

Statistical Fragmentation in pp & ep & ee Collisions

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Motivation

- Goal

*Hadronisation inside **fat jets***

- Proposed model

Statistical Model

- Suggestion

Parametrise fragmentation functions as

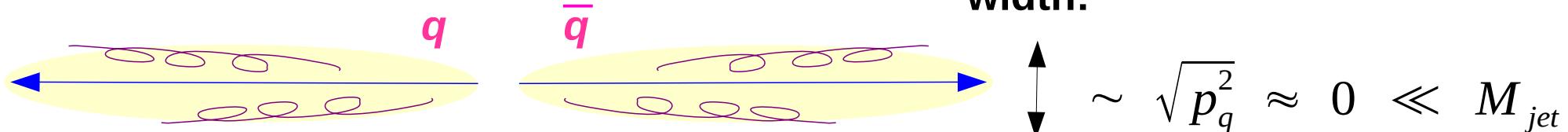
$$D \left[x = \frac{2 P_{\mu}^{jet} p_h^{\mu}}{M_{jet}^2}, Q^2 = M_{jet}^2 \right]$$

Energy fraction the hadron takes away in the *frame co-moving with the jet*

Fragmentation scale: *jet mass*

Outline

- 3D Statistical Jet fragmentation model
hadron distributions in jets in e^+e^- , ep , pp collisions
- Applications
 - *Transverse momentum spectra in pp collisions*
from a pQCD parton model calculation
 - *Spectra & anisotropy of hadrons in heavy-ion collisions*

Ideal world: **e^+e^- annihilations in the factorized picture****2 identical jets:**

$$p_{\mu}^{q,\bar{q}} = (\sqrt{s}/2, 0, 0, \pm\sqrt{s}/2)$$

Problem: $P^2 \sim 0$ quark produces a **heavy jet** of mass $M \sim [0.1 - 0.5] \sqrt{s}$

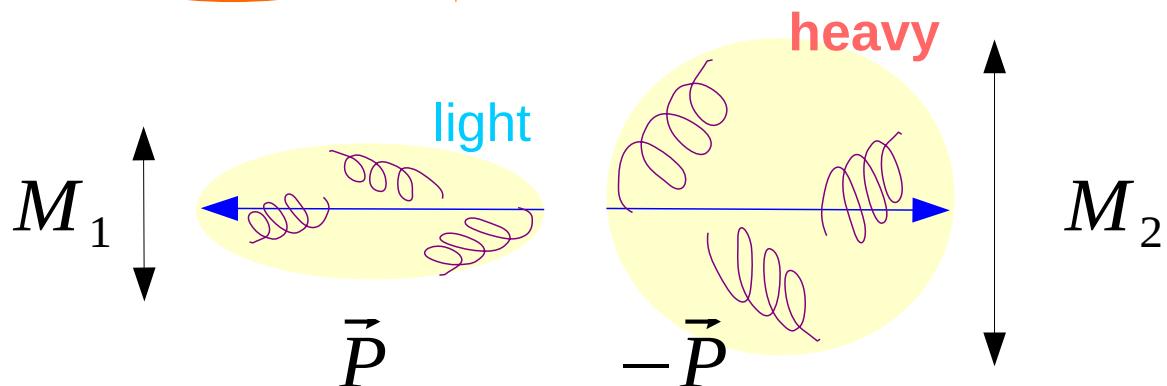
- **energy fraction** of the hadron takes away from the energy of the jet:
- **fragmentation scale:**

$$x = \frac{p_h^0}{\sqrt{s}/2}$$

$$Q \sim \sqrt{s}$$

Real world:

the 2 jets are *not identical*



Energy-momentum conservation:

$$P_1^u = (P^0, 0, 0, |\mathbf{P}|)$$

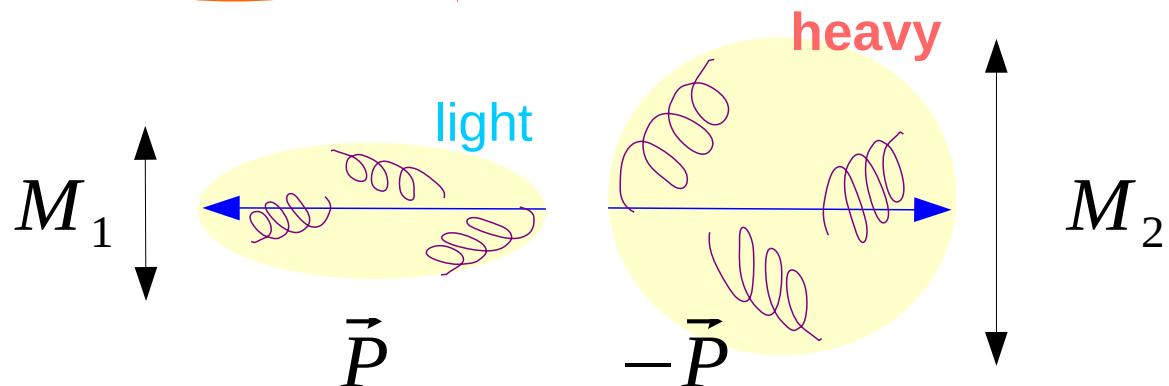
$$P_2^u = (\sqrt{s} - P^0, 0, 0, -|\mathbf{P}|)$$

Problems:

- **the energy of a jet** $P^0 \neq (\sqrt{s}/2)$, so $x = \frac{p_h^0}{\sqrt{s}/2}$ is **no longer the energy fraction**, the hadron takes away from the energy of the jet.
- **fragmentation scale** is **no longer** $\sqrt{s}/2$

Real world:

the 2 jets are not identical



Energy-momentum conservation:

$$P_1^\mu = (P^0, 0, 0, |\mathbf{P}|)$$

$$P_2^\mu = (\sqrt{s} - P^0, 0, 0, -|\mathbf{P}|)$$

We propose to use:

- the **real energy fraction** the hadron takes away from the energy of the jet in the **frame co-moving** with jet:
- the **jet mass** as **fragmentation scale**:

$$\chi = \frac{2 p_h^\mu P_\mu^{\text{jet}}}{M_{\text{jet}}^2}$$

$$Q \sim M_{\text{jet}}$$

These new variables, x and M_{jet} emerge naturally in a

Statistical Fragmentation Model

Statistical jet-fragmentation

The cross-section of the creation of hadrons h_1, \dots, h_N in a jet of N hadrons

$$d\sigma^{h_1, \dots, h_N} = |M|^2 \delta^{(4)} \left(\sum_i p_{h_i}^\mu - P_{tot}^\mu \right) d\Omega_{h_1, \dots, h_N}$$

If $|M| \approx \text{constans}$, we arrive at a *microcanonical ensemble*:

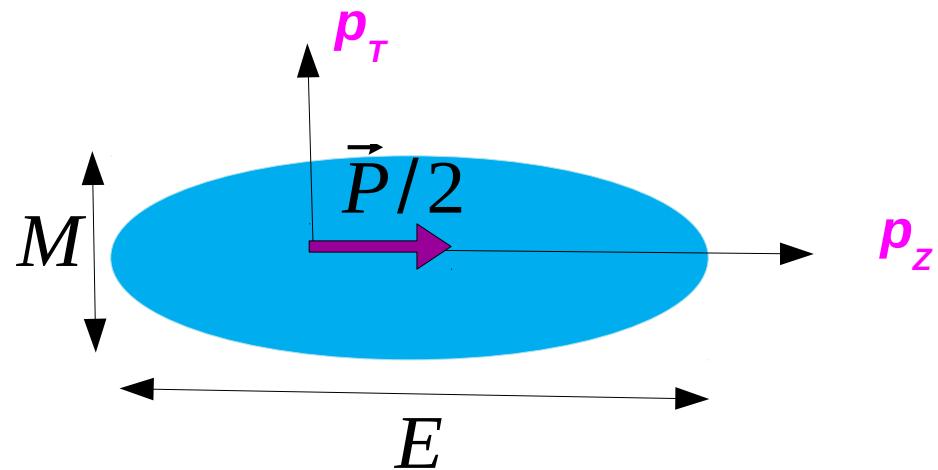
$$d\sigma^{h_1, \dots, h_n} \sim \delta \left(\sum_i p_{h_i}^\mu - P_{tot}^\mu \right) d\Omega_{h_1, \dots, h_n} \propto (P_\mu P^\mu)^{n-2} = M^{2n-4}$$

Thus, the haron distribution in a jet of n hadron is

$$p^0 \frac{d\sigma}{d^3 p} \stackrel{n=fix}{\propto} \frac{\Omega_{n-1} (P_\mu - p_\mu)}{\Omega_n (P_\mu)} \propto (1-x)^{n-3}, \quad x = \frac{P_\mu p^\mu}{M^2/2}$$

**Energy of the hadron
in the co-moving frame**

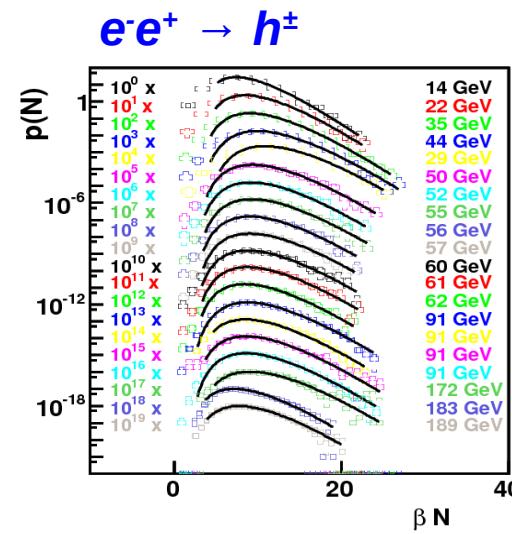
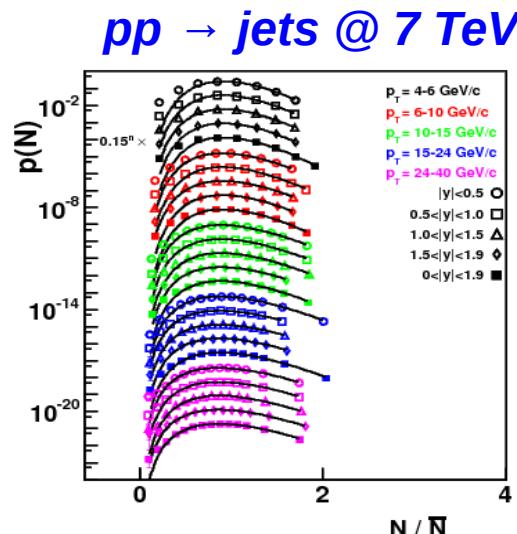
The hadron distribution in a jet of n hadron with total momentum \vec{P}



$$p^0 \frac{d\sigma}{d^3 p}{}^{n=fix} \propto (1-x)^{n-3}, \quad x = \frac{P_\mu p^\mu}{M^2/2}$$

Problems

- The **hadron multiplicity** in a jet **fluctuates**



$$P(n) = \binom{n+r-1}{r-1} \tilde{p}^n (1-\tilde{p})^r$$

Refs.:

Urmossy et.al., *PLB*, **701**: 111-116 (2011)

Urmossy et. al., *PLB*, **718**, 125-129, (2012)

Averaging over n fluctuations

The distribution in a jet with *fix n*

$$p^0 \frac{d\sigma}{d^3 p} \stackrel{n=fix}{\propto} (1-x)^{n-3}, \quad x = \frac{P_u p^u}{M^2/2}$$

The multiplicity distribution

$$P(n) = \binom{n+r-1}{r-1} \tilde{p}^n (1-\tilde{p})^r$$

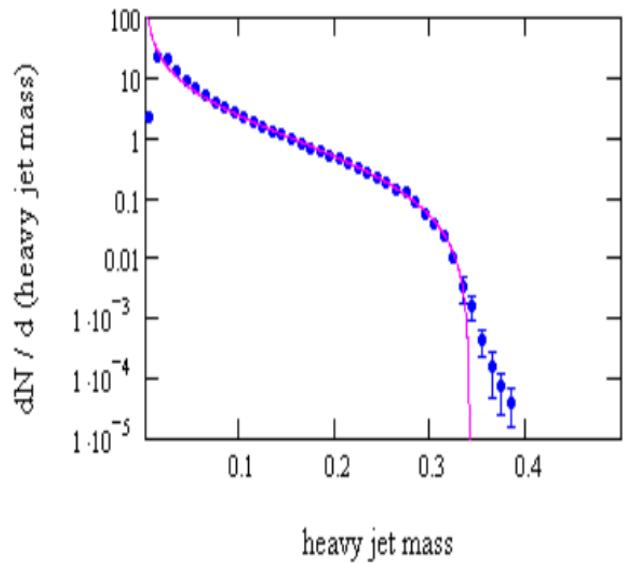
The *n-averaged* distribution

$$p^0 \frac{d\sigma}{d^3 p} = A \left[1 + \frac{q-1}{\tau} x \right]^{-1/(q-1)}$$

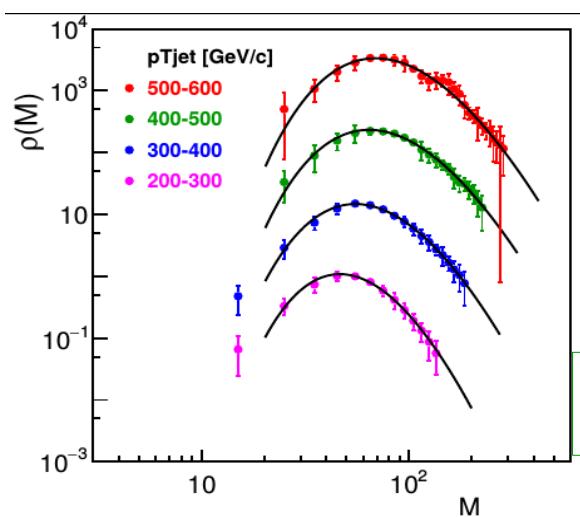
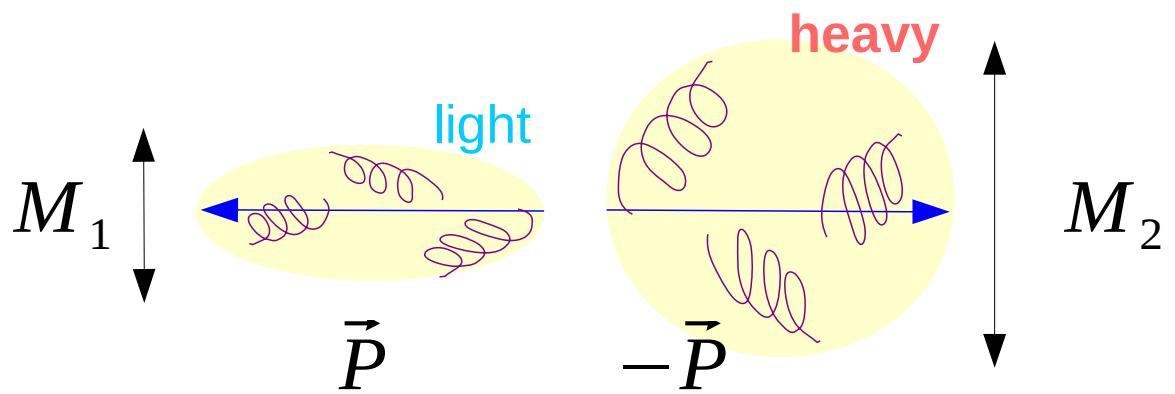
$$\tau = \frac{1-\tilde{p}}{\tilde{p}(r+3)}$$

$$q = 1 + \frac{1}{r+3}$$

Jet mass fluctuations

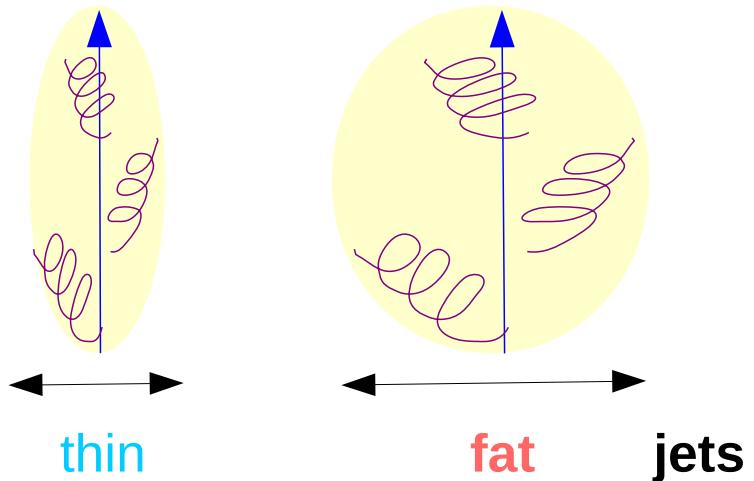


$e^+e^- \rightarrow 2 \text{ jet}$: both E and \vec{P} of the jets fluctuate

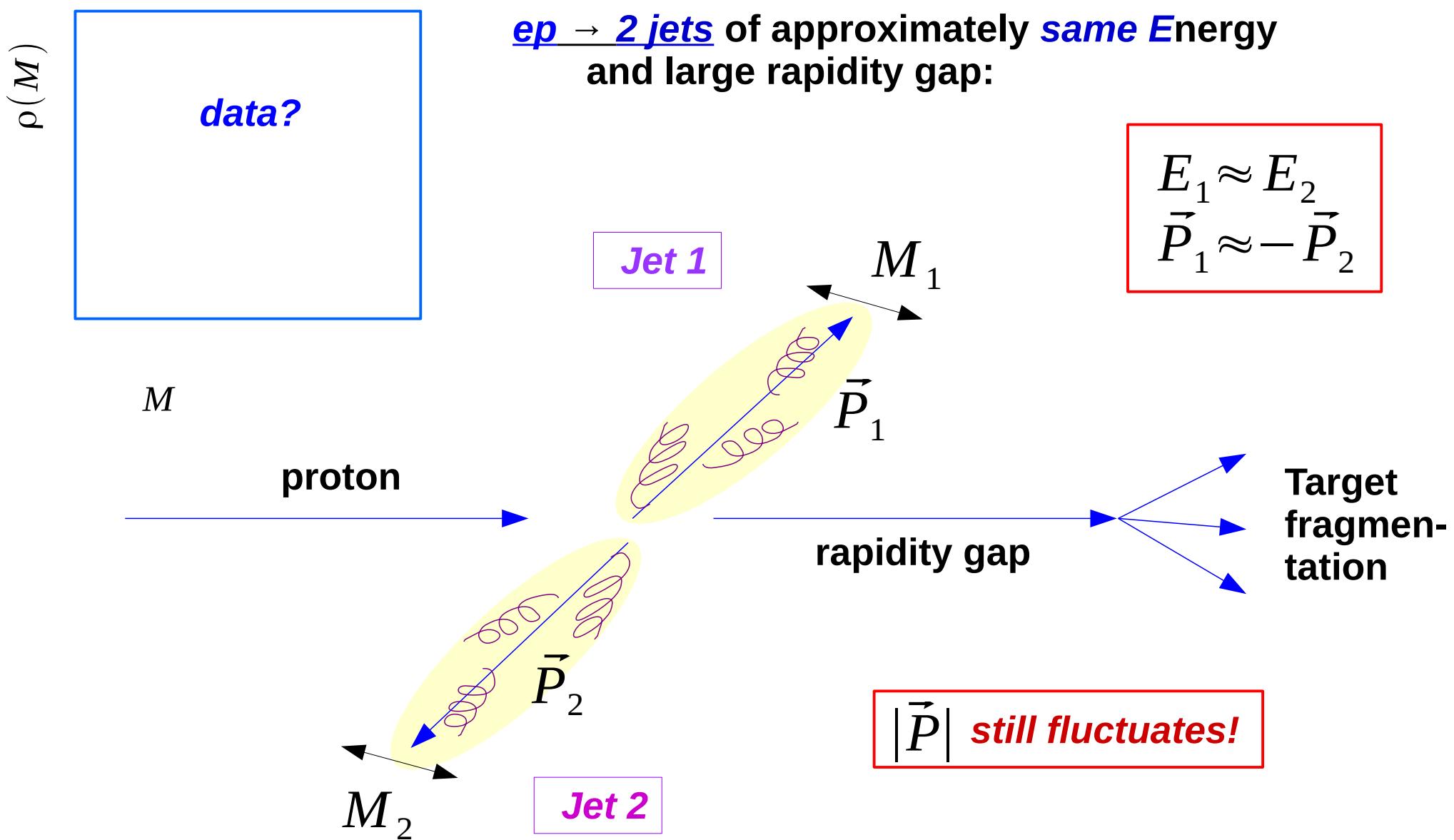


pp collisions: jet \vec{P} is measured, E, M fluctuates

K.U, Z. Xu,
arXiv:1605.06876



Problems



We have a haron distribution, which depends on $x = \frac{P_u}{M^2} p^u / 2$

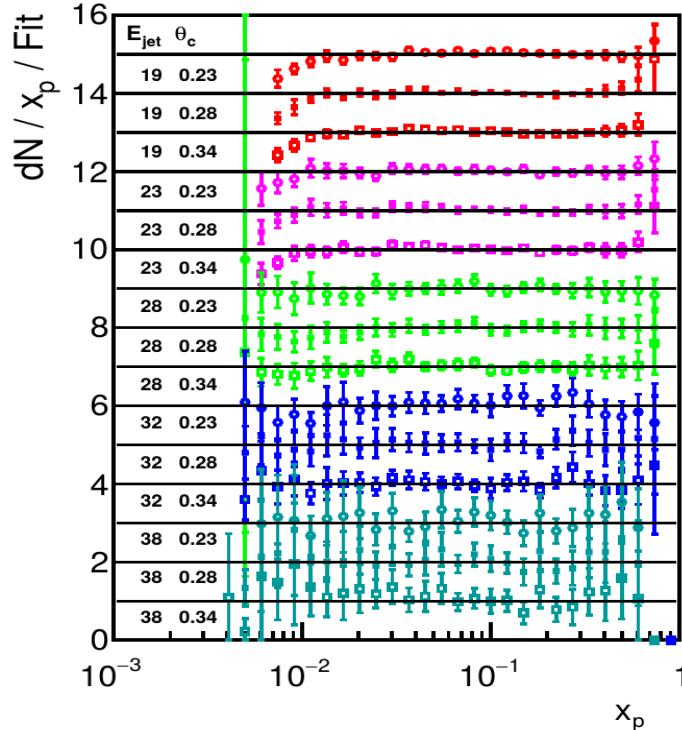
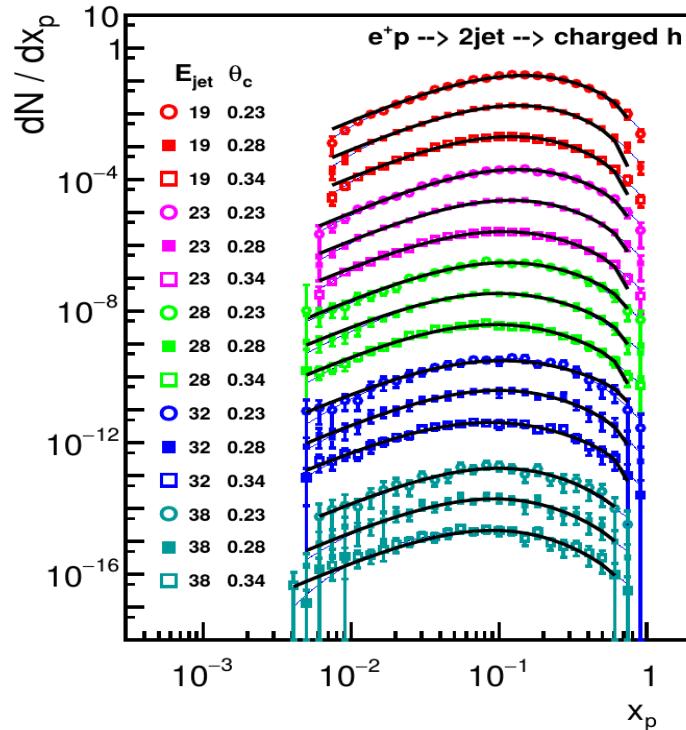
but, in case of available data, the *jet E or P fluctuate:*

- *pp collisions:* \vec{P} is measured, E fluctuates
- *$e^+e^- \rightarrow 2 \text{ jet}$:* both E and \vec{P} of the jets fluctuate
- *$e^+p \rightarrow 2 \text{ jet}$:* \vec{P} of the jets fluctuate

So, we *fit* a *characteristic/average jet mass* and extract the scale dependence of the parameters of the model

Results

$e^+ p \rightarrow 2 \text{ jets} \rightarrow \text{charged hadrons}$
with large rapidity gap



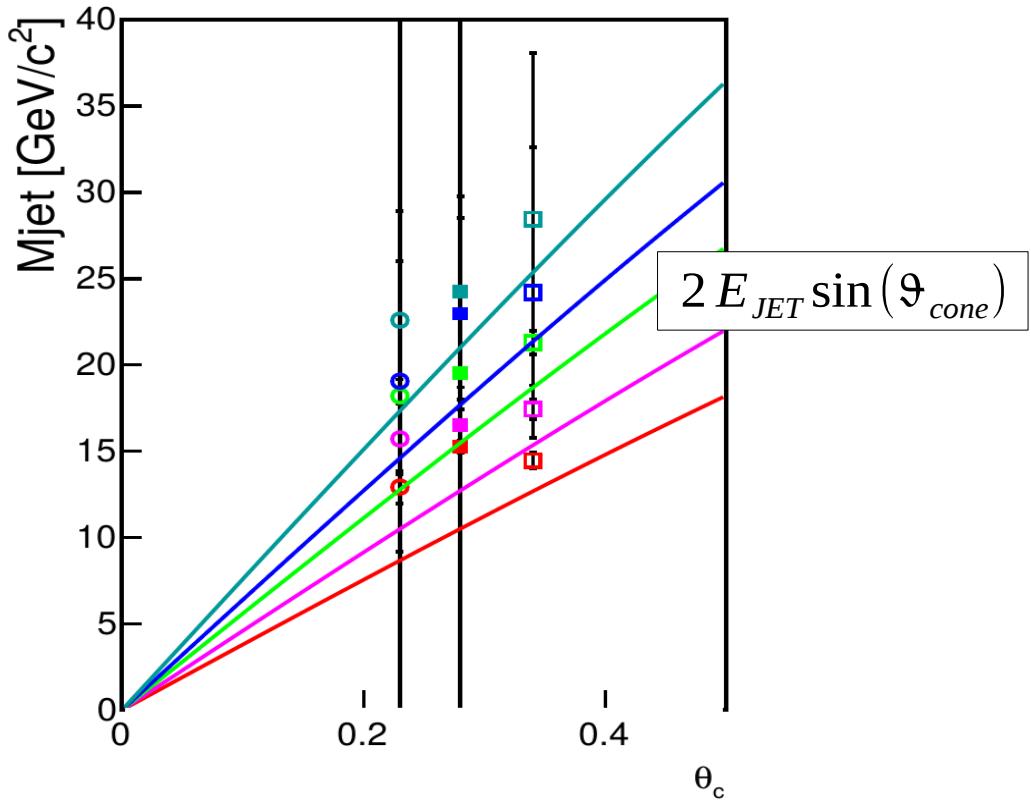
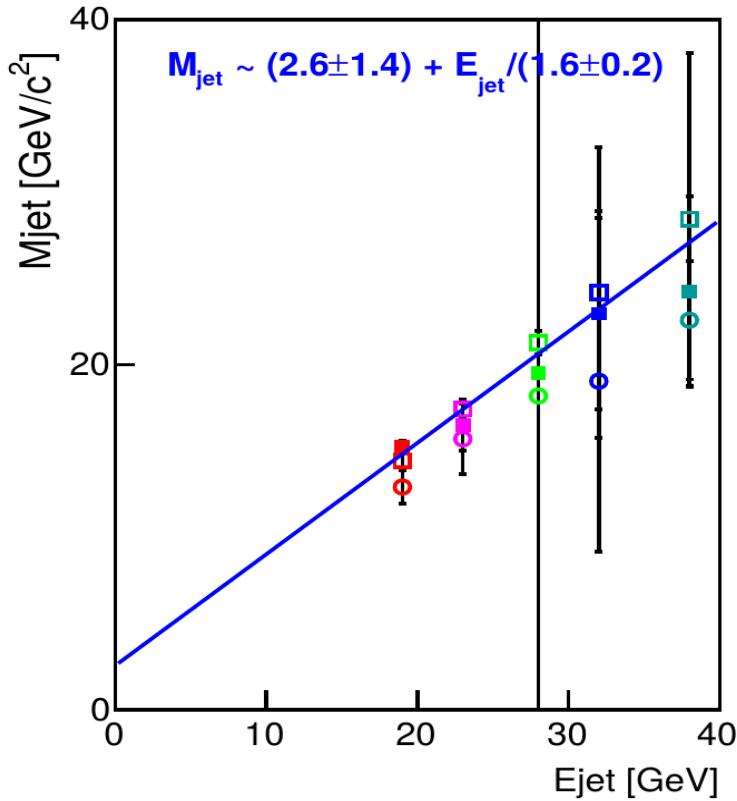
$$\frac{d\sigma}{dx_p} \sim x_p \left[1 + \frac{q-1}{\tau} x_p \right]^{-1/(q-1)}$$

$$x_p = 2p/M_{2JET}$$

$$M_{2JET} = \frac{E_1 + E_2}{2}$$

$$\frac{E_1}{E_2} = 1 \pm 0.2$$

Fitted average characteristic jet mass

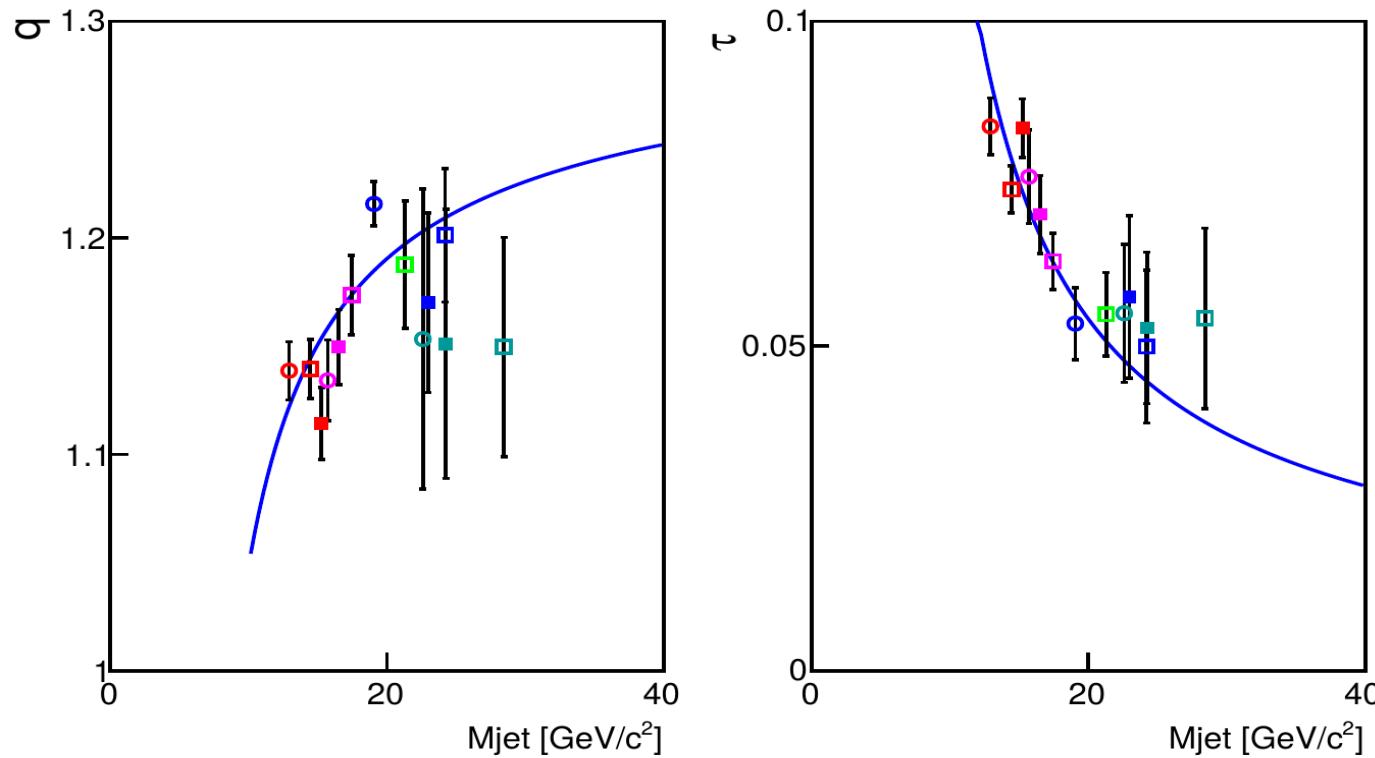


$$\text{fitted } \langle M_{JET} \rangle = M_0 + E_{JET}/E_0$$

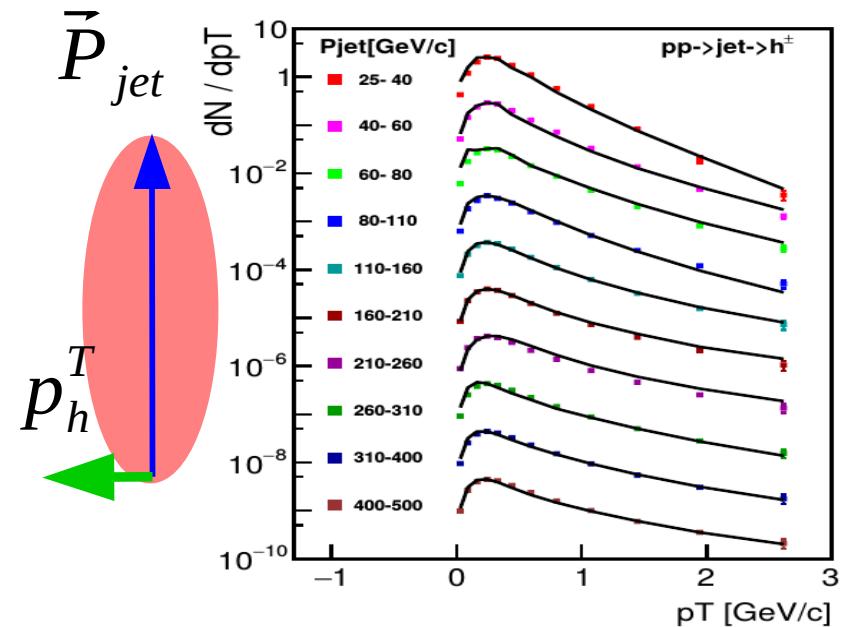
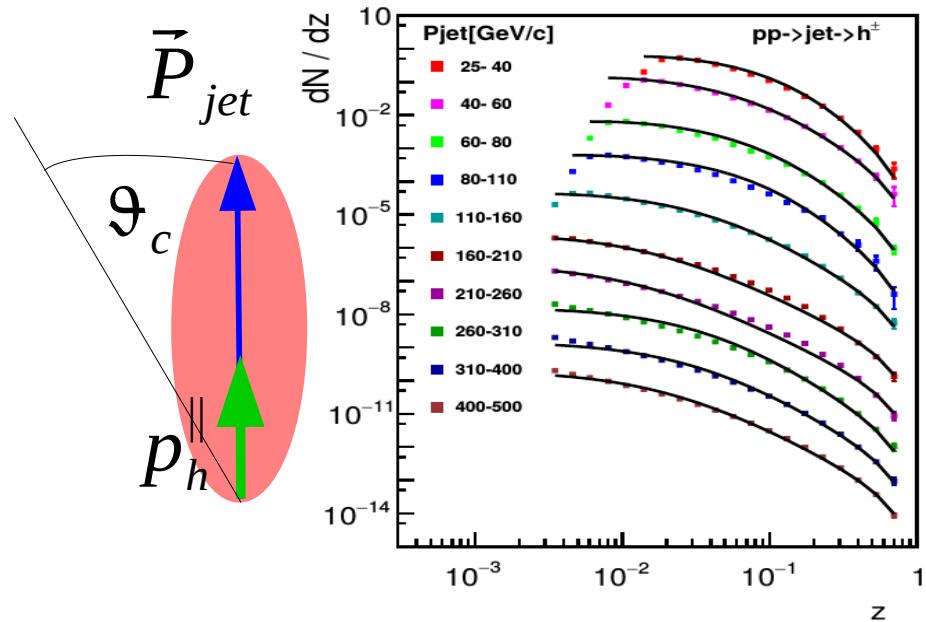
Fitted average jet mass is of the order of that used in DGLAP calcs.

$$\langle M_{JET} \rangle \sim 2 E_{JET} \sin(\theta_{cone})$$

Scale evolution of the fit parameters



PP → jet → charged hadrons



$$p^0 \frac{d\sigma}{d^3 p} \sim \left[1 + \frac{q-1}{\tau} x \right]^{-1/(q-1)}$$

$$x = \frac{2P_\mu^{jet} p^\mu}{M_{jet}^2}$$

What we have:

- an **approximate** formula for the **fragmentation function** which **does not solve DGLAP**

$$D(x) \sim \left[1 + \frac{q-1}{\tau} x \right]^{-1/(q-1)}$$

- Let us use **this ansatz** with **scale dependent parameters**

$$q, T \rightarrow q(t), T(t)$$

- along with some other conjectures

First step: in the Φ^3 theory

The Φ^3 theory case

Resummation of branchings with DGLAP

$$\frac{d}{dt} D(x,t) = g^2 \int_x^1 \frac{dz}{z} P(z) D(x/z, t), \quad t = \ln(Q^2/Q_0^2), \quad g^2 = 1/(\beta_0 t)$$

with **LO splitting function**: $P(z) = z(1-z) - \frac{1}{12} \delta(1-z)$

Let the non-perturbative input at starting scale Q_0 be:

$$D_0(x) = \left(1 + \frac{q_0 - 1}{\tau_0} x\right)^{-1/(q_0 - 1)}$$

The full solution is $D(x,t) = \int_x^1 \frac{dz}{z} f(z,t) D_0(x/z)$

with $f(x) \sim \delta(1-x) + \sum_{k=1}^{\infty} \frac{b^k}{k!(k-1)!} \sum_{j=0}^{k-1} \frac{(k-1+j)!}{j!(k-1-j)!} x \ln^{k-1-j} \left[\frac{1}{x} \right] [(-1)^j + (-1)^k x]$

$$b = \beta_0^{-1} \ln(t/t_0)$$

M. Grazzini,
Nucl. Phys. Proc. Suppl.
64: 147-151, 1998



Approximations

Let the FF preserve its form:

$$D_{apx}(x, t) = \left(1 + \frac{q(t)-1}{\tau(t)} x\right)^{-1/(q(t)-1)} \quad \text{with} \quad D(x, 0) = \left(1 + \frac{q_0-1}{\tau_0} x\right)^{-1/(q_0-1)}$$

From DGLAP:

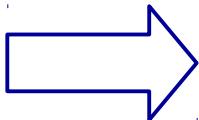
$$\tilde{D}(s, t) = \tilde{D}(s, 0) \exp\{b(t) \tilde{P}(s)\} \quad \text{with} \quad b(t) = \beta_0^{-1} \ln(t/t_0)$$

Let us prescribe the approximations:

$$\int D_{apx}(x, t) = \int D(x, t)$$

$$\int x D_{apx}(x, t) = \int x D(x, t) = 1 \quad (\text{by definition})$$

$$\int x^2 D_{apx}(x, t) = \int x^2 D(x, t)$$

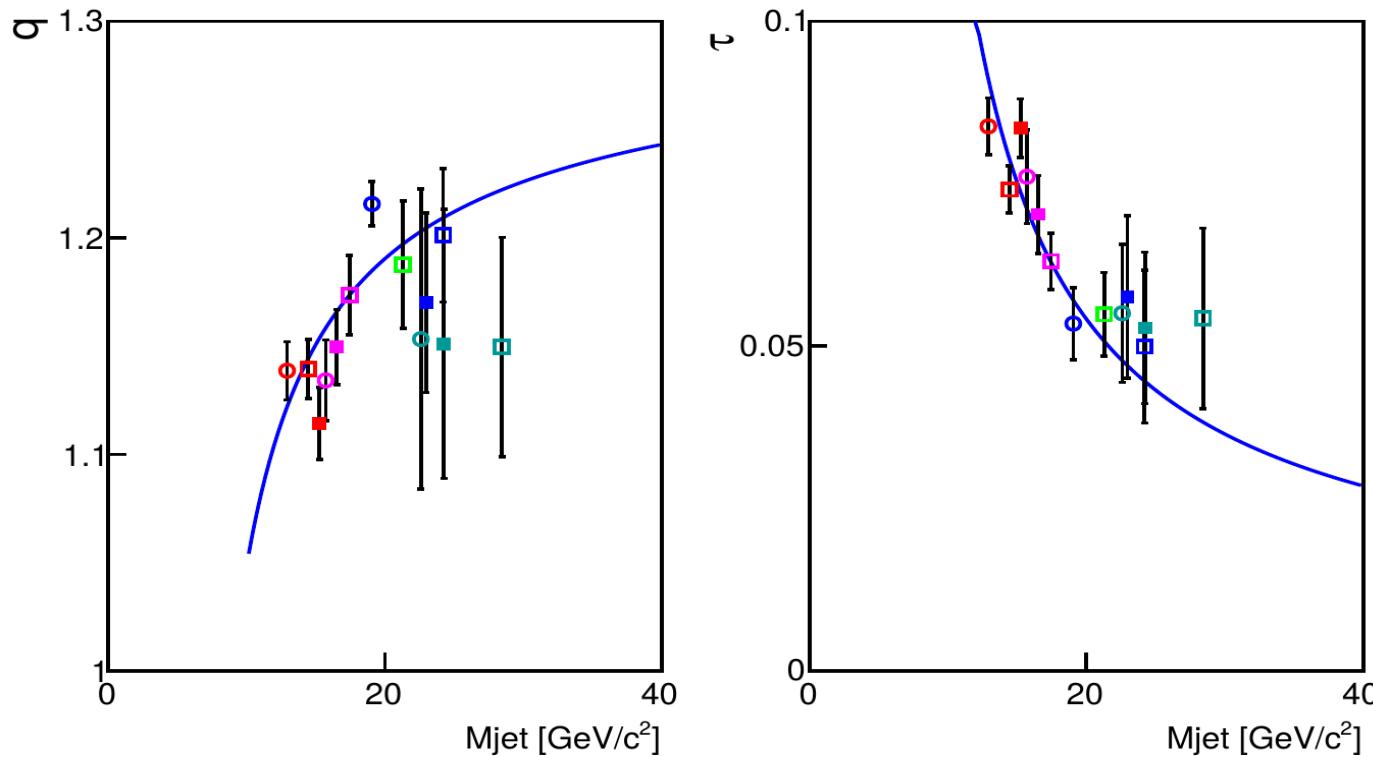


$$q(t) = \frac{\alpha_1(t/t_0)^{a1} - \alpha_2(t/t_0)^{-a2}}{\alpha_3(t/t_0)^{a1} - \alpha_4(t/t_0)^{-a2}}$$

$$\tau(t) = \frac{\tau_0}{\alpha_4(t/t_0)^{-a2} - \alpha_3(t/t_0)^{a1}}$$

$$a_1 = \tilde{P}(1)/\beta_0, \quad a_2 = \tilde{P}(3)/\beta_0$$

Scale evolution of the fit parameters



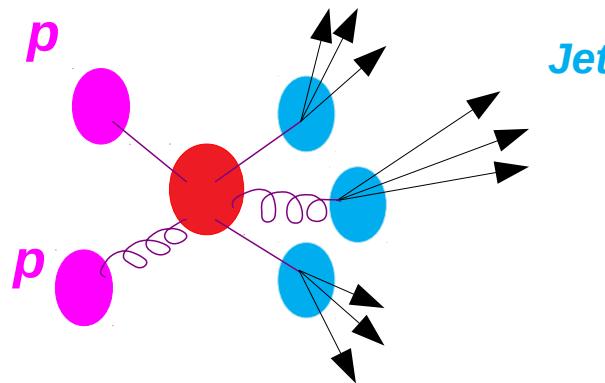
$$q(t) = \frac{\alpha_1(t/t_0)^{a1} - \alpha_2(t/t_0)^{-a2}}{\alpha_3(t/t_0)^{a1} - \alpha_4(t/t_0)^{-a2}}$$

$$\tau(t) = \frac{\tau_0}{\alpha_4(t/t_0)^{-a2} - \alpha_3(t/t_0)^{a1}}$$

$$t = \ln \left(\frac{M_{jet}^2}{\Lambda^2} \right)$$

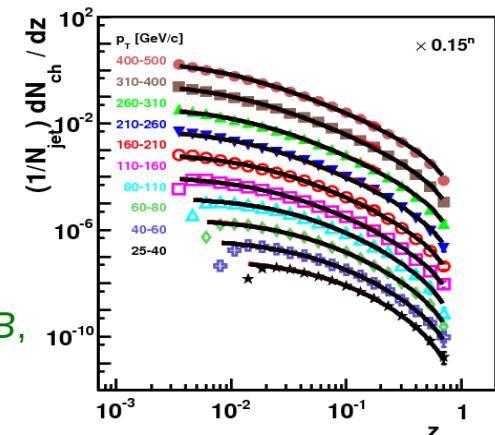
pp & ee collisions

pp → jets @LHC ($pT = 25\text{--}500 \text{ GeV}/c$)

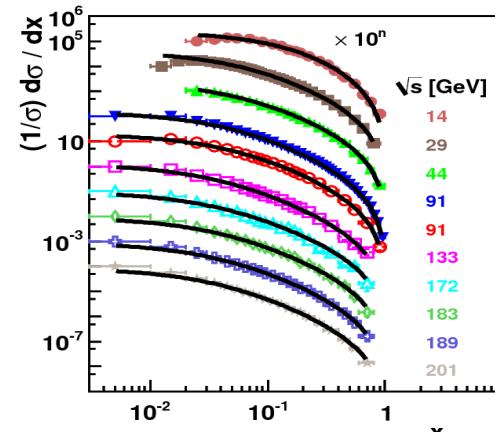
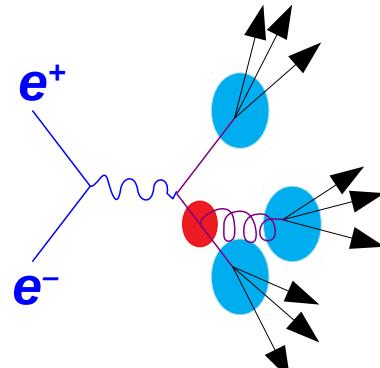


$$\frac{dN}{dz} \propto [1 - a \ln(1 - z)]^{-b}$$

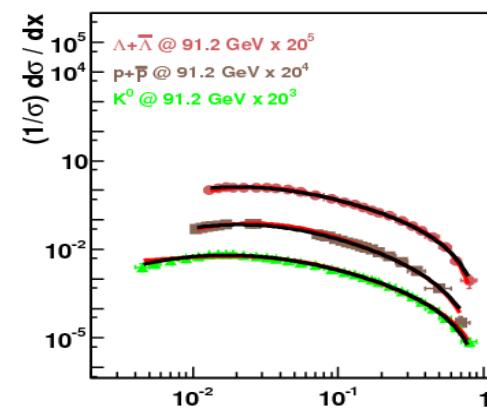
Urmossy et.al. *Phys. Lett. B*, **718**, 125-129, (2012)



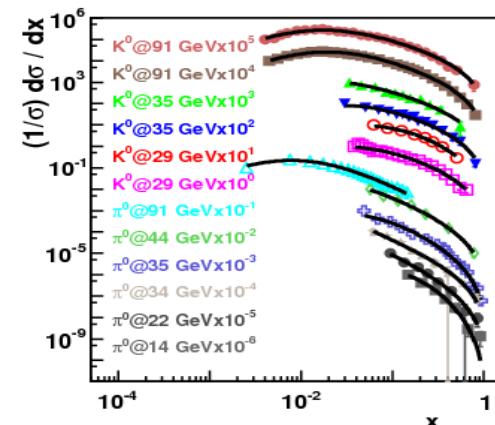
e⁺e⁻ annihilation @LEP ($\sqrt{s} = 14\text{--}200 \text{ GeV}$)



Urmossy et. al.,
Phys. Lett. B, **701**,
111-116 (2011)



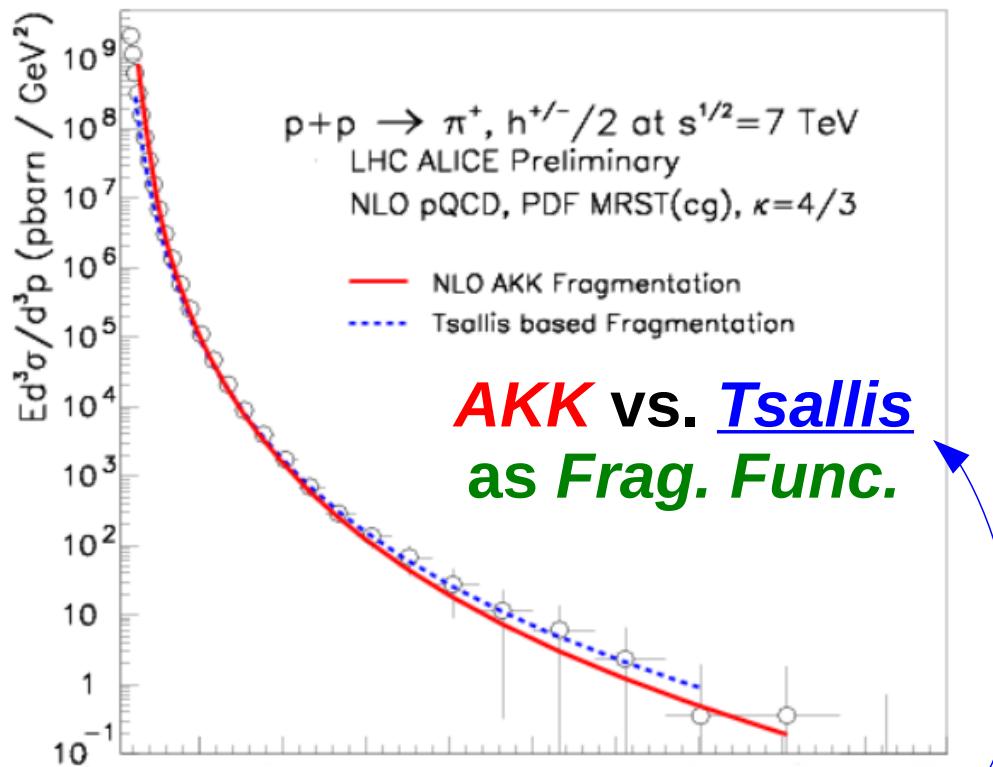
Urmossy et.al.,
Acta Phys. Polon. Supp. **5** (2012) 363-368



T. S. Biró et.al.,
Acta Phys. Polon. B,
43 (2012) 811-820

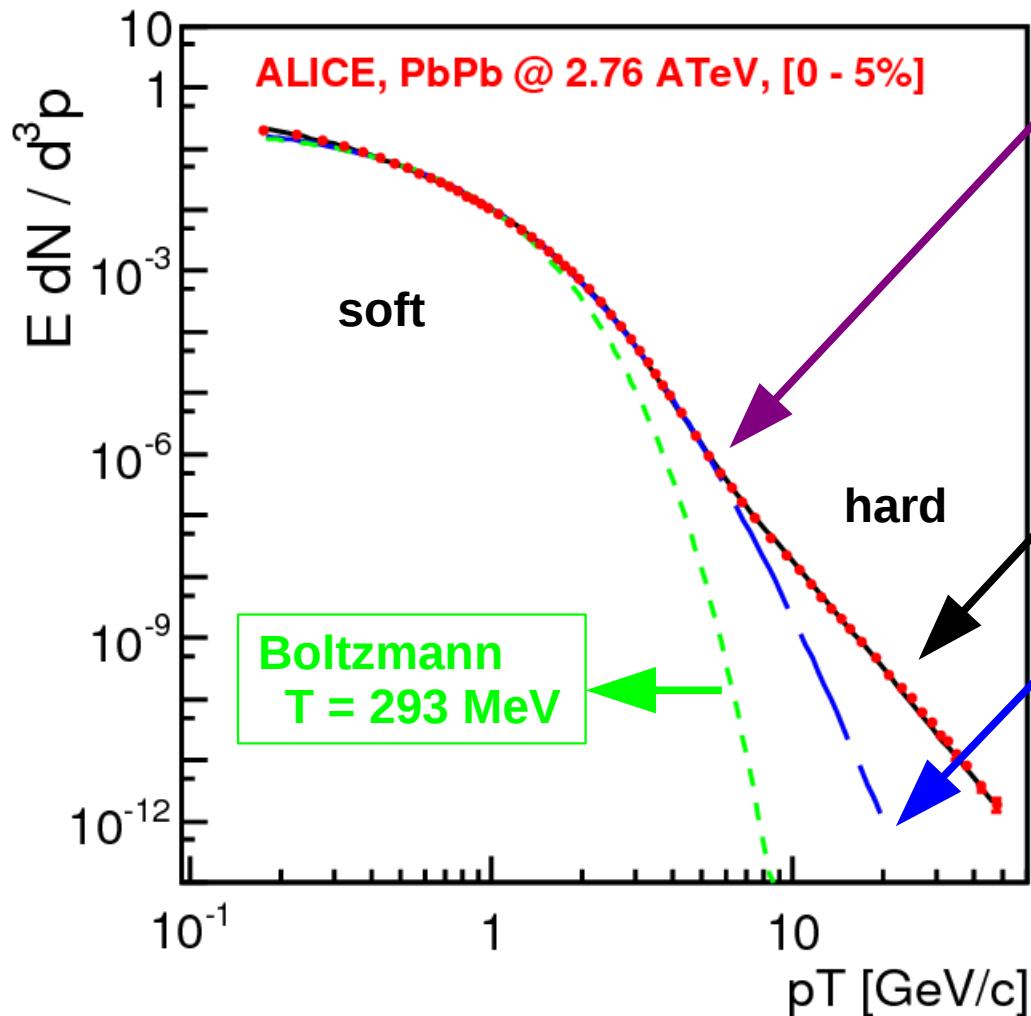
Application in a pQCD calculation

π^+ spectrum in $pp \rightarrow \pi^\pm X$ @ $\sqrt{s}=7$ TeV (NLO pQCD)



$$D_{p_i}^{\pi^+}(z) \sim (1 + (q_i - 1)z/T_i)^{-1/(q_i - 1)}$$

How about the *soft* part?



The power of the spectrum changes drastically at $p_T \sim 6 \text{ GeV}/c$.

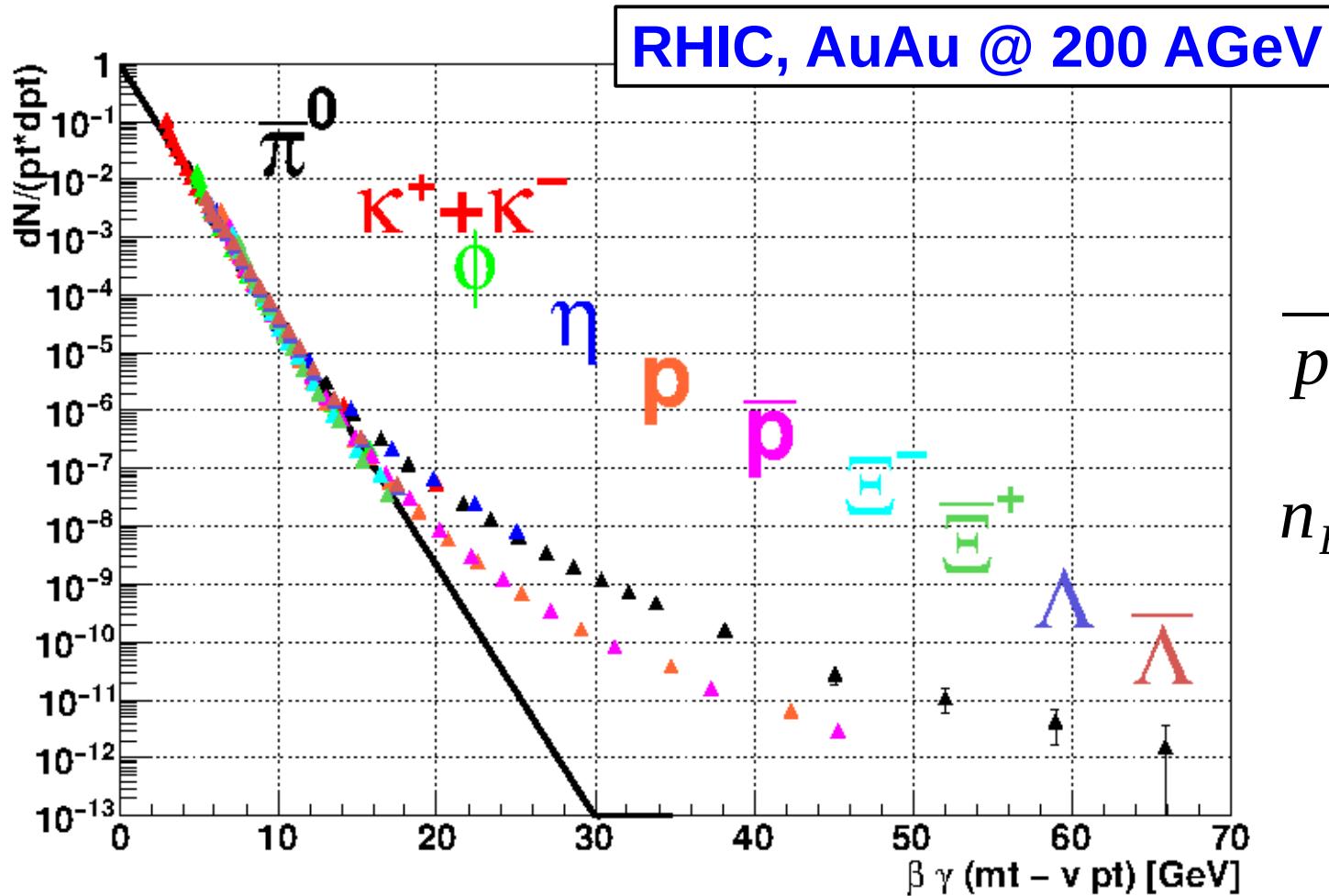
$$\sim p_T^{-6.08}$$

$$\text{Tsallis} \sim p_T^{-13.7}$$

A **hard + soft** model:

$$E \frac{dN}{d^3p} = E \frac{dN}{d^3p}^{\text{hard}} + E \frac{dN}{d^3p}^{\text{soft}}$$

Different q for baryons and mesons



$$\frac{dN}{p_T dp_T} \sim p_T^{-n}$$

$$n_{\text{Baryon}} \neq n_{\text{Meson}}$$

- Hadronisation: *rapid coalescence* of **thermal quarks** and **gluon fibres**:

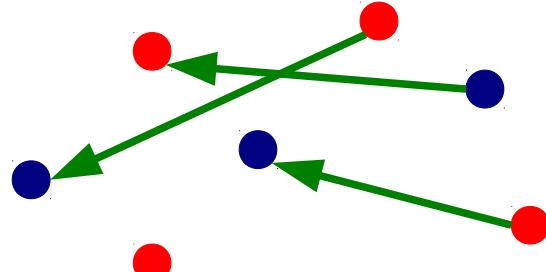
$$F_h(P_h, x) = f_q(x, p_{q1}) * \dots * f_q(x, p_{qn}) G(m) C(p_{qi}, m)$$

quarks: $f_q(x, \vec{p}_q) = \left(1 + \frac{q-1}{T} \epsilon_q\right)^{-1/(q-1)}$

gluon fibres: $G(m) = \exp\left(-\left[\Gamma(1+1/d) \frac{m}{\langle m \rangle}\right]^d\right)$

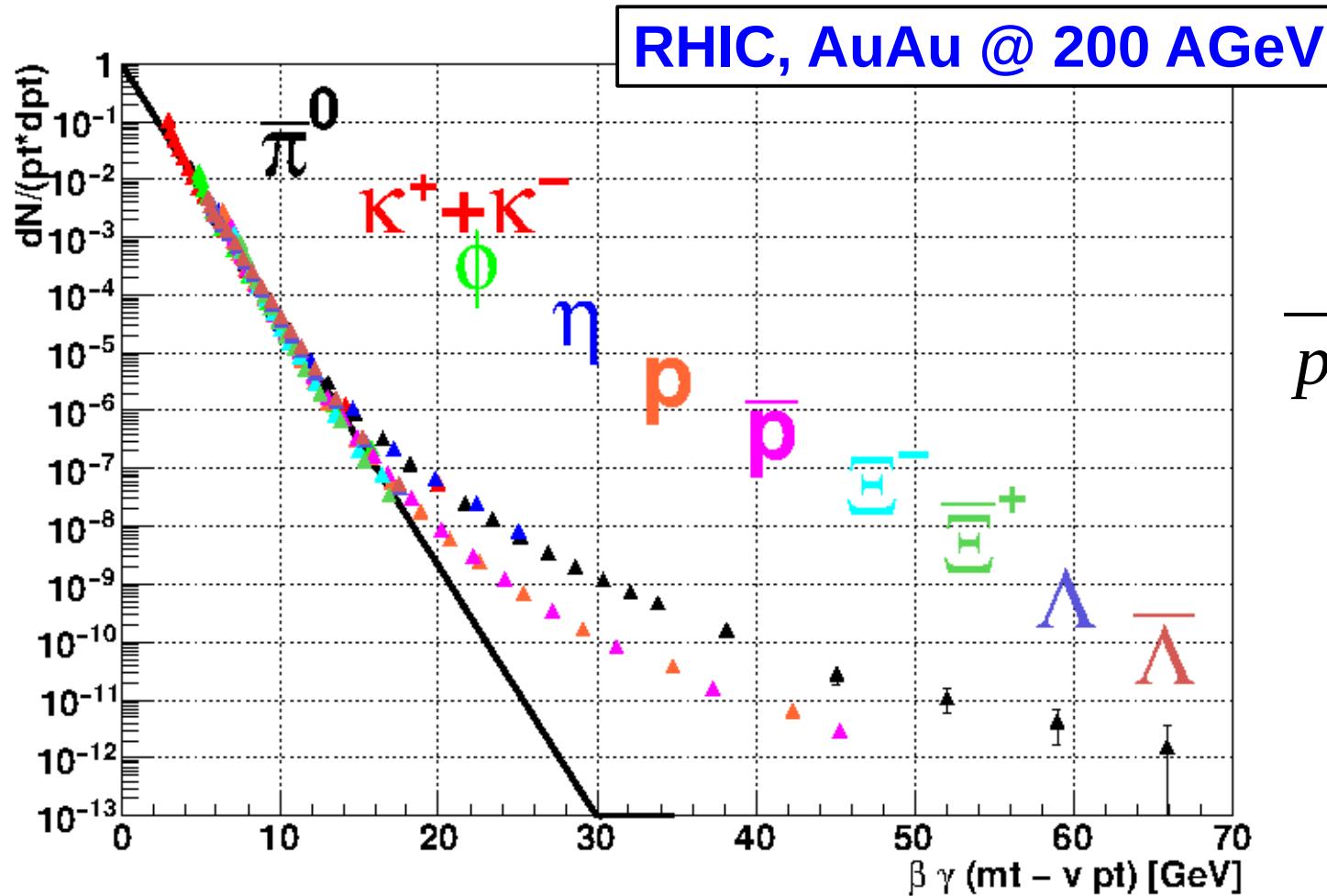
kernel: $C(x, p_{qi}) = \delta^3(\sum \vec{p}_i - \vec{P}_h) \prod_{i,j} \delta^3(\vec{p}_i - \vec{p}_j) \delta\left(\sum \epsilon_i + m - E_h\right)$

- The *distribution of the length of gluon fibres*: is the probability of finding two quarks at distance $l = \sigma/m$ in a homogenous quark sea with fractal dimension d .



T S Biró et al,
J. Phys. G-Nucl. Part. Phys., 37, 9, (2010)
J. Phys. G., G36, 064044, (2009)
Eur. Phys. J. A, 40, 325-340, (2009)

Different q for baryons and mesons



$$\frac{dN}{p_T dp_T} \sim p_T^{-n}$$

$$\frac{n_B}{2} \approx \frac{n_M}{3}$$

(1) Statistical description of hadron spectra:

$$E \frac{dN}{d^3 p} = \sum_{sources} f[u_\mu p^\mu]$$

(2) Space-time dependence only through $u_\mu(x)$ Bjorken + Blast Wave

$$u_\mu = (\gamma \cosh \zeta, \gamma \sinh \zeta, \gamma v \cos \alpha, \gamma v \sin \alpha), \quad \zeta = \frac{1}{2} \ln \left(\frac{t+z}{t-z} \right)$$

$$v(\alpha) = v_0 + \sum_1^N \delta v_m \cos(m\alpha)$$

Then, the spectrum and the v2 are

$$\frac{dN}{p_T dp_T dy}_{y=0} \propto f[E(v_0)] + O(\delta v^2)$$

$$v_2 \propto \delta v_2 (v_0 m_T - p_T) \frac{f'[E(v_0)]}{f[E(v_0)]} + O(\delta v^2) \quad E(v_0) = \gamma_0 (m_T - v_0 p_T)$$

Then, the spectrum and the v2 are

$$\frac{dN}{p_T dp_T dy} \Big|_{y=0} \propto f[E(v_0)] + O(\delta v^2) \quad E(v_0) = \gamma_0(m_T - v_0 p_T)$$

$$v_2 \propto \delta v_2 (v_0 m_T - p_T) \frac{f'[E(v_0)]}{f[E(v_0)]} + O(\delta v^2)$$

Boltzmann-distribution:

$$f[E(v_0)] \propto e^{-E(v_0)/T}$$

$$v_2 \propto p_T - v_0 m_T$$

Tsallis-distribution:

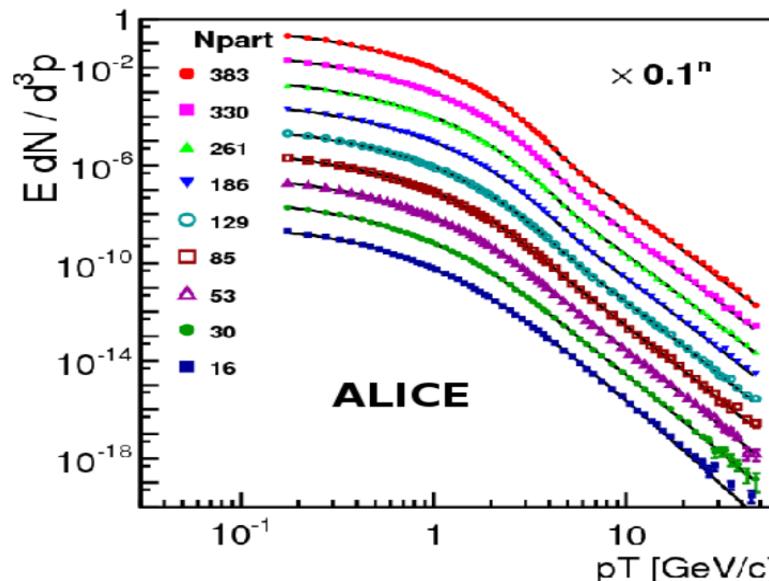
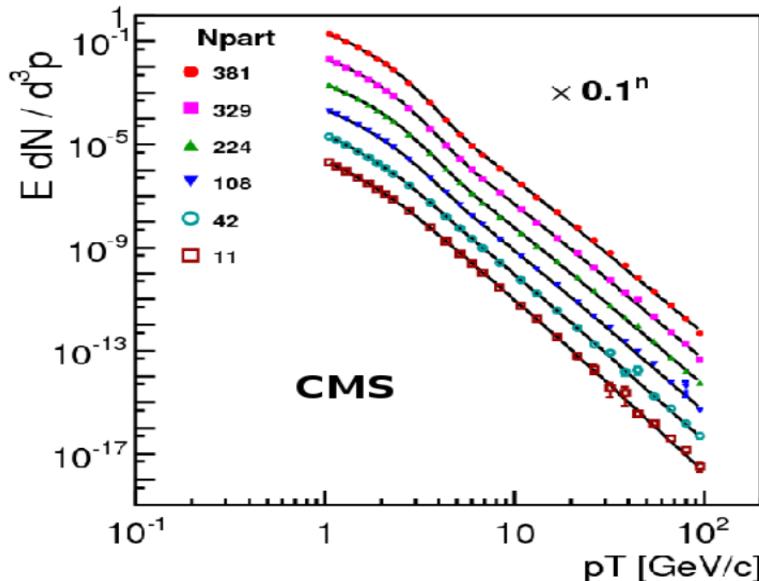
$$f[E(v_0)] \propto \left[1 + (q-1) \frac{E(v_0) - m}{T} \right]^{-1/(q-1)}$$

$$v_2 \propto \frac{p_T - v_0 m_T}{1 + \frac{q-1}{T} [\gamma_0(m_T - v_0 p_T) - m]}$$

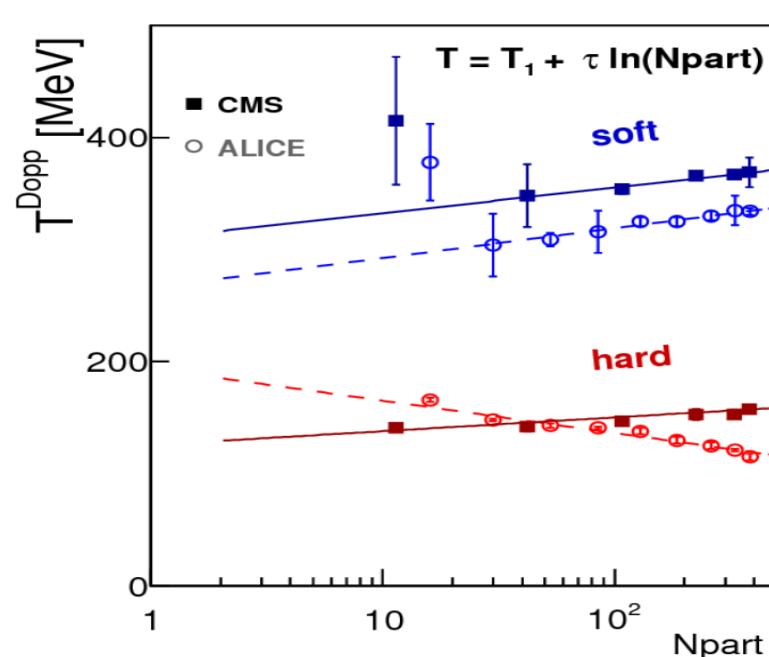
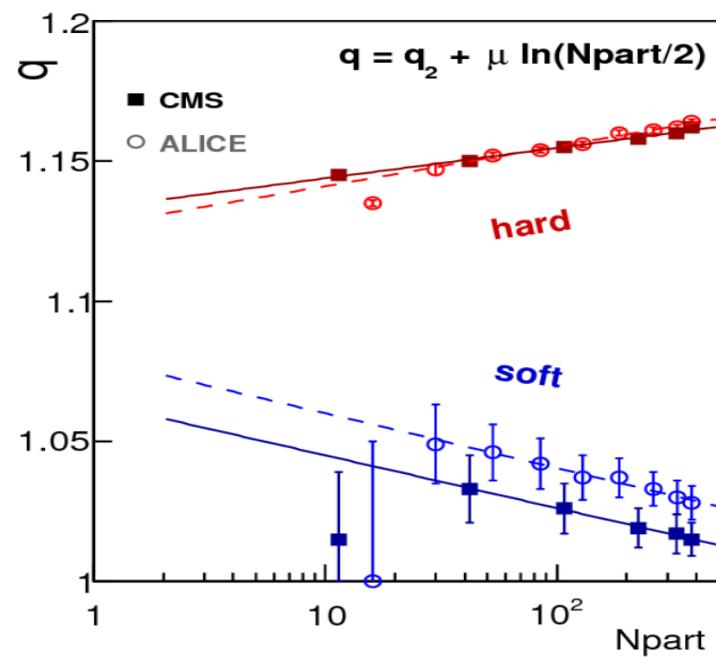
Barnaföldi et al, (Hot Quarks 2014) J. Phys. Conf. Ser. 612 (2015) 1, 012048

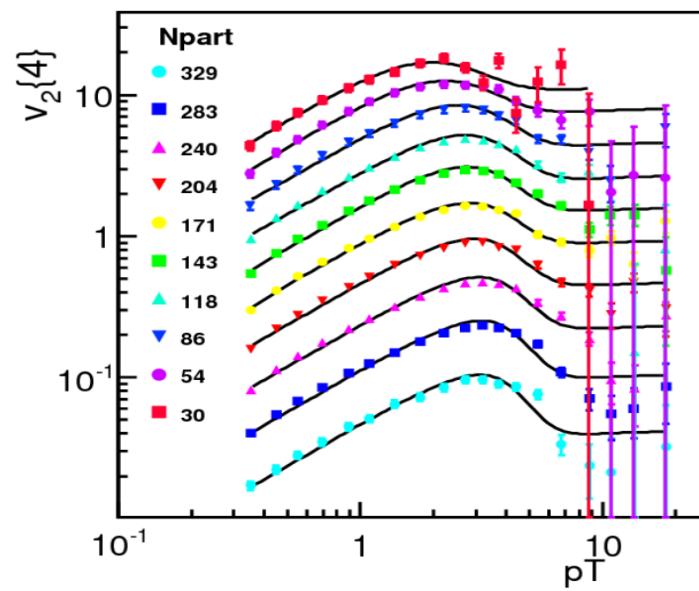
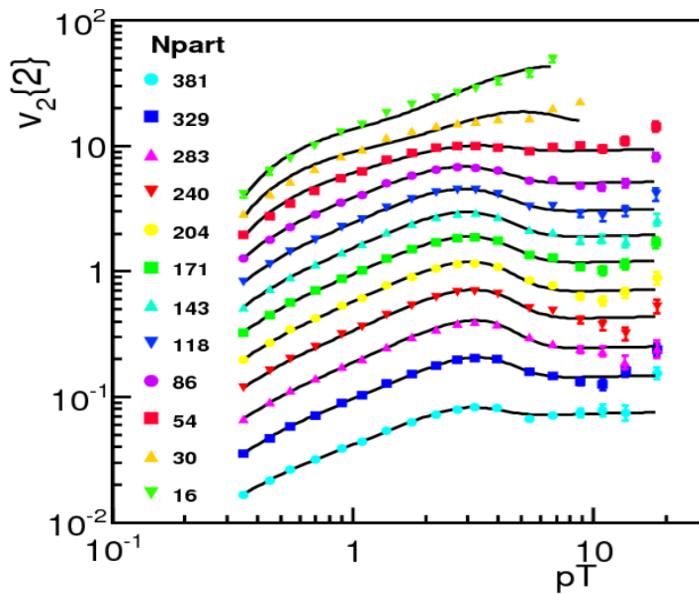
Urmossy et al, (WPCF 2014) arXiv:1501.05959, Conference: C14-08-25.8

Urmossy et al, (High-pT 2014), arXiv:1501.02352, arXiv:1405.3963

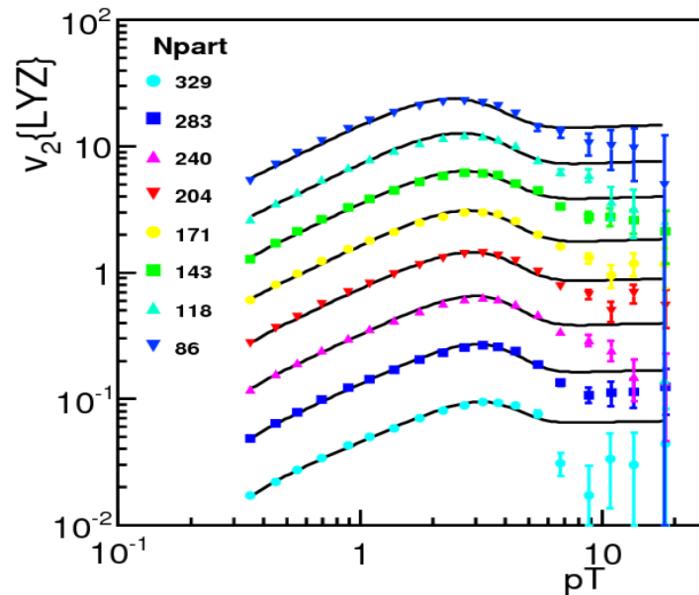
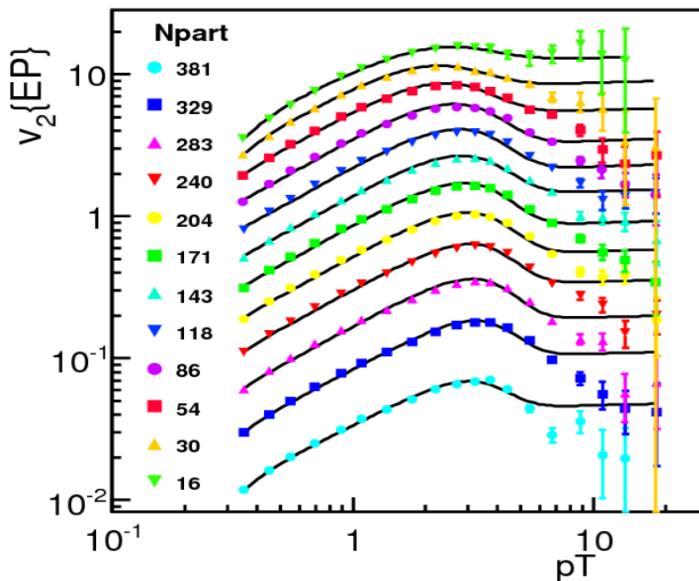


PbPb $\rightarrow h^\pm$
CMS

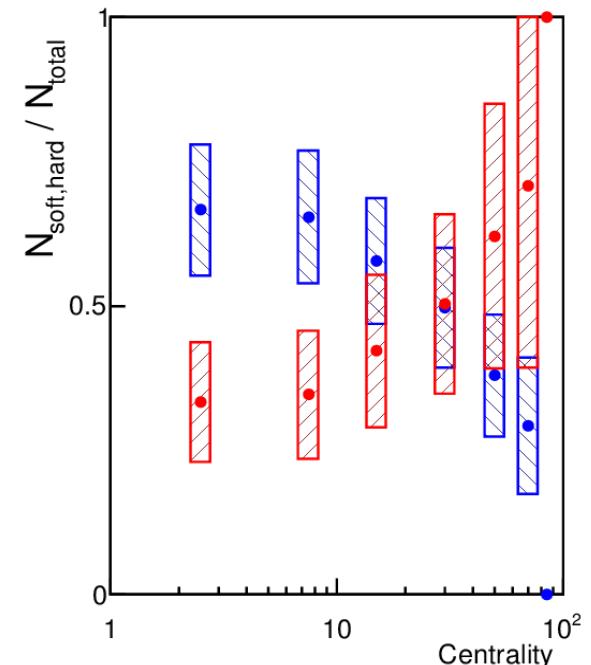
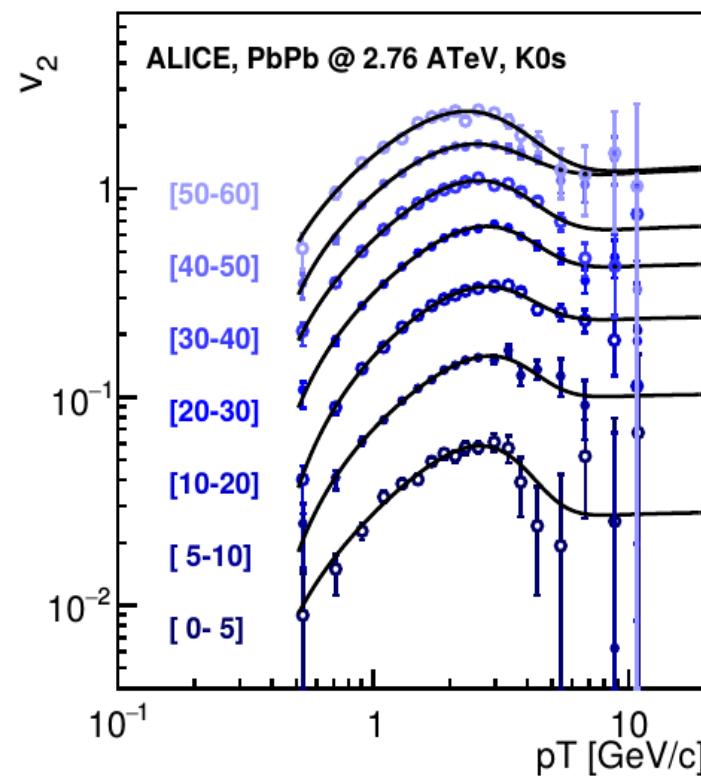
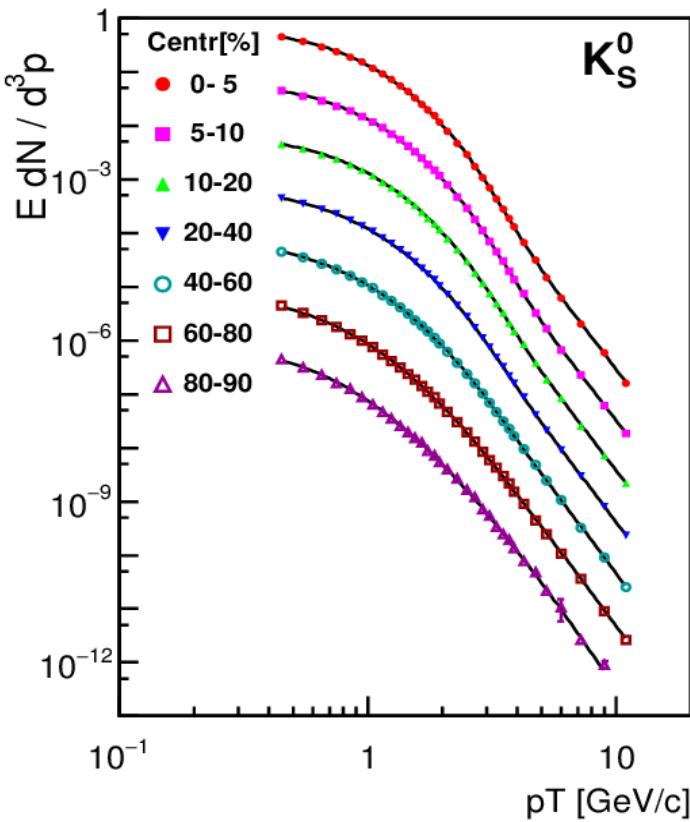




v2 of h^\pm

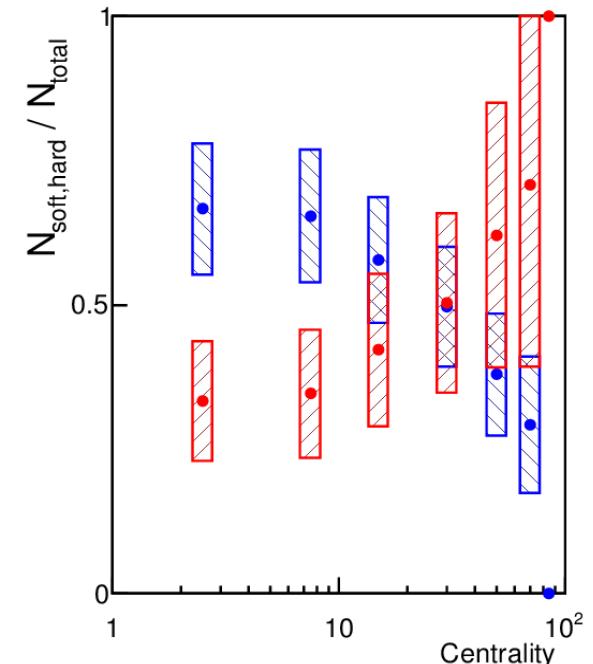
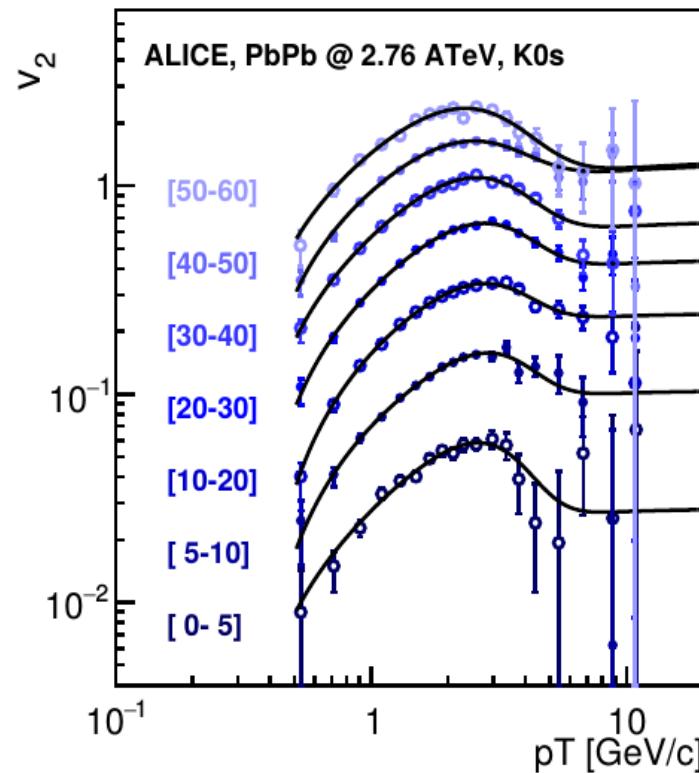
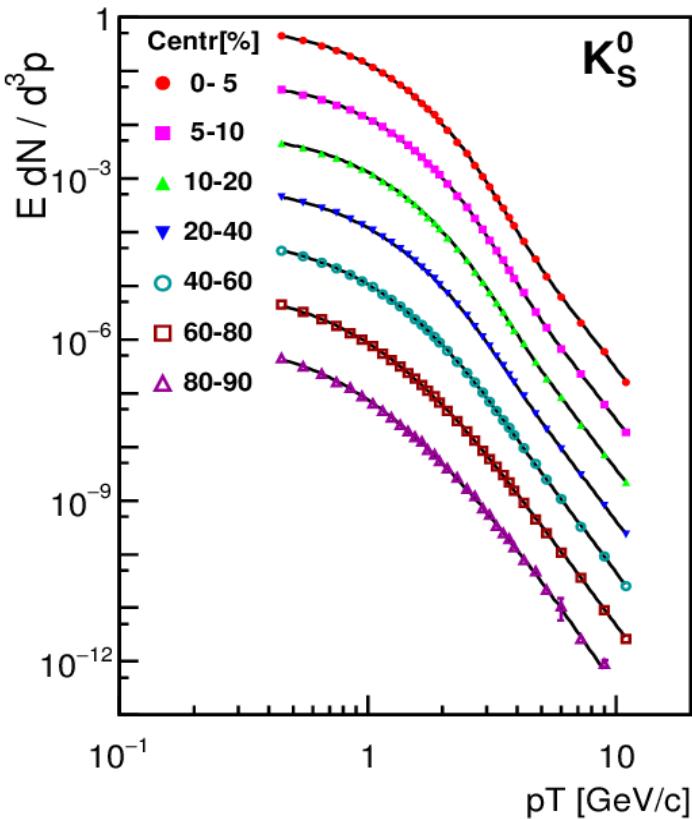


PbPb → K⁰s ALICE



Preliminary

Application in heavy-ion collisions



Preliminary

Conclusion

Jet-fragmentation might be statistical?

- **Suggestion**

It might be more suitable to

characterise JETs with their MASS

*instead of thier **P** or **E***

Conclusion

- Suggestion

Parametrise fragmentation functions as

$$D \left[x = \frac{2 P_\mu^{jet} p_h^\mu}{M_{jet}^2}, Q^2 = M_{jet}^2 \right]$$

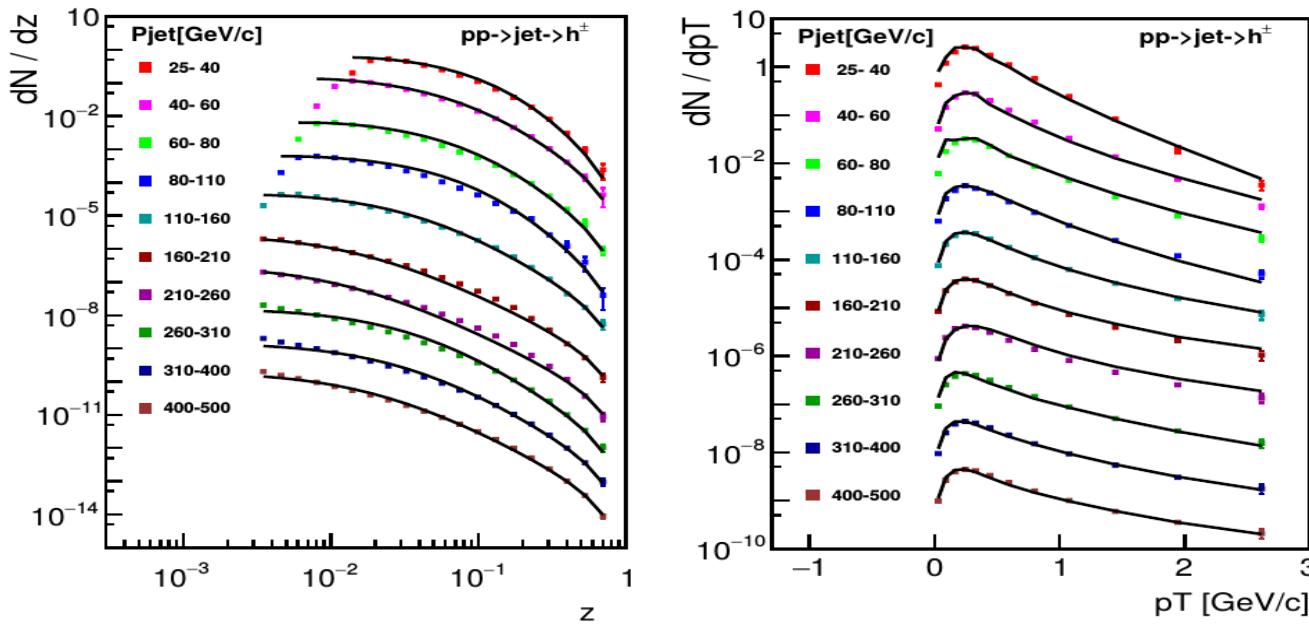
Energy fraction the hadron takes away in the frame co-moving with the jet

Fragmentation scale: jet mass

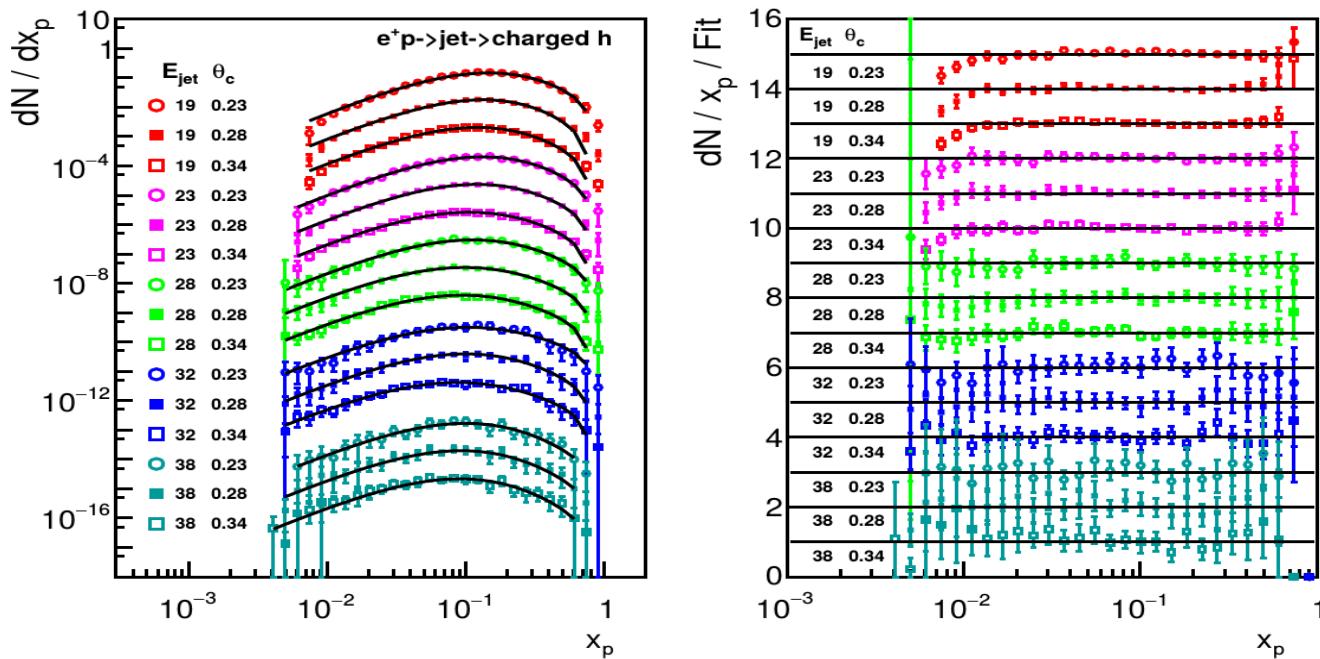
Thanks for the attention

Results

PP
collisions

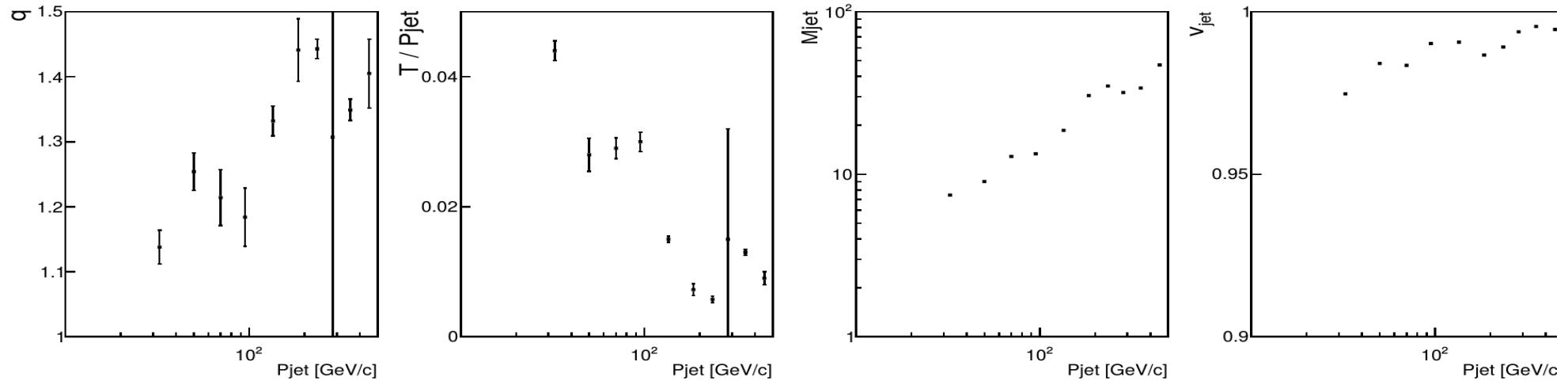


eP
collisions

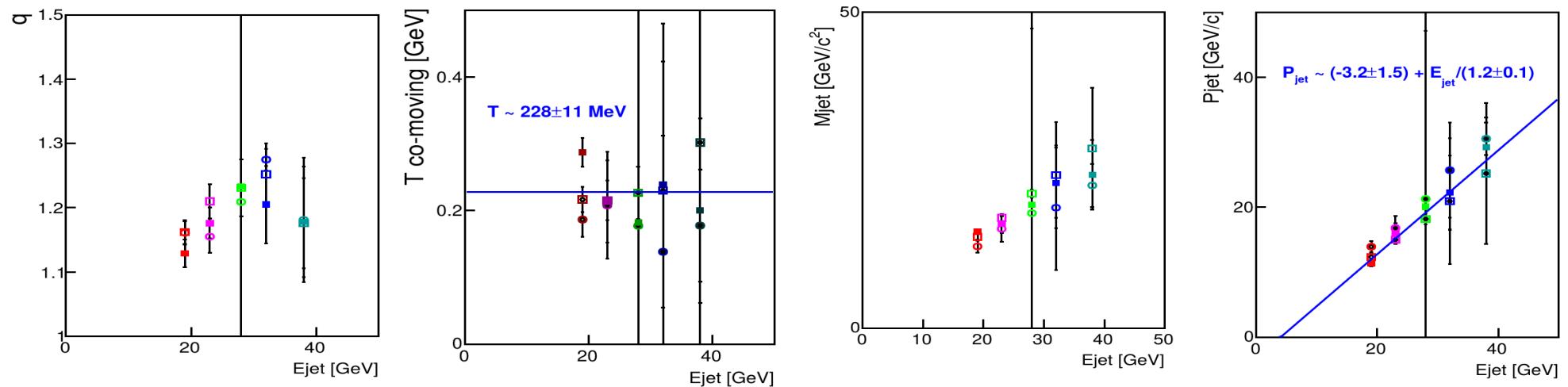


Results

PP



eP



Averaging over n fluctuations

The distribution in a jet with *fix n*

$$p^0 \frac{d\sigma}{d^3 p} \stackrel{n=fix}{\propto} (1-x)^{n-3}, \quad x = \frac{P_u p^u}{M^2/2}$$

The multiplicity distribution

$$P(n) = \binom{n+r-1}{r-1} \tilde{p}^n (1-\tilde{p})^r$$

The *n-averaged* distribution

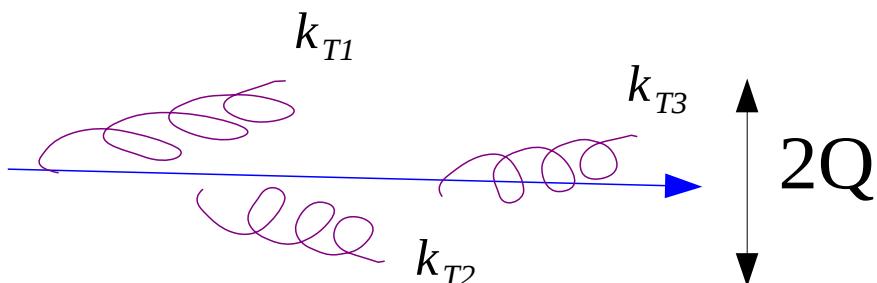
$$p^0 \frac{d\sigma}{d^3 p} = A \left\{ \left(1 + \frac{\tilde{p}}{1-\tilde{p}} x \right)^{-r-3} - \sum_3^{n_0-1} P(n) n f_n(x) \right\}$$

Q^2 Scale of the jet

- parton branching

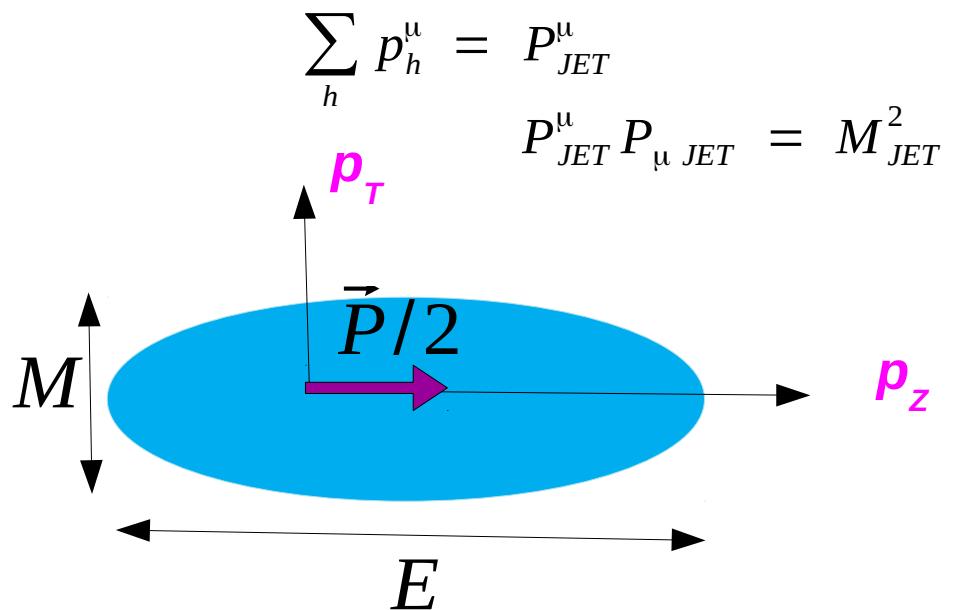
DGLAP for Fragmentation
Functions goes with Q^2

$$k_{Ti}^2 = k_{i\perp} k_i^\mu \leq Q^2$$



- Hadrons in the jet

Energy-momentum conservation



$$Q \sim M_{JET}/2$$

What is T ?

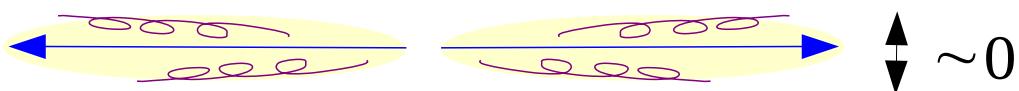
If in a single event / jet, we have equipartition:

$$1 \text{ event: } \frac{E_{\text{event}}}{N_{\text{event}}} = D T_{\text{event}}$$

On the average, we have:

$$\frac{E}{N} = \frac{\int \epsilon f_{TS}(\epsilon)}{\int f_{TS}(\epsilon)} = \frac{D T}{1 - (q-1)(D+1)}$$

($m \approx 0$ particles)

Ideal world: **e^+e^- annihilations in the factorized picture****2 identical jets:**

$$P_{\mu}^{1,2} = (\sqrt{s}/2, 0, 0, \pm\sqrt{s}/2)$$

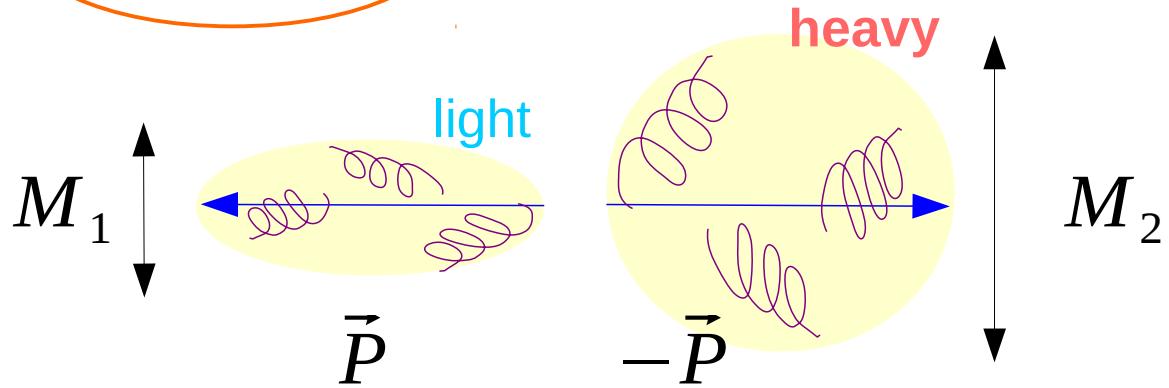
energy fraction of the hadron:

$$x = p_h^0 / (\sqrt{s}/2)$$

fragmentation scale:

$$Q \sim \sqrt{s}$$

Problem: $P^2 \sim 0$ quark produces a **heavy jet** of mass $M \sim [0.1 - 0.5]\sqrt{s}$

Real world:**the 2 jets are not identical**

$$P_1^\mu = (P^0, 0, 0, |\mathbf{P}|)$$

$$P_2^\mu = (\sqrt{s} - P^0, 0, 0, -|\mathbf{P}|)$$

We propose to use:

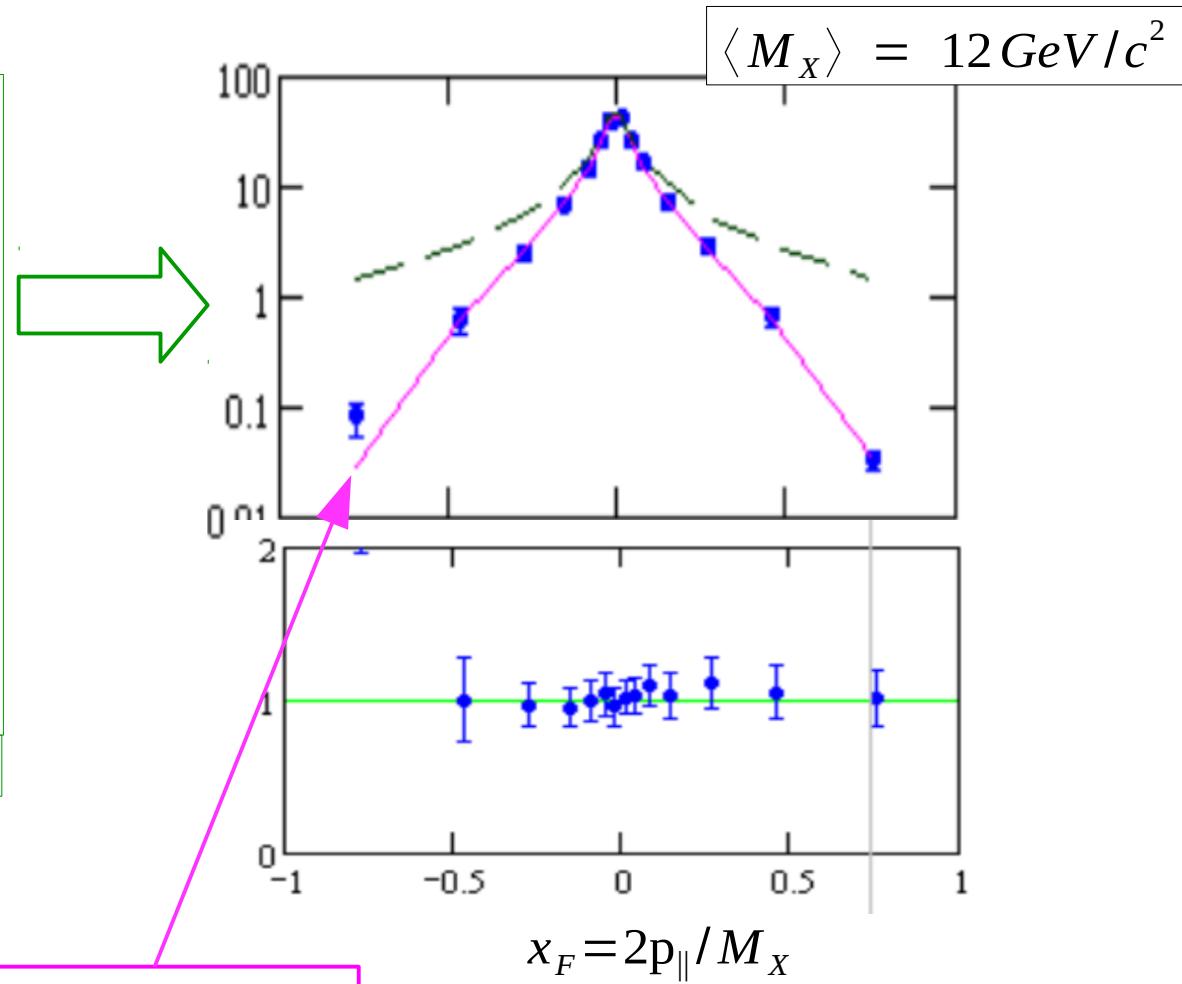
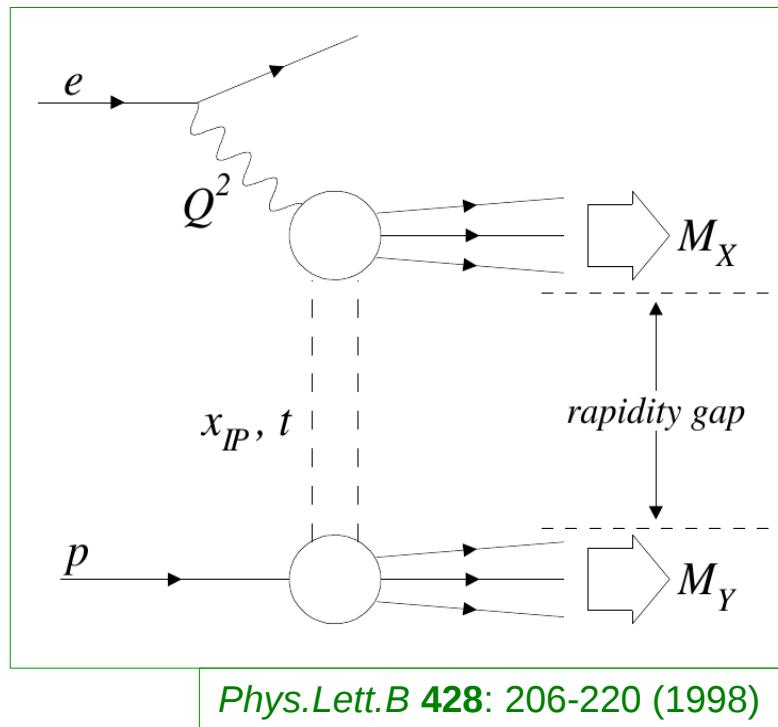
- **energy fraction** of the hadron in the **frame co-moving** with jet

$$x = 2 p_h^\mu P_\mu^{\text{jet}} / M_{\text{jet}}^2$$

- **fragmentation scale:**

$$Q \sim M_{\text{jet}}$$

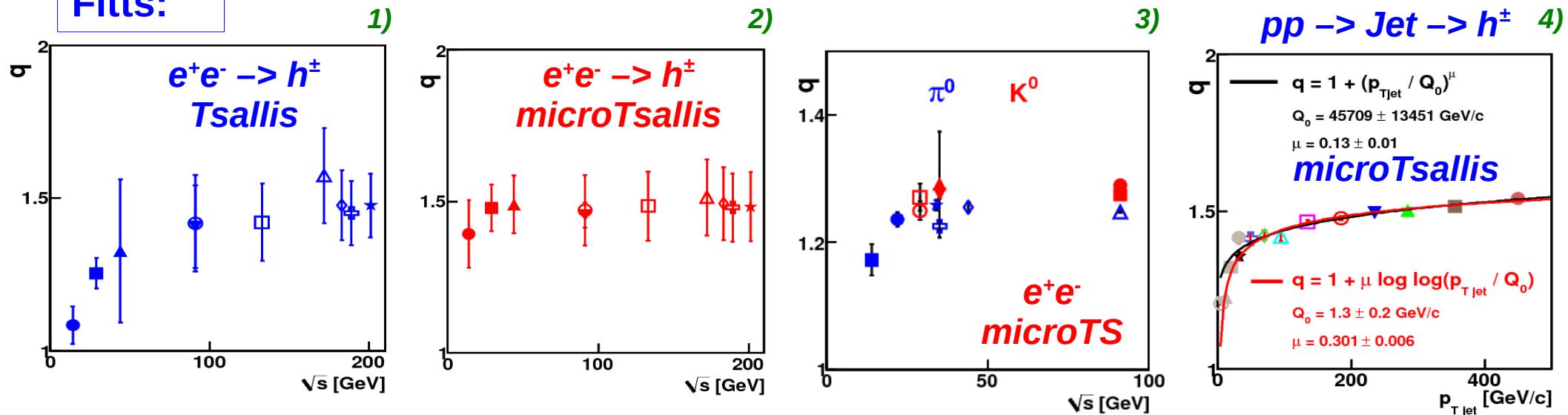
Charged hadrons from *diffractive eP* collisions in a frame co-moving with X



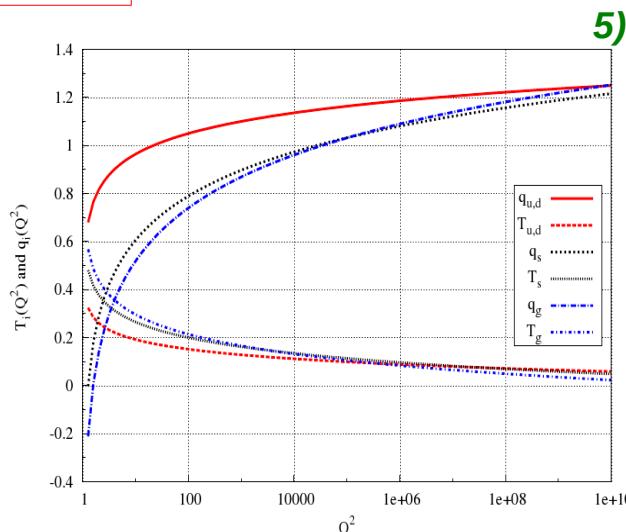
$$\frac{d\sigma}{x_F dx_F} \sim \left[1 + \frac{q-1}{\tau} x_F \right]^{-1/(q-1)}$$

Scale Evolution

Fits:



Theory: Scale evolution of q , T from fits to AKK Frag. Funcs:



$$D_{p_i}^{\pi^+}(z) \sim (1 + (q_i - 1)z/T_i)^{-1/(q_i - 1)}$$

$$q_i = q_{0i} + q_{1i} \ln(\ln(Q^2))$$

1-2) U.K. et al., *Phys.Lett. B*, **701** (2011) 111-116

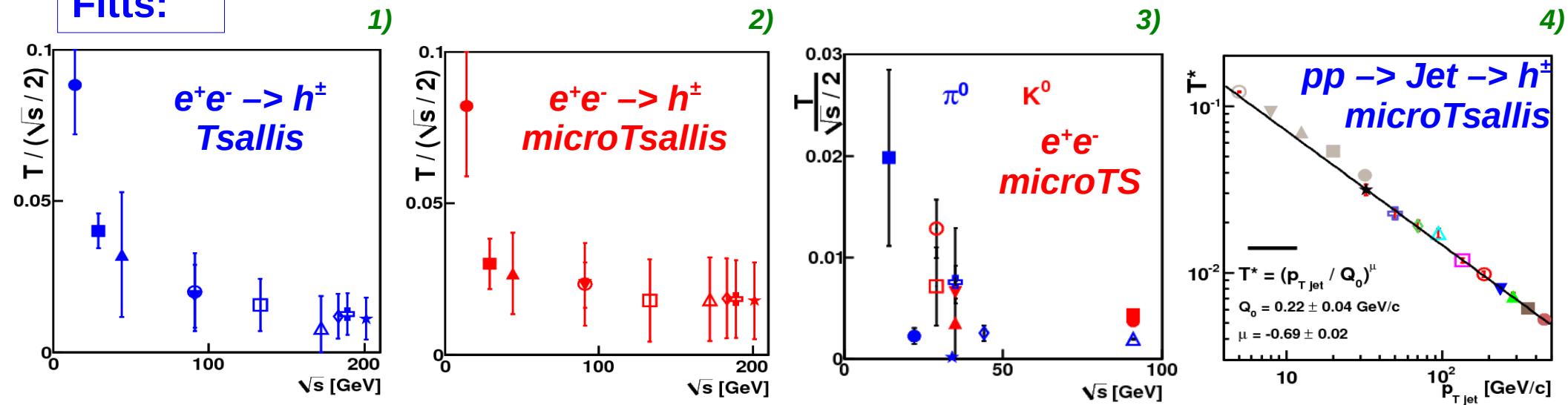
3) T. S. Biró et al., *Acta Phys.Polon. B*, **43** (2012) 811-820

4) U.K. et al., *Phys.Lett. B*, **718** (2012) 125-129

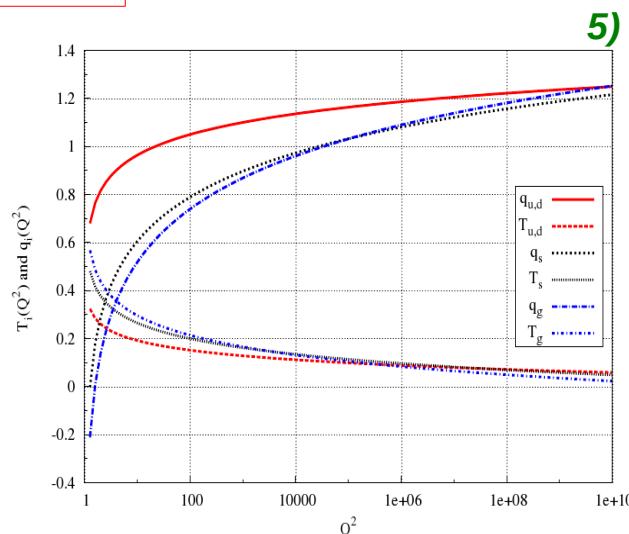
5) Barnaföldi et al., *Gribov-80 Conf: C10-05-26.1*, p.357-363

Scale Evolution

Fits:



Theory: Scale evolution of q , T from fits to AKK Frag. Funcs:



$$D_{p_i}^{\pi^+}(z) \sim (1 + (q_i - 1) z / T_i)^{-1/(q_i - 1)}$$

$$T_i = T_{0i} + T_{1i} \ln(\ln(Q^2))$$

1-2) U.K. et al., *Phys.Lett. B*, **701** (2011) 111-116

3) T. S. Biró et al., *Acta Phys.Polon. B*, **43** (2012) 811-820

4) U.K. et al., *Phys.Lett. B*, **718** (2012) 125-129

5) Barnaföldi et al., *Gribov-80 Conf: C10-05-26.1*, p.357-363