D^0-$\bar{D}^0$ mixing and CP violation in charm sector at Belle

Longke LI[1] (李龙科)

[1]University of Science and Technology of China (USTC)

中国物理学会高能物理分会第十二届全国粒子物理学术会议
8 月 22 日-26 日，中国科大，安徽合肥

Outline

Outline

Neutral meson mixing phenomena

- Mixing: particle changes to its anti-particle and vice versa,
- result from flavor eigenstates $\neq$ mass eigenstates:

$$\begin{pmatrix} |P^0(q\bar{q}')\rangle \\ |\bar{P}^0(\bar{q}q')\rangle \end{pmatrix} = \begin{pmatrix} p & +q \\ p & -q \end{pmatrix} \begin{pmatrix} |P_1(m_1, \Gamma_1)\rangle \\ |P_2(m_2, \Gamma_2)\rangle \end{pmatrix}$$

Neutral meson mixing phenomena

- Mixing: particle changes to its anti-particle and vice versa,
- result from flavor eigenstates $\neq$ mass eigenstates:

$$\begin{pmatrix} |P^0(q\bar{q}')\rangle \\ |\bar{P}^0(\bar{q}q')\rangle \end{pmatrix} = \begin{pmatrix} p & +q \\ p & -q \end{pmatrix} \begin{pmatrix} |P_1(m_1,\Gamma_1)\rangle \\ |P_2(m_2,\Gamma_2)\rangle \end{pmatrix}$$

- Mixing parameters:

$$\mathbf{x} \equiv 2\frac{m_1 - m_2}{\Gamma_1 + \Gamma_2}, \quad \mathbf{y} \equiv \frac{\Gamma_1 - \Gamma_2}{\Gamma_1 + \Gamma_2}$$

Neutral meson mixing phenomena

- Mixing: particle changes to its anti-particle and vice versa,
- result from flavor eigenstates $\neq$ mass eigenstates:

$$\begin{pmatrix} |P^0(q\bar{q}')\rangle \\ |\bar{P}^0(\bar{q}q')\rangle \end{pmatrix} = \begin{pmatrix} p & +q \\ p & -q \end{pmatrix} \begin{pmatrix} |P_1(m_1,\Gamma_1)\rangle \\ |P_2(m_2,\Gamma_2)\rangle \end{pmatrix}$$

- Mixing parameters:

$$\mathbf{x} \equiv 2\frac{m_1 - m_2}{\Gamma_1 + \Gamma_2}, \quad \mathbf{y} \equiv \frac{\Gamma_1 - \Gamma_2}{\Gamma_1 + \Gamma_2}$$

- Time evolution of an initial pure state $|P^0\rangle$(t=0)

$$P_{\text{no-mixing}}(t) = |\langle P^0|P^0(t)\rangle|^2 = e^{-\Gamma t}\frac{\cosh(y\Gamma t) + \cos(x\Gamma t)}{2}$$

$$P_{\text{mixing}}(t) = |\langle P^0|\bar{P^0}(t)\rangle|^2 = |\tfrac{q}{p}|^2 e^{-\Gamma t}\frac{\cosh(y\Gamma t) - \cos(x\Gamma t)}{2}$$

Neutral meson mixing phenomena

- Mixing: particle changes to its anti-particle and vice versa,
- result from flavor eigenstates $\neq$ mass eigenstates:

$$\begin{pmatrix} |P^0(q\bar{q}')\rangle \\ |\bar{P}^0(\bar{q}q')\rangle \end{pmatrix} = \begin{pmatrix} p & +q \\ p & -q \end{pmatrix} \begin{pmatrix} |P_1(m_1,\Gamma_1)\rangle \\ |P_2(m_2,\Gamma_2)\rangle \end{pmatrix}$$

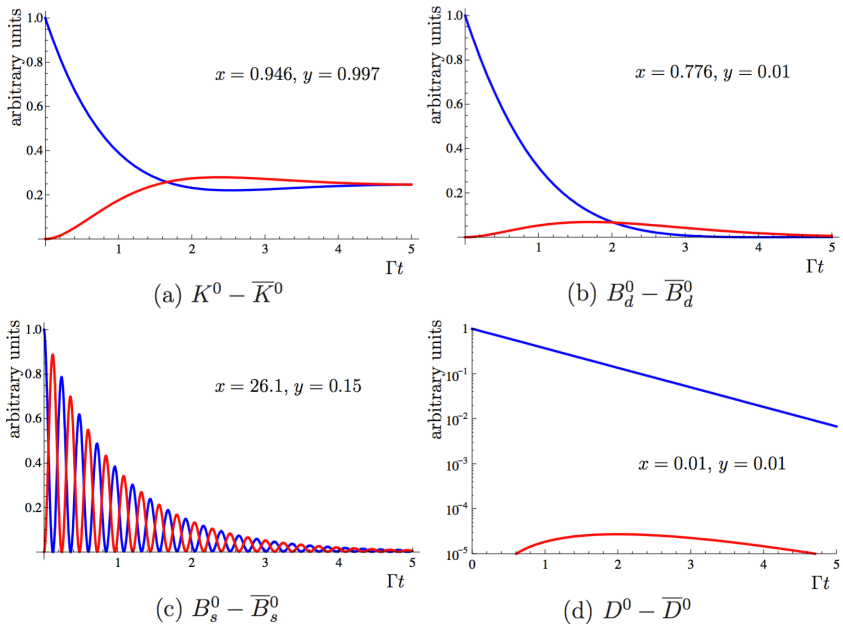
- Mixing parameters:

$$\mathbf{x} \equiv 2\frac{m_1 - m_2}{\Gamma_1 + \Gamma_2}, \quad \mathbf{y} \equiv \frac{\Gamma_1 - \Gamma_2}{\Gamma_1 + \Gamma_2}$$

- Time evolution of an initial pure state $|P^0\rangle$(t=0)

$$P_{\text{no-mixing}}(t) = |\langle P^0|P^0(t)\rangle|^2 = e^{-\Gamma t}\frac{\cosh(y\Gamma t) + \cos(x\Gamma t)}{2}$$

$$P_{\text{mixing}}(t) = |\langle P^0|\bar{P}^0(t)\rangle|^2 = |\tfrac{q}{p}|^2 e^{-\Gamma t}\frac{\cosh(y\Gamma t) - \cos(x\Gamma t)}{2}$$



(a) $K^0 - \overline{K}^0$ (b) $B_d^0 - \overline{B}_d^0$ (c) $B_s^0 - \overline{B}_s^0$ (d) $D^0 - \overline{D}^0$

Neutral meson mixing phenomena

- Mixing: particle changes to its anti-particle and vice versa,
- result from flavor eigenstates $\neq$ mass eigenstates:

$$\begin{pmatrix} |P^0(q\bar{q}')\rangle \\ |\bar{P}^0(\bar{q}q')\rangle \end{pmatrix} = \begin{pmatrix} p & +q \\ p & -q \end{pmatrix} \begin{pmatrix} |P_1(m_1,\Gamma_1)\rangle \\ |P_2(m_2,\Gamma_2)\rangle \end{pmatrix}$$

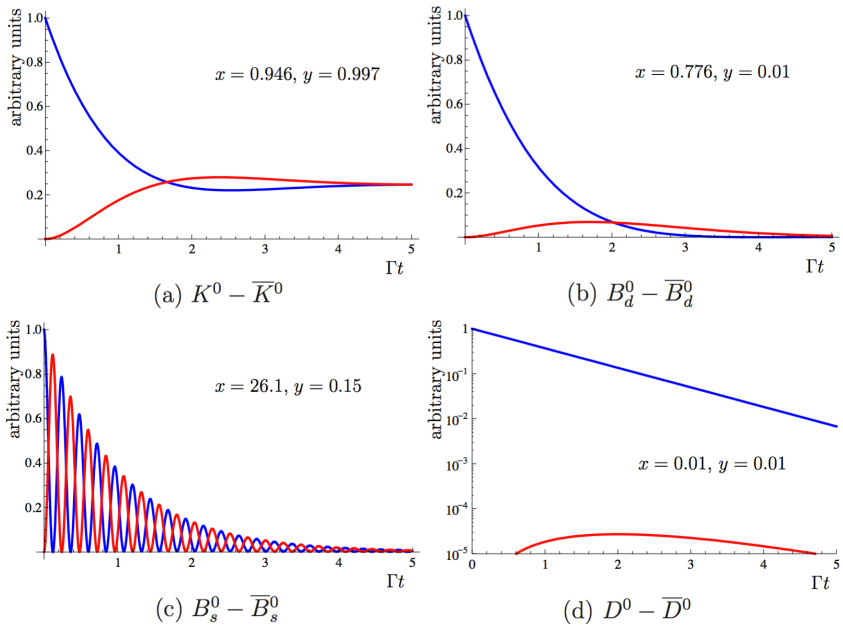
- Mixing parameters:

$$\mathbf{x} \equiv 2\frac{m_1 - m_2}{\Gamma_1 + \Gamma_2}, \quad \mathbf{y} \equiv \frac{\Gamma_1 - \Gamma_2}{\Gamma_1 + \Gamma_2}$$

- Time evolution of an initial pure state $|P^0\rangle$(t=0)

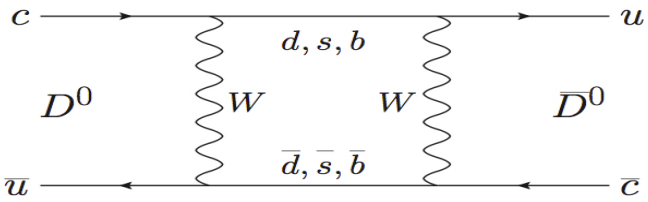
$$P_{\text{no-mixing}}(t) = |\langle P^0|P^0(t)\rangle|^2 = e^{-\Gamma t}\frac{\cosh(y\Gamma t) + \cos(x\Gamma t)}{2}$$

$$P_{\text{mixing}}(t) = |\langle P^0|\bar{P}^0(t)\rangle|^2 = |\tfrac{q}{p}|^2 e^{-\Gamma t}\frac{\cosh(y\Gamma t) - \cos(x\Gamma t)}{2}$$



(a) $K^0 - \overline{K}^0$ (b) $B_d^0 - \overline{B}_d^0$ (c) $B_s^0 - \overline{B}_s^0$ (d) $D^0 - \overline{D}^0$

- Standard Model(SM) prediction of D^0-$\bar{D}^0$ mixing: $|x|, |y|$(1%); CPV(0.01%).



(1) Short distance effect

(2) Long distance effect

CP violation (CPV)

- CP introduction:
  - C: charge-conjugated transform ($P \to \bar{P}$)
  - P: parity transform ($\vec{x} \to -\vec{x}$)
  - CP: C- P- combined transform (eg: $e_L^- \to e_R^+$).
- 1964, first observation of CPV in K^0 decays.

CP violation (CPV)

- CP introduction:
  - C: charge-conjugated transform ($P \to \bar{P}$)
  - P: parity transform ($\vec{x} \to -\vec{x}$)
  - CP: C- P- combined transform (eg: $e_L^- \to e_R^+$).
- 1964, first observation of CPV in K^0 decays.
- one of necessaries of three matter-anti-matter asymmetry conditions (Sakharov, 1967).
- SM: complex phase in CKM as CPV source

  CKM by Wolfenstein paratmerization

$$V = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

- New CPV source needed, search for New Physics
- 《*CP* 不守恒》杜东生 编著，北京大学出版社

CP violation (CPV)

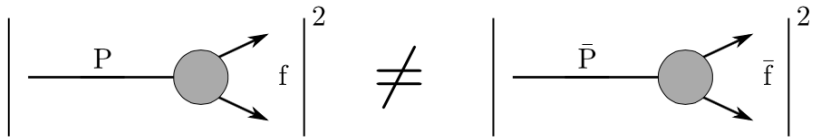
- CP introduction:
  - C: charge-conjugated transform ($P \to \bar{P}$)
  - P: parity transform ($\vec{x} \to -\vec{x}$)
  - CP: C- P- combined transform (eg: $e_L^- \to e_R^+$).
- 1964, first observation of CPV in K^0 decays.
- one of necessaries of three matter-anti-matter asymmetry conditions (Sakharov, 1967).
- SM: complex phase in CKM as CPV source

  CKM by Wolfenstein paratmerization

$$V = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

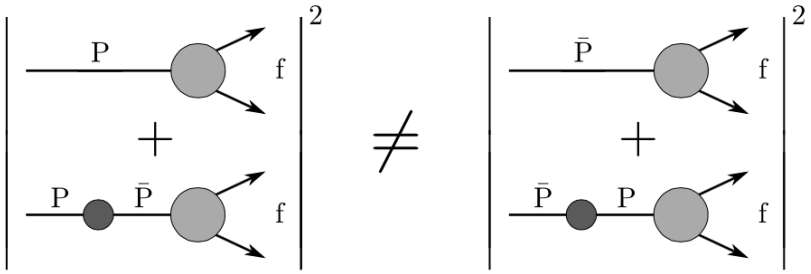
- New CPV source needed, search for New Physics
- 《*CP* 不守恒》杜东生 编著，北京大学出版社

- Charge-Parity transform
- in the decay (direct): $|\bar{A}_{\bar{f}}/A_f| \neq 1$;



- in mixing (indirect): $r_m = |q/p| \neq 1$;

- in the interference: $\arg(q/p) \neq 0$.



Available Charm samples from Charm factories, B-factories, hadron colliders

Experiment	Machine	Operation	C.M (GeV)	Lumin.	N(D)	efficiency
CLEO	CESR (e^+e^-)	2003-2008	3.77	0.8 fb^{-1}	2.9 M 2.3 M($D^\pm$)	~10-30%
			4.17	0.6 fb^{-1}	0.6 M	
BESIII	BEPC-II (e^+e^-)	2010-2011	3.77	2.92 fb^{-1}	10.5 M 8.4 M ($D^\pm$)	
		2016	4.18	3.0 fb^{-1}	3.0 M	
					★	★★★
BELLE	KEKB (e^+e^-)	1999-2008	10.58	1000 fb^{-1}	1.3 G	~5-10%
	PEP-II (e^+e^-)	1999-2008	10.58	500 fb^{-1}	0.65 G	
					★★	★★
CDF II	Tevatron ($p\bar{p}$)	2002-2011	1960	9.6 fb^{-1}	0.13 T	<0.5%
LHCb	LHC (pp)	2011	7000	1.0 fb^{-1}	5.0 T	
		2012	8000	2.0 fb^{-1}		
					★★★	★

here we used $\sigma(D^0\bar{D}^0@3.77\ GeV)$=3.61 nb, $\sigma(D^+D^-@3.77\ GeV)$=2.88 nb, $\sigma(D^*D_s@4.17\ GeV)$=0.967 nb, $\sigma(c\bar{c}@10.58\ GeV)$=1.3 nb, $\sigma(D^0@LHCb)$=1.661 nb. The table mainly refers to Int. J. Mod. Phys. A **29** (2014) 24, 14300518.

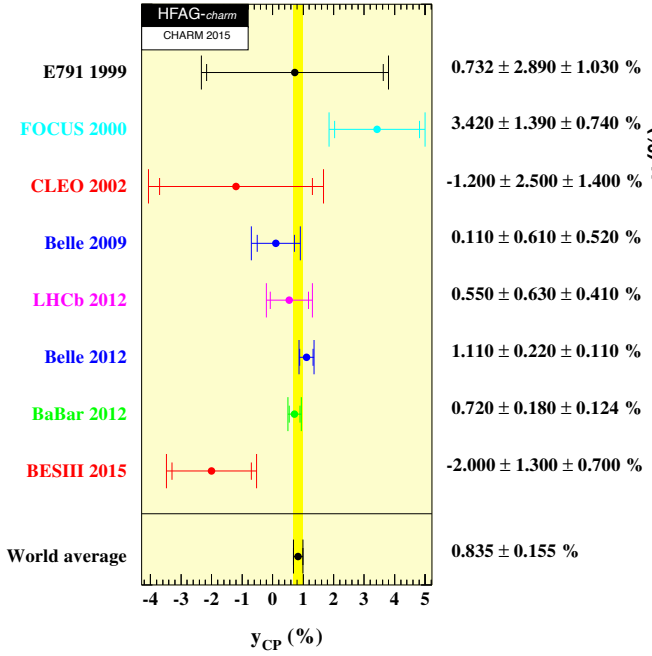
Available Charm samples from Charm factories, B-factories, hadron colliders

Experiment	Machine	Operation	C.M (GeV)	Lumin.	N(D)	efficiency
CLEO	CESR (e^+e^-)	2003-2008	3.77	0.8 fb^{-1}	2.9 M 2.3 M($D^\pm$)	~10-30%
			4.17	0.6 fb^{-1}	0.6 M	
BESIII	BEPC-II (e^+e^-)	2010-2011	3.77	2.92 fb^{-1}	10.5 M 8.4 M ($D^\pm$)	
		2016	4.18	3.0 fb^{-1}	3.0 M	
					★	★★★
BELLE	KEKB (e^+e^-)	1999-2008	10.58	1000 fb^{-1}	1.3 G	~5-10%
	PEP-II (e^+e^-)	1999-2008	10.58	500 fb^{-1}	0.65 G	
					★★	★★
CDF II	Tevatron ($p\bar{p}$)	2002-2011	1960	9.6 fb^{-1}	0.13 T	<0.5%
LHCb	LHC (pp)	2011	7000	1.0 fb^{-1}	5.0 T	
		2012	8000	2.0 fb^{-1}		
					★★★	★

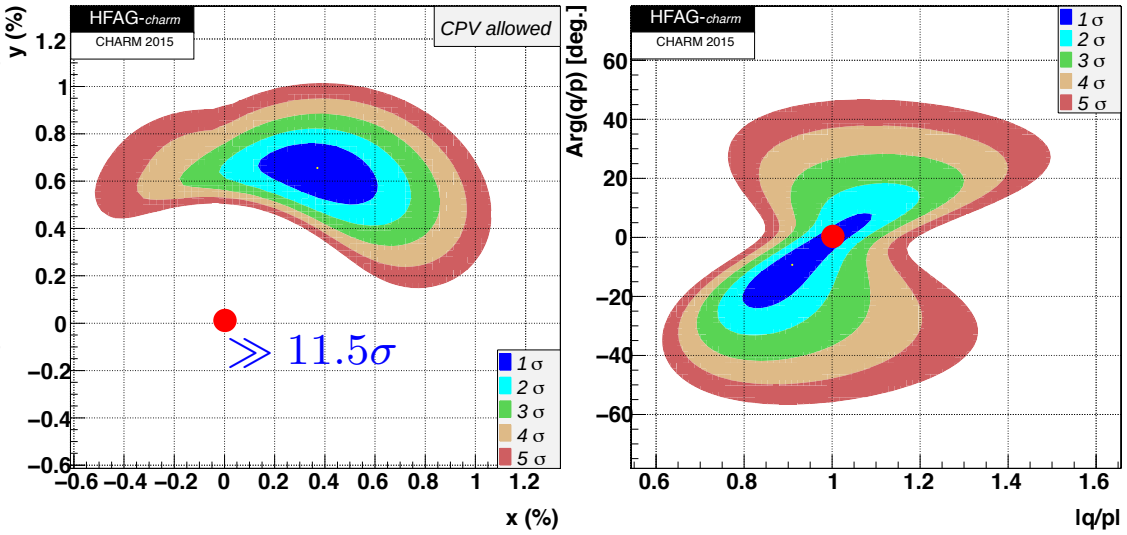
@B-factories

here we used $\sigma(D^0\bar{D}^0@3.77\ GeV)$=3.61 nb, $\sigma(D^+D^-@3.77\ GeV)$=2.88 nb, $\sigma(D^*D_s@4.17\ GeV)$=0.967 nb, $\sigma(c\bar{c}@10.58\ GeV)$=1.3 nb, $\sigma(D^0@LHCb)$=1.661 nb. The table mainly refers to Int. J. Mod. Phys. A **29** (2014) 24, 14300518.

Global fit for D^0-$\bar{D}^0$ mixing by HFAG: without CPV[link], with CPV [link]



contour-like 2-D plots



D^0-$\bar{D}^0$ mixing and CPV results from different experiments[mainly ref. charm physics at HFAG]

Decay Type	Final State	LHCb	BELLE	BABAR	CDF	CLEO	BESIII
DCS 2-body(WS)	$K^+\pi^-$	★	★	☆	★	✓	✓ $_{\delta K\pi}$
DCS 3-body(WS)	$K^+\pi^-\pi^0$		✓ A_{CP}	☆		✓ A_{CP}	
CP-eigenstates	K^+K^-, $\pi^+\pi^-$	☆ A_{CP} (a)	☆	☆	✓ A_{CP}	✓	
Self-conjugated 3-body decay	$K_S^0\pi^+\pi^-$		✓	✓	✓ A_{CP}	✓	
	$K_S^0K^+K^-$		✓ (b)	✓			
Self-conjugated SCS 3-body decay	$\pi^+\pi^-\pi^0$	✓ A_{CP}	✓ A_{CP}	✓ A_{CP}			
	$K^+K^-\pi^0$			✓ A_{CP}			
SCS 3-body	$K_S^0K^\pm\pi^\mp$	✓ $_{\delta K_S^0K\pi}$				✓ $_{\delta K_S^0K\pi}$	
Semileptonic decay	$K^+\ell^-\nu_\ell$		✓	✓		✓	
Multi-body(n≥4)	$\pi^+\pi^-\pi^+\pi^-$	✓ A_{CP}					
	$K^+\pi^-\pi^+\pi^-$	★	✓ A_{CP}	✓			
	$K^+K^-\pi^+\pi^-$	✓ A_{CP} (c)		✓ A_T		✓ A_{CP}	
$\psi(3770) \to D^0\bar{D}^0$ via correlations						✓ $_{\delta K\pi}$	✓ y_{CP}

★ for observation ($> 5\sigma$); ☆ for evidence ($> 3\sigma$); ✓ for measurement finished; Measurement on going not included.

The related references are linked under their corresponding signs.

(a) LHCb also give the measurement of indirect CP asymmetry in $D^0 \to h^-h^+$ decay in PRL **112**, 041801 (2014).

(b) Belle measured y_{CP} in $D^0 \to K_S^0\phi$ in PRD **80**, 052006 (2009), the amplitude analysis for mixing parameters (x, y) is on going.

(c) LHCb also search for CP violation using T-odd correlations in $D^0 \to K^+K^-\pi^+\pi^-$ decays in JHEP **10** (2014) 005.

D^0-$\bar{D}^0$ mixing and CPV results from different experiments[mainly ref. charm physics at HFAG]

Decay Type	Final State	LHCb	BELLE	BABAR	CDF	CLEO	BESIII
DCS 2-body(WS)	$K^+\pi^-$	★	★	☆	★	✓	✓ $_{\delta K\pi}$
DCS 3-body(WS)	$K^+\pi^-\pi^0$		✓ A_{CP}	☆		✓ A_{CP}	
CP-eigenstates	K^+K^-, $\pi^+\pi^-$	☆ $^{(a)}$ A_{CP}	☆	☆	✓ A_{CP}	✓	
Self-conjugated 3-body decay	$K^0_S\pi^+\pi^-$		✓	✓	✓ A_{CP}	✓	
	$K^0_S K^+K^-$		✓ $^{(b)}$	✓			
Self-conjugated SCS 3-body decay	$\pi^+\pi^-\pi^0$	✓ A_{CP}	✓ A_{CP}	✓ A_{CP}			
	$K^+K^-\pi^0$			✓ A_{CP}			
SCS 3-body	$K^0_S K^\pm\pi^\mp$	✓ $_{\delta K^0_S K\pi}$				✓ $_{\delta K^0_S K\pi}$	
Semileptonic decay	$K^+\ell^-\nu_\ell$		✓	✓		✓	
Multi-body(n≥4)	$\pi^+\pi^-\pi^+\pi^-$	✓ A_{CP}					
	$K^+\pi^-\pi^+\pi^-$	★	✓ A_{CP}	✓			
	$K^+K^-\pi^+\pi^-$	✓ $^{(c)}$ A_{CP}		✓ A_T		✓ A_{CP}	
$\psi(3770) \to D^0\bar{D}^0$ via correlations						✓ $_{\delta K\pi}$	✓ y_{CP}

★ for observation ($> 5\sigma$); ☆ for evidence ($> 3\sigma$); ✓ for measurement finished; Measurement on going not included.

The related references are linked under their corresponding signs.

(a) LHCb also give the measurement of indirect CP asymmetry in $D^0 \to h^-h^+$ decay in PRL **112**, 041801 (2014).

(b) Belle measured y_{CP} in $D^0 \to K^0_S\phi$ in PRD **80**, 052006 (2009), the amplitude analysis for mixing parameters (x, y) is on going.

(c) LHCb also search for CP violation using T-odd correlations in $D^0 \to K^+K^-\pi^+\pi^-$ decays in JHEP **10** (2014) 005.

Outline

Analysis techniques at B-factories

Analysis techniques at B-factories

- D^* inclusive sample: $e^+e^- \to \gamma^* \to c\bar{c} \to D^{*\pm}X^{\mp}$, $D^{*\pm} \to D^0\pi_s^+ / \bar{D}^0\pi_s^-$.

Analysis techniques at B-factories

- D^* inclusive sample: $e^+e^- \to \gamma^* \to c\bar{c} \to D^{*\pm}X^{\mp}$, $D^{*\pm} \to D^0\pi_s^+ / \bar{D}^0\pi_s^-$.
- veto signals from B decays and suppress background(BG): $p^*(D^*) > 2.5(\Upsilon(4S))$ / $3.1(\Upsilon(5S))$ GeV/c.
- multi-candidates using best candidate selection(BCS)
  - via the sum of D^0 and D^* vertex fitting qualities, eg: $D^0 \to K^-\pi^+$
  - via $\sum \chi^2$ of mass difference, eg: $D^0 \to K_S^0\pi^0\eta$

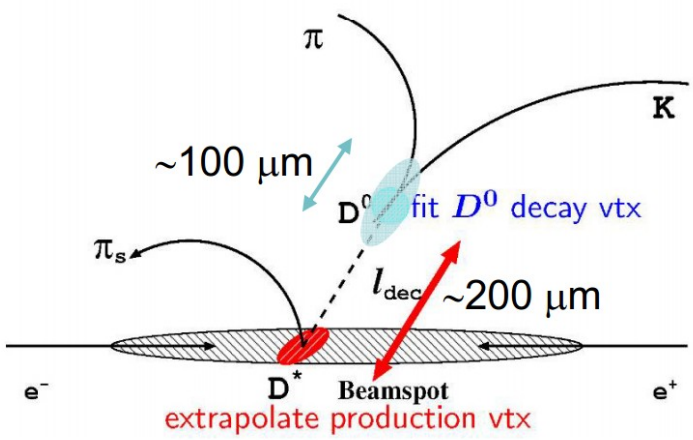
Analysis techniques at B-factories

- D^* inclusive sample: $e^+e^- \to \gamma^* \to c\bar{c} \to D^{*\pm}X^{\mp}$, $D^{*\pm} \to D^0\pi_s^+ / \bar{D}^0\pi_s^-$.
- veto signals from B decays and suppress background(BG): $p^*(D^*) > 2.5(\Upsilon(4S))$ / $3.1(\Upsilon(5S))$ GeV/c.
- multi-candidates using best candidate selection(BCS)
  - via the sum of D^0 and D^* vertex fitting qualities, eg: $D^0 \to K^-\pi^+$
  - via $\sum \chi^2$ of mass difference, eg: $D^0 \to K_S^0\pi^0\eta$
- time-dept. analyses: D^0 lifetime t_{D^0} and σ_t

$$t_{D^0} = \frac{m_{D^0}}{cp}(\vec{r_{dec}} - \vec{r_{IP}}) \cdot \frac{\vec{p}}{p}$$

$$\sigma_t^2 = \left(\frac{\partial t}{\partial \eta}\right)^T V_\eta \left(\frac{\partial t}{\partial \eta}\right)$$

$$V_\eta = \begin{pmatrix} V_{dec} & cov(dec, IP) & cov(dec, p) \\ cov(IP, dec) & V_{IP} & cov(IP, p) \\ cov(p, dec) & cov(p, IP) & V_p \end{pmatrix}$$



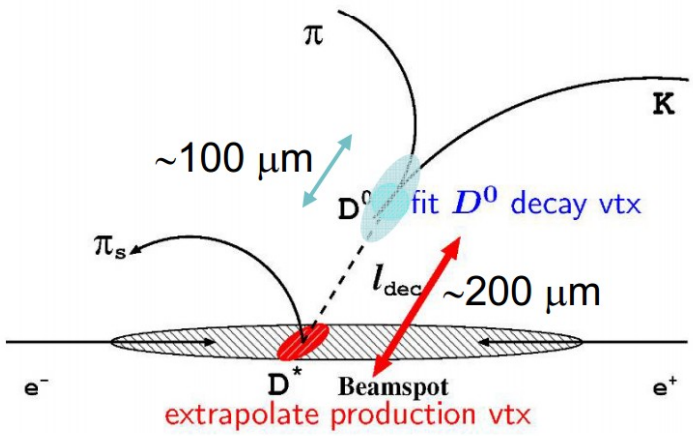
Analysis techniques at B-factories

- D^* inclusive sample: $e^+e^- \to \gamma^* \to c\bar{c} \to D^{*\pm}X^{\mp}$, $D^{*\pm} \to D^0\pi_s^+ / \bar{D}^0\pi_s^-$.
- veto signals from B decays and suppress background(BG): $p^*(D^*) > 2.5(\Upsilon(4S))$ / $3.1(\Upsilon(5S))$ GeV/c.
- multi-candidates using best candidate selection(BCS)
  - via the sum of D^0 and D^* vertex fitting qualities, eg: $D^0 \to K^-\pi^+$
  - via $\sum \chi^2$ of mass difference, eg: $D^0 \to K_S^0\pi^0\eta$
- time-dept. analyses: D^0 lifetime t_{D^0} and σ_t

$$t_{D^0} = \frac{m_{D^0}}{cp}(\vec{r_{dec}} - \vec{r_{IP}}) \cdot \frac{\vec{p}}{p}$$

$$\sigma_t^2 = \left(\frac{\partial t}{\partial \eta}\right)^T V_\eta \left(\frac{\partial t}{\partial \eta}\right)$$

$$V_\eta = \begin{pmatrix} V_{dec} & cov(dec, IP) & cov(dec, p) \\ cov(IP, dec) & V_{IP} & cov(IP, p) \\ cov(p, dec) & cov(p, IP) & V_p \end{pmatrix}$$



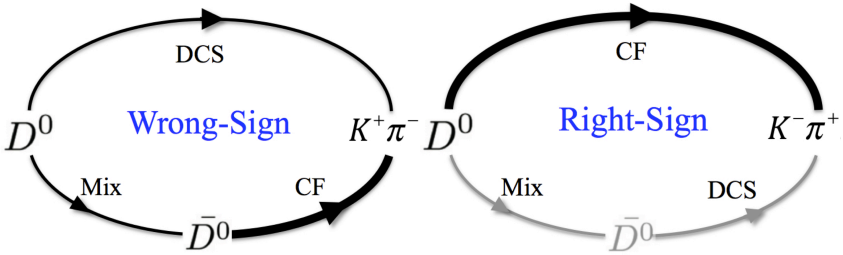
- extract $f_{sig,BG}$: $M = M_{D^0}$ and $Q = M_{D^*} - M_{D^0} - m_{\pi_s}$.

observation of D^0-$\bar{D}^0$ mixing in $D^0 \to K^+\pi^-$ [B.R. Ko *et al.* PRL **112**, 111801 (2014)]

- Time-dependent WS-to-RS decay rate ratio under CP conservation:

$$R_{WS}(t) = R_D + y'\sqrt{R_D}\Gamma t + \frac{x'^2 + y'^2}{4}\Gamma^2 t^2$$

with effective par. $x' = x\cos\delta_{K\pi} + y\sin\delta_{K\pi}$, $y' = y\cos\delta_{K\pi} - x\sin\delta_{K\pi}$



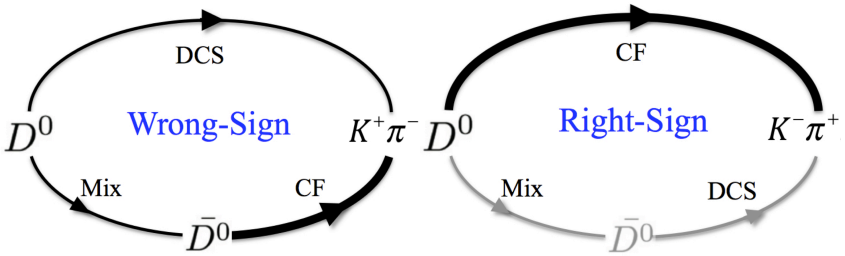
observation of D^0-$\bar{D}^0$ mixing in $D^0 \to K^+\pi^-$ [B.R. Ko *et al.* PRL **112**, 111801 (2014)]

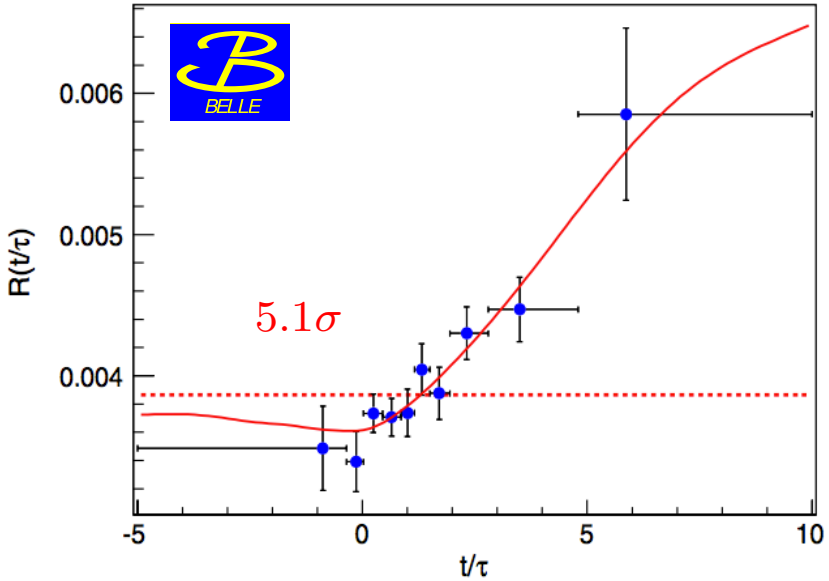
- Time-dependent WS-to-RS decay rate ratio under CP conservation:

$$R_{WS}(t) = R_D + y'\sqrt{R_D}\Gamma t + \frac{x'^2 + y'^2}{4}\Gamma^2 t^2$$

with effective par. $x' = x\cos\delta_{K\pi} + y\sin\delta_{K\pi}$, $y' = y\cos\delta_{K\pi} - x\sin\delta_{K\pi}$

- based on 976 fb^{-1} data
  - R_D=(0.353±0.013(stat+syst))%
  - x'^2=(0.09±0.22(stat+syst))×10^{-3}
  - y'=(4.6±3.4(stat+syst))×10^{-3}
  - (x'^2, y') with correlation -0.948
- first observation in e^+e^- collisions
- Belle II (50 ab^{-1}) estimation
  - scaled luminosity: $\sigma_{x'^2} = 0.09 \times 10^{-3}$, $\sigma_{y'} = 0.16\%$
  - ToyMC with improved $\sigma_t = 140$ *fs* $\sigma_{x'^2} = 0.044 \times 10^{-3}$, $\sigma_{y'} = 0.047\%$.





evidence of D^0-$\bar{D}^0$ mixing in $D^0 \to h^+h^-$ [M. Staric *et al.* PLB **753**, 412 (2016)]

- using CP eigenstates D^0 lifetime analysis relative to non-CP eigenstates.

$$y_{cp} = \frac{\tau_{K\pi}}{<\tau_{hh}>} - 1 \quad (h = K/\pi)$$

$$A_\Gamma = \frac{\tau(\bar{D}^0 \to h^-h^+) - \tau(D^0 \to h^+h^-)}{\tau(\bar{D}^0 \to h^-h^+) + \tau(D^0 \to h^+h^-)}$$

evidence of D^0-$\bar{D}^0$ mixing in $D^0 \to h^+h^-$ [M. Staric *et al.* PLB **753**, 412 (2016)]

- using CP eigenstates D^0 lifetime analysis relative to non-CP eigenstates.

$$y_{cp} = \frac{\tau_{K\pi}}{<\tau_{hh}>} - 1 \quad (h = K/\pi)$$

$$A_\Gamma = \frac{\tau(\bar{D}^0 \to h^-h^+) - \tau(D^0 \to h^+h^-)}{\tau(\bar{D}^0 \to h^-h^+) + \tau(D^0 \to h^+h^-)}$$

- with full dataset and tagged D^0 flavor by charge of π_s from D^*
- twice data (976 fb^{-1}) than first evidence result based on 540 fb^{-1} data:
  - (Belle 2007) $y_{CP} = (+1.31 \pm 0.32 \pm 0.25)\%$ [M. Staric *et al.* PRL **98**, 211801 (2007)]

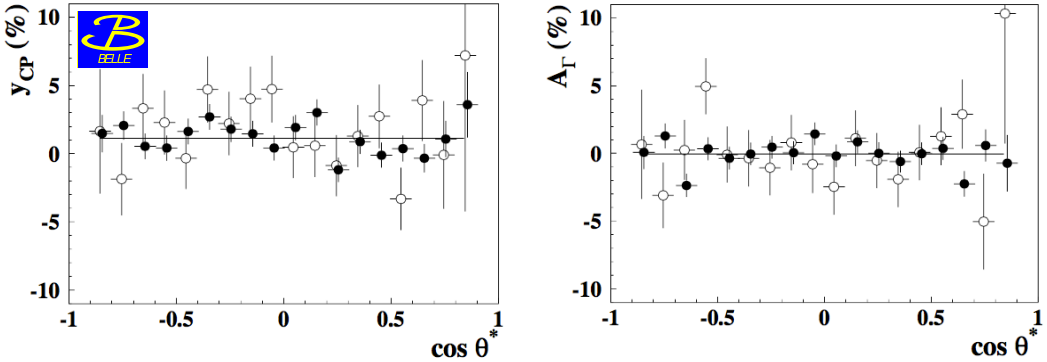
evidence of D^0-$\bar{D}^0$ mixing in $D^0 \to h^+h^-$ [M. Staric *et al.* PLB **753**, 412 (2016)]

- using CP eigenstates D^0 lifetime analysis relative to non-CP eigenstates.

$$y_{cp} = \frac{\tau_{K\pi}}{<\tau_{hh}>} - 1 \quad (h = K/\pi)$$

$$A_\Gamma = \frac{\tau(\bar{D}^0 \to h^-h^+) - \tau(D^0 \to h^+h^-)}{\tau(\bar{D}^0 \to h^-h^+) + \tau(D^0 \to h^+h^-)}$$

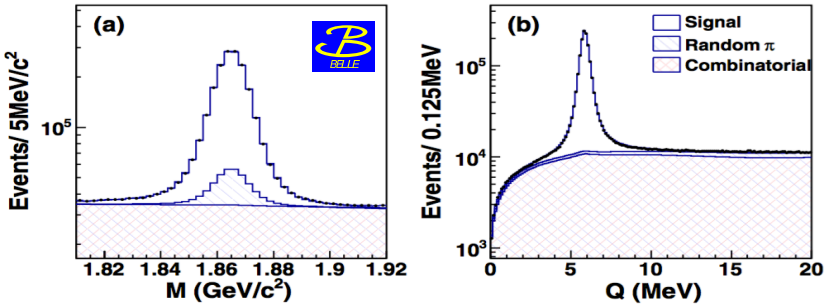
- with full dataset and tagged D^0 flavor by charge of π_s from D^*
- twice data (976 fb^{-1}) than first evidence result based on 540 fb^{-1} data:
  - (Belle 2007) $y_{CP} = (+1.31 \pm 0.32 \pm 0.25)\%$ [M. Staric *et al.* PRL **98**, 211801 (2007)]
- asymmetric time resolution function depends on D^* polar angle in CMS
- different configurations for SVD
  - y_{CP}=[1.11±0.22±0.09]%(4.7σ)
  - A_Γ=[-0.03±0.20±0.07]%
- Belle II: $\sigma_{y_{CP}} \approx 0.06\%$, $\sigma_{A_\Gamma} \approx 0.04\%$.



open(full) circles for 3(4)-layer SVD

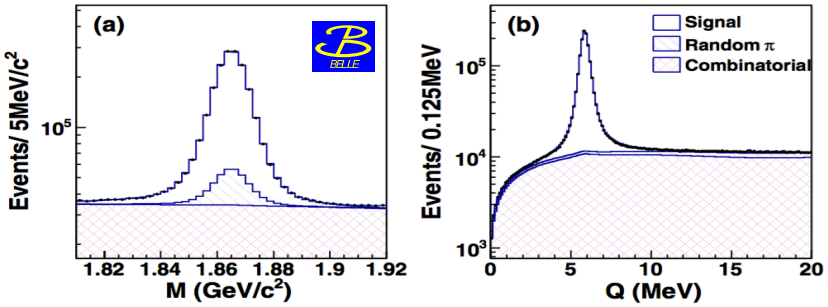
TDDA in Self-conjugated decay $D^0 \to K_S^0\pi^+\pi^-$ at Belle [PRD **89**, 091103(R)]

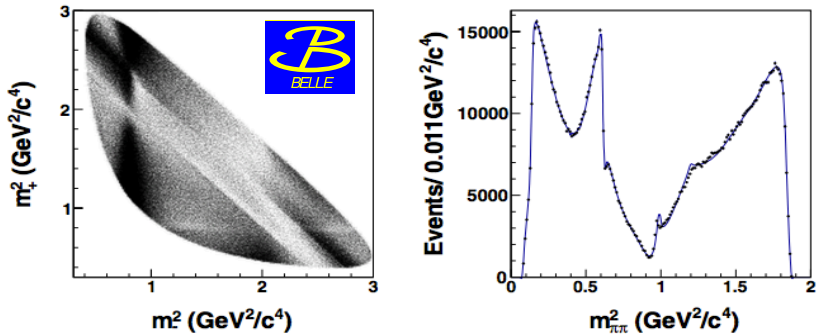
- 1.29M events with a purity of 95.6%.



TDDA in Self-conjugated decay $D^0 \to K_S^0\pi^+\pi^-$ at Belle [PRD **89**, 091103(R)]

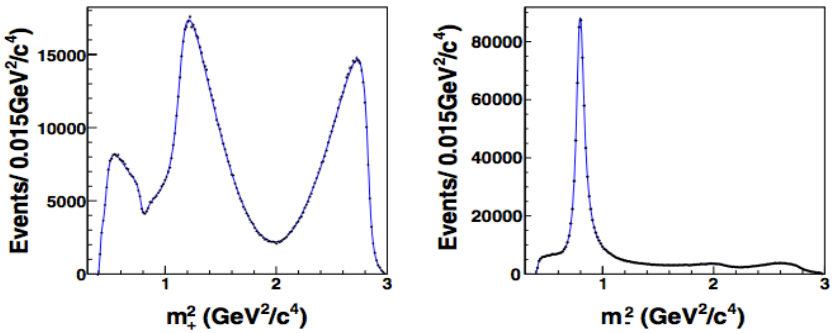
- 1.29M events with a purity of 95.6%.





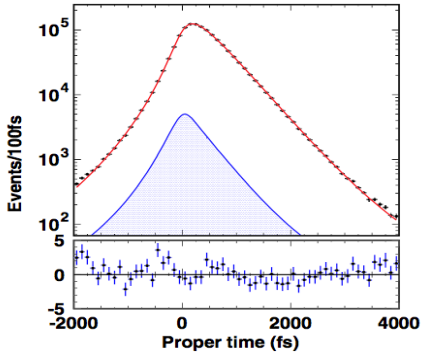
- Dalitz model: RBW(12 res.) + K-matrix($\pi\pi$ S-wave) + LASS($K\pi$ S-wave).
- with CP violation allowed

$$|\mathcal{M}(f,t)|^2 = \frac{e^{-\Gamma t}}{2}[(|\mathcal{A}_f|^2 + |\frac{q}{p}|^2|\mathcal{A}_{\bar{f}}|^2)\cosh(y\Gamma t)$$
$$+ (|\mathcal{A}_f|^2 - |\frac{q}{p}|^2|\mathcal{A}_{\bar{f}}|^2)\cos(x\Gamma t)$$
$$+ 2\Re[\frac{q}{p}\mathcal{A}_{\bar{f}}\mathcal{A}_f^*]\sinh(y\Gamma t) + 2\Im[\frac{q}{p}\mathcal{A}_{\bar{f}}\mathcal{A}_f^*]\sin(x\Gamma t)]$$

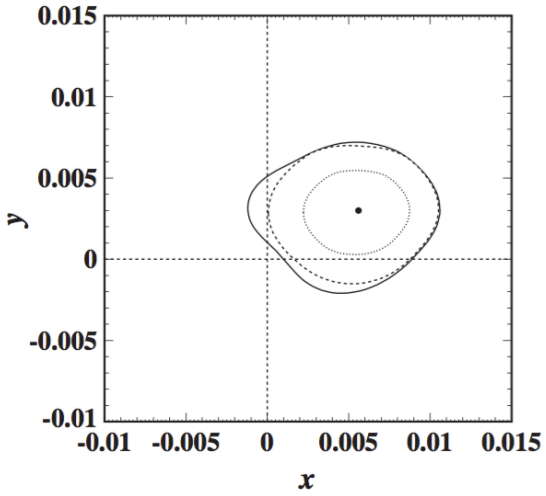


- if no CPV, D^0 and $\bar{D}^0$ have same formalism:

$$|\mathcal{M}(m_{12}^2, m_{13}^2, t)|^2 = [|\mathcal{A}_1|^2 e^{-y\Gamma t} + 2\Re[\mathcal{A}_1\mathcal{A}_2^*]\cos(x\Gamma t)$$
$$+ 2\Im[\mathcal{A}_1\mathcal{A}_2^*]\sin(x\Gamma t) + |\mathcal{A}_2|^2 e^{yt}]e^{-\Gamma t}$$



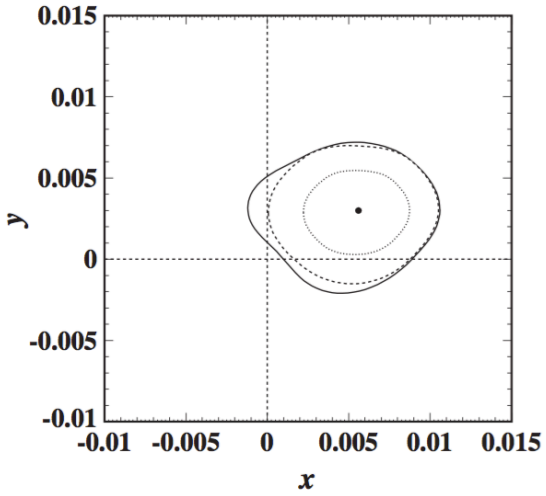
TDDA in Self-conjugated decay $D^0 \to K_S^0\pi^+\pi^-$ at Belle [PRD **89**, 091103(R)]



Observables	Belle (2014)	Belle II* 5 ab^{-1}	Belle II* 50 ab^{-1}	$\mathcal{L}_s^\dagger$ [ab^{-1}]	Year
x(%)	$0.56 \pm 0.19^{+0.07}_{-0.13}$	± 0.14	± 0.11	3	2019
y(%)	$0.30 \pm 0.15^{+0.05}_{-0.08}$	± 0.08	± 0.05	15	2021
$\lvert q/p \rvert$	$0.90^{+0.16+0.08}_{-0.15-0.06}$	± 0.10	± 0.07	5-6	2019
$\phi(^o)$	$-6 \pm 11^{+4}_{-5}$	± 6	± 4	10	2020

*refer to BELLE2-NOTE-0021, BELLE2-NOTE-PH-2015-002.
†$\mathcal{L}_s$ denotes the approximate integrated luminosity at which $\sigma_{stat.} \approx \sigma_{syst.}$.

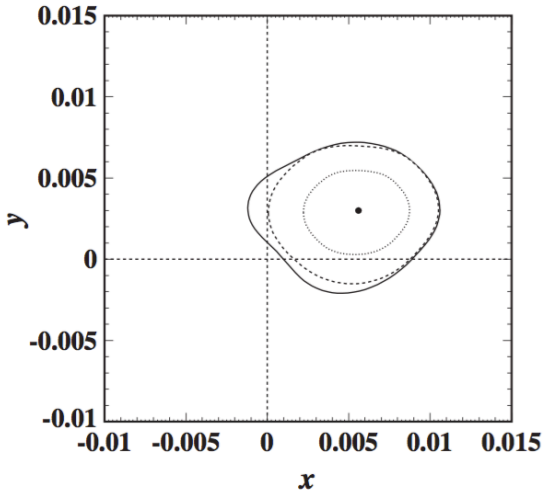
TDDA in Self-conjugated decay $D^0 \to K_S^0\pi^+\pi^-$ at Belle [PRD **89**, 091103(R)]



Observables	Belle (2014)	Belle II* 5 ab^{-1}	Belle II* 50 ab^{-1}	$\mathcal{L}_s^\dagger$ [ab^{-1}]	Year
$x(\%)$	$0.56 \pm 0.19^{+0.07}_{-0.13}$	± 0.14	± 0.11	3	2019
$y(\%)$	$0.30 \pm 0.15^{+0.05}_{-0.08}$	± 0.08	± 0.05	15	2021
$\lvert q/p \rvert$	$0.90^{+0.16+0.08}_{-0.15-0.06}$	± 0.10	± 0.07	5-6	2019
$\phi(^o)$	$-6 \pm 11^{+4}_{-5}$	± 6	± 4	10	2020

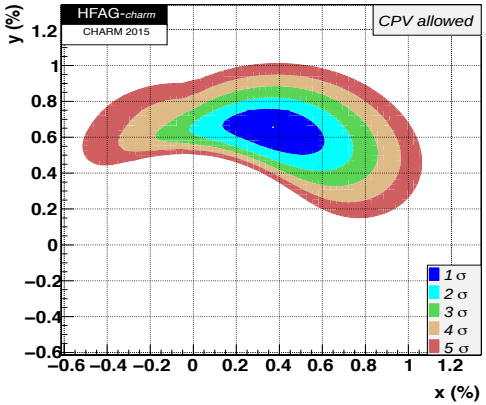
*refer to BELLE2-NOTE-0021, BELLE2-NOTE-PH-2015-002.
$^\dagger\mathcal{L}_s$ denotes the approximate integrated luminosity at which $\sigma_{stat.} \approx \sigma_{syst.}$.

TDDA in Self-conjugated decay $D^0 \to K_S^0\pi^+\pi^-$ at Belle [PRD **89**, 091103(R)]

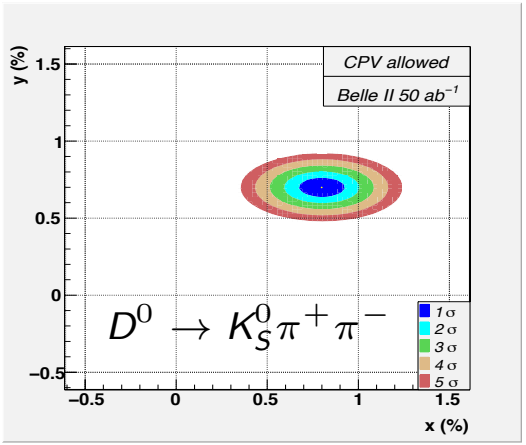


Observables	Belle (2014)	Belle II* 5 ab^{-1}	Belle II* 50 ab^{-1}	$\mathcal{L}_s^\dagger$ [ab^{-1}]	Year
$x(\%)$	$0.56 \pm 0.19^{+0.07}_{-0.13}$	± 0.14	± 0.11	3	2019
$y(\%)$	$0.30 \pm 0.15^{+0.05}_{-0.08}$	± 0.08	± 0.05	15	2021
$\lvert q/p \rvert$	$0.90^{+0.16+0.08}_{-0.15-0.06}$	± 0.10	± 0.07	5-6	2019
$\phi(^o)$	$-6 \pm 11^{+4}_{-5}$	± 6	± 4	10	2020

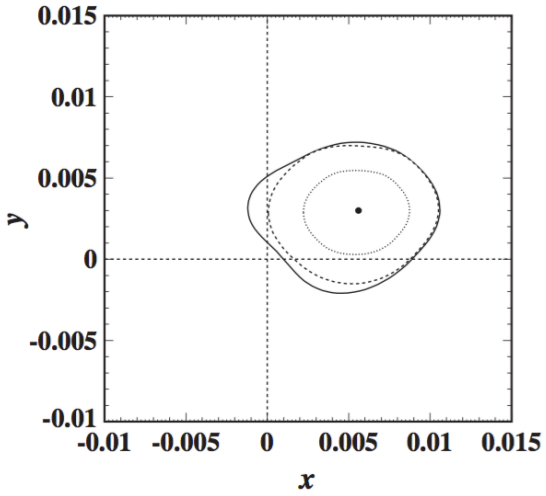
*refer to BELLE2-NOTE-0021, BELLE2-NOTE-PH-2015-002.
$^\dagger\mathcal{L}_s$ denotes the approximate integrated luminosity at which $\sigma_{stat.} \approx \sigma_{syst.}$.



$\Rightarrow$

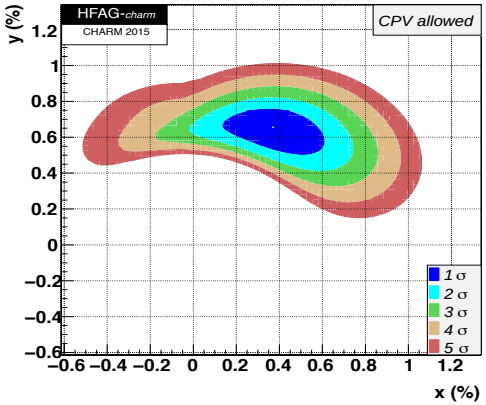


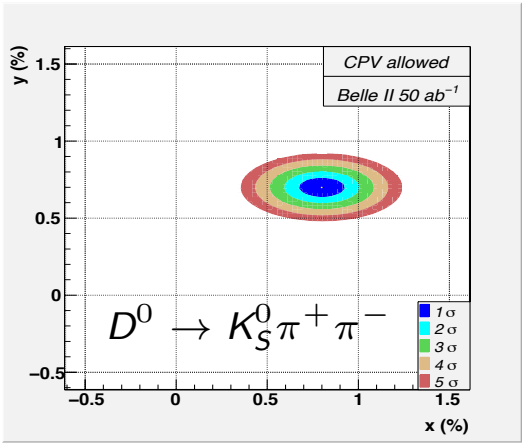
TDDA in Self-conjugated decay $D^0 \to K_S^0\pi^+\pi^-$ at Belle [PRD **89**, 091103(R)]

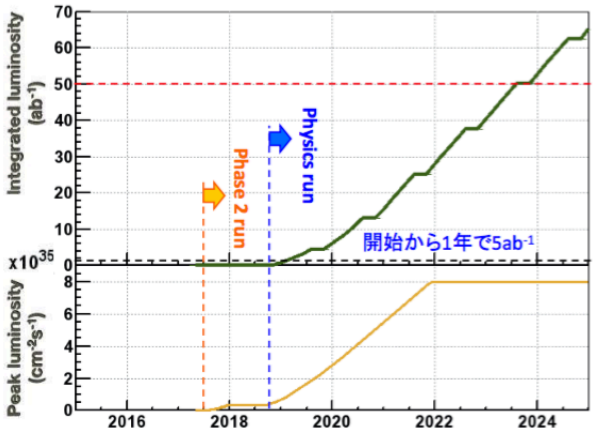


Observables	Belle (2014)	Belle II* 5 ab^{-1}	Belle II* 50 ab^{-1}	$\mathcal{L}_s^\dagger$ [ab^{-1}]	Year
$x(\%)$	$0.56 \pm 0.19^{+0.07}_{-0.13}$	± 0.14	± 0.11	3	2019
$y(\%)$	$0.30 \pm 0.15^{+0.05}_{-0.08}$	± 0.08	± 0.05	15	2021
$\lvert q/p \rvert$	$0.90^{+0.16+0.08}_{-0.15-0.06}$	± 0.10	± 0.07	5-6	2019
$\phi(^o)$	$-6 \pm 11^{+4}_{-5}$	± 6	± 4	10	2020

*refer to BELLE2-NOTE-0021, BELLE2-NOTE-PH-2015-002.
$^\dagger\mathcal{L}_s$ denotes the approximate integrated luminosity at which $\sigma_{stat.} \approx \sigma_{syst.}$.







Outline

search for CPV in SCS decay: $D^0 \to \pi^0\pi^0$ [N.K. Nisar *et al.* PRL **112**, 211601 (2014)]

Measured asymmetry A_{rec} and obtain A_{CP} based on 966 fb^{-1} of Belle data

$$A_{rec} = \frac{N_{rec}^{D^{*+}\to D^0\pi_s^+} - N_{rec}^{D^{*-}\to \bar{D}^0\pi_s^-}}{N_{rec}^{D^{*+}\to D^0\pi_s^+} + N_{rec}^{D^{*-}\to \bar{D}^0\pi_s^-}}$$

$$A_{rec}^{cor} = A_{rec} - A_{\epsilon}^{\pi_s} = A_{CP} + A_{FB}(\cos\theta^*)$$

$$A_{CP/FB} = [A_{rec}^{cor}(\cos\theta^*) \pm A_{rec}^{cor}(-\cos\theta^*)]/2$$

A_{CP}: indept of all kinematic variables;

A_{FB}: due to $\gamma - Z^0$ interference; an odd function of $\cos\theta^*$ in C.M. frame.

$A_{\epsilon}^{\pi_s}$: indept of final state; subtract from control data samples $D^0 \to K^-\pi^+$

search for CPV in SCS decay: $D^0 \to \pi^0\pi^0$ [N.K. Nisar *et al.* PRL **112**, 211601 (2014)]

Measured asymmetry A_{rec} and obtain A_{CP} based on 966 fb^{-1} of Belle data

$$A_{rec} = \frac{N_{rec}^{D^{*+}\to D^0\pi_s^+} - N_{rec}^{D^{*-}\to \bar{D}^0\pi_s^-}}{N_{rec}^{D^{*+}\to D^0\pi_s^+} + N_{rec}^{D^{*-}\to \bar{D}^0\pi_s^-}}$$

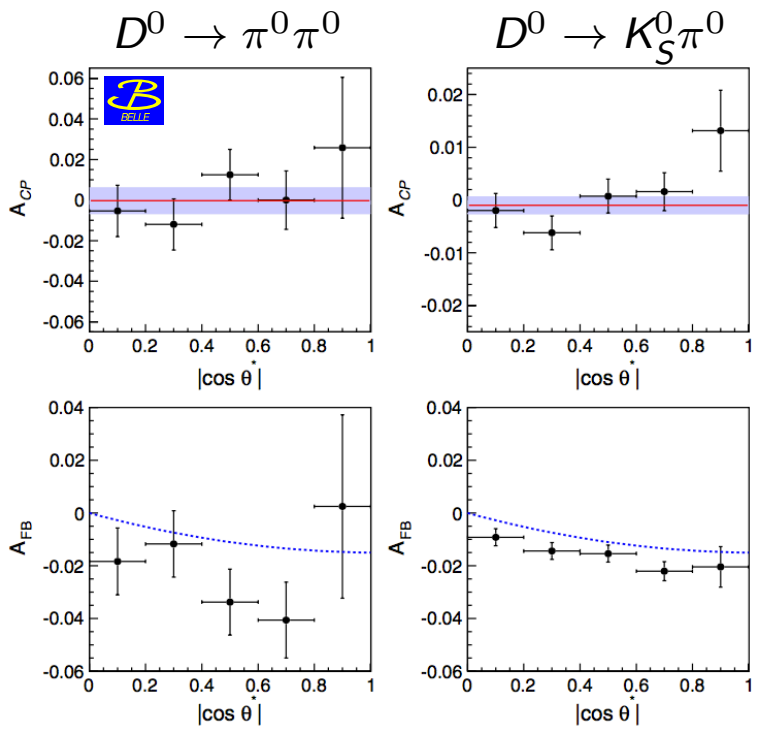
$$A_{rec}^{cor} = A_{rec} - A_\epsilon^{\pi_s} = A_{CP} + A_{FB}(\cos\theta^*)$$

$$A_{CP/FB} = [A_{rec}^{cor}(\cos\theta^*) \pm A_{rec}^{cor}(-\cos\theta^*)]/2$$

A_{CP}: indept of all kinematic variables;

A_{FB}: due to $\gamma - Z^0$ interference; an odd function of $\cos\theta^*$ in C.M. frame.

$A_\epsilon^{\pi_s}$: indept of final state; subtract from control data samples $D^0 \to K^-\pi^+$

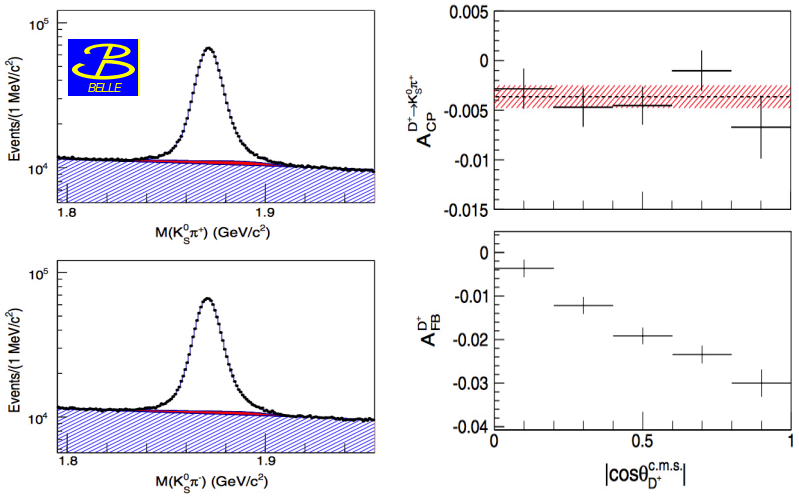


- $D^0(\bar{D}^0)$ yields from simultaneous fit to ΔM in 3D bins of $(\cos\theta^*, p_T^{\pi_s}, \cos\theta^{\pi_s})(10\times7\times8)$.
- $A_{CP}(D^0 \to \pi^0\pi^0) = (-0.03 \pm 0.64 \pm 0.10)\%$
  (1) an order of magnitude improvement.
  (2) no evidence for CP violation.
- $A_{CP}(D^0 \to K_S^0\pi^0) = (-0.21 \pm 0.16 \pm 0.07)\%$
- sensitivity estimation at Belle II:
  - 5 ab^{-1}: $\sigma(A_{CP}^{\pi^0\pi^0}) = 0.29\%$, $\sigma(A_{CP}^{K_S^0\pi^0}) = 0.08\%$.
  - 50 ab^{-1}: $\sigma(A_{CP}^{\pi^0\pi^0}) = 0.09\%$, $\sigma(A_{CP}^{K_S^0\pi^0}) = 0.03\%$.

direct CP violation in $D^+_{(s)}$ decays (no new measurement)

review A_{CP} in $D^+ \to K^0_S\pi^+$ [PRL **109**, 119903 (2012)]

- $A_{raw} = A_{CP} + A_{FB} + A^{\pi}_{\epsilon} + A^{K^0}_{mat}$
- Pion and K^0-material asymmetries corrected by event weighting
- in $\cos\theta^*_{D^+}$ bins using simultaneous fit to $M(K^0_S\pi^\pm)$
- $A_{CP}^{K^0_S\pi^+}$ =(-0.363±0.094±0.067)% (3.2σ)
- consistent with expected CPV due to K^0-mixing (-0.345±0.008)%



direct CP violation in $D^+_{(s)}$ decays (no new measurement)

review A_{CP} in $D^+ \to K^0_S\pi^+$ [PRL **109**, 119903 (2012)]

- $A_{raw} = A_{CP} + A_{FB} + A^{\pi}_{\epsilon} + A^{K^0}_{mat}$
- Pion and K^0-material asymmetries corrected by event weighting
- in $\cos\theta^*_{D^+}$ bins using simultaneous fit to $M(K^0_S\pi^\pm)$
- $A_{CP}^{K^0_S\pi^+}$ =(-0.363±0.094±0.067)% (3.2σ)
- consistent with expected CPV due to K^0-mixing (-0.345±0.008)%

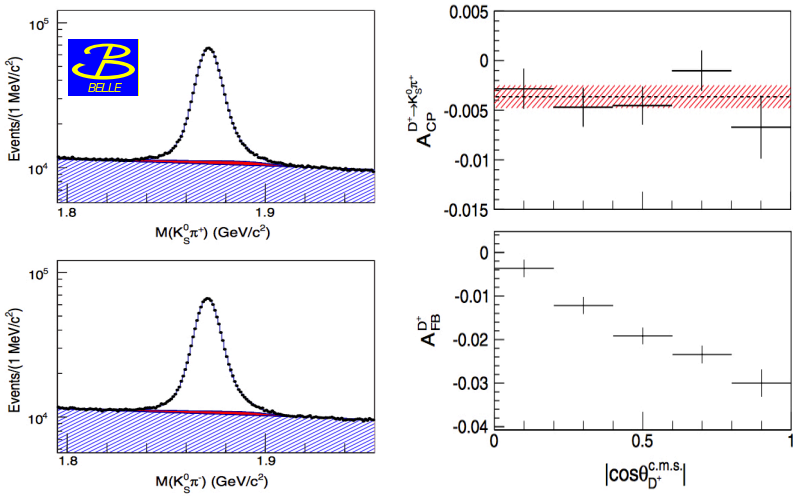


Table: A_{CP} measurements at Belle and sensitivity estimation at Belle II at 50 ab^{-1}.

meson	final	$\mathcal{L}$(fb^{-1})	A_{CP}(%)	Belle II(%)	references
$D^+ \to$	$\phi\pi^+$	955	$+0.51 \pm 0.28 \pm 0.05$	± 0.04	PRL **108**, 071801 (2012)
	$\eta\pi^+$	791	$+1.74 \pm 1.13 \pm 0.19$	± 0.14	PRL **107**, 221801 (2011)
	$\eta'\pi^+$	791	$-0.12 \pm 1.12 \pm 0.17$	± 0.14	PRL **107**, 221801 (2011)
	$K^0_S\pi^+$	977	$-0.36 \pm 0.09 \pm 0.07$	± 0.03	PRL **109**, 021601 (2012)
	$K^0_S K^+$	977	$-0.25 \pm 0.28 \pm 0.14$	± 0.05	JHEP **02**, 98 (2013)
$D^+_s \to$	$K^0_S\pi^+$	673	$+5.45 \pm 2.50 \pm 0.33$	± 0.29	PRL **104**, 181602 (2010)
	$K^0_S K^+$	673	$+0.12 \pm 0.36 \pm 0.22$	± 0.05	PRL **104**, 181602 (2010)

$$\sigma_{Belle\ II} = \sqrt{(\sigma^2_{stat} + \sigma^2_{sys})\frac{\mathcal{L}_{Belle}}{50\ ab^{-1}} + \sigma^2_{inred}}$$

Outline

Summary and Prospect

D^0-$\bar{D}^0$ mixing and CPV measurement at Belle and prospect at Belle II

- WS decay $D^0 \to K^+\pi^-$: (5.1σ):
  - x'^2=(0.09±0.22(stat+syst))×10^{-3}; y'=(4.6±3.4(stat+syst))×10^{-3}
- CP-eigenstate $D^0 \to h^+h^-$:
  - y_{CP}=(+1.11±0.22±0.09)%(4.7σ); A_Γ=(-0.03±0.20±0.07)%
- self-conjugated decay $D^0 \to K_S^0\pi^+\pi^-$:
  - w/o CPV (2.5σ): $x = (0.56 \pm 0.19^{+0.03+0.06}_{-0.09-0.09})\%$, $y = (0.30 \pm 0.15^{+0.04+0.03}_{-0.05-0.06})\%$
  - w/ CPV: $|q/p| = 0.90^{+0.16+0.05+0.06}_{-0.15-0.04-0.05}$, $\arg(q/p) = (-6 \pm 11 \pm 3^{+3}_{-4})^o$.
- sensitivity estimation at Belle II:
  - $D^0 \to K_S^0\pi^+\pi^-$: $\sigma_x = \pm 0.08\%$, $\sigma_y = \pm 0.05\%$; $\sigma_{|q/p|} = \pm 0.06$, $\sigma_{\arg(q/p)} = \pm 4^o$.
  - $D^0 \to K^+\pi^-\pi^0$ (with improved time resolution): $\sigma_{x'^2} = \pm 0.022\%$, $\sigma_{y'} = \pm 0.34\%$

Summary and Prospect

D^0-$\bar{D}^0$ mixing and CPV measurement at Belle and prospect at Belle II

- WS decay $D^0 \to K^+\pi^-$: (5.1σ):
  - x'^2=(0.09±0.22(stat+syst))$\times 10^{-3}$; y'=(4.6±3.4(stat+syst))$\times 10^{-3}$
- CP-eigenstate $D^0 \to h^+h^-$:
  - y_{CP}=(+1.11±0.22±0.09)%(4.7σ); A_Γ=(-0.03±0.20±0.07)%
- self-conjugated decay $D^0 \to K_S^0\pi^+\pi^-$:
  - w/o CPV (2.5σ): $x = (0.56 \pm 0.19^{+0.03+0.06}_{-0.09-0.09})\%$, $y = (0.30 \pm 0.15^{+0.04+0.03}_{-0.05-0.06})\%$
  - w/ CPV: $|q/p| = 0.90^{+0.16+0.05+0.06}_{-0.15-0.04-0.05}$, $\arg(q/p) = (-6 \pm 11 \pm 3^{+3}_{-4})^o$.
- sensitivity estimation at Belle II:
  - $D^0 \to K_S^0\pi^+\pi^-$: $\sigma_x = \pm 0.08\%$, $\sigma_y = \pm 0.05\%$; $\sigma_{|q/p|} = \pm 0.06$, $\sigma_{\arg(q/p)} = \pm 4^o$.
  - $D^0 \to K^+\pi^-\pi^0$ (with improved time resolution): $\sigma_{x'^2} = \pm 0.022\%$, $\sigma_{y'} = \pm 0.34\%$

direct CP violation in charm decays at Belle and prospect at Belle II

- $D^0 \to \pi^0\pi^0$: A_{CP}=(-0.03±0.64±0.10)%. Belle II: $\sigma(A_{CP}^{\pi^0\pi^0}) = 0.09\%$
- $D^+_{(s)}$ decays: almost statistical limited, $A_{CP}^{D^+\to K_S^0\pi^+} = (-0.363 \pm 0.094 \pm 0.067)\%$ (3.2σ)
- no evidence for CPV in charm sector.

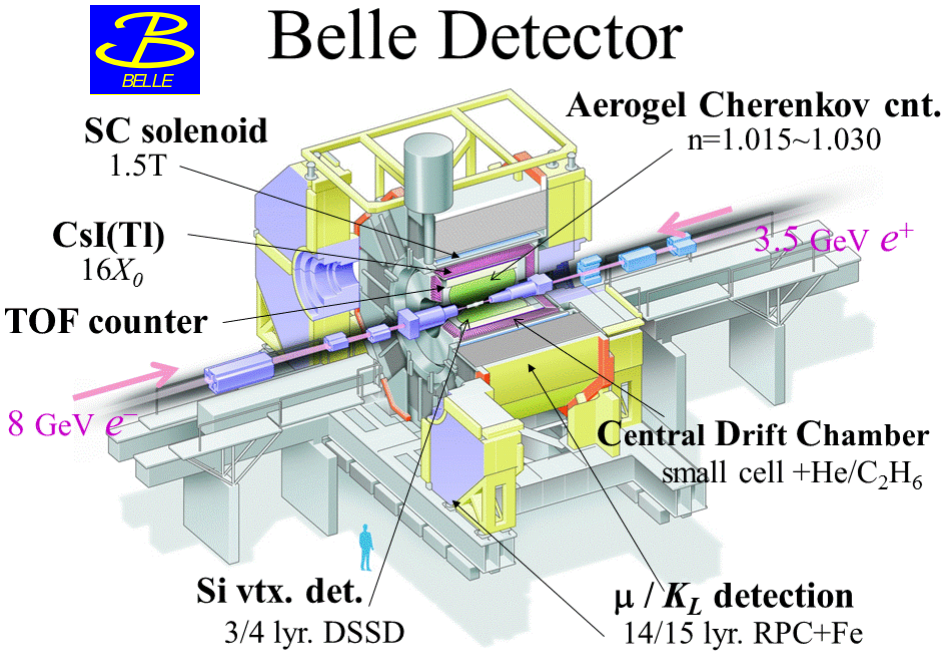
Back up

Thank you for your attention.

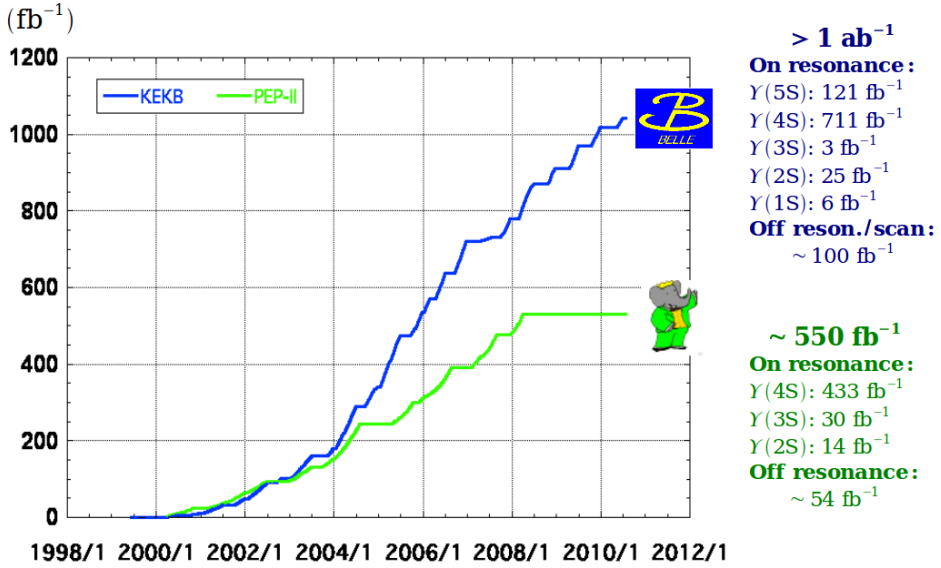
谢谢！

Longke LI (李龙科)
Rm 619, Dept. of Modern Physics
Univ. of Science and Technology of China (USTC)
No. 96, Jinzhai Road, Hefei City, 230026, P. R. China
Tel: (+86)-159-5693-4447
E-mail: lilongke@mail.ustc.edu.cn

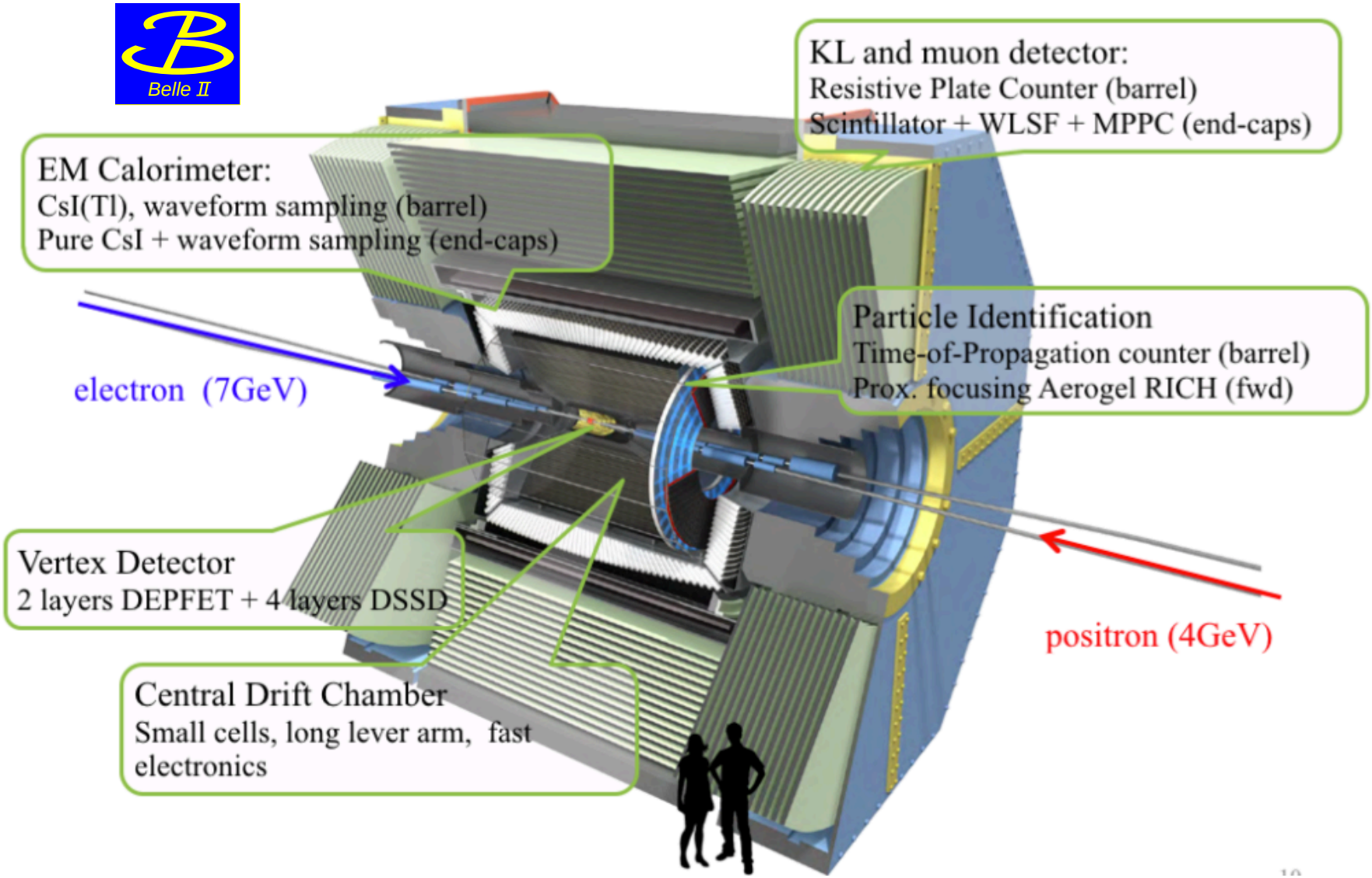
Outline



Integrated luminosity of B factories



- asymmetric-energy e^+e^- collider with 22 mrad cross angle for Belle.
- Belle has high peak luminosity $2.1 \times 10^{34}\ cm^{-2}s^{-1}$
- good momentum/vertex resolution (K/π separation up to 3.5 GeV/c)
- final state with $\gamma/K_S^0/\pi^0$ can be well reconstructed that are difficult/impractical to reconstruct at hadron machine
- low background(BKG), high trigger/rec. efficiencies, minimal decay time bias


Belle II
KL and muon detector:
Resistive Plate Counter (barrel)
Scintillator + WLSF + MPPC (end-caps)
EM Calorimeter:
CsI(Tl), waveform sampling (barrel)
Pure CsI + waveform sampling (end-caps)
Particle Identification
Time-of-Propagation counter (barrel)
Prox. focusing Aerogel RICH (fwd)
electron (7GeV)
Vertex Detector
2 layers DEPFET + 4 layers DSSD
positron (4GeV)
Central Drift Chamber
Small cells, long lever arm, fast
electronics

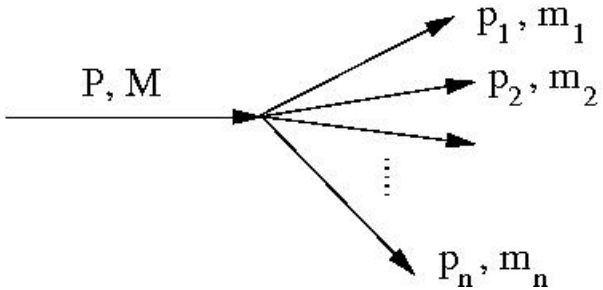
Outline

Dalitz analysis formalism

- R. H. Dalitz (1925-2006), Australian Physicsit, To study "$\tau \to 3\pi$"(kaon) decays. [Published: Philosophical Magazine Series 7, V. 44, Issue 357, Oct. 1953, p1068-1080.]

- Lorentz invariant phase space for n-body decay:

$$d\Phi_n(P; p_1, p_2, p_n) = \delta^4(P - \sum_{i=1}^{n} p_i) \prod_{i=1}^{n} \frac{d^3 p_i}{(2\pi)^3 2E_i}$$



- Degree of freedom:

Decay types	$P \to PPP$	$P \to PPPP$	$P \to VPP$
Examples	$D^0 \to K^-\pi^+\pi^0$	$D^0 \to 4\pi$	$B^0 \to \psi(2S)K^-\pi^+$
4-vectors	3×4	4×4	3×4
E-p const. laws	-4	-4	-4
final state mass	-3	-4	-3
arbitrary rotations	-3	-3	-1(2 vector helicity)
Total d.o.f	2	5	4

Dalitz analysis formalism in 3-body decays

- decays of $P_M \to P_1 P_2 P_3$:

$$m_{12}^2 + m_{13}^2 + m_{23}^2 = M^2 + m_1^2 + m_2^2 + m_3^2 = const.$$

- Standard form of Dalitz plot (DP):

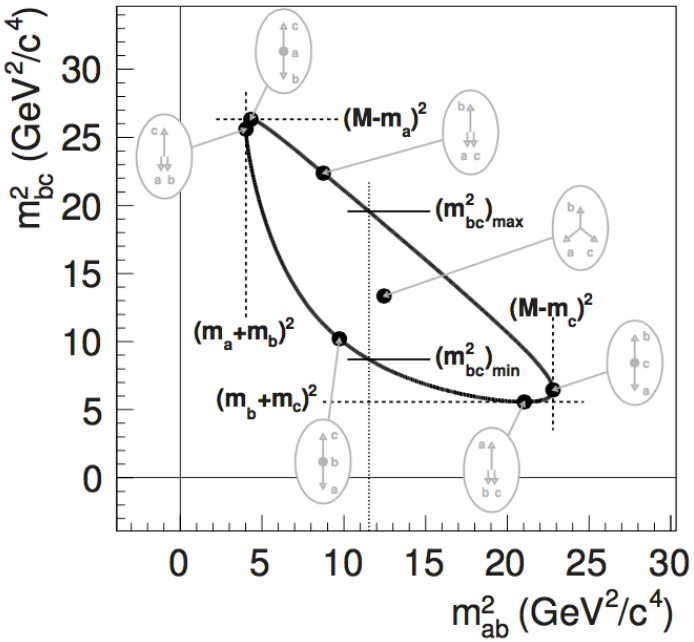
$$d\Gamma = \frac{1}{(2\pi)^3}\frac{1}{32M^3}\overline{|\mathcal{M}|^2}dm_{12}^2 dm_{23}^2$$

- DP kinematic limit: (eg: the form related to $\cos\theta_H$)

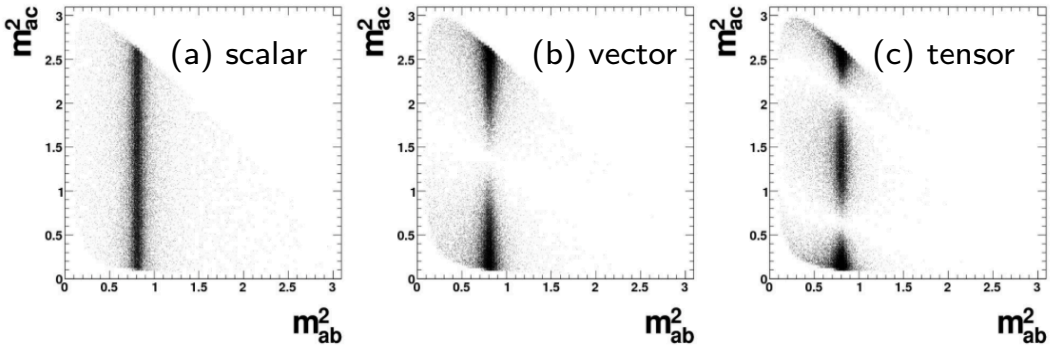
$$m_{23}^2 = (m_{23}^2)_{min}\frac{1+\cos\theta_{hel}}{2} + (m_{23}^2)_{max}\frac{1-\cos\theta_{hel}}{2}$$

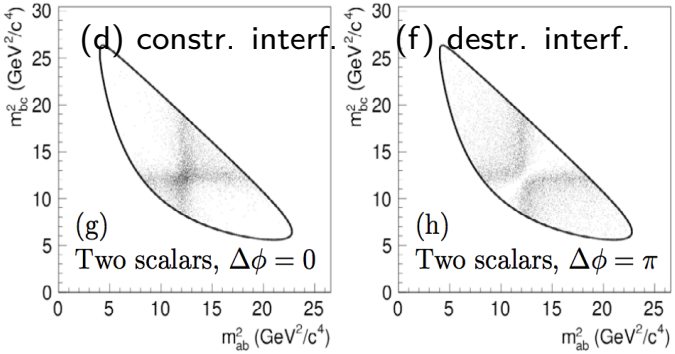
$$\cos\theta_{hel} = -1 \Longrightarrow (m_{23}^2)_{min}$$

$$\cos\theta_{hel} = +1 \Longrightarrow (m_{23}^2)_{max}$$



- DP structure of different spin-J particle





Isobar model for amplitude of 3-body decay

- 3-body decay includes qusai-two-body decays: CF decays, DCS decays, CP decays etc.
- matrix element as a coherent sum of processes where one daughter is spectator.

$$\mathcal{M}(m_{ab}^2, m_{bc}^2) = \sum_r a_r e^{i\phi_r} \mathcal{A}_r(m_{ab}^2, m_{bc}^2) + a_{NR} e^{i\phi_{NR}} \mathcal{A}_{NR}(m_{ab}^2, m_{bc}^2)$$

- amplitude of spin-J resonance: $\mathcal{A}_r = F_D \times F_r \times T_r \times W_r$.

- F_r, F_D: Blatt-Weisskopf Form Factors
  - $F_{J=0} = 1$;
  - $F_{J=1} = \dfrac{\sqrt{1 + R^2 q_r^2}}{\sqrt{1 + R^2 q_{ab}^2}}$
  - $F_{J=2} = \dfrac{\sqrt{9 + 3R^2 p_r^2 + R^4 p_r^4}}{\sqrt{9 + 3R^2 p_{ab}^2 + R^4 p_{ab}^4}}$

  here R is phenomenological factor, R_{D^0}=0 to10 GeV^{-1}, R_r=0 to 3 GeV^{-1}.

- T_r Resonance line-shape
  - narrow resonances: Breit-Wigner mode with mass-dependent width.
  - $a_0(980)$ and $f_0(980)$: Flatté mode.
  - $K\pi$ S-wave: LASS mode.
  - $\pi\pi$ S-wave: K-matrix mode;
  - $\pi\pi$ P-wave ρ: Gounaris-Sakurai mode.
  - non-resonance: constant (or exponential)
- W_r angular distribution
  - Zemach tensor formalism,
  - Helicity formalism.

Two forms of angular distribution description in Isobar model

- Zemach tensor form:

$$
\begin{aligned}
W_{J=0}(ABC|r) &= 1. \\
W_{J=1}(ABC|r) &= m_{ac}^2 - m_{bc}^2 + \frac{(M_D^2 - M_c^2)(M_b^2 - M_a^2)}{M_r^2}. \\
W_{J=2}(ABC|r) &= \left[m_{bc}^2 - m_{ac}^2 + \frac{(M_D^2 - M_c^2)(M_a^2 - M_b^2)}{M_r^2} \right]^2 \\
&\quad - \frac{1}{3} \left[m_{ab}^2 - 2M_D^2 - 2M_c^2 + \frac{(M_D^2 - M_c^2)^2}{M_r^2} \right] \times \\
&\quad \left[m_{ab}^2 - 2M_a^2 - 2M_b^2 + \frac{(M_a^2 - M_b^2)^2}{M_r^2} \right].
\end{aligned}
$$

- Helicity form:

$$
\begin{aligned}
W_{J=0}(ABC|r) &= 1. \\
W_{J=1}(ABC|r) &= -2(\vec{p} \cdot \vec{q}) = -2|p||q| \cos\theta_H. \\
W_{J=2}(ABC|r) &= \frac{4}{3} \left[3(\vec{p} \cdot \vec{q})^2 - (|p||q|)^2 \right] = \frac{4}{3} \left(3|p|^2|q|^2 \cos^2\theta_H - 1 \right)
\end{aligned}
$$

DP fitting method: maximum likelihood (ML)

- probability density function of signal:

$$p_{\rm sig}(m^2_{12,i}, m^2_{23,i}) = \frac{|\mathcal{M}(m^2_{12,i}, m^2_{23,i})|^2 \epsilon(m^2_{12,i}, m^2_{23,i})}{\oint_D dm^2_{12} dm^2_{23}\ |\mathcal{M}(m^2_{12}, m^2_{23})|^2 \epsilon(m^2_{12}, m^2_{23})}$$

- efficiency plane ϵ: large signal MC of 3-body decay produced at free PHSP.
- unbinned ML (X=signal and each background):

$$2\ln\mathcal{L} = 2\sum_{i=1}^{n} \ln\left[\sum_X f^i_X p_{\rm X}(m^2_{12,i}, m^2_{23,i})\right].$$

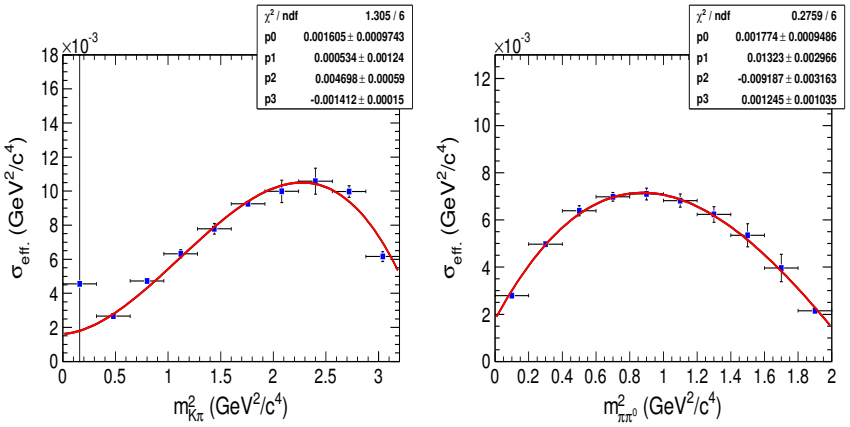
- signal and BG fractions f_X: extracted by M-Q fitting;
- fit fraction of intermediate resonance decays:

$$FF_r = \frac{\oint_{DP} |a_r e^{i\phi_r} \mathcal{M}_r(m^2_{12}, m^2_{23})|^2 dm^2_{12} dm^2_{23}}{\oint_{DP} |\mathcal{M}(m^2_{12}, m^2_{23})|^2 dm^2_{12} dm^2_{23}}$$

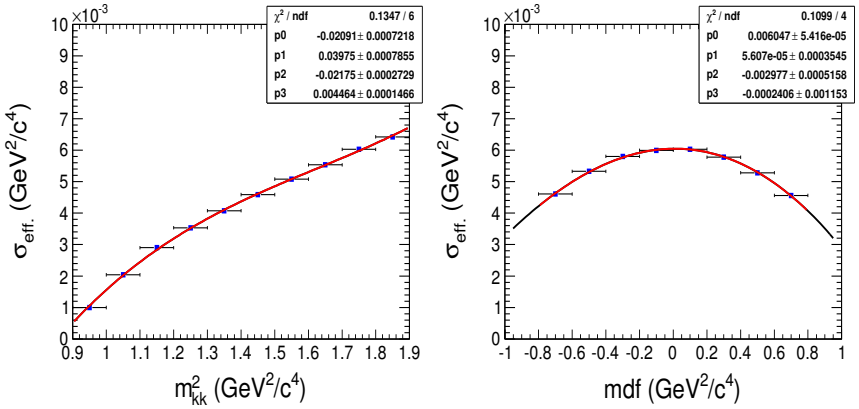
$$\sigma_{FF_{r\neq 1}} = 2\frac{\sigma_{a_r}}{a_r} FF_r; \quad \sigma_{FF_1} = \sqrt{\sum_{i=2}^{n_r} \sigma^2_{FF_i}}; \quad \sigma_{total} = \sqrt{2}\sigma_{FF_1}$$

mass resolution of DP

- in $D^0 \to K^+\pi^-\pi^0$: $\sigma_{DP} \ll 2m_r \cdot \Gamma_r$
  $\Gamma(K^*(892)^0) = 48.7\ MeV/c^2$



- in $D^0 \to K^0_S K^+ K^-$: $\sigma_{DP} \sim 2m_r \cdot \Gamma_r$
  $\Gamma(\phi(1020)) = 4.38\ MeV/c^2$.
  $m^2_{KK} = C - (m^2_+ + m^2_-)$ and $mdf = m^2_+ - m^2_-$.

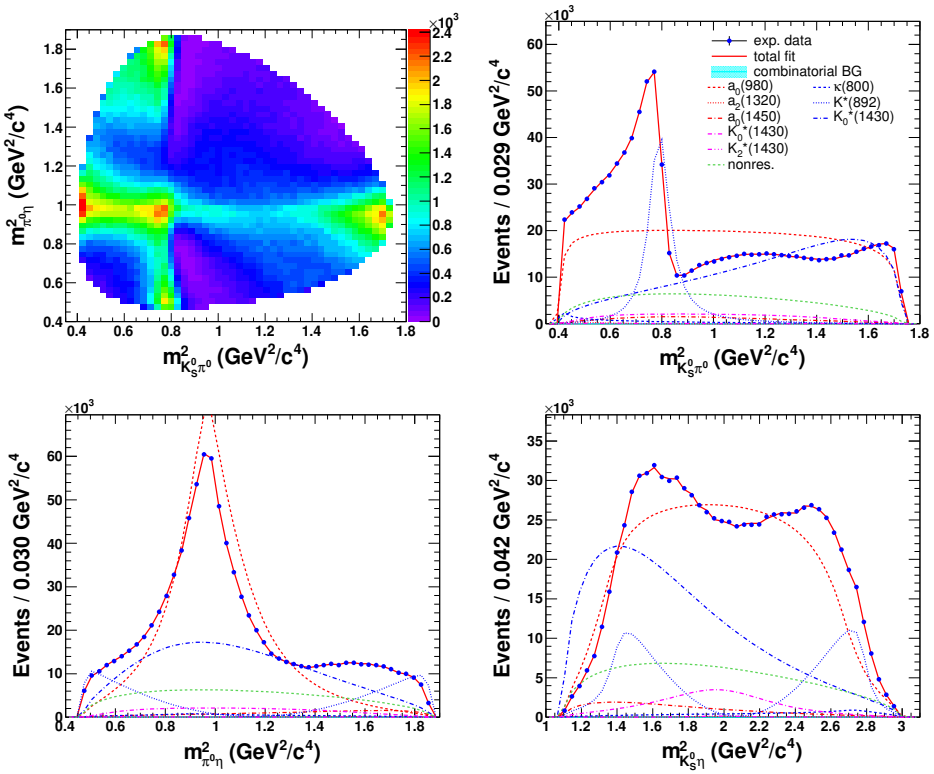


- exactly need a 2-dimensional integral for each event: time consuming
- grid size with Gaussian ($W_{lj} = e^{-(l^2+j^2)/2}$):

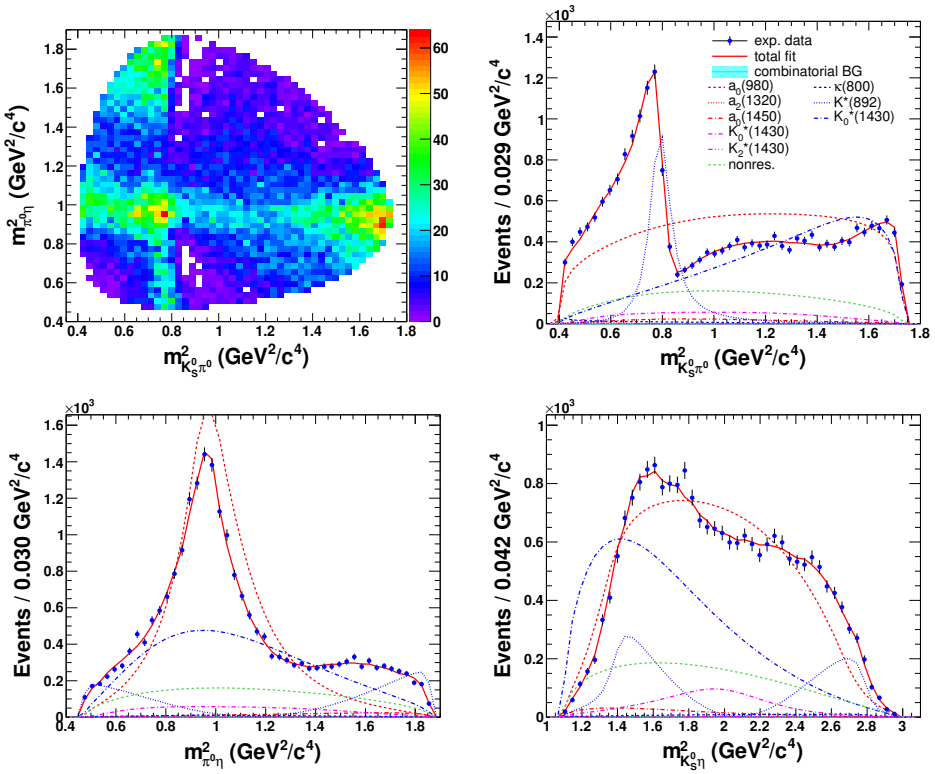
$$pdf_{resol}(x,y) = \sum_{l=-3,j=-3}^{3,3} pdf(x + l\sigma_x, y + j\sigma_y) \cdot W_{lj} / \sum_{l=-3,j=-3}^{3,3} W_{lj}$$

an example of Dalitz fit in $D^0 \to K_S^0 \pi^0 \eta$

- Dalitz plot fitting at generator level:



- Dalitz plot fitting at reconstruction level:



Outline

time-dependent Dalitz analysis (TDDA)

- signal and random BG lifetime:

$$p_{sig}(t;\tau_{D^0}) = \frac{1}{\tau_{D^0}} e^{-t/\tau_{D^0}}$$

- combinatorial BG lifetime:

$$p_{bg} = f_\delta \delta(t) + (1-f_\delta)\frac{1}{\tau_{cmb}} e^{-t/\tau_{cmb}}$$

- time resolution $R(t,\sigma_t)$:

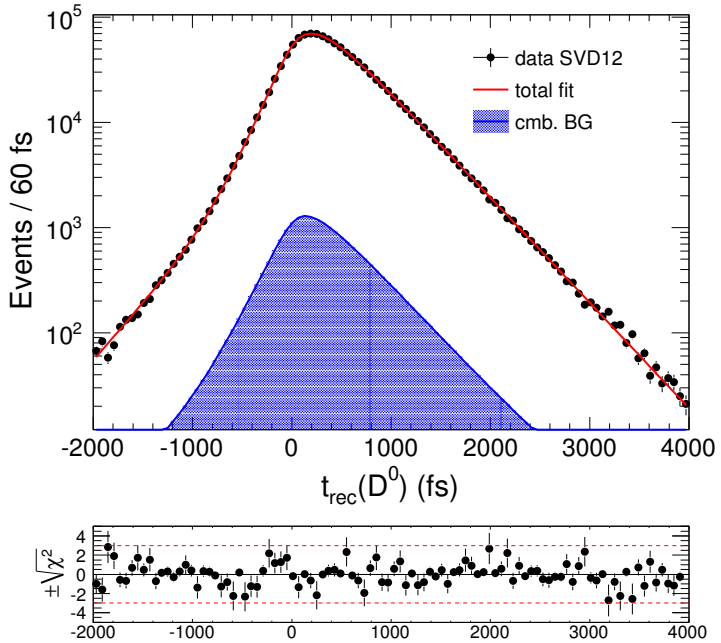
$$\begin{aligned} R(t,\sigma_t) =& f_0 G_2(t;\mu_1\sigma_t,\sigma_1\sigma_t,f_1,\mu_2\sigma_t,\sigma_2\sigma_t) \\ &+ (1-f_0)G_2(t;\mu_3,\sigma_3,f_3,\mu_4,\sigma_4) \end{aligned}$$

- reconstructed D^0 lifetime t

$$P_X(t) = p_X(t) \otimes_t R_X(t,\sigma_t) \ (X = sig, bkg)$$

- lifetime fitting with unbinned ML:

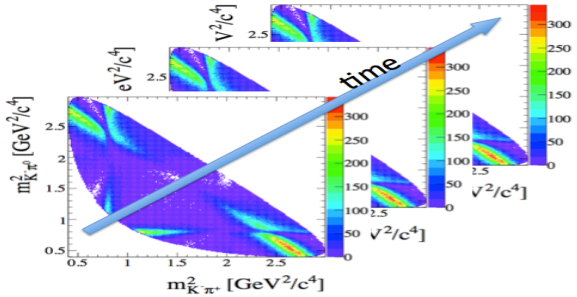
$$\begin{aligned} 2\ln\mathcal{L} =& 2\sum_i \ln(f^i_{sig}\cdot P_{sig}(t_i,\sigma^i_t;\tau_{D^0}) \\ &+ f^i_{bg}\cdot P_{bg}(t_i,\sigma^i_t)) \end{aligned}$$



time-dependent Dalitz amplitude analysis (TDDA)

- time evolution of DP for $D^0 \to f$ ($\lambda_f = \frac{q}{p}\frac{\bar{\mathcal{A}}_f}{\mathcal{A}_f}$):

$$\mathcal{M}^2 = |\mathcal{A}_f|^2 e^{-\Gamma t}[\frac{1+|\lambda_f|^2}{2}\cosh(y\Gamma t) - \Re(\lambda_f)\sinh(y\Gamma t) + \frac{1-|\lambda_f|^2}{2}\cos(x\Gamma t) + \Im(\lambda_f)\sin(x\Gamma t)]$$



- signal TDDP: $p_{\rm sig}(m^2_{12,i}, m^2_{23,i}, t_i, \sigma^i_t) = \frac{\int_0^{+\infty} dt' R_{\rm sig}(t_i - t', \sigma^i_t) \left|\mathcal{M}(m^2_{12,i}, m^2_{23,i}, t')\right|^2 \cdot \epsilon(m^2_{12,i}, m^2_{23,i})}{\int_0^{+\infty} dt \int_D dm^2_{12} dm^2_{23} \ \left|\mathcal{M}(m^2_{12}, m^2_{23}, t)\right|^2 \epsilon(m^2_{12}, m^2_{23})}$
- random BG TDDP:
  - $D^0 \to K^0_S hh$: $\mathcal{M}^2_{\rm rnd} = (1-f_w)|\mathcal{M}(m^2_+, m^2_-, t)|^2\epsilon(m^2_+, m^2_-) + f_w|\overline{\mathcal{M}}(m^2_-, m^2_+, t)|^2\epsilon(m^2_-, m^2_+)$
  - $D^0 \to K^+\pi^-\pi^0$: t-DP 3-dimensional histogram from RS in signal region.
- combinatorial BG TDDP:
  - $D^0 \to K^0_S hh$: DP from sideband $\mathcal{M}(m^2_+, m^2_-) \times p_{cmb}(t)$.
  - $D^0 \to K^+\pi^-\pi^0$: t-DP 3-dimension histogram from WS sideband.
- TDDP fitting with unbinned ML:

$$2\ln\mathcal{L} = 2\sum_i^n \ln(f^i_{\rm sig} p_{\rm sig}(m^2_{12,i}, m^2_{23,i}, t_i, \sigma^i_t; x, y) + \sum_{BG} f^i_{BG} p_{BG}(m^2_{12,i}, m^2_{23,i}, t_i))$$

- avoid the Punzi bias:

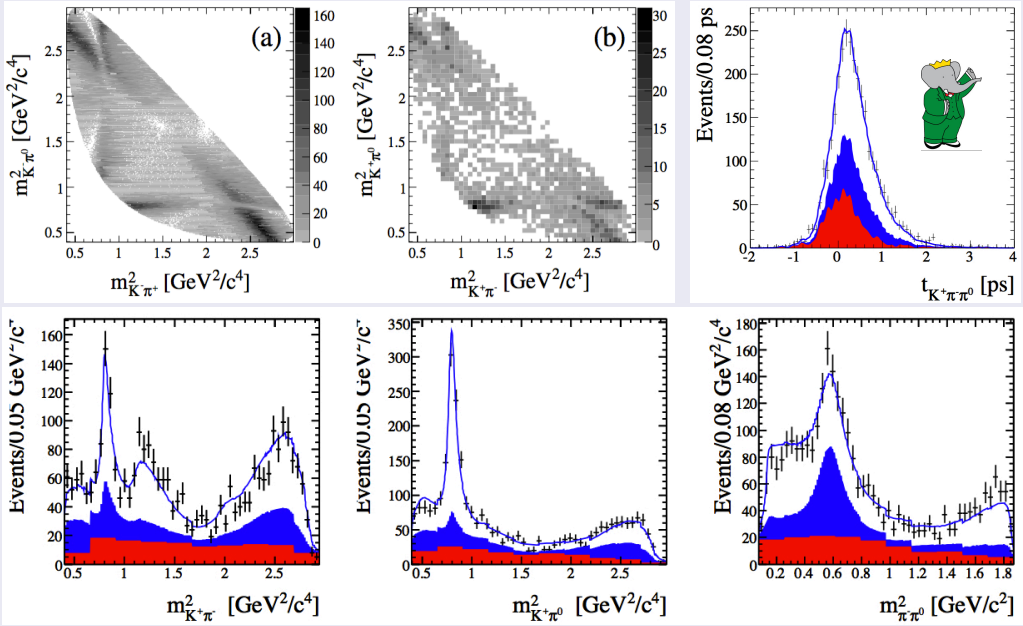
$$2\ln\mathcal{L} = 2\sum_i^n \ln(f^i_{\rm sig} p_{\rm sig}(m^2_{12,i}, m^2_{23,i}, t_i, \sigma^i_t; x, y) p^{nc}_{sig}(\sigma^i_t) + \sum_{BG} f^i_{BG} p_{BG}(m^2_{12,i}, m^2_{23,i}, t_i) p^{nc}_{bg}(\sigma^i_t))$$

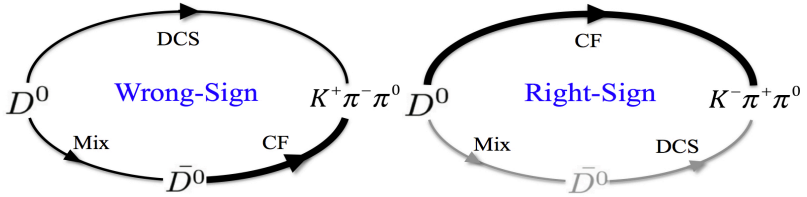
TDDA of WS decay $D^0 \to K^+\pi^-\pi^0$ at BABAR [PRL 103, 211801 (2009)]

TDDP fitting for WS

- RS(WS): 658,986(3009) events with a purity of 99%(50%)
- DCS: 7 res. (the largest fraction of conjugated channels in RS);
- CF: 12 res. in RS TIDA result with all par. fixed.
- Mixing par: ($c_1 = x'/r_0$, $c_2 = y'/r_0$) in

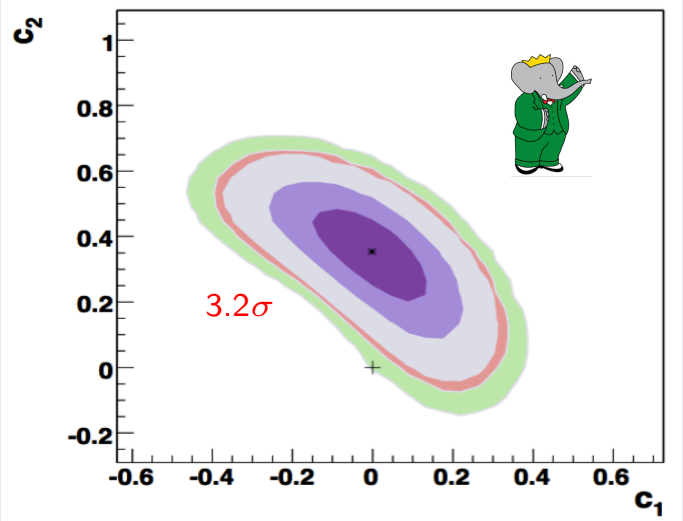
$$\mathcal{M}^2 = \frac{1}{N}[|A_f^{DCS}|^2 + \frac{c_1^2 + c_2^2}{4}|\bar{A}_f^{CF}|^2(\Gamma t)^2 + (c_1\Re(A_f^{*DCS}\bar{A}_f^{CF}) - c_2\Im(A_f^{*DCS}\bar{A}_f^{CF}))\Gamma t]e^{-\Gamma t}$$





mixing contour

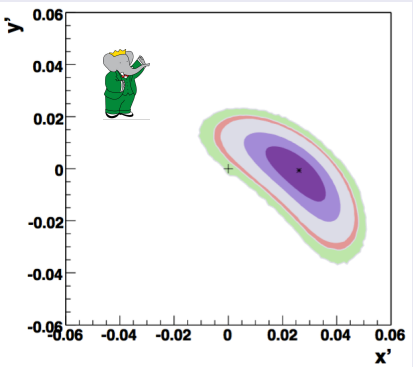
- ToyMC studies to correct the uncertainty bias: $+0.08\sigma_{c_2}$.
- $c_1 = -0.002 \pm 0.090 \pm 0.059$, $c_2 = +0.353 \pm 0.091 \pm 0.052$.
- significance: $-2\Delta \ln \mathcal{L} = 13.5\ (3.2\sigma)$.



TDDA of WS decay $D^0 \to K^+\pi^-\pi^0$ at BABAR [PRL **103**, 211801 (2009)]

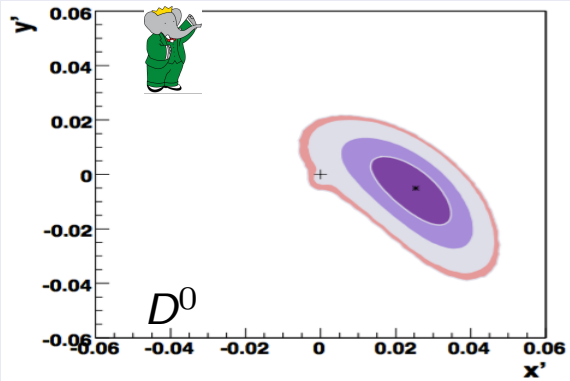
extraction of mixing parameters

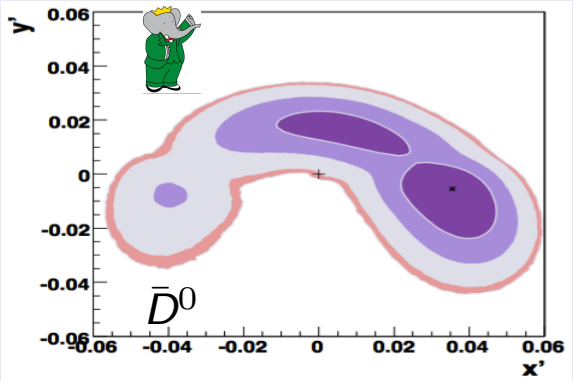
- obtain $r_0^2 = (5.25^{+0.25}_{-0.31} \pm 0.12) \times 10^{-3}$ using
  $r_0^2 = N_{WS}/[N_{RS}(1 + yA^2 - xB^2 + \frac{x^2+y^2}{2})]$ with
  $A^2(B^2) = \oint_{DP} \Re(\Im)(A_f^{DCS*} A_f^{CF}) dm_{12}^2 dm_{23}^2$.
- ToyMC 10^6 $(x'/r_0, y'/r_0)$ points in accordance with the fit covariance matrix.
- for each point, compute r_0 and then extract $x' = (2.61^{+0.57}_{-0.68} \pm 0.039)\%$ and $y' = (-0.06^{+0.55}_{-0.64} \pm 0.34)\%$ with $\rho = -0.75$.



search for CP violation in mixing or interference

- TDDA of WS D^0 and $\bar{D}^0$ events separately:
  D^0: $x'^+ = (2.53^{+0.54}_{-0.63} \pm 0.39)\%$, $y'^+ = (-0.05^{+0.63}_{-0.67} \pm 0.50)\%$, with $\rho = -0.69$;
  $\bar{D}^0$: $x'^- = (3.55^{+0.73}_{-0.83} \pm 0.65)\%$, $y'^- = (-0.54^{+0.40}_{-1.16} \pm 0.41)\%$, with $\rho = -0.66$.





sensitivity estimation of TDDA in $D^0 \to K^+\pi^-\pi^0$ at Belle II

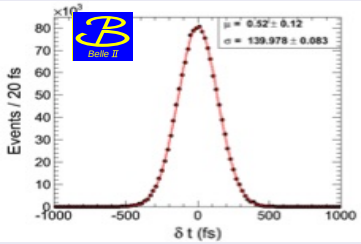
Belle II at SuperKEKB

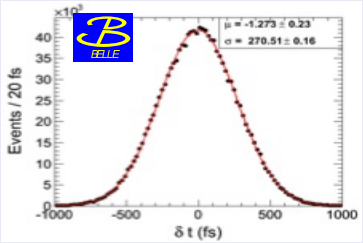
- Phase I: beam commissioning (2016 Feb.-Jun.)
- Phase II: collision tuning (2017, $10^{34}\ cm^{-2}s^{-1}$, 20 fb^{-1})
- Phase III: full Belle II commissioning (2018 starts)
- (40 times)$8 \times 10^{35}\ cm^{-2}s^{-1}$; 50 ab^{-1} dataset

We are really not in the future!!

Smearing D^0 lifetime

Belle II: time resolution σ=140 fs, twice better than Belle (270 fs)

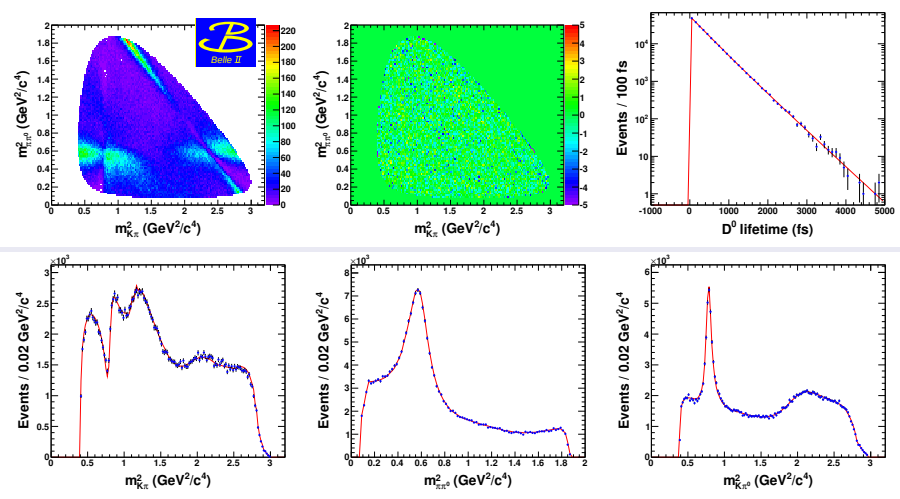




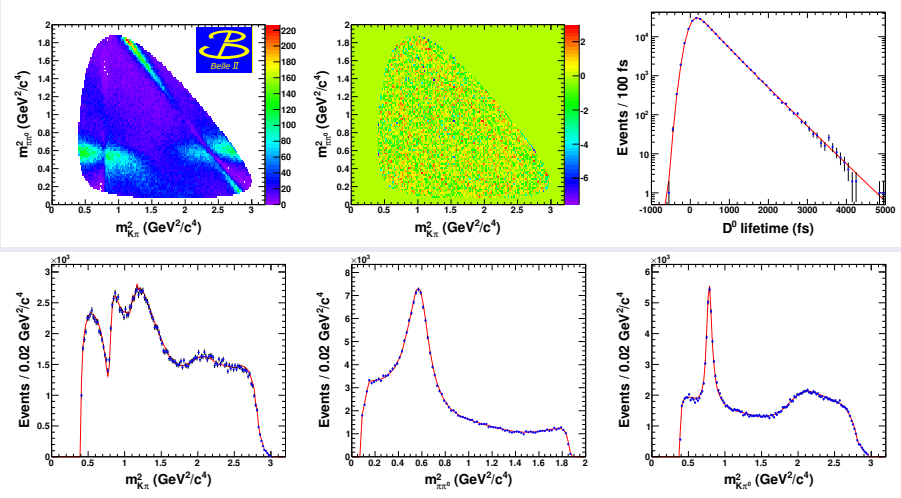
TDDA at generator level

with DCS res. and (x,y) floated;

with CF res. and lifetime and resolution(smear) par.s all fixed.



Sensitivity at TDDA with 225K signals



$\sigma_X = \pm 0.06\%$, $\sigma_Y = 0.049\%$ obtained from the average of 10 sets of signal MC samples (reported at 4th B2TiP workshop May 23-25, Pittsburgh)