

RGE improved analysis of $B \rightarrow \pi$ form factors from Light-Cone Sum Rules

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Based on work collaborated with

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arXiv:1607.08727

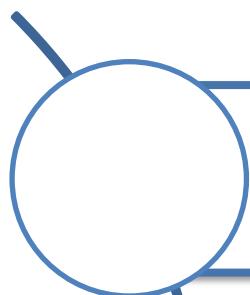
Motivation

B-meson LCSR

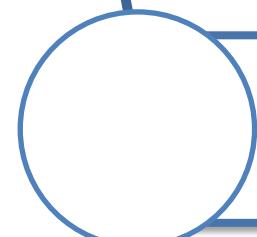
Numerical results

Conclusion

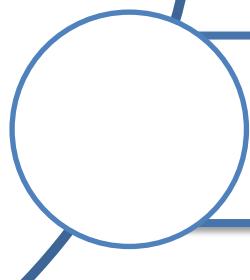
Motivation



**Semi-leptonic (*Anomaly, New Physics*)
and non-leptonic decay**



Determine CKM parameters



**Double check with π -meson LCSR,
full NLL resummation**

B-meson LCSR

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➤ Light-cone sum rules

P. Ball, J. High Energy Phys, 09 (1998) 005

A. Khodjamirian, T. Mannel, and N. Offen, Phys. Rev. D 75, 054013 (2007)

➤ PQCD approach

H. n. Li, Y. L. Shen, and Y. M. Wang, Phys. Rev. D 85, 074004 (2012)

C. D. Lü and M. Z. Yang, Eur. Phys. J. C 28, 515 (2003)

➤ Light-cone quark model

C. D. Lü, W. Wang, and Z. T. Wei, Phys. Rev. D 76, 014014 (2007)

➤ Covariant light front approach

H. Y. Cheng, C. K. Chua, and C. W. Hwang, Phys. Rev. D 69, 074025 (2004)

➤ Lattice QCD

E. Dalgic, A. Gray, M. Wingate, C. T. H. Davies, G. P. Lepage, and J. Shigemitsu, Phys. Rev. D 73, 074502 (2006); Phys. Rev. D 75, 119906 (2007)

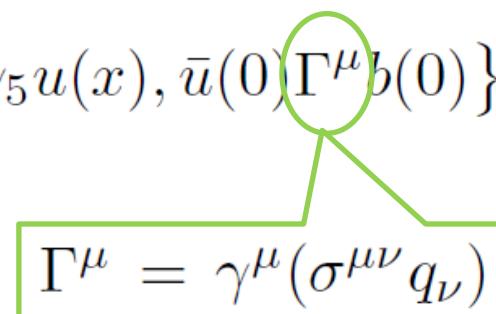
B-meson LCSR

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- QCD light-cone sum rules with B-meson distribution amplitudes
- Vacuum-to-B-meson correlation function defined with an interpolating current for pion

$$\Pi^\mu(p, q) = i \int d^4x e^{ip \cdot x} \langle 0 | T \{ \bar{d}(x) \gamma \not{v}_5 u(x), \bar{u}(0) \Gamma^\mu b(0) \} | \bar{B}(P_B) \rangle$$

- ◆ Hadronic level
- ◆ Partonic level


$$\Gamma^\mu = \gamma^\mu (\sigma^{\mu\nu} q_\nu)$$

Hadronic level

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π state

$$\Pi^\mu(p, q) = i \int d^4x e^{ip \cdot x} \langle 0 | T \{ \bar{d}(x) \gamma \not{v} \gamma_5 u(x), \bar{u}(0) \Gamma^\mu b(0) \} | \bar{B}(P_B) \rangle$$

vector form factors

$$\langle \pi(p) | \bar{u} \gamma_\mu b | \bar{B}(p_B) \rangle = f_{B\pi}^+(q^2) \left[p_B + p - \frac{m_B^2 - m_\pi^2}{q^2} q \right]_u + f_{B\pi}^0(q^2) \frac{m_B^2 - m_\pi^2}{q^2} q_\mu$$

$$\langle \pi(p) | \bar{d} \not{v} \gamma_5 u | 0 \rangle = -i n \cdot p f_\pi$$

$$\langle \pi(p) | \bar{q} \sigma^{\mu\nu} q_\nu b | B(p_B) \rangle = \frac{i f_{B\pi}^T(q^2)}{m_B + m_\pi} [(p_B + p)_\mu q^\nu - (m_B^2 - m_\pi^2) q^\mu]$$

tensor form factor

$$\Pi_T(p, q) = \frac{i(n \cdot p)^2 f_\pi f_{B\pi}^T(q^2)}{2(m_\pi^2 - p^2)} + \int_{\omega_s}^{\infty} d\omega \frac{\rho_h(\omega)}{\omega - \bar{n} \cdot p - i\epsilon}$$

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Partonic level

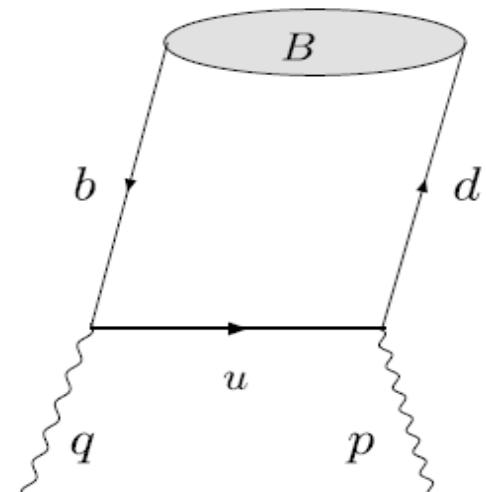
leading order

π momentum: p

$$n \cdot p \simeq \frac{m_B^2 + m_\pi^2 - q^2}{m_B} = 2E_\pi$$

$$\bar{n} \cdot p \sim \mathcal{O}(\Lambda_{\text{QCD}})$$

$$\begin{aligned} n^2 &= \bar{n}^2 = 0 \\ n \cdot \bar{n} &= 2 \end{aligned}$$



$$\Pi_\mu(p, q) = \Pi_T(n \cdot p, \bar{n} \cdot p) \epsilon^{\mu\nu} q_\nu$$

$$\epsilon^{\mu\nu} = (n^\mu \bar{n}^\nu - \bar{n}^\nu n^\mu)/2$$

$$\Pi_T(n \cdot p, \bar{n} \cdot p) = \tilde{f}_B(\mu) m_B \int_0^\infty d\omega' \frac{\phi_B^-(\omega')}{\omega' - \bar{n} \cdot p - i0} + \mathcal{O}(\alpha_s)$$

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$$\Pi_T^{hardonic} = \Pi_T^{partonic}$$

Quark-hadron duality

Dispersion relation

Borel transformation

$$f_{B\pi}^T(q^2) = \frac{f_B(m_B + m_\pi)}{n \cdot p f_\pi} e^{m_\pi^2 / (n \cdot p \omega_M)} \int_0^{\omega_s} d\omega e^{-\omega/\omega_M} \phi_-^B(\omega)$$

$$f_{B\pi}^+(q^2) = \frac{m_B}{n \cdot p} f_{B\pi}^0(q^2) = \frac{m_B}{m_B + m_\pi} f_{B\pi}^T(q^2)$$

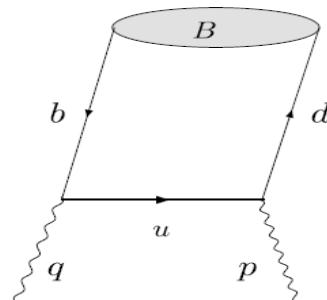
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◆ Diagrammatic factorization method

$$\begin{aligned}\Pi_{\mu, b\bar{d}} &= \Pi_{\mu, b\bar{d}}^{(0)} + \Pi_{\mu, b\bar{d}}^{(1)} + \dots = \Phi_{b\bar{d}} \otimes T \\ &= \Phi_{b\bar{d}}^{(0)} \otimes T^{(0)} + \left[\Phi_{b\bar{d}}^{(0)} \otimes T^{(1)} + \Phi_{b\bar{d}}^{(1)} \otimes T^{(0)} \right] + \dots\end{aligned}$$

DA of B-meson



Short-distance function

$$\Phi_{b\bar{d}}^{(0)} \otimes T^{(1)} = \Pi_{\mu, b\bar{d}}^{(1)} - \Phi_{b\bar{d}}^{(1)} \otimes T^{(0)}$$

Y. M. Wang and Y. L. Shen, Nucl. Phys. B **898** (2015) 563

◆ Soft cancellation

$$\Pi_{\mu, b\bar{d}} = \Phi_{b\bar{d}} \otimes T = C \cdot J \otimes \Phi_{b\bar{d}}$$

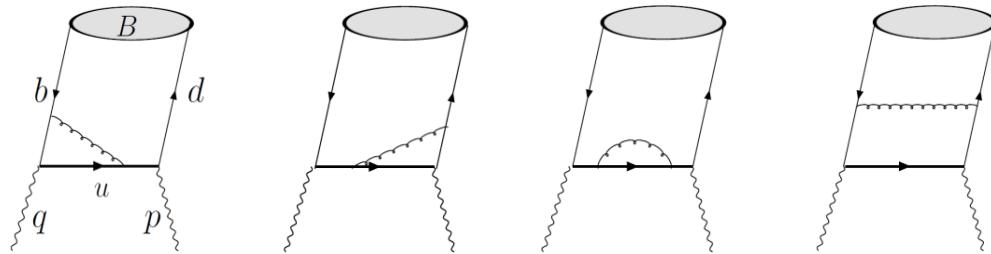
Hard region

Hard-collinear region

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NLO



method of region

- Simply calculation
- leading contribution regions are related to the external line quark momentum

Soft cancellation

- only concentrate on hard and/or hard-collinear regions

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Factorized formula of correlation function

$$\begin{aligned}\Pi_T(n \cdot p, \mu) = & \tilde{f}_B(\mu) m_B \sum_{k=\pm} C^{(k)}(n \cdot p, \mu) \\ & \times \int_0^\infty \frac{d\omega}{\omega - \bar{n} \cdot p} J^{(k)} \left(\frac{\mu^2}{n \cdot p \omega}, \frac{\omega}{\bar{n} \cdot p} \right) \phi_B^{(k)}(\omega, \mu)\end{aligned}$$

- Resummation of **hard function**, B-meson decay constant and **inverse moment of B-meson DAs.**
- Resummation of **hard function**, B-meson decay constant, **jet function** and **B-meson DAs.**

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RGE evolution

Hard function of tensor form factor is different from that of vector form factor

Renormalization scale

Factorization scale

$$C^{(-)}(n \cdot p, \mu, \nu) = C^{(-)}(n \cdot p, \mu) + \delta C^{(-)}(n \cdot p, \mu, \nu) \quad \delta C^{(-)}(n \cdot p, \mu, \nu) = -\frac{\alpha_s C_F}{2\pi} \ln \frac{\nu}{\mu}$$

$$\frac{d}{d \ln \mu} C^{(-)}(n \cdot p, \mu, \nu) = \Gamma_c(\mu) C^{(-)}(n \cdot p, \mu)$$

$$\frac{d}{d \ln \nu} C^{(-)}(n \cdot p, \mu, \nu) = \gamma_T(\alpha_s) C^{(-)}(n \cdot p, \mu, \nu)$$



$$C^{(-)}(n \cdot p, \mu, \nu) = e^{\int_{m_b}^{\nu} \frac{d\nu'}{\nu'} \gamma_T(\alpha_s)} C^{(-)}(n \cdot p, \mu, m_b)$$

$$C^{(-)}(n \cdot p, \mu) = U_1(n \cdot p, \mu_{h1}, \mu) C^{(-)}(n \cdot p, \mu_{h1})$$

Operator renormalization of tensor current

The ν dependent of the form factor must be cancelled by that of Wilson coefficient of tensor current

M. Beneke and J. Rohrwild, Eur. Phys. J. C **71** (2001) 1818

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$$\begin{aligned} \frac{d}{d \ln \mu} \phi_B^-(\omega, \mu) = & - \left[\Gamma_{\text{cusp}}(\alpha_s) \ln \frac{\mu}{\omega} + \gamma_+(\alpha_s) \right] \phi_B^-(\omega, \mu) \\ & - \int_0^\infty d\omega' \omega \Gamma(\omega, \omega', \mu) \phi_B^-(\omega', \mu) \end{aligned}$$

**The evolution kernel is not diagonal,
we can translated it into the “dual”
space with the integration kernel
being the eigenfunction of the
evolution function of the wave
function**

G. Bell, T. Feldmann, Y. M. Wang and M. W. Y. Yip,
JHEP **1311** (2013) 191

$$\rho_B^-(\omega', \mu) = \int_0^\infty \frac{d\omega}{\omega'} J_0(2\sqrt{\frac{\omega}{\omega'}}) \phi_B^-(\omega, \mu)$$

$$\frac{d}{d \ln \mu} \rho_B^-(\omega', \mu) = \Gamma_\rho(\mu) \rho_B^-(\omega', \mu)$$

$$\frac{d}{d \ln \mu} j(\hat{\omega}', \mu) = \Gamma_j j(\hat{\omega}', \mu) \quad \Pi_T(\mu, n \cdot p) = \tilde{f}_B(\mu) m_B \sum_{k=\pm} C^{(k)}(n \cdot p, \mu)$$

$$\begin{aligned} \frac{d}{d \ln \mu} \tilde{f}_B(\mu) = & \tilde{\gamma}(\alpha_s) \tilde{f}_B(\mu) \\ & \times \int_0^\infty \frac{d\omega'}{\omega'} j^{(k)} \left(\frac{\mu^2}{n \cdot p \hat{\omega}'}, \frac{\hat{\omega}'}{\bar{n} \cdot p} \right) \rho_B^{(k)}(\omega', \mu) \end{aligned}$$

B-meson LCSR

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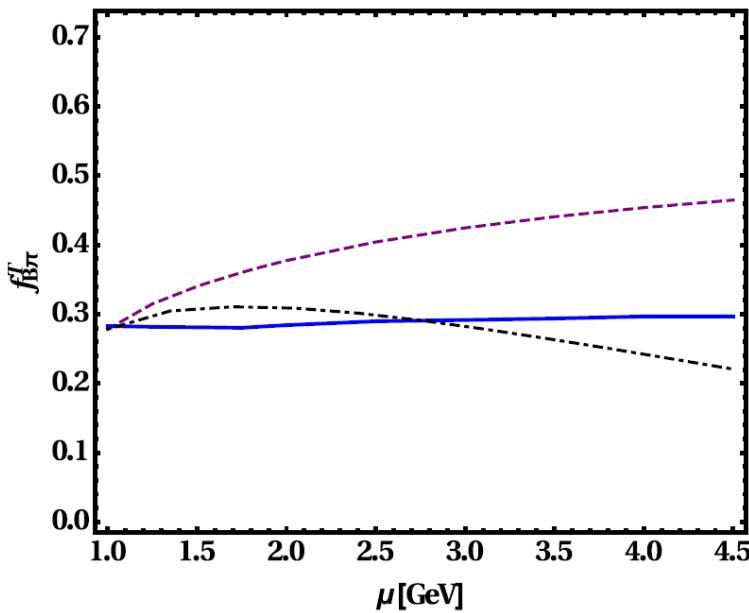
Quark-hadron duality

Dispersion relation

Borel transformation

$$\begin{aligned}
 f_\pi e^{-m_\pi^2 n \cdot p / \omega_M^2} f_{B\pi}^T(q^2) = & U_2(\mu_{h2}, \mu) \tilde{f}_B(\mu_{h2}) \int_0^{\omega_s} d\omega e^{-\omega/\omega_M} \left[\phi_{B,\text{eff}}^+(\omega) \right. \\
 & \left. + U_1(n \cdot p, \mu_{h1}, \mu) C^{(-)}(n \cdot p, \mu_{h1}) \rho_{B,\text{eff}}^-(\omega) \right] \\
 f_\pi e^{-m_\pi^2 / (n \cdot p \omega_M)} & \left\{ \frac{n \cdot p}{m_B} f_{B\pi}^+(q^2), f_{B\pi}^0(q^2) \right\} \\
 = & \left[U_2(\mu_{h2}, \mu) \tilde{f}_B(\mu_{h2}) \right] \int_0^{\omega_s} d\omega' e^{-\omega'/\omega_M} \left[r \phi_{B,\text{eff}}^+(\omega', \mu) \right. \\
 & \left. + \left[U_1(n \cdot p, \mu_{h1}, \mu) \tilde{C}^{(-)}(n \cdot p, \mu_{h1}) \right] \rho_{B,\text{eff}}^-(\omega', \mu) \right. \\
 & \left. \pm \frac{n \cdot p - m_B}{m_B} \left(\phi_{B,\text{eff}}^+(\omega', \mu) + C^{(-)}(n \cdot p, \mu) \phi_B^-(\omega', \mu) \right) \right]
 \end{aligned}$$

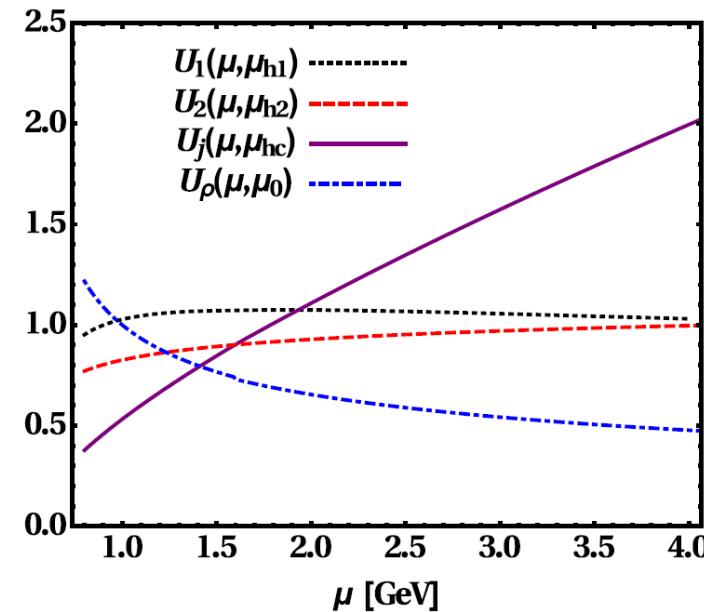
Numerical results



Evolution function cancellation between jet function and wave function evolution effect

Scale dependence of tensor form factor

blue solid line: complete RGE evolution
purple dashed line: RGE evolution only performed for hard function and f_B
Black dot-dashed line: additional evolution of B-meson DA inverse moment

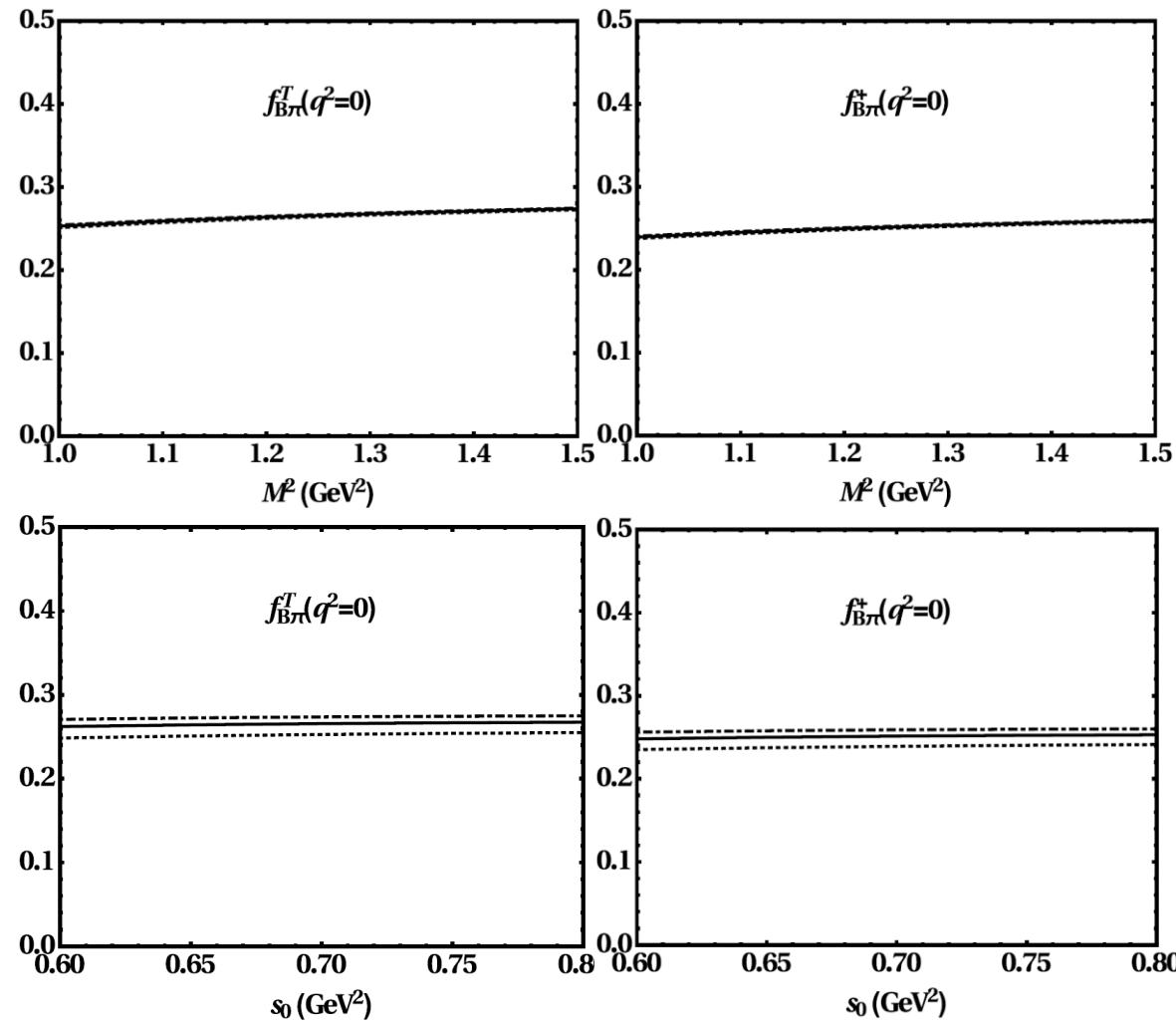


Numerical results

Up panel: dotted, solid and dot-dashed curve correspond to $S_0 = 0.65(\text{GeV}^2), 0.70(\text{GeV}^2), 0.75(\text{GeV}^2)$ respectively

Down panel: dotted, solid and dot-dashed curve correspond to $M^2 = 1.0(\text{GeV}^2), 1.25(\text{GeV}^2), 1.5(\text{GeV}^2)$ respectively

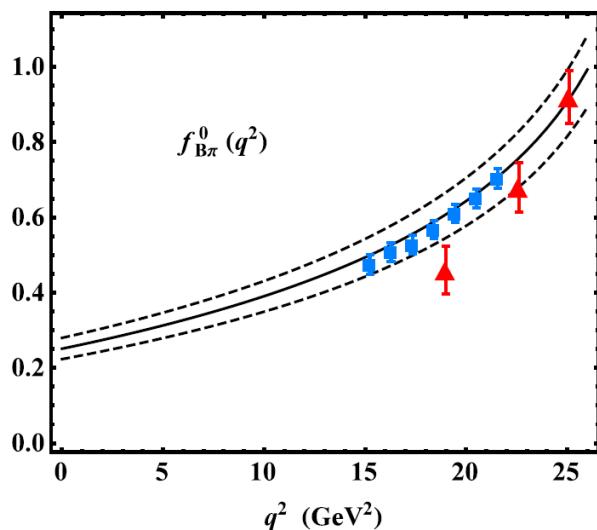
Parameter dependence



Numerical results

$$f_{B\pi}^T(0) = 0.267$$

$$f_{B\pi}^+(0) = f_{B\pi}^0(0) = 0.251$$

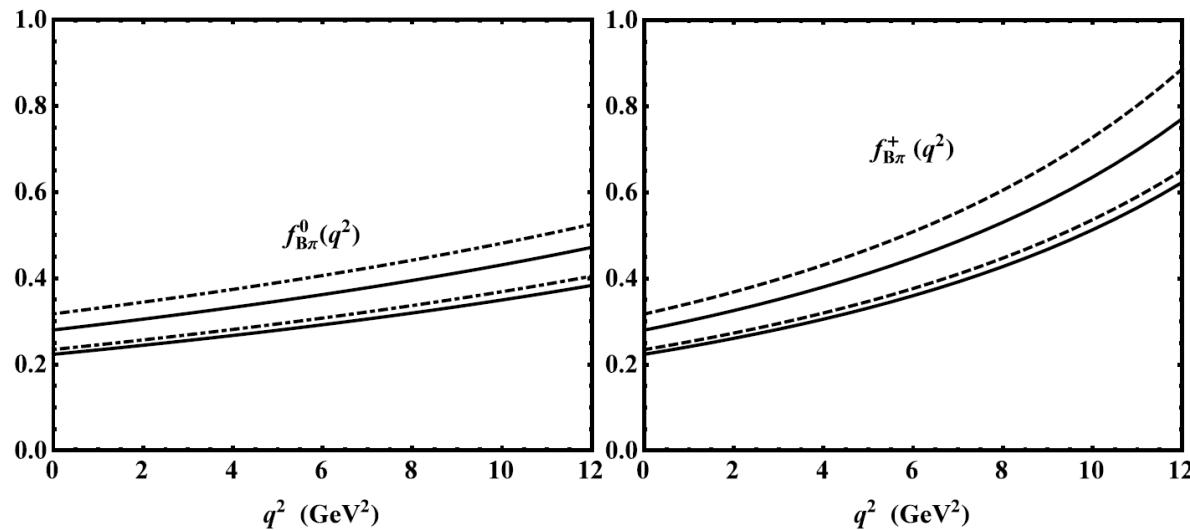


Black curves: our results

Blue squares: Lattice date from HPQCD Collaboration

Red triangles: Lattice date from RBC/UKQCD Collaborations

q^2 dependence: z parameter fit



Solid curves: our results

Dot-dashed curves: results of Y. M.
Wang and Y. L. Shen, Nucl. Phys. B 898
(2015) 563

Conclusion CONCLUSION

Thanks !

Method of region

- Transparent **separation of different leading regions** of one loop corrections

Complete RGE evolution

- Complete RGE evolution to the factorized correlation functions, solve the RGE of **B-meson DA** and **jet function** in “dual” space

Numerical results

- Check the behavior of each evolution kernel, illustrate nearly **scale independent** of the form factors, and compare with previous result

B-meson LCSR

Factorized formula of correlation function

$$\Pi_T(n \cdot p, \mu) = \tilde{f}_B(\mu) m_B \sum_{k=\pm} C^{(k)}(n \cdot p, \mu) \times \int_0^\infty \frac{d\omega}{\omega - \bar{n} \cdot p} J^{(k)} \left(\frac{\mu^2}{n \cdot p \omega}, \frac{\omega}{\bar{n} \cdot p} \right) \phi_B^{(k)}(\omega, \mu)$$

$$C^{(+)} = 1,$$

$$C^{(-)} = 1 - \frac{\alpha_s C_F}{4\pi} \left[2 \ln^2 \frac{\mu}{n \cdot p} + 7 \ln \frac{\mu}{m_b} - \ln^2 r - 2 \text{Li}_2 \left(-\frac{\bar{r}}{r} \right) - \frac{4r-2}{r-1} \ln r - \frac{\pi^2}{12} + 6 \right]$$

$$J^{(+)} = \frac{\alpha_s C_F}{4\pi} (1 + \eta) \ln(1 + \eta),$$

$$J^{(-)} = 1 + \frac{\alpha_s C_F}{4\pi} \left[\ln^2 \frac{\mu^2}{n \cdot p (\omega - \bar{n} \cdot p)} - 2 \ln \frac{\bar{n} \cdot p - \omega}{\bar{n} \cdot p} \ln \frac{\mu^2}{n \cdot p (\omega - \bar{n} \cdot p)} - \ln^2 \frac{\bar{n} \cdot p - \omega}{\bar{n} \cdot p} - \left(1 + \frac{2\bar{n} \cdot p}{\omega} \right) \ln \frac{\bar{n} \cdot p - \omega}{\bar{n} \cdot p} - \frac{\pi^2}{6} - 1 \right].$$

hard region
contribution

hard-collinear region
contribution

B-meson LCSR

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Quark-hadron duality

Dispersion relation

Borel transformation

$$\phi_{B,\text{eff}}^+(\Omega, \mu) = \frac{\alpha_s C_F}{4\pi} \int_\Omega^\infty \frac{d\omega}{\omega} \phi_B^+(\omega, \mu)$$

$$f_\pi e^{-m_\pi^2 n \cdot p / \omega_M^2} f_{B\pi}^T(q^2) = U_2(\mu_{h2}, \mu) \tilde{f}_B(\mu_{h2}) \int_0^{\omega_s} d\omega e^{-\omega/\omega_M} \left[\phi_{B,\text{eff}}^+(\omega) \right]$$

$$+ U_1(n \cdot p, \mu_{h1}, \mu) C^{(-)}(n \cdot p, \mu_{h1}) \rho_{eff}^-(\omega) \Big]$$

$$f_\pi e^{-m_\pi^2 / (n \cdot p \omega_M)} \left\{ \frac{n \cdot p}{m_B} f_{B\pi}^+(q^2), f_{B\pi}^0(q^2) \right\}$$

$$= \left[U_2(\mu_{h2}, \mu) \tilde{f}_B(\mu_{h2}) \right] \int_0^{\omega_s} d\omega' e^{-\omega'/\omega_M} \left[r \phi_{B,\text{eff}}^+(\omega', \mu) \right]$$

$$+ \left[U_1(n \cdot p, \mu_{h1}, \mu) \tilde{C}^{(-)}(n \cdot p, \mu_{h1}) \right] \rho_{B,\text{eff}}^-(\omega', \mu)$$

$$\pm \frac{n \cdot p - m_B}{m_B} \left(\phi_{B,\text{eff}}^+(\omega', \mu) + C^{(-)}(n \cdot p, \mu) \phi_B^-(\omega', \mu) \right)$$

B-meson LCSR

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$$\begin{aligned}
 \rho_{eff}^-(\Omega, \mu) = & \int_0^\infty \frac{d\omega'}{\omega'} \left\{ \left[1 + \frac{\alpha_s C_F}{4\pi} \left(\ln^2 \frac{\mu^2}{n \cdot p \Omega} - 2 \ln \frac{\mu^2}{n \cdot p \Omega} \ln \frac{\omega'}{\Omega} \right. \right. \right. \\
 & \left. \left. \left. + \frac{1}{2} \ln^2 \frac{\omega'}{\Omega} - \frac{3}{2} \ln \frac{\omega'}{\Omega} + \frac{\pi^2}{2} - 1 \right) \right] J_0 \left(2 \sqrt{\frac{\Omega}{\omega'}} \right) \right. \\
 & \left. + \frac{\alpha_s C_F}{4\pi} \left(\ln \frac{\omega'}{\Omega} + \frac{3}{2} \right) \pi N_0 \left(2 \sqrt{\frac{\Omega}{\omega'}} \right) \right. \\
 & \left. + \frac{\alpha_s C_F}{2\pi} J_0^{(2,0)} \left(2 \sqrt{\frac{\Omega}{\omega'}} \right) \right. \\
 & \left. + \frac{\alpha_s C_F}{2\pi} \int_{2\sqrt{\frac{\Omega}{\omega'}}}^\infty \frac{d\beta}{\beta} J_0(\beta) \right\} U_j(\mu_{hc}, \mu) U_\rho(\mu_0, \mu) \rho_B^{(-)}(\omega', \mu_0)
 \end{aligned}$$