

Renormalization group improved pQCD prediction for $\Upsilon(1S)$ leptonic decay

申建明

Chongqing University

August 24, 2016

Outline

- Renormalization scheme-scale ambiguities
- Renormalization Group Invariance
- Principle of Maximum Conformality
- $\Upsilon(1S)$ leptonic decay
- Summary

Renormalization scheme-scale ambiguities

QCD, Asymptotic Freedom

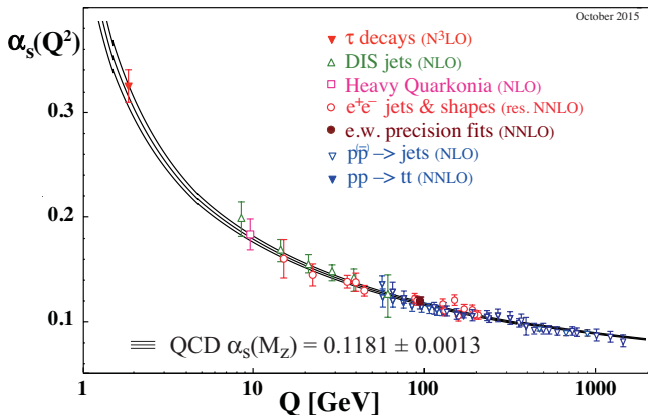


Figure: Summary of measurements of α_s as a function of the energy scale Q .

Renormalization scheme-scale ambiguities

Asymptotic Freedom, perturbative calculable

QCD Asymptotic Freedom \rightarrow a hep observable $\rho(Q)$ perturbative calculable,

$$\rho_n = \sum_{i=1}^n r_i(\mu_0^2/Q^2) \alpha_s^{\mathcal{R}}(\mu_0)^{p+i-1}, \quad (1)$$

where α_s satisfies RGE:

$$\mu^2 \frac{\partial}{\partial \mu^2} \left(\frac{\alpha_s}{4\pi} \right) = \beta^{\mathcal{R}}(\alpha_s) = - \sum_{i=0}^{\infty} \beta_i^{\mathcal{R}} \left(\frac{\alpha_s}{4\pi} \right)^{i+2}, \quad (2)$$

with $\alpha_s = g^2/(4\pi)$, $\beta_0 = 11 - \frac{2}{3}n_f$, $\beta_i = \dots$.

$$\rho_n = \rho_n(\mu_0, Q, \{\beta_i^{\mathcal{R}}\}), \quad \rho_{\infty} = \rho(Q).$$

conventional choice, $\mu_0 = Q$, $[1/2Q, 2Q]$.

Renormalization scheme-scale ambiguities

Scale ambiguities

Examples: $\Gamma_{\Upsilon(1S) \rightarrow e^+e^-}|_{\text{Exp.}} = 1.340(18) \text{ keV}$

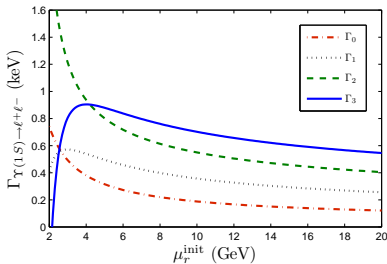


Figure: The $\Gamma_{\Upsilon(1S) \rightarrow \ell^+\ell^-}$ as a function of μ_r^{init} . Γ_n ($n = 0, 1, 2, 3$) stands for the decay rate with up to n_{th} -loop QCD corrections.

The Principle of Maximum Conformality (PMC) provides a systematic means of eliminating ambiguities in the renormalization scheme and in initial scale.

Renormalization Group Invariance

Guidance of PMC

RGI: the prediction for a physical observable $\rho(Q)$ should be independent of the choice of the renormalization scheme \mathcal{R} or initial scale μ , i.e.,

$$\frac{\partial \rho(Q)}{\partial \ln \mu^2} \equiv 0, \quad (3)$$

$$\frac{\partial \rho(Q)}{\partial \beta_i^{\mathcal{R}}} \equiv 0. \quad (4)$$

- $\rho(Q) \leftrightarrow$ An effective coupling $a_\rho(Q) \equiv [\rho(Q)/r_1]^{1/p}$.
 a_ρ satisfies the corresponding RGE,

$$\mu^2 \frac{\partial}{\partial \mu^2} a_\rho = \beta^\rho(a_\rho) = - \sum_{i=0}^{\infty} \beta_i^\rho a_\rho^{i+2},$$

$\beta^\rho(a_\rho)$ is related to $\beta^{\mathcal{R}}$ through the identity, $\beta^\rho = \frac{\partial a_\rho}{\partial a^{\mathcal{R}}} \beta^{\mathcal{R}}$.

Renormalization Group Invariance

extended RGE

- A universal coupling constant $\hat{a}(\tau_{\mathcal{R}}, \{c_i^{\mathcal{R}}\})$ satisfies the following extended RGEs,

$$\beta(\hat{a}, \{c_i^{\mathcal{R}}\}) = \frac{\partial \hat{a}}{\partial \tau_{\mathcal{R}}} = -\hat{a}^2 [1 + \hat{a} + c_2^{\mathcal{R}} \hat{a}^2 + c_3^{\mathcal{R}} \hat{a}^3 + \dots], \quad (5)$$

$$\beta_n(\hat{a}, \{c_i^{\mathcal{R}}\}) = \frac{\partial \hat{a}}{\partial c_n^{\mathcal{R}}} = -\beta(\hat{a}, \{c_i^{\mathcal{R}}\}) \int_0^{\hat{a}} \frac{x^{n+2} dx}{\beta^2(x, \{c_i^{\mathcal{R}}\})}, \quad (6)$$

where for any given \mathcal{R} -scheme,

$$\hat{a}(\tau_{\mathcal{R}}, \{c_i^{\mathcal{R}}\}) = \frac{\beta_1}{4\pi\beta_0} \alpha_s^{\mathcal{R}}(\tau_{\mathcal{R}}, \{c_i^{\mathcal{R}}\}),$$

$$\tau_{\mathcal{R}} = \frac{\beta_0^2}{\beta_1} \ln \mu^2 |_{\mathcal{R}}, \quad c_i^{\mathcal{R}} = \beta_i^{\mathcal{R}} \beta_0^{i-1} / \beta_1^i \quad (i = 2, 3, \dots).$$

General case for RGI,

$$\frac{\partial \hat{a}(\tau_{\mathcal{R}}, \{c_i^{\mathcal{R}}\})}{\partial \tau_{\mathcal{S}}} \equiv 0, \quad \frac{\partial \hat{a}(\tau_{\mathcal{R}}, \{c_i^{\mathcal{R}}\})}{\partial c_j^{\mathcal{S}}} \equiv 0. \quad (7)$$

Renormalization Group Invariance

Demonstration

Taylor Expansion, $\hat{a}_{\mathcal{R}} = \hat{a}(\tau_{\mathcal{R}}, \{c_i^{\mathcal{R}}\})$, $\hat{a}_{\mathcal{S}} = \hat{a}(\tau_{\mathcal{S}}, \{c_i^{\mathcal{S}}\})$

$$\hat{a}_{\mathcal{R}} = \hat{a}_{\mathcal{S}} + \frac{1}{1!} \left[\left(\frac{\partial \hat{a}}{\partial \tau} \right)_{\mathcal{S}} \bar{\tau} + \sum_i \left(\frac{\partial \hat{a}}{\partial c_i} \right)_{\mathcal{S}} \bar{c}_i \right] + \cdots, \quad (8)$$

where $\bar{\tau} = \tau_{\mathcal{R}} - \tau_{\mathcal{S}}$, $\bar{c}_i = c_i^{\mathcal{R}} - c_i^{\mathcal{S}}$.

Demonstration of RGI: For an n -th order estimate,

$$\frac{\partial \hat{a}_{\mathcal{R}}}{\partial \tau_{\mathcal{S}}} = \frac{\partial^{(n+1)} \hat{a}_{\mathcal{S}}}{\partial \tau_{\mathcal{S}}^{(n+1)}} \frac{\bar{\tau}^n}{n!} + \sum_i \frac{\partial^{(n+1)} \hat{a}_{\mathcal{S}}}{\partial c_i^{\mathcal{S}} \partial \tau_{\mathcal{S}}^{(n)}} \frac{\bar{\tau}^{n-1} \bar{c}_i}{(n-1)!} + \cdots, \quad (9)$$

If $n \rightarrow \infty$, then $\hat{a}(\tau_{\mathcal{R}}, \{c_i^{\mathcal{R}}\})$ is independent of $\tau_{\mathcal{S}}$.

Two key points:

- By summing all types of c_i -terms, $\hat{a}(\tau_{\mathcal{R}}, \{c_i^{\mathcal{R}}\})$ will be independent of any choice of scheme and scale;
- Residual scale dependence for a fixed-order estimate; e.g., if $n \neq \infty$, the right-hand of Eq.(9) is non-zero.

Principle of Maximum Conformality

MS-like scheme

How to summing $\{\beta_i\}$ terms ?

- PMC-I, PMC-BLM correspondence principle
- PMC-II, \mathcal{R}_δ -scheme

In MS-scheme one absorbs the $1/\epsilon$ poles appearing in loop integrals which come in powers of

$$\ln \frac{\mu^2}{\Lambda^2} + \frac{1}{\epsilon} + c ,$$

$c \leftarrow$ the finite part of the integral.

Subtracting any finite part as well with $1/\epsilon \Leftrightarrow$ Redefining μ .

e.g. The $\overline{\text{MS}}$ -scheme differs from the MS-scheme by an additional absorption, corresponds to redefining μ to:

$$\mu^2 = \mu_{\overline{\text{MS}}}^2 \exp(\ln 4\pi - \gamma_E) \quad (10)$$

MS-like scheme: MS-scheme, $\overline{\text{MS}}$ -scheme, G-scheme, etc.

Principle of Maximum Conformality

\mathcal{R}_δ -scheme

\mathcal{R}_δ -scheme transformation, $\mu_\delta^2 = \mu_{\overline{\text{MS}}}^2 \exp(\delta)$

$$a(\mu_0) = a(\mu_\delta) + \sum_{n=1}^{\infty} \frac{1}{n!} \frac{d^n a(\mu)}{(d \ln \mu^2)^n} \Big|_{\mu=\mu_\delta} (-\delta)^n,$$

where $\ln \mu_0^2 / \mu_\delta^2 = -\delta$. In any \mathcal{R}_δ -scheme,

$$\begin{aligned} \rho_\delta(Q) = & \textcolor{red}{r}_1 a_1(\mu_\delta)^n + [\textcolor{red}{r}_2 + n\beta_0 r_1 \delta_1] a_2(\mu_\delta)^{n+1} + \left[\textcolor{red}{r}_3 + n\beta_1 r_1 \delta_1 \right. \\ & \left. + (n+1)\beta_0 r_2 \delta_2 + \frac{n(n+1)}{2} \beta_0^2 r_1 \delta_1^2 \right] a_3(\mu_\delta)^{n+2} + \left[\textcolor{red}{r}_4 + n\beta_2 r_1 \delta_1 \right. \\ & \left. + (n+1)\beta_1 r_2 \delta_2 + (n+2)\beta_0 r_3 \delta_3 + \frac{n(3+2n)}{2} \beta_0 \beta_1 r_1 \delta_1^2 \right. \\ & \left. + \frac{(n+1)(n+2)}{2} \beta_0^2 r_2 \delta_2^2 + \frac{n(n+1)(n+2)}{3!} \beta_0^3 r_1 \delta_1^3 \right] a_4(\mu_\delta)^{n+3} + \dots, \end{aligned}$$

where $r_i = r_{i,0} + \mathcal{O}(\{\beta_i\})$,

Principle of Maximum Conformality

\mathcal{R}_δ -scheme

General case up to α^{n+3} ,

$$\begin{aligned}
 \rho_\delta(Q) = & r_{1,0}a(Q)^n + [r_{2,0} + \underline{n\beta_0 r_{2,1}}]a(Q)^{n+1} + \left[r_{3,0} \right. \\
 & + \underline{n\beta_1 r_{2,1}} + \underline{(n+1)\beta_0 r_{3,1}} + \underline{\frac{n(n+1)}{2}\beta_0^2 r_{3,2}} \left. \right] a(Q)^{n+2} \\
 & + \left[r_{4,0} + \underline{n\beta_2 r_{2,1}} + \underline{(n+1)\beta_1 r_{3,1}} + \underline{(n+2)\beta_0 r_{4,1}} \right. \\
 & + \underline{\frac{n(3+2n)}{2}\beta_0\beta_1 r_{3,2}} + \underline{\frac{(n+1)(n+2)}{2}\beta_0^2 r_{4,2}} \\
 & \left. + \underline{\frac{n(n+1)(n+2)}{3!}\beta_0^3 r_{4,3}} \right] a(Q)^{n+3} + \mathcal{O}(a^{n+4}) , \quad (11)
 \end{aligned}$$

β_i terms, absorbed into the LO term, determine Q_1 ;

β_i terms, absorbed into the NLO term, determine Q_2 ;

β_i terms, absorbed into the NNLO term, determine Q_3 ;

Principle of Maximum Conformality

The final result up to $N^n\text{LO}$,

$$\rho(Q) = \sum_{i=0}^n r_{i+1,0} a(Q_i)^{p+i} + \mathcal{O}(a^{p+n+1}), \quad (12)$$

the k_{th} -order PMC scale $Q_{k,N^{n-k}\text{LLO}}$,

$$\ln \frac{Q_k^2}{Q^2} = \frac{s_{k,1} + \Delta_k^{(1)} s_{k,2} + \Delta_k^{(2)} s_{k,3}}{1 + \Delta_k^{(1)} s_{k,1} + \Delta_k^{(1)2} (s_{k,2} - s_{k,1}^2) + \Delta_k^{(2)} s_{k,1}^2} = -\frac{r_{k+1,1}}{r_{k,0}} + \mathcal{O}(a).$$

where the short-hand notation,

$$s_{k,j} = (-1)^j \frac{r_{k+j,j}}{r_{k,0}},$$

$$\Delta_k^{(1)}(a) = \frac{1}{2} \left[\frac{\partial \beta}{\partial a} + (k-1) \frac{\beta}{a} \right],$$

$$\Delta_k^{(2)}(a) = \frac{1}{3!} \left[\beta \frac{\partial^2 \beta}{\partial a^2} + \left(\frac{\partial \beta}{\partial a} \right)^2 + 3(k-1) \frac{\beta}{a} \frac{\partial \beta}{\partial a} + (k-1)(k-2) \frac{\beta^2}{a^2} \right],$$

...

Principle of Maximum Conformality

Basic procedure for PMC scale setting

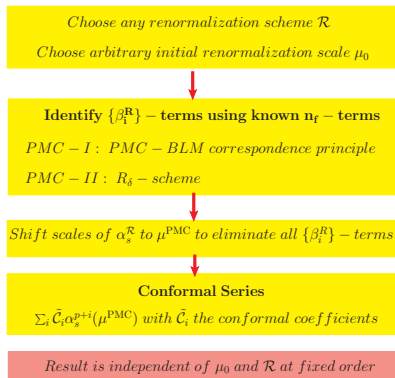


Figure: Basic procedure for PMC scale setting. Two ways, namely, PMC-I and PMC-II, are suggested to absorb the $\{\beta_i\}$ terms and the final result is conformal and independent of μ_0 and \mathcal{R} .

Principle of Maximum Conformality

Features of PMC

Features of PMC:

- Satisfying RGI
- Predictions are scale- and scheme- independent
- Matches conformal series
- Convergence pQCD series
- Insensitive residual scale dependence
- Accurate estimation for Low Order Terms

Reference: Prog.Part.Nucl.Phys.72,44; Rept.Prog.Phys.78,126201;
PRD89,014027; PRL110,192001; PRL109,042002; PRD86,054018;
PRD86,014021; PRD85,114040; Front.Phys.China11,111201;
NPB876,731; JHEP10(2013)117; EPJC74,2825; JPG41,075010;
Chin.Phys.Lett.31,051202; PRD89,116001; PRD90,037503;
PRD90,034025; PLB748,422; PRD90,114034; JHEP06(2015)169;
JPG43,075001; PRD93,014004; arXiv:1605.02572; PRD89,014018;

$\Upsilon(1S)$ leptonic decay

The decay rate of $\Upsilon(1S) \rightarrow \ell^+ \ell^-$,

$$\Gamma_{\Upsilon(1S) \rightarrow \ell^+ \ell^-} = \frac{4\pi\alpha^2}{9m_b^2} Z_1, \quad (13)$$

$\alpha \leftarrow$ the fine structure constant, $m_b \leftarrow$ the b -quark pole mass,
 $Z_1 \leftarrow$ the residue of the $1S$ -wave two-point correlation function near $(b\bar{b})$ -threshold,

$$Z_1 = |\psi_1(0)|^2 c_v \left[c_v - \frac{E_1}{m_b} \left(c_v + \frac{d_v}{3} \right) + \cdots \right], \quad (14)$$

c_v and d_v , the matching coefficients of the leading and sub-leading $(b\bar{b})$ -currents within the NRQCD framework; $|\psi_1(0)| \leftarrow$ the renormalized wavefunction at the origin, $E_1 \leftarrow$ the binding energy of $\Upsilon(1S)$, $|\psi_1(0)|$ and E_1 corresponding to the bound-state contributions, the perturbative corrections from high-order heavy quark potentials and dynamical gluon effect.

References: PRL112(2014)151801, PRD89(2014)034027,
PLB658(2008)222, NPB714(2005)67, NPB716(2005)303,
PLB653(2007)53, PLB668(2008)143.

$\Upsilon(1S)$ leptonic decay

$$c_v = 1 + \sum_{k=1}^n c_k a_s^k, \quad d_v = 1 + \sum_{k=1}^n d_k a_s^k,$$

$$E_1 = E_1^{(0)} \left(1 + \sum_{k=1}^n e_k a_s^k \right), \quad |\psi_1(0)|^2 = |\psi_1^{(0)}(0)|^2 \left(1 + \sum_{k=1}^n f_k a_s^k \right),$$

$$e_i = e_i^C + e_i^{\text{nC}} + e_i^{\text{us}}, \quad f_i = f_i^C + f_i^{\text{nC}} + f_i^{\text{us}},$$

$$e_1^{\text{nC}} = e_1^{\text{us}} = 0, \quad f_1^{\text{nC}} = f_1^{\text{us}} = f_2^{\text{us}} = 0,$$

$$\left| \psi_1^{(0)}(0) \right|^2 = \frac{(m_b C_F \alpha_s)^3}{8\pi}, \quad E_1^{(0)} = -\frac{1}{4} m_b (C_F \alpha_s)^2,$$

To reconstruct all the coefficients with full renormalization scale dependence, we use,

$$\begin{aligned} a(\mu_0)^k &= a(\mu_\delta)^k + k\beta_0\delta a(\mu_\delta)^{k+1} + k \left[\beta_1\delta + \frac{k+1}{2}\beta_0^2\delta^2 \right] a(\mu_\delta)^{k+2} \\ &+ k \left[\beta_2\delta + \frac{2k+3}{2}\beta_0\beta_1\delta^2 + \frac{(k+1)(k+2)}{3!}\beta_0^3\delta^3 \right] a(\mu_\delta)^{k+3} + \dots \end{aligned}$$

$\Upsilon(1S)$ leptonic decay

PMC scales for $\Gamma_{\Upsilon(1S) \rightarrow \ell^+ \ell^-}$

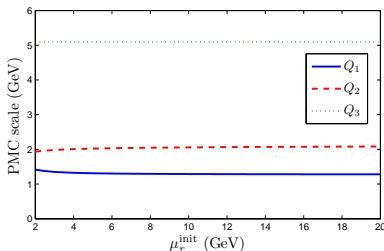


Figure: The PMC scales Q_i with $i = (1, 2, 3)$ at each perturbative order versus the initial renormalization scale μ_r^{init} . The solid, the dashed, and the dotted lines are for Q_1 , Q_2 , and Q_3 , respectively.

$Q_1 \simeq 1.31$ GeV, $Q_2 \simeq 2.02$ GeV, $Q_3 \simeq 5.10$ GeV.

Different from the conventional choice m_b , or the guessed value ~ 3.5 GeV that leads to maximum value.

$\Upsilon(1S)$ leptonic decay

Decay width

$$\Gamma_{\Upsilon(1S) \rightarrow \ell^+ \ell^-} |_{\text{PMC}} = 1.270^{+0.130+0.043}_{-0.182-0.042} \pm 0.015 \text{ keV} = 1.270^{+0.137}_{-0.187} \text{ keV},$$

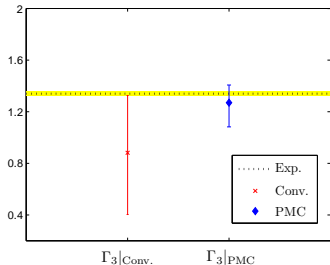
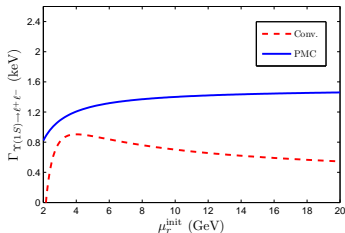


Figure: (left) The $\Gamma_{\Upsilon(1S) \rightarrow e^+ e^-}$ versus μ_r^{init} before and after the PMC scale setting. (right) A comparison of $\Gamma_{\Upsilon(1S) \rightarrow e^+ e^-}$ together with its pQCD errors before and after the PMC scale setting.

Summary

Advantages of BLM/PMC:

- Predictions are scale-independent at fixed-order (residual)
- Accurate estimation for Low Order Terms (residual)
- Accurate α_s behavior of each order
- Achieve convergence pQCD series
- Improve pQCD prediction
- Maximal sensitivity to new physics !

Outlook:

- A physical scheme (MOM or V-scheme) is better than MS-like schemes in some cases. Why ? vertex type ?
- How to use PMC more automatically ?
- The running behavior of α_s in Low energy.

Thank you for your attention !