第十二届全国粒子物理学术会议

Quasi parton distribution functions and evolution

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2016.8.25, 中国科学技术大学, 合肥

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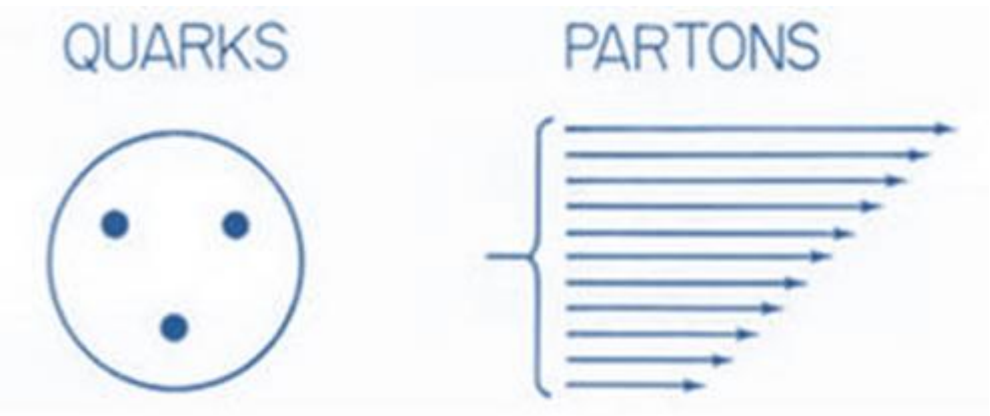
1. Parton distribution functions

- Hadron structure
- Quark model
- Parton and parton model : Feynman in 1969

- QCD: Asymptotic freedom
  parton: quarks and gluons

- Parton distributions function
- The key to understand hadron structure



- Basic input parameter for particle physics at hadron colliders

- Define pdfs in QCD (see *Foundations of perturbative QCD* by Collins)

$$f_{j/H}(\xi) = \int \frac{dw^-}{2\pi}\, e^{-i\xi P^+ w^-} \langle P|\overline{\psi}_j(0, w^-, \mathbf{0}_T)\, \frac{\gamma^+}{2}\, \psi_j(0)|P\rangle_c.$$

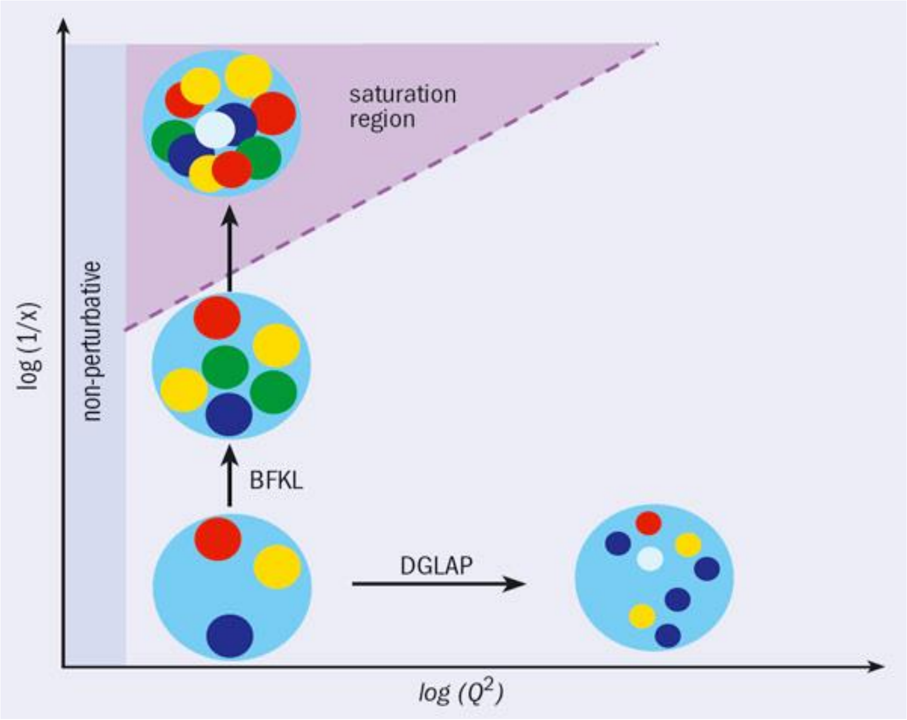
- light-cone correlations
- nonperturbative

- Factorization theorems:
- Foundation of applying perturbation theory in QCD.
- for DIS

$$d\sigma \sim C_q(x, Q, \mu) \bigotimes f_{q/P}(x, \mu) + \cdots$$

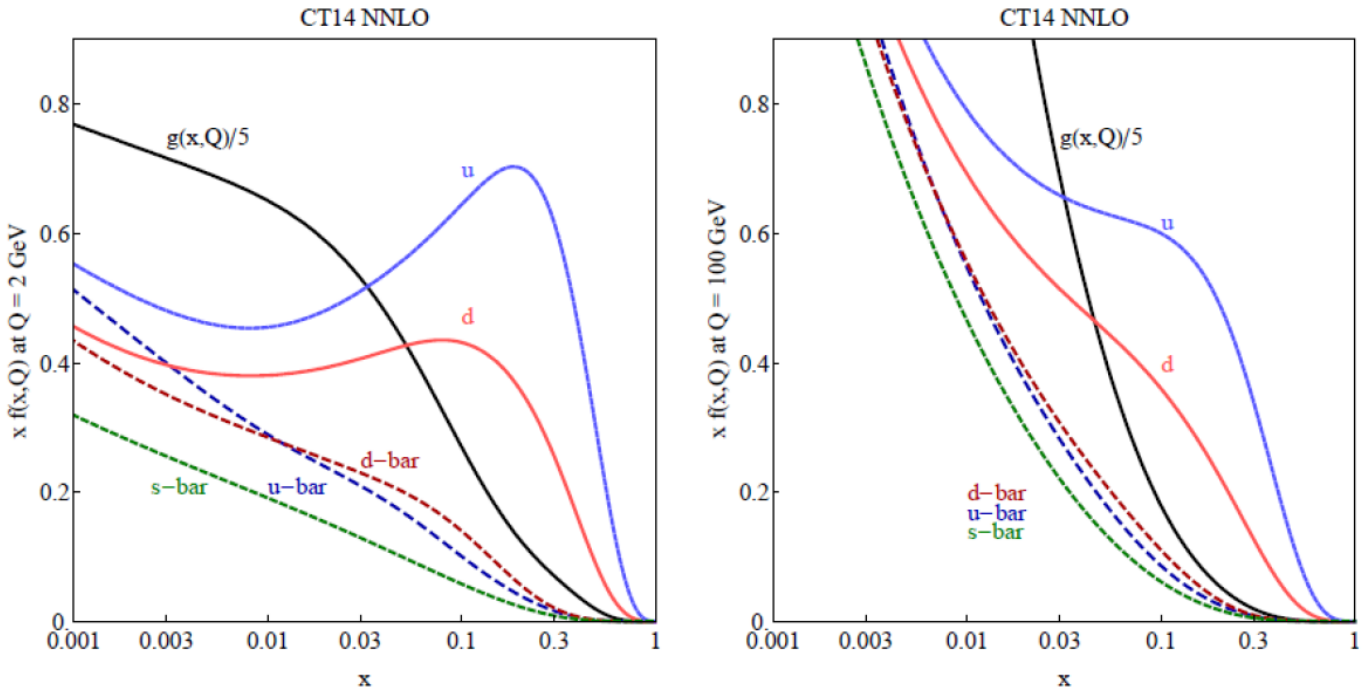
- Scale dependence: DGLAP equations

$$\frac{d}{d\ln\mu} f_{j/H}(\xi;\mu) = 2\sum_{j'} \int_\xi^1 \frac{dz}{z}\, P_{jj'}(z;\alpha_s(\mu)) f_{j'/H}(\xi/z;\mu),$$



- Extract pdfs from experiment data

CTEQ (Dulat *et al*. arxiv: 1506.07443)
NNPDF, MSTW…



QUESTION: Can we calculate pdfs from QCD?

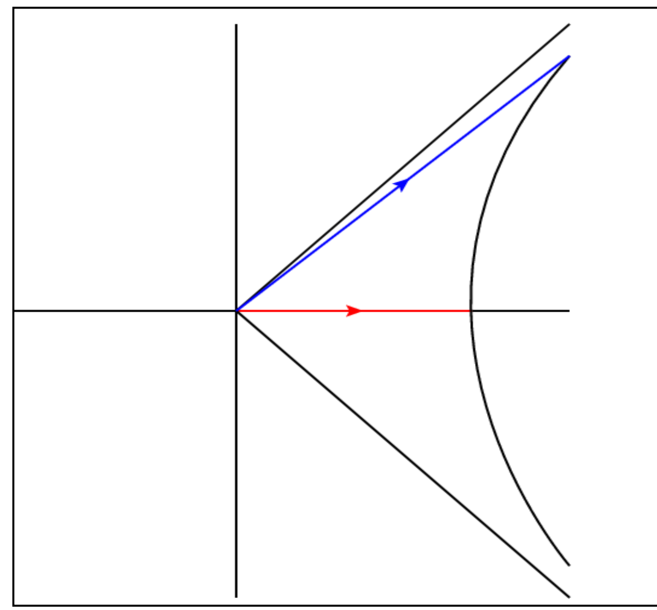
2. Quasi pdfs

● Calculating pdfs from QCD theory

- Nonperturbative method: Calculating on Euclidean lattice.
- The non-local light-cone correlators are time-dependent and Minkowskian.
- Only the moments can be calculated.

● Quasi quark pdf(Ji,2013,2014)

$$
\begin{aligned}
q(x,\mu^2,P^z) \;=\; & \int \frac{dz}{4\pi} e^{izk^z} \langle P|\overline{\psi}(z)\gamma^z \\
& \times \exp\left(-ig\int_0^z dz' A^z(z')\right)\psi(0)|P\rangle \\
& +\mathcal{O}\left(\Lambda^2/(P^z)^2, M^2/(P^z)^2\right) \;,
\end{aligned}
$$



- Large momentum effective theory (LaMET)

- When taking $p^z \to \infty$ first in F before an UV regularization, one recovers f.
- On the other hand, the lattice matrix element is calculated at large p^z, with UV regularization imposed first.
- The different limits do not change the infrared physics. The factorization in terms of Feynman diagrams can be proved order by order (Ma,Qiu,2014)

LaMET: $$F(P^z/\Lambda) = Z(P^z/\Lambda, \Lambda/\mu) f(\mu) + \mathcal{O}(1/(P^z)^2) + \ldots .$$

HQET: $$O(m_b/\Lambda) = Z(m_b/\Lambda, \Lambda/\mu) o(\mu) + \mathcal{O}(1/m_b) + \ldots$$

- Matching condition:

$$\tilde{q}(x,\mu^2,P^z) = \int_{-1}^{1} \frac{dy}{|y|} Z\left(\frac{x}{y}, \frac{\mu}{P^z}\right) q(y,\mu^2) + \mathcal{O}\left(\Lambda^2/(P^z)^2, M^2/(P^z)^2\right) \ ,$$

$$Z(x,\mu/P^z) = \delta(x-1) + \frac{\alpha_s}{2\pi} Z^{(1)}(x,\mu/P^z) + \ldots$$

Quasi pdfs: finite p^z, from “full theory”

Standard pdfs : $p^z \to \infty$, then renormalization, from LaMET.

Z: matching coefficient, the difference of the UV physics, can be calculated in perturbation theory.

- Quasi physical observables:

GPD

$$F(x,\xi,t,P^z) = \int \frac{dz}{2\pi} e^{-izk^z} \langle P'|\overline{\psi}(-z/2)\gamma^z \times L(-z/2,z/2)\psi(z/2)|P\rangle$$

TMD

$$q(x,k_\perp,P_z,\mu^2) = \int \frac{dz}{4\pi} d^2\vec{r}_\perp e^{i(zk^z+\vec{k}_\perp \vec{r}_\perp)} \langle P|\overline{\psi}(\vec{r}_\perp,z) \times L^\dagger(\pm\infty;(\vec{r}_\perp,z))\gamma^z L(\pm\infty;0)\psi(0)|P\rangle$$

Wigner

$$W(x,k_\perp,b_\perp,P^z,\mu^2) = \int \frac{dz}{4\pi} d^2\Delta_\perp d^2\vec{r}_\perp e^{-i\Delta_\perp\cdot b_\perp} e^{i(zk^z+\vec{k}_\perp\vec{r}_\perp)} \times \langle P'|\overline{\psi}(\vec{r}_\perp,z)L^\dagger(\pm\infty;(\vec{r}_\perp,z))\gamma^z L(\pm\infty;0)\psi(0)|P\rangle$$

LCDA

$$\phi_0(x,P^z)2P^z = \int \frac{dz}{2\pi} e^{izk^z} \langle 0|\overline{d}(0)\gamma^z\gamma_5 L(0,z)u(z)|\pi^+(P)\rangle \ .$$

……

●The way to study parton observables on lattice

- Start from a particular parton observable f which is an operator made of light-cone fields.

- Construct an Euclidean version F which, under an infinite Lorentz boost, goes to f.

- Calculate the lattice matrix element of F in a hadron with large momentum Pz

- Use matching equation to extract the parton physics f.

Some progresses

- One loop matching for quark (Xiong, Ji, Zhang, Zhao,2013)
- Renormalization (Ji,Zhang,2014)
- Quasi GPD (Ji ,Schafer, Xiong ,Zhang, 2015)
- Quasi TMD and soft factor subtraction (Ji,Sun,Xiong,Yuan,2015)
- “Lattice cross section” approach (Ma, Qiu, 2014)
- Lattice calculation (Lin, Chen, Cohen,Ji, 2014; Chen, Cohen, Ji, Lin , Zhang ,2016)
- Quasi distribution amplitude of Heavy Quarkonia (Jia, Xiong,2015)
- Non-dipolar Wilson line (Li,2016)
- diquark spectator model (Gamberg, Kang, Vitev, Xing)

……

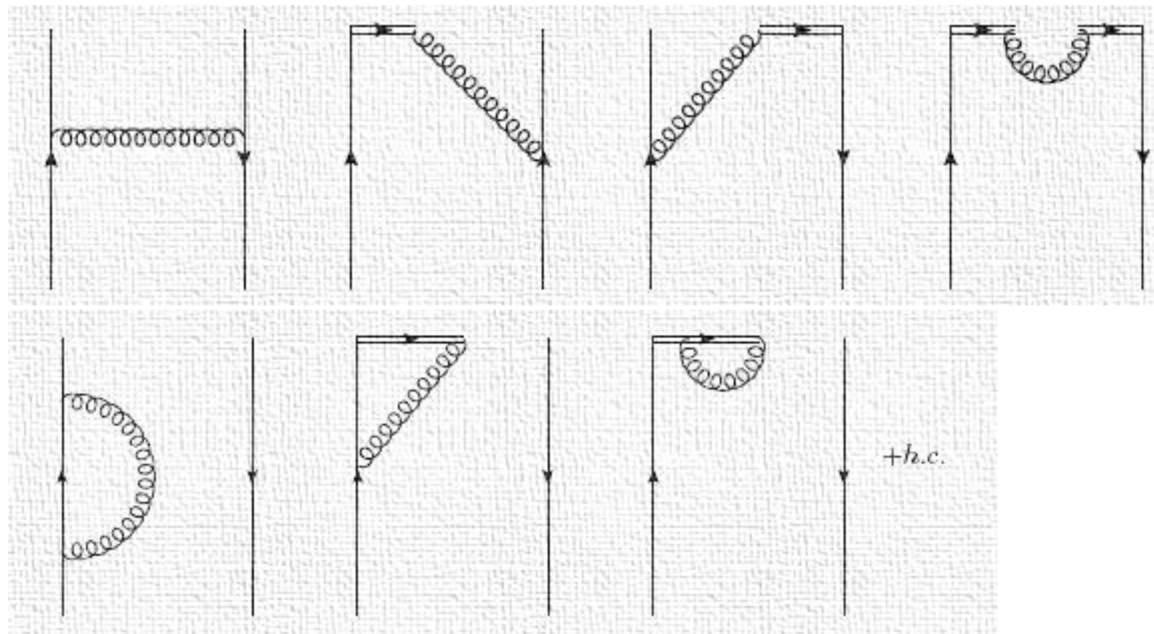
3. One loop matching

- For quark: (Xiong,Ji,Zhang,Zhao,2013)

  - Tree level:

$$\tilde{q}^{(0)}(x) = q^{(0)}(x) = \delta(1-x)$$

  - One loop level:



$$\tilde{q}^{(1)}(x,p^z) = \frac{\alpha_s C_F}{2\pi}\begin{cases} \frac{1+x^2}{x-1}\ln\frac{x-1}{x}+1, & x>1 \\ \frac{1+x^2}{1-x}\ln\frac{4xp_z^2}{(1-x)m^2}+\frac{1-5x}{1-x}, & 0<x<1 \\ \frac{1+x^2}{1-x}\ln\frac{x-1}{x}-1, & x<0 \end{cases}$$

$$q^{(1)}(x,\mu) = \frac{\alpha_s C_F}{2\pi}\begin{cases} 0, & x>1,\quad x<0 \\ \frac{1+x^2}{1-x}\ln\frac{\mu^2}{m^2(1-x)^2}-\frac{1+x^2}{1-x}, & 0<x<1 \end{cases}$$

- **Gluon: dominant in high energy hadron**

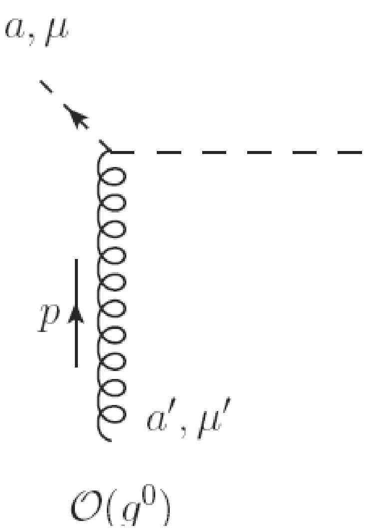
- Gluon pdf

$$g(x,\mu)=\int_{-\infty}^{+\infty}\frac{d\xi^-}{2\pi xP^+}e^{-ix\xi^-P^+}\langle P|F^{+\mu}(\xi^-)W(\xi^-,0)F_\mu{}^+(0)|P\rangle,$$

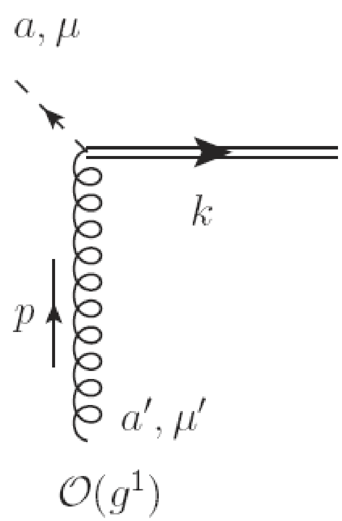
- Quasi gluon pdf

$$\tilde{g}(x,\mu,P^z)=\int_{-\infty}^{+\infty}\frac{dz}{2\pi xP^z}e^{izk^z}\langle P|F^{3\mu}(z)W(z,0)F_\mu{}^3(0)|P\rangle,$$

$$W(z_2,z_1)=\mathcal{P}e^{-ig\int_{z_1}^{z_2}dz'A^z(z')}$$



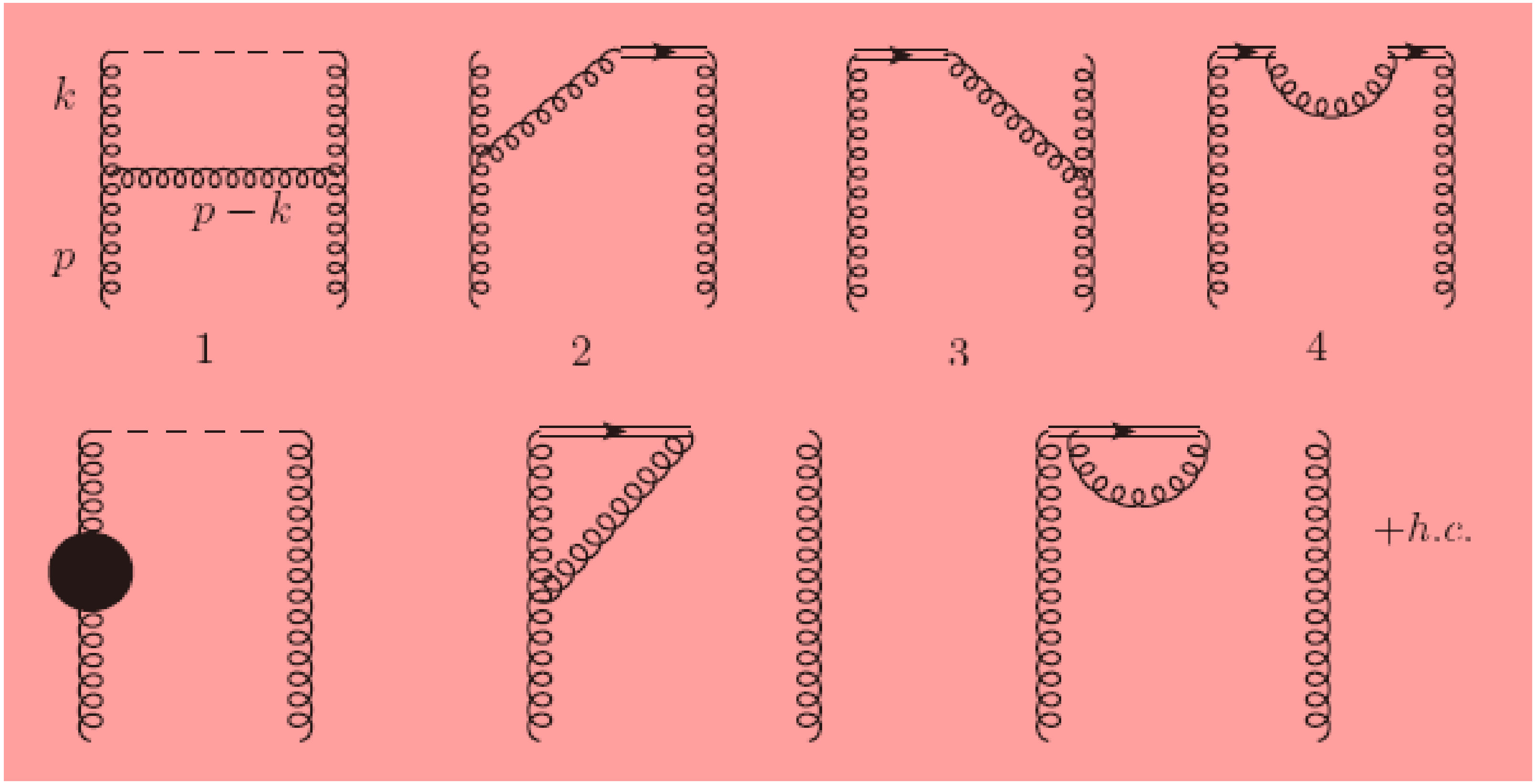
$$=i(p^z g_{\mu\mu'}-p_\mu n_{\mu'})\delta^{aa'}$$



$$=i((p^z-k^z)g_{\mu\mu'}-p_\mu n_{\mu'})\delta^{aa'}$$

Matching calculation for quasi gluon pdf

- Tree level: $\delta(1-x)$
- One loop level:



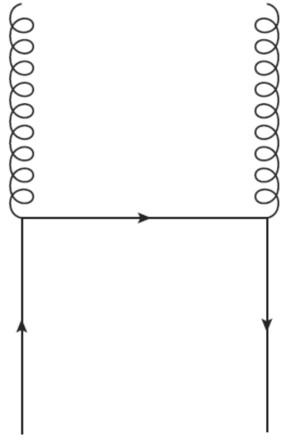
$$\tilde{g}^{(1)}(x,p_z)=\frac{\alpha_s N_c}{\pi}\begin{cases}\frac{(1-x+x^2)^2}{x(1-x)}\ln\frac{x}{x-1}+x-\frac{3}{4}+\frac{3}{2x}, & x>1\\ \frac{(1-x+x^2)^2}{x(1-x)}\ln\frac{4x(1-x)p_z^2}{(1-x+x^2)m^2} & \\ +\frac{9x}{4}+x^3+\frac{x}{2(x-1)}-\frac{3x}{2(1-x+x^2)}, & 0<x<1\\ \frac{(1-x+x^2)^2}{x(1-x)}\ln\frac{x-1}{x}-x+\frac{3}{4}-\frac{3}{2x}, & x<0\end{cases}$$

$$g^{(1)}(x,\mu)=\frac{\alpha_s N_c}{\pi}\begin{cases}0, & x>1,\quad x<0\\ \frac{(1-x+x^2)^2}{x(1-x)}\ln\frac{\mu^2}{m^2(1-x+x^2)} & \\ +\frac{x+7}{2}-\frac{1}{x(1-x)}-\frac{3}{2(1-x+x^2)}, & 0<x<1\end{cases}$$

- Quark in gluon and gluon in quark

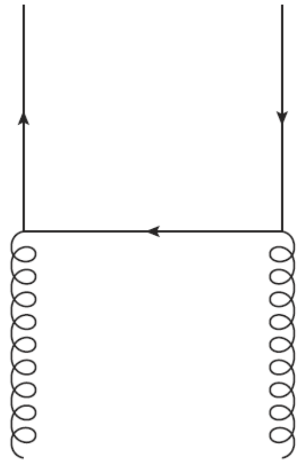
$$\widetilde{\Gamma}_{gq} = \frac{\alpha_s C_F}{2\pi}\begin{cases} \frac{1+(1-x)^2}{x}\ln\frac{x}{x-1} - \frac{x-3}{x}, & x>1 \\ \frac{1+(1-x)^2}{x}\ln\frac{4(1-x)}{x\Delta^2} + \frac{3x-1}{x}, & 0<x<1 \\ \frac{1+(1-x)^2}{x}\ln\frac{x-1}{x} + \frac{x-3}{x}, & x<0 \end{cases}$$

$$\Gamma_{gq} = \frac{\alpha_s C_F}{2\pi}\begin{cases} 0, & x>1,\ \ x<0 \\ -\frac{1+(1-x)^2}{x}\left(\ln\frac{m^2x^2}{\mu^2}+1\right), & 0<x<1 \end{cases}$$



$$\widetilde{\Gamma}_{qg} = \frac{\alpha_s T_F}{2\pi}\begin{cases} (x^2+(1-x)^2)\ln\frac{x}{x-1} - 2x+1, & x>1 \\ (x^2+(1-x)^2)\ln\frac{4x(1-x)}{(1-x+x^2)\Delta^2} + 1+2x-4x^2, & 0<x<1 \\ (x^2+(1-x)^2)\ln\frac{x-1}{x} + 2x-1. & x<0 \end{cases}$$

$$\Gamma_{qg} = \frac{\alpha_s T_F}{2\pi}\begin{cases} 0, & x>1,\ \ x<0 \\ (x^2+(1-x)^2)\ln\frac{\mu^2}{m^2(1-x+x^2)} + 2x-2x^2, & 0<x<1 \end{cases}$$



$$\Delta \equiv \frac{m}{p_z}.$$

- Matching coefficients:

$$Z_q\left(\xi,\frac{\mu}{p^z}\right)=\delta(\xi-1)+\frac{\alpha_s}{\pi}Z_q^{(1)}\left(\xi,\frac{\mu}{p^z}\right)+\cdots,$$
$$Z_g\left(\xi,\frac{\mu}{p^z}\right)=\delta(\xi-1)+\frac{\alpha_s}{\pi}Z_g^{(1)}\left(\xi,\frac{\mu}{p^z}\right)+\cdots.$$

$$Z_q^{(1)}\left(\xi,\frac{\mu}{p^z}\right)=\frac{C_F}{2}\begin{cases}\frac{1+\xi^2}{\xi-1}\ln\frac{\xi-1}{\xi}+1, & \xi>1\\ \frac{1+\xi^2}{1-\xi}\ln\frac{4\xi(1-\xi)p_z^2}{\mu^2}+\frac{2-5\xi+\xi^2}{1-\xi}, & 0<\xi<1\\ \frac{1+\xi^2}{1-\xi}\ln\frac{\xi-1}{\xi}-1. & \xi<0\end{cases}$$

$$Z_g^{(1)}\left(\xi,\frac{\mu}{p^z}\right)=\frac{C_A}{2}\begin{cases}\frac{2(1-\xi^2+\xi^2)^2}{\xi(1-\xi)}\ln\frac{\xi}{\xi-1}+2\xi-\frac{3}{2}+\frac{3}{\xi}, & \xi>1,\\ \frac{2(1-\xi+\xi^2)^2}{\xi(1-\xi)}\ln\frac{4\xi(1-\xi)p_z^2}{\mu^2} & \\ +\frac{1}{(1-\xi)}+\frac{3(1-\xi)}{1-\xi+\xi^2}+\frac{2}{\xi}-6+\frac{7\xi}{2}+2\xi^3, & 0<\xi<1\\ -\frac{2(1-\xi^2+\xi^2)^2}{\xi(1-\xi)}\ln\frac{\xi}{\xi-1}-2\xi+\frac{3}{2}-\frac{3}{\xi}, & \xi<0,\end{cases}$$

4. Evolution with p^z

- Return to the matching equation:

$$F(P^z/\Lambda) = Z(P^z/\Lambda, \Lambda/\mu) f(\mu) + \mathcal{O}(1/(P^z)^2) + \dots .$$

$$\gamma(\alpha_s) = \frac{1}{Z}\frac{\partial Z}{\partial \ln P^z} .$$

$$\frac{\partial F(P^z)}{\partial \ln P^z} = \gamma(\alpha_s) F(P^z) + \mathcal{O}(1/(P^z)^2) ,$$

- Quasi physical observables depend on the hadron momentum p^z
- The large logarithms of p^z needs to be summed.
- An evolution equation with p^z is needed.

- p^z evolution equations at one loop:

$$\begin{cases} \frac{d}{d\ln p^z}\tilde{q}(x,p^z) = \frac{\alpha_s}{\pi}\int\frac{dy}{y}\left(\tilde{P}_{q\leftarrow q}(y)\tilde{q}\left(\frac{x}{y},\frac{\mu}{p^z}\right)+\tilde{P}_{q\leftarrow g}(y)\tilde{g}\left(\frac{x}{y},\frac{\mu}{p^z}\right)\right), \\ \frac{d}{d\ln p^z}\tilde{\bar{q}}(x,p^z) = \frac{\alpha_s}{\pi}\int\frac{dy}{y}\left(\tilde{P}_{q\leftarrow q}(y)\tilde{\bar{q}}\left(\frac{x}{y},\frac{\mu}{p^z}\right)+\tilde{P}_{q\leftarrow g}(y)\tilde{g}\left(\frac{x}{y},\frac{\mu}{p^z}\right)\right), \\ \frac{d}{d\ln p^z}\tilde{g}(x,p^z) = \frac{\alpha_s}{\pi}\int\frac{dy}{y}\left[\tilde{P}_{g\leftarrow g}(y)\tilde{q}\left(\frac{x}{y},\frac{\mu}{p^z}\right)+\tilde{P}_{g\leftarrow q}(y)\sum_{n_f}\left(\tilde{q}\left(\frac{x}{y},\frac{\mu}{p^z}\right)+\tilde{\bar{q}}\left(\frac{x}{y},\frac{\mu}{p^z}\right)\right)\right]. \end{cases}$$

The same as DGLAP eqs for standard pdf !

Evolution kernels

$$
\begin{aligned}
P_{q\leftarrow q}(y) &= C_F\left(\frac{1+y^2}{(1-y)_+}+\frac{3}{2}\delta(1-y)\right),\\
P_{q\leftarrow g}(y) &= T_F(y^2+(1-y^2)),\\
P_{g\leftarrow q}(y) &= C_F\frac{1+(1-y)^2}{y},\\
P_{g\leftarrow g}(y) &= C_A\frac{2(1-y+y^2)^2}{y(1-y)_+}+\left(\frac{11}{6}C_A-\frac{2}{3}n_fT_F\right)\delta(1-y),
\end{aligned}
$$

Identical to the DGLAP kernel (at one loop)!

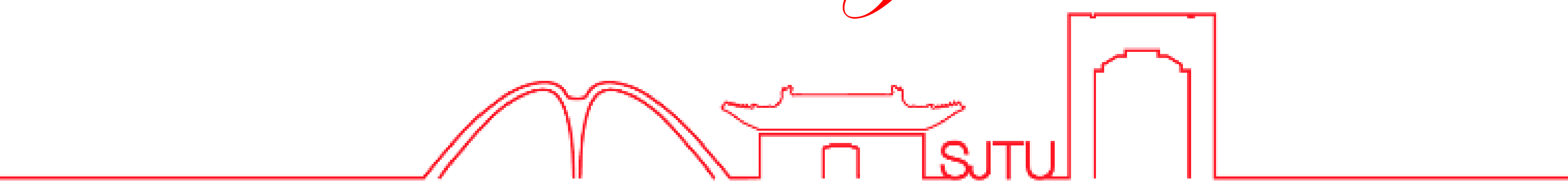
But high orders ??? ……

5.Summary

- LaMET offers a way to compute light-cone observables on Euclidean lattice.
- Matching coefficient for quasi gluon pdf is obtained at one loop level, which is free of infrared divergence and can be calculated perturbatively.
- The p^z evolution equations have the same form and share the same evolution kernel with DGLAP equations.
- Still a lot of questions to be answered
- Lattice calculation
- Match to lattice gauge theory
- Linear UV divergence
- High order evolution equations

……

Thank you!



Back up

- Self energy diagrams (quark)

$$Z_F^{(1)} = -\frac{\alpha_s C_F}{2\pi} \int dy \begin{cases} (y-1)\ln\frac{y-1}{y} + 1, & y > 1 \\ (1-y)\ln\frac{4y}{(1-y)\Delta^2} + \frac{1-3y}{1-y}, & 0 < y < 1 \\ (1-y)\ln\frac{y-1}{y} - 1, & y < 0 \end{cases}$$

$$Z_F^{(1)} = -\frac{\alpha_s C_F}{2\pi} \int_0^1 dy \left((1-y)\ln\frac{\mu^2}{m^2(1-y)^2} - \frac{1+y^2}{1-y} \right).$$

$$\tilde{Z}_V^{(1)} = -\frac{\alpha_s C_F}{2\pi} \int dy \begin{cases} \frac{2y}{1-y}\ln\frac{y}{y-1} - \frac{1}{1-y}, & y > 1 \\ \frac{2y}{1-y}\ln\frac{4y}{(1-y)\Delta^2} + \frac{1-2y}{1-y}, & 0 < y < 1 \\ \frac{2y}{1-y}\ln\frac{y-1}{y} + \frac{1}{1-y}, & y < 0 \end{cases}$$

$$Z_V^{(1)} = -\frac{\alpha_s C_F}{2\pi} \int_0^1 dy \frac{2y}{1-y} \ln\frac{\mu^2}{m^2(1-y)^2}.$$

$$\tilde{Z}_W^{(1)} = -\frac{\alpha_s C_F}{2\pi}\begin{cases} \frac{1}{1-y}, & y>1 \\ \frac{1}{y-1}, & 0<y<1 \\ \frac{1}{y-1}, & y<0 \end{cases}$$

$$Z_W^{(1)} = 0.$$

- Self energy diagrams (gluon)

$$\underset{\sim\sim\sim}{\tilde{Z}_{1,\mathrm{quark}}^{(1)}} = -\frac{\alpha_s n_f T_F}{\pi}\int dy \begin{cases} 2y(1-y)\ln\frac{y}{y-1} - 2y + 1, & y>1 \\ 2y(1-y)\ln\frac{4y(1-y)}{\Delta^2} - 1, & 0<y<1 \\ 2y(1-y)\ln\frac{y-1}{y} + 2y - 1, & y<0 \end{cases}$$

$$\underset{\sim\sim\sim}{Z_{1,\mathrm{quark}}^{(1)}} = -\frac{\alpha_s n_f T_F}{\pi}\int dy \begin{cases} 0, & y>1, \quad y<0 \\ 2y(1-y)\ln\frac{\mu^2}{m^2}, & 0<y<1 \end{cases}$$

$$\underset{\sim\sim\sim}{Z^{(1)}_{1,\text{quark}}} = -\frac{\alpha_s n_f T_F}{\pi}\int dy \begin{cases} 0, & y>1, \quad y<0 \\ 2y(1-y)\ln\dfrac{\mu^2}{m^2}, & 0<y<1 \end{cases}$$

$$\underset{\sim\sim\sim}{\tilde{Z}^{(1)}_{1,3-\text{vertex}}} = -\frac{\alpha_s N_c}{4\pi}\int dy \begin{cases} (1+5y(1-y))\ln\dfrac{y-1}{y} - 2y + 1, & y>1 \\ -(1+5y(1-y))\ln\dfrac{4y(1-y)}{\Delta^2} - 1, & 0<y<1 \\ (1+5y(1-y))\ln\dfrac{y}{y-1} + 2y - 1, & y<0 \end{cases}$$

$$\underset{\sim\sim\sim}{Z^{(1)}_{1,3-\text{vertex}}} = \frac{\alpha_s N_c}{4\pi}\int dy \begin{cases} 0, & y>1, \quad y<0 \\ (1+5y(1-y))\ln\dfrac{\mu^2}{m^2}, & 0<y<1 \end{cases}$$

$$\tilde{Z}^{(1)}_{1,4-\text{vertex}} = -\frac{3\alpha_s N_c}{2\pi}\int dy \begin{cases} y-1, & y>1 \\ 1-y, & 0<y<1 \\ 1-y, & y<0 \end{cases}$$

$$Z^{(1)}_{1,4-\text{vertex}} = 0$$

$$\underset{\sim\sim\sim}{\tilde{Z}^{(1)}_{1,\text{ghost}}} = \frac{\alpha_s N_c}{4\pi} \int dy \begin{cases} y(1-y)\ln\frac{y-1}{y}, & y>1 \\ y(y-1)\ln\frac{4y(1-y)}{\Delta^2}, & 0<y<1 \\ y(1-y)\ln\frac{y}{y-1}, & y<0 \end{cases}$$

$$\underset{\sim\sim\sim}{Z^{(1)}_{1,\text{ghost}}} = \frac{\alpha_s N_c}{4\pi} \int dy \begin{cases} 0, & y>1 \\ y(y-1)\ln\frac{\mu^2}{m^2}, & 0<y<1 \end{cases}$$

$$\tilde{Z}^{(1)}_2 = -\frac{\alpha_s N_c}{2\pi} \int dy \begin{cases} \frac{1+y}{1-y}\ln\frac{y}{y-1} + \frac{1-2y}{1-y}, & y>1, \\ \frac{1+y}{1-y}\ln\frac{4y(1-y)}{(1-y+y^2)\Delta^2} - \frac{2y^2-2y+1}{1-y}, & 0<y<1 \\ \frac{1+y}{1-y}\ln\frac{y-1}{y} + \frac{2y-1}{1-y}, & y<0 \end{cases}$$

$$Z^{(1)}_2 = -\frac{\alpha_s N_c}{2\pi} \int_0^1 dy \frac{1+y}{1-y}\left(\ln\frac{\mu^2}{m^2(1-y+y^2)} - 1\right).$$

$$Z_3 = -\frac{\alpha_s N_c}{2\pi}\int dy \begin{cases} \frac{y^2}{1-y}, & y>1 \\ \frac{y^2}{y-1}, & 0<y<1 \\ \frac{y^2}{y-1}, & y<0 \end{cases}$$

- Matching coefficients at ξ=1

$$Z_q^{(1)}\left(\xi=1,\frac{\mu}{p^z}\right) = -\frac{C_F}{2}\int d\eta \begin{cases} \frac{1+\eta^2}{\eta-1}\ln\frac{\eta-1}{\eta}+1, & \eta>1 \\ \frac{1+\eta^2}{1-\eta}\ln\frac{4\eta(1-\eta)p_z^2}{\mu^2}+\frac{2-5\eta+\eta^2}{1-\eta}, & 0<\eta<1 \\ \frac{1+\eta^2}{1-\eta}\ln\frac{\eta-1}{\eta}-1, & \eta<0 \end{cases}$$

$$Z_g^{(1)}\left(\xi=1,\frac{\mu}{p^z}\right) = \int d\eta \begin{cases} -n_f T_F\left(2\eta(\eta-1)\ln\frac{\eta-1}{\eta}+1-2\eta\right) \\ +N_c\left(\frac{2\eta^2-5\eta+3}{4(1-\eta)}+\frac{1-\eta+8\eta^2-4\eta^3}{4(1-\eta)}\ln\frac{\eta-1}{\eta}\right), & \eta>1, \\ -n_f T_F\left(2\eta(\eta-1)\ln\frac{4\eta(1-\eta)p_z^2}{\mu^2}-1\right) \\ +N_c\left(\frac{9\eta-5}{4(1-\eta)}+\frac{1-\eta+8\eta^2-4\eta^3}{4(1-\eta)}\ln\frac{4\eta(1-\eta)p_z^2}{\mu^2}\right), & 0<\eta<1, \\ +n_f T_F\left(2\eta(\eta-1)\ln\frac{\eta-1}{\eta}+1-2\eta\right) \\ -N_c\left(\frac{2\eta^2-5\eta+3}{4(1-\eta)}+\frac{1-\eta+8\eta^2-4\eta^3}{4(1-\eta)}\ln\frac{\eta-1}{\eta}\right) & \eta<0, \end{cases}$$