格点QCD计算在稀有K介子衰变方面的进展

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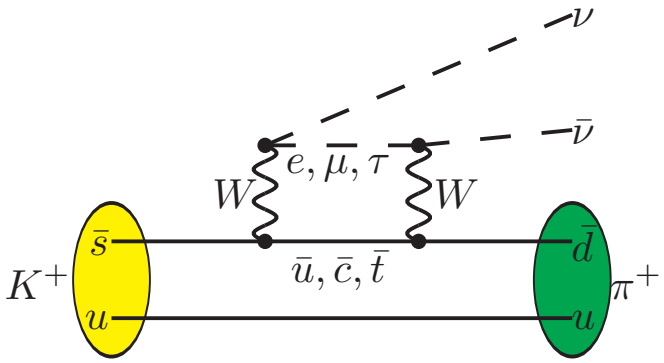
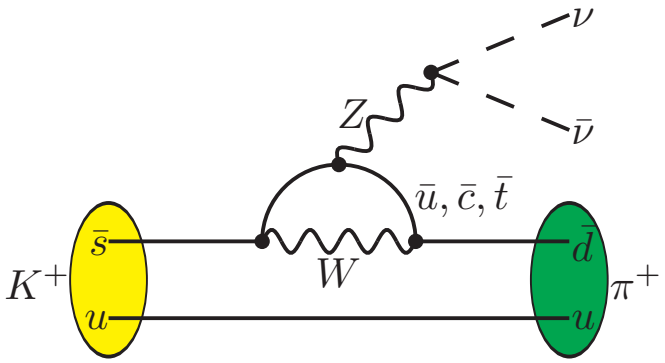
$K^+ \to \pi^+ \nu\bar{\nu}$: 实验 vs 标准模型

作为味道改变中性流过程, $K \to \pi\nu\bar{\nu}$ 必须通过二阶电弱相互作用来实现



W-W



Z-exchange

标准模型贡献被高阶压低 → 探索新物理的理想实验场

过去的实验测量值是标准模型预言值的2倍

$$\mathrm{Br}(K^+ \to \pi^+ \nu\bar{\nu})_{\mathrm{exp}} = 1.73^{+1.15}_{-1.05} \times 10^{-10} \qquad \text{arXiv:0808.2459}$$

$$\mathrm{Br}(K^+ \to \pi^+ \nu\bar{\nu})_{\mathrm{SM}} = 9.11 \pm 0.72 \times 10^{-11} \qquad \text{arXiv:1503.02693}$$

但因为实验误差$> 60\%$，所以实验和理论还是符合的

新一代实验应运而生

新一代实验: NA62 at CERN aims at

- 2-3年内把观测到的$K^+ \to \pi^+\nu\bar{\nu}$事例由7个提高到$O(100)$个
- 把Br($K^+ \to \pi^+\nu\bar{\nu}$)的精度提高到10%

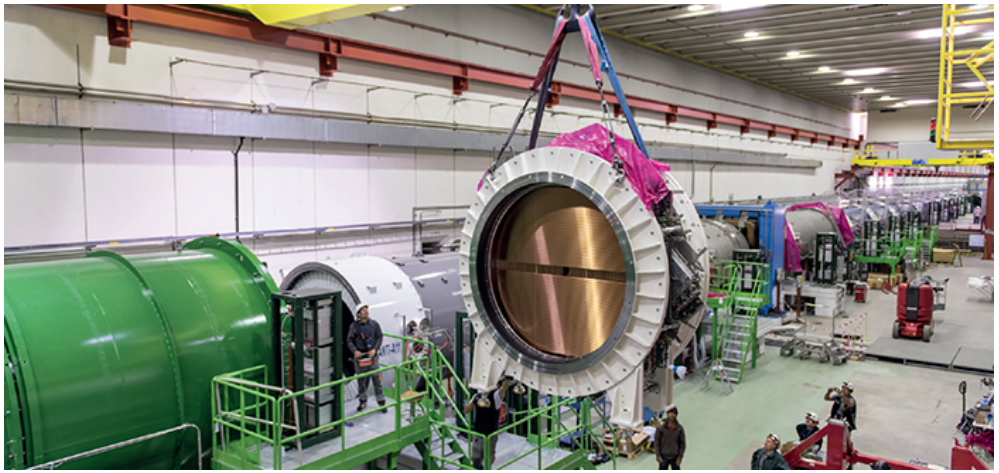
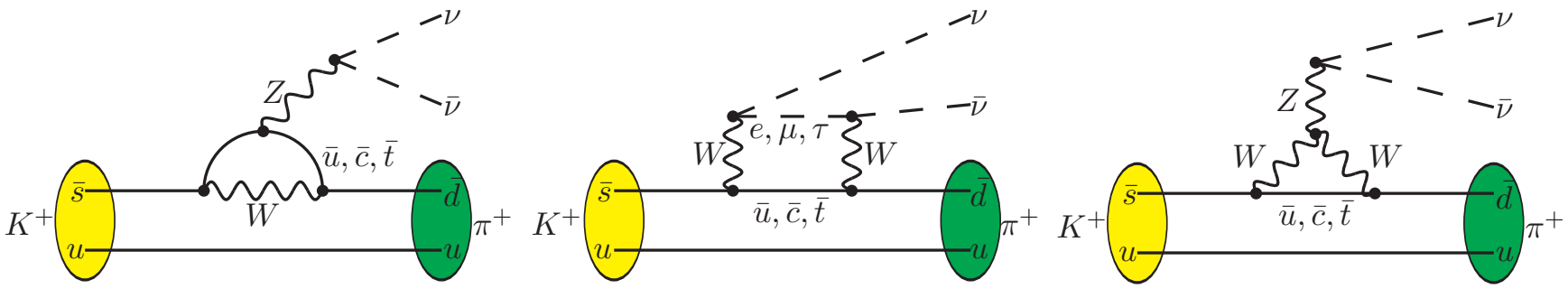


Fig: 09/2014, the final straw-tracker module is lowered into position in NA62

$K_L \to \pi^0\nu\bar{\nu}$

- 实验上更困难：因为初末态都是中性粒子
- 没有观测到事例，only upper bound set by KEK E391a in 2010
- 新一代 J-PARC KOTO实验，就是为了寻找K_L衰变而设计运行

标准模型当中的$K^+ \to \pi^+ \nu \bar{\nu}$



$\frac{1}{M_W^4}$ 或者 $\frac{1}{M_W^2 M_Z^2}$ 的因子意味着 quadratic GIM mechanism

因为 $m_t = 173$ GeV, $m_c = 1.3$ GeV, $m_u = 2.3$ MeV, 所以

- top quark 贡献占主导 ~ $\lambda_t \frac{m_t^2}{M_W^2}$
- short-distance (SD) charm quark 贡献次之 ~ $\lambda_c \frac{m_c^2}{M_W^2} \ln \frac{m_c^2}{M_W^2}$
  - 有一个 $\frac{m_c^2}{m_t^2}$ 因子的压低, 但有一个 $\frac{\lambda_c}{\lambda_t}$ 因子的增强. Here $\lambda_q = V_{qs}^* V_{qd}$
- 余下的是long-distance (LD)贡献 ~ $\lambda \frac{m_c^2}{M_W^2}$, $\lambda \frac{\Lambda_{\rm QCD}^2}{M_W^2}$

Branching ratio

Branching ratio for $K^+ \to \pi^+ \nu\bar{\nu}$ [Buras et.al. JHEP11(2015)033]

$$\mathrm{Br} = \kappa_+(1+\Delta_{\mathrm{EM}}) \cdot \Big[\Big(\underbrace{\frac{\mathrm{Im}\,\lambda_t}{\lambda^5} X(x_t)}_{0.270\times 1.481(9)} \Big)^2 + \Big(\underbrace{\frac{\mathrm{Re}\,\lambda_c}{\lambda} P_c}_{-0.974\times 0.405(23)} + \underbrace{\frac{\mathrm{Re}\,\lambda_t}{\lambda^5} X(x_t)}_{-0.533\times 1.481(9)} \Big)^2 \Big]$$

- $X(x_t)$: top quark 贡献; P_c: charm 和 LD 贡献

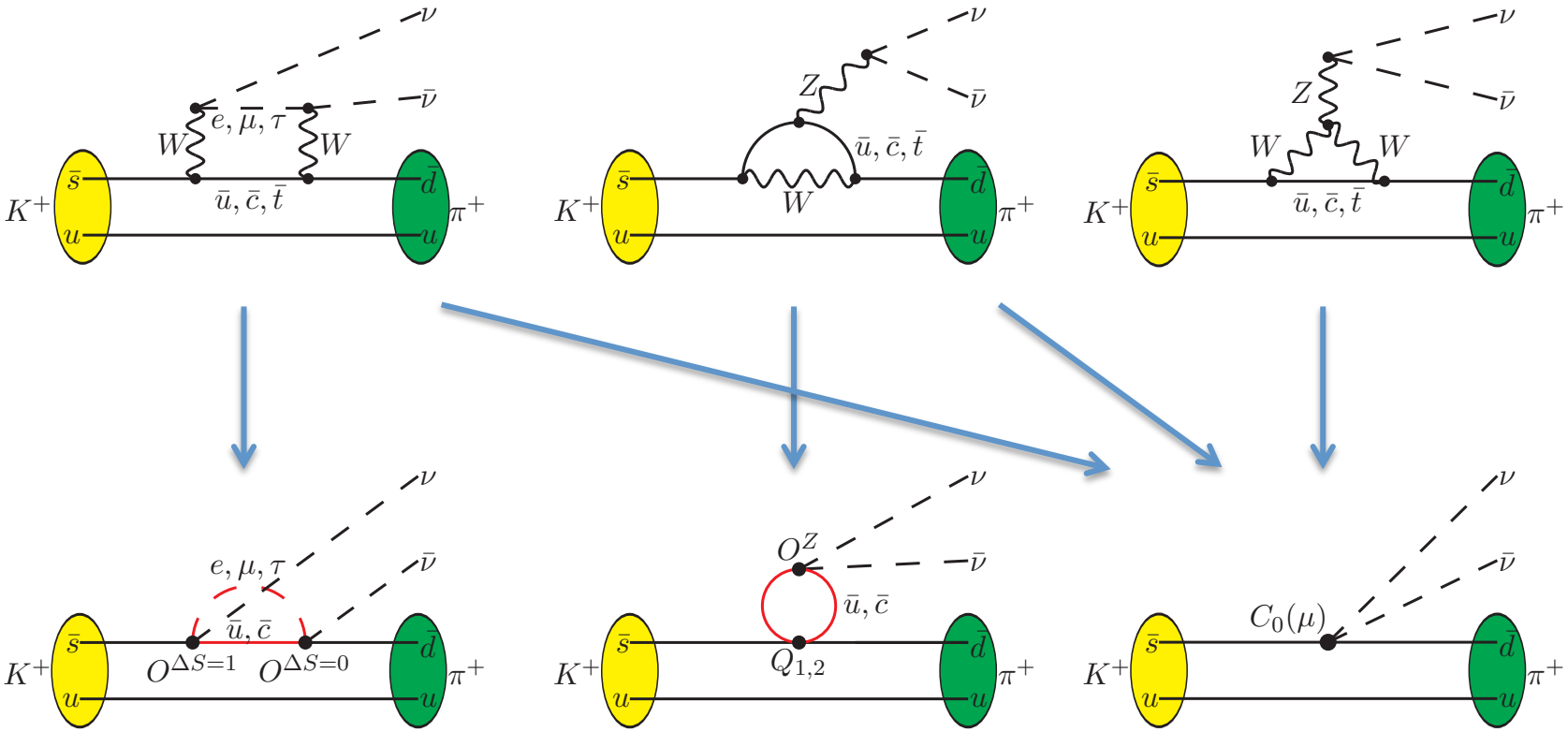
没有 P_c 这一部分, branching ratio 变小 50%

主要理论误差来源

- 最主要的来源是 CKM factor λ_t
- 一旦 CKM factor 给定, 那么 P_c 是最主要的误差来源
  - P_c 的误差主要来自long-distance (LD)的这一部分

如何精确计算long-distance QCD对P_c的贡献非常重要

OPE: integrate out heavy fields Z, W, t, $\cdots$



Bilocal LD contribution

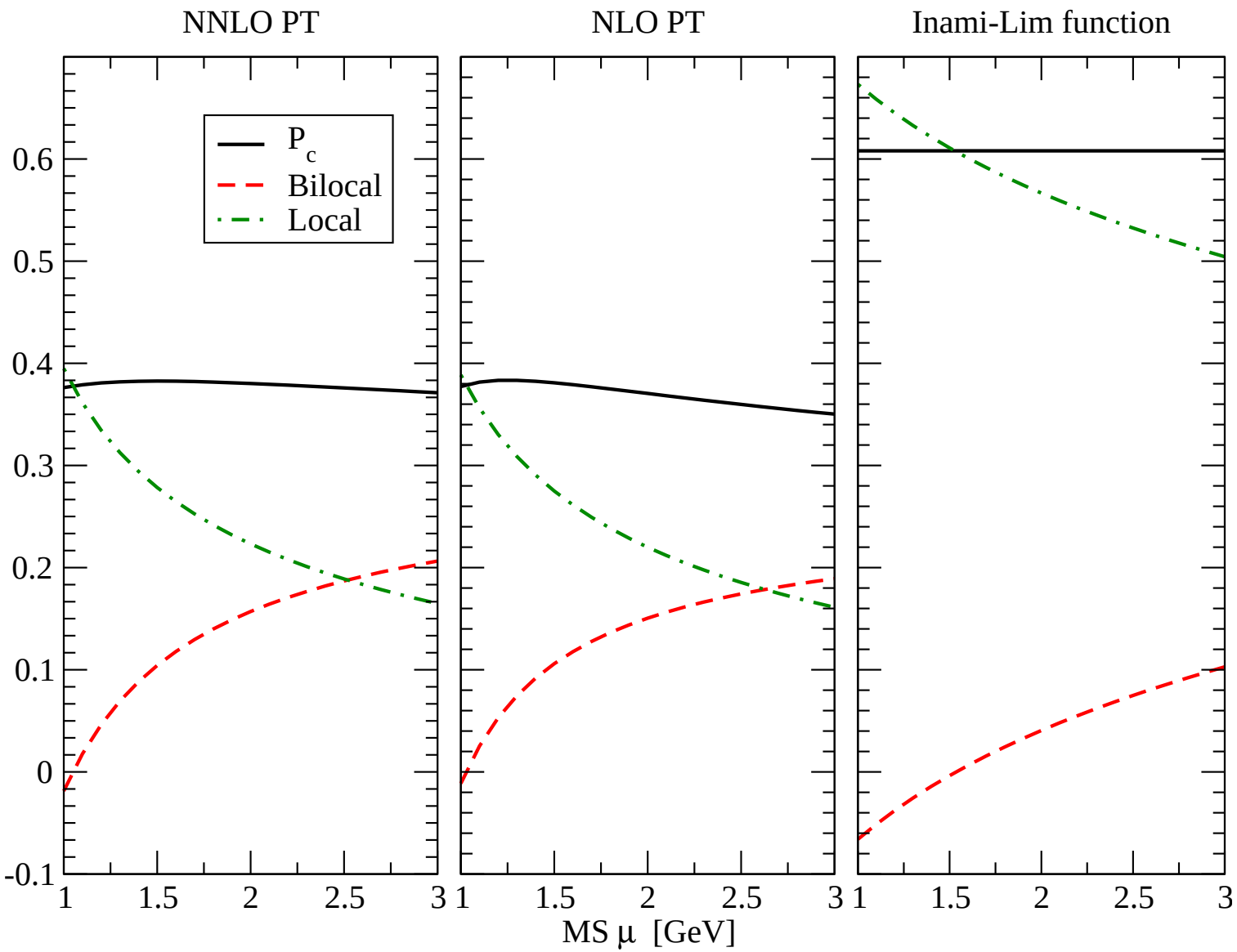
Local SD contribution

Hadronic part known: $\langle \pi^+ | V_\mu | K^+ \rangle$

$\langle \pi^+ \nu \bar{\nu} | Q_A(x) Q_B(0) | K^+ \rangle$: need lattice QCD

Bilocal contribution vs local contribution

Bilocal $C_A^{\overline{\rm MS}}(\mu)C_B^{\overline{\rm MS}}(\mu)r_{AB}^{\overline{\rm MS}}(\mu)$ vs Local $C_0^{\overline{\rm MS}}(\mu)$, hep-ph/0603079



At μ = 2.5 GeV, 50% charm quark 贡献来自于 bilocal 项

Lattice methodology

欧氏时空下的非物理项

2阶电弱过程的强子矩阵元

$$\int_{-T}^{T} dt \, \langle \pi^+ \nu \bar{\nu} | T \left[Q_A(t) Q_B(0) \right] | K^+ \rangle$$
$$= \sum_n \left\{ \frac{\langle \pi^+ \nu \bar{\nu} | Q_A | n \rangle \langle n | Q_B | K^+ \rangle}{M_K - E_n} + \frac{\langle \pi^+ \nu \bar{\nu} | Q_B | n \rangle \langle n | Q_A | K^+ \rangle}{M_K - E_n} \right\} \left(1 - e^{(M_K - E_n) T} \right)$$

- For $E_n > M_K$, at large T, $e^{(M_K - E_n) T}$ 指数衰减
- For $E_n < M_K$, $e^{(M_K - E_n) T}$ 指数增加，必须做减除
- $\sum_n$: branch-cut主值积分被有限体积下的态求和所取代
  - 可能会导致大的有限体积修正，尤其在$E_n \to M_K$的时候

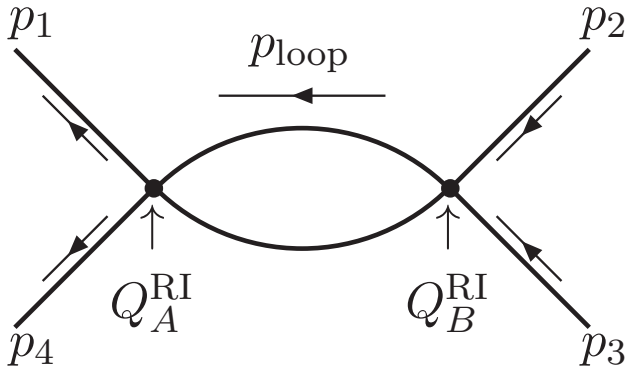
    [N. Christ, XF, G. Martinelli, C. Sachrajda, arXiv:1504.01170]

Short-distance 发散

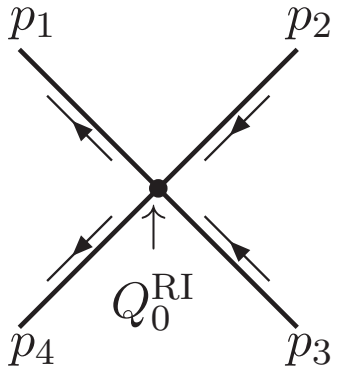
在bilocal $Q_A(x)Q_B(0)$系统中，当 $x \to 0$，SD 发散

- 引进抵消项 $X \cdot Q_0$ 来去除SD发散

$$\langle\{Q_A Q_B\}^{\mathrm{RI}}\rangle\Big|_{p_i^2=\mu_0^2} =$$



$$- X(\mu_0, a) \times$$



$$= 0$$

系数 X 可以在RI/SMOM scheme下得到

- Bilocal operator in the $\overline{\mathrm{MS}}$ scheme 可以表示成

$$\left\{\int d^4x\, T[Q_A^{\overline{\mathrm{MS}}}(x) Q_B^{\overline{\mathrm{MS}}}(0)]\right\}^{\overline{\mathrm{MS}}}$$

$$= Z_A Z_B \left\{\int d^4x\, T[Q_A^{\mathrm{lat}} Q_B^{\mathrm{lat}}]\right\}^{\mathrm{lat}} + \left(-X^{\mathrm{lat}\to\mathrm{RI}} + Y^{\mathrm{RI}\to\overline{\mathrm{MS}}}\right) Q_0(0)$$

  - $X^{\mathrm{lat}\to\mathrm{RI}}$ 可以用非微扰重整化来计算，$Y^{\mathrm{RI}\to\overline{\mathrm{MS}}}$ 可以用微扰论来计算

Lattice results

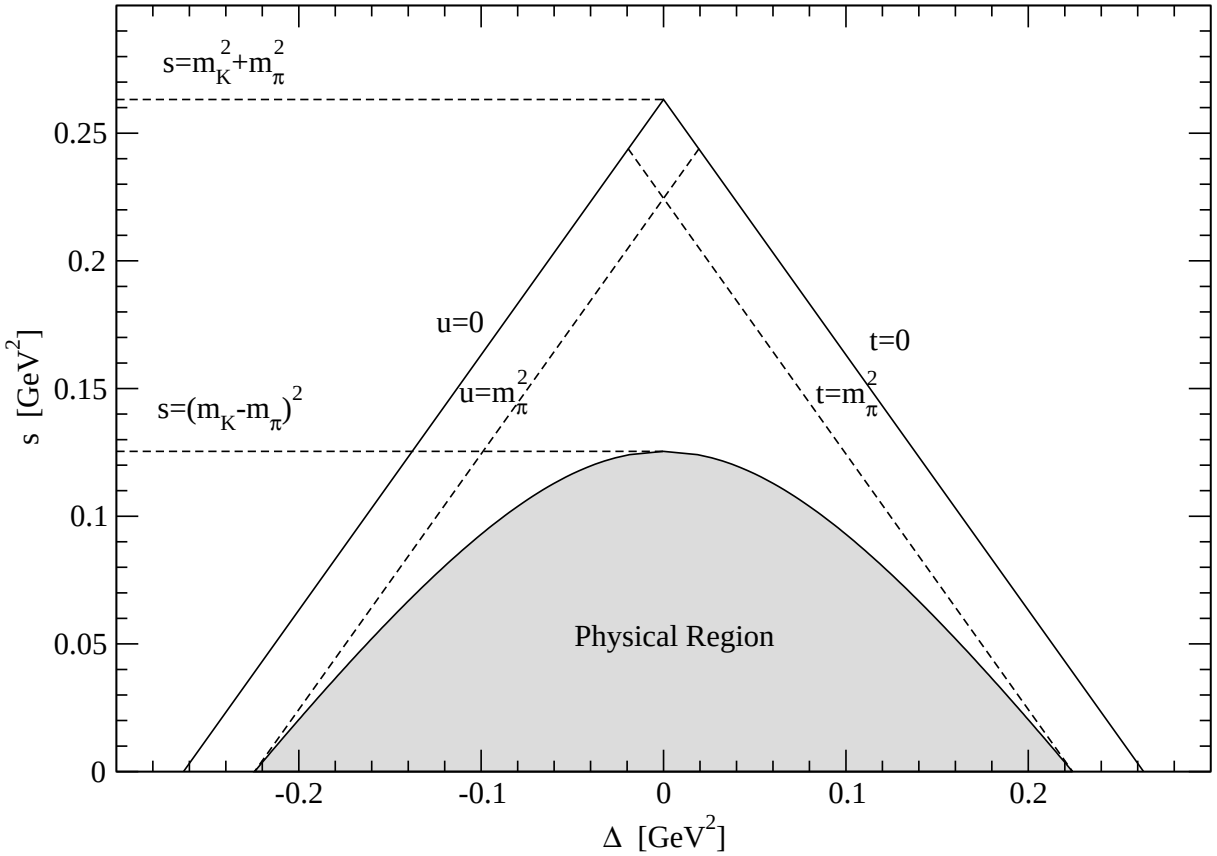
Scalar amplitude

所有的计算结果写成scalar amplitude的形式

$$\int d^4x \langle \pi^+ \nu \bar{\nu} | T[Q_A(x) Q_B(0)] | K^+ \rangle = F(s, \Delta) \cdot \bar{u}(p_\nu) \not{p}_K (1 - \gamma_5) v(p_{\bar{\nu}})$$

where s and Δ are Lorentz invariant variables

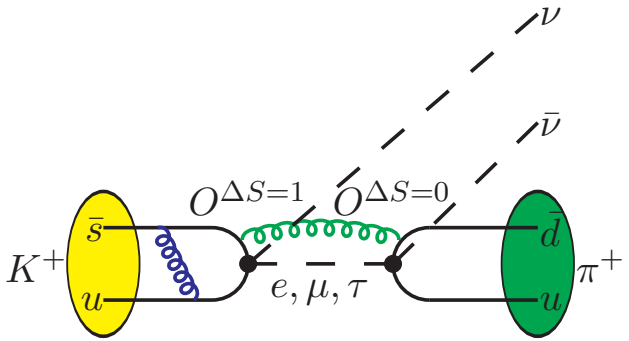
$$s = (p_K - p_\pi)^2, \quad \Delta = (p_K - p_\nu)^2 - (p_K - p_{\bar{\nu}})^2$$

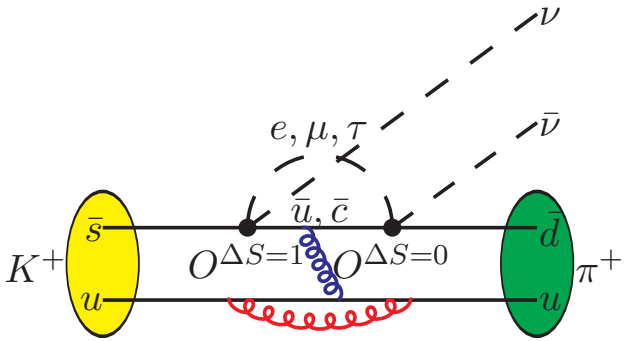


Summary of diagrams

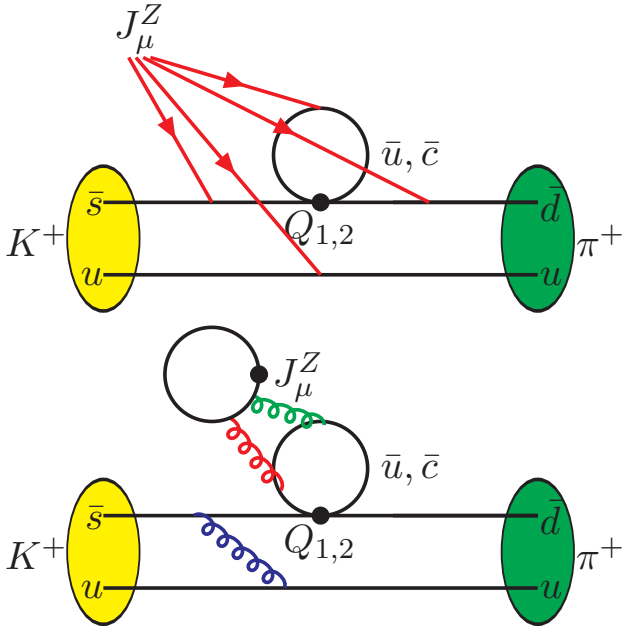
计算了所有的图

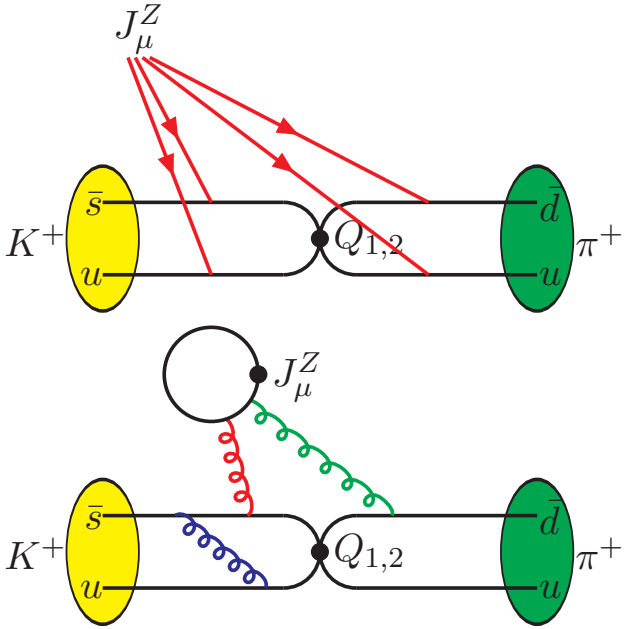
- W-W diagram:



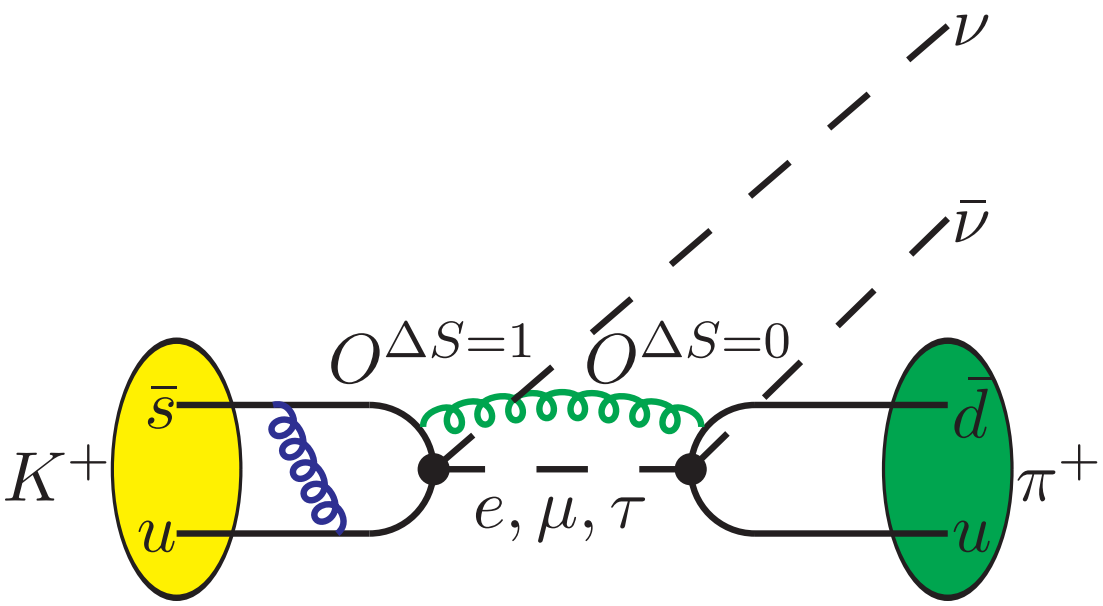


- Z-exchange diagram:

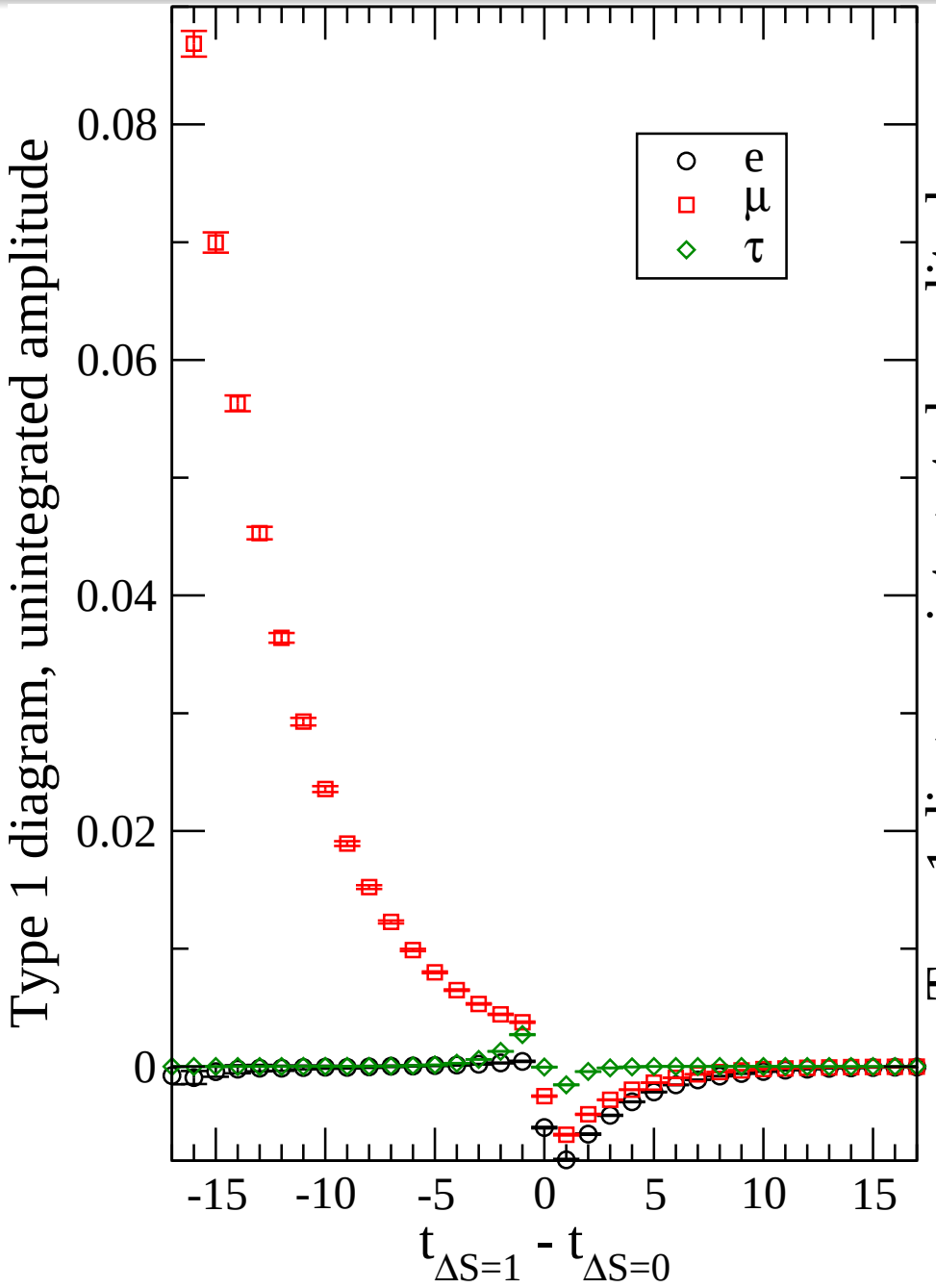


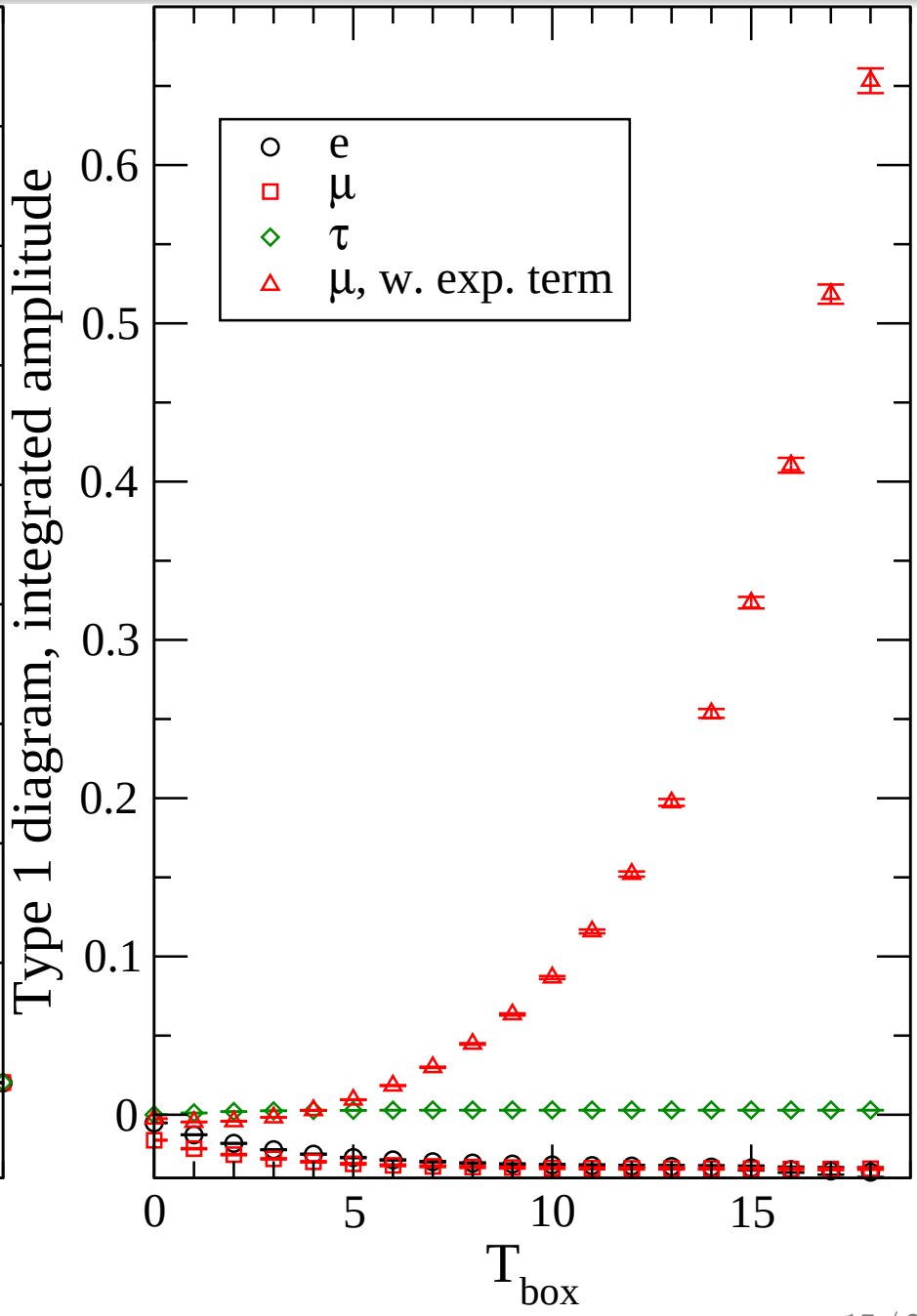


Type 1 diagram



Unintegrated scalar amplitude 的时间依赖关系





F_{WW} for Type 1 diagram

F_{WW}	Type 1	model
e	$-1.685(47)\times 10^{-2}$	$-1.740(6)\times 10^{-2}$
μ	$-1.818(40)\times 10^{-2}$	$-1.822(6)\times 10^{-2}$
τ	$1.491(36)\times 10^{-3}$	$1.471(5)\times 10^{-3}$

- 与模型做对比：模型假设中间态中只有单轻子基态有贡献

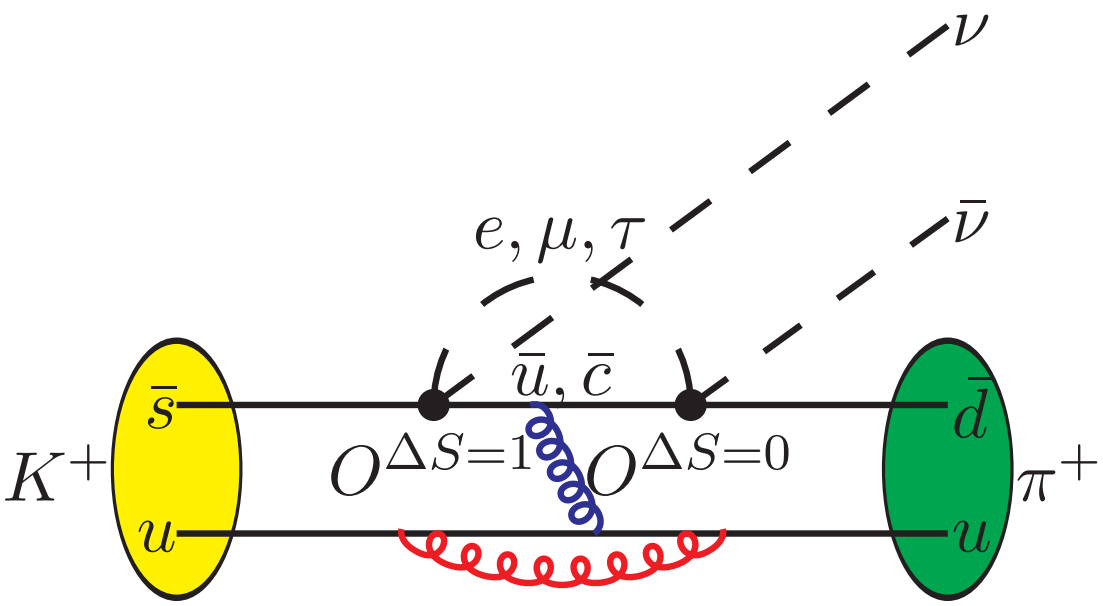
$$-f_K\bar{u}(p_\nu)\not{p}_K(1-\gamma_5)\frac{\not{q}}{q^2-m_\ell^2}\not{p}_\pi(1-\gamma_5)v(p_{\bar{\nu}})f_\pi$$

$$=-f_K f_\pi\frac{2q^2}{q^2-m_\ell^2}\bar{u}(p_\nu)\not{p}_K(1-\gamma_5)v(p_{\bar{\nu}})$$

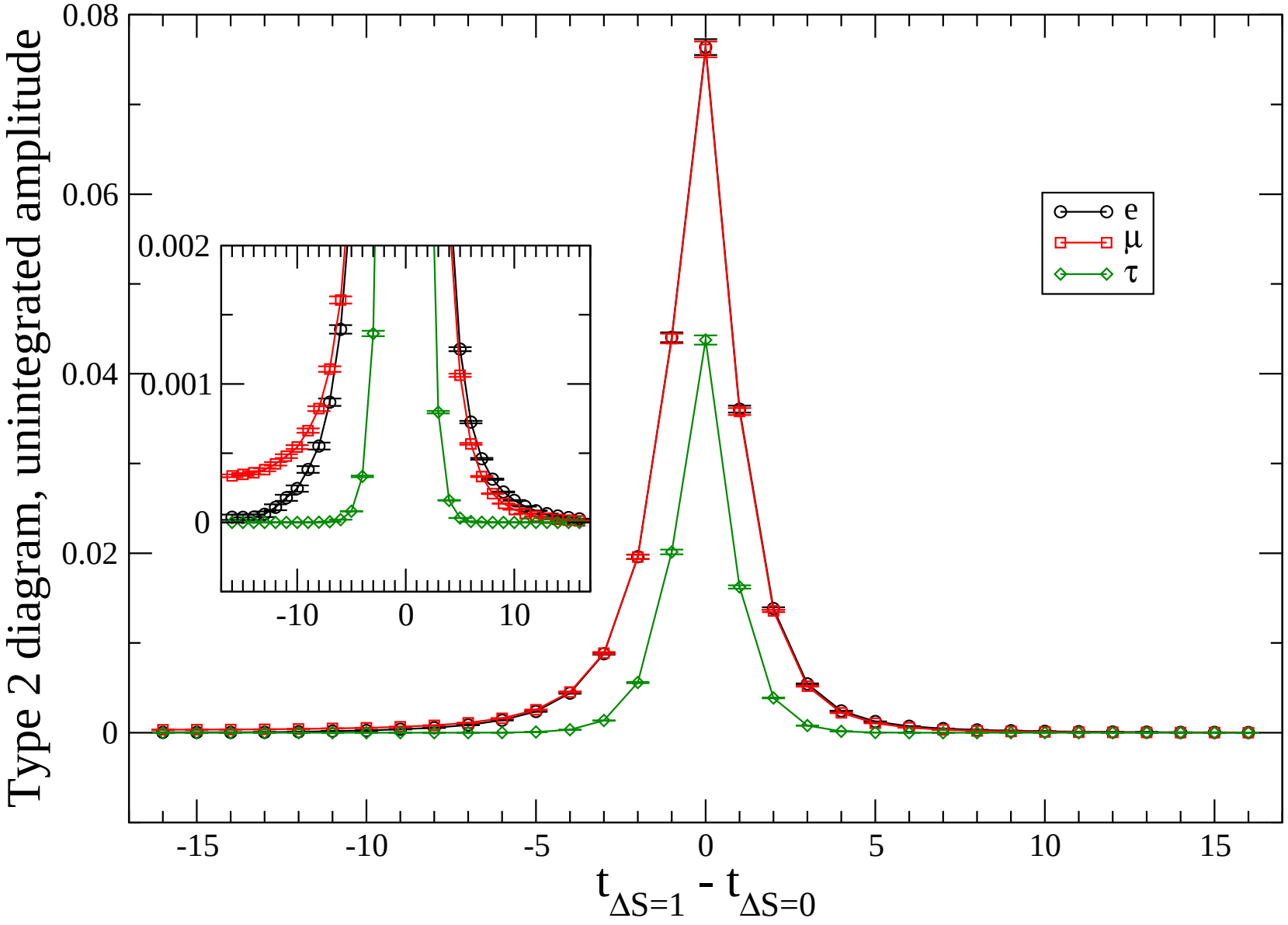
with $q=p_K-p_\nu=p_\pi+p_{\bar{\nu}}$

- 格点和模型对比结果说明激发态的贡献很小

Type 2 diagram



Unintegrated scalar amplitude 的时间依赖关系



Summary results for W-W diagram

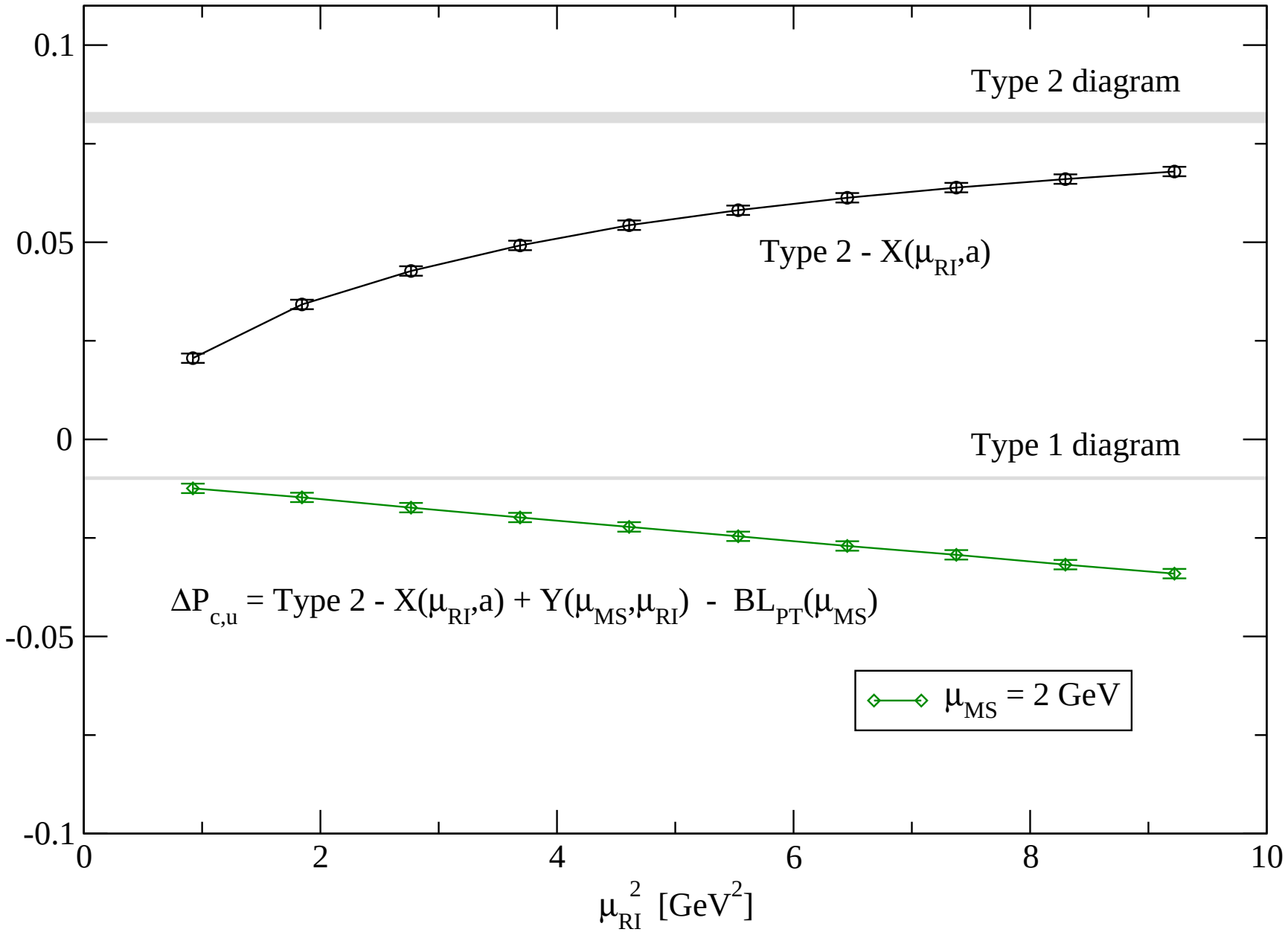
Scalar amplitude for W-W diagram

F_{WW}	Type 1	model	Type 2
e	$-1.685(47)\times10^{-2}$	$-1.740(6)\times10^{-2}$	$1.123(17)\times10^{-1}$
μ	$-1.818(40)\times10^{-2}$	$-1.822(6)\times10^{-2}$	$1.194(18)\times10^{-1}$
τ	$1.491(36)\times10^{-3}$	$1.471(5)\times10^{-3}$	$4.690(77)\times10^{-2}$

Type 2 图的贡献大大超过 Type 1, 但是

Type 2 图包含了一个大的格点截断效应，因为Type 2 图是 SD 发散的

Contribution from W-W diagram



Results for charm quark contribution

Charm quark contribution P_c

$$P_c = P_c^{\mathrm{SD}} + \delta P_{c,u}$$

NNLO PT [Buras et.al, hep-ph/0603079]:

$$P_c^{\mathrm{SD}} = 0.365(12)$$

Phenomenological ansatz [Isidori et.al, hep-ph/0503107]:

$$\delta P_{c,u} = 0.040(20)$$

Preliminary Lattice results

$$\Delta P_{c,u} = \underbrace{-0.007(2)}_{WW:-0.032(1),Z:+0.025(1)} \begin{pmatrix} +7 \\ -11 \end{pmatrix}_{\mathrm{RI}} \begin{pmatrix} +5 \\ -21 \end{pmatrix}_{\mathrm{MS}}$$

$$\Delta P_{c,u} = \mathrm{Lattice} - X(\mu_{\mathrm{RI}}, a) + Y(\mu_{\overline{\mathrm{MS}}}, \mu_{\mathrm{RI}}) - \mathrm{Bilocal}_{\mathrm{PT}}(\mu_{\overline{\mathrm{MS}}})$$

我们离最终目标还有多远?

现在还不能把格点计算结果直接用于理论预言

- 目前参数还是非物理的: $16^3 \times 32$, $m_\pi = 420$ MeV, $m_c = 860$ MeV
- 物理的 pion and charm quark 质量 $\Rightarrow$ 采用大规模格点进行计算
- 需要控制 $\mu_{\overline{\mathrm{MS}}}$ 和 μ_{RI} dependence 带来的系统误差

我们发展了格点QCD的计算方法，已经能够处理稀有K衰变，剩下的任务是控制系统误差

下一步

- USQCD project: 采用大格子 $32^3 \times 64$ 以及 $m_\pi = 170$ MeV
  - 成功申请到 2700万 BGQ core hours, 格点数据已采集，分析中
- Move to $1/a = 2.38$ GeV, $64^3 \times 128$ 以及物理的 m_π and m_c
  - currently a USQCD Incite proposal: 3年共1亿 BGQ core hours

展望

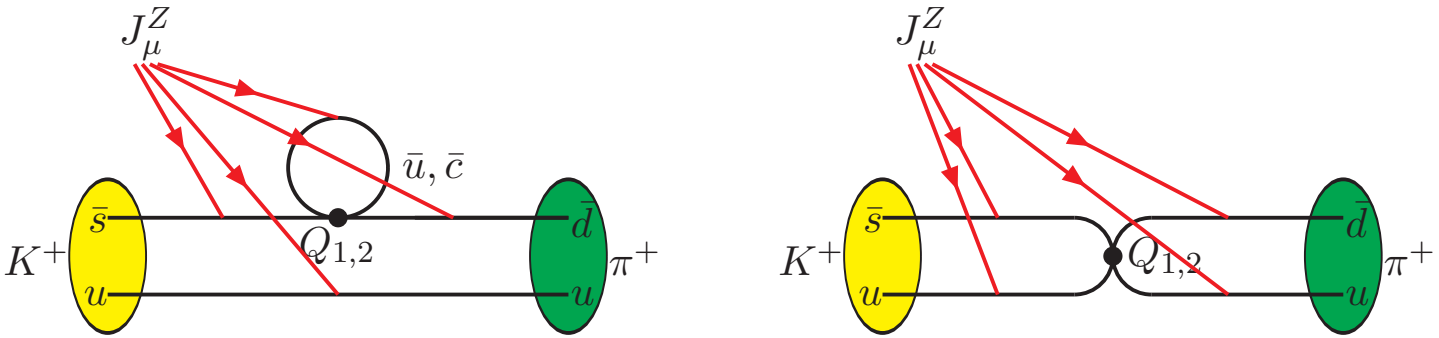
- Calculation of the non-local matrix element is highly non-trivial
- Our exploratory study sheds light on the feasibility of lattice calculation of $K^+ \to \pi^+ \nu \bar{\nu}$
- Other interesting bilocal system
  - K^0-$\bar{K}^0$ and D^0-$\bar{D}^0$ mixing
  - other rare decays: $K_S \to \pi^0 \ell^+ \ell^-$, $B \to K^* \ell^+ \ell^-$, $\cdots$
  - electromagnetic correction to hadron mass and leptonic decay width
  - nucleon double beta decay: $0\nu\beta\beta$
  - ...

Bilocal system: an exciting and new area for lattice QCD!

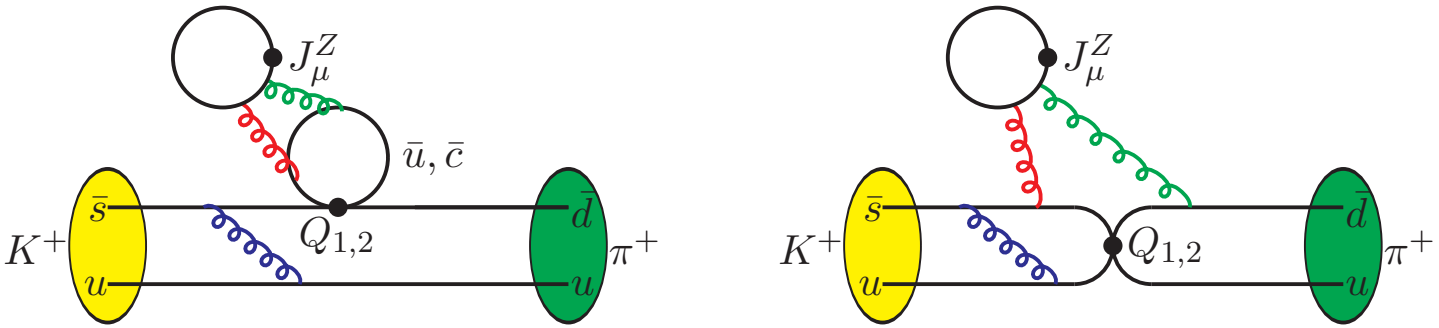
Backup slides

Summary of Z-exchange diagrams

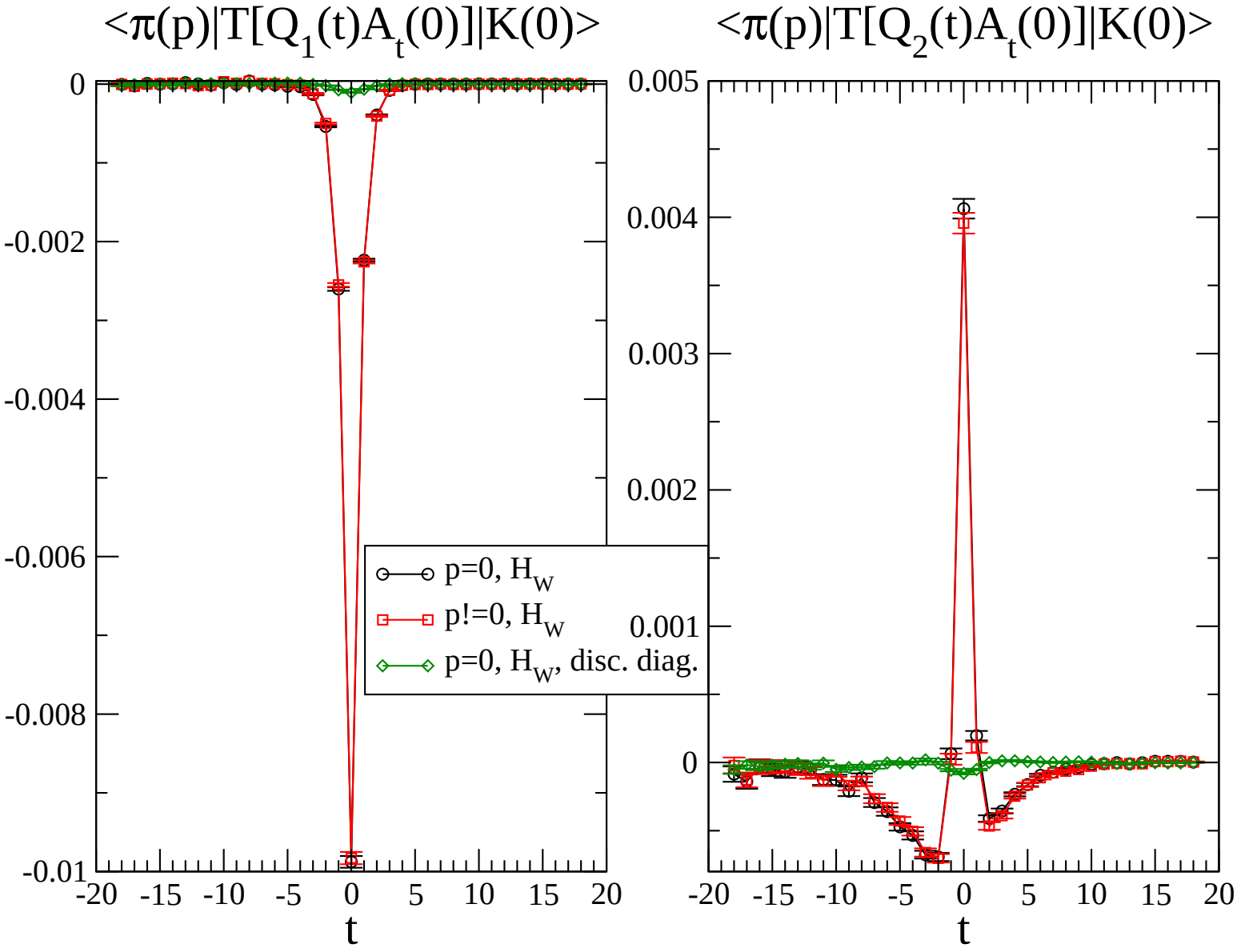
Connected diagrams, J_μ^Z can be inserted into all the possible quark line



Disconnected diagrams (difficult since they are noisy)

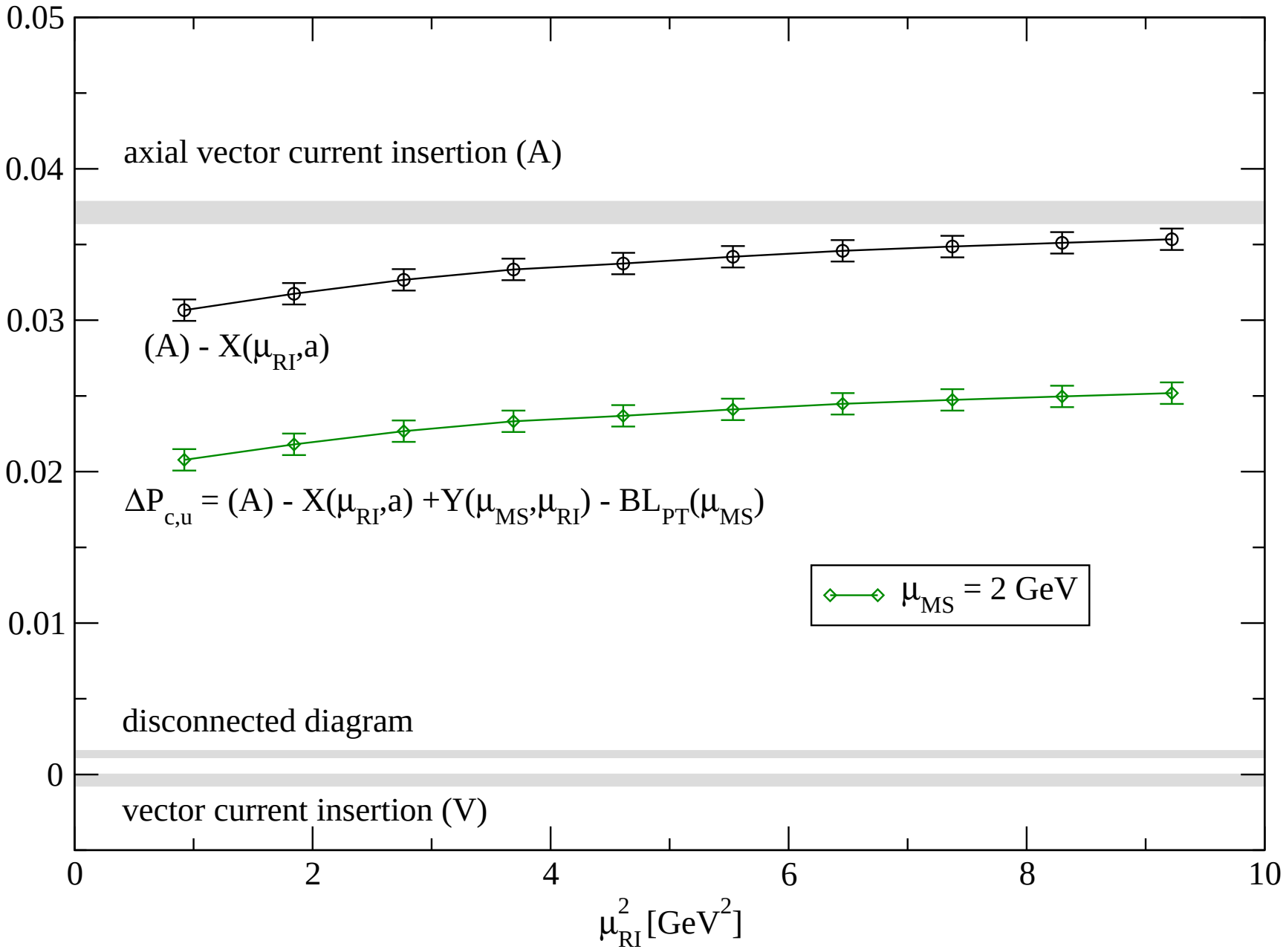


Z-exchange diagram: unintegrated matrix element



Unintegrated matrix element for Z-exchange diagram

Contribution from Z-exchange diagram

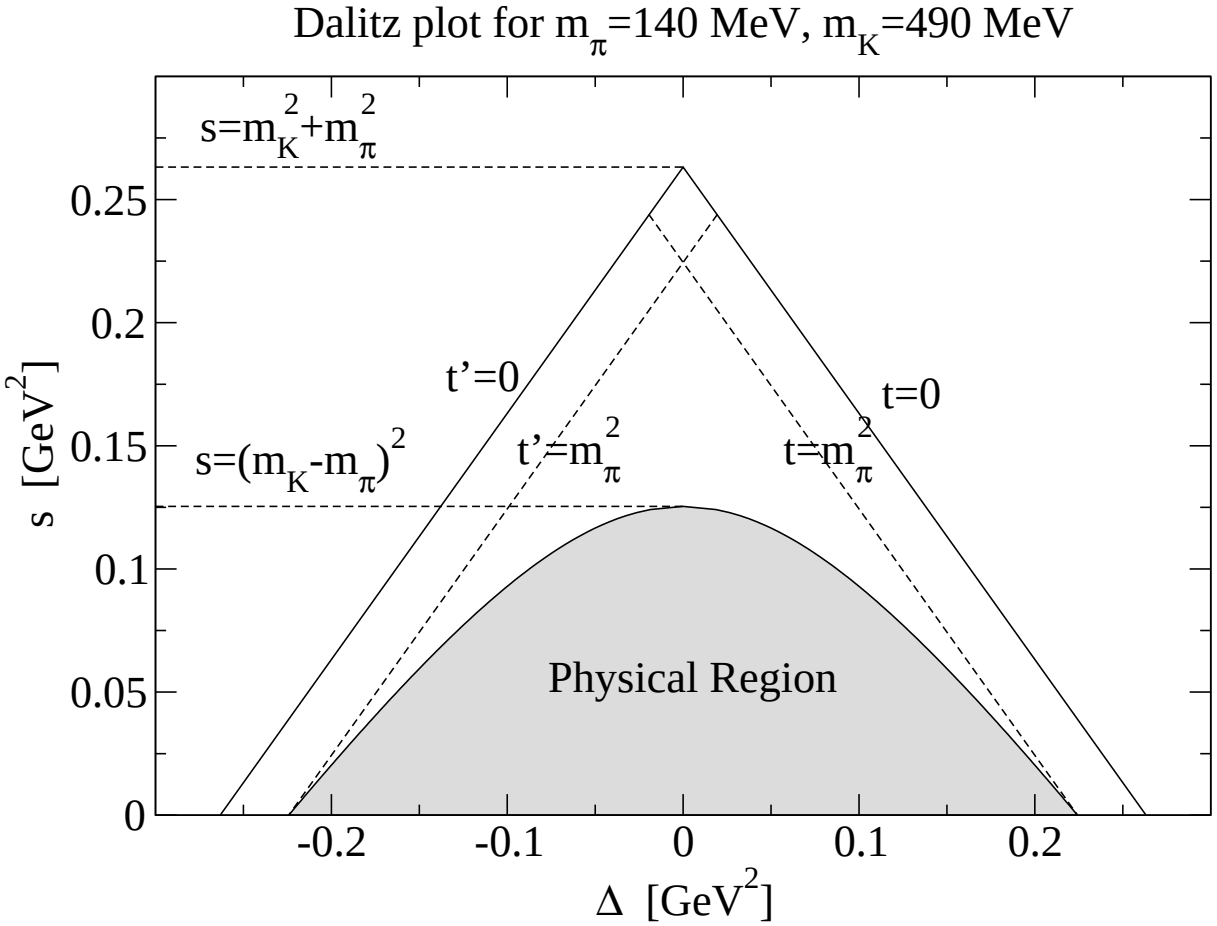


Dalitz plot

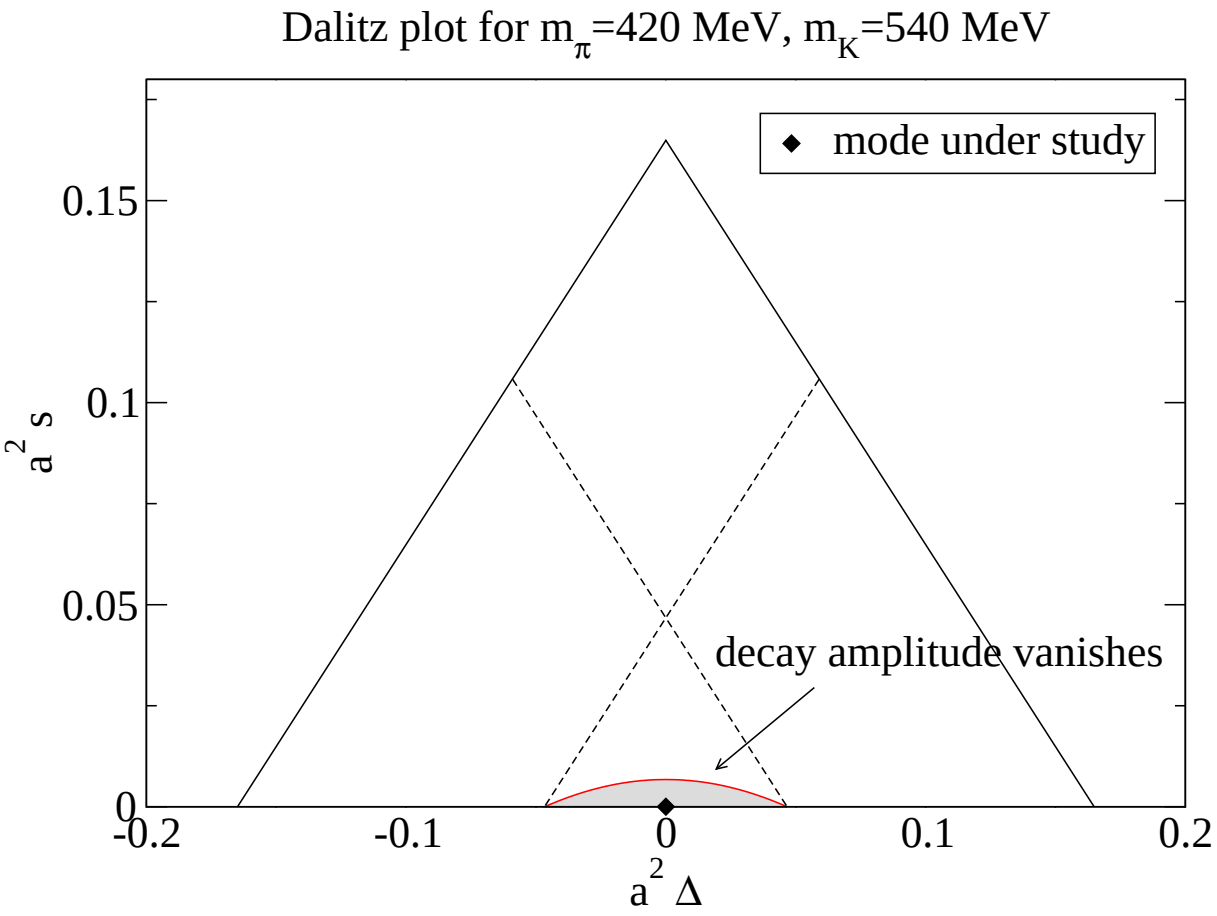
Three Lorentz invariants s, t, t'

$$s = (p_K - p_\pi)^2 = (p_\nu + p_{\bar{\nu}})^2, \quad t = (p_K - p_\nu)^2 = (p_{\bar{\nu}} + p_\pi)^2$$
$$t' = (p_K - p_{\bar{\nu}})^2 = (p_\nu + p_\pi)^2, \quad s + t + t' = m_K^2 + m_\pi^2$$

Two independent variables: s and $\Delta = t' - t$



Momentum mode under study

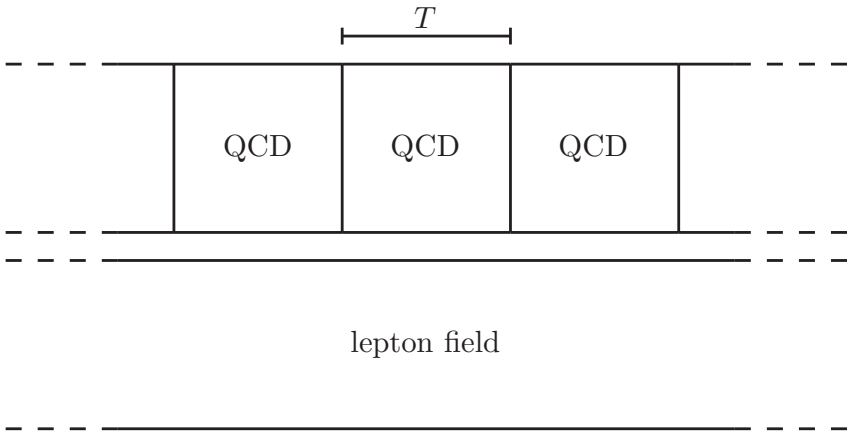


- Allowed momentum region highly suppressed at $m_\pi = 420$ MeV
- On-shell massless neutrinos → modulus of decay amplitude vanishes at the edge of the Dalitz plot
- Away from edge $(\Delta, s) = (0, 0)$ $\Rightarrow$ $\vec{p}_\nu = \vec{p}_{\bar{\nu}}$, $\vec{p}_\pi = -\vec{p}_\nu - \vec{p}_{\bar{\nu}}$

Evaluation of non-local matrix element

$$\int dt \, \langle \pi^+ \nu \bar{\nu} | T\{ O^{\Delta S=1}(t) O^{\Delta S=0}(0) \} | K^+ \rangle$$

- Construct 4-point correlator $\langle \phi_\pi(t_\pi) O^{\Delta S=1}(t_1) O^{\Delta S=0}(t_0) \phi_K^\dagger(t_K) \rangle$
- Perform time translation average → statistical error reduced by $\sqrt{T}$
  - propagators generated on all time slices, quite a lot of cost
  - use low-mode deflation w. 100 low-lying eigenvectors to accelerate CG
  - time required to generate light quark propagators is reduced to 10%
- Use overlap fermion for lepton propagator
  - time extent for lepton is infinite



Evaluation of non-local matrix element

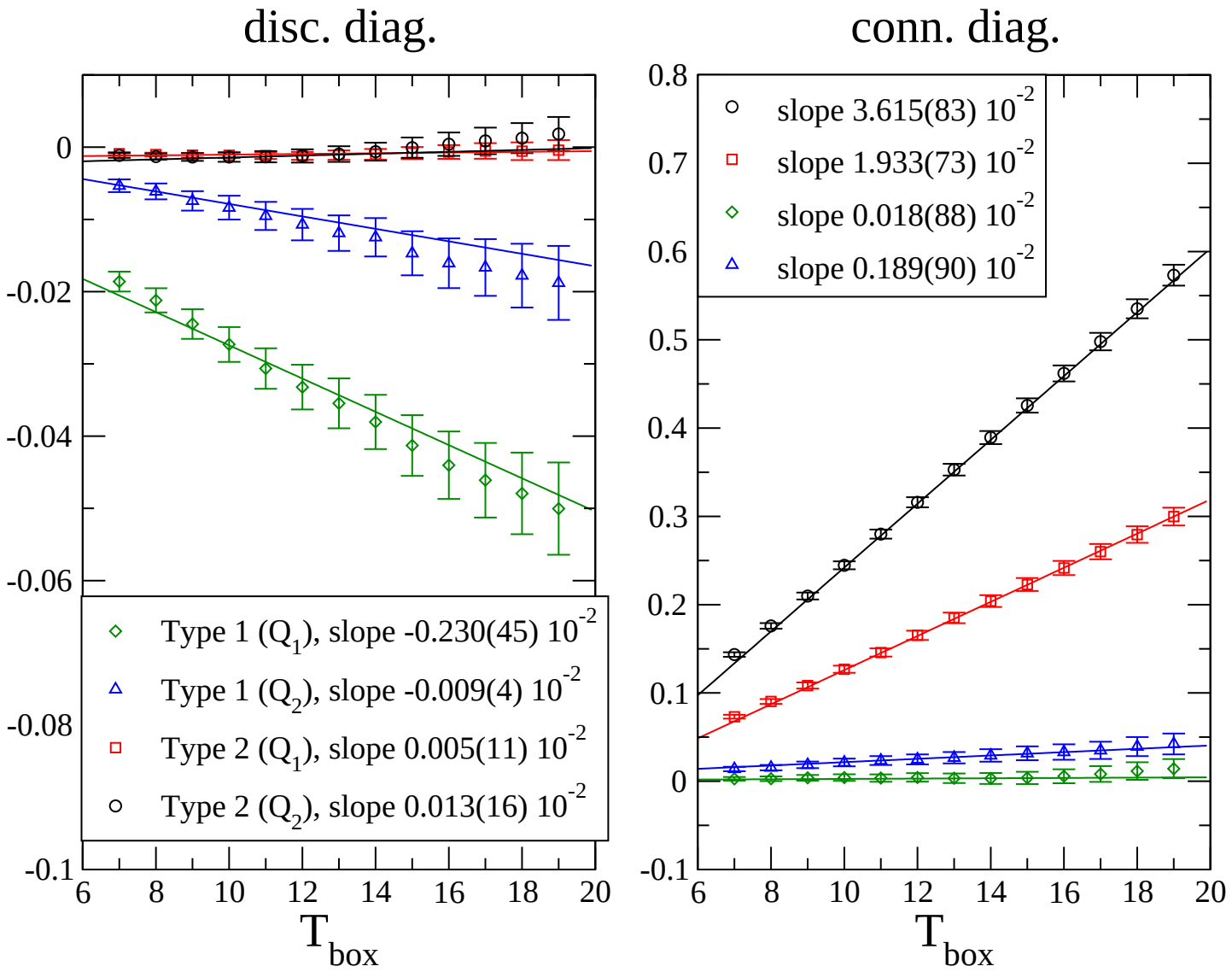
$$T_\mu^Z = \int dt \, \langle \pi^+ | T\{ Q_{1,2}(t) J_\mu^Z(0) \} | K^+ \rangle$$

- Z-exchange diagrams do not require on-shell neutrinos
  - we use $\vec{p}_K = \vec{p}_\pi = 0, \quad J_\mu^Z, \ \mu = t$
- Hadronic current J_μ^Z has vector and axial vector component
  - for the vector current, according to Ward identity (WI), we have

$$T_\mu^{Z,V} = F^{Z,V}(q^2) \left(q^2 (p_K + p_\pi)_\mu - (m_K^2 - m_\pi^2) q_\mu \right), \quad q = p_K - p_\pi$$

  - with $\vec{p}_K = \vec{p}_\pi = 0 \quad \Rightarrow \quad q^2 (p_K + p_\pi)_\mu - (m_K^2 - m_\pi^2) q_\mu = 0$
  - WI suggests $T_\mu^{Z,V} = 0$, this is confirmed by our numerical calculation
- In the following, I will present the results for axial vector current

Integrated matrix element for Z-exchange



Disc. diag. is relatively noisy, but its contribution is small. Adding the disc. part does not affect the conn. part significantly