Relativistic Corrections to Electromagnetic Heavy Quarkonium Production

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Outline:

- Motivation
- NRQCD factorization
- Charmonium production
- Charmonium decay
- Summary

Motivation

- *Heavy quarkonia probe all the energy regimes of QCD, and are thus an ideal, and to some extent, unique laboratory to test our understanding of QCD (both perturbative and nonperturbative aspects). (Brambilla et al., 2005).*
- *NRQCD provides an elegant approach to separate relativistic physics of annihilation from the nonrelativistic physics of quarkonium structure; (BBL, 1995)*
- *Large discrepancies are found between leading order (LO) theoretical prediction and the experimental measurements for the process* $e^+e^- \to J/\Psi + H$ *at B-factory, Belle, and BaBar. (Brambilla, et al., 2011)*

	$J/\psi + \eta_c$	$J/\psi + \chi_{c0}$	$J/\psi + \eta_c(2S)$
$\sigma \times B_{>2}$ (fb) (Belle)	$25.6 \pm 2.8 \pm 3.4$	$6.4 \pm 1.7 \pm 1.0$	$16.5 \pm 3.0 \pm 2.4$
$\sigma \times B_{>2}$ (fb) (BABAR)	$17.6 \pm 2.8^{+1.5}_{-2.1}$	$10.3 \pm 2.5^{+1.4}_{-1.8}$	$16.4 \pm 3.7^{+2.4}_{-3.0}$
σ (fb) (Liu, He, Chao)	5.5	6.9	3.7
σ (fb) (Braaten, Lee)	3.78 ± 1.26	2.40 ± 1.02	1.57 ± 0.52

Motivation

- *Corrections of higher order in both α_s and v can bring theory into agreement with experiment. (Zhang, Gao, and Zhao (2005), Gong, and Wang (2007), He, Fan, and Chao (2007), Zhang, Ma, and Chao (2008))*
- $v^2 \approx 0.3, \quad \alpha_s(m) \approx 0.24$, *thus relativistic correction is as significant as radiation correction. And contribution of colour octet states is as important as that of colour singlet states.*

Motivation

- Cross section for $e^+e^- \to H_{c\bar{c}} + \gamma$ can be larger than that for $e^+e^- \to J/\Psi + \mathrm{H}$ by 2 orders, and thus could be observed at B-factory. (Chung, Lee, and Yu, 2008)

$$e^+ + e^- \to H_{c\bar{c}} + \gamma$$

Previous works:

- Chung, Lee, and Yu (2008):
  LO for S-wave and P-wave

- Sang, and Chen (2010):
  NLO in α_s^0 for S-wave and P-wave, NLO in v for η_c;

- Li, Xu, Liu, and Zhang (2013);
  NLO in both α_s^0 and v for S-wave and P-wave

(Contributions of color octet operators were not considered);

NRQCD Factorization

Hamiltonian:

$$H_{eff} = -\frac{1}{2m}\bar{\Psi}\boldsymbol{D}^2\Psi + \frac{1}{2}\boldsymbol{B}^2 + \frac{1}{2}\boldsymbol{E}^2 + \cdots$$
$$-\frac{1}{2m}g\bar{\Psi}\boldsymbol{B}\cdot\sigma\Psi - \frac{1}{8m^2}g\bar{\Psi}\gamma^0(\boldsymbol{D}\cdot\boldsymbol{E} - \boldsymbol{E}\cdot\boldsymbol{D})\Psi + \cdots, \qquad (1)$$

where

$$\Psi = \begin{pmatrix}\psi \\ \chi\end{pmatrix}$$

field annihilates a heavy quark

field creates an heavy antiquark

(Newton-Wigner representation)

$$\sigma^i = \gamma^5\gamma^0\gamma^i, \qquad [\sigma^i, \sigma^j] = 2i\varepsilon^{ijk}\sigma^j.$$

Particle number densities:

$$n_\pm = \frac{1}{2}\Psi^\dagger(1 \pm \gamma^0)\Psi$$

conserved individually

NRQCD Factorization

Factorization

Decay rate:

$$\Gamma(H) = \sum_n \frac{2\mathrm{Im}\, f_n(\Lambda)}{m^{d_n-4}} \langle H|\mathcal{O}_n(\Lambda)|H\rangle \qquad (2)$$

$f_n(\Lambda)$: effective coupling constant

$\mathcal{O}_n(\Lambda)$: four-fermion interactions

Inclusive cross section:

$$\sigma(H) = \sum_n F_n(\Lambda)\langle 0|\mathcal{O}_n^H(\Lambda)|0\rangle \qquad (3)$$

$F_n(\Lambda)$: short-distance coefficients (SDCs)

Perturbative & process-dependent

$\langle 0|\mathcal{O}_n^H(\Lambda)|0\rangle$: long-distance matrix elements (LDMEs)

Nonperturbative & process-independent

NRQCD Factorization

Factorization at amplitude level

(for electromagnetic decay or exclusive electromagnetic production)

$$\mathcal{M}(H) = \sum_n C_n \langle H | \bar{\Psi} \mathcal{K}_n \Psi \, | 0 \rangle \qquad (4)$$

Colour octet

Contribution of colour octet state and that of colour single state are of the same order in v for P-wave states.

Considering h_c for instance, taking $\mathcal{K}n = D^i$,

arising from amplitude for Fock state $|c\bar{c}g\rangle$

$$\langle c\bar{c} | \bar{\Psi} D^i \Psi | 0 \rangle \sim v \, , \qquad v \, \langle c\bar{c}g | \bar{\Psi} D^i \Psi | 0 \rangle \sim v$$

NRQCD Factorization

Velocity-scaling rules

Operator	Estimate	Description
α_s	v	effective quark-gluon coupling constant
ψ	$(Mv)^{3/2}$	heavy-quark (annihilation) field
χ	$(Mv)^{3/2}$	heavy-antiquark (creation) field
D_t	Mv^2	gauge-covariant time derivative
$\mathbf{D}$	Mv	gauge-covariant spatial derivative
$g\mathbf{E}$	M^2v^3	chromoelectric field
$g\mathbf{B}$	M^2v^4	chromomagnetic field
$g\phi$ (in Coulomb gauge)	Mv^2	scalar potential
$g\mathbf{A}$ (in Coulomb gauge)	Mv^3	vector potential

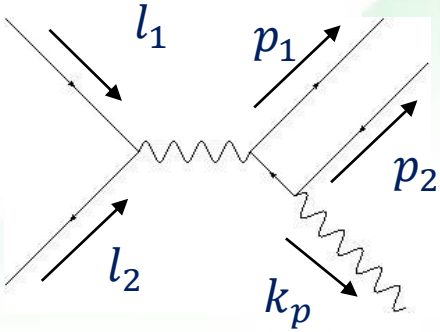
"The sum in the factorization formula is a v expansion"

Charmonium Production

$$e^+ + e^- \to H_{c\bar{c}} + \gamma$$

QCD amplitude:

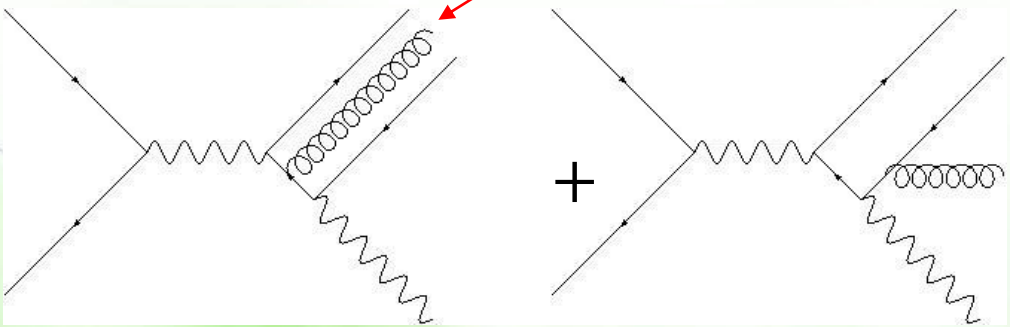
colour singlet



$+ \quad \cdots$

soft gluon

colour octet



$+ \quad \cdots$

Charmonium Production

QCD amplitude:

The QCD amplitudes are calculated in the standard way.

$$\mathcal{M} = -\frac{i}{s} V_\mu \mathcal{M}'^\mu = \frac{i}{s}\left(\delta^{ij} - \frac{Q^i Q^j}{(Q^0)^2}\right) V^i \mathcal{M}'^i \equiv \frac{i}{s} V'^i \mathcal{M}'^i, \qquad (5)$$

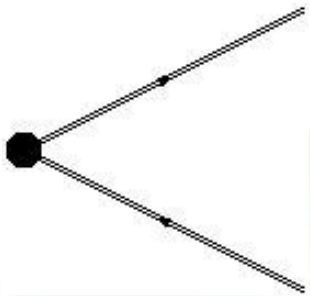
where $Q = l_1 + l_2,\ s = Q^2,\ V^\mu = \bar{v}(l_2)\gamma^\mu u(l_1)$, T'^μ is the amplitude for the decay of a virtual photon. We stay in the rest frame of the hadron. Thus $\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{k}_g = \mathbf{0}$, where k_g is the momentum of the soft gluon.

(The final result is too lengthy to be presented here.)

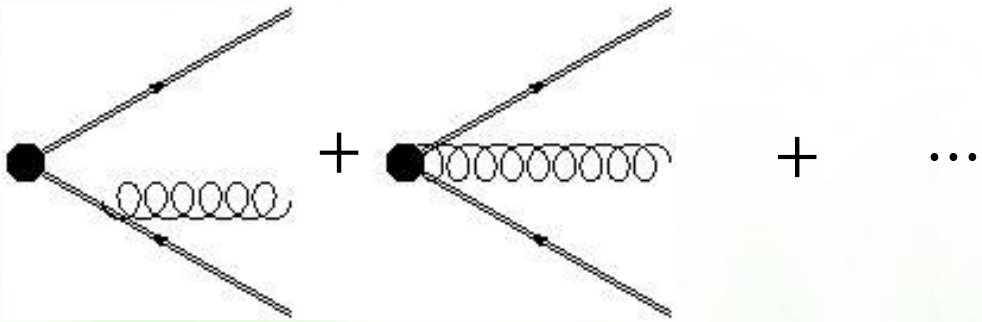
Charmonium Production

NRQCD amplitude:

colour singlet



colour octet



Charmonium Production

more NRQCD vertexes :

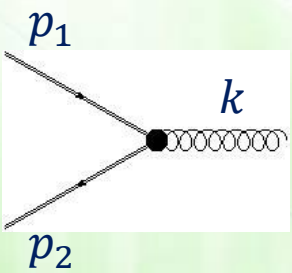
$$\frac{1}{8m^2} ig\bar{\Psi}\gamma^0(\boldsymbol{E}\times\boldsymbol{D}-\boldsymbol{D}\times\boldsymbol{E})\cdot\sigma\Psi,$$

$$-\frac{1}{8m^2}\bar{\Psi}\boldsymbol{D}^4\Psi, \qquad -\frac{1}{8m^3}g\bar{\Psi}(\boldsymbol{D}^2\boldsymbol{B}\cdot\sigma+\boldsymbol{B}\cdot\sigma\boldsymbol{D}^2)\Psi$$

NRQCD Feynman rules:

$$\frac{i}{p^0-E(p,m)}$$

on-shell energy



$$-\frac{1}{2m}\boldsymbol{D}^2 \quad , \quad -\frac{1}{2m}g\boldsymbol{B}\cdot\sigma \quad , \quad \frac{1}{8m^2}ig\gamma^0(\boldsymbol{E}\times\boldsymbol{D}-\boldsymbol{D}\times\boldsymbol{E})\cdot\sigma\cdots$$

$$\frac{i}{2m}(\boldsymbol{p}_1+\boldsymbol{p}_2)\cdot\boldsymbol{\epsilon} \quad , \quad \frac{1}{2m}g(\boldsymbol{k}\times\boldsymbol{\epsilon})\cdot\boldsymbol{\sigma} \quad , \quad \frac{1}{8m^2}g[(\boldsymbol{p}_1+\boldsymbol{p}_2)\times\boldsymbol{\epsilon}]\cdot\sigma \qquad \cdots$$

Charmonium Production

NRQCD operators:

η_c $\quad \frac{1}{m}\bar{\Psi}\Psi, \quad -\frac{1}{m^3}\bar{\Psi}\mathbf{D}^2\Psi, \quad \frac{1}{m^3}\bar{\Psi}\mathbf{B}\cdot\boldsymbol{\gamma}\Psi$

χ_{c0} $\quad -\frac{i}{m^2}\bar{\Psi}\mathbf{D}\cdot\boldsymbol{\gamma}\Psi, \quad \frac{i}{m^2}\bar{\Psi}\mathbf{D}\cdot\boldsymbol{\gamma}\mathbf{D}^2\Psi, \quad \frac{i}{m^3}\bar{\Psi}\gamma^0\mathbf{E}\cdot\boldsymbol{\gamma}\Psi$

χ_{c1} $\quad -\frac{i}{m^2}\bar{\Psi}(\boldsymbol{\gamma}\times\mathbf{D})^{\boldsymbol{i}}\Psi, \quad \frac{i}{m^4}\bar{\Psi}(\boldsymbol{\gamma}\times\mathbf{D})^{\boldsymbol{i}}\mathbf{D}^2\Psi, \quad \frac{i}{m^3}\bar{\Psi}\gamma^0(\boldsymbol{\gamma}\times\mathbf{E})^{\boldsymbol{i}}\Psi$

χ_{c2} $\quad -\frac{i}{m^2}\bar{\Psi}\gamma^{(i}D^{j)}\Psi, \quad \frac{i}{m^4}\bar{\Psi}\gamma^{(i}D^{j)}\mathbf{D}^2\Psi, \quad \frac{i}{m^3}\bar{\Psi}\gamma^0\gamma^{(i}E^{j)}\Psi,$

$$\frac{i}{m^4}\bar{\Psi}\left(\gamma^{(i}D^{j)}\mathbf{D}\cdot\boldsymbol{\gamma}-\frac{\mathbf{2}}{\mathbf{5}}\gamma^{(i}D^{j)}\mathbf{D}^2\right)\Psi$$

? $\quad \frac{1}{m^3}\bar{\Psi}(\boldsymbol{\gamma}\times\boldsymbol{B})^{\boldsymbol{i}}\Psi$

Charmonium Production

matching

To do matching, spinors in QCD amplitude are substituted by

$$u(p) = \sqrt{\frac{E(\mathbf{p},m)+m}{2E(\mathbf{p},m)}}\begin{pmatrix}\xi \\ \frac{\mathbf{p}\cdot\sigma}{E(\mathbf{p},m)+m}\xi\end{pmatrix}, \qquad (5)$$

$$v(p) = \sqrt{\frac{E(\mathbf{p},m)+m}{2E(\mathbf{p},m)}}\begin{pmatrix}\frac{\mathbf{p}\cdot\sigma}{E(\mathbf{p},m)+m}\eta \\ \eta\end{pmatrix}, \qquad (6)$$

The QCD amplitude thus obtained are expanded in power series of v.

Contribution of a specific angular momentum state is extracted by decomposing the amplitude into spherical tensors (Coope, and Snider, 1970):

$$A^iB^j = A^{(i}B^{j)} + A^{[i}B^{j]} + \frac{1}{3}\boldsymbol{A}\cdot\boldsymbol{B}\delta^{ij} \qquad (7)$$

$$\vdots$$

Charmonium Production

The short-distance coefficients are obtained by matching QCD amplitude and NRQCD amplitude.

SDCs for amplitude:

(The superscript corresponds to the order of the corresponding operator listed in page 13)

$$c_{\eta_c}^{(1)} = 4\pi\alpha Q_c^2 \mathbf{V}' \times \epsilon_{\mathbf{p}}^* \cdot \hat{\mathbf{k}}_{\mathbf{p}}$$

$$c_{\chi_{c0}}^{(1)} = -\frac{4}{3} i(2a-1)\pi\alpha Q_c^2 \dot{V}' \epsilon_p^*$$

$$c_{\eta_c}^{(2)} = -\frac{8}{3}\pi\alpha Q_c^2 \mathbf{V}' \times \epsilon_{\mathbf{p}}^* \cdot \hat{\mathbf{k}}_{\mathbf{p}}$$

$$c_{\chi_{c0}}^{(2)} = i\left(\frac{32}{15}a - \frac{6}{5}\right)\pi\alpha Q_c^2 \dot{V}' \epsilon_p^*$$

$$c_{\eta_c}^{(3)} = 4(1+a)\pi\alpha Q_c^2 \mathbf{V}' \times \epsilon_{\mathbf{p}}^* \cdot \hat{\mathbf{k}}_{\mathbf{p}}$$

$$c_{\chi_{c0}}^{(3)} = \frac{2}{3}(4a^2+6a+1)\pi\alpha Q_c^2 V' \times \boldsymbol{\epsilon}_p^* \cdot \hat{k}_p$$

$$c_{\chi_{c1}}^{(1)} = 2i\pi\alpha Q_c^2 \left[2(1+a)(\mathbf{V}' \times \epsilon_{\mathbf{p}}^*)^i + \dot{V}'\hat{k}_p(\epsilon_{\mathbf{p}}^* \times \hat{\mathbf{k}}_{\mathbf{p}})^i\right]$$

$$c_{\chi_{c1}}^{(2)} = -\frac{2}{5} i\pi\alpha Q_c^2 \left[2(8+3a)(\mathbf{V}' \times \epsilon_{\mathbf{p}}^*)^i + 4\dot{V}'\hat{k}_p(\epsilon_{\mathbf{p}}^* \times \hat{\mathbf{k}}_{\mathbf{p}})^i\right]$$

Charmonium Production

SDCs for amplitude :

$$c^{(3)}_{\chi_{c1}} = 2\pi\alpha Q_c^2\left[(2a^2+3a+2)\left(V'\times\boldsymbol{\epsilon}_p^*\right)^i + (1+a)V'\cdot\hat{k}_p\left(\boldsymbol{\epsilon}_p^*\times\hat{k}_p\right)^i\right]$$

$$c^{(1)}_{\chi_{c2}} = 2i\pi\alpha Q_c^2(\mathbf{V}'\cdot\epsilon^*_{\mathbf{p}}\hat{k}_p^i\hat{k}_p^j - 2aV'^i\epsilon_p^{*j} - \dot{V}'\hat{k}_p\hat{k}_p^i\epsilon_p^{*j}) + i \rightleftharpoons j;$$

$$c^{(2)}_{\chi_{c2}} = i\pi\alpha Q_c^2(2aV'^i\epsilon_p^{*j} - \frac{6}{5}\mathbf{V}'\cdot\epsilon^*_{\mathbf{p}}\hat{k}_p^i\hat{k}_p^j - 2\dot{V}'\hat{k}_p\hat{k}_p^i\epsilon_p^{*j}) + i \rightleftharpoons j$$

$$c^{(3)}_{\chi_{c2}} = \pi\alpha Q_c^2\left[(4a^2+2a)V'^i\epsilon_p^{*j} - 2(1+a)\mathbf{V}'\cdot\epsilon^*_{\mathbf{p}}\hat{k}_p^i\hat{k}_p^j + 2(1+a)\dot{V}'\hat{k}_p\hat{k}_p^i\epsilon_p^{*j}\right] + i \rightleftharpoons j;$$

$$c^{(4)}_{\chi_{c2}} = i\pi\alpha Q_c^2\left[\frac{50}{21}aV'^i\epsilon_p^{*j} - (\frac{20}{s1}a+\frac{1}{7})\mathbf{V}'\cdot\epsilon^*_{\mathbf{p}}\hat{k}_p^i\hat{k}_p^j + (\frac{5}{21}-\frac{40}{21}a)\dot{V}'\hat{k}_p\hat{k}_p^i\epsilon_p^{*j}\right] + i \rightleftharpoons j$$

$$c^{(1)}_{?} = \pi\alpha Q_c^2\left[2(1+2a)\dot{V}'\hat{k}_p\epsilon_p^{*i} - 4(1+a)\dot{V}'\epsilon_p^*\hat{k}_p^i\right]$$

where ϵ_p^* is the polarization vector for the photon, and $a = \frac{m}{|\mathbf{k}_p|}$.

Charmonium Production

cross section

The cross section is obtained by summing the squared norm of amplitude over polarizations, and integrating over phase space.

$$\sigma = \frac{(2\pi)^4}{2s} \int d\Pi \sum |\mathcal{M}|^2 \, \delta^4\left(Q - \sum_i p_i\right), \qquad (8)$$

where the summation of $|\mathcal{M}|^2$ is taken over all the polarizations of particles in the final state, $d\Pi \equiv \prod_i \frac{d^3\mathbf{p}_i}{(2\pi)^3 2E(\mathbf{p}_i)}$, and $\sum_i p_i$ is the total momentum of the final state.

Hadron mass M arising from phase space integration can be eliminated by applying Gremm-Kapustin relation, which is a direct consequence of a more general relation:

$$(M - 2m)\langle 0|\mathcal{O}_n|H\rangle = \langle 0|[\mathcal{O}_n, H]|H\rangle$$

Charmonium Production

cross section

(To save writing, here I only explicitly show the result for χ_{c0})

$$\sigma = \frac{8\pi^2\alpha^3 {Q_c}^4 (s-M^2)(s-4m^2)^2}{9s^3m^2(s-4m^2)^2}\langle 0|\mathcal{O}_1(\,^3P_0)|0\rangle$$

$$+\frac{8\pi^2\alpha^3 {Q_c}^4 (s-M^2)(s-4m^2)(96sm^2-560m^4-9s^2)}{45s^3m^2(s-4m^2)^3}\langle 0|\mathcal{P}_1(\,^3P_0)|0\rangle$$

$$+\frac{4\pi^2\alpha^3 {Q_c}^4 (s-M^2)(s-4m^2)}{9s^3m^2(s-4m^2)^3}\langle 0|\mathcal{T}_8(\,^3P_0)|0\rangle$$

where

$$\mathcal{O}_1(\,^3P_0) = -\frac{1}{3}\bar{\Psi}\mathbf{D}\cdot\boldsymbol{\gamma}\Psi\bar{\Psi}\mathbf{D}\cdot\boldsymbol{\gamma}\Psi;$$

$$\langle 0|\mathcal{P}_1(\,^3P_0)|0\rangle = \frac{1}{6}\bar{\Psi}\mathbf{D}\cdot\boldsymbol{\gamma}\Psi\bar{\Psi}\mathbf{D}\cdot\boldsymbol{\gamma}\mathbf{D}^2\Psi + \frac{1}{6}\bar{\Psi}\mathbf{D}\cdot\boldsymbol{\gamma}\mathbf{D}^2\Psi\bar{\Psi}\mathbf{D}\cdot\boldsymbol{\gamma}\Psi$$

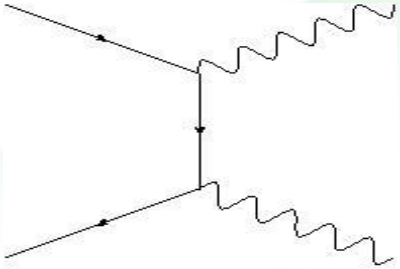
$$\langle 0|\mathcal{T}_8(\,^3P_0)|0\rangle = \frac{1}{6}\bar{\Psi}\mathbf{D}\cdot\boldsymbol{\gamma}\Psi\bar{\Psi}\gamma^0\mathbf{E}\cdot\boldsymbol{\gamma}\Psi + \frac{1}{6}\bar{\Psi}\gamma^0\mathbf{E}\cdot\boldsymbol{\gamma}\Psi\bar{\Psi}\mathbf{D}\cdot\boldsymbol{\gamma}\Psi$$

Charmonium Decay

$$H_{c\bar{c}} \to \gamma + \gamma$$

QCD amplitude:

colour singlet



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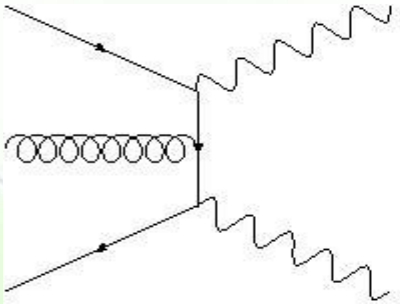
colour octet



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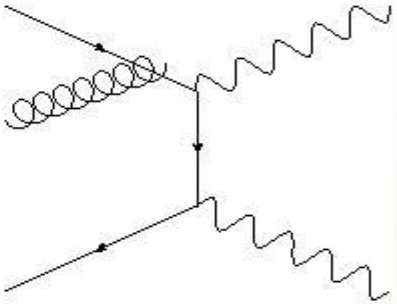

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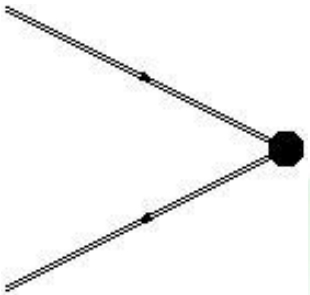
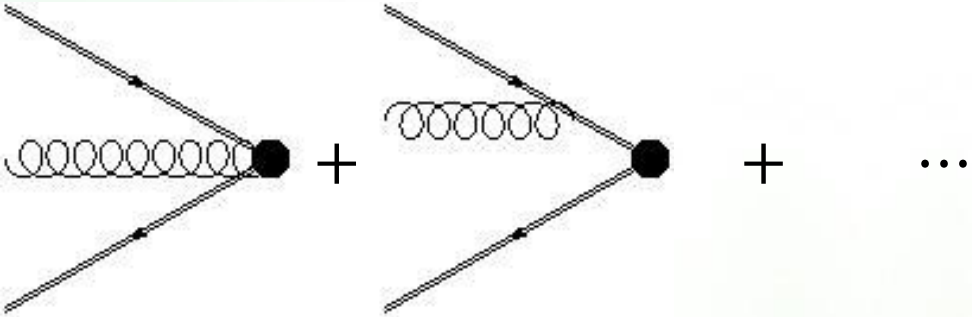
Charmonium Decay

NRQCD amplitude:

colour singlet



colour octet



NRQCD amplitude for decay is just the complex conjugate of that for production with different SDCs.

Charmonium Decay

matching

Similar to the case of production, the SDCs are obtained by matching QCD amplitude and NRQCD amplitude. The decay rate is obtained by summing the squared norm of amplitude over polarizations, and integrating over phase space.

$$\Gamma = \frac{(2\pi)^4}{2M} \int d\Pi \sum |\mathcal{M}|^2 \, \delta^4 \left(P - \sum_i p_i \right), \qquad (7)$$

(The detail of the result is too lengthy to be presented here, and is thus omitted.)

The SDCs for amplitude are exactly the same as those in Sang (2013), which are obtained by FWT transformation. And SDCs for decay rate are the same as those in Brambilla, Mereghetti, and Vairo (2006), which are obtained by matching decay rate directly.

Summary

We calculated the relativistic corrections for the production of charmonia with $J \leq 2$, for which, contributions of colour octet operators were considered.

We verified that the SDCs were independent of the hadron state.

The LO results were consistent with those in Chung, Lee, and Yu (2008), and Sang and Chen (2010).

The relativistic corrections differed from those in Li, Xu, Liu, and Zhang (2013).

Thanks for your attention!