

“晨光杯” 论文答辩

NNLO QCD Corrections to  
Quarkonium Decay and Production  
within NRQCD Framework

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# Based on:

- arXiv:1505.02665 (PRL115, 222001)

Can Nonrelativistic QCD Explain the  $\gamma\gamma^* \rightarrow \eta_c$  Transition Form Factor Data?

- arXiv:1511.06288 (Under Review)

Next-to-next-to-leading-order QCD corrections to  $\chi_{c0,2} \rightarrow \gamma\gamma$

In Corroboration with  
贾宇 & 桑文龙

# Contents:

- Motivation.
- NNLO QCD correction to  $\gamma\gamma^* \rightarrow \eta_c$  form factor and confront the *BaBar* data.
- NNLO QCD correction to  $\chi_{c0,2} \rightarrow \gamma\gamma$  and confront with *BESIII* data.
- Summary & Outlook.

# Motivation

- Nowadays, NRQCD becomes an important approach to tackle various quarkonium production and decay processes.
- Lots of Calculations @LO & @NLO.
- How about the status @NNLO?

# Motivation

- Status @NNLO

$$\Gamma(J/\psi \rightarrow \ell\ell) = \Gamma^{(0)} \left\{ 1 - \frac{8}{3} \frac{\alpha_s}{\pi} - (44.55 - 0.41n_f) \frac{\alpha_s^2}{\pi^2} + (-2091 + 120.66n_f - 0.82n_f^2) \frac{\alpha_s^3}{\pi^3} \right\}$$

$$\Gamma(B_c \rightarrow \ell\nu) = \Gamma^{(0)} \left\{ 1 - 1.39 \frac{\alpha_s}{\pi} - 23.7 \frac{\alpha_s^2}{\pi^2} + \mathcal{O}(\alpha_s^3) \right\}$$

$$\Gamma(\eta_c \rightarrow \gamma\gamma) = \Gamma^{(0)} \left\{ 1 - 1.69 \frac{\alpha_s}{\pi} - 56.52 \frac{\alpha_s^2}{\pi^2} + \mathcal{O}(\alpha_s^3) \right\}$$

# Motivation

- Status @NNLO

$$\Gamma(J/\psi \rightarrow \ell\ell) = \Gamma^{(0)} \left\{ 1 - \frac{8}{3} \frac{\alpha_s}{\pi} - (44.55 - 0.41n_f) \frac{\alpha_s^2}{\pi^2} + (-2091 + 120.66n_f - 0.82n_f^2) \frac{\alpha_s^3}{\pi^3} \right\}$$

S-Wave

$$\Gamma(B_c \rightarrow \ell\nu) = \Gamma^{(0)} \left\{ 1 - 1.39 \frac{\alpha_s}{\pi} - 23.7 \frac{\alpha_s^2}{\pi^2} + \mathcal{O}(\alpha_s^3) \right\}$$

Decay

$$\Gamma(\eta_c \rightarrow \gamma\gamma) = \Gamma^{(0)} \left\{ 1 - 1.69 \frac{\alpha_s}{\pi} - 56.52 \frac{\alpha_s^2}{\pi^2} + \mathcal{O}(\alpha_s^3) \right\}$$

# Motivation:

- S-wave Production:  $\gamma\gamma^* \rightarrow \eta_c$

TABLE III. The  $Q^2$  interval and the weighted average  $Q^2$  value ( $\overline{Q^2}$ ), the  $e^+e^- \rightarrow e^+e^-\eta_c$  cross section multiplied by  $\mathcal{B}(\eta_c \rightarrow K\bar{K}\pi)$  [ $d\sigma/dQ^2(\overline{Q^2})$ ], and the normalized  $\gamma\gamma^* \rightarrow \eta_c$  transition form factor ( $|F(\overline{Q^2})/F(0)|$ ). The statistical and systematic errors are quoted separately for the cross section, but are combined in quadrature for the form factor. Only  $Q^2$ -dependent systematic errors are quoted; the  $Q^2$ -independent error is 6.6% for the cross section and 4.3% for the form factor.

$Q^2$ interval (GeV <sup>2</sup> )	$\overline{Q^2}$ (GeV <sup>2</sup> )	$d\sigma/dQ^2(\overline{Q^2})$ (fb/GeV <sup>2</sup> )	$ F(\overline{Q^2})/F(0) $
2–3	2.49	$18.7 \pm 4.2 \pm 0.8$	$0.740 \pm 0.085$
3–4	3.49	$10.6 \pm 2.1 \pm 0.8$	$0.680 \pm 0.073$
4–5	4.49	$6.62 \pm 1.18 \pm 0.19$	$0.629 \pm 0.057$
5–6	5.49	$4.00 \pm 0.80 \pm 0.10$	$0.555 \pm 0.056$
6–8	6.96	$3.00 \pm 0.43 \pm 0.17$	$0.563 \pm 0.043$
8–10	8.97	$1.58 \pm 0.30 \pm 0.08$	$0.490 \pm 0.049$
10–12	10.97	$0.72 \pm 0.17 \pm 0.05$	$0.385 \pm 0.048$
12–15	13.44	$0.55 \pm 0.13 \pm 0.03$	$0.395 \pm 0.047$
15–20	17.35	$0.34 \pm 0.07 \pm 0.01$	$0.385 \pm 0.038$
20–30	24.53	$0.084 \pm 0.026 \pm 0.004$	$0.261 \pm 0.041$
30–50	38.68	$0.019 \pm 0.009 \pm 0.001$	$0.204 \pm 0.049$

(BaBar 2010)

# Motivation:

- P-wave Decay:  $\chi_{c0,2} \rightarrow \gamma\gamma$

$$\Gamma_{\gamma\gamma}(\chi_{c0}) = (2.33 \pm 0.20 \pm 0.13 \pm 0.17) \text{ keV}$$

$$\mathcal{R} = \frac{\Gamma_{\gamma\gamma}(\chi_{c2})}{\Gamma_{\gamma\gamma}(\chi_{c0})} = 0.271 \pm 0.029 \pm 0.013 \pm 0.027$$

$$f_{0/2} = \frac{\Gamma_{\gamma\gamma}^{\lambda=0}(\chi_{c2})}{\Gamma_{\gamma\gamma}^{\lambda=2}(\chi_{c2})} = 0.00 \pm 0.02 \pm 0.02$$

( BESIII 2012 )

# Basic Procedures:

## 1. Feynman Diagrams & Amplitudes

- FEYNARTS - MATHEMATICA Package  
<http://www.feynarts.de>
- QGRAF - FORTRAN Program  
<http://cfif.ist.utl.pt/~paulo/qgraf.html>

## 2. Color- & Spin-Traces

- FEYNCALC - MATHEMATICA Package  
<https://github.com/FeynCalc>
- FEYNCALC/FORMLINK - MATHEMATICA Package  
<https://github.com/FormLink>

# Basic Procedures:

## 3. Partial Fragmentation & IBP Reduction

- APART - MATHEMATICA Package  
<https://github.com/F-Feng>
- FIRE - MATHEMATICA Program & C++ version  
<http://science.sander.su>

## 4. Master Integrals – Numerical Evaluation

- FIESTA - MATHEMATICA Package  
<http://science.sander.su>
- CUBPACK - FORTRAN Code  
<http://nines.cs.kuleuven.be/software/CUBPACK/>

# Feynman Diagrams:

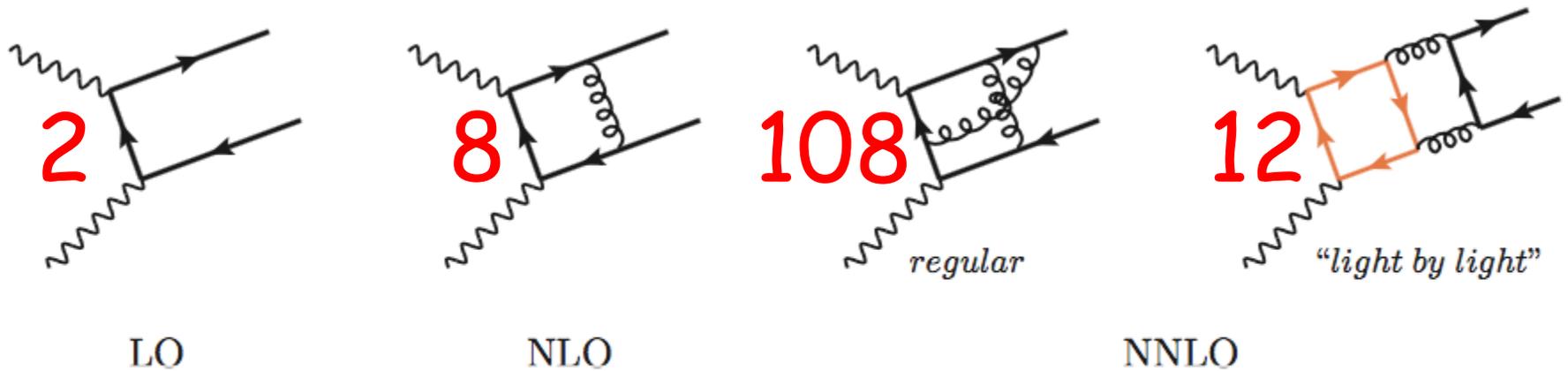


FIG. 1: Sample Feynman diagrams for  $\gamma^* \gamma \rightarrow c\bar{c}({}^1S_0^{(1)})$ .

$$\gamma\gamma^* \rightarrow \eta_c \quad \& \quad \chi_{c0,2} \rightarrow \gamma\gamma$$

$$\gamma\gamma^* \rightarrow \eta_c :$$

- Form Factor

$$\langle \eta_c(p) | J^\mu | \gamma(k, \varepsilon) \rangle = ie^2 \epsilon^{\mu\nu\rho\sigma} \varepsilon_\nu q_\rho k_\sigma F(Q^2)$$

- NRQCD Factorization

$$F(Q^2) = C(Q, m, \mu_R, \mu_\Lambda) \frac{\langle \eta_c | \psi^\dagger \chi(\mu_\Lambda) | 0 \rangle}{\sqrt{m}} + \mathcal{O}(v^2)$$

- Form Factor Ratio

$$\left| \frac{F(Q^2)}{F(0)} \right| = \left| \frac{C(Q, m, \mu_R, \mu_\Lambda)}{C(0, m, \mu'_R, \mu_\Lambda)} \right| + \mathcal{O}(v^2)$$

$$\gamma\gamma^* \rightarrow \eta_c :$$

## ● Perturbative Expansion

$$C(Q, m, \mu_R, \mu_\Lambda) = C^{(0)}(Q, m) \left\{ 1 + C_F \frac{\alpha_s(\mu_R)}{\pi} f^{(1)}(\tau) + \frac{\alpha_s^2}{\pi^2} \left[ \frac{\beta_0}{4} \ln \frac{\mu_R^2}{Q^2 + m^2} C_F f^{(1)}(\tau) - \pi^2 C_F \left( C_F + \frac{C_A}{2} \right) \times \ln \frac{\mu_\Lambda}{m} + f^{(2)}(\tau) \right] + \mathcal{O}(\alpha_s^3) \right\},$$

where  $\tau = \frac{Q^2}{m^2}$  and  $f^{(2)}(\tau) = f_{\text{reg}}^{(2)}(\tau) + f_{\text{lbl}}^{(2)}(\tau)$

$$\gamma\gamma^* \rightarrow \eta_c :$$

● Result @LO

$$C^{(0)}(Q, m) = \frac{4e_c^2}{Q^2 + 4m^2}$$

● Result @NLO

$$f^{(1)}(\tau) = \frac{\pi^2(3 - \tau)}{6(4 + \tau)} - \frac{20 + 9\tau}{4(2 + \tau)} - \frac{\tau(8 + 3\tau)}{4(2 + \tau)^2} \ln \frac{4 + \tau}{2}$$

$$+ 3\sqrt{\frac{\tau}{4 + \tau}} \tanh^{-1} \sqrt{\frac{\tau}{4 + \tau}} + \frac{2 - \tau}{4 + \tau} \left( \tanh^{-1} \sqrt{\frac{\tau}{4 + \tau}} \right)^2$$

$$- \frac{\tau}{2(4 + \tau)} \text{Li}_2 \left( -\frac{2 + \tau}{2} \right)$$

$$\gamma\gamma^* \rightarrow \eta_c :$$

## ● Numerical Result @ $Q^2 = 0$

$$f_{\text{reg}}^{(2)}(0) = C_F^2 f_A + C_A C_F f_{\text{NA}} + N_L C_F T_R f_L + N_H C_F T_R f_H$$

$$\begin{cases} f_A = -21.10789797(4) \\ f_{\text{NA}} = -4.79298000(3) \\ f_L = -\frac{1}{144} \left( 42\zeta(3) - 164 + 13\pi^2 + 96 \ln 2 \right) \\ f_H = 0.223672013(2) \end{cases}$$

- A. Czarnecki and K. Melnikov, “Charmonium decays:  $J/\psi \rightarrow e^+e^-$  and  $\eta_c \rightarrow \gamma\gamma$ ”, Phys. Lett. B **519**, 212 (2001) [hep-ph/0109054].

$$\gamma\gamma^* \rightarrow \eta_c :$$

- Numerical Result @  $Q^2 = 0$

$$s_A(\mu) = -21.0 - \pi^2 \left( \frac{1}{4\epsilon} + \ln \frac{\mu}{m} \right),$$

$$s_{NA}(\mu) = -4.79 - \frac{\pi^2}{2} \left( \frac{1}{4\epsilon} + \ln \frac{\mu}{m} \right),$$

$$s_L = \frac{41}{36} - \frac{13}{144}\pi^2 - \frac{2}{3}\ln 2 - \frac{7}{24}\zeta_3 \simeq -0.565,$$

$$s_H = 0.22. \tag{4}$$

- A. Czarnecki and K. Melnikov, “Charmonium decays:  $J/\psi \rightarrow e^+e^-$  and  $\eta_c \rightarrow \gamma\gamma$ ”, Phys. Lett. B **519**, 212 (2001) [hep-ph/0109054].

$$\gamma\gamma^* \rightarrow \eta_c :$$

## ● Numerical Result @ $Q^2 = 0$

$$f_{\text{reg}}^{(2)}(0) = C_F^2 f_A + C_A C_F f_{\text{NA}} + N_L C_F T_R f_L + N_H C_F T_R f_H$$

$$\left\{ \begin{array}{l} f_A = -21.10789797(4) \\ f_{\text{NA}} = -4.79298000(3) \\ f_L = -\frac{1}{144} \left( 42\zeta(3) - 164 + 13\pi^2 + 96 \ln 2 \right) \\ f_H = 0.223672013(2) \end{array} \right. \quad \text{Precision } \sim 10^{-9}$$

- A. Czarnecki and K. Melnikov, “Charmonium decays:  $J/\psi \rightarrow e^+e^-$  and  $\eta_c \rightarrow \gamma\gamma$ ”, Phys. Lett. B **519**, 212 (2001) [hep-ph/0109054].

$$\gamma\gamma^* \rightarrow \eta_c :$$

- Numerical Result @  $Q^2 = 0$

$$f_{1b1}^{(2)}(0) = \left( 0.64696557 + 2.07357556 i \right) n_H C_F T_f + \left( 0.73128459 + \frac{\pi}{9} (\pi^2 - 15) i \right) C_F T_f \sum_{f=\text{light quark}} \frac{e_f^2}{e_Q^2}$$

Missing Part

- A. Czarnecki and K. Melnikov, “Charmonium decays:  $J/\psi \rightarrow e^+e^-$  and  $\eta_c \rightarrow \gamma\gamma$ ”, Phys. Lett. B **519**, 212 (2001) [hep-ph/0109054].

$$\gamma\gamma^* \rightarrow \eta_c :$$

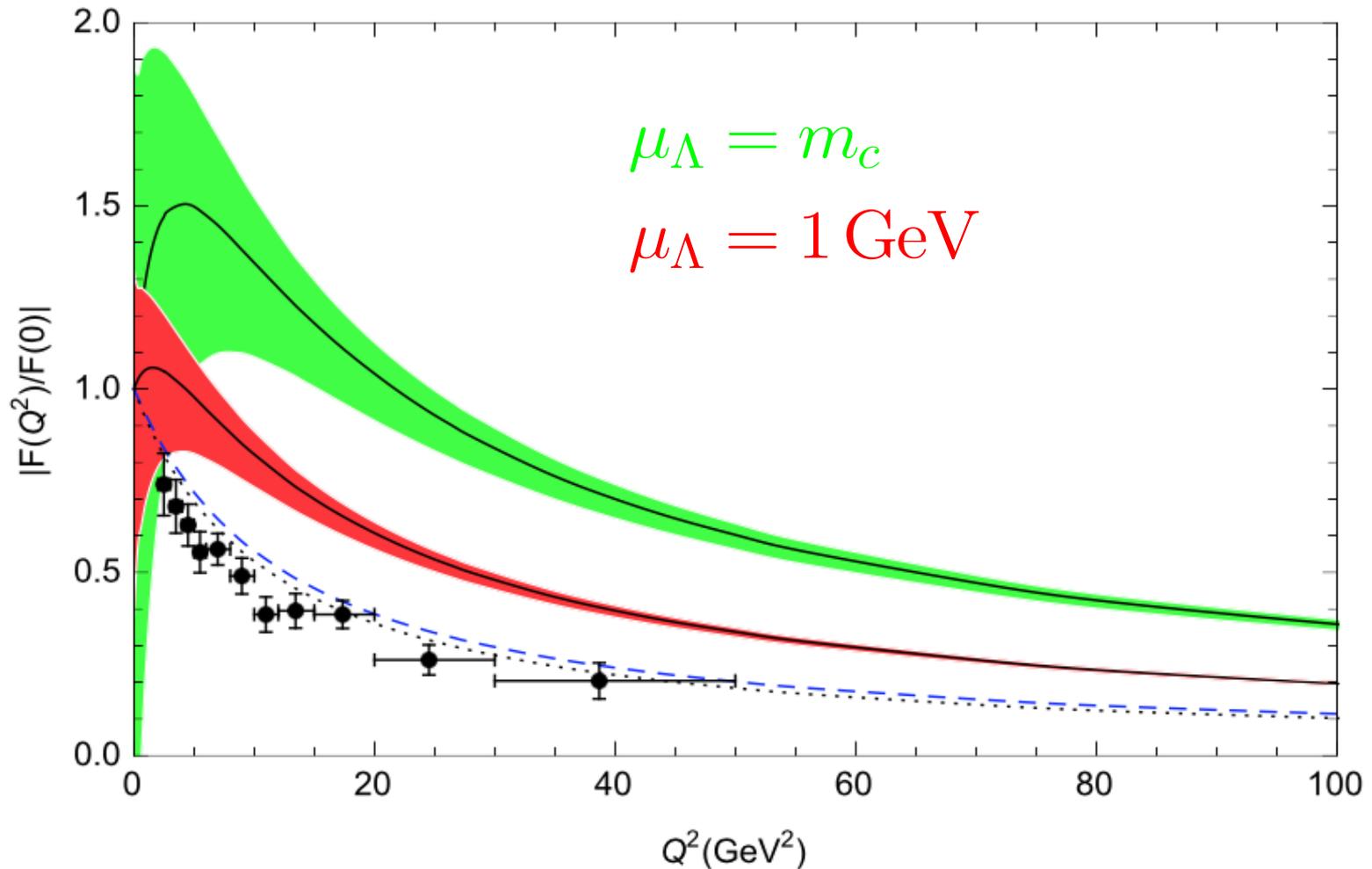
## ● Numerical Result @NNLO

$\tau$	1	5	10	25	50
$f_{\text{reg}}^{(2)}$	-59.420(6)	-61.242(6)	-61.721(7)	-61.843(8)	-61.553(8)
$f_{\text{lbl}}^{(2)}$	0.49(4)	-0.48(5)	-1.09(5)	-2.12(6)	-3.10(6)
	-0.65(3) $i$	-0.72(4) $i$	-0.71(4) $i$	-0.69(4) $i$	-0.68(4) $i$
$f_{\text{reg}}^{(2)}$	-59.636(6)	-61.278(6)	-61.716(7)	-61.864(8)	-61.668(8)
$f_{\text{lbl}}^{(2)}$	0.8(1)	-5.6(2)	-9.4(2)	-15.3(2)	-20.3(2)
	-12.44(8) $i$	-13.5(2) $i$	-13.8(2) $i$	-14.0(2) $i$	-14.1(2) $i$

$f_{\text{reg}}^{(2)}(\tau)$  and  $f_{\text{lbl}}^{(2)}(\tau)$  at some typical values of  $\tau$ .  
 The first two rows for  $\eta_c$  and the last two for  $\eta_b$

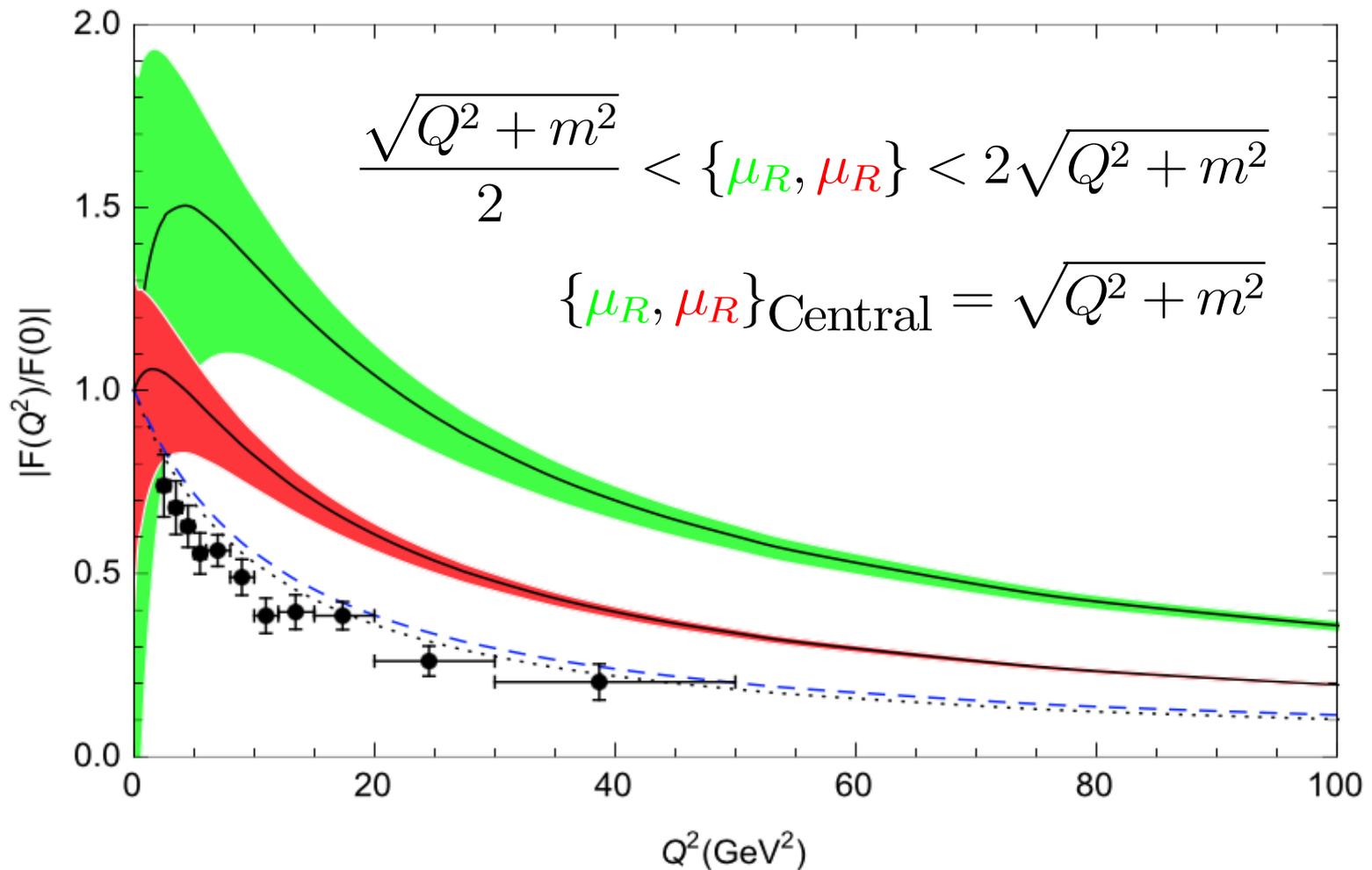
$$\gamma\gamma^* \rightarrow \eta_c :$$

- Confront the *BaBar* Data



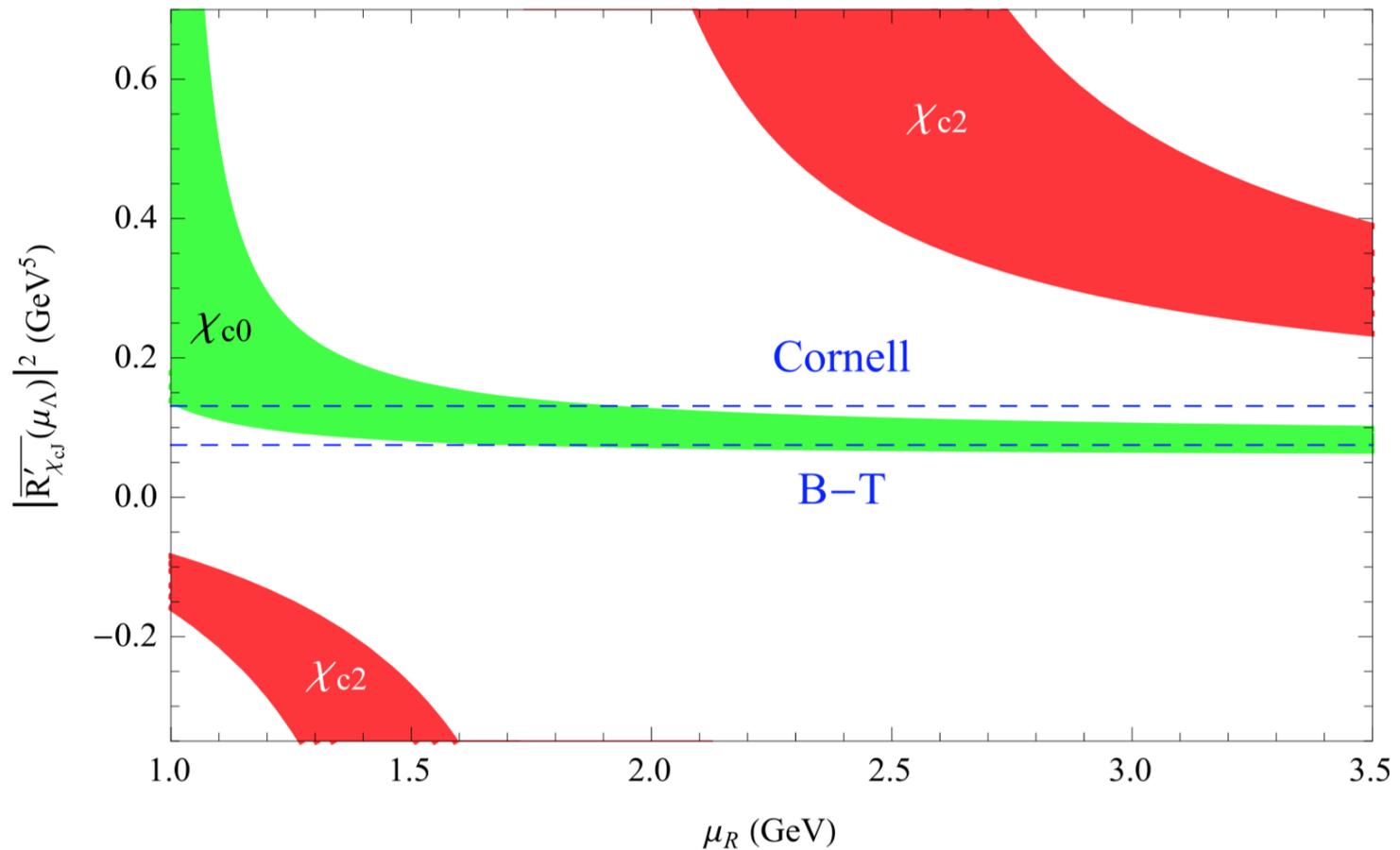
$$\gamma\gamma^* \rightarrow \eta_c :$$

● Confront the *BaBar* Data



$\chi_{c0,2} \rightarrow \gamma\gamma$  :

- Determine  $\left| \overline{R'_{\chi_{c0,2}}}(\mu_\Lambda) \right|^2$  from the *BESIII* Data



# Summary & Outlook:

- The NNLO NRQCD predictions to  $\gamma\gamma^* \rightarrow \eta_c$  form factor and  $\chi_{c0,2} \rightarrow \gamma\gamma$  fail to explain the experimental data!
- Higher corrections needed, including N<sup>3</sup>LO QCD corrections and relativistic corrections to those processes.
- Only exclusive processes considered, how about **Inclusive Decay?**

$\eta_c \rightarrow$  Light Hadrons

# Innovations:

- **1st NNLO Calculation in S-wave Quarkonium Production** process.
- **1st NNLO Calculation in P-wave Quarkonium Decay** process.
- **1st complete result for  $\eta_{c,b} \rightarrow \gamma\gamma$  including Light-by-Light in very High Precision.**

# Contributions:

- Combine **FORM** and **FeynCalc** in FeynCalc / FormLink package.
- Implement **CubPack** in FIESTA with **Quadruple Precision**.
- Key algorithm for **Optimization** in High Precision Computations using FIESTA & CubPack.

Thanks for your attention!