

# Scaling behaviour of the $p_T$ spectra for identified hadrons in pp collisions

张文超  
(陕西师范大学, 西安)

August 22-26, 2016, Hefei, China

[W. C. Zhang, J. Phys. G, **43**, 015003, 2016]

# Outline

- Introduction
- Method to search for the scaling behaviour
- Scaling behaviour of identified  $\pi$ ,  $K$  and  $p$
- Colour string percolation model
- Conclusions

# Introduction

- One of the most important observations in high energy collisions is the  $p_T$  spectra for different species of final state particles.
- From the spectra, we can learn a lot about the regularities of the particle productions
- In many studies, searching for a scaling behaviour of the  $p_T$  spectra is helpful to reveal these regularities.

# Introduction

- Bjorken scaling in DIS

PDFs depend not on the energy of DIS but on the ratio of the energy to the momentum transfer in the scattering.

[ J. D. Bjorken and E. A. Paschos, Phys. Rev. 185, 1975, 1969.]

- KNO (Koba-Nielsen-Olesen) scaling

The multiplicity distributions  $P_n$  collapse on a universal scaling curve when presented as a function of  $z = n/\langle n \rangle$ :  $\Psi(z) = \langle n \rangle P_n$

[ Z. Koba, H. B. Nielsen and P. Olesen, Nucl. Phys. B 40, 317, 1972.]

# Introduction

- Scaling behaviour in the **pion  $p_T$  spectra** with different centralities at **midrapidity** in Au+Au collisions at RHIC.

[ R. C. Hwa and C. B. Yang, Phys. Rev. Lett. 90, 212301, 2003]

- This scaling behaviour of pions was also found in **noncentral** regions in Au+Au and d+Au collisions.

[ L. L. Zhu and C. B. Yang, Phys. Rev. C 75, 044904, 2007]

- An analogous scaling behaviour in the **proton** and **anti-proton**  $p_T$  spectra with different centralities at **midrapidity** in Au+Au collisions.

[ W. C. Zhang et al. Phys. Rev. C 76, 044910, 2007]

# Introduction

- Recently, we observed a scaling behaviour in the  $p_T$  spectra of **inclusive charged hadrons** in  $pp(p\bar{p})$  collisions at  $\sqrt{s} = 0.9, 2.76$  and 7 (0.63, 1.8 and 1.96) TeV.

[W. C. Zhang, C. B. Yang, J. Phys. G, **41**, 105006, 2014]

- In this talk, we will show the scaling behaviour in the  $p_T$  spectra of identified **pions, kaons and protons** in pp collisions at  $\sqrt{s} = 0.9, 2.76$  and 7 TeV.

[W. C. Zhang, J. Phys. G, **43**, 015003, 2016]

# Introduction

- These identified charged hadron  $p_T$  spectra were published by the ALICE collaboration. [\[Eur. Phys. J. C, 71, 1655, 2011\]](#)  
[\[Phys. Lett. B, 736, 196, 2014\]](#)
- $\pi$ :  $(\pi^+ + \pi^-)/2$ ,  $K$ :  $(K^+ + K^-)/2$ ,  $p$ :  $(p + \bar{p})/2$  [\[Eur. Phys. J. C, 75, 226, 2015\]](#)

- The  $p_T$  coverage for pions:

0.9 TeV: 0.1-2.6 GeV/ c

2.76 TeV : 0.1-20.0 GeV/ c ← The largest coverage

7 TeV: 0.1-3.0 GeV/ c

## Method to search for the scaling behaviour

- Take the **pion**  $p_T$  spectra as an example
- Define  $z = p_T/K$  and  $\Phi(z) = A \cdot (2\pi p_T)^{-1} d^2N/dp_T dy|_{p_T=Kz}$
- By choosing proper  $K$  and  $A$ , the scaled  $p_T$  spectra at different energies can be **put into one curve**.
- Here, we set  $K$  and  $A$  for the collisions at 2.76 TeV to be **1**.
- For the collisions at 0.9 and 7 TeV,  $K$  and  $A$  are determined by the **quality factor (QF) method**.



## Method to search for the scaling behaviour

- QF is defined as

$$QF(K, A) = \left[ \sum_i \frac{(v^i - v^{i-1})^2}{(u^i - u^{i-1})^2 + \varepsilon^2} \right]^{-1}$$

$v^i = \log(A \cdot (2\pi p_T^i)^{-1} d^2 N^i / dp_T^i dy^i)$

$u^i = p_T^i / K$

$\varepsilon = 0.01$

- Before entering the QF formula,  $(u^i, v^i)$  has been rescaled so that  $0 \leq u^i, v^i \leq 1$ , and  $u^i$  are ordered.
- Two successive data points being **close in  $u$**  and **far in  $v$**  will give **a large contribution** to the **sum** in QF.

## Method to search for the scaling behaviour

- As a result, a set of data points with a **small sum** ( thus a **large QF**) is expected to lie close to a **unique curve**.
- For the scaling parameters at 0.9 (7) TeV, we utilize the data points at 0.9 ( 7) and 2.76 TeV to determine them.
- The best set of  $(K, A)$  for 0.9 (7) TeV is chosen to be the one which globally **maximizes the QF**.

# Method to search for the scaling behaviour

## Pions

$\sqrt{s}$ (TeV)	$K$	$A$
0.9	$0.93 \pm 0.04$	$1.11 \pm 0.14$
2.76	1	1
7	$1.06 \pm 0.02$	$0.89 \pm 0.07$

## Kaons

$\sqrt{s}$ (TeV)	$K$	$A$
0.9	$0.913 \pm 0.007$	$1.05 \pm 0.01$
2.76	1	1
7	$1.14 \pm 0.04$	$1.05 \pm 0.14$

## Protons

$\sqrt{s}$ (TeV)	$K$	$A$
0.9	$0.926 \pm 0.001$	$1.08 \pm 0.02$
2.76	1	1
7	$1.108 \pm 0.006$	$1.04 \pm 0.02$

## Scaling behaviour of identified $\pi$ , $K$ and $p$

- Tsallis distributions to parameterize the spectra at 2.76 TeV

$$\Phi(z) = C_q \left[ 1 - (1 - q) \frac{\sqrt{m^2 + z^2} - m}{z_0} \right]^{\frac{1}{1-q}}$$

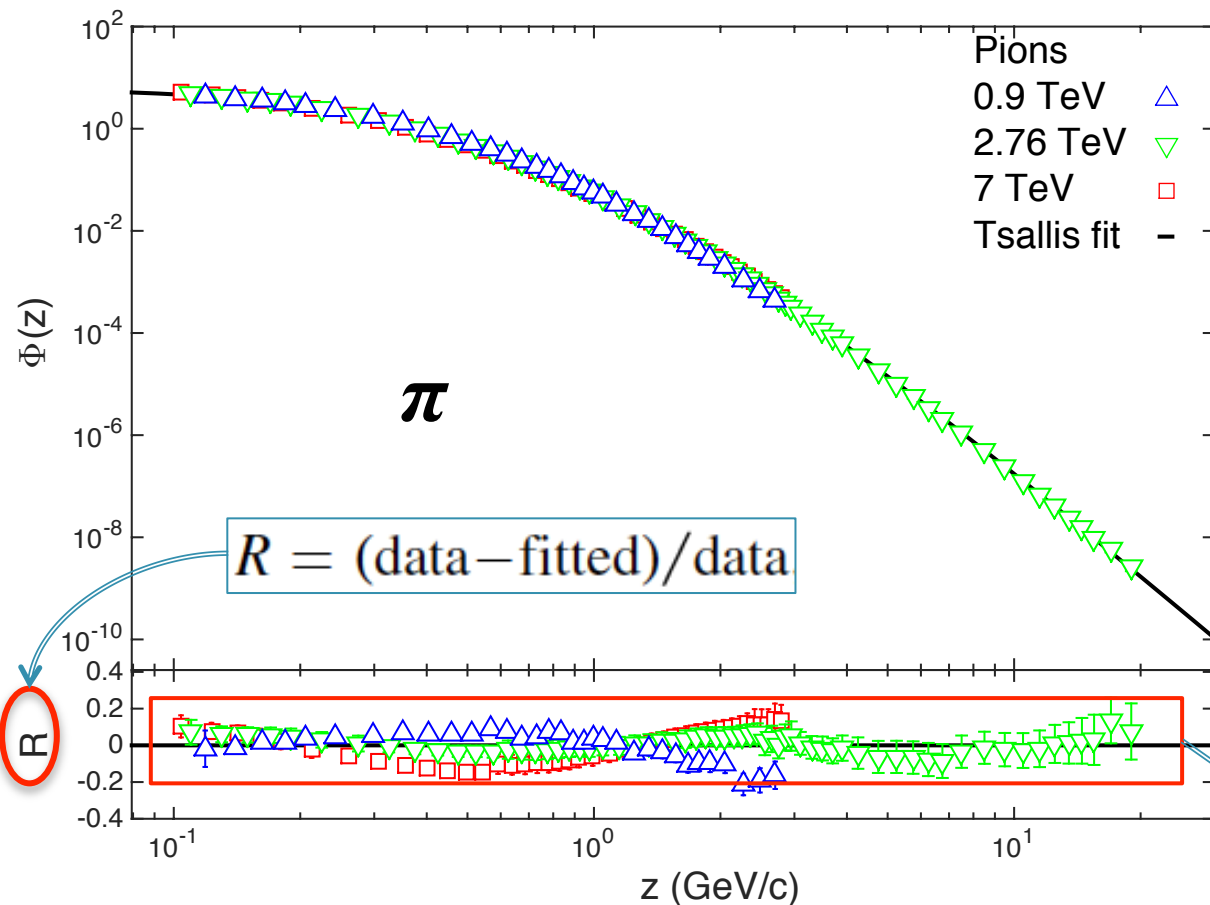
Diagram illustrating the Tsallis distribution function  $\Phi(z)$  and its parameters:

- Fit parameters:**  $C_q$ ,  $q$ ,  $z_0$  (indicated by arrows pointing to the boxed text)
- The mass of  $\pi$ ,  $K$ ,  $p$**  (indicated by an arrow pointing to the boxed text)

0.1(0.1 and 0.1) GeV/c at 2.76 TeV

0.14(0.494 and 0.938) GeV/c<sup>2</sup>

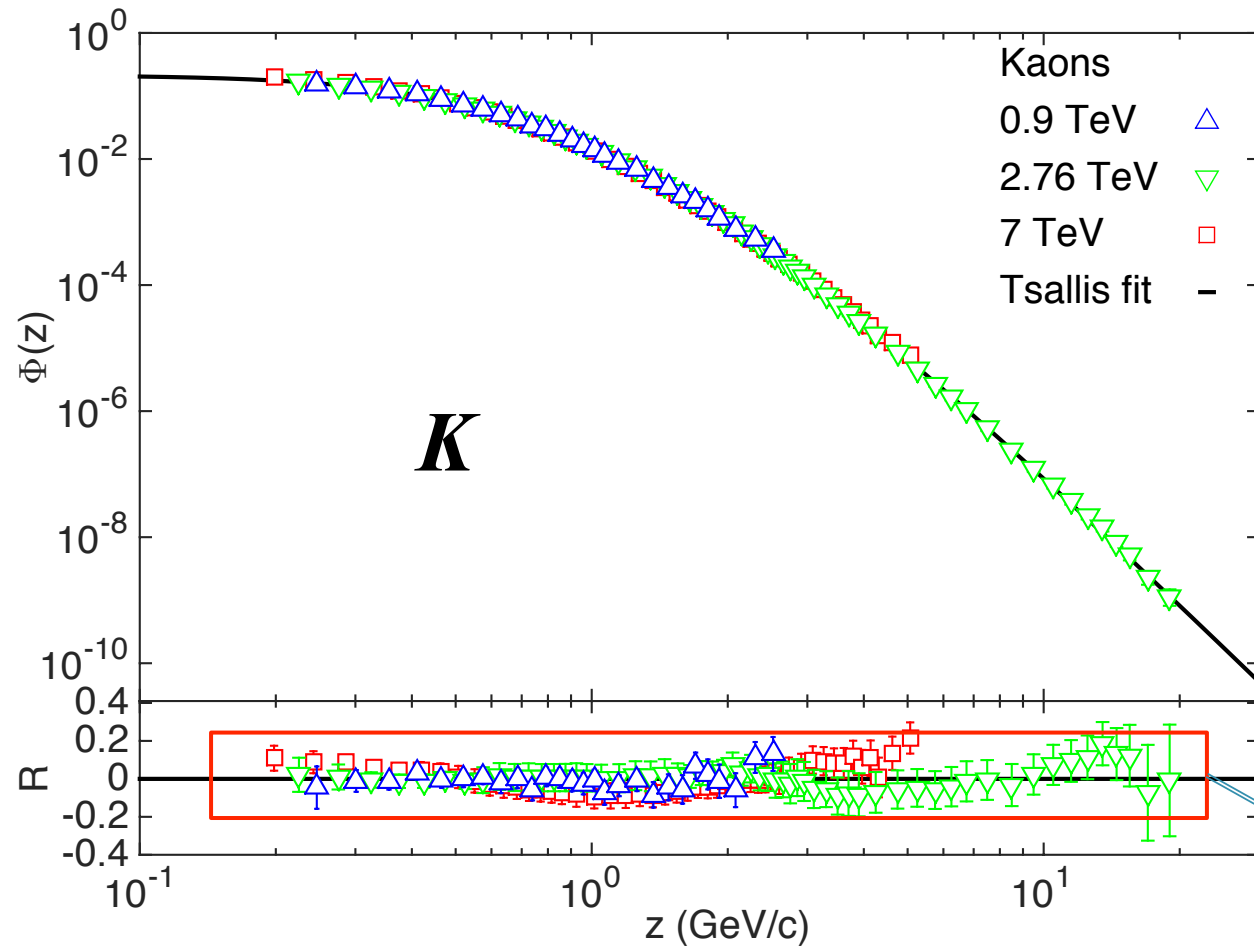
# Scaling behaviour of identified $\pi$ , $K$ and $p$



Pions	
$C_q$	$6.01 \pm 0.08$
$q$	$1.1416 \pm 0.0007$
$z_0 \text{ (GeV/c)}$	$0.1308 \pm 0.0008$
$\chi^2/\text{dof}$	0.49

agreement is within 20%.

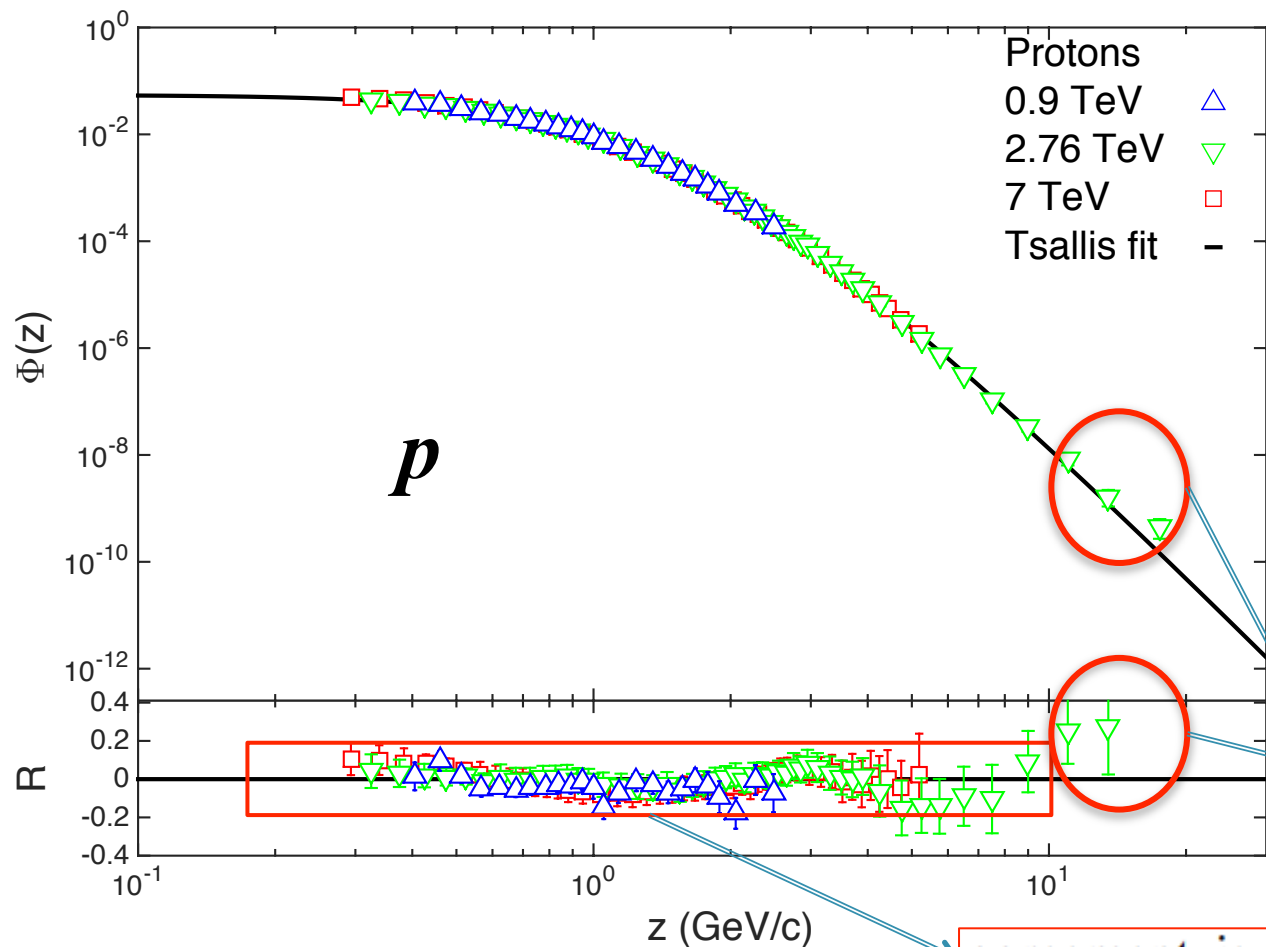
# Scaling behaviour of identified $\pi$ , $K$ and $p$



Kaons	
$C_q$	$0.214 \pm 0.002$
$q$	$1.1402 \pm 0.0004$
$z_0$ (GeV/c)	$0.193 \pm 0.001$
$\chi^2/\text{dof}$	0.23

agreement is within 20%.

# Scaling behaviour of identified $\pi$ , $K$ and $p$



Protons	
$C_q$	$0.0546 \pm 0.0008$
$q$	$1.115 \pm 0.002$
$z_0$ (GeV/c)	$0.220 \pm 0.002$
$\chi^2/\text{dof}$	0.40

The yield and exp. errors have the same order.

agreement is within 20%.

# Colour string percolation model

- Colour strings are stretched between the partons of protons in pp collisions.
- These strings then decay into new ones with the emission of  $q\bar{q}$  pairs.
- Identified hadrons are produced through the hadronization of these new strings.

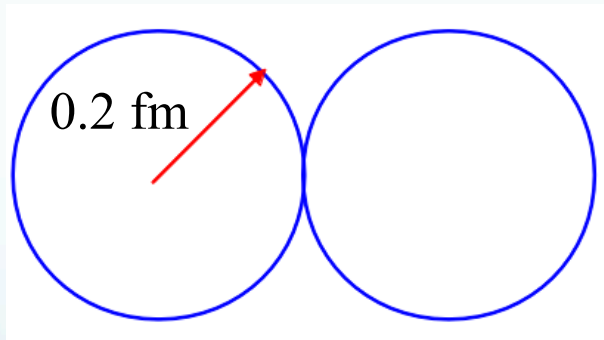
[L. Cunqueiro et al, Eur. Phys. J. C, 53, 585–9, 2008]

[J. Dias de Deus J et al, Eur. Phys. J. C, 41 229–41, 2005]



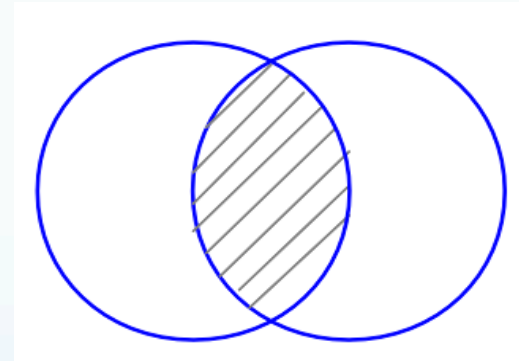
# Colour string percolation model

- These strings are viewed as small areas in the transverse plane.
- When there are  $n$  strings, they start to overlap and form clusters with transverse areas of  $S_n$ .



get in touch,  $S_n = nS_1$ ,  $nS_1/S_n = 1$

degree of string overlap



maximumly overlap,  $S_n = S_1$ ,  $nS_1/S_n = n$

$$\langle p_T^2 \rangle_{ni} = \sqrt{nS_1 / S_n} \langle p_T^2 \rangle_{1i}$$

# Colour string percolation model

- The  $p_T$  spectra of identified particles produced in pp collisions:

$$\frac{d^2 N}{2\pi p_T dp_T dy} = C \int_0^\infty W(x) f(x, p_T) dx$$

normalization const.

$$W(x) = \frac{\gamma}{\Gamma(k)} (\gamma x)^{k-1} \exp(-\gamma x)$$

cluster size distribution,  $\gamma$ ,  $k$  free para.

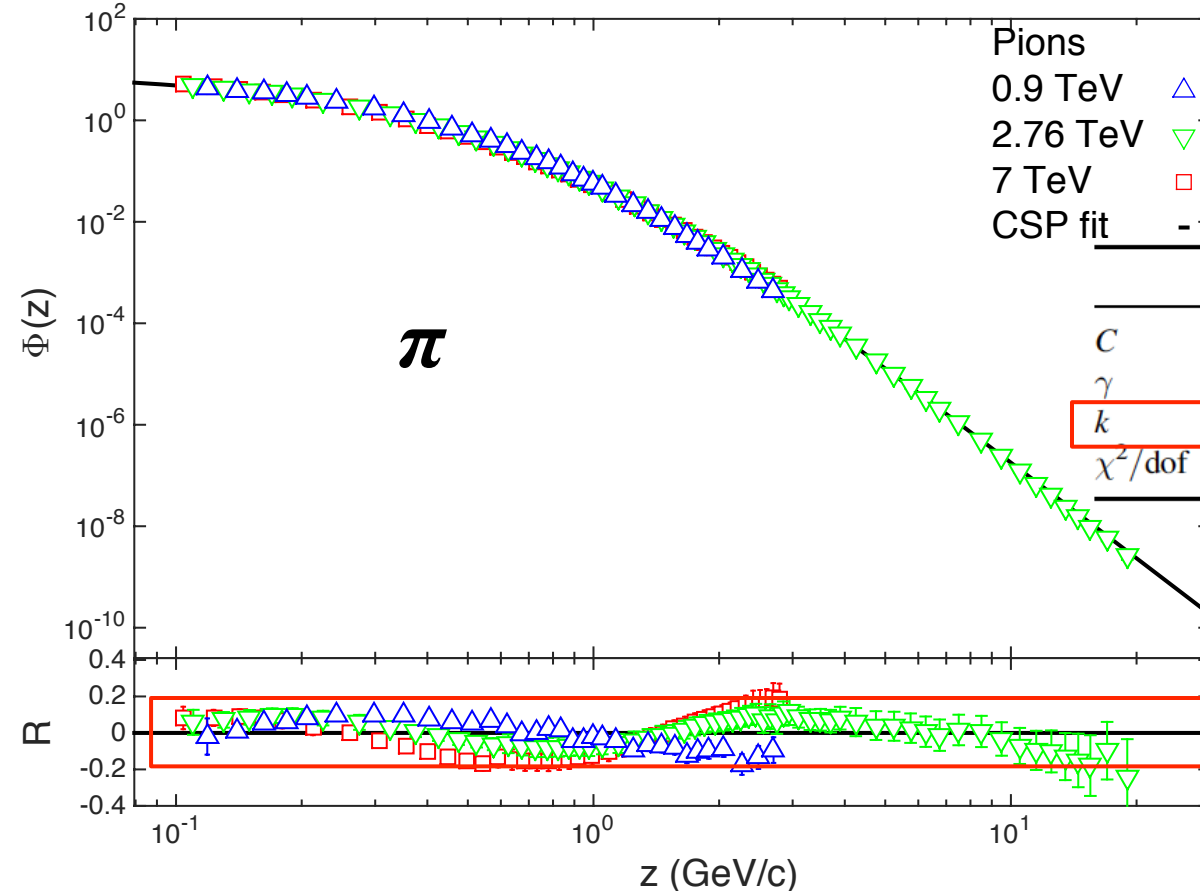
$$f(x, p_T) = \exp(-\sqrt{2x} p_T)$$

spectra produced by one cluster

# Colour string percolation model

- The CSP model can describe the scaling behaviour presented

in  $z$  well.



Pions

0.9 TeV

2.76 TeV

7 TeV

CSP fit

	Pions	Kaons	Protons
$C$	$9.1 \pm 0.2$	$0.53 \pm 0.01$	$0.160 \pm 0.007$
$\gamma$	$0.140 \pm 0.003$	$0.342 \pm 0.008$	$0.91 \pm 0.07$
$k$	$3.15 \pm 0.02$	$3.33 \pm 0.02$	$5.0 \pm 0.2$
$\chi^2/\text{dof}$	1.53	0.91	1.91

agreement is within 20%.

# Colour string percolation model

- Under the transformation

$$x \rightarrow x' = \lambda x, \gamma \rightarrow \gamma' = \gamma / \lambda, p_T \rightarrow p'_T = p_T / \sqrt{\lambda}$$

$$W(x) = \frac{\gamma}{\Gamma(k)} (\gamma x)^{k-1} \exp(-\gamma x)$$

$$f(x, p_T) = \exp(-\sqrt{2x} p_T)$$

$$\frac{d^2 N}{2\pi p_T dp_T dy} = C \int_0^\infty W(x) f(x, p_T) dx$$

Invariant

The scaling behaviour  
we are looking for.

# Colour string percolation model

- Compare the  $p_T$  transformation in the CSP model  $p'_T \rightarrow p'_T \sqrt{\lambda}$  with the one in the search for the scaling behaviour  $p_T \rightarrow p_T / K$

$$K \sim 1 / \sqrt{\lambda}$$

$$\lambda = \langle S_n / nS_1 \rangle^{1/2}$$

$$K \sim \langle nS_1 / S_n \rangle^{1/4}$$

J. Dias de Deus J and C. Pajares,  
Phys. Lett. B, 642, 455–8, 2006

- $nS_1/S_n$  grows with energy,  $K$  should also increase with energy

Pions

$\sqrt{s}$ (TeV)	$K$	$A$
0.9	$0.93 \pm 0.04$	$1.11 \pm 0.14$
2.76	1	1
7	$1.06 \pm 0.02$	$0.89 \pm 0.07$

Kaons

$\sqrt{s}$ (TeV)	$K$	$A$
0.9	$0.913 \pm 0.007$	$1.05 \pm 0.01$
2.76	1	1
7	$1.14 \pm 0.04$	$1.05 \pm 0.14$

Protons

$\sqrt{s}$ (TeV)	$K$	$A$
0.9	$0.926 \pm 0.001$	$1.08 \pm 0.02$
2.76	1	1
7	$1.108 \pm 0.006$	$1.04 \pm 0.02$

That's indeed what we observed!

# Summary

- We observed the scaling behaviour in the spectra of identified particles at 0.9, 2.76 and 7 TeV.
- This scaling behaviour is exhibited when  $p_T \rightarrow z = p_T / K$ .
- $K$  is determined by the QF method.
- The scaling behaviour could be explained by the CSP model in a quantitative way.

## Thanks for your attention!

# Back up slides

## Scaling behaviours of identified $\pi$ , $K$ and $p$

- With different choices of  $K$  and  $A$ , the scaling functions  $\Phi(z)$  are different.

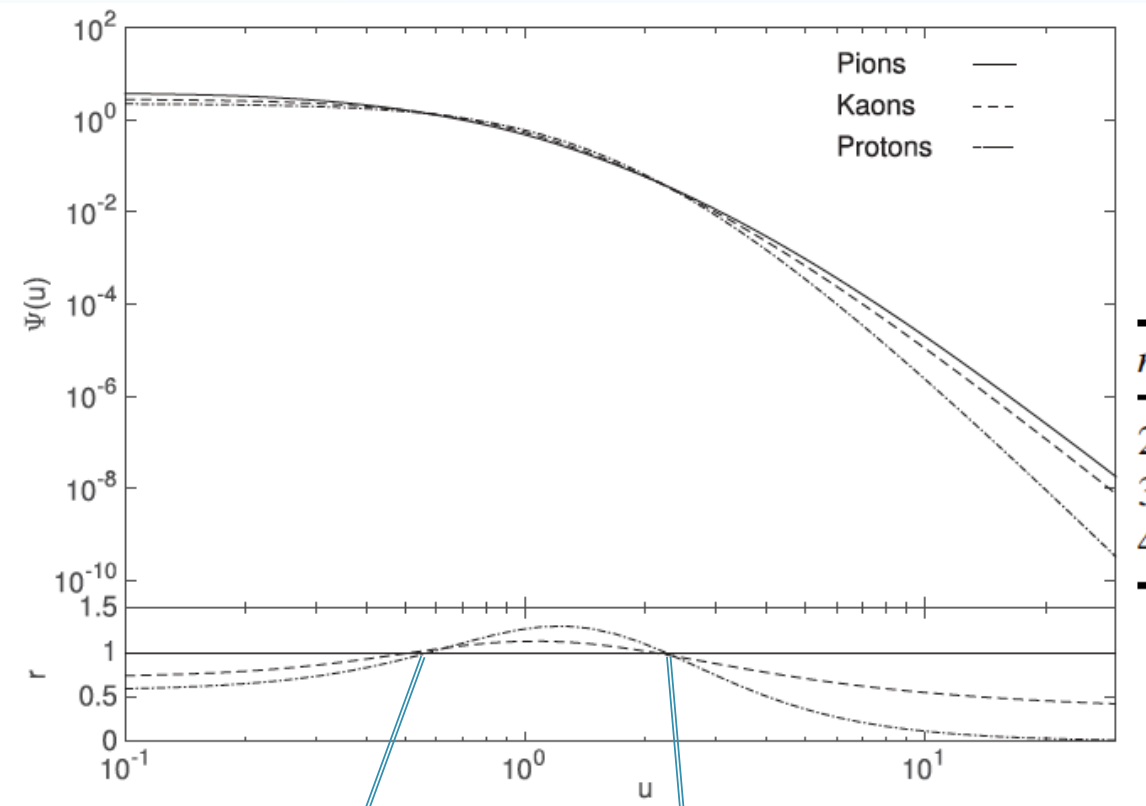
- In order to get rid of this arbitrariness, we introduce another scaling variable,  $u = z/\langle z \rangle = p_T/\langle p_T \rangle$ , and the corresponding

normalized scaling function  $\Psi(u) = \langle z \rangle^2 \Phi(\langle z \rangle u) / \int_0^\infty \Phi(z) z dz$

- Here  $\langle z \rangle = \int_0^\infty z \Phi(z) z dz / \int_0^\infty \Phi(z) z dz$



# Comparison among the scaling functions of $\pi$ , $K$ and $p$



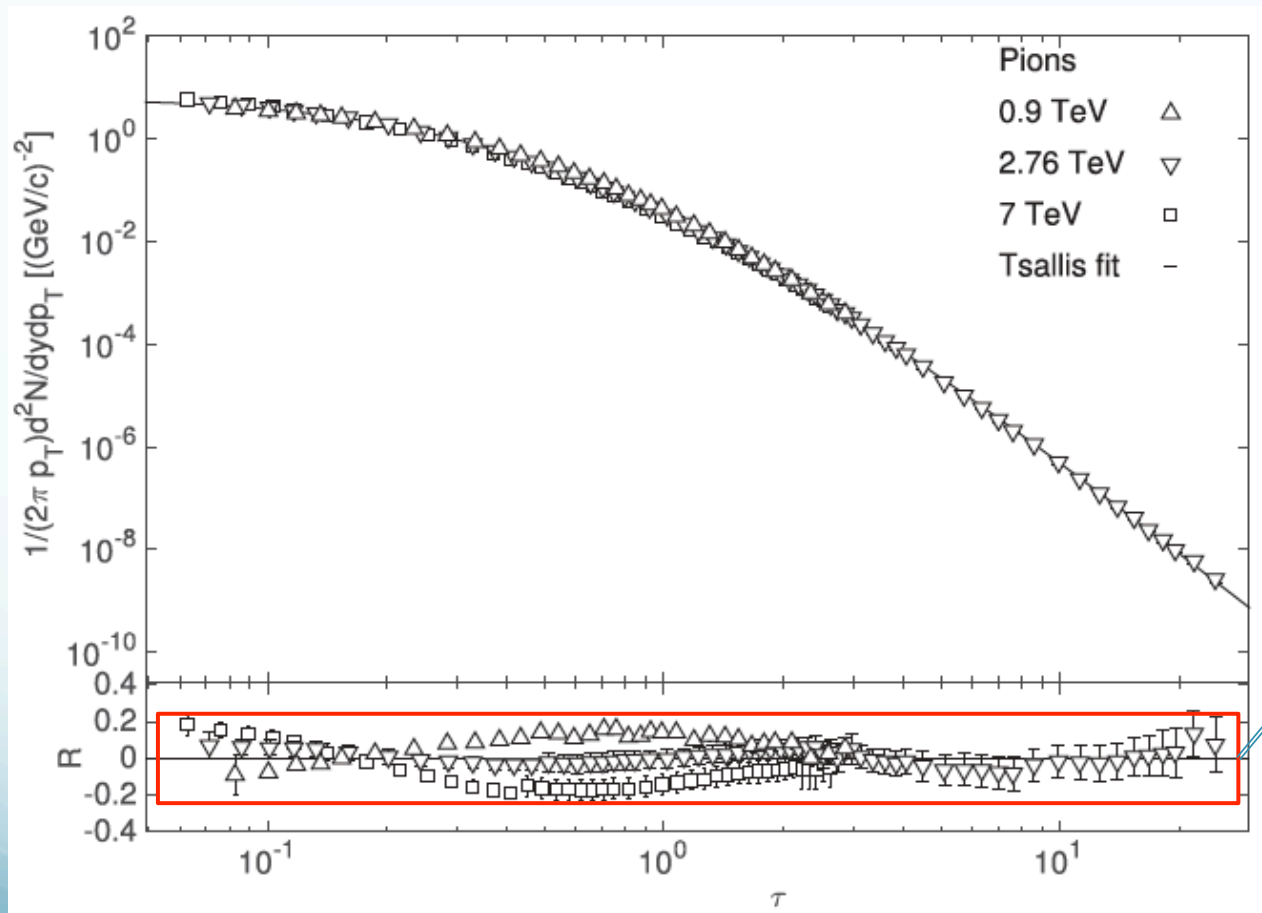
$$\frac{\langle p_T^n \rangle}{\langle p_T \rangle^n} = \int_0^\infty u^n \Psi(u) u du$$

$n$	Pions	Kaons	Protons
2	$1.9 \pm 0.2$	$1.7 \pm 0.1$	$1.5 \pm 0.2$
3	$6.5 \pm 1.0$	$4.9 \pm 0.3$	$3.3 \pm 0.5$
4	$44.8 \pm 8.2$	$26.8 \pm 2.2$	$10.7 \pm 2.2$

$u \approx 0.6$

$u \approx 2$

# Comparison to the scaling behaviour presented in $\tau$



$$\tau = p_T / Q_0 (p_T / \sqrt{s})^{\lambda/2}$$

$$Q_0 = 1 \text{ GeV}/c$$

$$\lambda \approx 0.27.$$

agreement is within 20%.

Comparable!

# Colour string percolation model

- The fragmentation function for the cluster is chosen as the Schwinger formula  $f(x) = \exp(-p_T^2 x)$
- However, this function only describes the spectra well at the soft  $p_T$  region.
- In order to describe the spectra at the high  $p_T$  region as well, the Schwinger formula should be replaced by  $f(x, p_T) = \exp(-\sqrt{2x} p_T)$

# Colour string percolation model

- The rate of the  $K$  values increasing with energy for pions is different with the one for kaons or protons.
- The ratio between the values of  $K$  at different energies should be equal to the ratio between the values of  $\langle p_T \rangle$  at different energies.

$$\langle p_T \rangle = \frac{\int_0^\infty \int_0^\infty W(x) f(x, p_T) p_T^2 dx dp_T}{\int_0^\infty \int_0^\infty W(x) f(x, p_T) p_T dx dp_T}$$

$f(x, p_T)$  are the same for pions (kaons or protons) at different energies.

The cluster's size distributions  $W(x)$  are different for different particles.