

Exotic Z_b states and the more debatable $X(5568)$

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- 1 General results for Z_b from their pole positions Xian-Wei Kang, Zhi-Hui Guo and J.A.Oller, PRD2016
- 2 the state $X(5568)$ observed by D0 collaboration Xian-Wei Kang, J.A.Oller, PRD2016

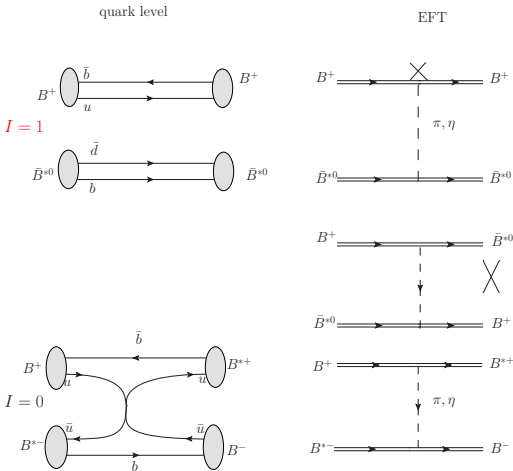
- A series of exotic hadron candidate XYZ were and are observed, cannot be accommodated by potential model — kinematical effects, molecular, quark-gluon hybrid, et al..
- Typically different scenarios predicts different constituents, in practice, may involve several mechanism
- Stay close to threshold of meson pairs: only 2-3 MeV above $\bar{B}^{(*)}B^*$ threshold
- Quantum numbers: $I^G(J^P) = 1^+(1^+)$, and thus for the electrical neutral isotopic state $I_3 = 0$, $J^{PC} = 1^{+-}$. $\bar{B}B^*$ stands for $G = +1$ combination $B\bar{B}^* - \bar{B}B^*$.

Effective range expansion

- Effective range expansion (ERE) could be a proper tool to study the physics in the vicinity of threshold. **elastic isovector S -wave $\bar{B}B^*$ scattering**. No specific dynamics is assumed.
- However, at pole position, momentum (modulus of complex values) is around m_π (pion mass). Left-hand branch point happens from $im_\pi/2$, inside the circle of the convergence radius, which prevents the validity of ERE.
- However, the pion exchange are suppressed. Specific calculation exists [Valderrama PRD2012; Hong-Wei Ke et al. JHEP2012, Dias PRD2015; Dian-Yong Chen and Xiang Liu, PRD2011]
- Chiral EFT power counting also shows coupled-channel effects are also suppressed. [Valderrama PRD2012]
- Also supported by Belle measurement: large branching ratio for $Z_b^{(\prime)} \rightarrow \bar{B}B^{(*)}$, around 80% [arXiv:1209.6450 [hep-ex]]
- **Uncoupled-channel ERE works, at least, as a first approximation approach**

OZI suppression

- The light meson ($q\bar{q}$) exchange violates OZI rule for $I = 1$ sector. Two-meson exchange contributes as the lowest-order.
- $B\bar{B}\pi$ vertex does not exist, but $B\bar{B}\rho$ and $B\bar{B}\omega$ do. Total contribution vanishes if ρ and ω are taken as equal mass. [Dias et al., PRD2015]



Compositeness for resonance

- Weinberg compositeness condition: wave function renormalization constant $Z=0$. In fact, $Z = 1 - X$, where $X = \gamma^2 \frac{dG(s_R)}{s_R}$ quantifies the weight of constituents. γ is the residue for $t(s)$ in the 1st sheet at the pole, and G is the two-point loop function. **only applied to bound state** — model independent relation for deuteron [Weinberg 1963; 1965]
- For resonance case, as long as $\sqrt{\text{Re}E_R^2}$ larger than the lightest threshold, $X = |\gamma^2 \frac{dG(s_R)}{s_R}|$, γ residue in the 2nd sheet [Guo and Oller, PRD2015],
- **Adapted to non-relativistic case, since here we use ERE.** criterion: $M_R > M_{\text{th}}$, consistent with above
- $X = \gamma_k^2 = -\frac{k_i}{k_r} = (2r/a - 1)^{-1/2}$, where the resonant momentum $k_R = k_r + ik_i$, γ_k is the residue in the variable k , a and r are scattering length and range.
- In terms of resonance property

$$\begin{aligned} X &= -\frac{2(M_R - M_{\text{th}})}{\Gamma_R} + \sqrt{1 + \left[\frac{2(M_R - M_{\text{th}})}{\Gamma_R}\right]^2} \\ &= 1 - \frac{2(M_R - M_{\text{th}})}{\Gamma_R} + 2\left[\frac{(M_R - M_{\text{th}})}{\Gamma_R}\right]^2 + \dots \end{aligned}$$

	$Z_b(10610)$	$Z_b(10650)$
a (fm)	-1.03 ± 0.17	-1.18 ± 0.26
r (fm)	-1.49 ± 0.20	-2.03 ± 0.38
$X = \gamma_k^2$	0.75 ± 0.15	0.67 ± 0.16
g^2 (GeV ²)	362 ± 71	263 ± 63

TABLE I. From top to bottom and left to right, we give the scattering lengths (a) and effective ranges (r) of the $B\bar{B}^*$ (Z_b) and $B^{(*)}\bar{B}^*$ (Z_b') systems, in order. In the last two lines compositeness (X), which is equal to γ_k^2 , and couplings squared g^2 for the $Z_b^{(\prime)}$ resonances are collected.

- g^2 : the absolute squared coupling constant for calculating the width in the standard PDG form
- *Real* values for a and r are obtained so as to reproduce the pole position. negative a and r can be realized by local contact or square well potential
- A distinct feature: r is in natural size

- Only right-hand cut without crossed-channel effect [Oller and Oset, PRD1999]

$$t(E) = \left[\sum_i \frac{g_i}{E_i - M_{\text{CDD}}} - ik \right]^{-1}$$

- Expansion of $\text{Re } t(E)^{-1}$ in powers of k^2 is equivalent to ERE, but worry for the small scale $[M_{\text{CDD}} - M_{\text{th}}]$, which restricts the validity range.
- M_{CDD} far away from M_{th} , then modulu of r is around 1 fm, otherwise r is very large.

$$\delta a = -\frac{M_{\text{th}} - M_{\text{CDD}}}{g_i}, \quad \delta r = -\frac{g_i}{\mu(M_{\text{th}} - M_{\text{CDD}})^2} \quad (1)$$

- M_{CDD} close to M_{th} , then small X , i.e., containing also other important components, e.g., compact quark-gluon states; M_{CDD} far from M_{th} , then the two meson constitute dominates
- First looking at r , how large it is

Width and compositeness: consistency check I

- In ERE, real a and r are fixed by reproducing pole position, single channel saturates, thus the width of Z_b s should correspond to partial decay width to $\bar{B}B^{(*)}$ [g^2 input from ERE].

$$\begin{aligned}
 \Gamma^{(1)} &= \frac{k(M_R)g^2}{8\pi M_R^2} \\
 \Gamma^{(2)} &= \frac{g^2}{16\pi^2} \int_{M_{\text{th}}}^{+\infty} \frac{dW k(W)}{W^2} \frac{\Gamma_R}{(M_R - W)^2 + \Gamma_R^2/4} \\
 &= \frac{X|k_R|M_R^2}{\pi\mu} \int_{M_{\text{th}}}^{+\infty} \frac{dW k(W)}{W^2} \frac{\Gamma_R}{(M_R - W)^2 + \Gamma_R^2/4} \quad (2)
 \end{aligned}$$

- In $\Gamma^{(2)}$, integrating up to ∞ is denoted by subscript $>$, and up to $M_R + n\Gamma_R$ ($n = 8$ is used) by $<$

Γ (MeV)	$Z_b(10610)$	$Z_b(10650)$
$\Gamma^{(1)}$	14.9 ± 2.3	9.5 ± 2.1
$\Gamma_{>}^{(2)}$	21.9 ± 3.3	13.4 ± 2.8
$\Gamma_{<}^{(2)}$	18.5 ± 2.4	11.3 ± 2.1

Experiment: $\Gamma_{Z_b} = 18.4 \pm 2.4$, $\Gamma_{Z'_b} = 11.5 \pm 2.2$ in unit of MeV

Consistency check II

- Again, from the working assumption – single channel saturates, fixing $\Gamma^{(2)} = \Gamma_R$, one gets g^2 and further the residue $|\gamma_k^2| = X$ (X compositeness)
- Branching ratio from Belle measurement can be taken into account, $\Gamma^{(2)} = \Gamma_R \times \text{Br}$ [arXiv:1209.6450[hep-ex]]

$$\text{Br}(Z_b(10610)^+ \rightarrow \bar{B}B^*) = (86.0 \pm 3.6)\%,$$

$$\text{Br}(Z_b(10650)^+ \rightarrow \bar{B}^*B^*) = (73.4 \pm 3.6)\%$$

	$Z_b(10610)$	$Z_b(10650)$
$\Gamma = \Gamma_R$	$X = 0.76 \pm 0.12$	$X = 0.69 \pm 0.14$
$\Gamma = \Gamma_R \cdot \text{Br}$	$X_{\text{ex}} = 0.66 \pm 0.11$	$X_{\text{ex}} = 0.51 \pm 0.10$

- $\bar{B}B^{(*)}$ weight is dominant but other non- $\bar{B}B^{(*)}$ components also play important role

ERE, Flatté, and Breit-Wigner parameterization

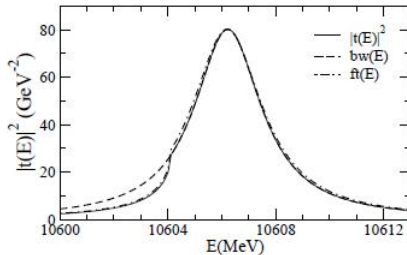
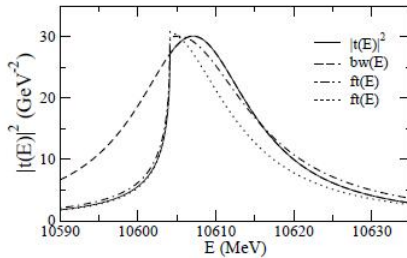
- Discuss the applicability of Breit-Wigner functions in the experimental analyses [see e.g., Cleven et al., EPJA 2011]
- ERE is adequate to check BW or Flatté, since it is not affected by threshold singularity

$$bw(E) = |t(M_R)|^2 \frac{\Gamma_R^2/4}{(E - M_R)^2 + \Gamma_R^2/4}$$

$$ft(E) = \frac{f_0}{|E - M'_R + i\Gamma_R(E)/2|^2}$$

$$\begin{aligned}\Gamma_R(E) &= \frac{2\gamma_k^2}{\mu} k(E)|k_R|, \quad E \geq M_R, \\ &= i \frac{2\gamma_k^2}{\mu} |k(E)k_R|, \quad E < M_R\end{aligned}$$

$$M'_R = M_R + \gamma_k^2 k_i |k_R| / \mu$$

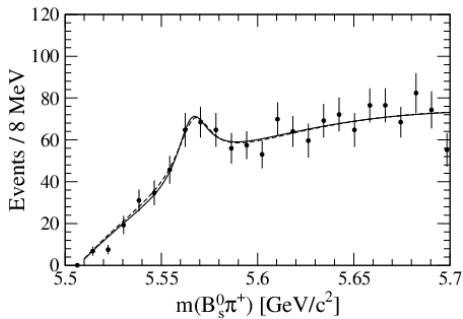
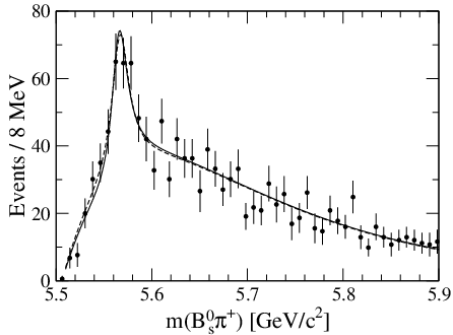


- top channel: $\Gamma_R = 18.4$ MeV, where dotted line corresponds to $M'_R = M_{\text{th}} + |k_R|^2/2\mu$, $M_R - M_{\text{th}} \approx 2$ MeV, and the large Γ_R softens the energy dependence of $t(E)$
- bottom channel: $\Gamma_R = 3.0$ MeV

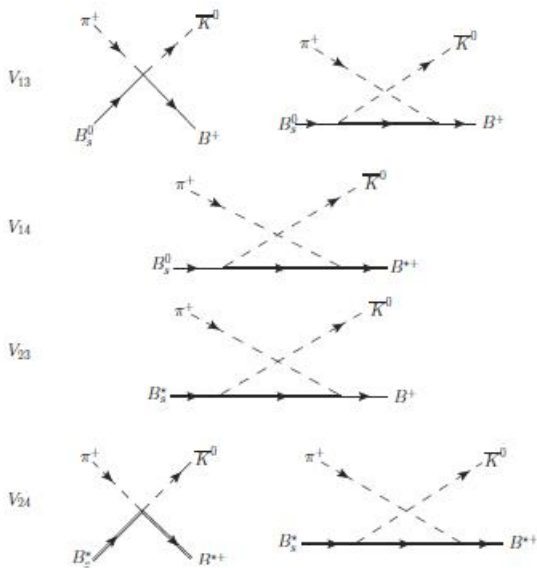
$X(5568)$: Motivation and idea

- D0 Collaboration has reported a narrow peak in the $B_s^0 \pi^{+-}$ mass spectrum, denoted as $X(5568)$.
- In fact, the quantum numbers J^P for the resonance have not been determined unambiguously yet.
- S-wave $B_s \pi - B \bar{K}$ transition was studied [Albaladejo et al., PLB2016]
- *Note:* $X(5568)$ is very close to $B_s^* \pi$ (above only 10 MeV), from a “molecular” perspective, it is suggestive to look for a coupled-channel-scattering scenario involving both $B_s \pi$ and $B_s^* \pi$. In this case, at least P -wave
- 4×4 coupled channel: (1) $B_s \pi$, (2) $B_s^* \pi$, (3) $B \bar{K}$, (4) $B^* \bar{K}$
- Indeed, interpretation of D0 data by P -wave scattering can not be excluded, will be shown below
- BW parameterization with orbital angular momentum L :

$$BW_L(m) \propto \frac{M_X^2 \Gamma_X(m)}{(M_X - m)^2 + M^2 \Gamma_X^2(m)}$$
$$\Gamma(m) = \Gamma_X \left(\frac{k(m)}{k(M_X)} \right)^{2L+1}$$



Potential



Results and discussion

- Chiral Unitary Approach: the potential is treated on shell and the Bethe-Salpeter equation can then be solved algebraically [Oller and Oset, NPA1997]
- $T^{-1}(s) = V^{-1}(s) + G(s)$, with G two-point Green function, the determinant $\Delta(s) = \det[V^{-1}(s) + G(s)]$. pole exists then $\Delta(s_R) = 0$
- $m_{B_S^0} + m_\pi < m_X < m_B + m_K$, and
 $G(s) = \text{diag}\{G_1^{\prime\prime}(s), G_2^{\prime\prime}(s), G_3(s), G_4(s)\}$ //: second (unphysical) sheet
- G -function regularized by dimensional regularization, $G_{i,\alpha}$; by cutoff, $G_{i,\Lambda}$. channel label: i , and μ renormalization scale
- u -channel crossing symmetry constraint $\Rightarrow \alpha$, also very close to the natural size estimation $\alpha_i(\mu_i) = -2 \log \left(1 + \sqrt{1 + m_{2i}^2/\mu_i^2} \right)$ with m_{2i} the heavier meson mass in channel i ; take $\Lambda = 770$ MeV or 1 GeV
- With these fixed values, no pole corresponding to $X(5568)$ is found
- Release the subtraction constants such that the pole $X(558)$ is imposed, then no physical interpretation (hindering the P -wave dynamically generated resonance)
- We have checked potential is too weak to “bound” two mesons
- Combining both S - and P -wave study, the molecular assignment is not plausible