Two-body non-leptonic heavy-to-heavy decays at NNLO in QCD factorization approach

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based on **1606.02888**

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Outline

Why B physics:

- Motivation of B physics:

  - to test the CKM mechanism of CP violation, to search for NP signals beyond the SM;

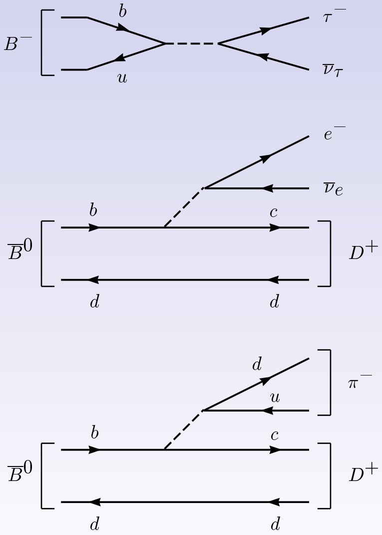
    $\hookrightarrow$ complementary to EWP tests @ (LEP, Tevatron) and direct NP searches @ (LHC)

  - to understand how quarks and gluons are confined into hadrons, i.e., the non-pert. aspects of QCD;

    $\hookrightarrow$ operator product expansion, QCD effective field theories, factorization theorems

- Three different classes: depending on the different final states, B-hadron weak decays can be divided into three classes:

  leptonic, semi-leptonic, $\boxed{\text{non-leptonic}}$



Simple quark-line diagrams

Non-leptonic B decays:

- Play a crucial role in testing and qualifying the CKM mechanism of quark flavour mixing:

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$

  - α: from time-dep. CP asym. in $B \to \pi\pi$, $\pi\rho$ and $\rho\rho$ decays;

$$(90.4^{+2.0}_{-1.0})^\circ$$

  - β: from $B \to J/\psi K_S$ and other charmonium modes;

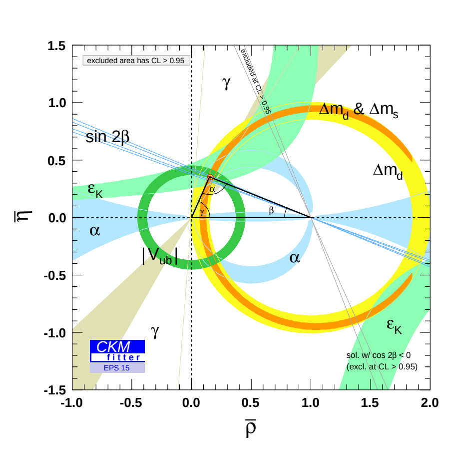
$$(22.62^{+0.44}_{-0.42})^\circ$$

  - γ: from $B \to DK$, $B \to K\pi\pi$, $B \to KKK$ decays;

$$(67.01^{+0.88}_{-1.99})^\circ$$

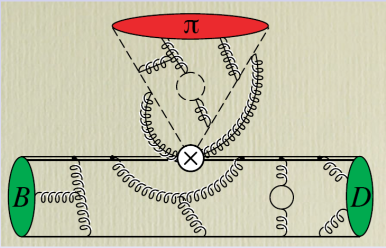
  - β_s: from $B_s \to J/\psi\phi$ and $B_s \to \phi\phi$ decays, $\cdots$;

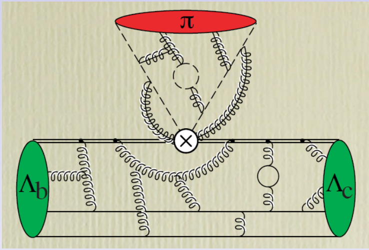
$$(0.01882^{+0.00036}_{-0.00042})\,rad$$



taken from CKMfitter group as of Summer 2015.

Why $\bar{B} \to D^{(*)+}L^-$ and $\Lambda_b \to \Lambda_c^+ L^-$ decays: Class-I





- At the quark-level: they are mediated by the weak decay $b \to c\bar{u}d(s)$, where b- and c-quark are massive and the light quarks massless;

- Physical picture relatively simpler: only current-current operators involved; spectator-scattering and annihilation effects power-suppressed; much simpler than $\bar{B} \to \pi^+\pi^-$;

- Exp. status: thanks to BaBar, Belle, Tevatron and LHCb, as well as future Belle-II, more data available and the precision further improved; [*HFAG, 1412.7515*]

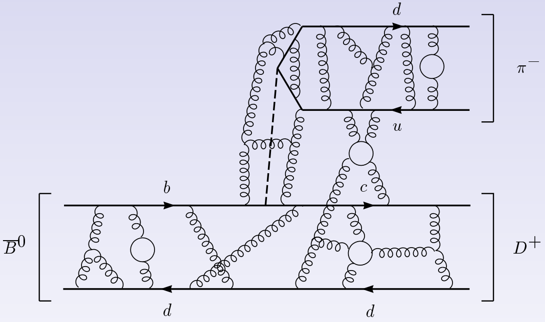
- To catch up with the precise exp. measurements, it is now very necessary and urgent to further improve the theoretical calculation! $\hookrightarrow$ this is our motivation for this project!

Difficulties in non-leptonic B decays:

- For a non-leptonic decay: both initial- and final-states are hadrons, involving very complicated QCD effect together with weak interaction, theoretically very difficult;

  $\hookrightarrow$the simplicity of weak interaction overshadowed by complex strong interaction!

$\bar{B}^0 \to D^+\pi^-$ decay:



- Non-leptonic B decay: a multi-scale problem with highly hierarchical interaction scales;

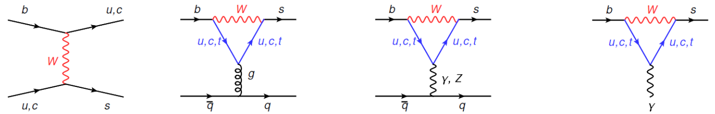
EW interaction scale	$\gg$	ext. mom'a in B rest frame	$\gg$	QCD-bound state effects
$m_W \sim 80$ GeV $m_Z \sim 91$ GeV	$\gg$	$m_b \sim 5$ GeV	$\gg$	$\Lambda_{\rm QCD} \sim 1$ GeV

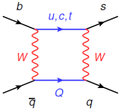
Effective weak Hamiltonion for non-leptonic B decays:

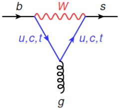
- The starting point: $\mathcal{L}_{\text{eff}}$ obtained by integrating out the heavy d.o.f. $(m_W, m_Z, m_t \gg m_b)$;
  [*BBL basis: Buras, Buchalla, Lautenbacher '96;* *CMM basis: Chetyrkin, Misiak, Münz '98*]

$$\mathcal{L}_{\text{eff}} \sim G_F\, V_{\text{CKM}} \times \Big[\sum_{p=u,c} \sum_{i=1,2} C_i\, \mathcal{O}_i^p + \sum_{3,\ldots,6} C_i\, \mathcal{O}_i + \sum_{7,\ldots,10} C_i\, \mathcal{O}_i + \sum_{7\gamma,8g} C_i\, \mathcal{O}_i \Big]$$

charged current | QCD-penguin | EW-penguin | electro- & chromo-mgn

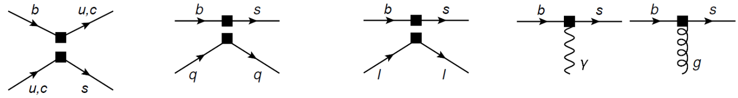






- C_i: containing physics above $\mu \sim m_b$;
  pert. calculable; NNLO program complete;
  [Buras, Buchalla, Lautenbacher '96; Gorbahn, Haisch '04]

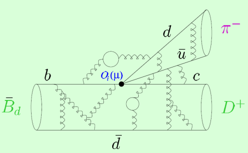
- $\mathcal{Q}_i$: local dim-6 operators; $\langle \mathcal{Q}_i \rangle$ containing physics below $\mu \sim m_b$;



Calculation of the hadronic matrix elements of Q_i:

- $\langle M_1 M_2 | \mathcal{Q}_i | \bar{B} \rangle$: depends on the spin and parity of $M_{1,2}$; final-state re-scattering determines strong phases, and hence direct CP asymmetries;

  $\hookrightarrow$ *still a multi-scale, strong-interaction problem!*



- Effective theories/Factorization theorem/Approximate symmetries of QCD/$\cdots$:

  express $\langle M_1 M_2 | Q_i | \bar{B} \rangle$ in terms of (few) universal non-perturbative hadronic quantities;

  PQCD, $\boxed{\text{QCDF}}$, SCET, LCSR, lattice QCD,

  Isospin, U-Spin, V-Spin, and flavour SU(3) symmetries, $\cdots$

- $\langle D^+ L^- | \mathcal{Q}_i | \bar{B} \rangle$ in QCDF: in the heavy-quark limit, it obeys the factorization formula [*BBNS'99-'04*]

$$\langle D^+ L^- | \mathcal{Q}_i | \bar{B} \rangle = \sum_j F_j^{B \to D}(m_L^2) \int_0^1 du\, T_{ij}(u) \Phi_L(u) + \mathcal{O}(1/m_b)$$

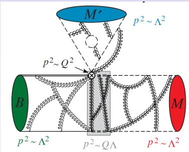
  - $F_j^{B \to D}$: $B \to D$ transition form factors; contains non-pert. long-distance effects;
  - Φ_L: the LCDA of the light meson; contains non-pert. long-distance effects;
  - T_{ij}: the hard-scattering kernels, perturbatively calculable order-by-order in α_s;

- QCDF: a systematic framework to all orders in α_s, but limited by $1/m_b$ corrections.

Factorization formula from the SCET point of view:

- SCET: an EFT of QCD designed to describe processes involving energetic hadrons/jets; [Bauer, Flemming, Pirjol, Stewart, '01; Beneke, Chapovsky, Diehl, Feldmann, '02; Becher, Broggio, Ferroglia '14]
- In a two-body $B \to MM'$ decay: relevant degrees of freedom including

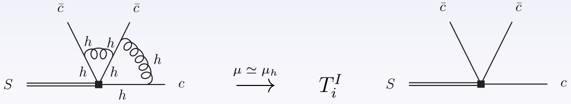
  - low-virtuality modes:
    - ⋆ HQET fields: $p - m_b v \sim \mathcal{O}(\Lambda)$
    - ⋆ soft spectators in B meson: $p_s^\mu \sim \Lambda \ll m_b, \quad p_s^2 \sim \mathcal{O}(\Lambda^2)$
    - ⋆ collinear quarks and gluons in pion: $E_c \sim m_b, \quad p_c^2 \sim \mathcal{O}(\Lambda^2)$
  - high-virtuality modes:
    - ⋆ hard modes: $(\text{heavy quark} + \text{collinear})^2 \sim \mathcal{O}(m_b^2)$
    - ⋆ hard-collinear modes: $(\text{soft} + \text{collinear})^2 \sim \mathcal{O}(m_b \Lambda)$



- In SCET, factorization established because various types of fields with differing kinematics decouple at the level of the $\mathcal{L}_{\text{tot}} = \mathcal{L}_n + \mathcal{L}_{\bar{n}} + \mathcal{L}_s$;

  [Bauer, Flemming, Pirjol, Stewart, '01; Beneke, Chapovsky, Diehl, Feldmann, '02; Becher, Broggio, Ferroglia '14]

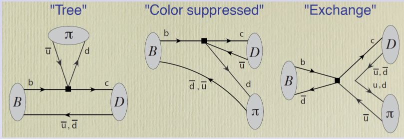
- For T_{ij}: perform the one-step matching from QCD onto $\text{SCET}_\text{I}(hc, c, s)$;

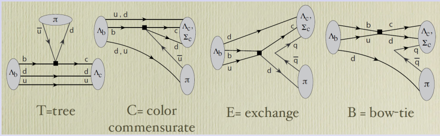


- SCET: field-theoretical basis for QCDF, equiv. to Feynman diagrammatic factorization;
  $\hookrightarrow$ SCET factorization is exactly the same as QCDF; [Beneke '15]

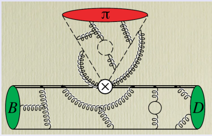
Features for heavy-light final states in QCDF:

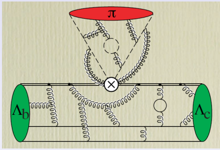
- Relevant Feynman diagrams for heavy-light final states:





- Only colour-allowed tree amplitude, no colour-suppressed tree nor penguin contributions;





- Only vertex kernels to T_{ij}, spectator-scattering and weak annihilation are power-suppressed;
  [*Beneke, Buchalla, Neubert, Sachrajda, '00; Bauer, Pirjol, Stewart, '01; Leibovich, Ligeti, Stewart, Wise, '03*]

- Factorization theorem well established in these class-I decays;
  [*Beneke, Buchalla, Neubert, Sachrajda, '00; Bauer, Pirjol, Stewart, '01; Leibovich, Ligeti, Stewart, Wise, '03*]

- Motivation for NNLO: NLO result colour-suppressed alongside with small WC; At NNLO colour suppression lifted and large WC re-enters; $\hookrightarrow$ how about the NNLO corrections?

Factorization formula for $\bar{B}_{(s)} \to D_{(s)}^{(*)+} L^-$:

- In the heavy-quark limit, the decay amplitude for $\bar{B}^0 \to D^+\pi^-$ is given by: [*BBNS, '00*]

$$\langle D^+\pi^-|Q_i|\bar{B}^0\rangle = \sum_j F_j^{B\to D}(m_\pi^2)\int_0^1 du\, T_{ij}^I(u)\phi_L(u)$$

- Demonstration of factorization based on Feynman diagrams at two-loop order: [*BBNS, '00*]

$$F^{(0)}_{B\to D}\cdot T^{(0)} * \Phi^{(0)}_\pi = A^{(0)}$$

$$F^{(0)}_{B\to D}\cdot T^{(1)} * \Phi^{(0)}_\pi = A^{(1)} - F^{(1)}_{B\to D}\cdot T^{(0)} * \Phi^{(0)}_\pi - F^{(0)}_{B\to D}\cdot T^{(0)} * \Phi^{(1)}_\pi$$

$$\begin{aligned} F^{(0)}_{B\to D}\cdot T^{(2)} * \Phi^{(0)}_\pi = A^{(2)} &- F^{(0)}_{B\to D}\cdot T^{(1)} * \Phi^{(1)}_\pi - F^{(1)}_{B\to D}\cdot T^{(1)} * \Phi^{(0)}_\pi \\ &- F^{(2)}_{B\to D}\cdot T^{(0)} * \Phi^{(0)}_\pi - F^{(0)}_{B\to D}\cdot T^{(0)} * \Phi^{(2)}_\pi - F^{(1)}_{B\to D}\cdot T^{(0)} * \Phi^{(1)}_\pi \end{aligned}$$

- Proof within SCET: factorization $\Leftrightarrow$ separation of scales and decoupling $\Leftrightarrow$ $Q_i = Q_c \times Q_s$ at the Langrangian level $\mathcal{L} = \mathcal{L}_c^0 + \mathcal{L}_s^0$; [*Bauer, Pirjol, Stewart, '01*]

$$\langle D\pi|(\bar{c}b)(\bar{u}d)|B\rangle = N\,\xi(v\cdot v')\int_0^1 dx\, T(x,\mu)\,\phi_\pi(x,\mu)$$

Universal functions:

$$\langle D^{(*)}|O_s|B\rangle = \xi(v\cdot v')$$

$$\langle \pi|O_c(x)|0\rangle = f_\pi \phi_\pi(x)$$

Calculate T, $\alpha_s(Q)$

$Q = E_\pi, m_b, m_c$

corrections will be $\Lambda/m_c \sim 30\%$

The operator basis in QCD and SCET:

- The relevant weak Hamiltonian: [*Buras, Buchalla, Lautenbacher '96; Chetyrkin, Misiak, Münz '98*]

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* \left(C_1 \mathcal{Q}_1 + C_2 \mathcal{Q}_2 \right) + \text{h.c.}$$

- CMM operator basis in full QCD:

$$\mathcal{Q}_1 = \bar{c}\gamma^{\mu}(1-\gamma_5)T^A b \; \bar{d}\gamma_{\mu}(1-\gamma_5)T^A u$$

$$\mathcal{Q}_2 = \bar{c}\gamma^{\mu}(1-\gamma_5) b \; \bar{d}\gamma_{\mu}(1-\gamma_5) u$$

$+$ four evanescent operators

- Evanescent operators in QCD: although vanish in 4-dim., but needed to complete the operator basis under renormalization! [Gorbahn, Haisch 04; Gorbahn, Haisch, Misiak 05]

- Nonlocal SCET operator basis:

$$\mathcal{O}_1 = \bar{\chi}\frac{\not{n}_-}{2}(1-\gamma_5)\chi \; \bar{h}_{v'}\not{n}_+(1-\gamma_5)h_v$$

$$\mathcal{O}_2 = \bar{\chi}\frac{\not{n}_-}{2}(1-\gamma_5)\gamma_\perp^\alpha\gamma_\perp^\beta\chi \; \bar{h}_{v'}\not{n}_+(1-\gamma_5)\gamma_{\perp\beta}\gamma_{\perp\alpha}h_v$$

$$\mathcal{O}_3 = \bar{\chi}\frac{\not{n}_-}{2}(1-\gamma_5)\gamma_\perp^\alpha\gamma_\perp^\beta\gamma_\perp^\gamma\gamma_\perp^\delta\chi \; \bar{h}_{v'}\not{n}_+(1-\gamma_5)\gamma_{\perp\delta}\gamma_{\perp\gamma}\gamma$$

$$\mathcal{O}_1' = \bar{\chi}\frac{\not{n}_-}{2}(1-\gamma_5)\chi \; \bar{h}_{v'}\not{n}_+(1+\gamma_5)h_v$$

$$\mathcal{O}_2' = \bar{\chi}\frac{\not{n}_-}{2}(1-\gamma_5)\gamma_\perp^\alpha\gamma_\perp^\beta\chi \; \bar{h}_{v'}\not{n}_+(1+\gamma_5)\gamma_{\perp\alpha}\gamma_{\perp\beta}h_v$$

$$\mathcal{O}_3' = \bar{\chi}\frac{\not{n}_-}{2}(1-\gamma_5)\gamma_\perp^\alpha\gamma_\perp^\beta\gamma_\perp^\gamma\gamma_\perp^\delta\chi \; \bar{h}_{v'}\not{n}_+(1+\gamma_5)\gamma_{\perp\alpha}\gamma_{\perp\beta}\gamma$$

- Express QCD matrix elements $\langle \mathcal{Q}_i \rangle$ as a linear combination of SCET ones $\langle \mathcal{O}_a^{(\prime)} \rangle$:

$$\langle \mathcal{Q}_i \rangle = \sum_{a=1}^{3} \left[H_{ia} \langle \mathcal{O}_a \rangle + H_{ia}' \langle \mathcal{O}_a' \rangle \right],$$

H_{ia} and H_{ia}' are the matching coefficients!

Matching calculation from QCD onto SCET$_{\mathrm{I}}$: I

- The matching formula from full QCD onto SCET: $\langle \mathcal{Q}_i \rangle = \sum_{a=1}^{3} \left[H_{ia} \langle \mathcal{O}_a \rangle + H'_{ia} \langle \mathcal{O}'_a \rangle \right]$

- Renormalized on-shell matrix elements $\langle \mathcal{Q}_i \rangle$ up to 2-loop order: in five-flavour theory!

$$\begin{aligned}
\langle \mathcal{Q}_i \rangle = \Big\{ & A_{ia}^{(0)} + \frac{\alpha_s}{4\pi} \left[A_{ia}^{(1)} + Z_{ext}^{(1)} A_{ia}^{(0)} + Z_{ij}^{(1)} A_{ja}^{(0)} \right] \\
& + \left(\frac{\alpha_s}{4\pi} \right)^2 \Big[A_{ia}^{(2)} + Z_{ij}^{(1)} A_{ja}^{(1)} + Z_{ij}^{(2)} A_{ja}^{(0)} + Z_{ext}^{(1)} A_{ia}^{(1)} + Z_{ext}^{(2)} A_{ia}^{(0)} + Z_{ext}^{(1)} Z_{ij}^{(1)} A_{ja}^{(0)} \\
& + (-i)\delta m_b^{(1)} A_{ia}^{*(1)} + (-i)\delta m_c^{(1)} A_{ia}^{**(1)} + Z_{\alpha}^{(1)} A_{ia}^{(1)} \Big] + \mathcal{O}(\alpha_s^3) \Big\} \langle \mathcal{O}_a \rangle^{(0)} \\
& + (A \leftrightarrow A') \langle \mathcal{O}'_a \rangle^{(0)}
\end{aligned}$$

- Renormalized on-shell matrix elements $\langle \mathcal{O}_a^{(\prime)} \rangle$ up to 2-loop order: in three-flavour theory!

$$\begin{aligned}
\langle \mathcal{O}_a \rangle = \Big\{ & \delta_{ab} + \frac{\hat{\alpha}_s}{4\pi} \left[M_{ab}^{(1)} + Y_{ext}^{(1)} \delta_{ab} + Y_{ab}^{(1)} \right] \\
& + \left(\frac{\hat{\alpha}_s}{4\pi} \right)^2 \Big[M_{ab}^{(2)} + Y_{ext}^{(1)} M_{ab}^{(1)} + Y_{ac}^{(1)} M_{cb}^{(1)} + \hat{Z}_{\alpha}^{(1)} M_{ab}^{(1)} + Y_{ext}^{(2)} \delta_{ab} \\
& + Y_{ext}^{(1)} Y_{ab}^{(1)} + Y_{ab}^{(2)} \Big] + \mathcal{O}(\hat{\alpha}_s^3) \Big\} \langle \mathcal{O}_b \rangle^{(0)} \\
= \Big\{ & \delta_{ab} + \frac{\hat{\alpha}_s}{4\pi} Y_{ab}^{(1)} + \left(\frac{\hat{\alpha}_s}{4\pi} \right)^2 Y_{ab}^{(2)} + \mathcal{O}\left(\hat{\alpha}_s^3 \right) \Big\} \langle \mathcal{O}_b \rangle^{(0)}
\end{aligned}$$

In the DR scheme, $Y_{ext} = 1$, and $M_{ab}^{(1)} = M_{ab}^{(2)} = 0$ because in SCET only scaleless integrals involved.

Matching calculation from QCD onto SCET$_{\rm I}$: II

- To extract T_{ij} from the matching procedure, introduce two factorized QCD operators:

$$Q^{(\prime)\mathrm{QCD}} = \left[\bar{q}\frac{\not{n}_-}{2}(1-\gamma_5)q\right]\left[\bar{c}\not{n}_+(1\mp\gamma_5)b\right] = C_{\bar{q}q}C_{FF}^{\rm D}O_1^{(\prime)} + C_{\bar{q}q}C_{FF}^{\rm ND}O_1^{(\prime)}$$

$$C_{\bar{q}q} = 1+\mathcal{O}(\alpha_s^2), \quad C_{FF}^{\rm D} = 1+\mathcal{O}(\alpha_s), \quad C_{FF}^{\rm ND} = \mathcal{O}(\alpha_s)$$

$\hookrightarrow$ their matrix element is the product of a light-meson LCDA and the full heavy-to-heavy form factor;

- The final matching formula from QCD onto SCET rewritten as:

$$\boxed{\langle \mathcal{Q}_i\rangle = T_i\langle\mathcal{O}^{\rm QCD}\rangle + T_i'\langle\mathcal{O}'^{\rm QCD}\rangle + \textstyle\sum_{a>1}\left[H_{ia}\langle\mathcal{O}_a\rangle + H_{ia}'\langle\mathcal{O}_a'\rangle\right]}$$

$$\hookrightarrow \quad \begin{pmatrix}\hat{T}_i\\ \hat{T}_i'\end{pmatrix} = \begin{pmatrix}C_{\bar{q}q}C_{FF}^{\rm D} & C_{\bar{q}q}C_{FF}^{\rm ND}\\ C_{\bar{q}q}C_{FF}^{\rm ND} & C_{\bar{q}q}C_{FF}^{\rm D}\end{pmatrix}^{-1}\begin{pmatrix}H_{i1}\\ H_{i1}'\end{pmatrix}$$

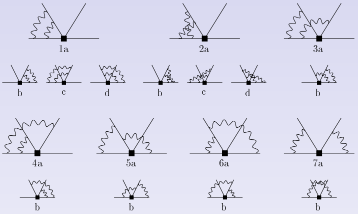
- Final master formulas for the hard scattering kernels:

$$T_i^{(0)} = A_{i1}^{(0)}, \qquad T_i^{(1)} = A_{i1}^{(1)nf} + Z_{ij}^{(1)}A_{j1}^{(0)}$$

$$T_i^{(2)} = A_{i1}^{(2)nf} + Z_{ij}^{(1)}A_{j1}^{(1)} + Z_{ij}^{(2)}A_{j1}^{(0)} + Z_{\alpha}^{(1)}A_{i1}^{(1)nf} - \hat{T}_i^{(1)}\left[C_{FF}^{\rm D(1)} + Y_{11}^{(1)} - Z_{ext}^{(1)}\right]$$
$$- C_{FF}^{\rm ND(1)}\hat{T}_i'^{(1)} + (-i)\delta m_b^{(1)}A_{i1}^{*(1)nf} + (-i)\delta m_c^{(1)}A_{i1}^{**(1)nf} - \sum_{b\neq 1}H_{ib}^{(1)}Y_{b1}^{(1)}$$

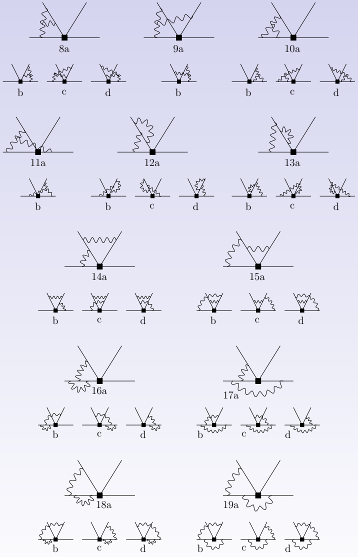
Explicit calculation of NNLO vertex corrections to T^I:

- Two-loop non-factorizable Feynman diagrams contributing to $A_{i1}^{(2)nf}$: [*BBNS '01*]



  - about 70 two-loop diagrams;
  - Laporta reduction based on IBP;
  - 39 new MIs and solved using DEs in a canonical basis;

- Both UV and IR div. cancelled analytically, thus factorization established!



Multi-loop calculations in a nutshell: I

- Adopt the DR scheme with $D = 4 - 2\epsilon$, to regulate both the UV and IR div.; at two-loop order, UV and IR poles appear up to $1/\epsilon^2$ and $1/\epsilon^4$, respectively.

- Basis strategy and procedure:

  - perform the general tensor reduction via Passarino-Veltman ansatz, $\Longrightarrow$thousands of scalar integrals, [*Passarino,Veltman '79*]

  - reduce them to Master Integrals via Laporta algorithm based on IBP identities $\Longrightarrow$totally 42 MIs, [*Tkachov '81; Chetyrkin,Tkachov '81; Laporta '01; Anastasiou,Lazopoulos '04*]

  - calculate these MIs, very challenging as we need analytical results.

- Techniques used to calculate MIs: developed very rapidly in recent years;

  - standard Feynman/Schwinger parameterisation, only for very simpler MIs;

  - method of differential equations; [*Kotikov '91; Remiddi '97; Henn '13*]

  - Mellin-Barnes techniques; [*Smirnov '99; Tausk '99*]

  - method of sector decomposition, for numerical check! [*Binoth, Heinrich 00*]

Calculate the MIs in a canonical basis:

- Besides the known ones, 39 new MIs found and computed based on the DE approach in a canonical basis; [*Huber, Kränkl '15*]

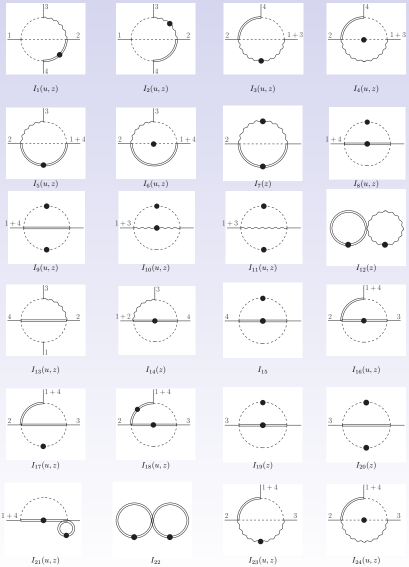
- Choose an "optimal" basis of MIs, so that the DEs decouple order-by-order in ϵ expansion, and the dependence of MIs on the kinematic variables is factorised from that on the ϵ: [*Henn, 1304.1806*]

$$\frac{\partial}{\partial x_m}\,\vec{M}(\epsilon, x_n) = \epsilon\, A_m(x_n)\,\vec{M}(\epsilon, x_n)$$

- The above simplified form of DEs trivial to solve in terms of iterated integrals; [*Bell, Huber '14*]

- Together with boundary conditions, analytic results of the MIs obtained in terms of generalised HPLs (or Goncharov polylogarithms); [*Maitre, 0703052*]

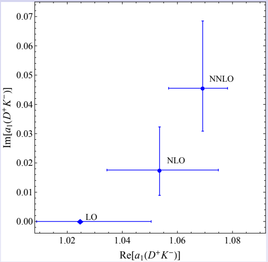
- The analytic results make it much easier to handel the threshold at $\bar{u}m_b^2 = 4m_c^2$ and the convolution integral $\int_0^1 du T^I(u)\phi(u)$; [*Bell, Beneke, Huber, Li '15*]



Predictions for $a_1(D^{(*)+}L^-)$:

- Convolution with the LCDA: $\boxed{a_1(D^+L^-) = \sum_{i=1}^{2} C_i(\mu) \int_0^1 du \left[\hat{T}_i(u,\mu) + \hat{T}_i'(u,\mu)\right] \Phi_L(u,\mu)}$

- Numerical results for $a_1(D^+K^-)$:



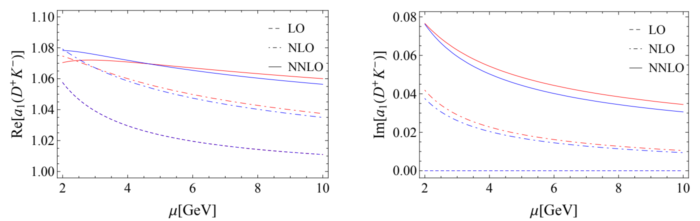
$$a_1(D^+K^-) = 1.025 + [0.029 + 0.018i]_{\rm NLO} + [0.016 + 0.028i]_{\rm NNLO}$$

$$= (1.069^{+0.009}_{-0.012}) + (0.046^{+0.023}_{-0.015})i$$

$\sim 2\%$ correction to real part, $\sim 60\%$ to imaginary part.

both the NLO and NNLO contribute constructively to the LO result.

- Dependence on μ and quark-mass scheme: pole (blue) and $\overline{\rm MS}$ running (red) for $m_{b,c}$;



Considerable stabilization for the real part, but less for the imaginary part.

Predictions for class-I decays:

- Brs ($\times 10^{-3}$ for $b \to c\bar{u}d$ and $\times 10^{-4}$ for $b \to c\bar{u}s$ transitions of $\bar{B}_{(s)} \to D_{(s)}^{(*)+} L^-$ decays:

Decay mode	LO	NLO	NNLO	Exp.
$\bar{B}_d \to D^+ \pi^-$	3.58	$3.79\,^{+0.44}_{-0.42}$	$3.93\,^{+0.43}_{-0.42}$	2.68 ± 0.13
$\bar{B}_d \to D^{*+} \pi^-$	3.15	$3.32\,^{+0.52}_{-0.49}$	$3.45\,^{+0.53}_{-0.50}$	2.76 ± 0.13
$\bar{B}_d \to D^+ \rho^-$	9.51	$10.06\,^{+1.25}_{-1.19}$	$10.42\,^{+1.24}_{-1.20}$	7.5 ± 1.2
$\bar{B}_d \to D^{*+} \rho^-$	8.45	$8.91\,^{+0.74}_{-0.71}$	$9.24\,^{+0.72}_{-0.71}$	6.0 ± 0.8
$\bar{B}_s \to D_s^+ \pi^-$	4.00	$4.24\,^{+1.32}_{-1.15}$	$4.39\,^{+1.36}_{-1.19}$	3.04 ± 0.23
$\bar{B}_s \to D_s^{*+} \pi^-$	2.05	$2.16\,^{+0.54}_{-0.49}$	$2.24\,^{+0.56}_{-0.50}$	2.0 ± 0.5
$\bar{B}_s \to D_s^+ \rho^-$	10.31	$10.91\,^{+3.46}_{-3.02}$	$11.30\,^{+3.56}_{-3.11}$	7.0 ± 1.5
$\bar{B}_s \to D_s^{*+} \rho^-$	5.86	$6.18\,^{+1.38}_{-1.28}$	$6.41\,^{+1.42}_{-1.31}$	10.2 ± 2.5
$\bar{B}_d \to D^+ K^-$	2.74	$2.90\,^{+0.33}_{-0.31}$	$3.01\,^{+0.32}_{-0.31}$	1.97 ± 0.21
$\bar{B}_d \to D^{*+} K^-$	2.37	$2.50\,^{+0.39}_{-0.36}$	$2.59\,^{+0.39}_{-0.37}$	2.14 ± 0.16
$\bar{B}_d \to D^+ K^{*-}$	4.79	$5.07\,^{+0.65}_{-0.62}$	$5.25\,^{+0.65}_{-0.63}$	4.5 ± 0.7
$\bar{B}_d \to D^{*+} K^{*-}$	4.30	$4.54\,^{+0.41}_{-0.40}$	$4.70\,^{+0.40}_{-0.39}$	–

- Our predictions generally come out higher than the exp. data, especially for $\bar{B}_d \to D^{(*)+}\pi^-$ and $\bar{B}_d \to D^{(*)+}\rho^-$;
- For $\bar{B}_s$ decays, our predictions still plagued by larger uncertainties from $B_s \to D_s^{(\prime)}$ transition form factors.

Test of factorization in class-I decays:

- Free from FFs uncertainties and particularly clean: [*Bjorken, '89; Neubert and Stech, '97*]

$$R_L^{(*)} \equiv \frac{\Gamma(\bar{B}_d \to D^{(*)+} L^-)}{d\Gamma(\bar{B}_d \to D^{(*)+} \ell^- \bar{\nu}_\ell)/dq^2 \mid_{q^2=m_L^2}} = 6\pi^2 \, |V_{ij}|^2 f_L^2 \, |a_1(D^{(*)+} L^-)|^2 \, X_L^{(*)}$$

$X_V = X_V^* = 1$ for a vector or axial-vector meson, for a pseudoscalar $X_L^{(*)}$ deviates from 1 below the percent level;

$\lvert a_1(D^{(*)+} L^-)\rvert$	LO	NLO	NNLO	Exp.
$\lvert a_1(D^+ \pi^-)\rvert$	1.025	$1.054^{+0.022}_{-0.020}$	$1.073^{+0.012}_{-0.014}$	0.89 ± 0.05
$\lvert a_1(D^{*+} \pi^-)\rvert$	1.025	$1.052^{+0.020}_{-0.018}$	$1.071^{+0.013}_{-0.014}$	0.96 ± 0.03
$\lvert a_1(D^+ \rho^-)\rvert$	1.025	$1.054^{+0.022}_{-0.019}$	$1.072^{+0.012}_{-0.014}$	0.91 ± 0.08
$\lvert a_1(D^{*+} \rho^-)\rvert$	1.025	$1.052^{+0.020}_{-0.018}$	$1.071^{+0.013}_{-0.014}$	0.86 ± 0.06
$\lvert a_1(D^+ K^-)\rvert$	1.025	$1.054^{+0.022}_{-0.019}$	$1.070^{+0.010}_{-0.013}$	0.87 ± 0.06
$\lvert a_1(D^{*+} K^-)\rvert$	1.025	$1.052^{+0.020}_{-0.018}$	$1.069^{+0.010}_{-0.013}$	0.97 ± 0.04
$\lvert a_1(D^+ K^{*-})\rvert$	1.025	$1.054^{+0.022}_{-0.019}$	$1.070^{+0.010}_{-0.013}$	0.99 ± 0.09
$\lvert a_1(D^+ a_1^-)\rvert$	1.025	$1.054^{+0.022}_{-0.019}$	$1.072^{+0.012}_{-0.014}$	0.76 ± 0.19

- Our predictions result in an essentially universal value of $|a_1(D^{(*)+} L^-)| \simeq 1.07\ (1.05)$ at NNLO (NLO), being consistently higher than the central values favoured by the current exp. data!

Test of factorization and SU(3)symmetry:

- Ratios of $\bar{B}_{d,s} \to D_{s,d}^{(*)+} L^-$ decay rates: [*Neubert, Stech, '97; Fleischer, Serra, Tuning, '04, '12*]

$$\mathcal{A}(\bar{B}_d^0 \to D^{(*)+}\pi^-) = \text{Tree} + \text{W-exchange}\,, \quad \mathcal{A}(\bar{B}_d^0 \to D^{(*)+}K^-) = \text{Tree}$$

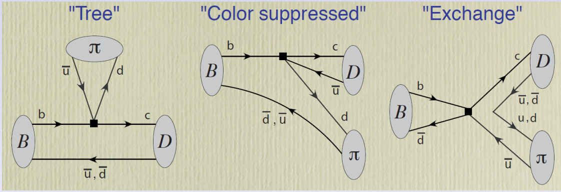
$\hookrightarrow$ useful to gain information on W-exchange contribution, to test factorization hypothesis and the $\mathrm{SU}(3)$ relations;

Ratios	LO	NLO	NNLO	Exp.
$\frac{\mathrm{Br}(\bar{B}_d \to D^+\rho^-)}{\mathrm{Br}(\bar{B}_d \to D^+\pi^-)}$	2.654	$2.653^{+0.163}_{-0.158}$	$2.653^{+0.163}_{-0.158}$	2.80 ± 0.47
$\frac{\mathrm{Br}(\bar{B}_d \to D^+K^{*-})}{\mathrm{Br}(\bar{B}_d \to D^{*+}K^-)}$	2.019	$2.026^{+0.404}_{-0.358}$	$2.023^{+0.403}_{-0.358}$	2.103 ± 0.363
$\frac{\mathrm{Br}(\bar{B}_d \to D^+K^-)}{\mathrm{Br}(\bar{B}_d \to D^+\pi^-)}$	0.077	$0.077^{+0.002}_{-0.002}$	$0.077^{+0.002}_{-0.002}$	0.074 ± 0.009
$\frac{\mathrm{Br}(\bar{B}_d \to D^{*+}K^-)}{\mathrm{Br}(\bar{B}_d \to D^{*+}\pi^-)}$	0.075	$0.075^{+0.002}_{-0.002}$	$0.075^{+0.002}_{-0.002}$	0.078 ± 0.007
$\frac{\mathrm{Br}(\bar{B}_s \to D_s^+\pi^-)}{\mathrm{Br}(\bar{B}_d \to D^+K^-)}$	14.67	$14.67^{+1.34}_{-1.28}$	$14.67^{+1.34}_{-1.28}$	15.43 ± 2.02
$\frac{\mathrm{Br}(\bar{B}_s \to D_s^+\pi^-)}{\mathrm{Br}(\bar{B}_d \to D^+\pi^-)}$	1.120	$1.120^{+0.109}_{-0.104}$	$1.120^{+0.109}_{-0.104}$	1.134 ± 0.102

- General consistency indicates small impact of the W-exchange topology and of nonfac. $\mathrm{SU}(3)$-breaking effects!
- With LQCD for $B_{(s)} \to D_{(s)}$ FFs, the last two allow precise measurement of fragmentation functions f_s/f_d!

Comments on the power correction in class-I decays:

- There exist power-suppressed corrections from spectator-scattering and W-exchange annihilation:



- Our findings: our predictions for non-lep. to semi-lep ratios larger than the data, while for non-lep. ratios agree well with data;

- Possibility I: non-negligible power correction stemming from spectator-scattering and W-exchange annihilation that is negative in sign and 10 – 15% in size on the amplitude level;

  $\hookrightarrow$ render the factorization test via non-lep. to semi-lep ratios better, but cancel out in the non-lep. ratios;

- Possibility II: to reduce the values of $|V_{cb}| \times$ FFs by $\sim 10\%$;

  $\hookrightarrow$ render the Brs close to the current data, while keep the non-lep. ratios unchanged;

Predictions for $\Lambda_b \to \Lambda_c^+ L^-$ decays:

- At the LHC, Λ_b production constitutes $\sim 20\%$ of b-hadrons; [*LHCb, arXiv:1111.2357*]
- Due to $S = \frac{1}{2}$, its decays complementary to B-meson decays; $\hookrightarrow$ a new testing ground for different QCD models and factorization assumptions used in B-meson case.

Decay mode	LO	NLO	NNLO	Exp.
$\Lambda_b \to \Lambda_c^+ \pi^-$	2.60	$2.75\,^{+0.53}_{-0.53}$	$2.85\,^{+0.54}_{-0.54}$	$4.30\,^{+0.36}_{-0.35}$
$\bar{B}_d \to D^+ \pi^-$	3.58	$3.79\,^{+0.44}_{-0.42}$	$3.93\,^{+0.43}_{-0.42}$	2.68 ± 0.13
$\Lambda_b \to \Lambda_c^+ K^-$	2.02	$2.14\,^{+0.40}_{-0.39}$	$2.21\,^{+0.40}_{-0.40}$	3.42 ± 0.33
$\bar{B}_d \to D^+ K^-$	2.74	$2.90\,^{+0.33}_{-0.31}$	$3.01\,^{+0.32}_{-0.31}$	1.97 ± 0.21
$\frac{\mathrm{Br}(\Lambda_b \to \Lambda_c^+ \mu^- \bar{\nu})}{\mathrm{Br}(\Lambda_b \to \Lambda_c^+ \pi^-)}$	18.88	$17.87\,^{+2.31}_{-2.33}$	$17.25\,^{+2.19}_{-2.18}$	$16.6\,^{+4.1}_{-4.7}$
$\frac{\mathrm{Br}(\Lambda_b \to \Lambda_c^+ K^-)}{\mathrm{Br}(\Lambda_b \to \Lambda_c^+ \pi^-)}$ (%)	7.77	$7.77\,^{+0.19}_{-0.18}$	$7.77\,^{+0.19}_{-0.18}$	7.31 ± 0.23
$\frac{\mathrm{Br}(\Lambda_b \to \Lambda_c^+ \pi^-)}{\mathrm{Br}(\bar{B}_d \to D^+ \pi^-)}$	0.73	$0.73\,^{+0.16}_{-0.15}$	$0.73\,^{+0.16}_{-0.15}$	3.3 ± 1.2

- For mesonic decays, larger than data, but for baryonic decays, lower than data, and NNLO has a right directions!
- From the ratios, non-fact. effects should be small in these Λ_b decays;

Conclusion and outlook

- In QCDF/SCET framework, the 2-loop vertex corrections to colour-allowed tree topology a_1 for class-I decays $\bar{B}_{(s)} \to D_{(s)}^{(*)+} L^-$ and $\Lambda_b \to \Lambda_c^+ L^-$ were calculate;
- For the colour-allowed tree amplitude a_1, the NNLO contributions yield a positive shift, sizable for its imaginary part, but small for its real part and its magnitude;
- The dependence on μ gets reduced for the real part, but does not occur for the imaginary part; a quasi-universal $|a_1|$ is predicted in QCDF even up to the NNLO accuracy;
- For $\bar{B}_d$ decays, the central values are in general higher compared to the exp. data; For $\bar{B}_s$ decays, our predictions are still plagued by large uncertainties from form factors;
- For the baryonic decays, our predictions turn out to be $20-30\%$ smaller than the exp. data; Interesting to understand the reason for this difference in the $\bar{B}_d$ and the Λ_b decays;
- $\Lambda_b \to \Lambda_c^+ L^-$ decays provide another testing ground for different QCD models and factorization assumptions used in B-meson case;

谢 谢 大 家!