^2) \\ f_g(\xi,Q^2) \end{pmatrix}$$

$$P_{qq}(z) = 4/3(\frac{1+z^2}{1-z} + \frac{3}{2}\delta(1-z)) \qquad P_{gq}(z) = \frac{4}{3}\frac{1+(1-z)^2}{z}$$

$$\frac{F_2}{x} =$$

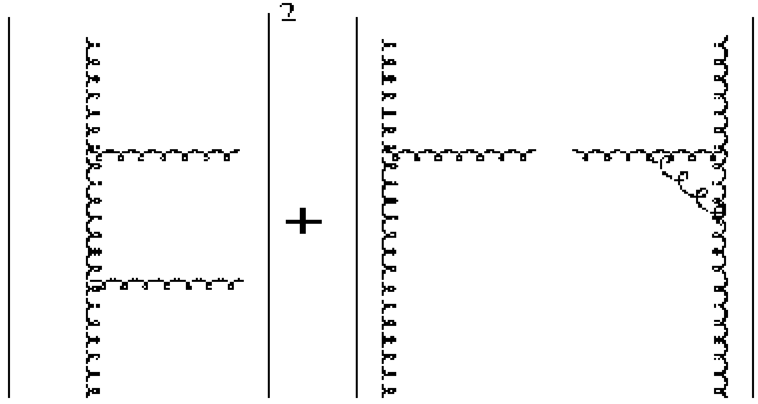


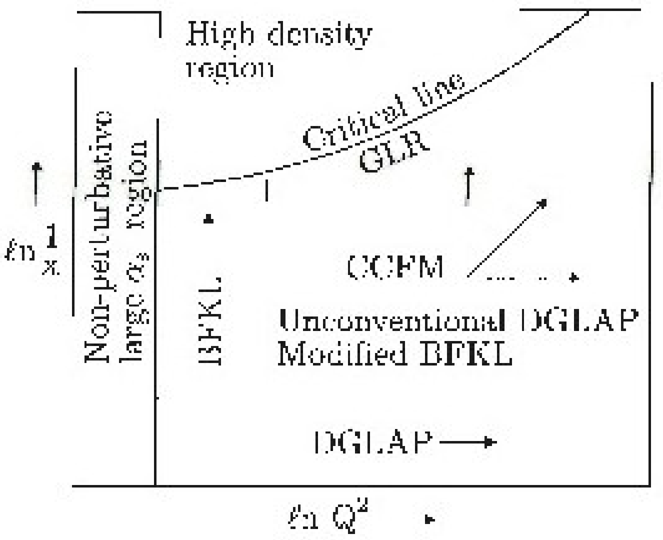
- The ladders- physics,picture
- Sums logs $\alpha_S ln(Q^2)$



BFKL

- Ladders , phase space approximation
- BK and exponential ends
- Switch regimes, different

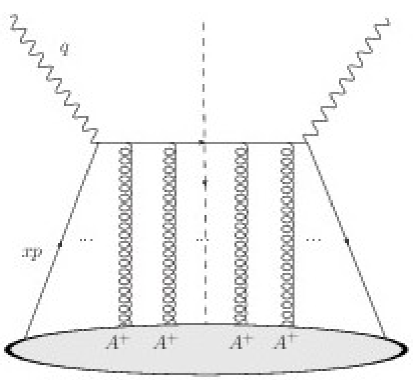


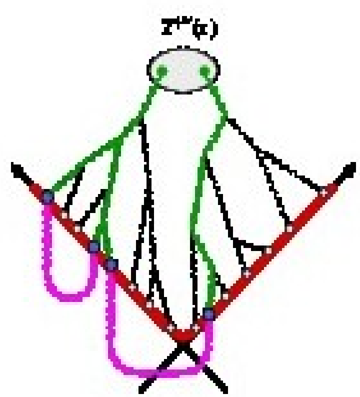


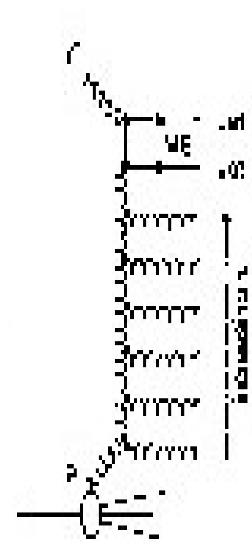
$$\partial/\partial log(x)G(x,b_{01}^2) = -\frac{N\alpha s}{2\pi^2}\int d^2b_2\frac{b_{10}^2}{b_{20}^2b_{21}^2}(G(x,b_{21}^2)+G(x,b_{20}^2)-G(x,b_{10}^2))$$

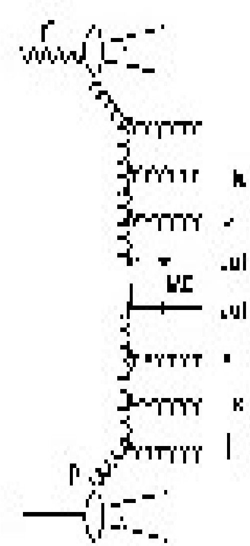
Higher twists, transverse space etc.

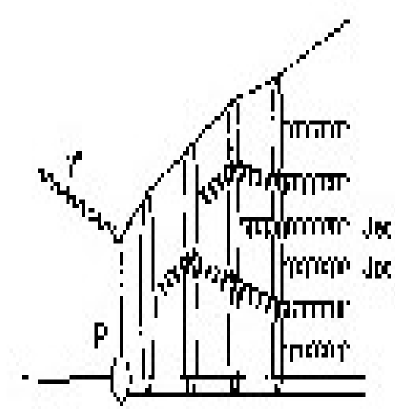
- Higher twists $f(x_1, x_2, Q^2) = < P|\bar{\psi}(x_1 p)\hat{A}(x_2 p)\psi(0)|P >$
- Small x functional eqn $H_{JIMWLK} = \int d^2x d^2x' \frac{\delta}{\delta A(\epsilon, x)} \eta(x, x') \frac{\delta}{\delta A(\epsilon, x)}$
- Exclusivity:interference between final states
- TMD





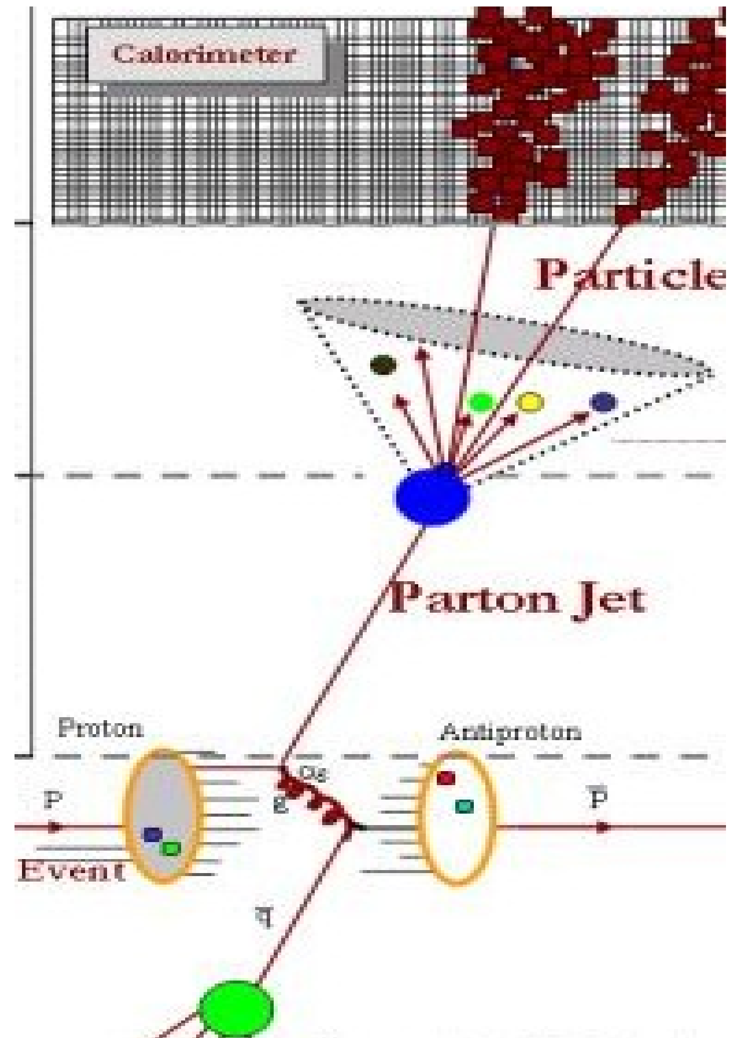






Jets

- The probability is concentrated near diagonals: partons are clustered
- Jets, picture, q g jets.
- B-tagging: new physics
- Jet substructure: new physics

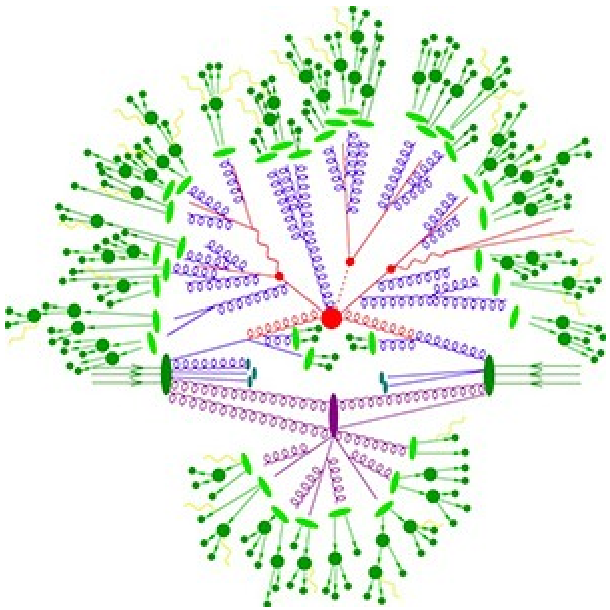


Event generators

- Pythia picture
- Several stages:

one is I.C. for the next

- Non-local objects: strings
- Analogy of Wilson lines



Factorization "theorem"

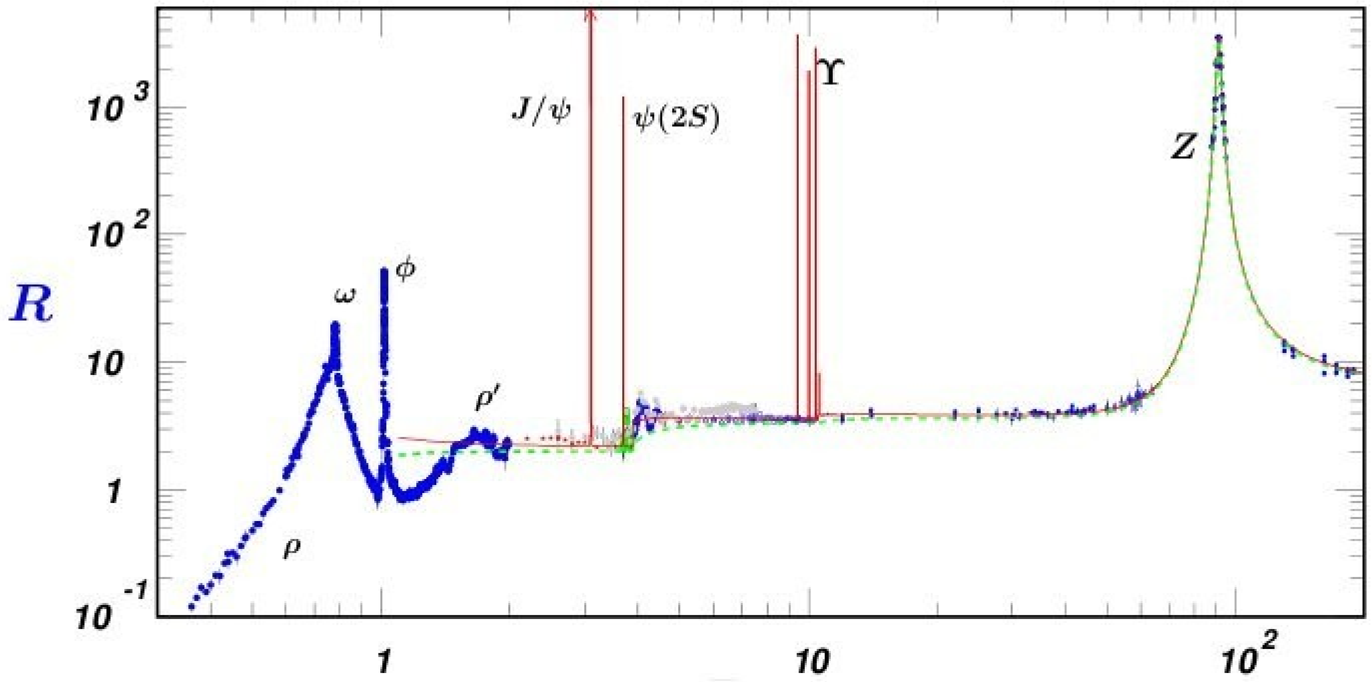
- Distribution and Fragmentation functions
- Scale space, L QCD, its geometry. Terms and ends of manifolds
- The formula $d\sigma/dp_1...dp_n\{G1, G2; F1, .., Fn\} = \sum_{m,k,l_i} \int G1_{x_1,...,x_m} G2_{y_1,...,y_k} K(x, y; z) F^{l1}(z_1^1, ...z_{l1}^1)...F^{ln}(z_1^n, ..., z_{ln}^n)$
- Typical calculation.
- What is the algebra behind it? It is a type of universal algebra (infinity of operations, of increasing arity)
- Ordering of space times. Causality.

Bound states

- Understanding of what they are conceptually
- The mass gap problem
- Clay Institute Millennium problem: Witten and Jaffe paper
- Essentially multi-particle problem. Define expansion space geometry.
- M_p interpret in terms of function space geometry.
- Scale space is large dimensional L_QCD/m_q~200
- Picks up quarks and gluons from vacuum

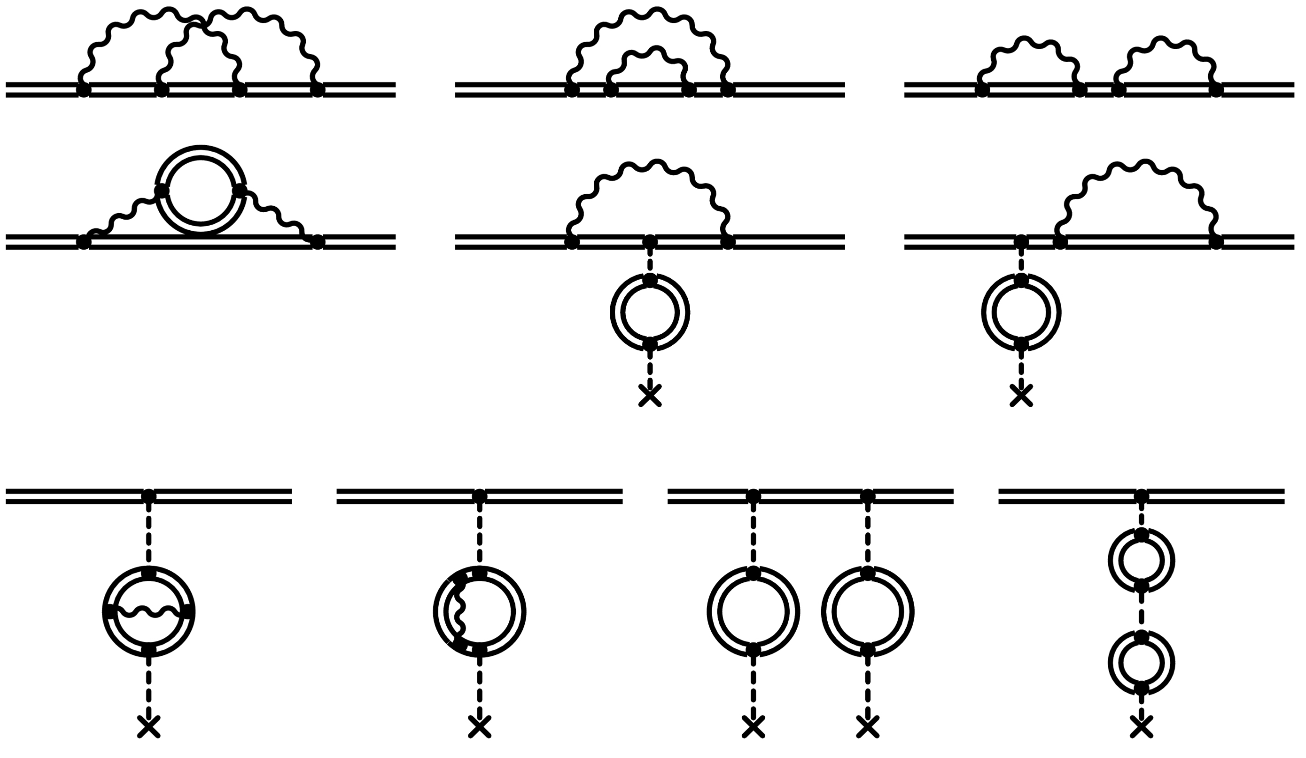
Bound states in experiment

- Pole in complex domain



Bound state equation

- Dirac equation with radiative corrections resums diagrams



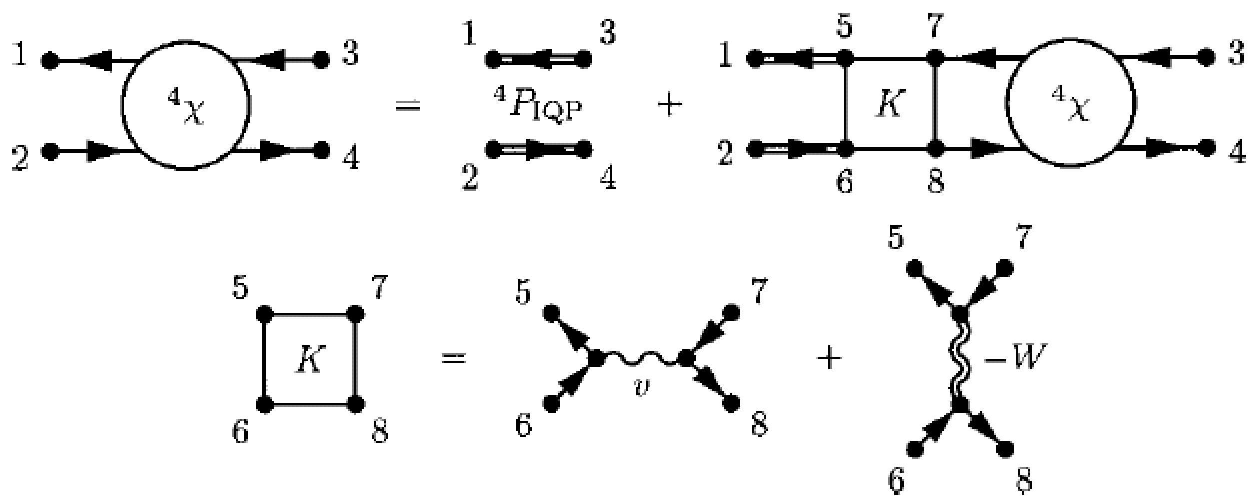
Bound states: the Dirac atom

- The Dirac equation.
- The two interacting lines, limit in p.T
- Recoil $(i\hat{\partial} - eA\ (x) - m)\psi(x) + e^2 \int dx'\Sigma(x, x')\psi(x') = 0$
- Bethe logarithm $m(Z\alpha)^4\alpha log(Z\alpha)^2$

2-body problem

Bethe Salpeter equation, resummation

$$(i\nabla\!\!\!/_1 - m_a)(i\nabla\!\!\!/_2 - m_b)\psi_{ab}(x_1, x_2) = -\int d^4x_3 d^4x_4 \overline{K}^{ab}(x_1, x_2; x_3, x_4)\psi_{ab}(x_3, x_4)$$



- Essentially two par
- Exponential decay of high energy states

Attempts of axiomatization

- Whightman
- Osterwalder-Schraeder
- Haag
- Segal (2d)
- Atiyah (topological)
- Seiberg Witten (a model of confinement)

“Pinching” and singularities

- For the evaluation of the integrals it is important to understand how the topology of the space of external momenta changes as we vary the parameters
- Zoo of pinches: non isolated singularities
- Vanishing cycles
- Residue forms
- Mild generalization of Grassmannians: the vanishing cycles in QFT.

Stratification of external momentum space

- The degeneracy conditions
- Elimination theory
- Toric geometry
- Examples: self energy` $p^4 - 2p^2(m_1^2 + m_0^2) + (m_1^2 - m_0^2)^2 = 0$
- Triangle, leading singularity

$$\begin{cases} q^2 = 0 \\ (q+p)^2 + m^2 = 0 \\ (q+p')^2 + m^2 = 0 \end{cases}$$

$$q = \alpha p + \beta p'$$

Local structure of amplitudes near multi-discriminant loci

- Steenbrink-Danilov-Varchenko
- Emergence of logarithms in 1d deformations of isolated singularity
- Holomorphic representations of the fundamental groups of the complements
- Local fundamental group.

Holonomic D-modules

- GZK approach.
- Pull back
- Poly logarithm on varieties. Beyond iterated integrals.
- Moduli spaces of singularities
- Series and special points
- Relate algebra of coefficients to the combinatorics of the graph

Wave functionals

- Generating functionals for correlators
- Laplace equation on the function space
- Boundary conditions. Spheres in function spaces. Possible non-contractibility of spheres. Does it matter for the set up of BV problem?
- Formal fundamental solution.
- Wave equation. Wave fronts. Light cone in FS.
- Generating functional for collinear parton distribution functions.
- The “fire ball” in QCD, diagonals

Ansatz choice for functionals

- The exp* poly ansatz $F[f] = exp(\int G_{xy} f(x) f(y)) \sum_n \int F^n(x_1, ..., x_n) f(x_1) ... f(x_n)$
- The ansatz for expansion coefficients $F[f] = lim F(f(f(f(....f(t_0))...)))$
- Functionals in Riemannian geometry: formal solutions.
- Difficulties: ambiguity in the definition, the meaning of branch loci.
- Non local Riemannian geometry
- FPDE translates into ordinary PDE
- Infinitely generated fundamental group.
- Ends of complexes: Weisberger, Yu, Ye, Ranicki, Quinn, Friedmann

Functional PDEs on spaces

- Spaces: Imm, Emb, Jet, Sobolev with constructive measures. Frechet-Borel measure theory
- Sequence spaces. Series solutions on the Hilbert space
- Tangent bundles for function manifolds
- Topology of functional manifolds. Differential topology is believed to be trivial: Atiyah. But there are variants: how about the loop space?
- What is the analogy of Sobolev functional spaces?
- Frechet homology theory. Simplicial story is possible – but the dimensions are manifolds.

Local renormalization group, local time

- Multi-scale processes: need to define coupling in each channel
- Field of infrared scales at higher orders
- Tomonaga equation $\frac{\delta}{\delta t(x)} F[\phi(y)] = \hat{H}(\phi, \frac{\delta}{\delta \phi}) F_t[\phi]$
- Evolution of multi-parton correlators $\partial_i F(x_1, ..., x_n) = \int_\zeta H_i(x, z) f(z) dz$
- The various flavors of renormalization group. Local renormalization group 1. many parameters. 2. partial integration of the field. Graph in space

Matrix form $\sum_m \hat{Z}_{n,m} f_m = 0$

Evolution on function spaces

- Multiscale processes: how to fix coupling. Related to factorization: how many diagram types to resum
- Local renormalization group
- Local time
- Partial integration over infinite graph spaces
- There is “homotopy” between interaction picture and pure field representations
- Property T, mass gap, spectrum of bound states
- Bound states and strata in function spaces
- Integral over a unit ball in a semi-norm is a function of RG scales

$$\int_B Df = g(\mu_1, ..., \mu_n, ...)$$

Conclusions

- Lots of interesting problems. Conceptual work: definitions are missing. QFT is somewhat mysterious: we are in the unique position to use the ambiguity
- Facts: 1. evolution takes place in a functional space 2. Space time is an effective notion 3. Topological manifolds are a part of calculations(implicitly)
- Questions: what exactly is the FS? What dimension theory to use?
- QCD- Riemannian field theory F[g]. Gromov Cheeger moduli

A selection of Refs.

- R.Brock et al. [CTEQ Collaboration], ``Handbook of perturbative QCD: Version 1.0''
- G. Sterman, J. Collins, “Factorization in QCD”
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Recursive equations for families of diagrams

- Schwinger Dyson equations

Radiative corrections: m/M small

- Summation of diagrams.
- Bethe Salpeter equation
- Integral form of it