1-loop RGE of dim=7 Operators in SMEFT

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Aug 23, 2016

- From SM to SMEFT
- Basis for dim-7 operators
- 1-loop RGE of dim-7 operators
- Structure of anomalous dimension matrix γ_{ij}
- Proton decay with $\Delta L = -\Delta B = 1$
- Summary

References

⊙ Higher dimensional operators:
[Buchmuller & Wyler 1986], [B. Grzadkowski et al:1008.4884],
[L. Lehman:1410.4193].

⊙ RGE of dim-5 operator:
[K. S. Babu:9309223], [S. Antusch et al:0108005].

⊙ RGE of dim-6 operator:
[A. V. Manohar et al:1301.2588], [J. Elias-Miro et al:1302.5661],
[J. Elias-Miro et al:1308.1879], [A. V. Manohar et al:1310.4838],
[A. V. Manohar et al1312.2014], [A. V. Manohar et al:1405.0486].

⊙ RGE of dim-7 operator:
[Y, Liao et al:1607.07309].

Convention

SM field content: $H, Q, L, u, d, e, B_\mu, W^I_\mu, G^A_\mu$
Symmetry: Poincare $\otimes$ Gauge $= T_{1,3} \ltimes SO^{\uparrow}_{+}(1,3) \otimes SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$
The SM Lagrangian:

$$\mathcal{L}_{\text{SM}} = -\frac{1}{4}G^A_{\mu\nu}G^{A\mu\nu} - \frac{1}{4}W^I_{\mu\nu}W^{I\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + (D_\mu H)^\dagger(D^\mu H) + \lambda v^2(H^\dagger H)$$
$$- \lambda(H^\dagger H)^2 + \sum_{\Psi=Q,L,u,d,e} \bar{\Psi} i \not{D} \Psi - \left[\bar{Q}Y_u u\tilde{H} + \bar{Q}Y_d dH + \bar{L}Y_e eH + \text{h.c.}\right]$$

A and I: the adjoint indices of the $SU(3)_C$ and $SU(2)_L$ group;
$Y_u,\ Y_d,\ Y_e$: the Yukawa couplings in flavor space;

$$\tilde{H}_i = \epsilon_{ij}H^*_j, \quad D_\mu = \partial_\mu - ig_3 T^A G^A_\mu - ig_2 T^I W^I_\mu - ig_1 Y B_\mu,$$

$T^A,\ T^I,\ Y$: the generator matrices appropriate for the fields to be acted on.

From SM to SMEFT

- SMEFT is the one which extends the SM by including higher dimensional operators with SM field contents and admits that the NP scale Λ is on top of electroweak scale.

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \frac{1}{\Lambda^{D-4}} \sum_{D \geq 5} C_i^D \mathcal{O}_i^D$$

  Wilson coefficients C_i^D encode the contribution from unknown NP.
- SMEFT is powerful because it is model-independent.
- For a given specific model, the operator coefficients can easily be matched by integrating out the heavy degrees of freedom.

From SM to SMEFT

- The complete basis of higher dimensional operators have been determined up to dim-7.
- Dim-5: 1 $\rightarrow$ neutrino mass operator $\rightarrow$ unique in dim-D(odd):

$$\mathcal{O}^{D}_{\text{n.m.}} = \left[(L^{\mathsf{T}}\epsilon H)C(L^{\mathsf{T}}\epsilon H)^{\mathsf{T}}\right](H^{\dagger}H)^{\frac{D-5}{2}} + \text{h.c.}$$

- Dim-6: $59 + 4(\not{B} : \Delta B - \Delta L = 0)$
  - Higgs physics dominated by dim-6 operators: Higgs production at LHC $gg \rightarrow h$ and decay $h \rightarrow \gamma\gamma$, $h \rightarrow \gamma Z$, ...
  - Proton decay with $\Delta B - \Delta L = 0$.
  - ...
- Dim-7: $12 + 6(\not{B} : \Delta B - \Delta L = 2)$
  - Exotic proton decay with $\Delta B - \Delta L = 2$, etc.

Basis for dim-7 operators

	$\psi^2 H^4 + \text{h.c.}$		$\psi^2 H^3 D + \text{h.c.}$
$\mathcal{O}_{LH}$	$\epsilon_{ij}\epsilon_{mn}(L^i C L^m) H^j H^n (H^\dagger H)$	$\mathcal{O}_{LeHD}$	$\epsilon_{ij}\epsilon_{mn}(L^i C\gamma_\mu e) H^j H^m i D^\mu H^n$
	$\psi^2 H^2 D^2 + \text{h.c.}$		$\psi^2 H^2 X + \text{h.c.}$
$\mathcal{O}_{LHD1}$	$\epsilon_{ij}\epsilon_{mn}(L^i C D^\mu L^j) H^m (D_\mu H^n)$	$\mathcal{O}_{LHB}$	$\epsilon_{ij}\epsilon_{mn}(L^i C\sigma_{\mu\nu} L^m) H^j H^n B^{\mu\nu}$
$\mathcal{O}_{LHD2}$	$\epsilon_{im}\epsilon_{jn}(L^i C D^\mu L^j) H^m (D_\mu H^n)$	$\mathcal{O}_{LHW}$	$\epsilon_{ij}(\epsilon\tau^I)_{mn}(L^i C\sigma_{\mu\nu} L^m) H^j H^n W^{I\mu\nu}$
	$\psi^4 D + \text{h.c.}$		$\psi^4 H + \text{h.c.}$
$\mathcal{O}_{\bar{d}uLLD}$	$\epsilon_{ij}(\bar{d}\gamma_\mu u)(L^i C i D^\mu L^j)$	$\mathcal{O}_{\bar{e}LLLH}$	$\epsilon_{ij}\epsilon_{mn}(\bar{e}L^i)(L^j C L^m) H^n$
$\mathcal{O}_{\bar{L}QddD}$	$(\bar{L}\gamma_\mu Q)(dCiD^\mu d)$	$\mathcal{O}_{\bar{d}LQLH1}$	$\epsilon_{ij}\epsilon_{mn}(\bar{d}L^i)(Q^j C L^m) H^n$
$\mathcal{O}_{\bar{e}dddD}$	$(\bar{e}\gamma_\mu d)(dCiD^\mu d)$	$\mathcal{O}_{\bar{d}LQLH2}$	$\epsilon_{im}\epsilon_{jn}(\bar{d}L^i)(Q^j C L^m) H^n$
		$\mathcal{O}_{\bar{d}LueH}$	$\epsilon_{ij}(\bar{d}L^i)(uCe) H^j$
		$\mathcal{O}_{\bar{Q}uLLH}$	$\epsilon_{ij}(\bar{Q}u)(LCL^i) H^j$
		$\mathcal{O}_{\bar{L}dud\tilde{H}}$	$(\bar{L}d)(uCd)\tilde{H}$
		$\mathcal{O}_{\bar{L}dddH}$	$(\bar{L}d)(dCd) H$
		$\mathcal{O}_{\bar{e}Qdd\tilde{H}}$	$\epsilon_{ij}(\bar{e}Q^i)(dCd)\tilde{H}^j$
		$\mathcal{O}_{\bar{L}dQQ\tilde{H}}$	$\epsilon_{ij}(\bar{L}d)(QCQ^i)\tilde{H}^j$
$\mathcal{O}^{(2)}_{\bar{d}uLLD}$	$\epsilon_{ij}(\bar{d}\gamma_\mu u)(L^i C\sigma^{\mu\nu} D_\nu L^j)$	$\mathcal{O}_{\bar{L}dQdD}$	$(\bar{L}iD^\mu d)(QC\gamma_\mu d)$

$13(B) + 7(\not{B})$ given by [L. Lehman:1410.4193].

Dim-7 Majorana neutrino mass operator;

Baryon number violating operators with $\Delta B = -\Delta L = 1$;

Redundant operators.

Proof for redundancies: EoMs + Fierz identities

♠ EoMs from SM Lagrangian $\mathcal{L}_4$

$$\begin{aligned} i\not{D}L &= Y_e e H \\ i\not{D}d &= Y_d^\dagger H^\dagger Q \end{aligned}$$

♠ Feriz identities for charge conjugated fields

$$\begin{aligned} (\Psi_{1L} C\gamma_\mu \Psi_{2R})(\overline{\Psi_{3R}}\gamma^\mu \Psi_{4R}) &= 2(\overline{\Psi_{3R}}\Psi_{1L})(\Psi_{4R} C\Psi_{2R}) \\ (\overline{\Psi_{1L}}\gamma_\mu \Psi_{2L})(\Psi_{3R} C\Psi_{4R}) &= (\overline{\Psi_{1L}}\Psi_{3R})(\Psi_{2L} C\gamma_\mu \Psi_{4R}) + (\overline{\Psi_{1L}}\Psi_{4R})(\Psi_{2L} C\gamma_\mu \Psi_{3R}) \end{aligned}$$

where the notation $\Psi^C = C\bar{\Psi}^T$ is used in actual calcution, and the charge conjugated field is defined by $(\Psi C\chi) = \overline{\Psi^C}\chi$ with $(\Psi^C)^C = \Psi$, where the matrix C satisfies the relations $C^T = C^\dagger = -C$ and $C^2 = -1$.

♠ Linear dependent operators

$$\begin{aligned} \mathcal{O}^{(2)prst}_{\bar{d}uLLD} &= 2(Y_e)_{tu}\mathcal{O}^{psru}_{\bar{d}LueH} - \mathcal{O}^{prst}_{\bar{d}uLLD} \\ \mathcal{O}^{prst}_{\bar{L}dQdD} &= \mathcal{O}^{pstr}_{\bar{L}QddD} - (Y_d^\dagger)_{ru}\mathcal{O}^{ptsu}_{\bar{L}dQQ\tilde{H}} \end{aligned}$$

Number of independent operators with flavor indices added

Class	Operator	[Yi Liao:1607.07309]-Direct counting			[H. Murayama:1512.03433]-Hilbert series		
		n_f	1	3	n_f	1	3
$\psi^2 H^4$	$\mathcal{O}_{LH}$	$\frac{1}{2} n_f(n_f+1)$	1	6	$\frac{1}{2} n_f(n_f+1)$	1	6
$\psi^2 H^3 D$	$\mathcal{O}_{LeHD}$	n_f^2	1	9	n_f^2	1	9
$\psi^2 H^2 D^2$	$\mathcal{O}_{LHD1}$	$\frac{1}{2} n_f(n_f+1)$	1	6	$n_f(n_f+1)$	2	12
	$\mathcal{O}_{LHD2}$	$\frac{1}{2} n_f(n_f+1)$	1	6			
$\psi^2 H^2 X$	$\mathcal{O}_{LHB}$	$\frac{1}{2} n_f(n_f-1)$	0	3	$\frac{1}{2} n_f(3n_f-1)$	1	12
	$\mathcal{O}_{LHW}$	n_f^2	1	9			
$\psi^4 H$	$\mathcal{O}_{\bar{e}LLLH}$	$\frac{1}{3} n_f^2(2n_f^2+1)$	1	57	$\frac{1}{3} n_f^2(14n_f^2+1)$	5	381
	$\mathcal{O}_{\bar{d}LQLH1}$	n_f^4	1	81			
	$\mathcal{O}_{\bar{d}LQLH2}$	n_f^4	1	81			
	$\mathcal{O}_{\bar{d}LueH}$	n_f^4	1	81			
	$\mathcal{O}_{\bar{Q}uLLH}$	n_f^4	1	81			
$\psi^4 D$	$\mathcal{O}_{\bar{d}uLLD}$	$\frac{1}{2} n_f^3(n_f+1)$	1	54	$\frac{1}{2} n_f^3(n_f+1)$	1	54
Total: B		$\frac{31}{6} n_f^4 + \frac{1}{2} n_f^3 + \frac{13}{3} n_f^2 + n_f$	11	474	$\frac{31}{6} n_f^4 + \frac{1}{2} n_f^3 + \frac{13}{3} n_f^2 + n_f$	11	474
$\not{B}$: $\psi^4 H$	$\mathcal{O}_{\bar{L}dud\tilde{H}}$	n_f^4	1	81	$\frac{1}{6} n_f^2(17n_f^2-3n_f-2)$	2	213
	$\mathcal{O}_{\bar{L}dddH}$	$\frac{1}{3} n_f^2(n_f^2-1)$	0	24			
	$\mathcal{O}_{\bar{e}Qdd\tilde{H}}$	$\frac{1}{2} n_f^3(n_f-1)$	0	27			
	$\mathcal{O}_{\bar{L}dQQ\tilde{H}}$	n_f^4	1	81			
$\not{B}$: $\psi^4 D$	$\mathcal{O}_{\bar{L}QddD}$	$\frac{1}{2} n_f^3(n_f+1)$	1	54	$\frac{1}{6} n_f^2(4n_f^2+6n_f+2)$	2	84
	$\mathcal{O}_{\bar{e}dddD}$	$\frac{1}{6} n_f^2(n_f^2+3n_f+2)$	1	30			
Total: $\not{B}$		$\frac{7}{2} n_f^4 + \frac{1}{2} n_f^3$	4	297	$\frac{7}{2} n_f^4 + \frac{1}{2} n_f^3$	4	297
Total: $B + \not{B}$		$\frac{26}{3} n_f^4 + n_f^3 + \frac{13}{3} n_f^2 + n_f$	15	771	$\frac{26}{3} n_f^4 + n_f^3 + \frac{13}{3} n_f^2 + n_f$	15	771

Table: Comparison of operator counting: direct counting **vs** Hilbert series.

1-loop RGE of dim-7 operators

- RGE preserves the operator symmetry, e.g. $\not{B}$ operators do not mix with B-conserving operators.
- RG running of six dim-7 both $\not{L}$ and $\not{B}$ operators: all correction calculated.
- RG running of twelve dim-7 $\not{L}$ but B-conserving operators: part of them finished.
- Operator mixing is ubiquitous among different classes.
- Interesting cancellation takes place between different diagrams.
- The rich and non-trivial flavor mixing.

1-loop RGE of dim-7 operators

Dimensional regularizaton + $\overline{\text{MS}}$ scheme + R_ξ-gauge

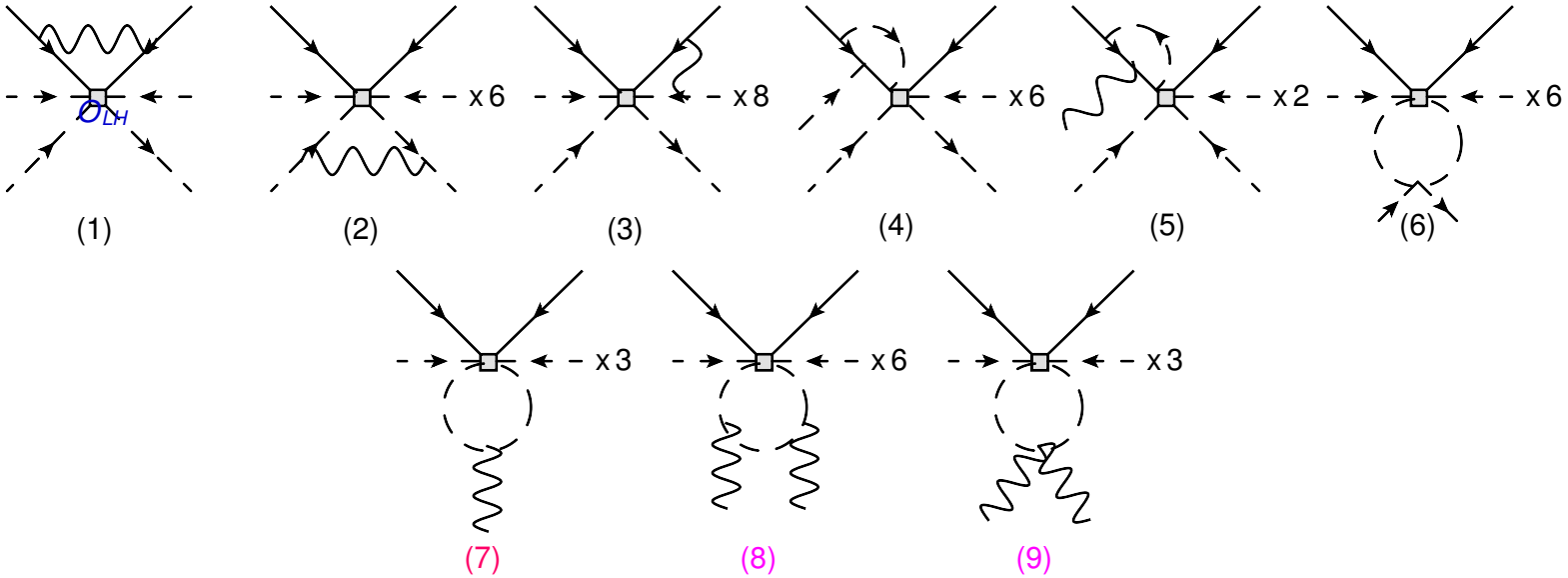
$$16\pi^2\beta_i = 16\pi^2\frac{d}{d\mu}C_i = \sum_j \gamma_{ij}C_j$$

- Calculate the β-function or the anomalous dimension matrix γ is boiling down to determine the counterterm.
- Extracting counterterm by computing the generated amplitude with an insertion of dim-7 operator at 1-loop order.
- Also needing to include field strength renormalization constants.
- ξ independent as a check for the calculation.
- Promote flavor symmetry as an another check.

1-loop RGE of six dim-7 both $\not{L}$ and $\not{B}$ operators(part)

$$
\begin{aligned}
\dot{C}_1^{prst} &= +\left(-4g_3^2-\frac{9}{4}g_2^2-\frac{17}{12}g_1^2+W_H\right)C_1^{prst}-\frac{10}{3}g_1^2C_1^{ptsr}-\frac{3}{2}(Y_eY_e^\dagger)_{pv}C_1^{vrst} \\
&+3(Y_d^\dagger Y_d)_{vr}C_1^{pvst}+3(Y_d^\dagger Y_d)_{vt}C_1^{prsv}+2(Y_u^\dagger Y_u)_{vs}C_1^{prvt}-2(Y_d^\dagger Y_u)_{vs}\left(C_2^{pvrt}+v\leftrightarrow r\right) \\
&+4(Y_e)_{pv}(Y_u)_{ws}C_3^{vwrt}-2\Big((Y_u)_{vs}(Y_d)_{wt}+s\leftrightarrow t\Big)C_4^{prvw}-\frac{1}{6}\left(11g_1^2+24g_3^2\right)(Y_u)_{vs}C_5^{pvrt} \\
&+\frac{1}{6}\left(13g_1^2+48g_3^2\right)(Y_u)_{vs}C_5^{pvtr}-\frac{3}{2}(Y_d)_{vt}(Y_d^\dagger Y_u)_{ws}C_5^{pvrw} \\
&-3(Y_u)_{vs}\Big((Y_d^\dagger Y_d)_{wt}C_5^{pvrw}-r\leftrightarrow t\Big)+\frac{3}{2}(Y_e)_{pv}(Y_d^\dagger Y_u)_{ws}C_6^{vrtw}, \\
\dot{C}_2^{prst} &= +\left(-4g_3^2-\frac{9}{4}g_2^2-\frac{13}{12}g_1^2+W_H\right)C_2^{prst}+\frac{5}{2}(Y_eY_e^\dagger)_{pv}C_2^{vrst} \\
&+2\Big((Y_d^\dagger Y_d)_{vr}C_2^{pvst}+(Y_d^\dagger Y_d)_{vs}C_2^{prvt}+(Y_d^\dagger Y_d)_{vt}C_2^{prsv}\Big) \\
&-\frac{1}{4}\Big[\Big((Y_u^\dagger Y_d)_{vs}C_1^{prvt}+(Y_u^\dagger Y_d)_{vr}C_1^{psvt}+(Y_u^\dagger Y_d)_{vs}C_1^{ptvr}\Big)-s\leftrightarrow t\Big] \\
&+\Big\{\Big[\Big(\frac{1}{3}(g_1^2-6g_3^2)(Y_d)_{vr}C_5^{pvst}-\frac{1}{4}g_1^2(Y_d)_{vs}C_5^{pvrt}-\frac{3}{4}(Y_d)_{vr}(Y_d^\dagger Y_d)_{wt}C_5^{pvsw}\Big)+r\leftrightarrow t\Big]-s\leftrightarrow t\Big\} \\
&+\frac{1}{2}(Y_e)_{pv}\Big\{\Big[g_1^2\left(C_6^{vrst}+r\leftrightarrow s\right)+\frac{3}{4}\Big((Y_d^\dagger Y_d)_{wt}\left(C_6^{vrsw}+r\leftrightarrow s\right)+(Y_d^\dagger Y_d)_{wr}C_6^{vtsw}\Big)\Big]-s\leftrightarrow t\Big\}, \\
\dot{C}_3^{prst} &= +\left(-4g_3^2-\frac{9}{4}g_2^2+\frac{11}{12}g_1^2+W_H\right)C_3^{prst} \\
&+\Big[\Big((Y_e^\dagger Y_e)_{pv}C_3^{vrst}+\frac{5}{4}(Y_uY_u^\dagger+Y_dY_d^\dagger)_{vr}C_3^{pvst}+3(Y_d^\dagger Y_d)_{vs}C_3^{prvt}-(Y_d^\dagger)_{wr}(Y_d)_{vs}C_3^{pvwt}\Big)-s\leftrightarrow t\Big] \\
&-\frac{1}{2}(Y_e^\dagger)_{pv}\Big[\Big((Y_u^\dagger)_{wr}C_1^{vtws}+2(Y_d)_{ws}C_4^{vtwr}+(Y_d)_{wt}C_4^{vsrw}+3g_1^2C_5^{vrst}+3(Y_d^\dagger Y_d)_{wt}C_5^{vrsw}\Big)-s\leftrightarrow t\Big] \\
&+\frac{1}{4}(g_1^2+12g_3^2)(Y_d^\dagger)_{vr}\Big[\left(C_6^{pvst}+C_6^{psvt}+C_6^{pstv}\right)-s\leftrightarrow t\Big] \\
&-\frac{3}{4}\Big\{\Big[(Y_d^\dagger Y_d)_{vs}(Y_d^\dagger)_{wr}\left(C_6^{ptvw}-r\leftrightarrow v\right)+(Y_d^\dagger Y_d)_{ws}(Y_d^\dagger)_{vr}\left(C_6^{ptvw}+2C_6^{pvtw}\right)\Big]-s\leftrightarrow t\Big\},
\end{aligned}
$$

Renormalize dim-7 neutrino mass operator



$$\mathcal{M}_{(7)} = 0, \quad \mathcal{M}_{(8)} + \mathcal{M}_{(9)} = \text{finite}$$

And the β-function is

$$16\pi^2 \mu \frac{d}{d\mu} C_{LH}^{pr} = (40\lambda + 4Y - \frac{3}{2}g_1^2 - \frac{15}{2}g_2^2) C_{LH}^{pr} - \frac{3}{2}\Big[(Y_e Y_e^\dagger)_{vp} C_{LH}^{vr} + p \to r\Big] + \ldots$$

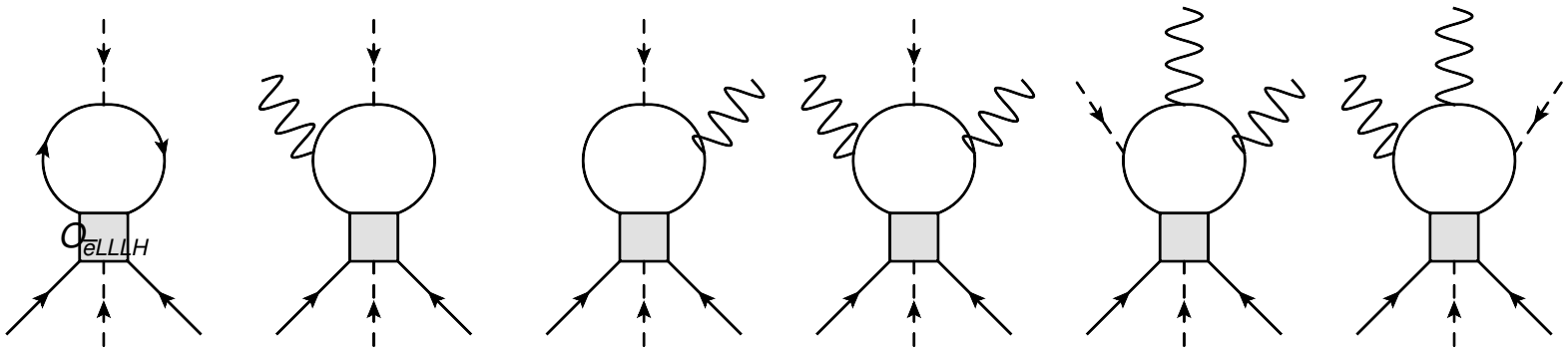
where ... stands for contributions from all other 11 $\not{L}$ but B-conserving operators. Comparing it with dim-5 neutrino mass operator[S. Antusch et al:0108005]

$$16\pi^2 \mu \frac{d}{d\mu} C_5^{pr} = (4\lambda + 2Y - 3g_2^2) C_5^{pr} - \frac{3}{2}\Big[(Y_e Y_e^\dagger)_{vp} C_5^{vr} + p \to r\Big]$$

$$Y \equiv \mathrm{Tr}\Big[3(Y_u^\dagger Y_u) + 3(Y_d^\dagger Y_d) + (Y_e^\dagger Y_e)\Big]$$

Operator mixing

Example: From class $\Psi^4 H$ to $\Psi^4 H$, $\Psi^2 H^4$ and $\Psi^2 H^2 X$
Consider the following diagrams with an insertion of the operator $\mathcal{O}_{\bar{e}LLLH}$



$$\sum \text{diagrams} \propto L^2 H D^2 H + \mathcal{O}_{LHX}$$
$$\propto L^2 H^4 + \Psi^4 H + L^2 H^2 X$$

EoM: $D^2 H = \lambda v^2 H - 2\lambda (H^\dagger H) H - \epsilon^{\mathsf{T}} \bar{Q} Y_u^\dagger u - \bar{d} Y_d Q - \bar{e} Y_e L$

Structure of γ-matrix

Based on nonrenormalization theorem[C. Cheung:1505.01844], one can understand the structure of anomalous dimension matrix γ_{ij}. First define the holomorphic and anti-holomorphic weights of an operator $\mathcal{O}$ by

$$\omega(\mathcal{O}) = n(\mathcal{O}) - h(\mathcal{O}),\ \bar{\omega}(\mathcal{O}) = n(\mathcal{O}) + h(\mathcal{O})$$

$n(\mathcal{O})$: the minimal number of fields the operator $\mathcal{O}$ generates;
$h(\mathcal{O})$: the total helicity of the operator $\mathcal{O}$.
And the SM field weights are as follows:

SM "fields"	D_μ	H	Ψ	$X_{\mu\nu}$	$\bar{X}_{\mu\nu}$	$\bar{\Psi}$
weights $(\omega, \bar{\omega})$	$(0,0)$	$(1,1)$	$(\frac{1}{2}, \frac{3}{2})$	$(0,2)$	$(2,0)$	$(\frac{3}{2}, \frac{1}{2})$

Left-handed and right-handed fermion fields: Ψ and $\bar{\Psi}$;
Dual field strength tensor: $\tilde{X}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}X^{\rho\sigma}$.

Structure of γ-matrix

The nonrenormalization theorem dictates that an operator $\mathcal{O}_i$ can only be renormalized by an operator $\mathcal{O}_j$ if $\omega_i \geq \omega_j$ and $\bar{\omega}_i \geq \bar{\omega}_j$, and absent of non-holomorphic Yukawa couplings.

	weights	$(3,5)$	$(3,5)$	$(3,7)$	$(4,6)$	$(4,6)$	$(5,7)$
weights	γ_{ij}	$\Psi^2 H^2 D^2$	$\bar{\Psi}\Psi^3 D$	$\Psi^2 H^2 X$	$\Psi^2 H^3 D$	$\bar{\Psi}\Psi^3 H$	$\Psi^2 H^4$
$(3,5)$	$\Psi^2 H^2 D^2$	$\surd$	$\surd$	0	0	0	0
$(3,5)$	$\bar{\Psi}\Psi^3 D$	$\surd$	$\surd$	0	0	0	0
$(3,7)$	$\Psi^2 H^2 X$	$\surd$	$\times$	$\surd$	0	$\emptyset y$	0
$(4,6)$	$\Psi^2 H^3 D$	$\surd$	$\times$	$\emptyset y$	$\surd$	$\surd$	0
$(4,6)$	$\bar{\Psi}\Psi^3 H$	$\surd$	$\surd$	$\emptyset y$	$\surd$	$\surd$	0
$(5,7)$	$\Psi^2 H^4$	$\surd$	$\times$	$\surd$	$\surd$	$\surd$	$\surd$

0: vanishing due to nonrenormalization theorem

$\times$: vanishing because no corresponding diagrams

$\emptyset y$: nonvanishing due to nonholomorphic Yukawa coupling y

$\surd$: surviving pieces

Proton decay with $\Delta L = -\Delta B = 1$

Consider the dominant operator relating with proton decay after symmetry breaking,

$$\tilde{\mathcal{O}}^{p111}_{\bar{L}dud\tilde{H}} = v(\bar{\nu}_p P_R d)(uCP_R d)$$
$$\tilde{\mathcal{O}}^{p111}_{\bar{L}dQQ\tilde{H}} = -v(\bar{\nu}_p P_R d)(uCP_L d)$$

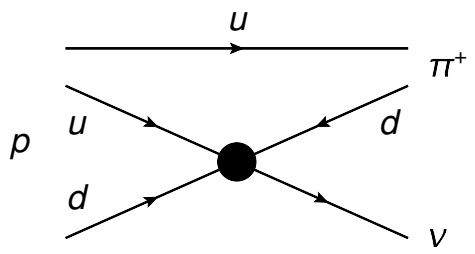


Figure: $p \to \pi^+\nu$ with $\Delta L = -\Delta B = 1$

Their Wilson coefficients decouple when we only keep gauge and top-Yukawa couplings,

$$\mu\frac{d}{d\mu}C^{p111}_{\bar{L}dud\tilde{H}} = \frac{1}{4\pi}[-4\alpha_3 - \frac{9}{4}\alpha_2 - \frac{57}{12}\alpha_1 + 3\alpha_t]C^{p111}_{\bar{L}dud\tilde{H}}$$
$$\mu\frac{d}{d\mu}C^{p111}_{\bar{L}dQQ\tilde{H}} = \frac{1}{4\pi}[-4\alpha_3 - \frac{27}{4}\alpha_2 - \frac{19}{12}\alpha_1 + 3\alpha_t]C^{p111}_{\bar{L}dQQ\tilde{H}}$$

where $\alpha_i \equiv g_i^2/4\pi$ and $\alpha_t \equiv y_t^2/4\pi$.

Proton decay with $\Delta L = -\Delta B = 1$

Estimating the coefficients from GUT scale(M 10^{15}GeV) to proton mass scale(m_p 1GeV), The overall RGE running results are

$$
\begin{aligned}
C^{p111}_{\bar{L}dud\tilde{H}}(m_p) &= (2.034)(1.158)(1.262)(0.787)\, C^{p111}_{\bar{L}dud\tilde{H}}(M) \\
&= 2.34\, C^{p111}_{\bar{L}dud\tilde{H}}(M), \\
C^{p111}_{\bar{L}dQQ\tilde{H}}(m_p) &= (2.034)(1.551)(1.081)(0.787)\, C^{p111}_{\bar{L}dQQ\tilde{H}}(M) \\
&= 2.68\, C^{p111}_{\bar{L}dQQ\tilde{H}}(M),
\end{aligned}
$$

$$g_3, \quad g_2, \quad g_1, \quad y_t$$

They receive an enhancement factor of about 2.

Summary

- Basis of dim-7 operator determined: $12 + 6(\not{B})$.
- 1-loop RGE of six dim-7 both $\not{L}$ and $\not{B}$ operators calculated.
- 1-loop RGE of twelve dim-7 $\not{L}$ but B-conserving operators calculated in part.
- The structure of anomalous dimensional matrix-γ determined.
- Proton exotic decay with $\Delta B - \Delta L = 2$ discussed.