Twist-3 T-odd fragmentation functions $G^{\perp}$ and $\tilde{G}^{\perp}$ in a spectator model

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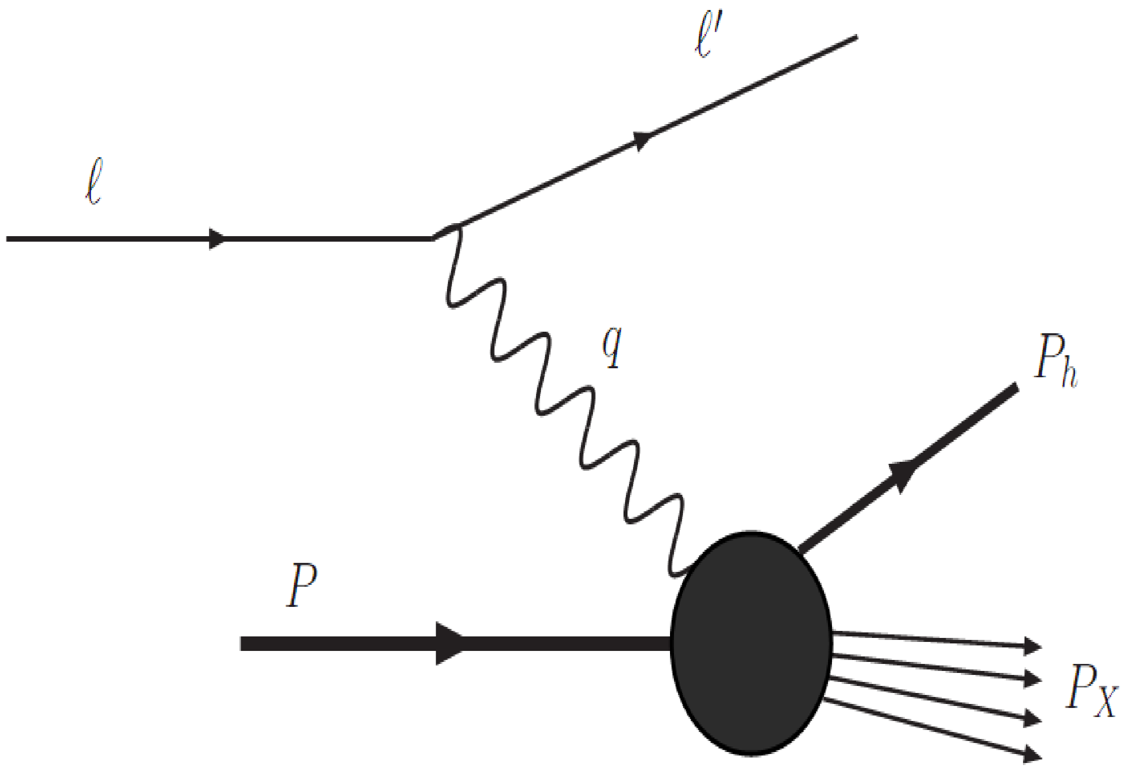
Outline

- Formalism
- Parameterization and a spectator model
- The calculation of $G^{\perp}$ and $\tilde{G}^{\perp}$
- The numerical calculation
- Summary

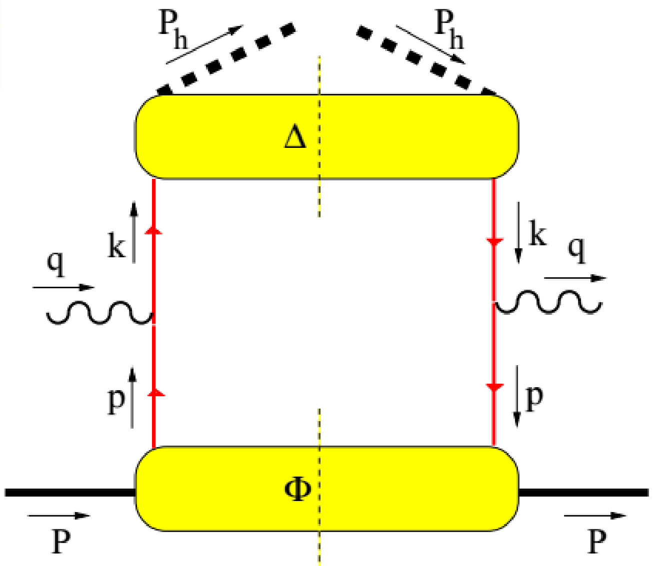
Formalism

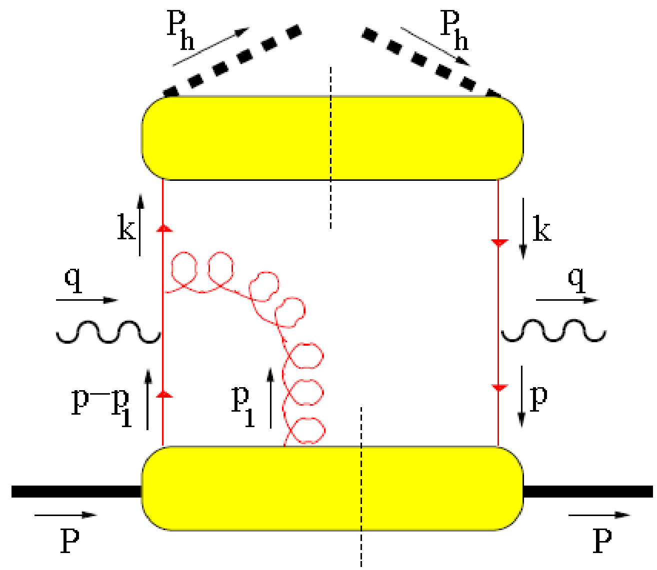
- Semi-inclusive DIS by lepton beam off nucleon target:

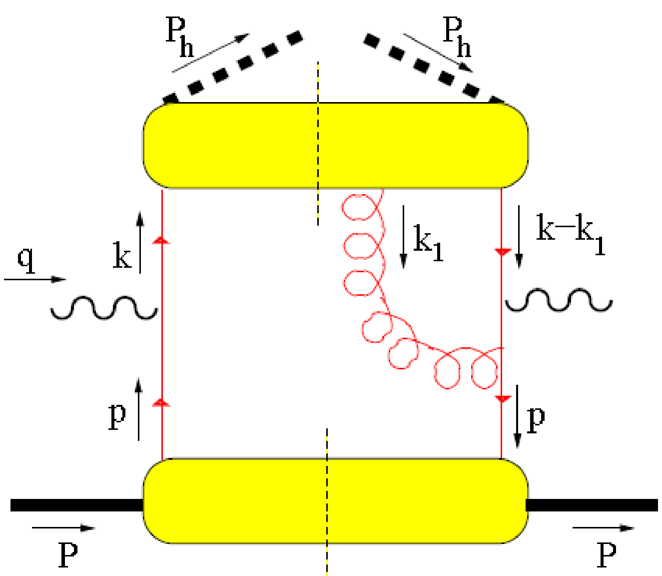
$$l(\ell) + N(P) \rightarrow l'(\ell') + h(P_h) + X(P_X)$$



Formalism





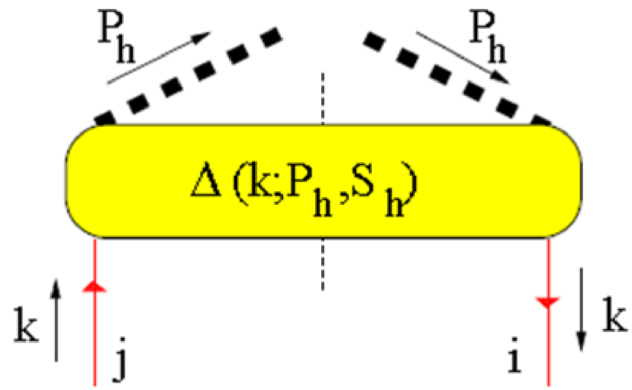


$$\frac{d\sigma}{dx\,dy\,d\psi\,dz\,d\phi_h\,dP_{h\perp}^2} = \frac{\alpha^2 y}{8z\,Q^4}\,2M\,W^{\mu\nu}\,L_{\mu\nu}$$

D. Boer *et al.*, Nucl. Phys. B 667, 201 (2003)

Parameterization

- The q-q fragmentation correlation function



$$\Delta_{ij}(z,k_T) = \frac{1}{2z}\sum_X \int \frac{d\xi^+ d^2\boldsymbol{\xi}_T}{(2\pi)^3}\, e^{ik\cdot\xi}\, \langle 0|\,\mathcal{U}^{n_+}_{(+\infty,\xi)}\,\psi_i(\xi)|h,X\rangle\langle h,X|\bar{\psi}_j(0)\,\mathcal{U}^{n_+}_{(0,+\infty)}|0\rangle\Big|_{\xi^-=}$$

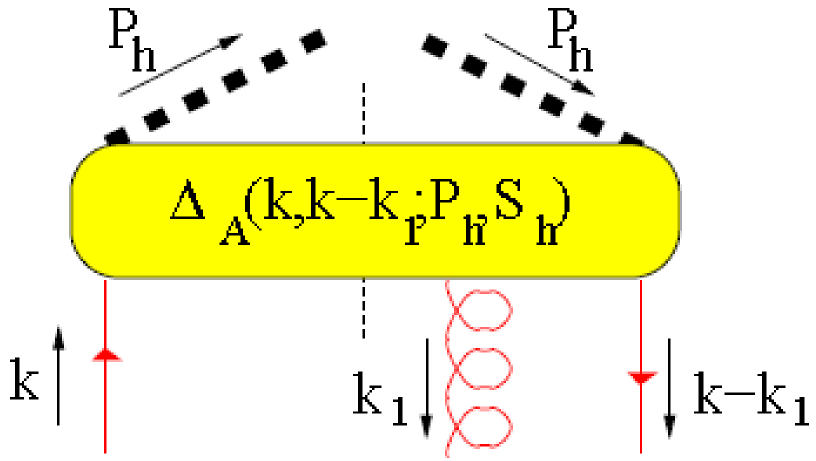
$$\Delta^{[-]}(z,k_T) \equiv \int dk^+ \Delta^{[-]}(P_h,k;n_+)$$

$$= z\left\{D_1\,\not{n}_- + \mathrm{i}H_1^\perp \frac{[\not{k}_T,\not{n}_-]}{2M_h}\right\}$$

$$+\frac{zM_h}{P_h^-}\left\{E + D^\perp\frac{\not{k}_T}{M_h} + \mathrm{i}H\frac{[\not{n}_-,\not{n}_+]}{2} + G^\perp\gamma_5\frac{\epsilon_T^{\rho\sigma}\gamma_\rho k_{T\sigma}}{M_h}\right\}.$$

Parameterization

- The q-g-q fragmentation correlation function



$$\tilde{\Delta}_A^{\alpha}(z,k_T) = \sum\!\!\!\!\!\!\!\int_X \frac{1}{2z}\int\frac{d\xi^+ d^2\xi_T}{(2\pi)^3}\int e^{ik\cdot\xi}\langle 0|\int_{\pm\infty^+}^{\xi^+} d\eta^+\,\mathcal{U}^{\xi_T}_{(\infty^+,\eta^+)}$$

$$\times\, gF_\perp^{-\alpha}(\eta)\,\mathcal{U}^{\xi_T}_{(\eta^+,\xi^+)}\psi(\xi)|P_h;X\rangle\langle P_h;X|\bar{\psi}(0)\,\mathcal{U}^{0_T}_{(0^+,\infty^+)}\mathcal{U}^{\infty^+}_{(0_T,\xi_T)}|0\rangle\Big|_{\eta^+=\xi^+=0}$$

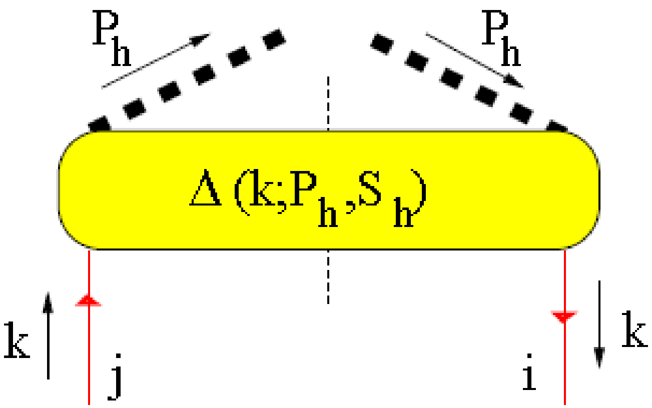
$$\tilde{\Delta}_A^{\alpha}(z,k_T) = \frac{M_h}{2z}\Big\{\big(\tilde{D}^\perp - i\,\tilde{G}^\perp\big)\frac{k_{T\rho}}{M_h}\big(g_T^{\alpha\rho} + i\epsilon_T^{\alpha\rho}\gamma_5\big)$$

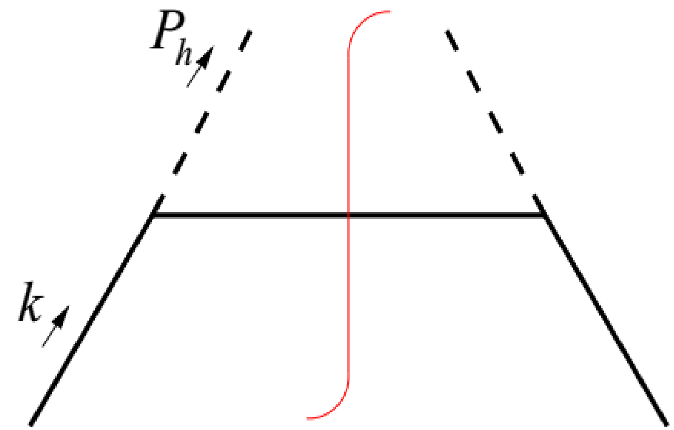
$$+\big(\tilde{H} + i\,\tilde{E}\big)\,i\gamma_T^{\alpha} + \ldots\big(g_T^{\alpha\rho} - i\epsilon_T^{\alpha\rho}\gamma_5\big)\Big\}\frac{\not{n}_-}{2}.$$

A. Bacchetta *et al.*, JHEP0702, 093 (2007)

The spectator model

- For the fragmentation correlator $q \to \pi X$





- The assumptions:
  - Hadron-quark vertex $\langle P_h; X|\bar{\psi}(0)|0\rangle \Longrightarrow \bar{U}(P_X)(i\gamma_5)\frac{i(\not{k}+m)}{k^2-m^2}$
  - The pion-quark vertex is $g_{hq}\,\gamma_5$
  - Assume coupling to be point-like
  - The mass of spectator is not equal to parent quark

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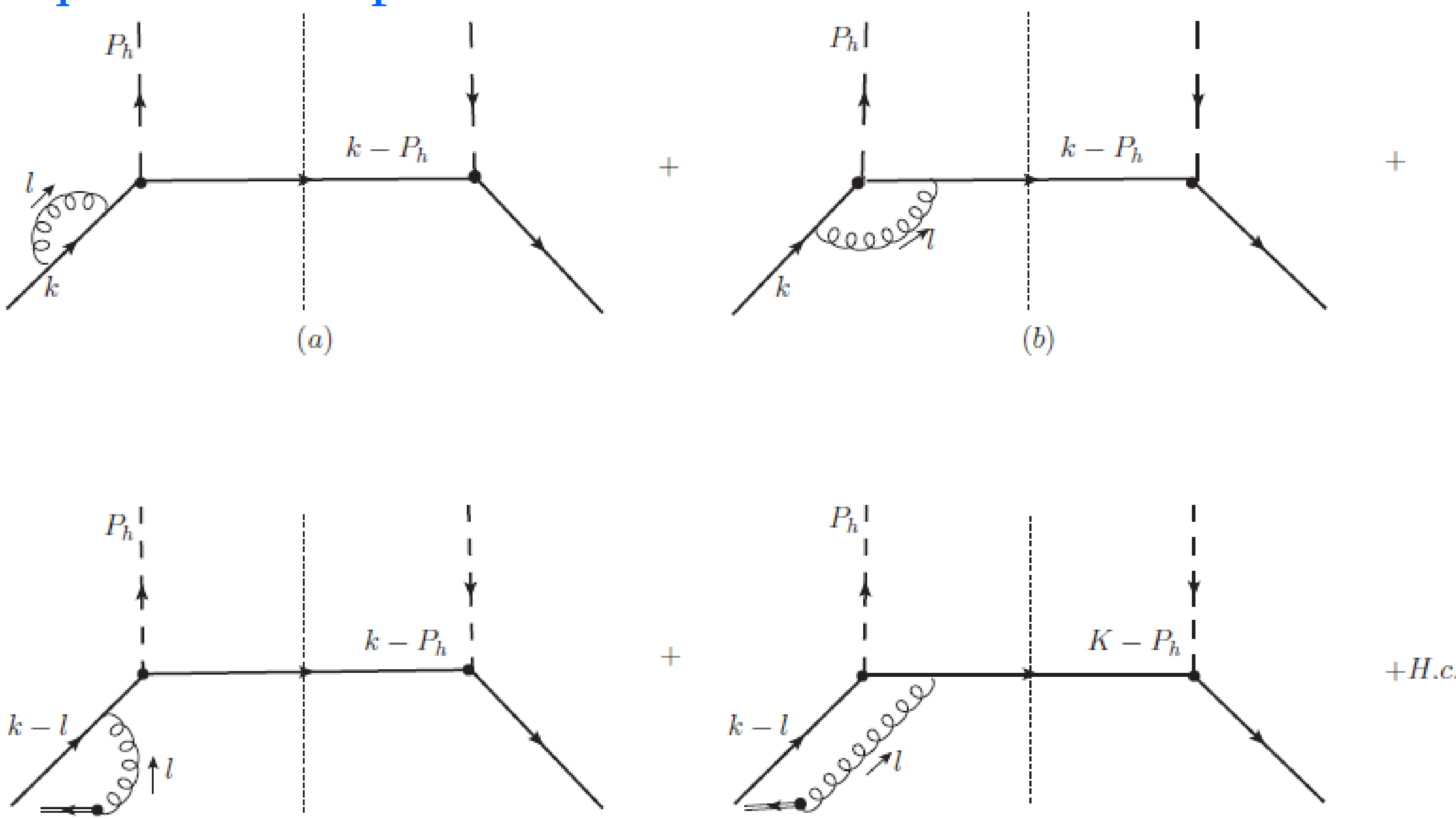
D. Amrath *et al*, Phys. Rev. D 71, 114018 (2005)
R. Jakob *et al*., Nucl. Phys. A 626, 937(1997)

The calculation of $G^{\perp}$

$$\frac{1}{P_h^-}\epsilon_T^{\alpha\beta}k_{T\beta}G^{\perp}(z,k_T^2)=\frac{1}{2}\mathrm{Tr}[\Delta(z,k_T)\gamma^{\alpha}\gamma_5]$$

- Single gluon-loop diagrams corrections to the fragmentation of a quark into a pion



The calculation of $G^{\perp}$

- The expression for each diagram are as follows:

$$\Delta_{(a)}(z,k_T) = i\frac{4C_F\alpha_s}{2(2\pi)^2(1-z)P_h^-}\frac{(\not{k}+m)}{(k^2-m^2)^3}g_{qh}\gamma_5\,(\not{k}-\not{P}_h+m_s)g_{qh}\gamma_5(\not{k}+m)$$

$$\times\int\frac{d^4l}{(2\pi)^4}\frac{\gamma^\mu\,(\not{k}-\not{l}+m)\,\gamma_\mu\,(\not{k}+m)}{((k-l)^2-m^2+i\varepsilon)(l^2+i\varepsilon)},$$

$$\Delta_{(b)}(z,k_T) = i\frac{4C_F\alpha_s}{2(2\pi)^2(1-z)P_h^-}\frac{(\not{k}+m)}{(k^2-m^2)^2}g_{qh}\gamma_5\,(\not{k}-\not{P}_h+m_s)$$

$$\times\int\frac{d^4l}{(2\pi)^4}\frac{\gamma^\mu(\not{k}-\not{P}_h-\not{l}+m_s)\,g_{qh}\gamma_5\,(\not{k}-\not{l}+m)\,\gamma_\mu\,((\not{k}+m)}{((k-P_h-l)^2-m_s^2+i\varepsilon)((k-l)^2-m^2+i\varepsilon)(l^2+i\varepsilon)},$$

$$\Delta_{(c)}(z,k_T) = i\frac{4C_F\alpha_s}{2(2\pi)^2(1-z)P_h^-}\frac{(\not{k}+m)}{k^2-m^2}g_{qh}\gamma_5\,(\not{k}-\not{P}_h+m_s)g_{qh}\gamma_5(\not{k}+m)$$

$$\times\int\frac{d^4l}{(2\pi)^4}\frac{\gamma^+\,(\not{k}-\not{l}+m)}{((k-l)^2-m^2+i\varepsilon)(-l^-\pm i\varepsilon)(l^2+i\varepsilon)},$$

$$\Delta_{(d)}(z,k_T) = i\frac{4C_F\alpha_s}{2(2\pi)^2(1-z)P_h^-}\frac{(\not{k}+m)}{k^2-m^2}g_{qh}\gamma_5\,(\not{k}-\not{P}_h+m_s)$$

$$\times\int\frac{d^4l}{(2\pi)^4}\frac{\gamma^+(\not{k}-\not{P}_h-\not{l}+m_s)\,g_{qh}\gamma_5\,(\not{k}-\not{l}+m)}{((k-P_h-l)^2-m_s^2+i\varepsilon)((k-l)^2-m^2+i\varepsilon)(-l^-\pm i\varepsilon)(l^2+i\varepsilon)}.$$

The calculation of $G^{\perp}$

- Using the Cutkosky cut rules

$$\frac{1}{l^2 + i\varepsilon} \to -2\pi i \delta(l^2), \qquad \frac{1}{(k-l)^2 + i\varepsilon} \to -2\pi i \delta((k-l)^2)$$

- Integrate loop momentum l

$$G^{\perp}(z, k_T^2) = \frac{2C_F \alpha_s g_{qh}^2}{(2\pi)^4 (1-z)} \frac{1}{(k^2 - m^2)} \left(G^{\perp}{}_{(c)}(z, k_T^2) + G^{\perp}{}_{(d)}(z, k_T^2) \right)$$

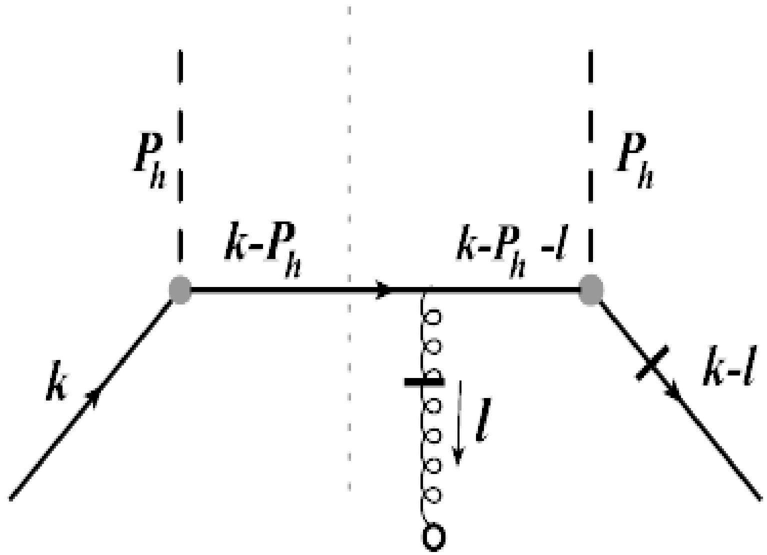
$$G^{\perp}{}_{(c)}(z, k_T^2) = 2zI_3 k^- + 2zI_1 \,,$$

$$G^{\perp}{}_{(d)}(z, k_T^2) = 2zI_1 + 2(k^2 + m^2)\mathcal{C} + 2(k^2 + m(m - 2m_s))\mathcal{D} + 2(1-z)\mathcal{E} P_h^-$$

- The result of $G^{\perp}$

$$G^{\perp}(z, k_T^2) = \frac{2C_F \alpha_s g_{qh}^2}{(2\pi)^4 (1-z)} \frac{1}{(k^2 - m^2)} \Big\{ 2zI_1 + 2\left(k^2 + m^2\right)\mathcal{C} + 2\left(k^2 + m(m - 2m_s)\right)\mathcal{D}$$
$$+ \frac{(1-z)}{zk_T^2} \left(\lambda(m_h, m_s) I_2 + \left((1-2z)k^2 + m_h^2 - m_s^2\right) I_{34} \right) + 2zI_{34}k^- \Big\} \,.$$

The calculation of $\tilde{G}^{\perp}$



$$\frac{z}{2}\mathrm{Tr}[\tilde{\Delta}_{A\rho}(z,k_T)(g_T^{\alpha\rho} - i\epsilon_T^{\alpha\rho}\gamma_5)\gamma^-] = k_T^\alpha(\tilde{D}^\perp(z,k_T^2) - i\tilde{G}(z,k_T^2))$$

$$\tilde{\Delta}_A^\alpha(z,k_T) = i\frac{C_F\alpha_s}{2(2\pi)^2(1-z)P_h^-}\frac{1}{k^2-m^2}$$

$$\int\frac{d^4l}{(2\pi)^4}\frac{(l^- g_T^{\alpha\mu} - l_T^\alpha g^{-\mu})(\not{k} - \not{l} + m)\, g_{qh}\gamma_5\,(\not{k} - \not{P}_h - \not{l} + m_s)\gamma_\mu(\not{k} - \not{P}_h + m_s)\, g_{qh}\gamma_5\,(\not{k} + m)}{(-l^- \pm i\varepsilon)((k-l)^2 - m^2 - i\varepsilon)((k-P_h-l)^2 - m_s^2 - i\varepsilon)(l^2 - i\varepsilon)}$$

The calculation of $\tilde{G}^{\perp}$

- We also apply cut rules to calculate $\tilde{G}^{\perp}$, the result is led to

$$\tilde{G}^{\perp}(z,k_T^2) = -\frac{C_F\alpha_s g_{qh}^2}{(2\pi)^4(1-z)}\frac{1}{(k^2-m^2)}\Bigg\{((m-m_s)^2-m_h^2)\Big[z\mathcal{A}+z\mathcal{B}-2I_2 + \left(2-\frac{2}{z}\right)\mathcal{C}\Big] + 4z(m^2\mathcal{A}+m(m-m_s)\mathcal{B}) + (k^2-m^2)[(z-1)\mathcal{C}-zI_2]-zI_1\Bigg\},$$

The calculation of $\tilde{G}^{\perp}$

- These functions originate from the following integrations:

$$\int d^4 l \frac{l^\mu \,\delta(l^2)\,\delta((k-l)^2-m^2)}{(k-P_h-l)^2-m_s^2} = \mathcal{A}\, k^\mu + \mathcal{B}\, P_h^\mu$$

$$\int d^4 l \frac{l^\mu \,\delta(l^2)\,\delta((k-l)^2-m^2)}{((k-P_h-l)^2-m_s^2)(-l\cdot n_+ + i\epsilon)} = \mathcal{C}\, k^\mu + \mathcal{D}\, P_h^\mu + \mathcal{E} n_+^\mu$$

$$\mathcal{A} = \frac{I_1}{\lambda(m_h, m_s)} \left(2k^2\left(k^2 - m_s^2 - m_h^2\right)\frac{I_2}{\pi} + \left(k^2 + m_h^2 - m_s^2\right)\right)$$

$$\mathcal{B} = -\frac{2k^2}{\lambda(m_h, m_s)} I_1 \left(1 + \frac{k^2 + m_s^2 - m_h^2}{\pi} I_2\right),$$

$$\mathcal{C} = \frac{I_{34} k^-}{2k_T^2} + \frac{1}{2zk_T^2}\left(-zk^2 + (2-z)\, m_h^2 + zm_s^2\right) I_2,$$

$$\mathcal{D} = \frac{-I_{34} k^-}{2zk_T^2} - \frac{1}{2zk_T^2}\left((1-2z)\, k^2 + m_h^2 - m_s^2\right) I_2,$$

$$\mathcal{E} = \frac{\lambda(m_h, m_s)}{4zP_h^- k_T^2} I_2 - \frac{1}{4z^2 k_T^2}\left((1-2z)\, k^2 + m_h^2 - m_s^2\right) I_{34} + \frac{k^2 - m^2}{2} I_4$$

The numerical result

- The fragmentation process : $u \to \pi^+$

$$G^{\perp\,(1/2)}(z) = \int d^2\boldsymbol{K}_T \frac{|\boldsymbol{k}_T|}{2m_h} G^{\perp}(z, k_T^2) = z^2 \int d^2\boldsymbol{k}_T \frac{|\boldsymbol{k}_T|}{2m_h} G^{\perp}(z, k_T^2)$$

$$\tilde{G}^{\perp\,(1/2)}(z) = \int d^2\boldsymbol{K}_T \frac{|\boldsymbol{k}_T|}{2m_h} \tilde{G}^{\perp}(z, k_T^2) = z^2 \int d^2\boldsymbol{k}_T \frac{|\boldsymbol{k}_T|}{2m_h} \tilde{G}^{\perp}(z, k_T^2)$$

$$H_1^{\perp(1/2)}(z) = \pi z^2 \int_0^{\infty} dk_T^2 \frac{|\boldsymbol{k}_T|}{2M_h} H_1^{\perp}(z, k_T^2)$$

Half-k_T moment

- The relation

$$\frac{G^{\perp}}{z} = \frac{\tilde{G}^{\perp}}{z} + \frac{m}{M_h} H_1^{\perp}$$

L. P. Gamberg *et al.*, Phys. Rev. D 68, 051501 (2003)
A. Bacchetta *et al.*, Phys. Lett. B 659, 234 (2008)
Z. Lu and I. Schmidt, Phys. Lett. B 747, 357 (2015)

The numerical result

- We adopt a Gaussian form factor for the coupling

$$g_{qh} \to g_{qh} \frac{e^{-\frac{k^2}{\Lambda^2}}}{z}. \qquad \Lambda^2 = \lambda^2 z^\alpha (1-z)^\beta$$

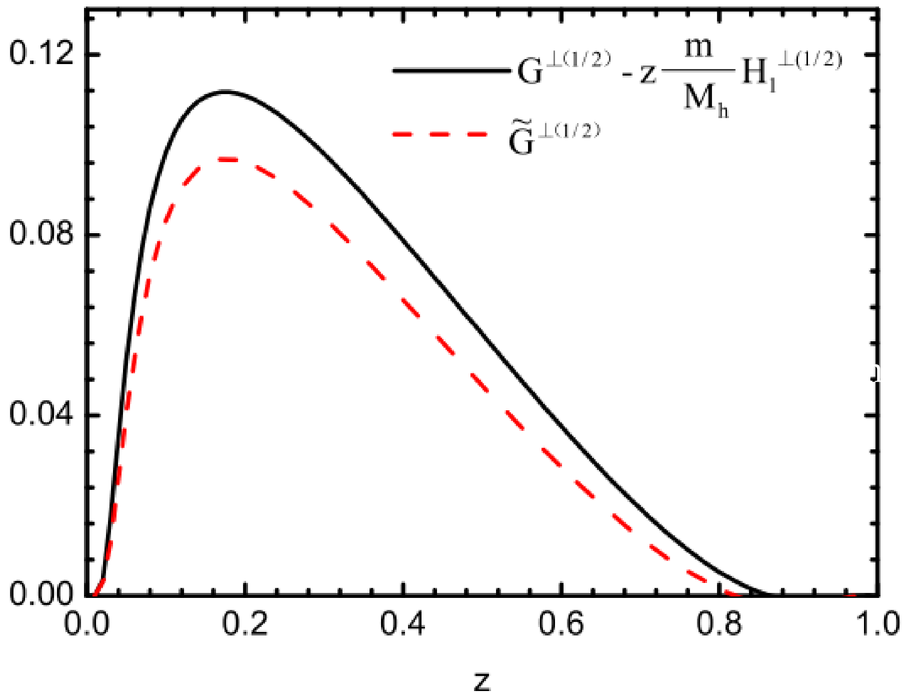
- The value of the parameters in this model

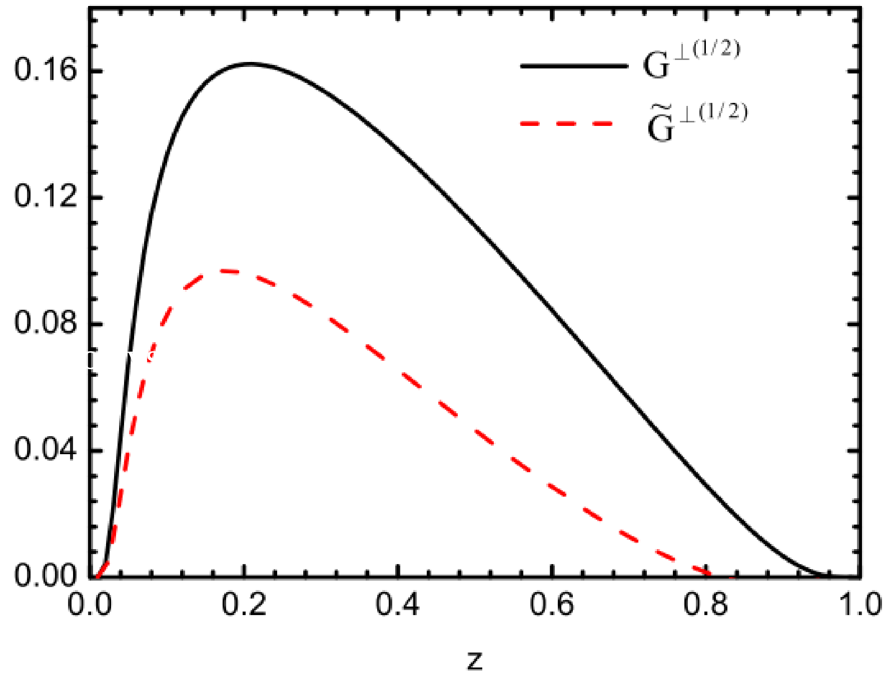
$$\alpha_s = 0.2, \qquad m_h = m_\pi = 0.135\,\mathrm{GeV}$$

$$\lambda = 2.18\ \mathrm{GeV},\ \alpha = 0.5\,(\mathrm{fixed}),\ \beta = 0\,(\mathrm{fixed}),$$
$$g_{qh} = g_{q\pi} = 5.09,\ m = 0.3\ \mathrm{GeV}\,(\mathrm{fixed}),\ m_s = 0.53\ \mathrm{GeV}\,;$$

The numerical result

- The collinear function vs z





- The dashed and solid lines represent $G^{\perp 1/2}(z)$ and $\tilde{G}^{\perp 1/2}(z)$ for the case $u \to \pi^+$. We find that their magnitudes are both sizable, although the size of $\tilde{G}^{\perp}$ is smaller than that of $G^{\perp}$.

Summary

- The twist-3 T-odd fragmentation function $G^{\perp}$ and $\tilde{G}^{\perp}$ were calculation in the spectator model.

- The result is free of light-cone divergence, and find that the relation holds approximately in the model.

- Our result implies that $\tilde{G}^{\perp}$ may provide considerable contributions to the longitudinal beam or target SSAs at the twist-3 level.

Thanks for your attention!

$$F_{LU}^{\sin\phi_h} = \frac{2M}{Q}\,\mathcal{C}\Big[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M_h}\Big(xe\,H_1^{\perp} + \frac{M_h}{M}\,f_1\frac{\tilde{G}^{\perp}}{z}\Big) + \frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M}\Big(xg^{\perp}D_1 + \frac{M_h}{M}\,h_1^{\perp}\frac{\tilde{E}}{z}\Big)\Big],$$

$$F_{UL}^{\sin\phi_h} = \frac{2M}{Q}\,\mathcal{C}\Big[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M_h}\Big(xh_L H_1^{\perp} + \frac{M_h}{M}\,g_{1L}\frac{\tilde{G}^{\perp}}{z}\Big) + \frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M}\Big(xf_L^{\perp}D_1 - \frac{M_h}{M}\,h_{1L}^{\perp}\frac{\tilde{H}}{z}\Big)\Big]$$

$$\frac{d\sigma}{dx\,dy\,d\psi\,dz\,d\phi_h\,dP_{h\perp}^2} =$$

$$\frac{\alpha^2}{xyQ^2}\,\frac{y^2}{2(1-\varepsilon)}\Big(1+\frac{\gamma^2}{2x}\Big)\Big\{F_{UU,T} + \varepsilon F_{UU,L} + \sqrt{2\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h}$$

$$+\,\varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h} + \lambda_e\sqrt{2\varepsilon(1-\varepsilon)}\,\sin\phi_h\,F_{LU}^{\sin\phi_h}$$

$$+\,S_{\parallel}\Big[\sqrt{2\varepsilon(1+\varepsilon)}\,\sin\phi_h\,F_{UL}^{\sin\phi_h} + \varepsilon\sin(2\phi_h)\,F_{UL}^{\sin 2\phi_h}\Big]$$

A. Bacchetta *et al.*, JHEP0702, 093 (2007)