NNLO QCD Corrections to Quarkonium Decay and Production within NRQCD Framework

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Based on:

- arXiv:1505.02665 （PRL115, 222001）

  Can Nonrelativistic QCD Explain the $\gamma\gamma^* \to \eta_c$ Transition Form Factor Data?

- arXiv:1511.06288

  Next-to-next-to-leading-order QCD corrections to $\chi_{c0,2} \to \gamma\gamma$

In Corroboration with
Y. JIA, W.-L. SANG & S.-R. LIANG

Contents:

- Brief introduction.
- NNLO QCD correction to $\gamma\gamma^* \to \eta_c$ form factor and confront the *BaBar* data.
- NNLO QCD correction to $\chi_{c0,2} \to \gamma\gamma$ and confront with *BESIII* data.
- Summary & Outlook.

Factorization:

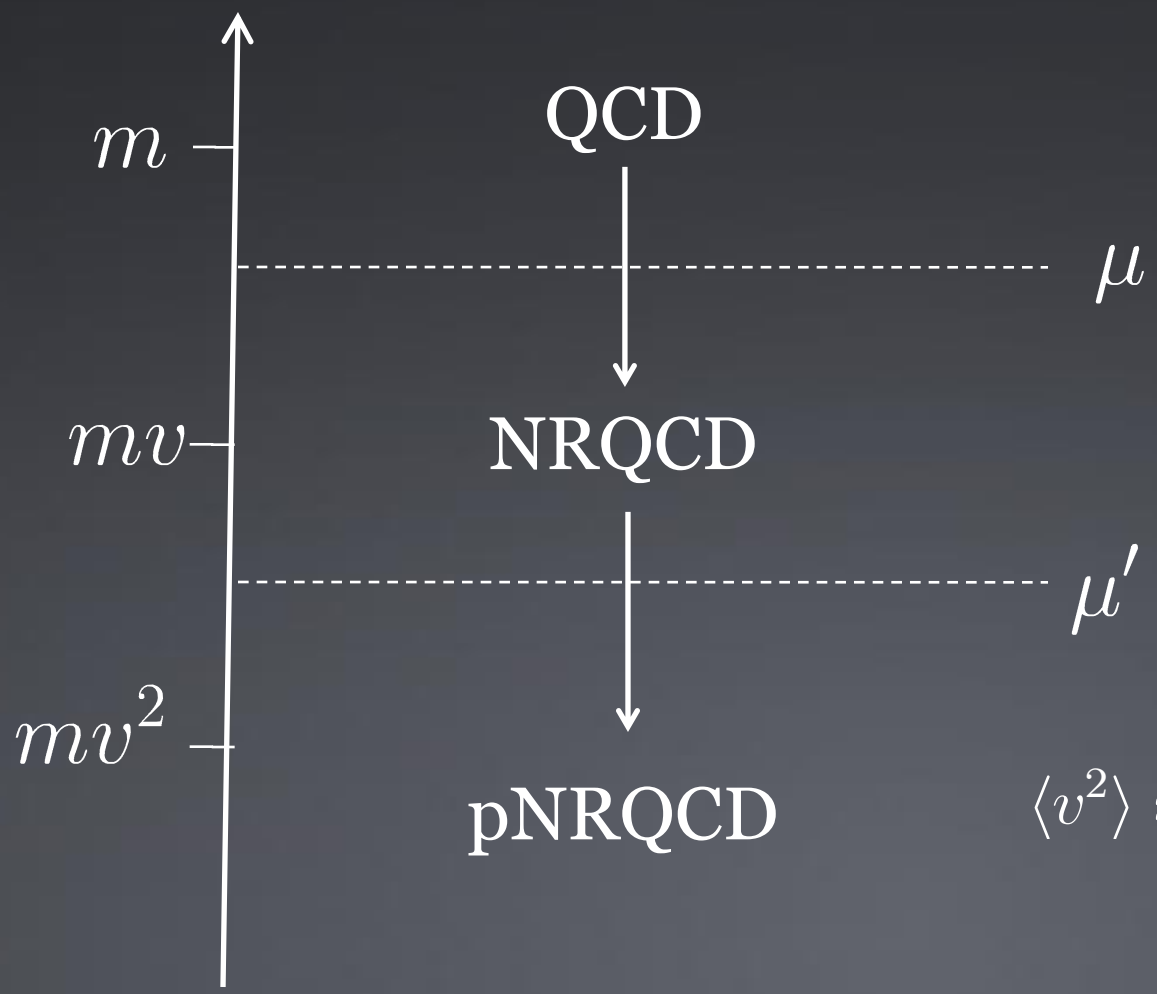
- Factorization: to separate "short-distance" and "long-distance"、"high-energy" and "low-energy" effects.

$$\mathcal{F} = \psi \otimes \mathcal{H} + \cdots$$

- Non-perturbative Part: "long-distance"、"low-energy" effects, due to confinement.

- Perturbative Part: "short-distance"、"high-energy" effects, due to asymptotic freedom.

Heavy Quarkonium:



This scale separation is usually referred to as **NRQCD Factorization**

$$\langle v^2 \rangle \approx \begin{cases} 0.3 & \text{Charmonium} \\ 0.1 & \text{Bottomonium} \end{cases}$$

NRQCD Status:

- Nowadays, NRQCD becomes an important approach to tackle various quarkonium production and decay processes.

- Large Corrections @NLO.

$e^+ + e^- \to J/\psi + \eta_c \qquad \mathcal{K} \approx 1.8 - 2.1$ Zhang *et.al.*

$p + p \to J/\psi + X \qquad \mathcal{K} \approx 2$ Campbell *et.al.*

$\ldots \qquad \ldots$

NRQCD Status:

- Large Corrections @NNLO.

$$\Gamma(J/\psi \to \ell\ell) = \Gamma^{(0)}\left\{1 - \frac{8}{3}\frac{\alpha_s}{\pi} - (44.55 - 0.41 n_f)\frac{\alpha_s^2}{\pi^2} + (-2091 + 120.66 n_f - 0.82\, n_f^2)\frac{\alpha_s^3}{\pi^3}\right\}$$

$$\Gamma(B_c \to \ell\nu) = \Gamma^{(0)}\left\{1 - 1.39\frac{\alpha_s}{\pi} - 23.7\frac{\alpha_s^2}{\pi^2} + \mathcal{O}(\alpha_s^3)\right\}$$

$$\Gamma(\eta_c \to \gamma\gamma) = \Gamma^{(0)}\left\{1 - 1.69\frac{\alpha_s}{\pi} - 56.52\frac{\alpha_s^2}{\pi^2} + \mathcal{O}(\alpha_s^3)\right\}$$

Motivation:

- S-wave Production: $\gamma\gamma^* \to \eta_c$

TABLE III. The Q^2 interval and the weighted average Q^2 value ($\overline{Q^2}$), the $e^+e^- \to e^+e^-\eta_c$ cross section multiplied by $\mathcal{B}(\eta_c \to K\bar{K}\pi)$ $[d\sigma/dQ^2(\overline{Q^2})]$, and the normalized $\gamma\gamma^* \to \eta_c$ transition form factor ($|F(\overline{Q^2})/F(0)|$). The statistical and systematic errors are quoted separately for the cross section, but are combined in quadrature for the form factor. Only Q^2-dependent systematic errors are quoted; the Q^2-independent error is 6.6% for the cross section and 4.3% for the form factor.

Q^2 interval (GeV^2)	$\overline{Q^2}$ (GeV^2)	$d\sigma/dQ^2(\overline{Q^2})$ (fb/GeV^2)	$\lvert F(\overline{Q^2})/F(0)\rvert$
2–3	2.49	$18.7 \pm 4.2 \pm 0.8$	0.740 ± 0.085
3–4	3.49	$10.6 \pm 2.1 \pm 0.8$	0.680 ± 0.073
4–5	4.49	$6.62 \pm 1.18 \pm 0.19$	0.629 ± 0.057
5–6	5.49	$4.00 \pm 0.80 \pm 0.10$	0.555 ± 0.056
6–8	6.96	$3.00 \pm 0.43 \pm 0.17$	0.563 ± 0.043
8–10	8.97	$1.58 \pm 0.30 \pm 0.08$	0.490 ± 0.049
10–12	10.97	$0.72 \pm 0.17 \pm 0.05$	0.385 ± 0.048
12–15	13.44	$0.55 \pm 0.13 \pm 0.03$	0.395 ± 0.047
15–20	17.35	$0.34 \pm 0.07 \pm 0.01$	0.385 ± 0.038
20–30	24.53	$0.084 \pm 0.026 \pm 0.004$	0.261 ± 0.041
30–50	38.68	$0.019 \pm 0.009 \pm 0.001$	0.204 ± 0.049

(BaBar 2010)

Motivation:

- P-wave Decay: $\chi_{c0,2} \longrightarrow \gamma\gamma$

(BESIII 2012)

$$\Gamma_{\gamma\gamma}(\chi_{c0}) = (2.33 \pm 0.20 \pm 0.13 \pm 0.17)\,\mathrm{keV}$$

$$\mathcal{R} \equiv \frac{\Gamma_{\gamma\gamma}(\chi_{c2})}{\Gamma_{\gamma\gamma}(\chi_{c0})} = 0.271 \pm 0.029 \pm 0.013 \pm 0.027$$

$$f_{0/2} \equiv \frac{\Gamma_{\gamma\gamma}^{\lambda=0}(\chi_{c2})}{\Gamma_{\gamma\gamma}^{\lambda=2}(\chi_{c2})} = 0.00 \pm 0.02 \pm 0.02$$

- Numerous theoretical works ...

Basic Procedures:

1. Feynman Diagrams & Amplitudes
   - FeynArts - Mathematica Package
     http://www.feynarts.de
   - QGRAF - Fortran Program
     http://cfif.ist.utl.pt/~paulo/qgraf.html
2. Color- & Spin-Traces
   - FeynCalc - Mathematica Package
     https://github.com/FeynCalc
   - FeynCalc/FormLink - Mathematica Package
     https://github.com/FormLink

Basic Procedures:

3. Partial Fragmentation & IBP Reduction
   - APart - Mathematica Package
     https://github.com/F-Feng
   - FIRE - Mathematica Program & C++ version
     http://science.sander.su
4. Master Integrals - Numerical
   - FIESTA - Mathematica Package
     http://science.sander.su
   - CubPack - Fortran Code
     http://nines.cs.kuleuven.be/software/cubpack/

Feynman Diagrams:

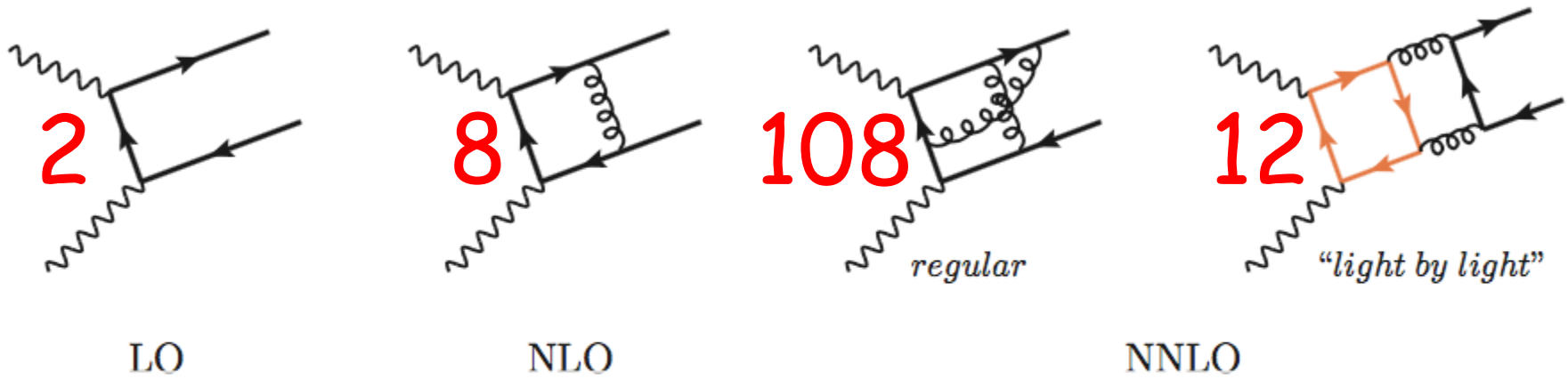


FIG. 1: Sample Feynman diagrams for $\gamma^*\gamma \to c\bar{c}(^1S_0^{(1)})$.

$\gamma\gamma^* \to \eta_c$ & $\chi_{c0,2} \to \gamma\gamma$

$\gamma\gamma^* \to \eta_c$:

- Form Factor

$$\langle \eta_c(p)|J^\mu|\gamma(k,\varepsilon)\rangle = ie^2\epsilon^{\mu\nu\rho\sigma}\varepsilon_\nu q_\rho k_\sigma F(Q^2)$$

- NRQCD Factorization

$$F(Q^2) = C(Q, m, \mu_R, \mu_\Lambda)\,\frac{\langle \eta_c|\psi^\dagger\chi(\mu_\Lambda)|0\rangle}{\sqrt{m}} + \mathcal{O}(v^2)$$

- Form Factor Ratio

$$\left|\frac{F(Q^2)}{F(0)}\right| = \left|\frac{C(Q, m, \mu_R, \mu_\Lambda)}{C(0, m, \mu'_R, \mu_\Lambda)}\right| + \mathcal{O}(v^2)$$

$\gamma\gamma^* \to \eta_c$:

- Perturbative Expansion

$$C(Q,m,\mu_R,\mu_\Lambda) = C^{(0)}(Q,m)\Bigg\{1 + C_F\frac{\alpha_s(\mu_R)}{\pi}f^{(1)}(\tau)$$

$$+\frac{\alpha_s^2}{\pi^2}\Bigg[\frac{\beta_0}{4}\ln\frac{\mu_R^2}{Q^2+m^2}C_F f^{(1)}(\tau) - \pi^2 C_F\left(C_F + \frac{C_A}{2}\right)$$

$$\times\ln\frac{\mu_\Lambda}{m} + f^{(2)}(\tau)\Bigg] + \mathcal{O}(\alpha_s^3)\Bigg\},$$

where $\tau = \frac{Q^2}{m^2}$ and $f^{(2)}(\tau) = f^{(2)}_{\rm reg}(\tau) + f^{(2)}_{\rm lbl}(\tau)$

$\gamma\gamma^* \to \eta_c$:

- Confront the *BaBar* Data

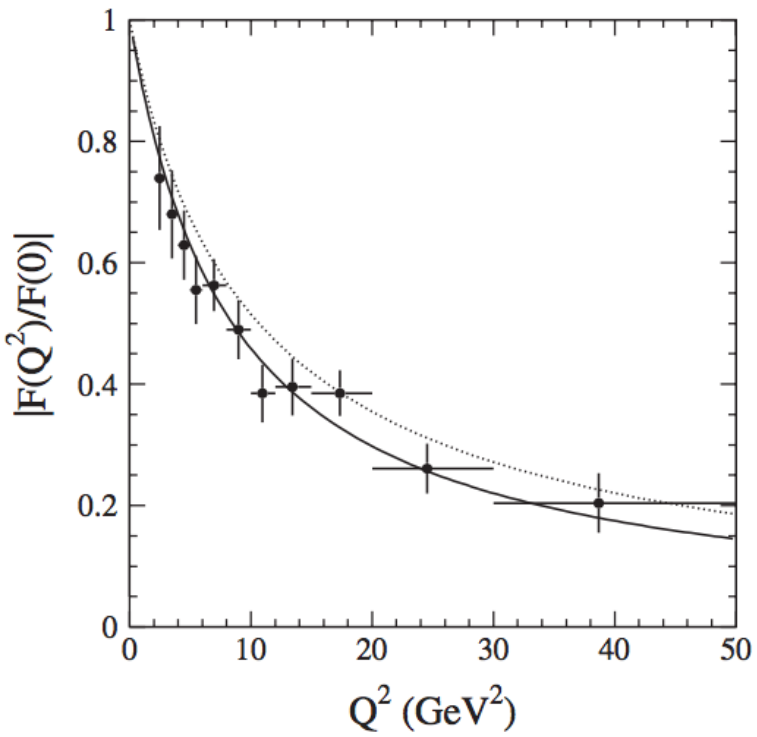


FIG. 26. The $\gamma\gamma^* \to \eta_c$ transition form factor normalized to $F(0)$ (points with error bars). The solid curve shows the fit to Eq. (22). The dotted curve shows the leading order pQCD prediction from Ref. [3].

$$\left|\frac{F(Q^2)}{F(0)}\right| = \frac{1}{1+Q^2/\Lambda}$$

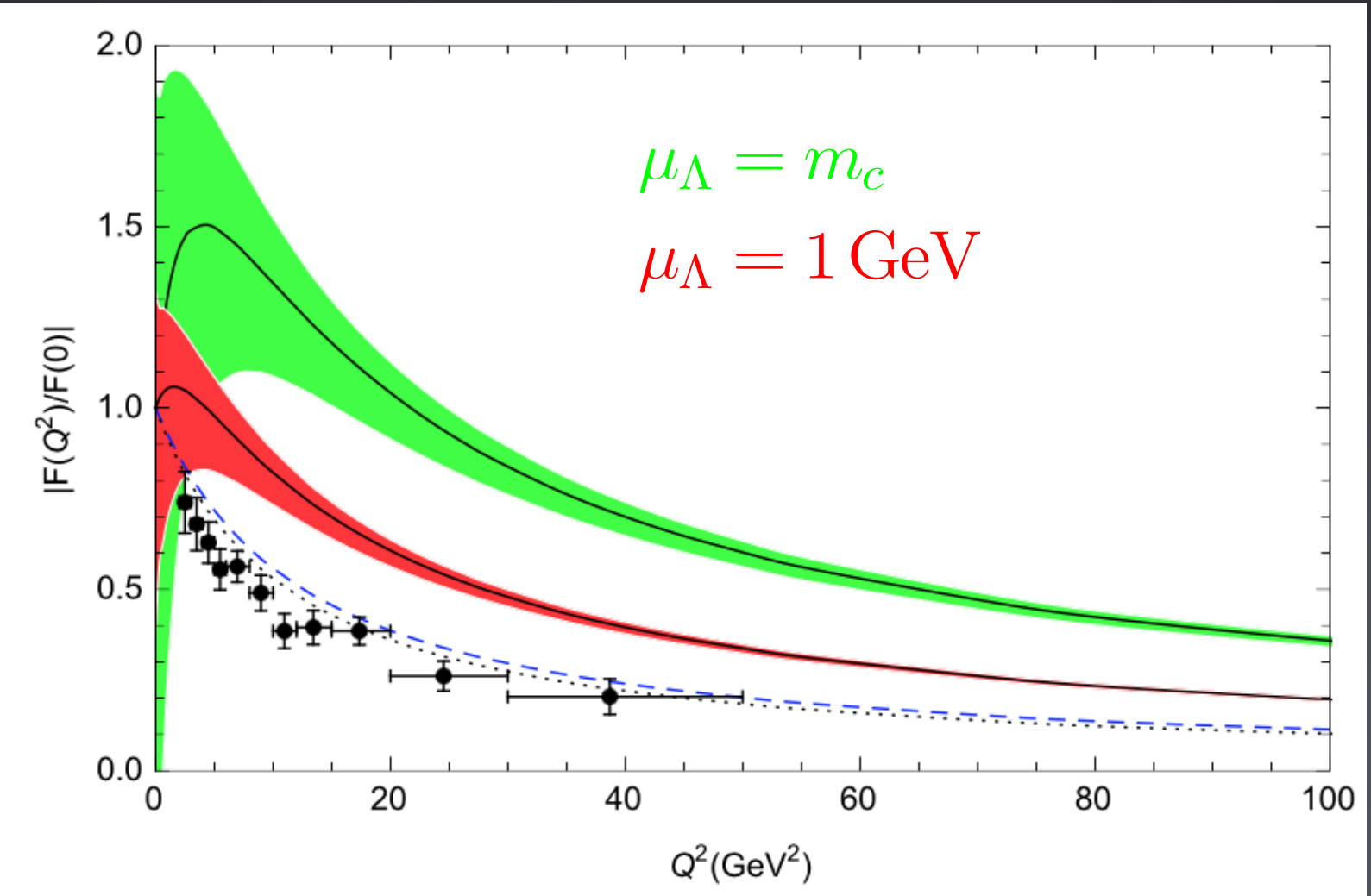
$$\Lambda = 8.5 \pm 0.6 \pm 0.7 \text{ GeV}^2$$

$$\left|\frac{F(Q^2)}{F(0)}\right| = \frac{4m_c^2}{Q^2+4m_c^2}$$

$$4m_c^2 \approx 4 \times 1.5^2 = 9\text{GeV}^2$$

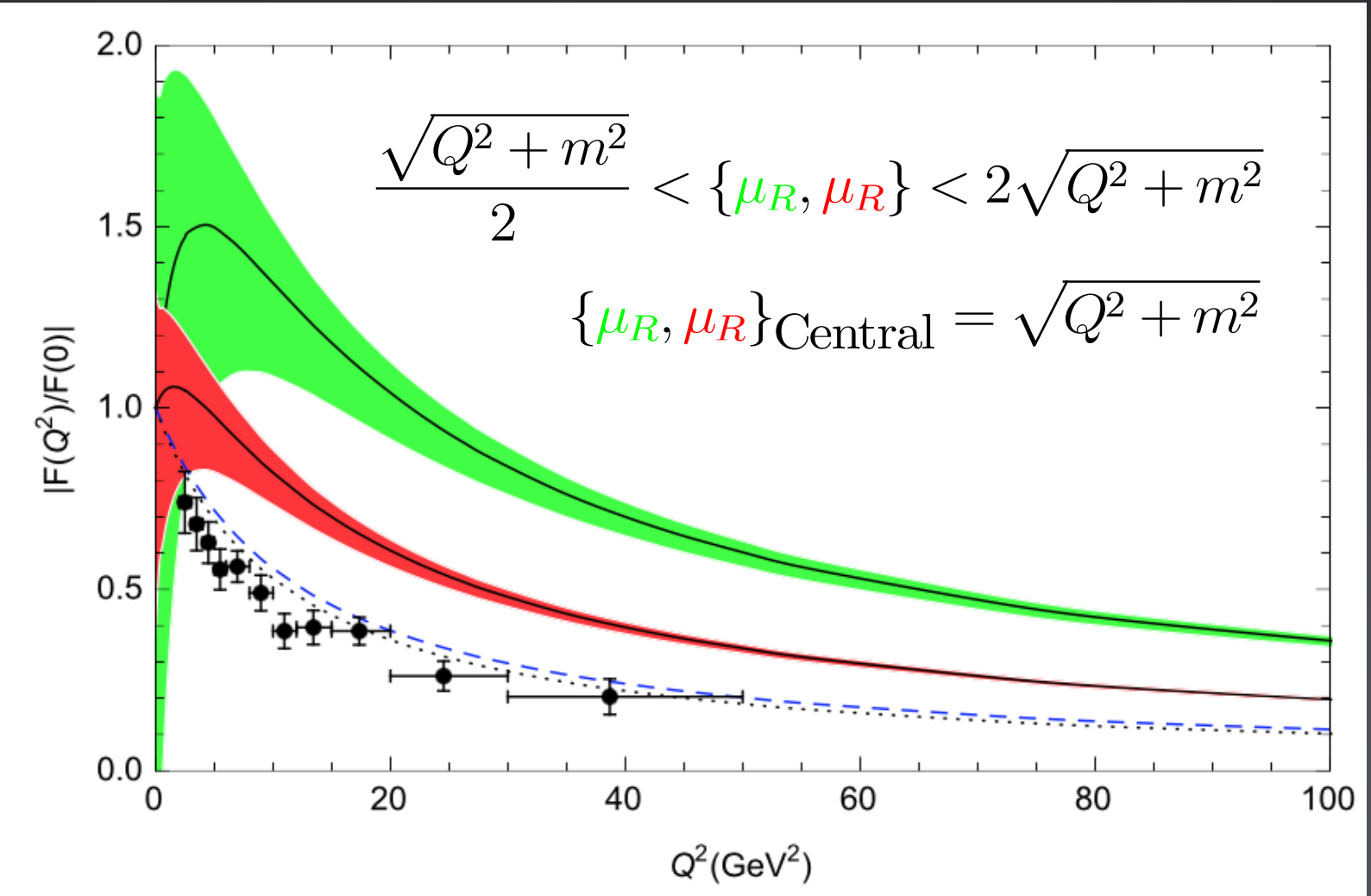
$\gamma\gamma^* \to \eta_c$:

- Confront the *BaBar* Data



$\gamma\gamma^* \to \eta_c$:

- Confront the *BaBar* Data



$\chi_{c0,2} \to \gamma\gamma$:

- NRQCD factorization

$$\mathcal{A}^{\chi_{0,2}}_{\lambda_1,\lambda_2} = \mathcal{C}^{\chi_{0,2}}_{\lambda}(m,\mu_R,\mu_\Lambda)\,\frac{\langle 0|\chi^\dagger \mathcal{K}_3 {}_{P_{0,2}}\psi(\mu_\Lambda)|\chi_{c0,2}\rangle}{m^{3/2}} + \mathcal{O}(v^2)$$

- Perturbative expansion

$$\mathcal{C}_\lambda = \mathcal{C}^{(0)}_\lambda \left\{ 1 + C_F \frac{\alpha_s}{\pi} f^{(1)}_\lambda + \frac{\alpha_s^2}{\pi^2} f^{(2)}_\lambda + \mathcal{O}(\alpha_s^3) \right\}$$

- Partial decay widths

$$\Gamma_{\gamma\gamma}(\chi_0) = \frac{1}{16\pi}\left(2|\mathcal{A}^{\chi_0}_{1,1}|^2\right)$$

$$\Gamma_{\gamma\gamma}(\chi_2) = \frac{1}{5}\frac{1}{16\pi}\left(2|\mathcal{A}^{\chi_2}_{1,1}|^2 + 2|\mathcal{A}^{\chi_2}_{1,-1}|^2\right)$$

$\chi_{c0,2} \to \gamma\gamma$:

- Perturbative Expansion

$$
\begin{aligned}
\mathcal{C}_0^{\chi_0} &= \frac{4\sqrt{3}\pi e_Q^2 \alpha}{\sqrt{m}} \Bigg\{ 1 + C_F \frac{\alpha_s}{\pi} \left(\frac{\pi^2}{8} - \frac{7}{6} \right) \\
&\quad + \frac{\alpha_s^2}{\pi^2} \left[C_F \frac{\beta_0}{4} \left(\frac{\pi^2}{8} - \frac{7}{6} \right) \ln \frac{\mu_R^2}{m^2} + \Delta_0^{\chi_0} \right] \Bigg\} \\
\mathcal{C}_0^{\chi_2} &= \frac{4\sqrt{6}\pi \alpha e_Q^2}{3\sqrt{m}} \Bigg\{ 0 + C_F \frac{\alpha_s}{\pi} \left(\frac{3\pi^2}{8} - 6\ln 2 + 1 \right) + \frac{\alpha_s^2}{\pi^2} \\
&\quad \times \left[C_F \frac{\beta_0}{4} \left(\frac{3\pi^2}{8} - 6\ln 2 + 1 \right) \ln \frac{\mu_R^2}{m^2} + \Delta_0^{\chi_2} \right] \Bigg\}, \\
\mathcal{C}_2^{\chi_2} &= -\frac{8\pi \alpha e_Q^2}{\sqrt{m}} \left[1 - 2C_F \frac{\alpha_s}{\pi} + \frac{\alpha_s^2}{\pi^2} \left(-2C_F \frac{\beta_0}{4} \ln \frac{\mu_R^2}{m^2} + \Delta_2^{\chi_2} \right) \right]
\end{aligned}
$$

$\chi_{c0,2} \to \gamma\gamma$:

$$\mathcal{C}_0^{\chi_0} = \frac{4\sqrt{3}\pi e_Q^2 \alpha}{\sqrt{m}} \Bigg[1 + 0.088\, \frac{\alpha_s}{\pi} + \frac{\alpha_s^2}{\pi^2} \times \left(-23.79 + 0.79\, i + 0.18 \ln \frac{\mu_R^2}{m^2} - 18.26 \ln \frac{\mu_\Lambda}{m} \right) \Bigg]$$

$$\mathcal{C}_0^{\chi_2} = \frac{4\sqrt{6}\pi \alpha e_Q^2}{3\sqrt{m}} C_F \frac{\alpha_s}{\pi} \left(\frac{3\pi^2}{8} - 6 \ln 2 + 1 \right) \times \left[1 + \left(6.38 + 0.072\, i + 2.07 \ln \frac{\mu_R^2}{m^2} \right) \frac{\alpha_s}{\pi} \right]$$

$$\mathcal{C}_2^{\chi_2} = -\frac{8\pi \alpha e_Q^2}{\sqrt{m}} \Bigg[1 - 0.85\, \alpha_s + \frac{\alpha_s^2}{\pi^2} \times \left(-30.79 - 1.14\, i - 5.53 \ln \frac{\mu_R^2}{m^2} - 10.36 \ln \frac{\mu_\Lambda}{m} \right) \Bigg]$$

$\chi_{c0,2} \to \gamma\gamma$:

- Confront the *BESIII* Data

$$\Omega \equiv \left| \frac{\overline{R'_{\chi_{c2}}}(\mu_\Lambda)}{\overline{R'_{\chi_{c0}}}(\mu_\Lambda)} \right|^2$$

$$\begin{aligned}
\mathcal{R} &= \frac{4}{15}\Omega\left[1 - \left(\frac{\pi^2}{3} + \frac{20}{9}\right)\frac{\alpha_s}{\pi}\right. \\
&- \left.\left(5.855 + 22.967\,\ln\frac{\mu_R}{m} + 15.791\,\ln\frac{m}{\mu_\Lambda}\right)\left(\frac{\alpha_s}{\pi}\right)^2\right], \\
f_{0/2} &= \frac{\alpha_s^2}{216\pi^2}\left(8 + 3\pi^2 - 48\ln 2\right)^2,
\end{aligned}$$

$\chi_{c0,2} \to \gamma\gamma$:

- Confront the *BESIII* Data

$$\Omega \equiv \left| \frac{\overline{R'_{\chi_{c2}}}(\mu_\Lambda)}{\overline{R'_{\chi_{c0}}}(\mu_\Lambda)} \right|^2$$

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f_{0/2} &= \frac{\alpha_s^2}{216\pi^2}\left(8 + 3\pi^2 - 48\ln 2\right)^2,
\end{aligned}$$

$$\mathcal{R} = (0.098^{+0.020}_{-0.013})\,\Omega(m_c), \qquad f_{0/2} = 0.0009^{+0.0009}_{-0.0004}$$

with $\mu_\Lambda = m_c$. The central values obtained by setting $\mu_R = m$, uncertainties are included by sliding the μ_R in the range $1\,\text{GeV} < \mu_R < 2m$.

$\chi_{c0,2} \to \gamma\gamma$:

- Confront the *BESIII* Data

(BESIII 2012)

$$\mathcal{R} \equiv \frac{\Gamma_{\gamma\gamma}(\chi_{c2})}{\Gamma_{\gamma\gamma}(\chi_{c0})} = 0.271 \pm 0.029 \pm 0.013 \pm 0.027$$

$$f_{0/2} \equiv \frac{\Gamma_{\gamma\gamma}^{\lambda=0}(\chi_{c2})}{\Gamma_{\gamma\gamma}^{\lambda=2}(\chi_{c2})} = 0.00 \pm 0.02 \pm 0.02$$

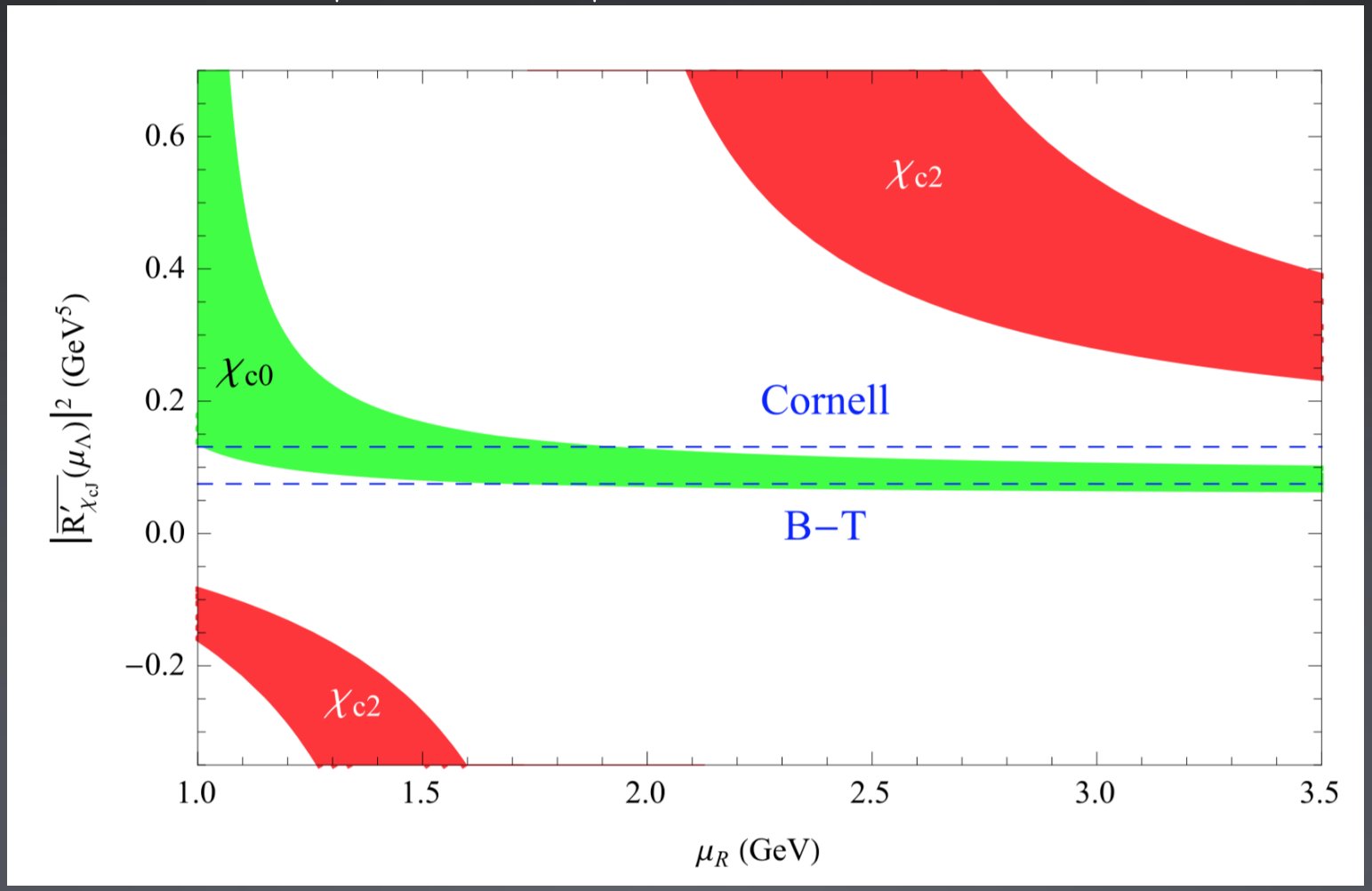
$$\Omega \equiv \left| \frac{\overline{R'_{\chi_{c2}}}(\mu_\Lambda)}{\overline{R'_{\chi_{c0}}}(\mu_\Lambda)} \right|^2$$

$$\mathcal{R} = (0.098^{+0.020}_{-0.013})\,\Omega(m_c), \qquad f_{0/2} = 0.0009^{+0.0009}_{-0.0004}$$

$\Omega = 1$, If heavy quark spin-symmetry(HQSS) holds
$\Omega < 1$, Considering spin-dependent forces within the Cornell potential model

$\chi_{c0,2} \to \gamma\gamma$:

- Determine $\left|\overline{R'_{\chi_{c0,2}}}(\mu_\Lambda)\right|^2$ from the *BESIII* Data



Summary & Outlook:

- The NNLO NRQCD predictions to $\gamma\gamma^* \to \eta_c$ form factor and $\chi_{c0,2} \to \gamma\gamma$ fail to explain the experimental data!

- Higher corrections needed, including N^3LO QCD corrections and relativistic corrections to those processes.

- Only exclusive processes considered, How about Inclusive Decay?

$$\eta_c \to \text{Light Hadrons}$$

Thanks for your attention!

Backup Slides

$\gamma\gamma^* \to \eta_c$:

- Result @LO

$$C^{(0)}(Q,m) = \frac{4e_c^2}{Q^2+4m^2}$$

- Result @NLO

$$f^{(1)}(\tau) = \frac{\pi^2(3-\tau)}{6(4+\tau)} - \frac{20+9\tau}{4(2+\tau)} - \frac{\tau(8+3\tau)}{4(2+\tau)^2}\ln\frac{4+\tau}{2}$$
$$+3\sqrt{\frac{\tau}{4+\tau}}\tanh^{-1}\sqrt{\frac{\tau}{4+\tau}} + \frac{2-\tau}{4+\tau}\left(\tanh^{-1}\sqrt{\frac{\tau}{4+\tau}}\right)^2$$
$$-\frac{\tau}{2(4+\tau)}\mathrm{Li}_2\left(-\frac{2+\tau}{2}\right)$$

$\gamma\gamma^* \to \eta_c$:

- **Numerical Result @$Q^2 = 0$**

$$f^{(2)}_{\rm reg}(0) = C_F^2 f_A + C_A C_F f_{\rm NA} + N_L C_F T_R f_L + N_H C_F T_R f_H$$

$$\begin{cases} f_A = -21.10789797(4) \\ f_{\rm NA} = -4.79298000(3) \\ f_L = -\frac{1}{144}\left(42\zeta(3) - 164 + 13\pi^2 + 96\ln 2\right) \\ f_H = 0.223672013(2) \end{cases}$$

- A. Czarnecki and K. Melnikov, "Charmonium decays: $J/\psi \to e^+e^-$ and $\eta_c \to \gamma\gamma$", Phys. Lett. B **519**, 212 (2001) [hep-ph/0109054].

$\gamma\gamma^* \to \eta_c$:

- Numerical Result @$Q^2 = 0$

$$s_A(\mu) = -21.0 - \pi^2\left(\frac{1}{4\epsilon} + \ln\frac{\mu}{m}\right),$$

$$s_{NA}(\mu) = -4.79 - \frac{\pi^2}{2}\left(\frac{1}{4\epsilon} + \ln\frac{\mu}{m}\right),$$

$$s_L = \frac{41}{36} - \frac{13}{144}\pi^2 - \frac{2}{3}\ln 2 - \frac{7}{24}\zeta_3 \simeq -0.565,$$

$$s_H = 0.22. \qquad (4)$$

- A. Czarnecki and K. Melnikov, "Charmonium decays: $J/\psi \to e^+e^-$ and $\eta_c \to \gamma\gamma$", Phys. Lett. B **519**, 212 (2001) [hep-ph/0109054].

$\gamma\gamma^* \to \eta_c$:

- **Numerical Result @$Q^2 = 0$**

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$\boxed{\text{Precision} \sim 10^{-9}}$

- A. Czarnecki and K. Melnikov, "Charmonium decays: $J/\psi \to e^+e^-$ and $\eta_c \to \gamma\gamma$", Phys. Lett. B **519**, 212 (2001) [hep-ph/0109054].

$\gamma\gamma^* \to \eta_c$:

- **Numerical Result @$Q^2 = 0$**

Missing Part

$$f_{\rm lbl}^{(2)}(0) = \left(0.64696557 + 2.07357556\, i\right) n_H C_F T_f$$
$$+ \left(0.73128459 + \frac{\pi}{9}(\pi^2 - 15)\, i\right) C_F T_f \sum_{f=\text{light quark}} \frac{e_f^2}{e_Q^2}$$

- A. Czarnecki and K. Melnikov, "Charmonium decays: $J/\psi \to e^+e^-$ and $\eta_c \to \gamma\gamma$", Phys. Lett. B **519**, 212 (2001) [hep-ph/0109054].

$\gamma\gamma^* \to \eta_c$:

- Numerical Result @NNLO

τ	1	5	10	25	50
$f^{(2)}_{\rm reg}$	-59.420(6)	-61.242(6)	-61.721(7)	-61.843(8)	-61.553(8)
$f^{(2)}_{\rm lbl}$	$0.49(4)$ $-0.65(3)i$	$-0.48(5)$ $-0.72(4)i$	$-1.09(5)$ $-0.71(4)i$	$-2.12(6)$ $-0.69(4)i$	$-3.10(6)$ $-0.68(4)i$
$f^{(2)}_{\rm reg}$	-59.636(6)	-61.278(6)	-61.716(7)	-61.864(8)	-61.668(8)
$f^{(2)}_{\rm lbl}$	$0.8(1)$ $-12.44(8)i$	$-5.6(2)$ $-13.5(2)i$	$-9.4(2)$ $-13.8(2)i$	$-15.3(2)$ $-14.0(2)i$	$-20.3(2)$ $-14.1(2)i$

$f^{(2)}_{\rm reg}(\tau)$ and $f^{(2)}_{\rm lbl}(\tau)$ at some typical values of τ.
The first two rows for η_c and the last two for η_b