

The $B \rightarrow D^{(*)} l \nu_l$ decays in the pQCD approach with the Lattice QCD input

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Outline

- ◆ **Introduction**
- ◆ **Theoretical Framework**
- ◆ **Results and Discussions**
- ◆ **Summary**

Introduction

- ◆ The **semileptonic decays of B meson** play an important role in testing the Standard Model (SM) and in searching for the New Physics (NP) beyond the SM.
- ◆ **B-decays with τ in the final state** offer possibilities to study NP effects not present in processes with light leptons.
- ◆ Popular NP test via

$$R(D) = \frac{\mathcal{B}(B \rightarrow D\tau\bar{\nu}_\tau)}{\mathcal{B}(B \rightarrow D\ell\bar{\nu}_\ell)}, \quad R(D^*) = \frac{\mathcal{B}(B \rightarrow D^*\tau\bar{\nu}_\tau)}{\mathcal{B}(B \rightarrow D^*\ell\bar{\nu}_\ell)} \quad (\ell = e, \mu)$$

in order to cancel/reduce theoretical uncertainties in V_{cb} /FF.

★ $B \rightarrow D^{(*)} l \nu_l$ semileptonic decays

	$R(D)$	$R(D^*)$
SM	0.296 ± 0.016	$0.252 \pm 0.003^{[1]}$
BaBar	$0.440 \pm 0.058 \pm 0.042$	$0.332 \pm 0.024 \pm 0.018^{[2]}$
Belle(2015)	$0.375 \pm 0.064 \pm 0.026$	$0.293 \pm 0.038 \pm 0.015^{[3]}$
Belle(2016)	--	$0.302 \pm 0.030 \pm 0.011^{[4]}$
LHCb	--	$0.336 \pm 0.027 \pm 0.030^{[5]}$
HFAG(2015)	$0.391 \pm 0.041 \pm 0.028$	$0.322 \pm 0.018 \pm 0.012^{[6]}$
HFAG(2016)	$0.397 \pm 0.040 \pm 0.028$	$0.316 \pm 0.016 \pm 0.010^{[7]}$



[1] *Phys. Rev. D* 85, 094025 (2012).

[2] *Phys. Rev. Lett.* 109, 101802 (2012).

[3] *Phys. Rev. D* 92, 072014 (2015).

[4] *arXiv:1603.06711; arXiv:1607.07923.*

[5] *Phys. Rev. Lett.* 115, 111803 (2015).

[6-7] *arXiv:1412.7515; http://www.slac.stanford.edu/xorg/hfag/semi.*

NP : such as the charged Higgs bosons in the 2HDMs

*Phys. Rev. D 86, 054014 (2012);
JHEP 01, 054 (2013);
Phys. Rev. Lett. 115, 181801 (2015);
Phys. Rev. Lett. 116, 081801 (2016);
Phys. Rev. D 93, 115009 (2016);*
...

SM : unquenched lattice QCD, the relativistic quark Model,
maximally employing the experimental information on the
relevant form factors from the data

*Phys. Rev. Lett. 109, 071802 (2012);
Phys. Rev. D 85, 114502 (2012);
Phys. Lett. B 716, 208 (2012);
Phys. Rev. D 92, 034506 (2015);
Phys. Rev. D 92, 114502 (2015)*
...

Talks : presented at recent conferences

*S. Stone, PoS ICHEP2012 (2013) 033,
arXiv: 1212.6374;
S. Fajfer, I. Nisandzic, arXiv: 1301.1167;
G. Ricciardi, PoS ConfinementX (2012)
145 , arXiv: 1301.4389;
A. Pich, arXiv: 1301.4474;*
...

$B \rightarrow D^{(*)} l \nu_l$ decays in pQCD approach

- ◆ The momentum can be chosen as:

$$P_1 = \frac{m_B}{\sqrt{2}}(1, 1, 0_{\perp}), \quad P_2 = \frac{rm_B}{\sqrt{2}}(\eta^+, \eta^-, 0_{\perp}),$$

$$\epsilon_L = \frac{1}{\sqrt{2}}(\eta^+, -\eta^-, 0_{\perp}), \quad \epsilon_T = (0, 0, 1),$$

$$k_1 = (0, x_1 \frac{m_B}{\sqrt{2}}, k_{1\perp}), \quad k_2 = \frac{m_B}{\sqrt{2}}(x_2 r \eta^+, x_2 r \eta^-, k_{2\perp}).$$

the factors $\eta^{\pm} = \eta \pm \sqrt{\eta^2 - 1}$ is defined in terms of the parameter

$$\eta = \frac{1}{2r} \left[1 + r^2 - \frac{q^2}{m_B^2} \right],$$

$r = m_D/m_B$ or m_{D^*}/m_B , and $q = p_1 - p_2$ is the lepton-pair momentum.

◆ The effective Hamiltonian:

$$\mathcal{H}_{eff}(b \rightarrow cl\bar{\nu}_l) = \frac{G_F}{\sqrt{2}} V_{cb} \bar{c}\gamma_\mu(1 - \gamma_5)b \cdot \bar{l}\gamma^\mu(1 - \gamma_5)\nu_l,$$

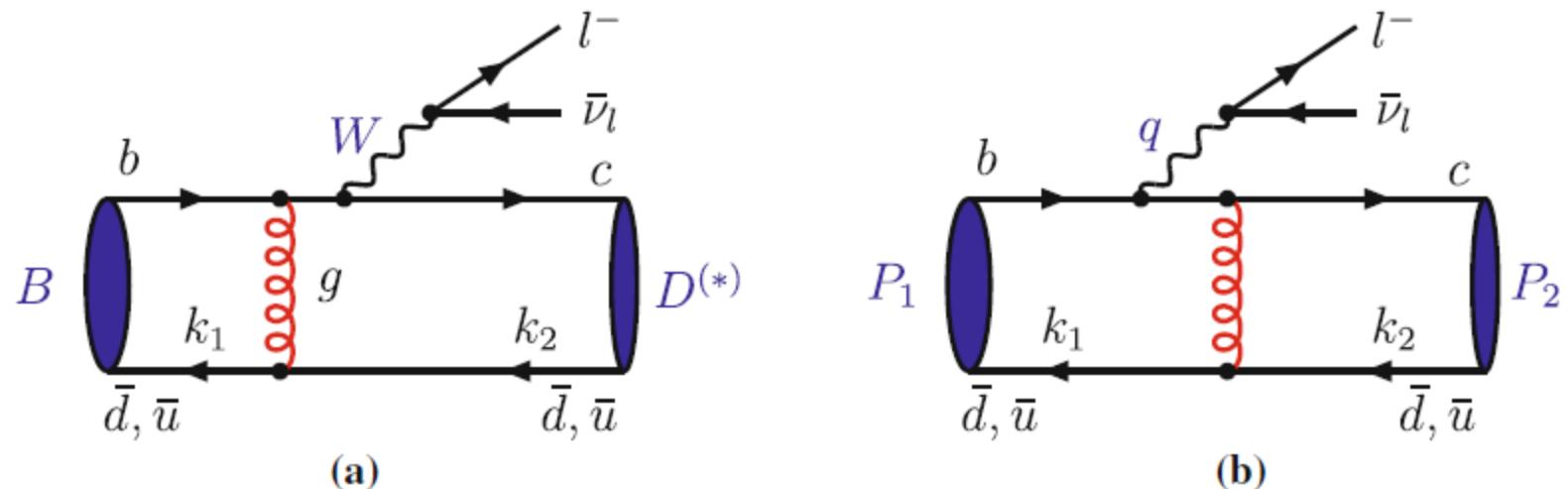


FIG. 1: The lowest order Feynman diagrams for the semileptonic decays $B \rightarrow D^{(*)} l^- \bar{\nu}_l$ in the pQCD approach, the winding curves are gluons.

◆ Form factors:

- the light cone QCD sum rules (LCSR_s)

*JHEP 09 (1998) 005, 10 (2001) 019
PRD 71, 014015 (2005)
JHEP 04 (2008) 014
PRD 83, 094031 (2011)*

- the lattice QCD (LQCD)

*PRD 73, 074502 (2006)
PRD 80, 034026 (2009)
PLB 486, 111 (2000)*

- the perturbative QCD (pQCD)

*PRD 65, 014007 (2001)
NPB 642, 263 (2002)
EPJC 23, 275 (2002) 28,515 (2003)*

The $B \rightarrow D$ form factors induced by vector currents are defined as

$$\begin{aligned}\langle D(p_2) | \bar{c}(0)\gamma_\mu b(0) | \bar{B}(p_1) \rangle &= F_+(q^2) \left[(p_1 + p_2)_\mu - \frac{m_B^2 - m_D^2}{q^2} q_\mu \right] \\ &\quad + F_0(q^2) \frac{m_B^2 - m_D^2}{q^2} q_\mu.\end{aligned}$$

$$\begin{aligned}F_+(q^2) &= \frac{1}{2\sqrt{r}} [(1+r)h_+(w) - (1-r)h_-(w)] \\ &= \frac{1+r}{2\sqrt{r}} \mathcal{G}(w),\end{aligned}$$

$$\begin{aligned}F_0(q^2) &= \sqrt{r} \left[\frac{1+w}{1+r} h_+(w) - \frac{w-1}{1-r} h_-(w) \right] \\ &= \frac{\sqrt{m_B m_D}}{m_B - m_D} \mathcal{G}(w) \Delta(w),\end{aligned}$$

where $r = m_D/m_B$, $w = (m_B^2 + m_D^2 - q^2)/(2m_B m_D)$ with $q^2 = (p_B - p_D)^2$.

Unlike the $B \rightarrow D$ transitions, the $B \rightarrow D^*$ transitions are

$$\langle D(p_2, \epsilon^*) | \bar{c}(0) \gamma_\mu b(0) | \bar{B}(p_1) \rangle = \frac{2iV(q^2)}{m_B + m_{D^*}} \epsilon_{\mu\nu\alpha\beta} \epsilon^{*\nu} p_1^\alpha p_2^\beta,$$

$$\begin{aligned} \langle D(p_2, \epsilon^*) | \bar{c}(0) \gamma_\mu \gamma_5 b(0) | \bar{B}(p_1) \rangle &= 2m_{D^*} A_0(q^2) \frac{\epsilon^* \cdot q}{q^2} q_\mu \\ &\quad + (m_B + m_{D^*}) A_1(q^2) \left(\epsilon_\mu^* - \frac{\epsilon^* \cdot q}{q^2} q_\mu \right) \\ &\quad - A_2(q^2) \frac{\epsilon^* \cdot q}{m_B + m_{D^*}} \left[(p_1 + p_2)_\mu - \frac{m_B^2 - m_{D^*}^2}{q^2} q_\mu \right]. \end{aligned}$$

$$V(q^2) = \frac{1+r}{2\sqrt{r}} h_V(w),$$

$$\begin{aligned} A_0(q^2) &= \frac{1}{2\sqrt{r}} [(1+w)h_{A_1}(w) - (1-wr)h_{A_2}(w) \\ &\quad + (r-w)h_{A_3}(w)], \end{aligned}$$

$$A_1(q^2) = \frac{\sqrt{r}}{1+r} (1+w)h_{A_1}(w),$$

$$A_2(q^2) = \frac{1+r}{2\sqrt{r}} (rh_{A_2}(w) + h_{A_3}(w)),$$

*Phys. Rev. D 65, 014007 (2001);
*Phys. Rev. D 85, 094025 (2012);
*Phys. Rev. D 89, 114504 (2014). 10***

◆ The differential decay widths:

$$\frac{d\Gamma(b \rightarrow cl\bar{\nu}_l)}{dq^2} = \frac{G_F^2 |V_{cb}|^2}{192\pi^3 m_B^3} \left(1 - \frac{m_l^2}{q^2}\right)^2 \frac{\lambda^{1/2}(q^2)}{2q^2} \cdot \left\{ 3m_l^2 (m_B^2 - m_D^2)^2 |F_0(q^2)|^2 + (m_l^2 + 2q^2) \lambda(q^2) |F_+(q^2)|^2 \right\},$$

$$\begin{aligned} \frac{d\Gamma_L(B \rightarrow D^* l\bar{\nu}_l)}{dq^2} &= \frac{G_F^2 |V_{cb}|^2}{192\pi^3 m_B^3} \left(1 - \frac{m_l^2}{q^2}\right)^2 \frac{\lambda^{1/2}(q^2)}{2q^2} \cdot \left\{ 3m_l^2 \lambda(q^2) A_0^2(q^2) \right. \\ &\quad \left. + \frac{m_l^2 + 2q^2}{4m^2} \cdot \left[(m_B^2 - m^2 - q^2)(m_B + m) A_1(q^2) - \frac{\lambda(q^2)}{m_B + m} A_2(q^2) \right]^2 \right\}, \end{aligned}$$

$$\begin{aligned} \frac{d\Gamma_{\pm}(B \rightarrow D^* l\bar{\nu}_l)}{dq^2} &= \frac{G_F^2 |V_{cb}|^2}{192\pi^3 m_B^3} \left(1 - \frac{m_l^2}{q^2}\right)^2 \frac{\lambda^{3/2}(q^2)}{2} \\ &\quad \times \left\{ (m_l^2 + 2q^2) \left[\frac{V(q^2)}{m_B + m} \mp \frac{(m_B + m) A_1(q^2)}{\sqrt{\lambda(q^2)}} \right]^2 \right\}, \end{aligned}$$

$$\frac{d\Gamma}{dq^2} = \frac{d\Gamma_L}{dq^2} + \frac{d\Gamma_+}{dq^2} + \frac{d\Gamma_-}{dq^2}.$$

Phys. Rev. D 79, 014013 (2009).
Phys. Rev. D 79, 054012 (2009).

Results and Discussions

★ **$B \rightarrow D^{(*)} l \nu$ semileptonic decays**

➤ Form factors:

$$F_0(q^2) = \frac{F_0(0)}{1 - a(q^2/m_B^2) + b(q^2/m_B^2)^2}$$

Phys. Rev. D 79, 054012 (2009).

lower q^2 region

$$m_l^2 \leq q^2 \leq m_\tau^2$$

large q^2 region

$$m_\tau^2 < q^2 \leq q_{max}^2,$$

$$q_{max}^2 = (m_B - m_D)^2$$

Phys. Rev. D 52, 3958 (1995);
Phys. Rev. D 67, 054028 (2003).

➤ Form factors:

lower q^2 region

$$m_l^2 \leq q^2 \leq m_\tau^2$$



pQCD

at the endpoint

$$q^2 = q_{\max}^2$$



Lattice QCD

other q^2 region

extrapolation



The pole model
para.

Phys Rev D 85:114502 (2012);

Phys Rev D 89:114504 (2014).

Lattice QCD:

$$\mathcal{G}(1) = 1.058 \pm 0.009,$$

$$h_{A_1}(1) = \mathcal{F}(1) = 0.906 \pm 0.004 \pm 0.012$$

$$F_0(q_{\max}^2) = 0.92 \pm 0.02, \quad F_+(q_{\max}^2) = 1.21 \pm 0.02,$$

$$V(q_{\max}^2) = 1.01 \pm 0.02, \quad A_0(q_{\max}^2) = 1.01 \pm 0.02,$$

$$A_1(q_{\max}^2) = 0.81 \pm 0.02, \quad A_2(q_{\max}^2) = 1.01 \pm 0.02,$$

where $q_{\max}^2 = (m_B - m_D)^2 = 11.63 \text{ GeV}^2$ for $B \rightarrow D$ transition,

$q_{\max}^2 = (m_B - m_{D^*})^2 = 10.69 \text{ GeV}^2$ for $B \rightarrow D^*$ transition.

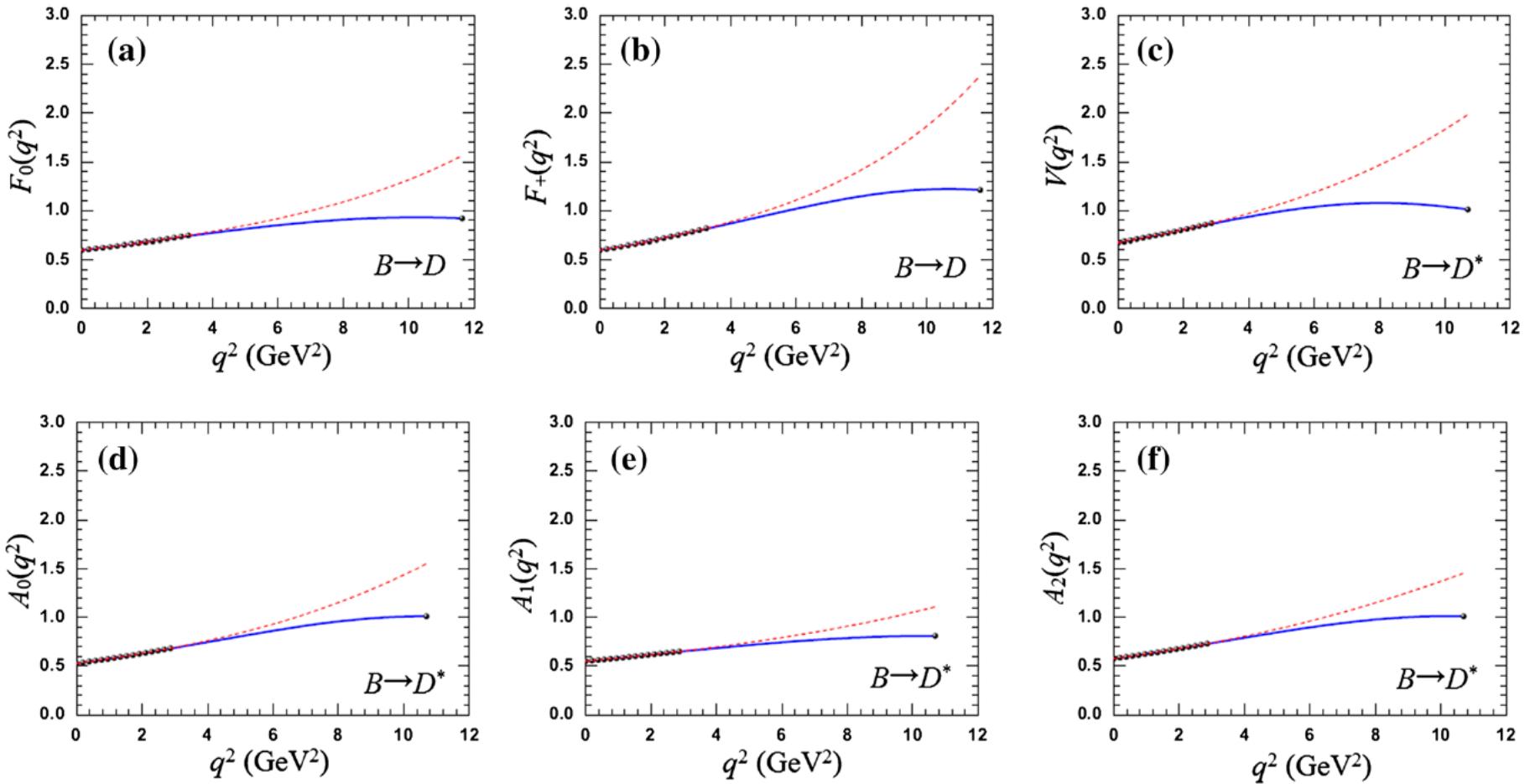


Fig. 1 (Color online) The theoretical predictions for the q^2 -dependence of the six form factors in the pQCD approach (the dashed curves), and the “pQCD + Lattice QCD” method (the solid curves)

➤ Branching ratios:

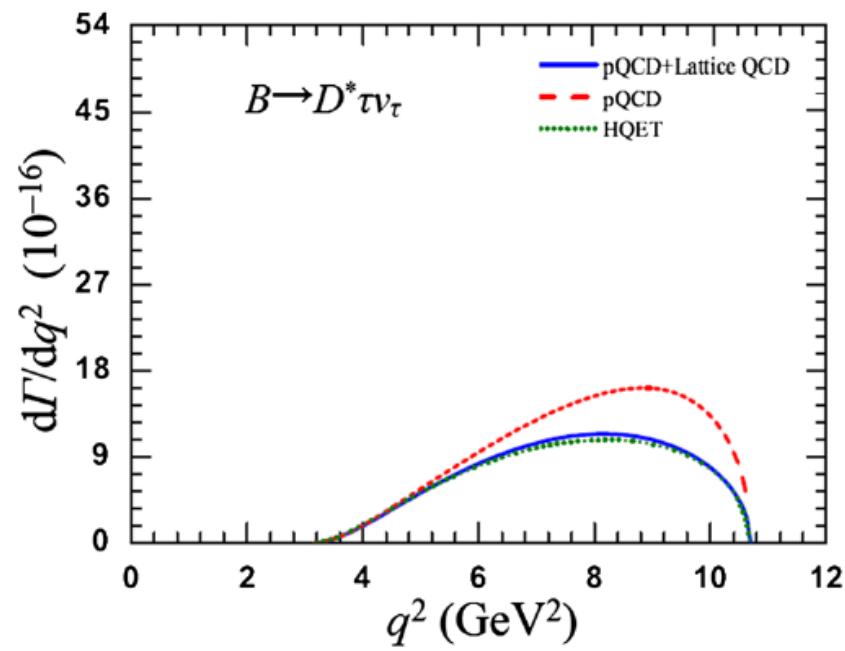
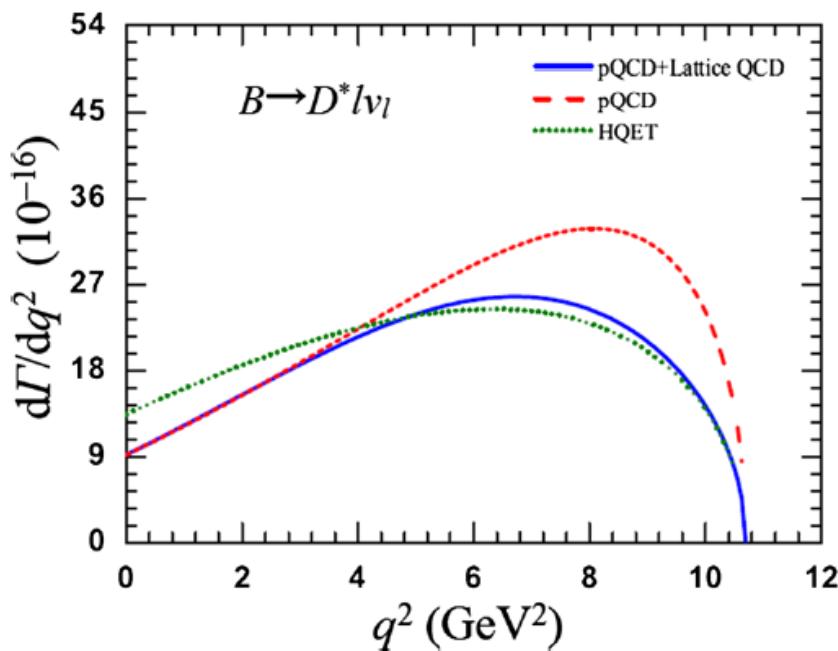
pQCD + Lattice QCD

&

pQCD

&

HQET



$$\mathcal{B}(B \rightarrow D^{(*)}l^-\bar{\nu}_l) = \frac{1}{2} [\mathcal{B}(B \rightarrow D^{(*)}e^-\bar{\nu}_e) + \mathcal{B}(B \rightarrow D^{(*)}\mu^-\bar{\nu}_\mu)] .$$

Channel	pQCD(%)[26]	pQCD+Lattice QCD(%)	HQET(%)[10]	PDG(%)[38]
$\bar{B}^0 \rightarrow D^+\tau^-\bar{\nu}_\tau$	$0.87^{+0.34}_{-0.28}$	$0.78^{+0.23}_{-0.20}$	$0.57 - 0.69$	1.03 ± 0.22
$\bar{B}^0 \rightarrow D^+l^-\bar{\nu}_l$	$2.03^{+0.92}_{-0.70}$	$2.31^{+1.05}_{-0.77}$	$2.13^{+0.19}_{-0.18}$	2.19 ± 0.12
$B^- \rightarrow D^0\tau^-\bar{\nu}_\tau$	$0.95^{+0.37}_{-0.31}$	$0.84^{+0.25}_{-0.21}$	$0.61 - 0.75$	0.77 ± 0.25
$B^- \rightarrow D^0l^-\bar{\nu}_l$	$2.19^{+0.99}_{-0.76}$	$2.48^{+1.12}_{-0.83}$	2.30 ± 0.20	2.27 ± 0.11
$\bar{B}^0 \rightarrow D^{*+}\tau^-\bar{\nu}_\tau$	$1.36^{+0.38}_{-0.37}$	$1.29^{+0.25}_{-0.24}$	$1.21 - 1.35$	1.84 ± 0.22
$\bar{B}^0 \rightarrow D^{*+}l^-\bar{\nu}_l$	$4.52^{+1.44}_{-1.31}$	$4.79^{+1.37}_{-1.18}$	4.94 ± 0.15	4.93 ± 0.11
$B^- \rightarrow D^{*0}\tau^-\bar{\nu}_\tau$	$1.47^{+0.43}_{-0.40}$	$1.40^{+0.27}_{-0.26}$	$1.31 - 1.48$	1.88 ± 0.20
$B^- \rightarrow D^{*0}l^-\bar{\nu}_l$	$4.87^{+1.60}_{-1.41}$	$5.18^{+1.49}_{-1.28}$	5.35 ± 0.16	5.69 ± 0.19

- [10] *Phys. Rev. D* 85, 094025 (2012);
 [26] *Sci. Bull.* 59, 125 (2014);
 [38] *Chin Phys C* 38, 090001(2014).

➤ Ratios : **R(D) and R(D*)**

$$\begin{aligned}
 R(D) &= 0.337 \pm 0.034 (\omega_B)^{+0.017}_{-0.014} (f_B) \\
 &= 0.337^{+0.038}_{-0.037}, \\
 R(D^*) &= 0.269 \pm 0.018 (\omega_B)^{+0.010}_{-0.009} (f_B) \\
 &= 0.269^{+0.021}_{-0.020},
 \end{aligned}$$

Errors~10%

Ratio	pQCD [26]	pQCD+Lattice	HQET [10]	HFAG [9]
$\mathcal{R}(D)$	$0.430^{+0.021}_{-0.026}$	$0.337^{+0.038}_{-0.037}$	0.296 ± 0.016	$0.391 \pm 0.041 \pm 0.028$
$\mathcal{R}(D^*)$	$0.301^{+0.013}_{-0.013}$	$0.269^{+0.021}_{-0.020}$	0.252 ± 0.003	$0.322 \pm 0.018 \pm 0.012$

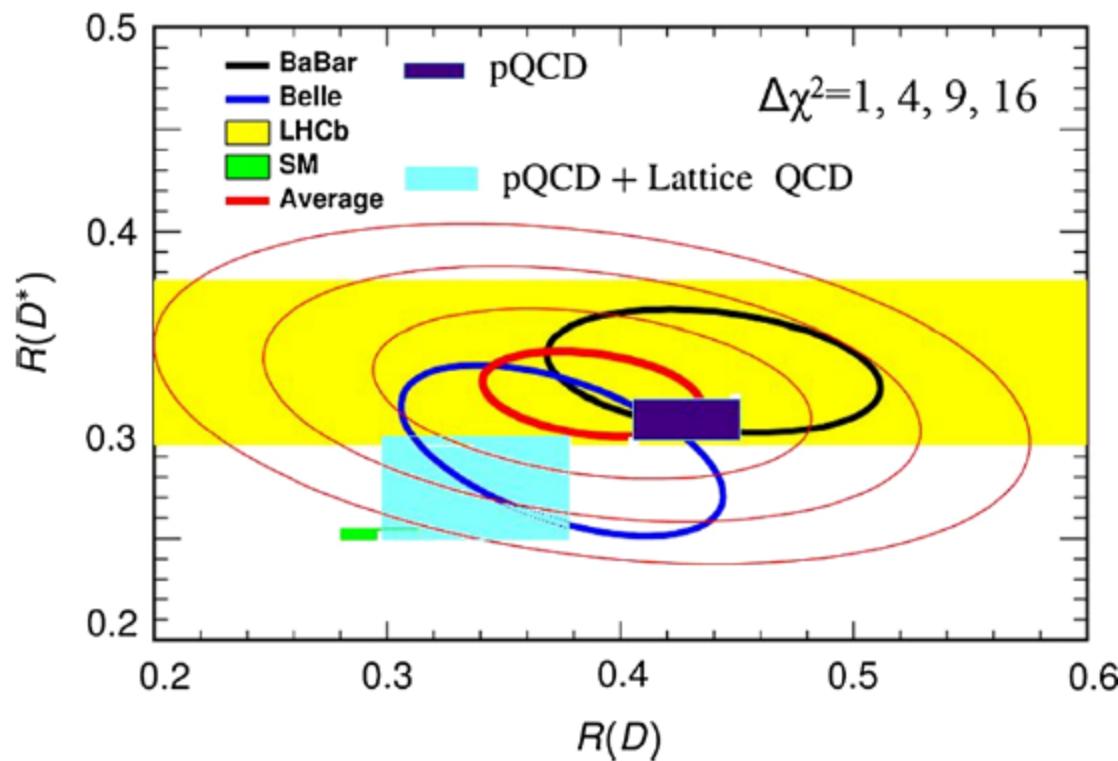


Fig. 3 The two-dimensional $R(D) - R(D^*)$ likelihood plane quoted directly from Z. Ligeti's talk (see footnote 1) with the inclusion of the new theoretical predictions obtained by using the pQCD approach (the purple box) or the “pQCD + Lattice QCD” method (the light-blue square)

Summary

We studied the semileptonic decays $B \rightarrow D^{(*)} l \nu_l$ in the framework of the SM by employing the “pQCD + Lattice QCD” method, and found that :

- ✓ By using the lattice QCD predictions as the input at the endpoint, the reliability of the extrapolation of the form factors from the low to the high q^2 region is improved effectively.
- ✓ The “pQCD + Lattice QCD” predictions for the branching ratios $\text{Br}(B \rightarrow D^{(*)} l \nu_l)$ agree well with the measured values with in one standard deviation.
- ✓ The “pQCD + Lattice QCD” predictions for the ratios $R(D^{(*)})$ are the following:

$$R(D) = 0.337^{+0.038}_{-0.037}, \quad R(D^*) = 0.269^{+0.021}_{-0.020}.$$

Thank you !



The $B \rightarrow D^{(*)} l \bar{v}_l$ decays in the pQCD approach with the Lattice QCD input

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Abstract In this paper, we studied the semileptonic decays $B \rightarrow D^{(*)} l^- \bar{v}_l$ by using the “pQCD + Lattice QCD” method. We made the extrapolation for the six relevant form factors by using the input values obtained from the pQCD factorization approach in the low q^2 region of $0 \leq q^2 \leq m_\tau^2$ and the lattice QCD input at the endpoint $q^2 = q_{\max}^2$. We then calculated the ratios $R(D)$ and $R(D^*)$ of the branching ratios $\mathcal{B}(B \rightarrow D^{(*)} l^- \bar{v}_l)$ and found numerically that (1) the “pQCD + Lattice QCD” predictions for the branching ratios $\mathcal{B}(B \rightarrow D^{(*)} l^- \bar{v}_l)$ agree well with the measured values within one standard deviation; and (2) the “pQCD + Lattice QCD” predictions for the ratios $R(D^{(*)})$ are $R(D) = 0.337^{+0.038}_{-0.037}$ and $R(D^*) = 0.269^{+0.021}_{-0.020}$; they agree with the data within 2σ deviation, in other words, one can explain the “ $R(D^{(*)})$ -puzzle” in the framework of the Standard Model.

Keywords “ $R(D^{(*)})$ -puzzle” · $B \rightarrow D^{(*)} l \bar{v}_l$ decays · The pQCD approach · Lattice QCD

the Belle and LHCb Collaborations this year with different methods [6–8]. For the ratios of the branching ratios: $R(D^{(*)}) \equiv \mathcal{B}(B \rightarrow D^{(*)} \tau \bar{v}_\tau) / \mathcal{B}(B \rightarrow D^{(*)} l \bar{v}_l)$ with $l = (e, \mu)$, the individual measurements [4–8] and the world averages [9] are the following:

$$\mathcal{R}(D)^{\text{exp}} = \begin{cases} 0.440 \pm 0.072, & \text{BaBar [4, 5],} \\ 0.375^{+0.064}_{-0.063} \pm 0.026, & \text{Belle [6, 7],} \\ 0.391 \pm 0.041 \pm 0.028, & \text{HFAG average [9],} \end{cases} \quad (1)$$

$$\mathcal{R}(D^*)^{\text{exp}} = \begin{cases} 0.332 \pm 0.024 \pm 0.018, & \text{BaBar [4, 5],} \\ 0.293^{+0.039}_{-0.037} \pm 0.015, & \text{Belle [6, 7],} \\ 0.336 \pm 0.027 \pm 0.030, & \text{LHCb [8],} \\ 0.322 \pm 0.018 \pm 0.012, & \text{HFAG average [9].} \end{cases} \quad (2)$$

On the theory side, the standard method to study the semileptonic $B \rightarrow D^{(*)} l \bar{v}_l$ decays is the heavy quark effective theory (HQET). The SM predictions based on the HQET as given in Ref. [10] are the following:

Semileptonic decays $B \rightarrow D^{(*)} l \bar{\nu}_l$ in the perturbative QCD factorization approach

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Abstract In this paper, we study the $B \rightarrow D^{(*)} l^- \bar{\nu}_l$ semileptonic decays and calculate the branching ratios $\mathcal{B}(B \rightarrow D^{(*)} l^- \bar{\nu}_l)$ and the ratios $R(D^{(*)})$ and $R_D^{l,\tau}$ by employing the perturbative QCD (pQCD) factorization approach. We find that (a) for $R(D)$ and $R(D^*)$ ratios, the pQCD predictions are $R(D) = 0.430^{+0.021}_{-0.026}$, $R(D^*) = 0.301 \pm 0.013$ and agree well with BaBar's measurements of $R(D^{(*)})$; (b) for the newly defined R_D^l and R_D^τ ratios, the pQCD predictions are $R_D^l = 0.450^{+0.064}_{-0.051}$ and $R_D^\tau = 0.642^{+0.081}_{-0.070}$, which may be more sensitive to the QCD dynamics of the considered semileptonic decays than $R(D^{(*)})$ and should be tested by experimental measurements.

Keywords B meson semileptonic decays · The pQCD factorization approach · Form factors · Branching ratios

where the isospin symmetry relations $\mathcal{R}(D^0) = \mathcal{R}(D^+) = \mathcal{R}(D)$ and $\mathcal{R}(D^{*0}) = \mathcal{R}(D^{*+}) = \mathcal{R}(D^*)$ have been imposed, and the statistical and systematic uncertainties have been combined in quadrature. These BaBar results are surprisingly larger than the standard model (SM) predictions given in Ref. [5]:

$$\mathcal{R}(D)^{\text{SM}} = 0.296 \pm 0.016, \quad \mathcal{R}(D^*)^{\text{SM}} = 0.252 \pm 0.003. \quad (2)$$

The combined BaBar results disagree with the SM predictions by 3.4σ [4, 6].

Since the report of BaBar measurements, this $R(D^{(*)})$ anomaly has been studied intensively by many authors, for example, in Refs. [7–18]. Some authors treat this 3.4σ deviation as the first evidence for new physics (NP) in semileptonic B meson decays to τ lepton [9–13], such as the NP contributions from the charged Higgs bosons in the Two-Higgs-Doublet models [10].

$$\begin{aligned}
F_+(q^2) &= \frac{1}{2} [f_1(q^2) + f_2(q^2)], \\
F_0(q^2) &= \frac{1}{2} f_1(q^2) \left[1 + \frac{q^2}{m_B^2 - m_D^2} \right] + \frac{1}{2} f_2(q^2) \left[1 - \frac{q^2}{m_B^2 - m_D^2} \right].
\end{aligned}$$

$$\begin{aligned}
f_1(q^2) &= 8\pi m_B^2 C_F \int dx_1 dx_2 \int b_1 db_1 b_2 db_2 \phi_B(x_1, b_1) \phi_D(x_2, b_2) \\
&\quad \times \left\{ [2r(1 - rx_2)] \cdot h_1(x_1, x_2, b_1, b_2) \cdot \alpha_s(t_1) \cdot \exp[-S_{ab}(t_1)] \right. \\
&\quad + \left[2r(2r_c - r) + x_1 r \left(-2 + 2\eta + \sqrt{\eta^2 - 1} - \frac{2\eta}{\sqrt{\eta^2 - 1}} + \frac{\eta^2}{\sqrt{\eta^2 - 1}} \right) \right] \\
&\quad \left. \cdot h_2(x_1, x_2, b_1, b_2) \cdot \alpha_s(t_2) \cdot \exp[-S_{ab}(t_2)] \right\},
\end{aligned}$$

$$\begin{aligned}
f_2(q^2) &= 8\pi m_B^2 C_F \int dx_1 dx_2 \int b_1 db_1 b_2 db_2 \phi_B(x_1, b_1) \phi_D(x_2, b_2) \\
&\quad \times \left\{ [2 - 4x_2 r(1 - \eta)] \cdot h_1(x_1, x_2, b_1, b_2) \cdot \alpha_s(t_1) \cdot \exp[-S_{ab}(t_1)] \right. \\
&\quad + \left[4r - 2r_c - x_1 + \frac{x_1}{\sqrt{\eta^2 - 1}}(2 - \eta) \right] \cdot h_2(x_1, x_2, b_1, b_2) \cdot \alpha_s(t_2) \cdot \exp[-S_{ab}(t_2)] \left. \right\},
\end{aligned}$$

$$\begin{aligned}
V(q^2) = & \ 8\pi m_B^2 C_F \int dx_1 dx_2 \int b_1 db_1 b_2 db_2 \phi_B(x_1, b_1) \phi_{D^*}^T(x_2, b_2) \cdot (1 + r) \\
& \times \left\{ [1 - rx_2] \cdot h_1(x_1, x_2, b_1, b_2) \cdot \alpha_s(t_1) \cdot \exp[-S_{ab}(t_1)] \right. \\
& \left. + \left[r + \frac{x_1}{2\sqrt{\eta^2 - 1}} \right] \cdot h_2(x_1, x_2, b_1, b_2) \cdot \alpha_s(t_2) \cdot \exp[-S_{ab}(t_2)] \right\},
\end{aligned}$$

$$\begin{aligned}
A_0(q^2) = & \ 8\pi m_B^2 C_F \int dx_1 dx_2 \int b_1 db_1 b_2 db_2 \phi_B(x_1, b_1) \phi_{D^*}^L(x_2, b_2) \\
& \times \left\{ [1 + r - rx_2(2 + r - 2\eta)] \cdot h_1(x_1, x_2, b_1, b_2) \cdot \alpha_s(t_1) \cdot \exp[-S_{ab}(t_1)] \right. \\
& \left. + \left[r^2 + r_c + \frac{x_1}{2} + \frac{\eta x_1}{2\sqrt{\eta^2 - 1}} + \frac{rx_1}{2\sqrt{\eta^2 - 1}} (1 - 2\eta(\eta + \sqrt{\eta^2 - 1})) \right] \right. \\
& \left. \cdot h_2(x_1, x_2, b_1, b_2) \cdot \alpha_s(t_2) \cdot \exp[-S_{ab}(t_2)] \right\},
\end{aligned}$$

$$\begin{aligned}
A_1(q^2) = & \ 8\pi m_B^2 C_F \int dx_1 dx_2 \int b_1 db_1 b_2 db_2 \phi_B(x_1, b_1) \phi_{D^*}^T(x_2, b_2) \cdot \frac{r}{1 + r} \\
& \times \left\{ 2[1 + \eta - 2rx_2 + r\eta x_2] \cdot h_1(x_1, x_2, b_1, b_2) \cdot \alpha_s(t_1) \cdot \exp[-S_{ab}(t_1)] \right. \\
& \left. + [2r_c + 2\eta r - x_1] \cdot h_2(x_1, x_2, b_1, b_2) \cdot \alpha_s(t_2) \cdot \exp[-S_{ab}(t_2)] \right\},
\end{aligned}$$

$$\begin{aligned}
A_2(q^2) = & \ \frac{(1+r)^2(\eta-r)}{2r(\eta^2-1)} \cdot A_1(q^2) - 8\pi m_B^2 C_F \int dx_1 dx_2 \int b_1 db_1 b_2 db_2 \phi_B(x_1, b_1) \\
& \cdot \phi_{D^*}^L(x_2, b_2) \cdot \frac{1+r}{\eta^2-1} \times \left\{ [(1+\eta)(1-r) - rx_2(1-2r+\eta(2+r-2\eta))] \right. \\
& \cdot h_1(x_1, x_2, b_1, b_2) \cdot \alpha_s(t_1) \cdot \exp[-S_{ab}(t_1)] \\
& + \left[r + r_c(\eta-r) - \eta r^2 + rx_1\eta^2 - \frac{x_1}{2}(\eta+r) + x_1 \left(\eta r - \frac{1}{2} \right) \sqrt{\eta^2-1} \right] \\
& \cdot h_2(x_1, x_2, b_1, b_2) \cdot \alpha_s(t_2) \exp[-S_{ab}(t_2)] \left. \right\},
\end{aligned}$$