



1st order transport coefficients of weakly coupled QGP

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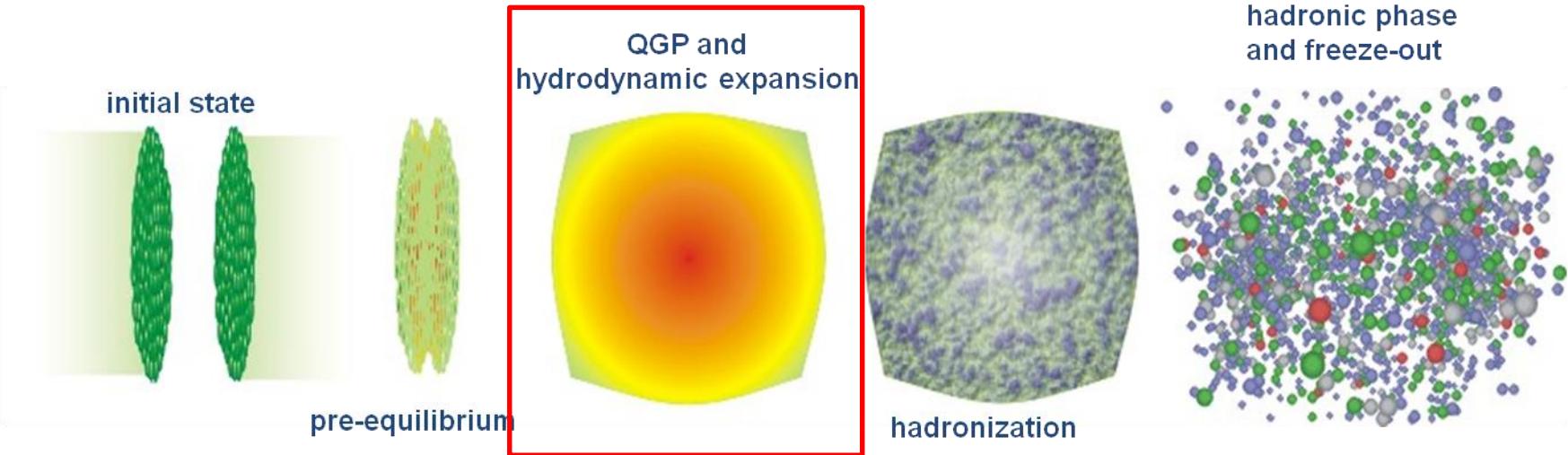
In collaboration with J. W. Chen, Y. F. Liu, S. Pu, Q. Wang, et al.

Outline

- Introduction
- Overview of effective kinetic theory
- Applications to non-zero chemical potential
- Numeric results of shear/bulk viscosity, conductivity
- Summary

Introduction

Evolution of Relativistic heavy ion collision (HIC)



- HIC mimics the Big Bang in the early universe
- Viscous hydrodynamics can be used to describe QGP phase successfully
- Shear/Bulk viscosity, Conductivity, 2nd order coef's ...

Basics of viscous hydrodynamics

- Hydrodynamic equation

$$\partial_\mu T^{\mu\nu} = 0$$

$$\partial_\mu j_a^\mu = 0$$

- 1st order viscous hydro

$$T^{\mu\nu} = (\epsilon + P)u^\mu u^\nu - Pg^{\mu\nu} - 2\eta \nabla_{\langle\mu} u_{\nu\rangle} - \zeta g^{\mu\nu} \nabla \cdot u$$

$$j_a^\mu = n_a u^\mu + \sum_b \lambda_{ab} \left[-\nabla \left(\frac{\mu_b}{T} \right) + Q_b \frac{\vec{E}}{T} \right]$$

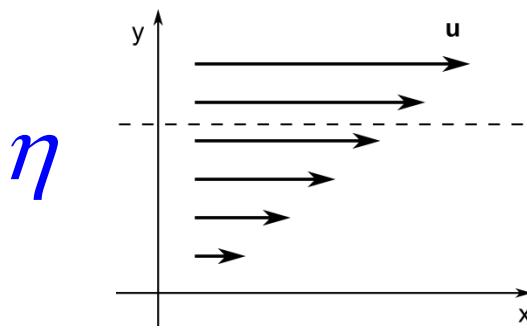
η – shear viscosity,

ζ – bulk viscosity,

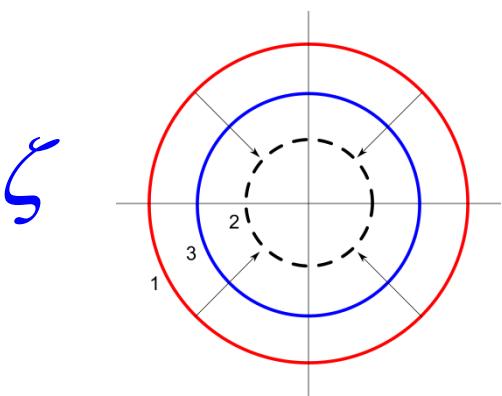
λ_{ab} – conductivity.

$\nabla_{\langle\mu} u_{\nu\rangle}$: symmetric traceless part of $\nabla_\mu u_\nu$

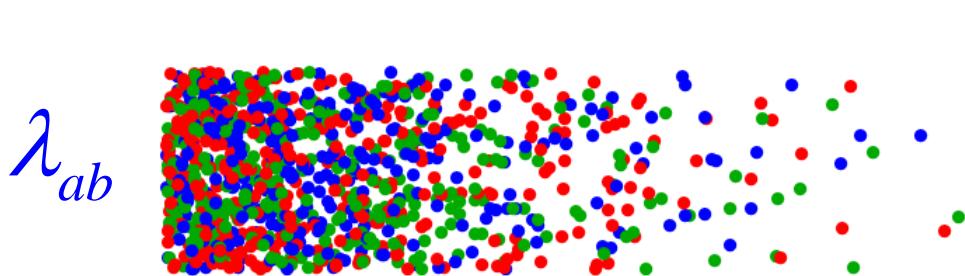
Physical interpretation



Response to flow velocity gradient
Extra momentum flux δT^{xy}



Response to expansion/compression
Extra pressure δP



Response to $\nabla \mu$, ∇T and E
Induced current

Kubo Formulae

- Linear response theory gives Kubo formulae

$$\eta = \frac{1}{20} \lim_{\omega \rightarrow 0} \frac{1}{\omega} \int d^4x e^{i\omega t} \langle [\pi_{lm}(t, \mathbf{x}), \pi_{lm}(0)] \rangle,$$

$$\zeta = \frac{1}{2} \lim_{\omega \rightarrow 0} \frac{1}{\omega} \int d^4x e^{i\omega t} \langle [\mathcal{P}(t, \mathbf{x}), \mathcal{P}(0)] \rangle, \quad \mathcal{P} = \frac{1}{3} (T^{11} + T^{22} + T^{33})$$

$$\pi_{lm} = T_{lm} - \delta_{lm} \mathcal{P}$$

$$\lambda_{ab} = \frac{T}{3} \lim_{\omega \rightarrow 0} \frac{1}{\omega} \int d^4x e^{i\omega t} \langle [j_a^i(t, \mathbf{x}), j_b^i(0)] \rangle.$$

- 1st principle calculation of transport coefficients
 - AdS/CFT
 - Lattice QCD
 - Perturbative QFT / effective kinetic theory

AdS/CFT: KSS bound

- Conjecture from AdS/CFT

P. Kovtun, D.Son, A. Starinets, PRL2005

Universal bound conjecture

$$\eta / s \geq \frac{1}{4\pi} \quad (T > 0, \mu = 0)$$

- AdS/CFT, finite chemical potential

- *O.Saremi, JHEP 2006*
- *K.Maeda, M.Natsuume, T.Okamura, PRD 2006*
- *P.Benincasa, A.Buchel, R.Naryshkin, PLB 2007*

$$\eta / s \geq \frac{1}{4\pi} \quad (T > 0, \mu > 0)$$

- Fit to Data gives

$$\frac{1}{4\pi} < \left(\frac{\eta}{s} \right)_{\text{QGP}} < \frac{2.5}{4\pi}$$

H. Song, NPA2013

Lattice QCD evaluation

- The Euclidean correlator is evaluated on lattice

$$C_s(x, \vec{p}) = L_0^5 \int d^3 \vec{x} e^{i\vec{p} \cdot \vec{x}} \langle T_{12}(0) T_{12}(x_0, \vec{x}) \rangle,$$

$$C_b(x, \vec{p}) = \frac{L_0^5}{9} \sum_{i,j=1}^3 \int d^3 \vec{x} e^{i\vec{p} \cdot \vec{x}} \langle T_{ii}(0) T_{jj}(x_0, \vec{x}) \rangle,$$

- The spectral functions are defined as

$$C_{s,b}(x_0, \vec{p}) = L_0^5 \int_0^\infty \rho_{s,b}(\omega, \vec{p}) \frac{\cosh \omega(L_0/2 - x_0)}{\sinh(\omega L_0/2)} d\omega$$

- Inverse above formulae to obtain $\rho_{s,b}(\omega, \vec{p})$

$$\eta(T) = \pi \lim_{\omega \rightarrow 0} \frac{\rho_s(\omega, \vec{0})}{\omega}, \quad \zeta(T) = \pi \lim_{\omega \rightarrow 0} \frac{\rho_b(\omega, \vec{0})}{\omega}$$

Examples of Lattice QCD results

Shear viscosity

$$\eta / s = \begin{cases} 0.134(33) & (T = 1.65T_c) \\ 0.102(56) & (T = 1.24T_c) \end{cases}$$

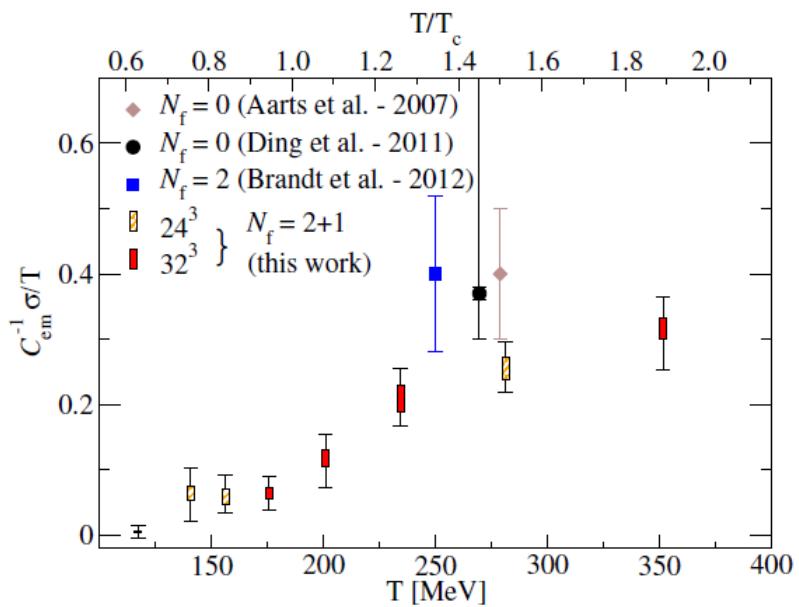
H.Meyer, PRD2007

Bulk viscosity

$$\zeta / s = \begin{cases} 0.008(7) \begin{bmatrix} 0.15 \\ 0 \end{bmatrix} & (T = 1.65T_c) \\ 0.065(17) \begin{bmatrix} 0.37 \\ 0.01 \end{bmatrix} & (T = 1.24T_c) \\ 0.73(3) \begin{bmatrix} 2.0 \\ 0.5 \end{bmatrix} & (T = 1.02T_c) \end{cases}$$

H.Meyer, PRD2008

Conductivity

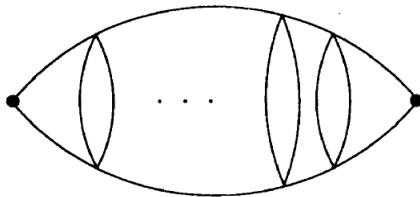


A.Amato, G.Aarts, P.Giudice, S.Hands, J.Skullerud, PRL2013

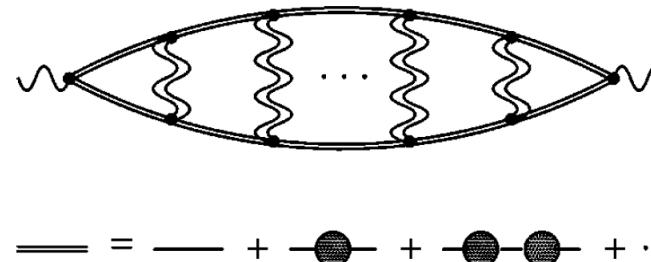
Near T_c , strong coupling region

Perturbative QFT

- Feynman diagrams to be summed



S. Jeon, PRD1995



$$\text{~~~~~} = \text{~~~~~} + \text{~~~~~} + \text{~~~~~} + \dots$$

P. Arnold, D. Son, L. Yaffe, PRD1999

- Infinite number of diagrams need to be summed, due to the **pinch singularities** (S.Jeon, PRD1995)
- Infinite Feynman diagrams summation
→ Effective vertex and propagators, kinetic equation.

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Effective kinetic theory

- **Summation of diagrams \leftrightarrow effective kinetic theory**

(S.Jeon, L.Yaffe, PRD1996; J.Gagnon, S.Jeon, PRD2007)

- **Phase space distribution** $\tilde{f}^g(x, p)$

$$E_p^g = \sqrt{\mathbf{p}^2 + m_g^2(q(x))}$$

$q(x)$: auxiliary field dependent on x

$$m_g^2(q(x)) = 2C_F g^2 \int \frac{d^3 p}{(2\pi)^3} \frac{\tilde{f}^g}{E^g},$$

$T(x)$ and $\mu_q(x)$ dependences enter

- **Energy-momentum tensor** (with only gluons)

$$T^{\mu\nu} = N_g \int \frac{d^3 p}{(2\pi)^3} \frac{\tilde{f}^g(p, x)}{E_a} \left[p^\mu p^\nu - g^{\mu\nu} U(q(x)) \right].$$

- **Boltzmann equation**

$$\frac{d\tilde{f}^g}{dt} = -\mathcal{C}[\tilde{f}]$$

Variational method

- Consider a system slightly away from equilibrium

$$\tilde{f} = f + \delta f, \quad f \equiv f_{\text{eq}}$$

Boltzmann equation $d\tilde{f} / dt = -\mathcal{C}[\tilde{f}]$ becomes,

$$\frac{d\tilde{f}}{dt} \approx \frac{\partial f}{\partial t} + \vec{v} \cdot \nabla f + \vec{F} \cdot \nabla_{\vec{p}} f \approx -\mathcal{C}^{(1)} \otimes \delta f$$

LHS involve the derivative of flow velocity u

The form of δf must be

$$\delta f = -f(1+f)\chi^g$$

$$\chi^g = \left[\frac{A^g(p)}{T} \nabla \cdot \mathbf{u} + \frac{B^g(p)}{T} \hat{p}_{\langle i} \hat{p}_{j\rangle} \nabla_{\langle i} u_{j\rangle} \right]$$

and the coefficients are constrained functions.

Variational method

- Constraints for **A's** and **B's**

$$p_{\langle i} p_{j\rangle} = \frac{E_g}{f^g F^g} \frac{1}{N_g} \times \frac{1}{2} \int d\Gamma_{gg \rightarrow gg} f_{k_1}^g f_{k_2}^g F_{k_3}^g F_p^g$$

$$\times \left[-B_{ij}^g(k_1) - B_{ij}^g(k_2) + B_{ij}^g(k_3) + B_{ij}^g(p) \right]$$


$$|S_\eta\rangle = \mathcal{C}_\eta |B\rangle$$

$$\frac{p^2}{3} - \left(E_g^2 - T^2 \frac{\partial m_g^2}{\partial T^2} \right) c_s^2 = \frac{E_g}{f^g F^g} \frac{1}{N_g} \times \frac{1}{2} \int_{k_1 k_2 k_3} d\Gamma_{gg \rightarrow gg} f_{k_1}^g f_{k_2}^g F_{k_3}^g F_p^g$$

$$\times \left[-A^g(k_1) - A^g(k_2) + A^g(k_3) + A^g(p) \right]$$


$$|S_\zeta\rangle = \mathcal{C}_\zeta |A\rangle$$

Expressions for viscosities

- Variation of energy-momentum tensor

$$\delta T^{\mu\nu} = \int \frac{d^3 p}{(2\pi)^3 E_p^g} f^g \left[1 + f^g \right] \chi^g(x, p) \times \left(p^\mu p^\nu - u^\mu u^\nu T^2 \frac{\partial m_g^2}{\partial T^2} \right)$$

- From viscosity **definitions** and χ^g expression

$$\delta T^{\mu\nu} = -2\eta \nabla_{\langle\mu} u_{\nu\rangle} - \zeta g^{\mu\nu} \nabla \cdot u$$

$$\chi^g = \left[\frac{A^g(p)}{T} \nabla \cdot \mathbf{u} + \frac{B^g(p)}{T} \hat{p}_{\langle i} \hat{p}_{j\rangle} \nabla_{\langle i} u_{j\rangle} \right]$$

one obtain

$$\eta = \frac{1}{10T} N_g \int \frac{d^3 p}{(2\pi)^3 E_a} f^g F^g B_{jk}^g(p) p_{[j} p_{k]} = \langle B | S_\eta \rangle,$$

$$\zeta = \frac{1}{T} \int \frac{d^3 p}{(2\pi)^3 E_a} f^g F^g A^g(p) \frac{p^2}{3} = \langle A | S_\zeta \rangle.$$

Solving η and ζ

- With Landau-Lifshitz condition $\delta T^{00} = 0$

$$\zeta = \langle \mathbf{A} | \mathbf{S}'_\zeta \rangle = \langle \mathbf{A} | \mathbf{S}_\zeta \rangle$$

- Final equations for viscosities

$$\eta = \langle \mathbf{B} | \mathbf{S}_\eta \rangle = \langle \mathbf{B} | \mathcal{C}_\eta | \mathbf{B} \rangle,$$

$$\zeta = \langle \mathbf{A} | \mathbf{S}'_\zeta \rangle = \langle \mathbf{A} | \mathbf{S}_\zeta \rangle = \langle \mathbf{A} | \mathcal{C}_\zeta | \mathbf{A} \rangle.$$

$$|\mathbf{S}_\eta\rangle = \mathcal{C}_\eta |\mathbf{B}\rangle$$

$$|\mathbf{S}_\zeta\rangle = \mathcal{C}_\zeta |\mathbf{A}\rangle$$

Solving above equations, one get η, ζ

$$\begin{cases} |\mathbf{B}\rangle = \mathcal{C}_\eta^{-1} |\mathbf{S}_\eta\rangle \\ |\mathbf{A}\rangle = \mathcal{C}_\zeta^{-1} |\mathbf{S}_\zeta\rangle \end{cases} \xrightarrow{\hspace{1cm}} \begin{cases} \eta = \langle \mathbf{S}_\eta | \mathcal{C}_\eta^{-1} | \mathbf{S}_\eta \rangle \\ \zeta = \langle \mathbf{S}_\zeta | \mathcal{C}_\zeta^{-1} | \mathbf{S}_\zeta \rangle \end{cases}$$

- Extension to system with quarks ($\mu_q = 0$)

(P.Arnold, G.Moore, L.Yaffe, 2000; P.Arnold, C.Dogan, G.Moore, 2006;
J.W.Chen, J.Deng, H.Dong, Q.Wang, 2011)

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Finite chemical potential

- Quark and gluon effective mass

$$m_a^2(q(x)), \quad a = g, q, \bar{q}$$

$$m_g^2 = 2g^2 \left(v_g \frac{C_A}{d_A} \int \frac{d^3 p}{(2\pi)^3} \frac{\tilde{f}^g}{2E^g} + v_q \sum_{a=1}^{N_f} \frac{C_F}{d_A} \int \frac{d^3 p}{(2\pi)^3} \frac{\tilde{f}^a}{2E^a} + v_{\bar{q}} \sum_{a=1}^{N_f} \frac{C_F}{d_A} \int \frac{d^3 p}{(2\pi)^3} \frac{\tilde{f}^{\bar{a}}}{2E^{\bar{a}}} \right),$$

$$m_q^2 = 2C_F g^2 \left(\int \frac{d^3 p}{(2\pi)^3} \frac{2\tilde{f}^g}{2E^g} + \int \frac{d^3 p}{(2\pi)^3} \frac{\tilde{f}^a}{2E^a} + \int \frac{d^3 p}{(2\pi)^3} \frac{\tilde{f}^{\bar{a}}}{2E^{\bar{a}}} \right).$$

- If we take $\tilde{f} = f_{eq}$, they will reduce to

$$m_g^2 = \frac{C_A}{6} g^2 T^2 + \sum_{a=1}^{N_f} \frac{t_F}{6} g^2 \left(T^2 + \frac{3}{\pi^2} \mu_a^2 \right),$$

$$m_q^2 = \frac{1}{4} C_F g^2 \left(T^2 + \frac{\mu_a^2}{\pi^2} \right).$$

Finite chemical potential

- Variation of quark and gluon distributions

$$f^g(x, p) = -f_{eq}^g(p)[1 + f_{eq}^g(p)]\chi^g(x, p),$$

$$f^q(x, p) = -f_{eq}^q(p)[1 - f_{eq}^q(p)]\chi^q(x, p),$$

$$f^{\bar{q}}(x, p) = -f_{eq}^{\bar{q}}(p)[1 - f_{eq}^{\bar{q}}(p)]\chi^{\bar{q}}(x, p)$$

- Energy momentum tensor

$$\delta T^{\mu\nu} = - \sum_{a=g,q,\bar{q}} \int \frac{d^3 p}{(2\pi)^3 E_p} f^a(p) \left[1 \pm f^a(p) \right] \chi^a(x, p)$$

$$\times \left(p^\mu p^\nu - u^\mu u^\nu T^2 \frac{\partial m_a^2}{\partial T^2} - u^\mu u^\nu \mu^2 \frac{\partial m_a^2}{\partial \mu^2} \right),$$

$$\kappa_a = \begin{cases} 0 & a = g \\ 1 & a = q \\ -1 & a = \bar{q} \end{cases}$$

$$\delta j_a^\mu = - \sum_{a=g,q,\bar{q}} \int \frac{d^3 p}{(2\pi)^3 E_p} f^a(p) [1 \pm f^a(p)] \chi^a(x, p) \left(\kappa_a \frac{p^\mu}{E^a} - u^\mu \frac{\partial E^a}{\partial \mu} \right).$$

Finite chemical potential

- The variation now rely on the gradient of μ and T

$$\chi^a(x, p) = \left[\frac{A^a(p)}{T} \nabla_i u_i + \frac{B^a(p)}{T} \hat{p}_{\langle i} \hat{p}_{j\rangle} \nabla_{\langle i} u_{j\rangle} + \frac{C^{ab}(p)}{T} \nabla \left(\frac{\mu_b}{T} \right) \right],$$

- Transport coefficients

$$\eta = \frac{1}{10T} \sum_{a=g,q,\bar{q}} N_a \int \frac{d^3 p}{(2\pi)^3 E^a} f^a F^a \mathbf{B}_{jk}^a \mathbf{p}_{\langle j} p_{k\rangle} = \langle \mathbf{B} | S_\eta \rangle,$$

$$\zeta = \frac{1}{T} \sum_{a=g,q,\bar{q}} \int \frac{d^3 p}{(2\pi)^3 E^a} f^a F^a \mathbf{A}^a \frac{\mathbf{p}^2}{3} = \langle \mathbf{A} | S_\zeta \rangle,$$

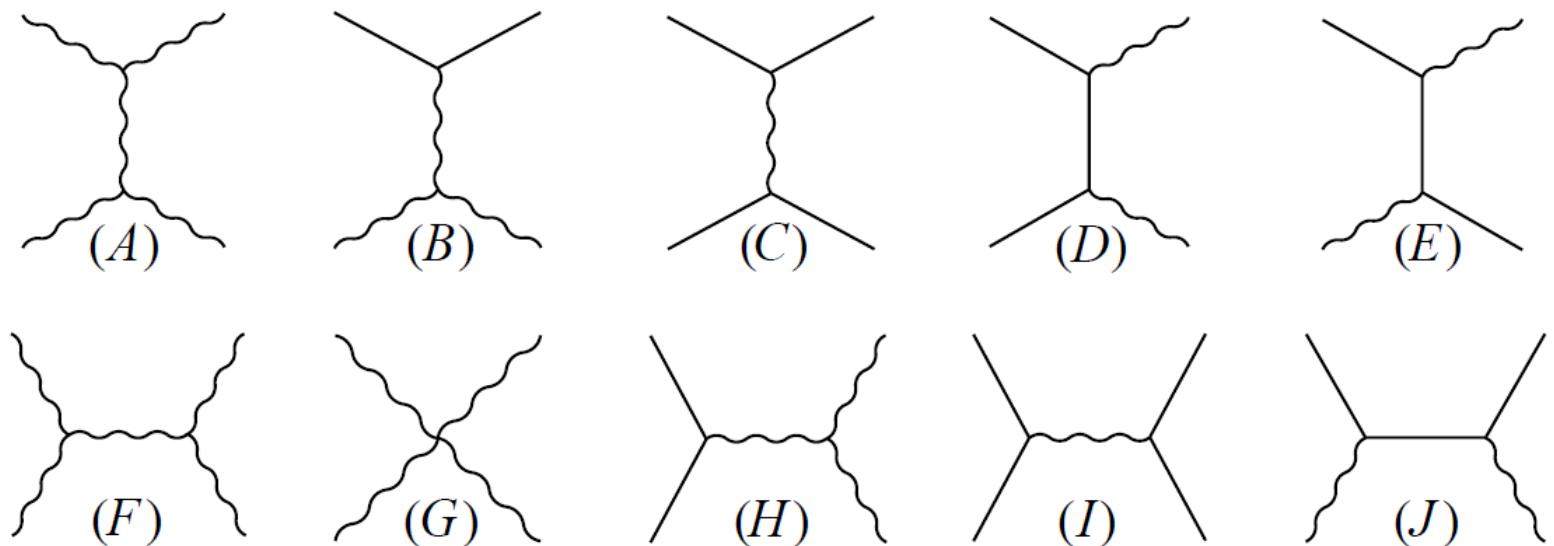
$$\lambda_{ab} = \frac{1}{T} \int \frac{d^3 p}{(2\pi)^3} \frac{1}{E_p} \left(f^a F^a \mathbf{p} \cdot \mathbf{C}^{ab} - f^{\bar{a}} F^{\bar{a}} \mathbf{p} \cdot \mathbf{C}^{\bar{a}b} \right) = \langle \mathbf{C} | S_\lambda \rangle$$

Boltzmann Equation

- Boltzmann equation

$$\frac{d\tilde{f}_p^a(x)}{dt} = -\mathcal{C}[\tilde{f}^a], \quad a = g, q, \bar{q}.$$

- Collision terms: $2 \rightarrow 2$ partonic scattering (**LL**)



Constraints from Boltzmann equation

η

$$p_{\langle i} p_{j \rangle} = \frac{E_g}{f^g F^g} \frac{1}{N_g} \times \frac{1}{2} \int d\Gamma_{gg \rightarrow gg} f_{k_1}^g f_{k_2}^g F_{k_3}^g F_p^g \left[-B_{ij}^g(k_1) - B_{ij}^g(k_2) + B_{ij}^g(k_3) + B_{ij}^g(p) \right] \\ +(gq \rightarrow gq) + (g\bar{q} \rightarrow g\bar{q}) + (q\bar{q} \rightarrow gg)$$

ζ

$$\frac{p^2}{3} - \left(E_g^2 - T^2 \frac{\partial m_g^2}{\partial T^2} - \mu^2 \frac{\partial m_g^2}{\partial \mu^2} \right) \left(\frac{\partial P}{\partial \epsilon} \right)_n + E_g \left(\frac{\partial E_g}{\partial \mu} - \kappa_g \right) \left(\frac{\partial P}{\partial n} \right)_\epsilon \\ = \frac{E_g}{f^g F^g} \frac{1}{N_g} \times \frac{1}{2} \int d\Gamma_{gg \rightarrow gg} f_{k_1}^g f_{k_2}^g F_{k_3}^g F_p^g \left[-A^g(k_1) - A^g(k_2) + A^g(k_3) + A^g(p) \right] \\ +(gq \rightarrow gq) + (g\bar{q} \rightarrow g\bar{q}) + (q\bar{q} \rightarrow gg)$$

λ_{ab}

$$\left(\frac{n_a}{\epsilon + P} - \frac{\kappa_g}{E^c} \delta^{ca} \right) \mathbf{p} \\ = \frac{E_g}{f^g F^g} \frac{1}{N_g} \times \frac{1}{2} \int d\Gamma_{gg \rightarrow gg} f_{k_1}^g f_{k_2}^g F_{k_3}^g F_p^g \left[-C^{ga}(k_1) - C^{ga}(k_2) + C^{ga}(k_3) + C^{ga}(p) \right] \\ +(gq \rightarrow gq) + (g\bar{q} \rightarrow g\bar{q}) + (q\bar{q} \rightarrow gg)$$

Transport coefficients

- Together with the Landau-Lifshitz condition

$$\eta = \langle \mathbf{B}^a | S_\eta^{\ a} \rangle = \langle \mathbf{B}^a | \mathcal{C}_\eta^{ab} | \mathbf{B}^b \rangle,$$

$$\zeta = \langle \mathbf{A}^a | S_\zeta^{\ a} \rangle = \langle \mathbf{A}^a | S_\zeta^{\ a} \rangle = \langle \mathbf{A}^a | \mathcal{C}_\zeta^{ab} | \mathbf{A}^b \rangle,$$

$$\lambda_{ab} = \langle S^{\prime ia} | C^{ib} \rangle = \langle S^{ia} | C^{ib} \rangle = \langle C^{ia} | \mathcal{C}_{\lambda,ij} | C^{jb} \rangle.$$

- By solving above equations, one get

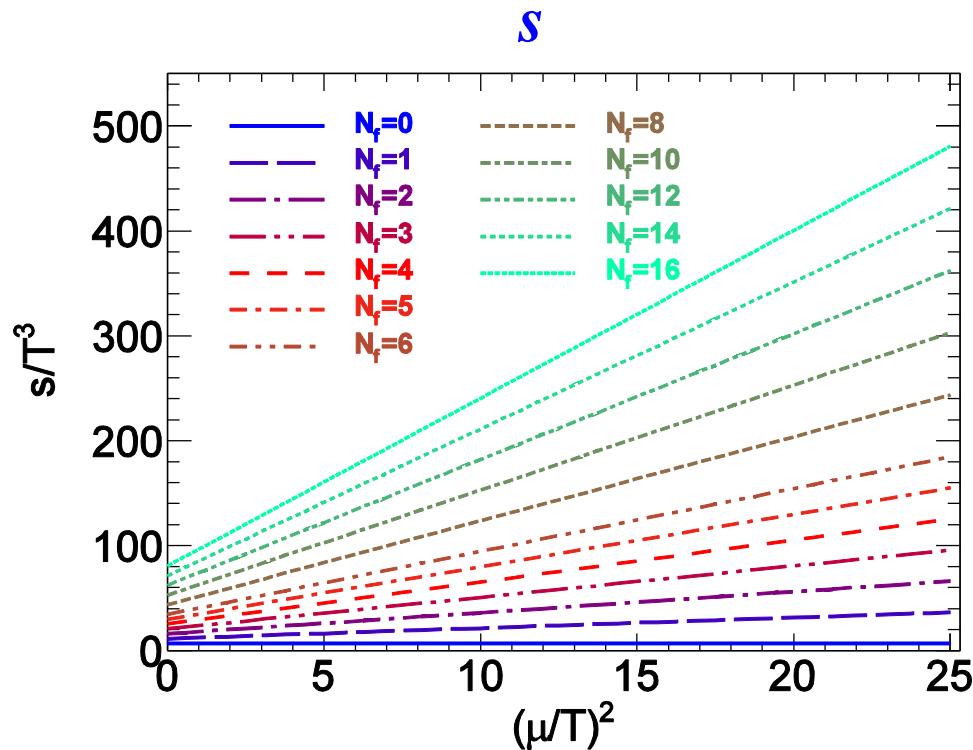
$$\begin{cases} |\mathbf{B}^a\rangle = (\mathcal{C}_\eta^{-1})^{ab} |S_\eta^{\ b}\rangle \\ |\mathbf{A}^a\rangle = (\mathcal{C}_\zeta^{-1})^{ab} |S_\zeta^{\ b}\rangle \\ |\mathbf{C}^{ia}\rangle = (\mathcal{C}_\lambda^{-1})_{ij} |S_\lambda^{ja}\rangle \end{cases} \xrightarrow{\hspace{1cm}} \begin{cases} \eta = \langle S_\eta^{\ a} | (\mathcal{C}_\eta^{-1})^{ab} | S_\eta^{\ b} \rangle \\ \zeta = \langle S_\zeta^{\ a} | (\mathcal{C}_\zeta^{-1})^{ab} | S_\zeta^{\ b} \rangle \\ \lambda_{ab} = \langle S_\lambda^{ia} | (\mathcal{C}_\lambda^{-1})_{ij} | S_\lambda^{jb} \rangle \end{cases}$$

- The rest of the work is numeric.

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Entropy density



$$s = N_g \frac{16\pi^5}{45} \mathbf{T}^3 + N_f N_q \left(\frac{28\pi^5}{45} \mathbf{T}^3 + \frac{4\pi^3}{3} \boldsymbol{\mu}^2 \mathbf{T} \right)$$

- $\mu \ll T, s \propto T^3$
- $\mu \gg T, s \propto \boldsymbol{\mu}^2 \mathbf{T} \sim \text{Size of the transitional zone near Fermi sphere}$

Shear viscosity $\mu_u = \mu_d = \dots = \mu$

- Comparing with AMY (P.Arnold, G.Moore, L. Yaffe, JHEP2000)

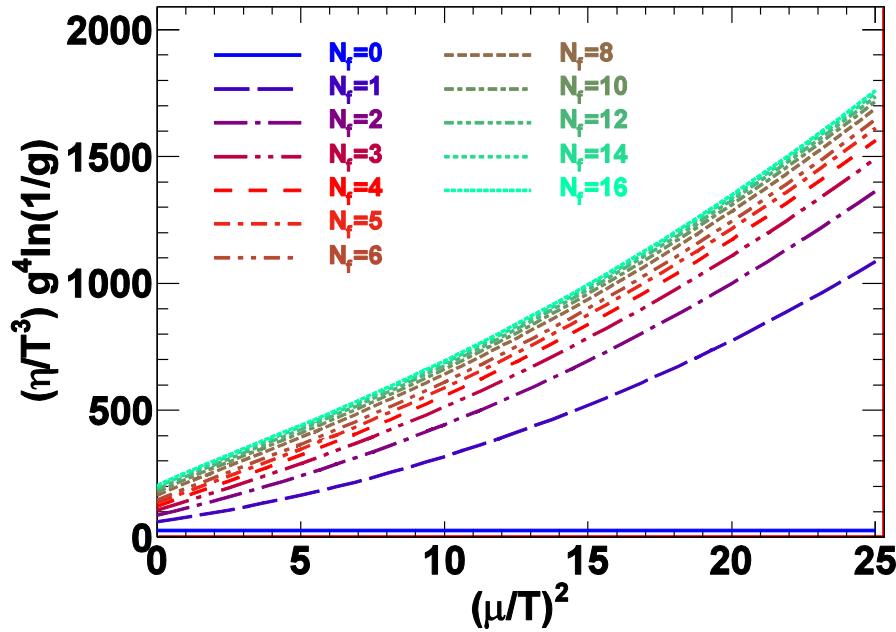
$$\tilde{\eta} = a_\eta + b_\eta (\mu/T)^2 + \dots$$

N_f	a_η	b_η	N_f	a_η	b_η
0	27.125	0	9	172.564	50.381
1	60.808	16.619	10	178.839	51.301
2	86.472	27.281	11	184.389	52.028
3	106.664	34.454	12	189.333	52.608
4	122.957	39.459	13	193.764	53.074
5	136.380	43.055	14	197.760	53.450
6	147.627	45.703	15	201.380	53.755
7	157.47	47.690	16	204.675	54.003
8	165.412	49.207			

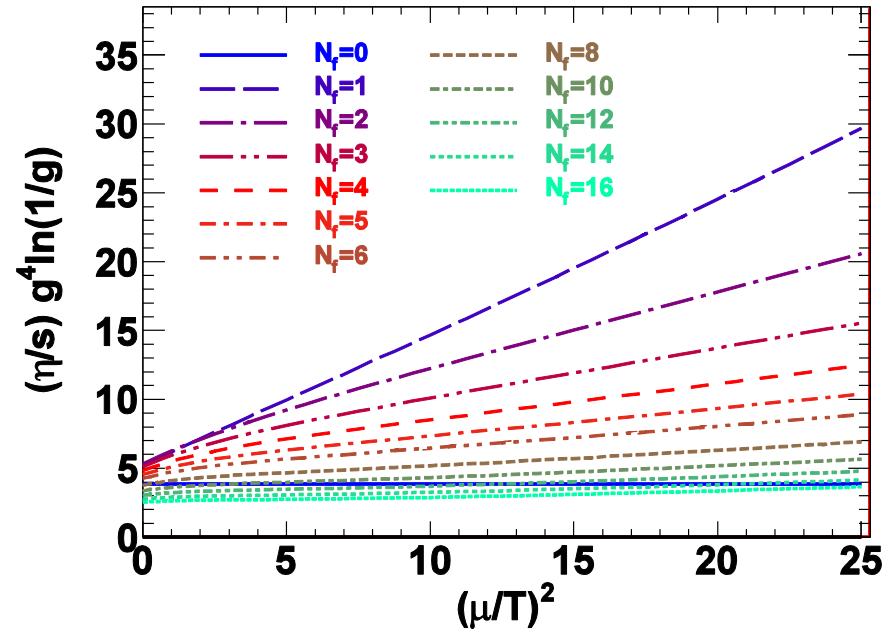
Consistent with AMY

Shear viscosity $\mu_u = \mu_d = \dots = \mu$

η



η / s



- η and η/s are minimal at $\mu=0$
- η/s scale as $(\mu/T)^2$ at large μ , why?

$$\begin{aligned} \eta &\sim m \bar{v} N l \\ m \bar{v} &\sim \mu, \quad N \sim \mu^2 T, \quad l \sim \mu T^{-2} \end{aligned} \quad \left. \right\} \Rightarrow \eta \sim \mu^4 / T$$

Bulk viscosity $\mu_u = \mu_d = \dots = \mu$

- Comparing with ADM (P.Arnold, C.Dogan, G.Moore, PRD2006)

$$\tilde{\zeta} = a_\zeta + b_\zeta (\mu/T)^2 + \dots$$

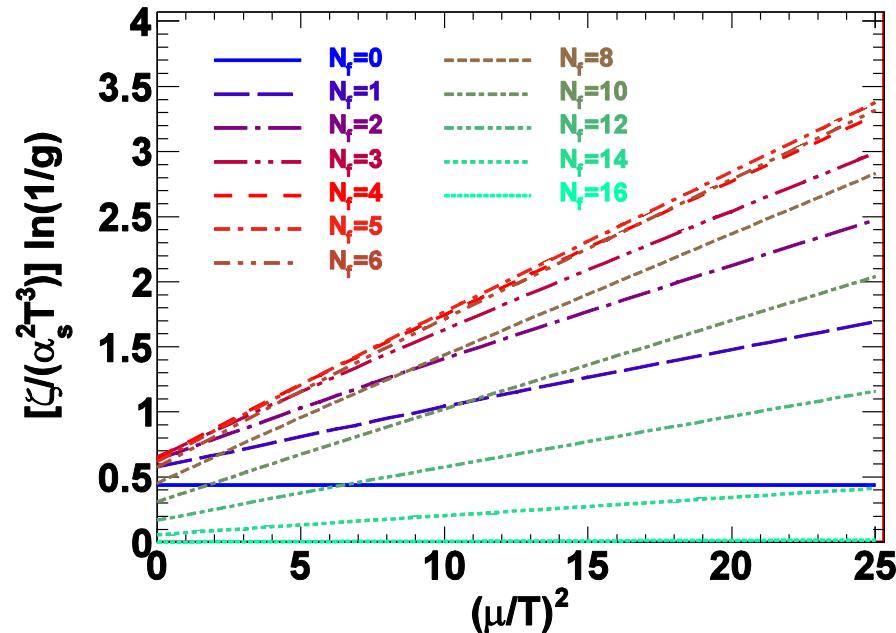
N_f	a_ζ	b_ζ	N_f	a_ζ	b_ζ
0	0.4430	0	9	0.3847	0.0873
1	0.5816	0.0393	10	0.3113	0.0725
2	0.6379	0.0747	11	0.2389	0.0569
3	0.6568	0.0981	12	0.1706	0.0414
4	0.6495	0.1116	13	0.1096	0.0270
5	0.6218	0.1172	14	0.0592	0.0148
6	0.5778	0.1163	15	0.0225	0.0057
7	0.5213	0.1103	16	0.0026	0.0007
8	0.4558	0.1003			

Consistent with ADM LL results

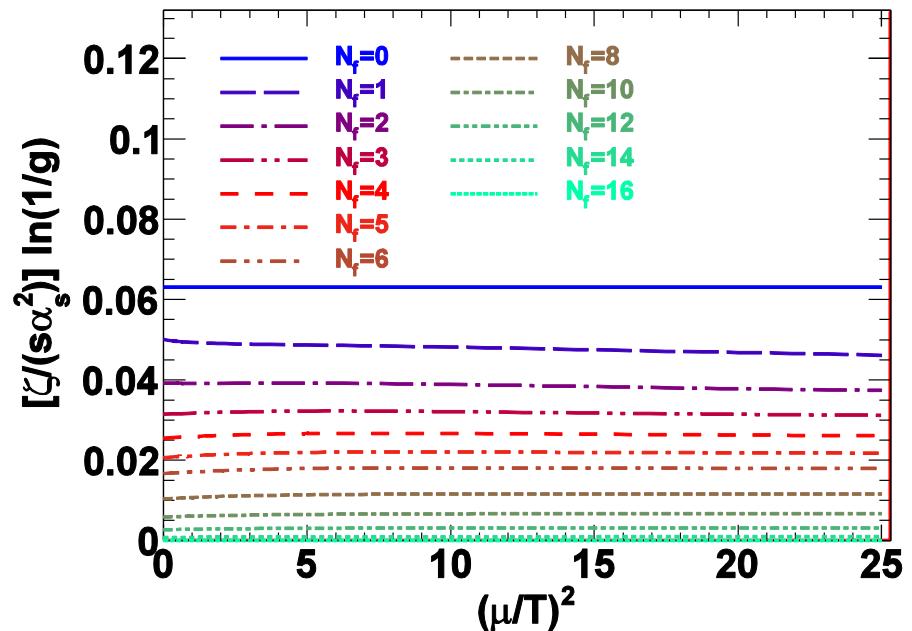
Bulk viscosity

$$\mu_u = \mu_d = \dots = \mu$$

ζ



ζ / s

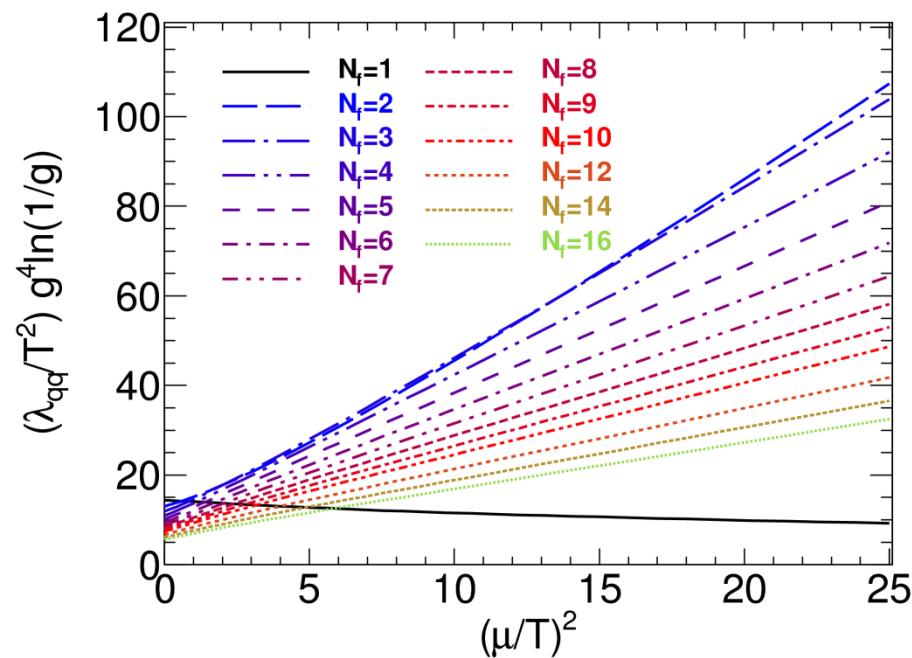


- ζ is minimal at $\mu=0$
- ζ/s is almost constant with increasing μ

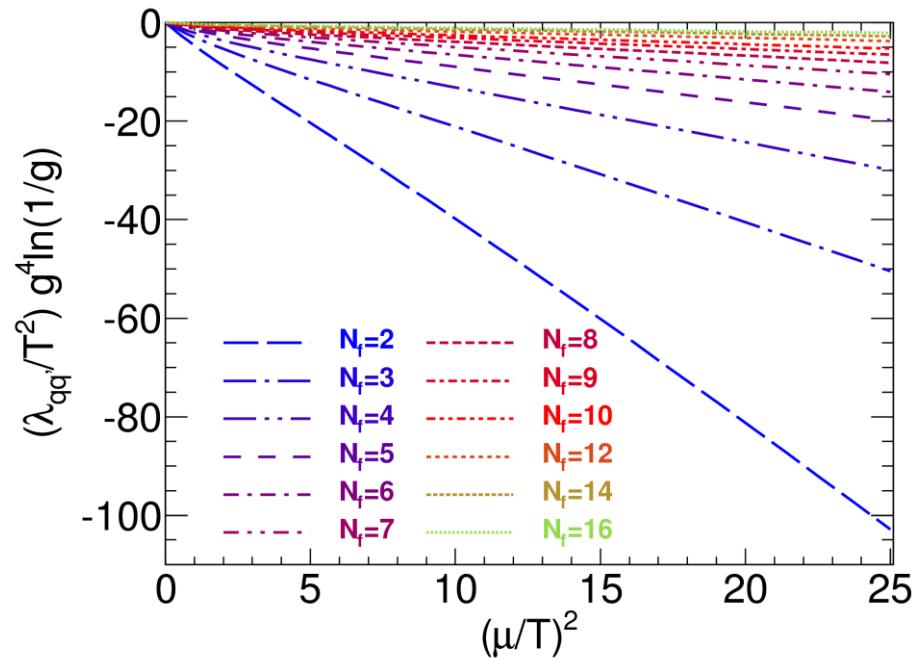
Conductivities

$$\mu_u = \mu_d = \dots = \mu$$

λ_{aa}



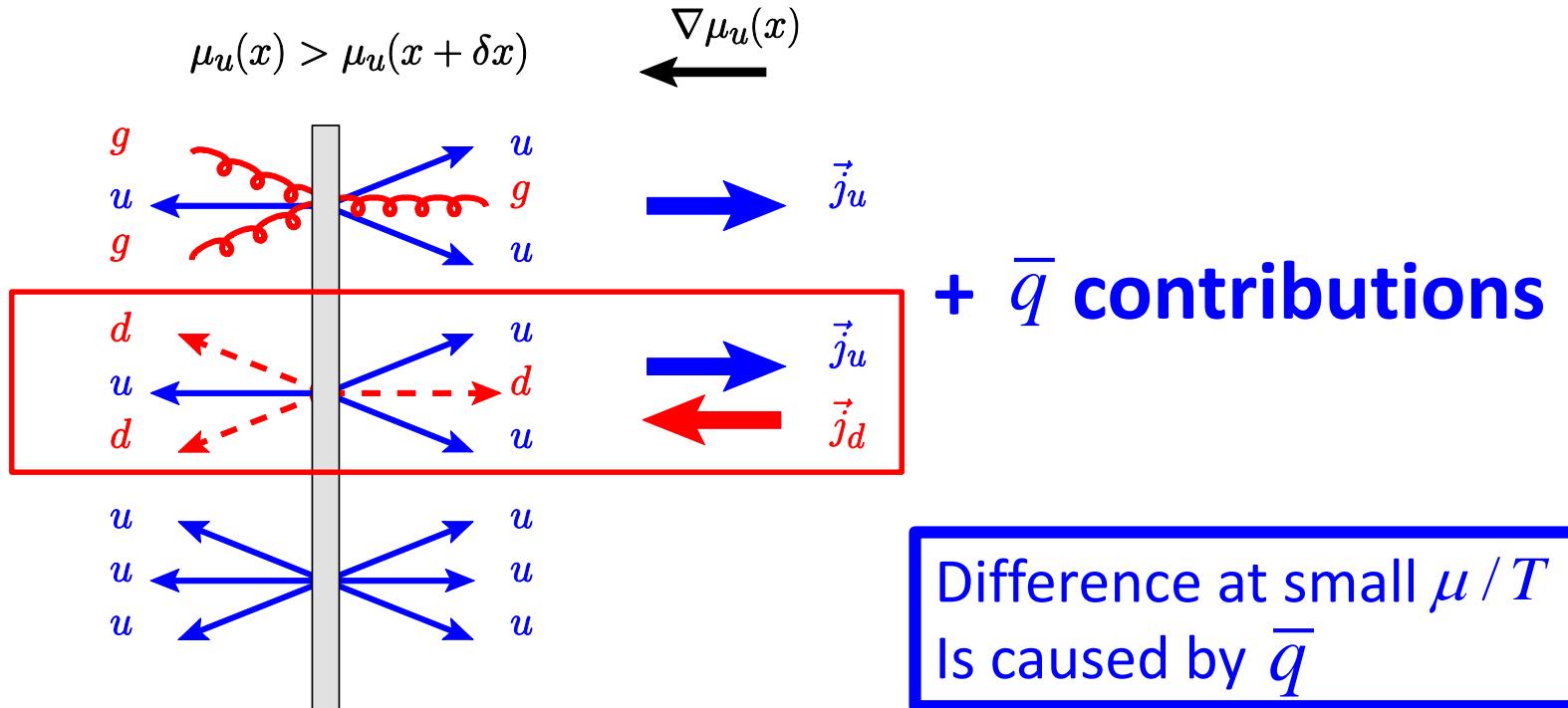
λ_{ab}



- λ_{ab} is negative, zero at $\mu/T = 0$
- $\lambda_{aa}, \lambda_{ab}$ scale as $(\mu/T)^2$ for large μ/T

Physical origin of the negative λ_{ud}

- Consider the case $\nabla \mu_u \neq 0, \nabla \mu_d = \dots = 0$
- Pauli blocking \rightarrow Preferable emission direction



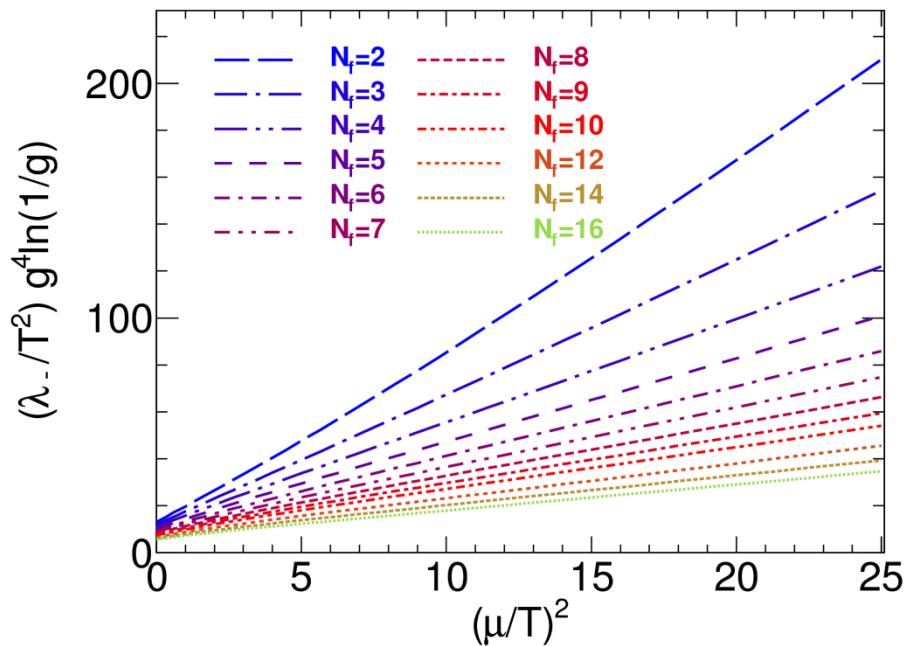
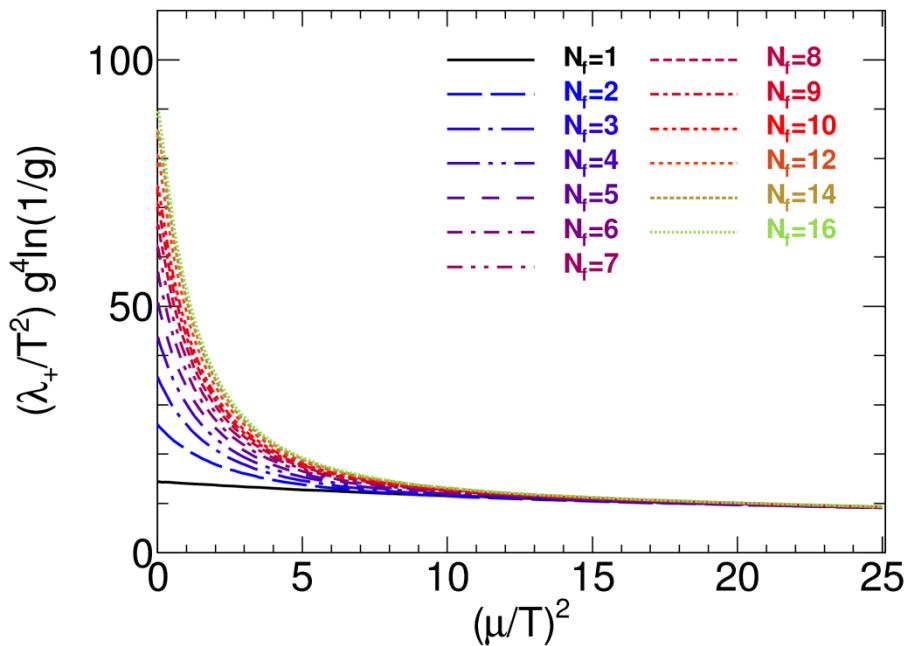
- $\mu_u = \mu_d = \dots \gg T$, only quark contribute

$$\lambda_+ \equiv \lambda_{uu} + \lambda_{du} + \dots \rightarrow \lambda_{uu} \Big|_{N_f=1}$$

Conductivities

$$\mu_u = \mu_d = \dots = \mu$$

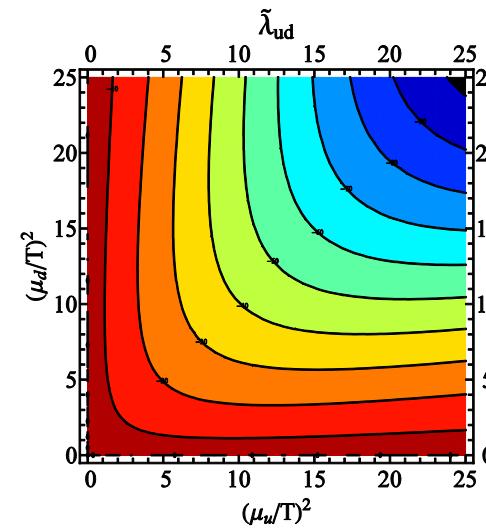
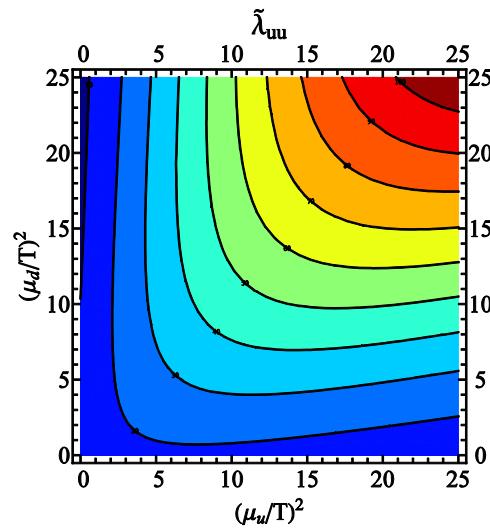
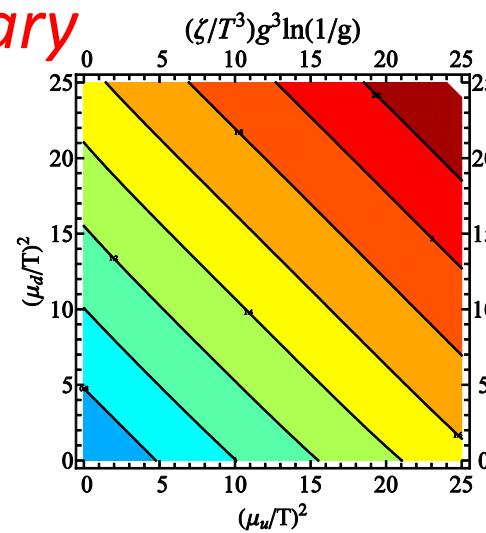
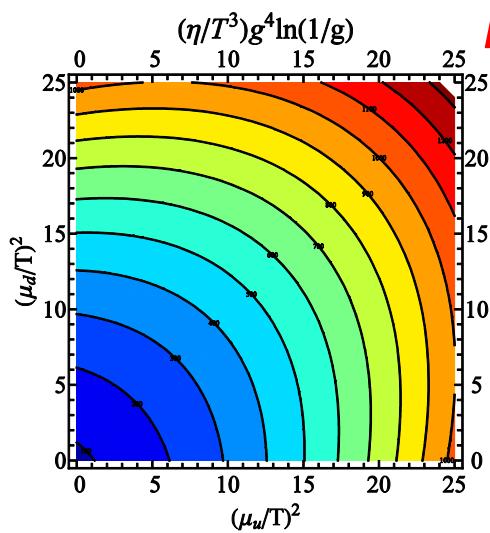
$$\lambda_+ = N_f \left(\lambda_{aa} + (N_f - 1)\lambda_{ab} \right)$$



- λ_+ scale like $(\mu / T)^{-2}$ for large μ / T , and become independent of N_f

Contour lines for transport coeffs.

Preliminary



Conclusion and outlook

- 1st order transport coefficients are calculated with effective kinetic theory at finite chemical potential
- η/s is minimal at zero chemical potential, and linearly rise with increasing $(\mu/T)^2$
- ζ/s is almost constant with increasing μ/T
- Negative off-diagonal conductivity λ_{ab} is obtained, and this phenomena can be understand with simple parton scattering picture
- Transport coefficients with asymmetric quark flavor are, CESE coefficient, et al.

Thanks for your attention!