Shallow S-wave pion-baryon resonances

龙炳蔚
(Bingwei Long)

四川大学
(Sichuan U., Chengdu, China)

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$\Lambda_c^+(2595)$

$I(J^P) = 0(\frac{1}{2}^-)$ $I(J^P) = 1(\frac{1}{2}^+)$

$$\Lambda_c^+(2595) \to \pi\Sigma_c(2455)$$

$\Lambda_c^+(2595)$ as an S-wave resonance in $\pi\Sigma_c$ channel

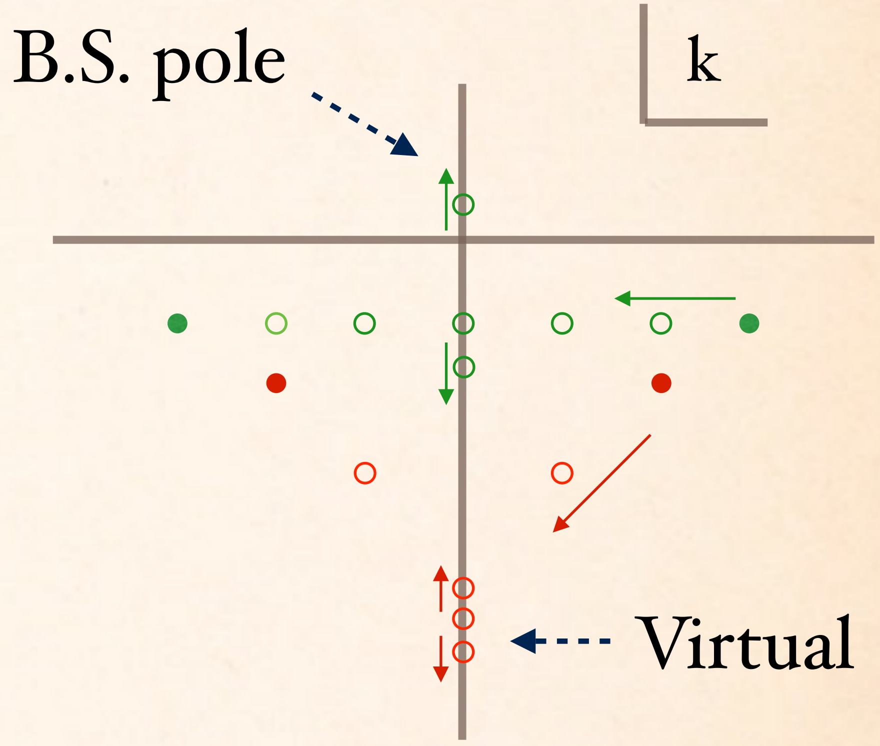
- Width ~ 2MeV — narrow
- ≤ 4 MeV above threshold — extremely shallow

Other channels

1. can be integrated out, for their threshold being much further away;
2. contribute at subleading orders (quantum-numbers)

S-wave resonances

— from the perspective of potential models

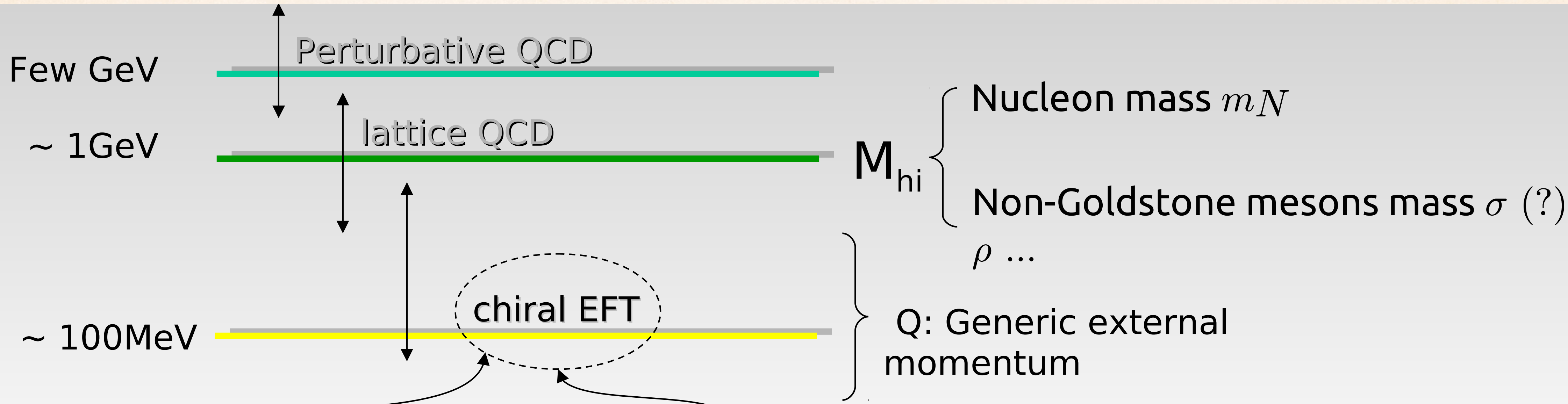


- Motion of poles when potential-strength tuned
- Large value of a — close to threshold (shallow)
- Large value of r — narrow resonance (as opposed to broad resonance)
- Two fine-tunings seem to be needed for a potential model to form a narrow, shallow S-wave resonance.

- naturally sized r
- unnaturally large r

$$f^{(0)} = \frac{1}{-\frac{1}{a} + \frac{r}{2}k^2 - ik}$$

Chiral Lagrangian for nuclear physics



D.o.f.s

Nucleons

Pions

chiral symmetry of QCD

$SU(2)_L \times SU(2)_R$

- spontaneously broken
- approximate $m_q > 0$

(Weinberg '69, '90)

$$\mathcal{L}_{int} = N^\dagger \left(-\frac{1}{4f_\pi^2}\epsilon_{abc}\tau_a\pi_b\dot{\pi}_c - \frac{g_A}{2f_\pi}\tau_a\vec{\sigma}\cdot\vec{\nabla}\pi_a \right) N - \frac{1}{2}C_S(N^\dagger N)^2 - \frac{1}{2}C_T(N^\dagger\vec{\sigma}N)^2$$

$\pi\pi NN$ πNN NNNN

+ ...

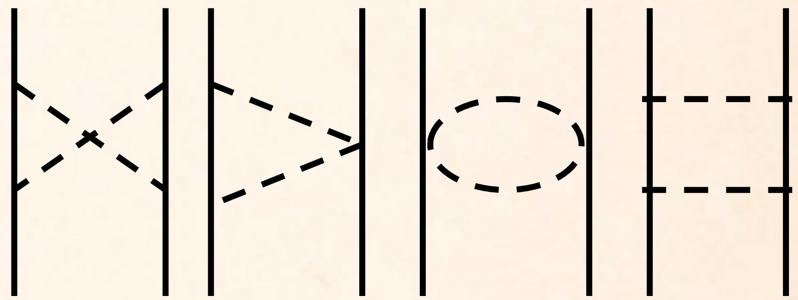
There are ∞ operators

chiral Lagrangian

Power counting for pion loops (HBChPT)

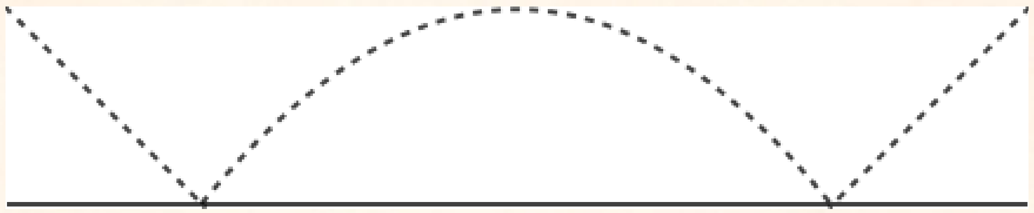
- Nucleon propagator — $1/Q$
- Pion propagator — $1/Q^2$
- Loop integral — $Q^4/(16\pi^2)$
- A vertex from $\mathcal{L}^{(\nu)}$ — Q^ν
- A pion loop brings a suppression factor of $\left(\frac{Q}{4\pi f_\pi}\right)^2$

Two-pion exchanges of nuclear forces



- Cutoff independence assumed → counting free of Λ

Generated by Weinberg-Tomozawa?



Weinberg-Tomozawa $\qquad \frac{i}{f_\pi^2}\Sigma^{a\dagger}\left(\pi^a\dot{\pi}^b - \pi^b\dot{\pi}^a\right)\Sigma^b$

Non-linear realization of chiral symmetry → fixed coupling

- S-wave interaction of pi-Sigma system
- But a pion loop always suppressed by $\left(\frac{Q}{4\pi f_\pi}\right)^2$ (Weinberg, '79)
- It's unlikely that W.T. alone can generate a S-wave resonance
- "Subleading"$_{\pi\pi\Sigma\Sigma}$ highly enhanced — fine-tuning

What we can do with ChPT then?

- Q: With chiral symmetry, are double fine-tuning still needed as in potential models?

  A: No, a single fine-tuning is sufficient

- Q: mutil pions + Σ_c ?

  A: likely to form more resonances

Explicit field of the excited baryon

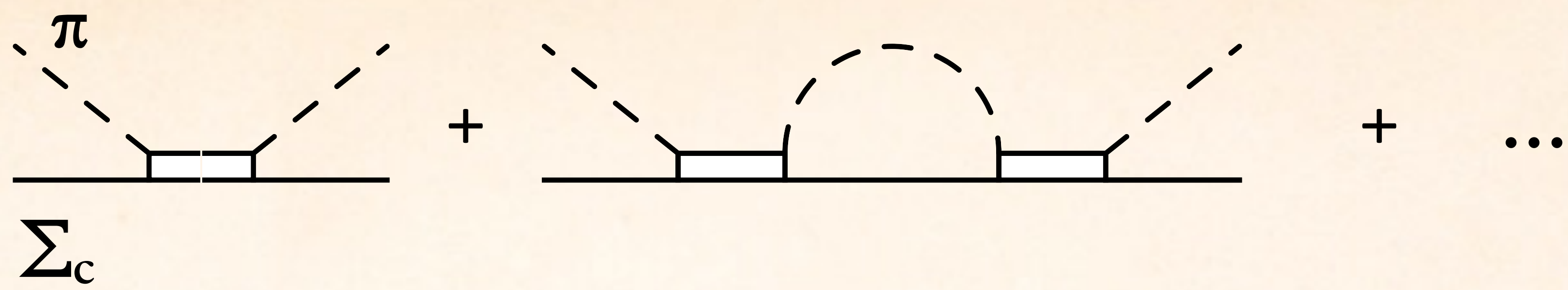
$$\Psi^\dagger \left(i\partial_0 - \Delta\right)\Psi + \frac{h}{\sqrt{3}f_\pi}\left(\Sigma^{a\dagger}\dot{\pi}^a\Psi + h.c.\right)$$

Ψ: $\Lambda_c^+(2595)$

h: O(1)

# of flavor of pions

- Ψ coupled to the S wave → time derivative on π, required by chiral symmetry (crucial)
- Nonrelativistic pion → coupling $\propto m_\pi$!
- Mass splitting $\Delta \sim m_\pi$ → near threshold



$$a = \frac{h^2 m_\pi^2}{4\pi f_\pi^2}\frac{1}{m_\pi - \Delta} \sim \left(\frac{140\mathrm{MeV}}{328\mathrm{MeV}}\right)^2 \frac{1}{4\mathrm{MeV}}$$

$$r = -\frac{4\pi f_\pi^2}{h^2 m_\pi^3} \sim \left(\frac{328\mathrm{MeV}}{140\mathrm{MeV}}\right)^2 \frac{1}{140\mathrm{MeV}}$$

$$f^{(0)} = \frac{1}{-\frac{1}{a} + \frac{r}{2}k^2 - ik}$$

h = 0.65
r = -19 fm
a = -10 fm

- r can be quite large when $\Delta \ll \sqrt{4\pi} f_\pi = 328\mathrm{MeV}$
- a single fine-tuning $\Delta - m_\pi \to 0$ makes both a and r large

→ Chiral symmetry helps $\Lambda_c^+(2595)$ remain narrow

Breakdown of universality

$$m_\pi^\star = \Delta$$

- Universality : observables expected to scale w/ $m_\pi^\star - m_\pi \to 0$
- Additional large length scale of r → universality relations break down sooner than expected

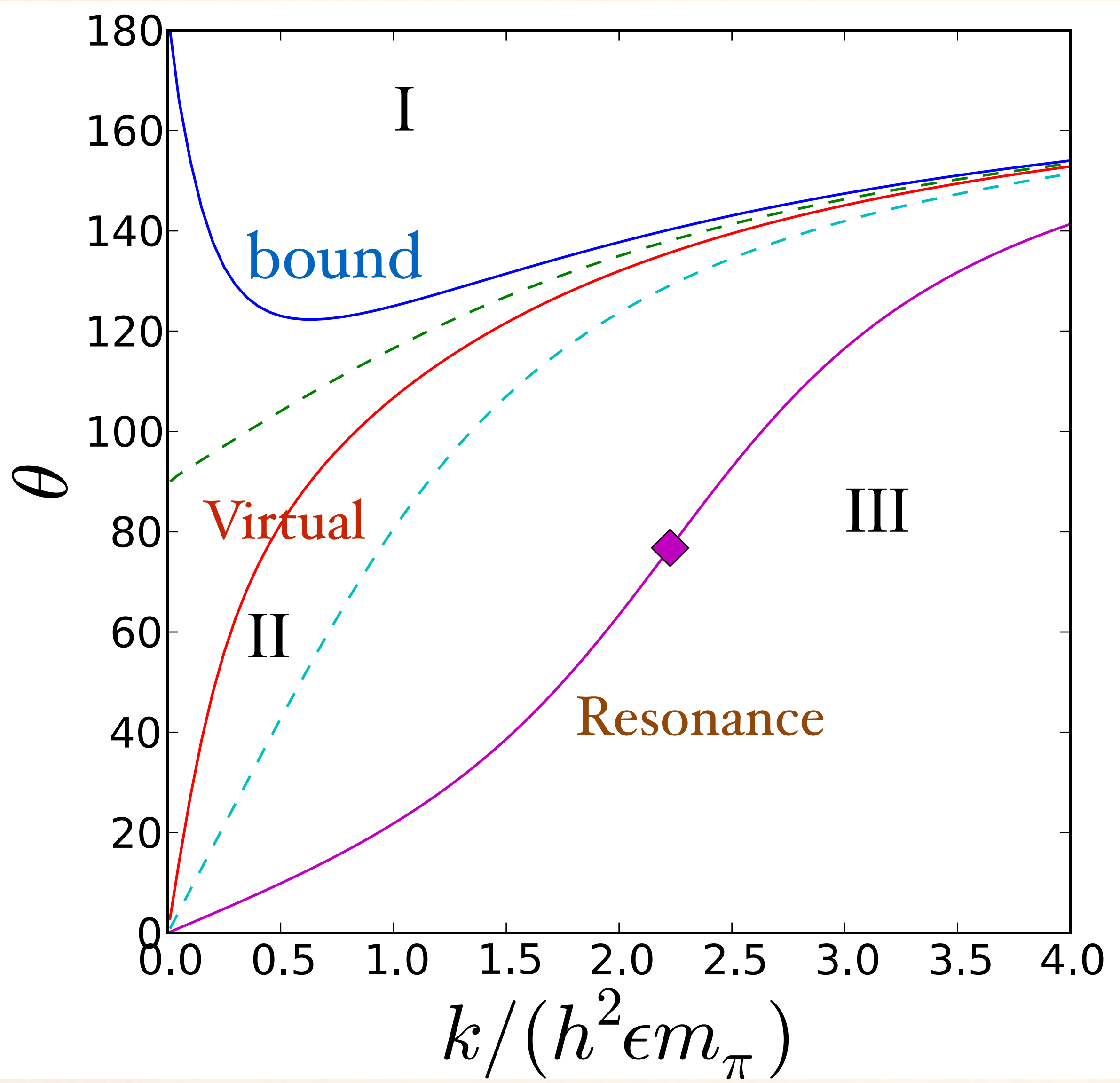
E.g., binding energy when $m_\pi > m_\pi^\star$

$$B_0(\delta; m_\pi) = \frac{h^4}{2}\epsilon^2 m_\pi \left(\sqrt{1 - \frac{2\delta}{h^4\epsilon^2 m_\pi}} - 1\right)^2 \qquad \delta = m_\pi^\star - m_\pi \qquad \epsilon = \left(\frac{m_\pi}{\sqrt{4\pi} f_\pi}\right)^2$$

Universality recovered only in a tiny window

$$B = \frac{\delta^2}{h^4\epsilon^2 m_\pi}\left[1 + \mathcal{O}\left(\frac{\delta}{h^2\epsilon^2 m_\pi}\right)\right] \text{ for } \left|\frac{m_\pi - m_\pi^\star}{m_\pi}\right| \ll \left(\frac{m_\pi^\star}{328\text{MeV}}\right)^4$$

The phase shifts



$$\epsilon = \left(\frac{m_\pi}{\sqrt{4\pi} f_\pi}\right)^2$$

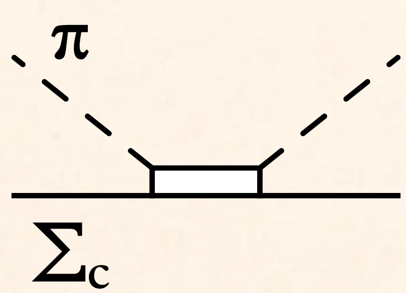
$$\tilde{\delta} \equiv \left(\frac{\sqrt{4\pi} f_\pi}{h m_\pi}\right)^4 (m_\pi^\star - m_\pi)$$

From top down

$$\tilde{\delta} = -0.2,\ 0.2,\ \text{and}\ 3$$

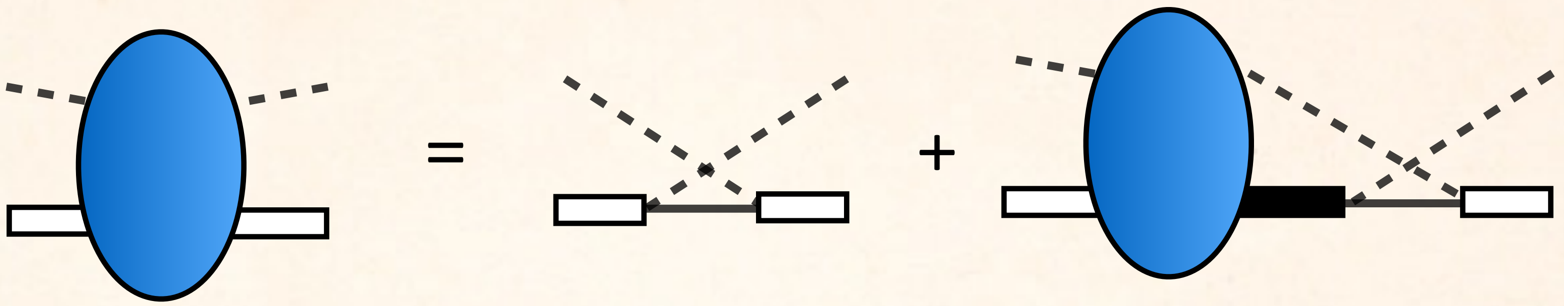
Two pions + Σ_c

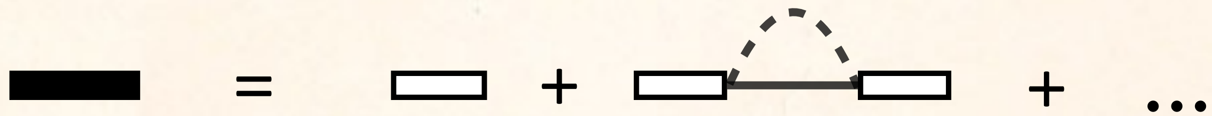
- $\pi\pi$ interactions are subleading
- $\pi\Sigma_c$ "potential" is energy-dependent → total binding energy is not simple addition of individual pions
- Searching 3-body states by finding poles of $\pi\Lambda_c^+(2595)$ scattering amplitude



$$\frac{h^2 m_\pi^2}{f_\pi^2 (E-\delta)}$$

$\pi\Lambda_c^+(2595)$ scattering amplitude





$$t(q;E,B_2) = \frac{8\pi}{3}\frac{|r|}{q^2+2m_\pi B_2}$$
$$+\frac{2}{3\pi}\int dl \frac{l^2}{l^2+q^2-2m_\pi E - i0}\,\frac{t(l;E,B_2)}{-\frac{1}{a}-\frac{|r|}{2}(2m_\pi E - l^2)+\sqrt{l^2-2m_\pi E - i0}}$$

q : pion momentum

B_2 : $\pi\Sigma_c$ binding energy

E : total energy

$\Lambda_c^+(2765)$?

quantum numbers uncertain

Summary

- Chiral symmetry ensures an S-wave pion-baryon to remain narrow close to threshold
- Large value taken by the effective range, ruining universality relations
- 2 pions + Σ_c