

Gluon TMDs in polarized high energy processes and small-x limit

Yajin Zhou

D. Boer, S. Cotogno, T. Daal, P. J. Mulders, A. Signori, YZ,
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周雅瑾

Outline

- 1 Introduction
- 2 Gluon TMDs for hadrons of spin ≤ 1
- 3 Wilson loop TMDs for hadrons of spin ≤ 1
- 4 Summary and prospect

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1 Introduction

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TMDs

Process involving hadron states in initial or final states

Cross section= Hard part * PDFs/PFFs

TMDs

Process involving hadron states in initial or final states

Cross section= Hard part * PDFs/PFFs

Cross section= Hard part * PDFs/PFFs/**TMDs**

TMDs

Process involving hadron states in initial or final states

Cross section= Hard part * PDFs/PFFs

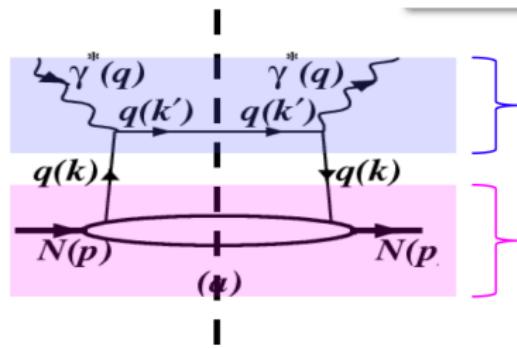
Cross section= Hard part * PDFs/PFFs/TMDs

TMDs: Transverse momentum dependent PDFs/PFFs



$$\text{parton momentum: } k^\mu = xP^\mu + k_T^\mu + \sigma n^\mu$$

quark quark correlator



The calculable hard part

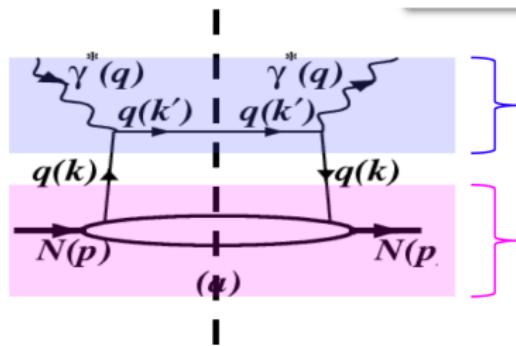
The quark-quark correlator, Φ

$$\Phi(x, p) = \int \frac{d^4 \xi}{(2\pi)^4} e^{ip \cdot \xi} \langle P | \bar{\Psi}(0) \Psi(\xi) | P \rangle \quad \star \text{ Unintegrated}$$

$$\Phi(x, p_T) = \int \frac{d\xi^- d^2 \xi_T}{(2\pi)^4} e^{ip \cdot \xi} \langle P | \bar{\Psi}(0) \Psi(\xi) | P \rangle_{\xi \cdot n = \xi^+ = 0} \quad \star \text{ TMD}$$

$$\Phi(x) = \int \frac{d\xi^-}{(2\pi)^4} e^{ip \cdot \xi} \langle P | \bar{\Psi}(0) \Psi(\xi) | P \rangle_{\xi \cdot n = 0, \xi^T = 0} \quad \star \text{ collinear}$$

quark quark correlator



The calculable hard part

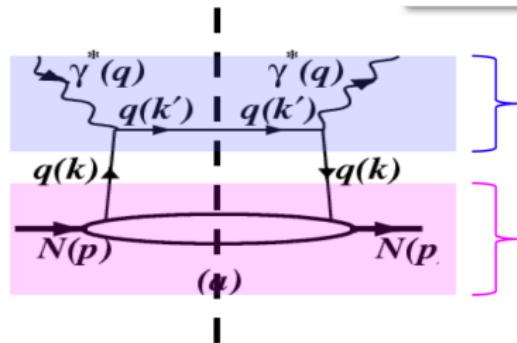
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quark quark correlator



The calculable hard part

The quark-quark correlator, Φ

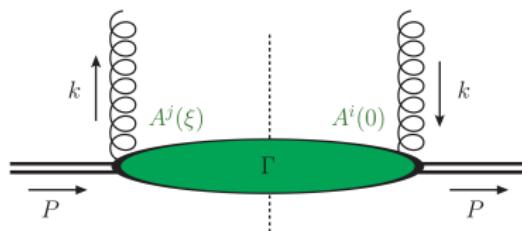
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$$\Phi(x) = \int \frac{d\xi^-}{(2\pi)^4} e^{ip\cdot\xi} \langle P | \bar{\Psi}(0)\Psi(\xi) | P \rangle_{\xi\cdot n = 0, \xi^T = 0} \quad \star \text{ collinear}$$

$$\Phi(x, p_T) = \int \frac{d\xi^- d^2\xi_T}{(2\pi)^4} e^{ip\cdot\xi} |P\rangle \bar{\Psi}(0) U_{[0,\xi]} \Psi(\xi) |P\rangle_{\xi\cdot n = 0} \quad \text{Gauge invariant TMD}$$

gluon-gluon correlator



$$\Gamma^{[U,U']\mu\nu;\rho\sigma}(k;P,n) \equiv \int \frac{d^4\xi}{(2\pi)^4} e^{ik\cdot\xi} \langle P| F^{\mu\nu}(0) U_{[0,\xi]} F^{\rho\sigma}(\xi) U'_{[\xi,0]} |P\rangle,$$

$$\Gamma^{[U,U']\mu\nu;\rho\sigma}(x, \mathbf{k}_T; P, n) \equiv \int \frac{d\xi \cdot P d^2\xi_T}{(2\pi)^3} e^{ik\cdot\xi} \langle P| F^{\mu\nu}(0) U_{[0,\xi]} F^{\rho\sigma}(\xi) U'_{[\xi,0]} |P\rangle \Big|_{\xi \cdot n = 0}$$

extracted parton

target pol.

	quark	gluon
unpolarized	😊	😊
vector polarised	😊	😊
tensor polarised	😊	?

Tangerman, Mulders 1994;
Boer, Mulders 1997;
Bacchetta, Mulders 2000;
Mulders, Rodrigues 2001

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parameterization of gluon correlator

$$\Gamma^{[U,U']\mu\nu;\rho\sigma}(x, \mathbf{k}_T; P, n) \equiv \int \frac{d\xi \cdot P d^2\xi_T}{(2\pi)^3} e^{ik \cdot \xi} \langle P, S, T | F^{\mu\nu}(0) U_{[0,\xi]} F^{\rho\sigma}(\xi) U'_{[\xi,0]} | P, S, T \rangle \Big|_{\xi \cdot n = 0}$$

- parameterization in terms of Lorentz structure: $g_{\mu\nu}, \epsilon_{\mu\nu\rho\sigma}$ vectors/tensors k, P, S, T, n

$$S^\mu = S_L \frac{P^\mu}{M} + S_T^\mu - M S_L n^\mu,$$

$$T^{\mu\nu} = \frac{1}{2} \left[\frac{2}{3} S_{LL} g_T^{\mu\nu} + \frac{4}{3} S_{LL} \frac{P^\mu P^\nu}{M^2} + \frac{S_{LT}^{\{\mu} P^{\nu\}}}{M} + S_{TT}^{\mu\nu} - \frac{4}{3} S_{LL} P^{\{\mu} n^{\nu\}} - M S_{LT}^{\{\mu} n^{\nu\}} + \frac{4}{3} M^2 S_{LL} n^\mu n^\nu \right],$$

- respecting Hermicity and parity

$$\Gamma^{[U,U']\rho\sigma;\mu\nu*}(k, P, S, T, n) = \Gamma^{[U,U']\mu\nu;\rho\sigma}(k, P, S, T, n),$$

$$\Gamma^{[U,U']\mu\nu;\rho\sigma}(k, P, S, T, n) = \Gamma_{\mu\nu;\rho\sigma}^{[U,U']}(\bar{k}, \bar{P}, -\bar{S}, \bar{T}, \bar{n})$$

- T-odd functions are allowed

$$\Gamma^{[U,U']\mu\nu;\rho\sigma*}(k, P, S, T, n) = \Gamma_{\mu\nu;\rho\sigma}^{[U^T,U'^T]}(\bar{k}, \bar{P}, \bar{S}, \bar{T}, \bar{n})$$

parameterization of gluon correlator

Tips:

- The leading twist contribution from $\Gamma^{[U,U']ni;nj}(x, \mathbf{k}_T; P, n)$, denoted as $\Gamma^{ij}(x, \mathbf{k}_T)$
- All the terms including n vector is twist-3
- Usually the expansion is in constant tensors $g_T^{\mu\nu}$ and $\epsilon_T^{\mu\nu}$

$$\Gamma_O^{ij}(x, \mathbf{k}_T) = \frac{x}{2} \frac{P^+}{M} \left[-g_T^{ij} G(x, \mathbf{k}_T^2) + \left(\frac{k_T^i k_T^j}{M^2} + g_T^{ij} \frac{\mathbf{k}_T^2}{2M^2} \right) H^\perp(x, \mathbf{k}_T^2) \right],$$

$$\Gamma_L^{ij}(x, \mathbf{k}_T) = \frac{x}{2} \frac{P^+}{M} \left[-i\epsilon_T^{ij} S_L \Delta G_L(x, \mathbf{k}_T^2) + \frac{\epsilon_T^{k_T \{i} k_T^{j\}}}{2M^2} S_L \Delta H_L^\perp(x, \mathbf{k}_T^2) \right],$$

$$\begin{aligned} \Gamma_T^{ij}(x, \mathbf{k}_T) = & \frac{x}{2} \frac{P^+}{M} \left[-g_T^{ij} \frac{\epsilon_T^k S_T}{M} G_T(x, \mathbf{k}_T^2) - i\epsilon_T^{ij} \frac{\mathbf{k}_T \cdot \mathbf{S}_T}{M} \Delta G_T(x, \mathbf{k}_T^2) \right. \\ & + \frac{\epsilon_T^{k_T \{i} k_T^{j\}}}{2M^2} \frac{\mathbf{k}_T \cdot \mathbf{S}_T}{M} \Delta H_T^\perp(x, \mathbf{k}_T^2) \\ & \left. + \frac{\epsilon_T^{k_T \{i} S_T^{j\}} + \epsilon_T^{S_T \{i} k_T^{j\}}}{4M} \left[\Delta H_T(x, \mathbf{k}_T^2) - \frac{\mathbf{k}_T^2}{2M^2} \Delta H_T^\perp(x, \mathbf{k}_T^2) \right] \right], \end{aligned}$$

$$\begin{aligned} \Delta \Gamma^{ij}(x, \mathbf{k}_T) = & \frac{x}{2} \left[\frac{g_T^{ij} \epsilon_T^{k_T S_T}}{M} f_{1T}^\perp(x, \mathbf{k}_T^2) + i\epsilon_T^{ij} g_{1s}(x, \mathbf{k}_T^2) \right. \\ & - \frac{\epsilon_T^{k_T \{i} S_T^{j\}} + \epsilon_T^{S_T \{i} k_T^{j\}}}{4M} h_{1T}(x, \mathbf{k}_T^2) - \left. \frac{\epsilon_T^{k_T \{i} k_T^{j\}}}{2M^2} h_{1s}^\perp(x, \mathbf{k}_T^2) \right] \end{aligned}$$

parameterization of gluon correlator

Definite rank TMDs

- Expansion in constant tensors in transverse momentum space

$$g_T^{\mu\nu} = g^{\mu\nu} - P^{\{\mu} n^{\nu\}} \quad \epsilon_T^{\mu\nu} = \epsilon^{Pn\mu\nu} = \epsilon^{-+\mu\nu}$$

- ... or traceless symmetric tensors (of definite rank)

$$k_T^i$$

$$k_T^{ij} = k_T^i k_T^j - \frac{1}{2} k_T^2 g_T^{ij}$$

$$k_T^{ijk} = k_T^i k_T^j k_T^k - \frac{1}{4} k_T^2 \left(g_T^{ij} k_T^k + g_T^{ik} k_T^j + g_T^{jk} k_T^i \right)$$

- Simple azimuthal behavior: $k_T^{i_1 \dots i_m} \longleftrightarrow |k_T| e^{\pm im\varphi}$
functions showing up in $\cos(m\phi)$ or $\sin(m\phi)$ asymmetries (wrt e.g. ϕ_T)
- Simple Bessel transform to b-space (relevant for evolution):

$$F_m(x, k_T) = \int_0^\infty b db \ J_m(k_T b) F_m(x, b)$$

$$F_m(x, b) = \int_0^\infty k_T dk_T \ J_m(k_T b) F_m(x, k_T)$$

leading twist spin-1 gluon TMDs: result

$$\Gamma^{ij}(x, \mathbf{k}_T) = \Gamma_U^{ij}(x, \mathbf{k}_T) + \Gamma_L^{ij}(x, \mathbf{k}_T) + \Gamma_T^{ij}(x, \mathbf{k}_T) + \Gamma_{LL}^{ij}(x, \mathbf{k}_T) + \Gamma_{LT}^{ij}(x, \mathbf{k}_T) + \Gamma_{TT}^{ij}(x, \mathbf{k}_T),$$

where

$$\Gamma_U^{ij}(x, \mathbf{k}_T) = \frac{x}{2} \left[-g_T^{ij} f_1(x, \mathbf{k}_T^2) + \frac{k_T^{ij}}{M^2} h_1^\perp(x, \mathbf{k}_T^2) \right],$$

$$\Gamma_L^{ij}(x, \mathbf{k}_T) = \frac{x}{2} \left[i\epsilon_T^{ij} S_L g_1(x, \mathbf{k}_T^2) + \frac{\epsilon_{T\alpha}^{[i} k_T^{j]\alpha} S_L}{2M^2} h_{1L}^\perp(x, \mathbf{k}_T^2) \right],$$

$$\Gamma_T^{ij}(x, \mathbf{k}_T) = \frac{x}{2} \left[-\frac{g_T^{ij} S_T k_T}{M} f_{1T}^\perp(x, \mathbf{k}_T^2) + \frac{i\epsilon_T^{ij} \mathbf{k}_T \cdot \mathbf{S}_T}{M} g_{1T}(x, \mathbf{k}_T^2) - \frac{\epsilon_T^{[i} S_T^{j]} + \epsilon_T^{S_T[i} k_T^{j]}}{4M} h_1(x, \mathbf{k}_T^2) - \frac{\epsilon_{T\alpha}^{[i} k_T^{j]\alpha} S_T}{2M^3} h_{1T}^\perp(x, \mathbf{k}_T^2) \right],$$

$$\Gamma_{LL}^{ij}(x, \mathbf{k}_T) = \frac{x}{2} \left[-g_T^{ij} S_{LL} f_{1LL}(x, \mathbf{k}_T^2) + \frac{k_T^{ij} S_{LL}}{M^2} h_{1LL}^\perp(x, \mathbf{k}_T^2) \right],$$

$$\Gamma_{LT}^{ij}(x, \mathbf{k}_T) = \frac{x}{2} \left[-\frac{g_T^{ij} \mathbf{k}_T \cdot \mathbf{S}_{LT}}{M} f_{1LT}(x, \mathbf{k}_T^2) + \frac{i\epsilon_T^{ij} S_{LT} k_T}{M} g_{1LT}(x, \mathbf{k}_T^2) + \frac{S_{LT}^{[i} k_T^{j]}}{M} h_{1LT}(x, \mathbf{k}_T^2) + \frac{k_T^{j\alpha} S_{LT\alpha}}{M^3} h_{1LT}^\perp(x, \mathbf{k}_T^2) \right],$$

$$\begin{aligned} \Gamma_{TT}^{ij}(x, \mathbf{k}_T) = \frac{x}{2} & \left[-\frac{g_T^{ij} k_T^{\alpha\beta} S_{TT\alpha\beta}}{M^2} f_{1TT}(x, \mathbf{k}_T^2) + \frac{i\epsilon_T^{ij} \epsilon_{Ty}^\beta k_T^{\gamma\alpha} S_{TT\alpha\beta}}{M^2} g_{1TT}(x, \mathbf{k}_T^2) \right. \\ & \left. + S_{TT}^{ij} h_{1TT}(x, \mathbf{k}_T^2) + \frac{S_{TT\alpha}^{[i} k_T^{j]\alpha}}{M^2} h_{1TT}^\perp(x, \mathbf{k}_T^2) + \frac{k_T^{ij\alpha\beta} S_{TT\alpha\beta}}{M^4} h_{1TT}^{\perp\perp}(x, \mathbf{k}_T^2) \right]. \end{aligned}$$

gluon TMDs: result

	$-g_T^{ij}$	$i\epsilon_T^{ij}$	k_T^i, k_T^{ij} , etc.
U	f_1		h_1^\perp
L		g_1	h_{1L}^\perp
T	f_{1T}^\perp	g_{1T}	h_1, h_{1T}^\perp
LL	f_{1LL}		h_{1LL}^\perp
LT	f_{1LT}	g_{1LT}	h_{1LT}, h_{1LT}^\perp
TT	f_{1TT}	g_{1TT}	$h_{1TT}, h_{1TT}^\perp, h_{1TT}^{\perp\perp}$

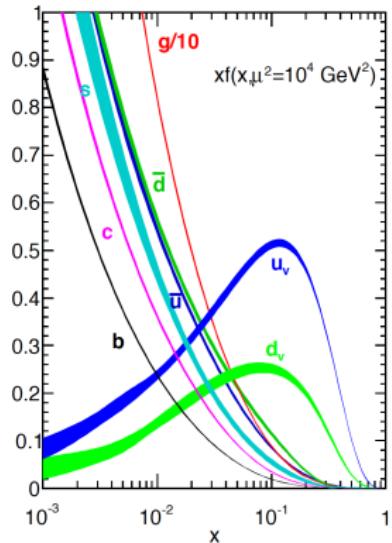
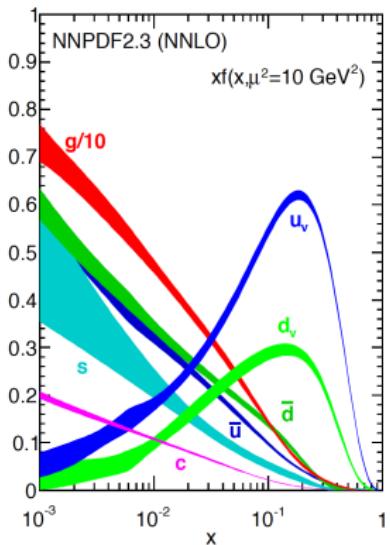
target pol.

extracted parton

	quark	gluon
unpolarized	😊	😊
vector polarised	😊	😊
tensor polarised	😊	😊

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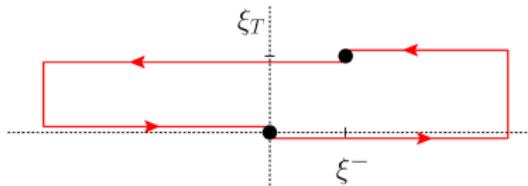


- gluons dominant over quarks at small x
- what happens to the gluon TMDs as $x \rightarrow 0$?
try to consider "empty" correlator (without gluons)

parameterization of Wilson loop

$$\Gamma^{[U,U']\mu\nu;\rho\sigma}(x, \mathbf{k}_T; P, n) \equiv \int \frac{d\xi \cdot P d^2\xi_T}{(2\pi)^3} e^{ik \cdot \xi} \langle P, S, T | F^{\mu\nu}(0) \mathbf{U}_{[0,\xi]} F^{\rho\sigma}(\xi) \mathbf{U}'_{[\xi,0]} | P, S, T \rangle \Big|_{\xi \cdot n = 0}$$

$$\Gamma_0^{[U,U']}(x, \mathbf{k}_T; P, n) \equiv \int \frac{d\xi \cdot P d^2\xi_T}{(2\pi)^3} e^{ik \cdot \xi} \langle P, S, T | \mathbf{U}_{[0,\xi]} \mathbf{U}'_{[\xi,0]} | P, S, T \rangle \Big|_{\xi \cdot n = 0}$$



gauge links: process dependent.

choose **Wilson loop** as an example,
"dipole" type, $U^{[\square]} \equiv U_{[0,\xi]}^{[+]} U_{[\xi,0]}^{[-]}$.

an example for dipole type gluon TMD: $pA \rightarrow \gamma^* jet + X$, the only clean process to test $h_{1,DP}^{\perp g}$, ongoing work collaborate with D.Boer, P. Mulders and Jian Zhou

parameterization of Wilson loop: result

$$\Gamma_0^{[U,U']}(\mathbf{k}_T) = \Gamma_{0U}^{[U,U']}(\mathbf{k}_T) + \Gamma_{0L}^{[U,U']}(\mathbf{k}_T) + \Gamma_{0T}^{[U,U']}(\mathbf{k}_T) + \Gamma_{0LL}^{[U,U']}(\mathbf{k}_T) + \Gamma_{0LT}^{[U,U']}(\mathbf{k}_T) + \Gamma_{0TT}^{[U,U']}(\mathbf{k}_T),$$

where

$$\Gamma_{0U}^{[U,U']}(\mathbf{k}_T) = \frac{\pi L}{M^2} e(\mathbf{k}_T^2),$$

$$\Gamma_{0L}^{[U,U']}(\mathbf{k}_T) = 0,$$

$$\Gamma_{0T}^{[U,U']}(\mathbf{k}_T) = \frac{\pi L}{M^2} \frac{\epsilon_T^{S_T k_T}}{M} e_T(\mathbf{k}_T^2),$$

$$\Gamma_{0LL}^{[U,U']}(\mathbf{k}_T) = \frac{\pi L}{M^2} S_{LL} e_{LL}(\mathbf{k}_T^2),$$

$$\Gamma_{0LT}^{[U,U']}(\mathbf{k}_T) = \frac{\pi L}{M^2} \frac{\mathbf{k}_T \cdot \mathbf{S}_{LT}}{M} e_{LT}(\mathbf{k}_T^2),$$

$$\Gamma_{0TT}^{[U,U']}(\mathbf{k}_T) = \frac{\pi L}{M^2} \frac{k_T^{\alpha\beta} S_{TT\alpha\beta}}{M^2} e_{TT}(\mathbf{k}_T^2).$$

relations between gluon TMDs and Wilson loop TMDs

$$\begin{aligned}
 k_T^i k_T^j \Gamma_0^{[\square]}(\mathbf{k}_T) &= 4 \int \frac{d^2 \xi_T}{(2\pi)^2} e^{ik_T \cdot \xi_T} \langle P | G_T^i(0) U_{[0,\xi]}^{[+]} G_T^j(\xi) U_{[\xi,0]}^{[-]} | P \rangle \Big|_{\xi \cdot n = 0} \quad \text{Wilson loop correlator} \\
 &= \int \frac{d\eta \cdot P d\eta' \cdot P d^2 \xi_T}{(2\pi)^2} e^{ik_T \cdot \xi_T} \langle P | F^{ni}(\eta') U_{[\eta',\eta]}^{[+]} F^{nj}(\eta) U_{[\eta,\eta']}^{[-]} | P \rangle \Big|_{\substack{\eta' \cdot n = \eta \cdot n = 0, \\ \eta'_T = 0_T, \eta_T = \xi_T}} \\
 &= 2\pi L \int \frac{d\xi \cdot P d^2 \xi_T}{(2\pi)^3} e^{ik \cdot \xi} \langle P | F^{ni}(0) U_{[0,\xi]}^{[+]} F^{nj}(\xi) U_{[\xi,0]}^{[-]} | P \rangle \Big|_{\xi \cdot n = k \cdot n = 0} \\
 &= 2\pi L \Gamma^{[+,-]ij}(0, \mathbf{k}_T) \quad \text{dipole type correlator}
 \end{aligned}$$

longitudinal extent of the Wilson loop:
 $L \equiv \int d\xi \cdot P = 2\pi \delta(0)$

$$\Gamma^{[+,-]ij}(x, \mathbf{k}_T) \xrightarrow{x \rightarrow 0} \frac{k_T^i k_T^j}{2\pi L} \Gamma_0^{[\square]}(\mathbf{k}_T) \Big|_{k_T^2 = t}$$

f, g, h - type TMDs, e - type TMDs

match f, g, h - type to e-type TMDs

match every kind of target polarization, by using the previous relation

e.g. transverse polarized ones:

$$\begin{aligned} \Delta\Gamma_{T\text{sym}}^{ij}(x, \mathbf{k}_T) &= \frac{x}{2} \left[-\frac{g_T^{ij} \epsilon_T^{S_T k_T}}{M} f_{1T}^\perp(x, \mathbf{k}_T^2) - \frac{\epsilon_T^{k_T \{i} S_T^{j\}} + \epsilon_T^{S_T \{i} k_T^{j\}}}{4M} h_1(x, \mathbf{k}_T^2) - \frac{\epsilon_{T\alpha}^{i\} k_T^{j\} \alpha S_T}}{2M^3} h_{1T}^\perp(x, \mathbf{k}_T^2) \right] \\ &\xrightarrow{x \rightarrow 0} \frac{k_T^i k_T^j}{2M^2} \frac{\epsilon_T^{S_T k_T}}{M} e_T(\mathbf{k}_T^2) \\ &= \frac{1}{2} \left[-\frac{g_T^{ij} \epsilon_T^{S_T k_T}}{M} \frac{\mathbf{k}_T^2}{2M^2} e_T(\mathbf{k}_T^2) - \frac{\epsilon_T^{k_T \{i} S_T^{j\}} + \epsilon_T^{S_T \{i} k_T^{j\}}}{4M} \frac{\mathbf{k}_T^2}{2M^2} e_T(\mathbf{k}_T^2) + \frac{\epsilon_{T\alpha}^{i\} k_T^{j\} \alpha S_T}}{2M^3} e_T(\mathbf{k}_T^2) \right], \end{aligned}$$

implies that

$$\lim_{x \rightarrow 0} x f_{1T}^\perp(x, \mathbf{k}_T^2) = \lim_{x \rightarrow 0} x h_1(x, \mathbf{k}_T^2) = -\frac{\mathbf{k}_T^2}{2M^2} \lim_{x \rightarrow 0} x h_{1T}^\perp(x, \mathbf{k}_T^2) = \frac{1}{2} \lim_{x \rightarrow 0} x h_{1T}(x, \mathbf{k}_T^2) = \frac{\mathbf{k}_T^2}{2M^2} e_T(\mathbf{k}_T^2),$$

in agreement with ref.[PRL116,122001, D.Boer, M.G.Echevarria, P.Mulders, J. Zhou]

Relations between gluon TMDs in the limit $x \rightarrow 0$ and the Wilson loop TMDs

U	$xf_1 = \frac{\mathbf{k}_T^2}{2M^2} x h_1^\perp = \frac{\mathbf{k}_T^2}{2M^2} e$
L	$xg_1 = xh_{1L}^\perp = 0$
T	$xf_{1T}^\perp = xh_1 = -\frac{\mathbf{k}_T^2}{2M^2} x h_{1T}^\perp = \frac{\mathbf{k}_T^2}{2M^2} e_T, \quad xg_{1T} = 0$
LL	$xf_{1LL} = \frac{\mathbf{k}_T^2}{2M^2} x h_{1LL}^\perp = \frac{\mathbf{k}_T^2}{2M^2} e_{LL}$
LT	$xf_{1LT} = xh_{1LT} = -\frac{\mathbf{k}_T^2}{4M^2} x h_{1LT}^\perp = \frac{\mathbf{k}_T^2}{4M^2} e_{LT}, \quad xg_{1LT} = 0$
TT	$xf_{1TT} = \frac{2M^2}{3\mathbf{k}_T^2} x h_{1TT} = -\frac{1}{2} x h_{1TT}^\perp = \frac{\mathbf{k}_T^2}{6M^2} x h_{1TT}^{\perp\perp} = \frac{\mathbf{k}_T^2}{6M^2} e_{TT}, \quad xg_{1TT} = 0$

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Summary

- introduced new results for spin-1 gluon TMDs, completed the spin ≤ 1 TMDs table

Quarks	γ^+	$\gamma^+ \gamma^5$	$i\sigma^{i+} \gamma^5$
U	f_1		h_1^\perp
L		g_1	h_{1L}^\perp
T	f_{1T}^\perp	g_{1T}	h_1, h_{1T}^\perp
LL	f_{1LL}		h_{1LL}^\perp
LT	f_{1LT}	g_{1LT}	h_{1LT}, h_{1LT}^\perp
TT	f_{1TT}	g_{1TT}	h_{1TT}, h_{1TT}^\perp

Gluons	$-g_T^{ij}$	ie_T^{ij}	k_{iT}^i, k_{iT}^{ij} , etc.
U	f_1		h_1^\perp
L		g_1	h_{1L}^\perp
T	f_{1T}^\perp	g_{1T}	h_1, h_{1T}^\perp
LL	f_{1LL}		h_{1LL}^\perp
LT	f_{1LT}	g_{1LT}	h_{1LT}, h_{1LT}^\perp
TT	f_{1TT}	g_{1TT}	h_{1TT}, h_{1TT}^\perp

- parameterized the Wilson loop correlator, and relate it to the small-x gluon TMDs

U	$xf_1 = \frac{\vec{k}_T^2}{2M^2} xh_1^\perp = \frac{\vec{k}_T^2}{2M^2} e$
L	$xg_1 = xh_{1L}^\perp = 0$
T	$xf_{1T}^\perp = xh_1 = -\frac{\vec{k}_T^2}{2M^2} xh_{1T}^\perp = \frac{\vec{k}_T^2}{2M^2} e_T, \quad xg_{1T} = 0$
LL	$xf_{1LL} = \frac{\vec{k}_T^2}{2M^2} xh_{1LL}^\perp = \frac{\vec{k}_T^2}{2M^2} e_{LL}$
LT	$xf_{1LT} = xh_{1LT} = -\frac{\vec{k}_T^2}{4M^2} xh_{1LT}^\perp = \frac{\vec{k}_T^2}{4M^2} e_{LT}, \quad xg_{1LT} = 0$
TT	$xf_{1TT} = \frac{2M^2}{3\vec{k}_T^2} xh_{1TT} = -\frac{1}{2} xh_{1TT}^\perp = \frac{\vec{k}_T^2}{6M^2} xh_{1TT}^\perp = \frac{\vec{k}_T^2}{6M^2} e_{TT}, \quad xg_{1TT} = 0$

Prospect

- we (with P. Mulders, T. Daal, A. Mukherjee) are considering a follow up project: construct gauge links for some diffractive processes, maybe related to "pomeron"