Quark mass effect on axial charge dynamics

Shu Lin 林树

Sun Yat-Sen University

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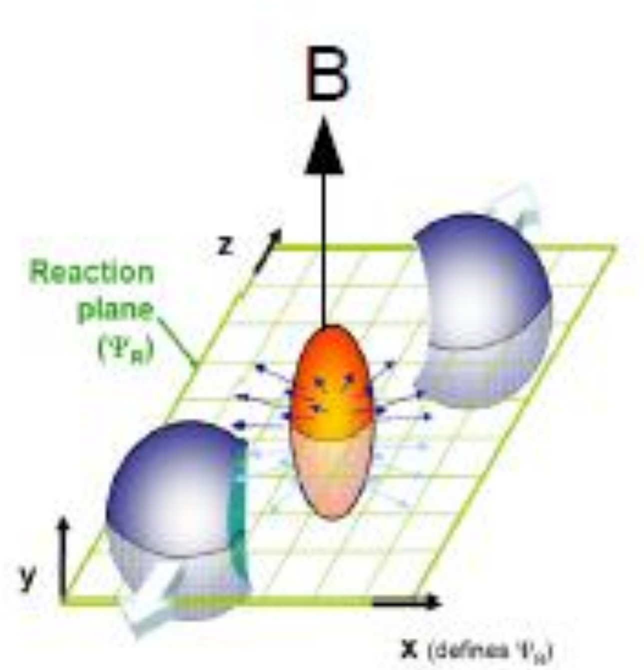
Guo, SL. 1602.03952, PRD 2016

1609.XXXXX

Outline

- Introduction
- Signature of chiral magnetic wave
- Quark mass effect on dynamics of axial charge
- Quark mass effect on CSE (static)
- Summary

Local parity violation in heavy ion collisions



Chiral Magnetic Effect (CME)

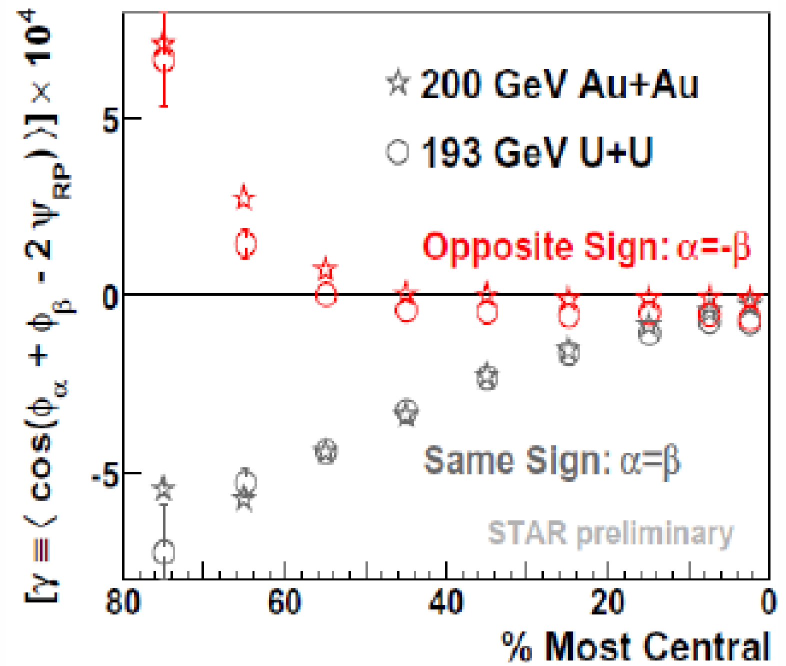
$$j_V = \frac{N_c \mu_A}{2\pi^2} eB$$

QED anomaly

μ_A: chiral imbalance in QGP
$eB \sim {m_\pi}^2$: strong magnetic field in heavy ion collisions

Kharzeev, Zhitnitsky, NPA 2007
Kharzeev, McLerran, Warringa, NPA 2008

Experimental signature of CME



$$j_V = \frac{N_c \mu_A}{2\pi^2} eB$$

$$\langle N_A \rangle = 0, \langle {N_A}^2 \rangle \neq 0$$

Origin of chiral imbalance: QCD anomaly

$$\partial_\mu {j_A}^\mu = -\frac{g^2 N_f}{8\pi^2} tr(G\tilde{G})$$

Same charge correlation enhanced than opposite charge correlation due to CME

Measurement on an event-by-event basis

STAR collaboration, PRL (2014), 1404.1433

Analog of CME: Chiral separation effect

$$j_V = \frac{N_c \mu_A}{2\pi^2} eB \quad \xRightarrow{V \leftrightarrow A} \quad j_A = \frac{N_c \mu_V}{2\pi^2} eB$$

CME

CSE

Metlitski, Zhitnitsky, PRD (2005)

$$\langle N_V \rangle \neq 0$$

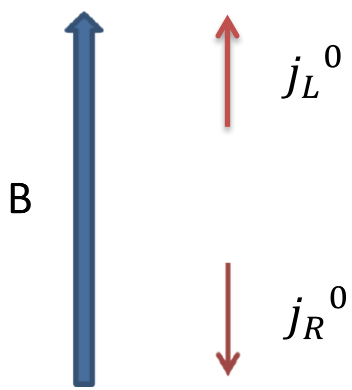
Yet experimentally not accessible

Chiral magnetic wave: an interplay of CME and CSE

$$j_V{}^1 = \frac{N_c \mu_A}{2\pi^2} eB - D_L \partial_1 j_V{}^0$$ Chiral magnetic effect + diffusion

$$j_A{}^1 = \frac{N_c \mu_V}{2\pi^2} eB - D_L \partial_1 j_A{}^0$$ Chiral separation effect + diffusion

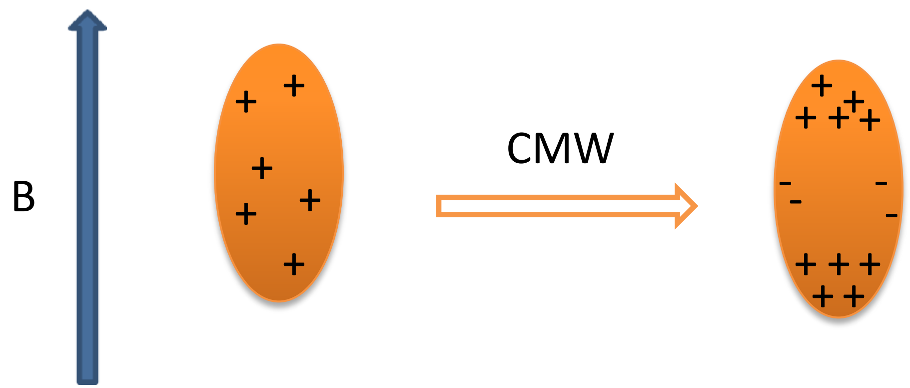
$$\begin{aligned} \partial_\mu j_V{}^\mu &= 0 \\ \partial_\mu j_A{}^\mu &= 0 \end{aligned} \quad \Longrightarrow \quad \left(\partial_0 \mp \frac{N_c e B \alpha}{2\pi^2}\partial_1 - D_L \partial_1^2\right) j^0_{L,R} = 0$$



$$v_\chi = \frac{N_c e B \alpha}{2\pi^2} = \frac{N_c e B}{4\pi^2}\left(\frac{\partial \mu_L}{\partial j^0_L}\right) = \frac{N_c e B}{4\pi^2}\left(\frac{\partial \mu_R}{\partial j^0_R}\right)$$

Kharzeev, Yee, PRD (2011)

Charge dependent flow from CMW



Negative charges have larger pressure gradient than positive charges

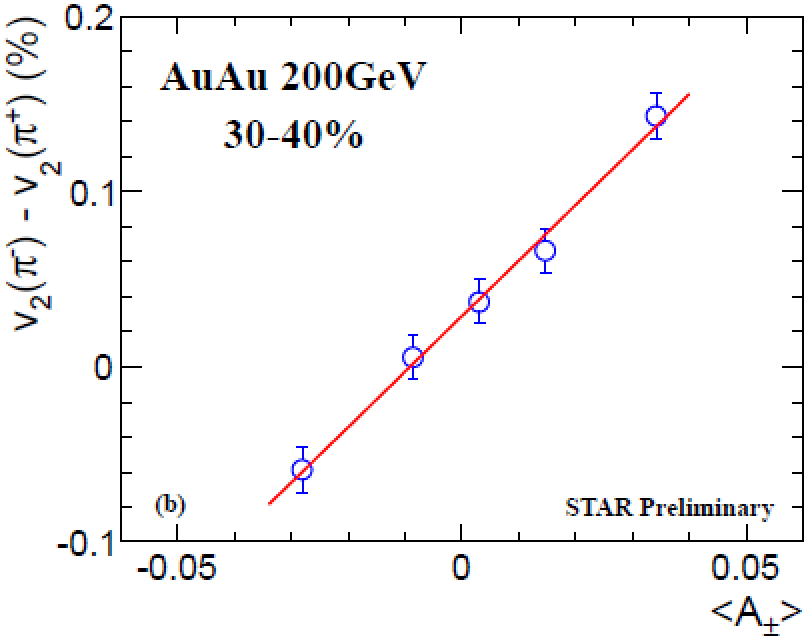
$$v_2(\pi^+) < v_2(\pi^-)$$

Charge dependent flow survives even after event averaging!

$$v_2^{\pm} = v_2 \mp \left(\frac{q_e}{\bar{\rho}_e}\right) A_{\pm} \qquad A_{\pm} \equiv (\bar{N}_+ - \bar{N}_-)/(\bar{N}_+ + \bar{N}_-)$$

charge asymmetry

Experimental signature of CMW



Hongwei Ke, J.Phys.Conf.Ser (2012)

Further confirmation from Kaon flow?

Quark mass effect

$$\partial_\mu j_5^\mu = 2im\bar{\psi}\gamma^5\psi - \frac{e^2}{16\pi^2}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma} - \frac{g^2}{16\pi^2}\mathrm{tr}\epsilon^{\mu\nu\rho\sigma}G_{\mu\nu}G_{\rho\sigma},$$

when $m \ll T$, neglect mass term above

HIC at RHIC, $T \lesssim 350MeV$
Strange quark mass $m \sim 100MeV$

Quark mass effects:

- modify fluctuation/dissipation of axial charge.
- modify CSE, though not modify CME

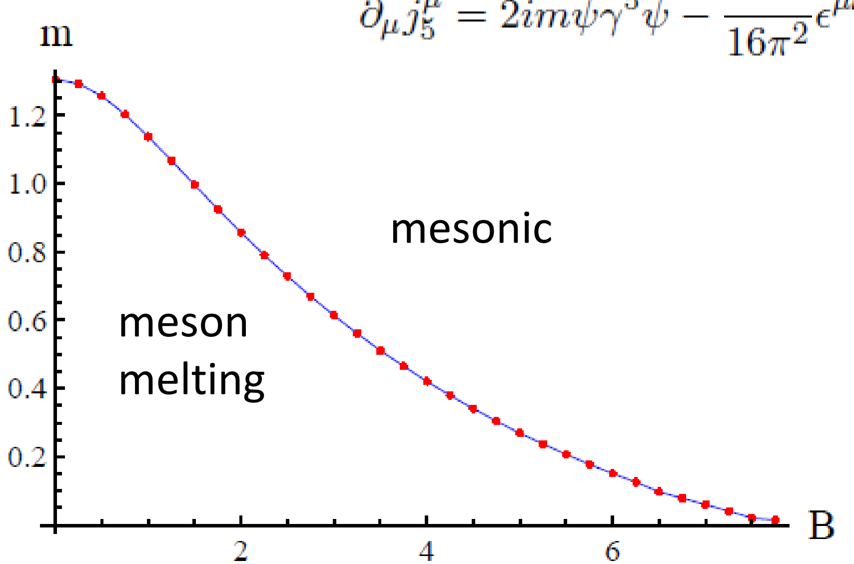
The D3/D7 model

N_c D3 branes + N_f D7 branes

gluons quarks

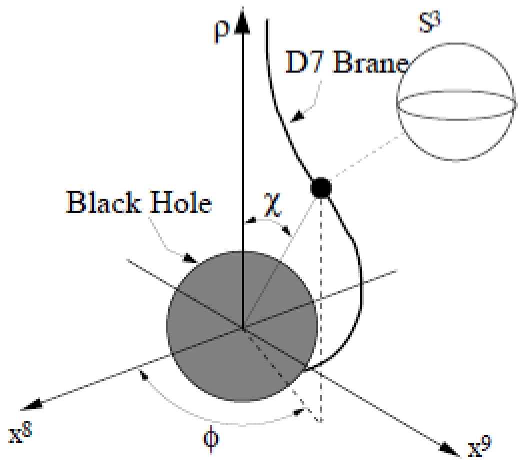
Pro: quark mass explicit (as compare to D4/D8)
Con: w/o QCD anomaly, but with QED anomaly and mass term

$$\partial_\mu j_5^\mu = 2im\bar{\psi}\gamma^5\psi - \frac{e^2}{16\pi^2}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma} - \cancel{\frac{g^2}{16\pi^2}\mathrm{tr}\epsilon^{\mu\nu\rho\sigma}G_{\mu\nu}G_{\rho\sigma}};$$



Mateos et al, PRL 2006
Filev et al, JHEP 2007
Erdmenger et al, JHEP 2007

Axial anomaly in D3/D7 model



axial-symmetry realized as rotation in x8-x9 plane

	x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
D3	×	×	×	×						
D7	×	×	×	×	×	×	×	×		

$$S = \mathcal{N}\int d^5x \left(-\frac{1}{2}\sqrt{-G}G^{MN}\partial_M\phi\partial_N\phi - \frac{1}{4}\sqrt{-H}F^2\right) - \mathcal{N}\kappa\int d^5x\Omega\epsilon^{MNPQR}F_{MN}F_{PQ}\partial_R\phi$$

$$\partial_\mu\left(\frac{\delta S}{\delta\partial_\mu\phi}\right) + \partial_\rho\left(\frac{\delta S}{\delta\partial_\rho\phi}\right) = 0$$

$$J^\mu_R = \int d\rho \tfrac{\delta S}{\delta\partial_\mu\phi} \qquad \partial_\mu J^\mu_R + \frac{\delta S}{\delta\partial_\rho\phi}\Big|^\infty_{\rho=\rho_h} = 0$$

ϕ dual to $\quad mi\bar{\psi}\gamma^5\psi + \cdots + \mathcal{N}E\cdot B$

Hoyos et al, JHEP (2011)

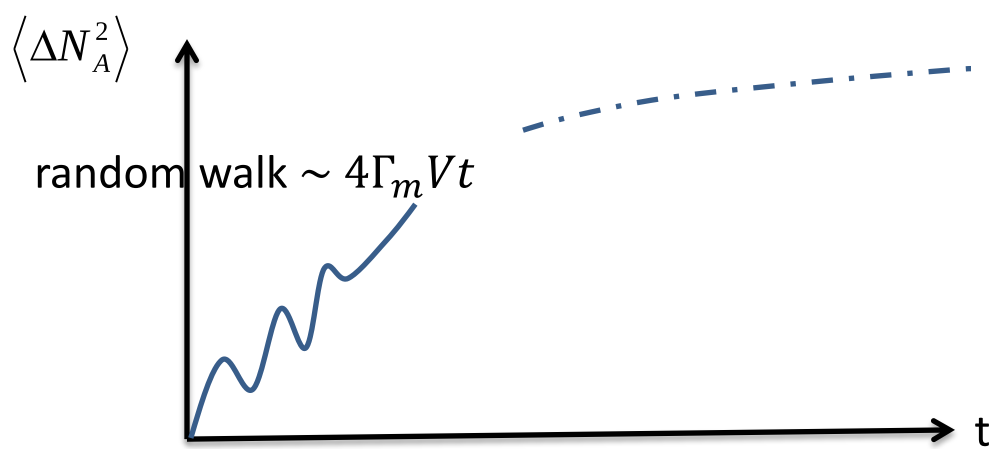
Mass diffusion rate

$$O_\eta = m i \bar{\psi} \gamma^5 \psi + \cdots$$

$$G_{\eta\eta}(\omega) = \int dt \langle [O_\eta(t), O_\eta(0)] \rangle \Theta(t) e^{i\omega t} \sim \frac{-i\omega\Gamma_m}{2T} \qquad \text{as } \omega \to 0$$

Γ_m analogous to Γ_{CS}

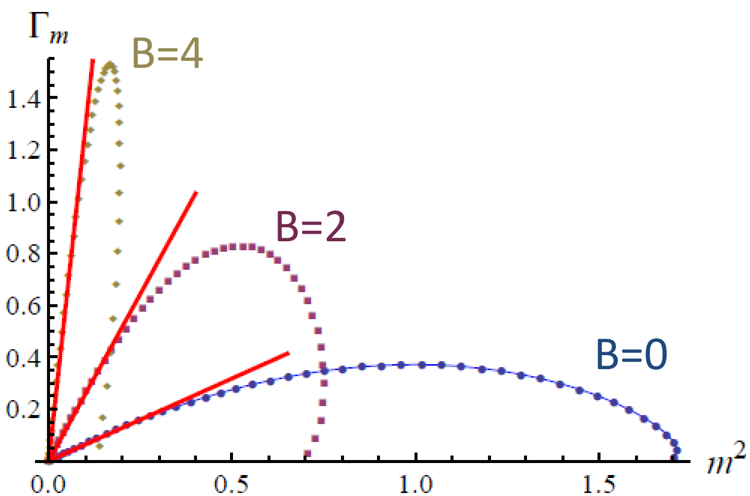
mass diffusion rate CS diffusion rate (absent in D3/D7)

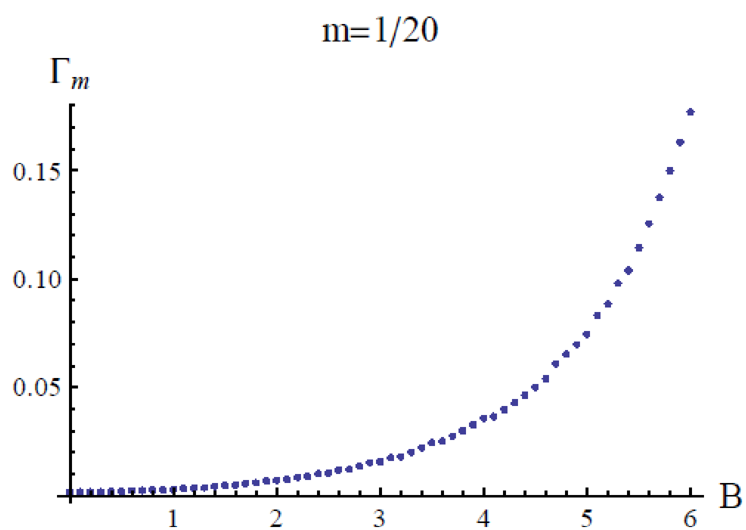


enhance axial charge fluctuation

Guo, SL, PRD (2016)

Mass diffusion rate





$$\Gamma_m \sim m^2 F(B)$$

Measure of helicity flipping rate

Magnetic field enhances helicity flipping rate

Guo, SL, PRD (2016)

$$B = m_\pi{}^2, T = 300MeV, M = M_s, N_f = 1$$

$$\Gamma_m \sim 6\Gamma_{cs}$$

Mass diffusion significant compared to Chern-Simon diffusion

Dynamical susceptibility from CME

Define dynamical axial chemical potential using CME $J(\omega) = C\mu_A(\omega)B(\omega)$

susceptibility $\chi(\omega) = \frac{n_A(\omega)}{\mu_A(\omega)}$

$\chi \sim O(\omega^{-1})$ as $\omega \to 0$ divergent suseptibility Guo, SL, PRD (2016)

- Spontaneous generation of axial charge costs no energy (diffusion)
- Leakage of axial charge from quarks to adjoint reservoir (specific to D3/D7 model)

while m=0 has a finite χ as $\omega \to 0$

Phenomenology? finite m versus ω

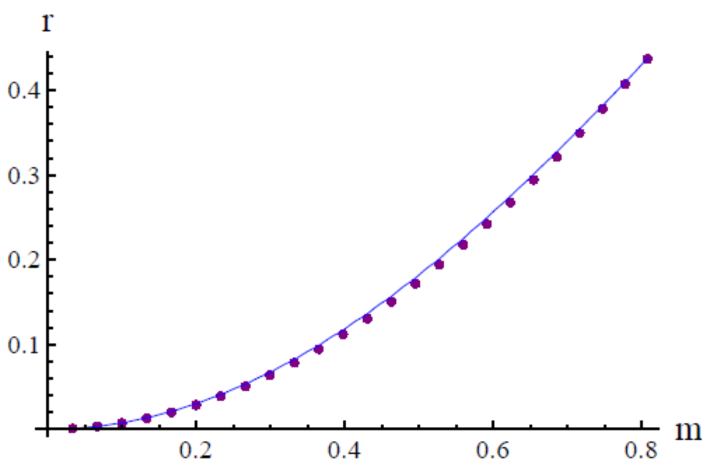
Mass dissipation rate

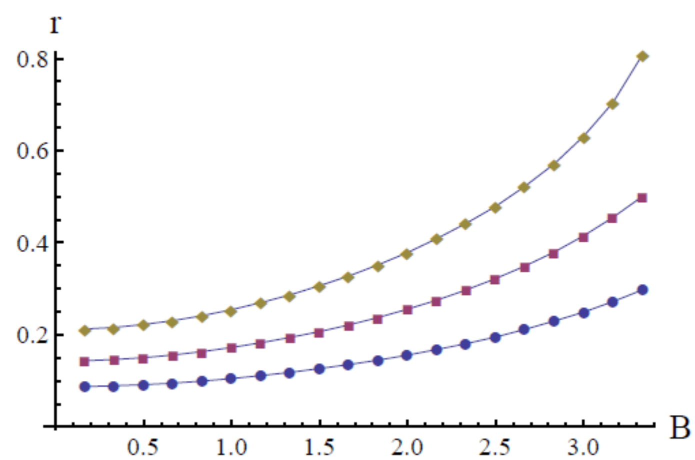
$$\partial_\mu j_5^\mu = 2im\bar{\psi}\gamma^5\psi - \frac{e^2}{16\pi^2}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma} - \cancel{\frac{g^2}{16\pi^2}\mathrm{tr}\epsilon^{\mu\nu\rho\sigma}G_{\mu\nu}G_{\rho\sigma}},$$

O_η $\quad E\cdot B$

$$r = \frac{O_\eta}{\mathcal{N} E\cdot B}.$$

Produce axial charge by setting up parallel E and B fields for $\omega \to 0$





Mass term more effective in dissipating axial charge at large m and B

r<1: axial charge survives the hydro limit for massive quarks?

Guo, SL, PRD (2016)

Mass dissipation rate

One unit of axial charge
$E \cdot B$ fields

1-r

Adjoint reservoir

$\rho = \rho_h$

r

Dissipates through mass term

$$\partial_\mu J^\mu_R + \frac{\delta S}{\delta \partial_\rho \phi}\Big|^{\infty}_{\rho=\rho_h} = 0$$

$\rho = \infty$

Effectively only r unit of axial charge is produced, all dissipates through the mass term in hydro limit

Guo, SL, PRD (2016)

Consistent with relaxation time approximation   Landsteiner et al, JHEP 2015

τ_{rel} increases with B, decreases with m

Phenomenology? finite τ_{hydro} versus τ_{rel}

Quark mass correction to CSE

e.g. modified CMW

$$j_V = \frac{N_c \mu_A}{2\pi^2} eB$$

Non-renormalization

$$j_A = \frac{N_c \mu_V}{2\pi^2} eB + O(m^2)$$

Can be studied reliably in D3/D7 model, no axial charge exchange between quarks and adjoint matter

structure of correction to CSE

$$\nabla \cdot \mathbf{j}_5 = C\mathbf{E}\cdot\mathbf{B} + 2M_q i\bar{\psi}\gamma^5\psi \quad \equiv \sigma_5$$

massless case: $\nabla \cdot \mathbf{j}_5 = -\nabla\cdot(C\mu\mathbf{B}) \Rightarrow \mathbf{j}_5 = -C\mu_q\mathbf{B}$

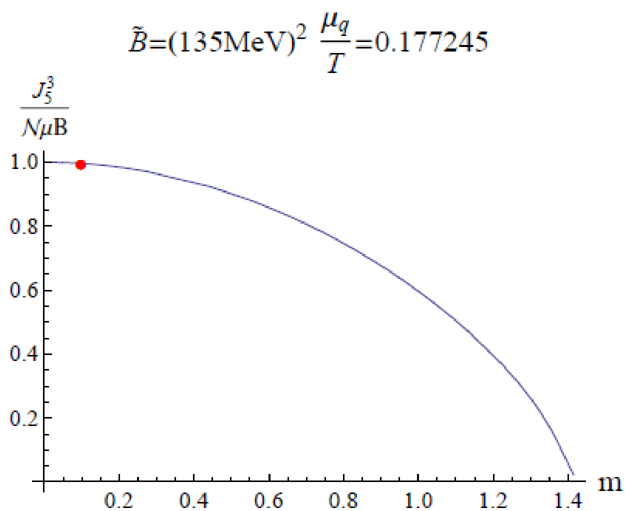
massive case: σ_5 P odd, T odd, while B P even, T odd, μ_q P even, T even

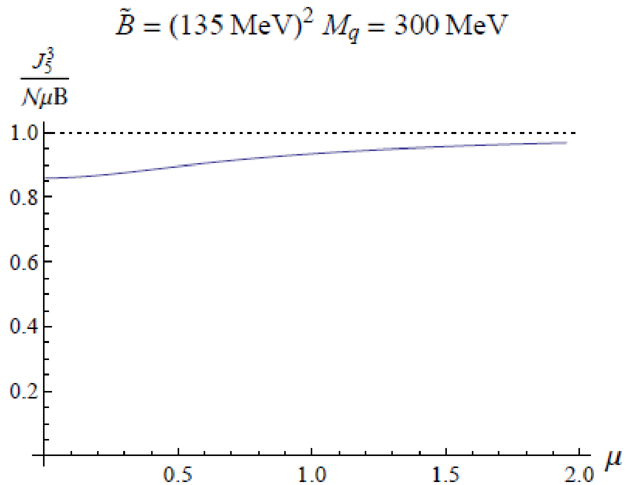
$$\sigma_5 = g(M_q^2, T, \mu, B)\mathbf{B}\cdot\nabla\mu_q \Longrightarrow \mathbf{j}_5 = -C\mu\mathbf{B} + g(M_q^2, T, \mu_q, B)\mu\mathbf{B}.$$

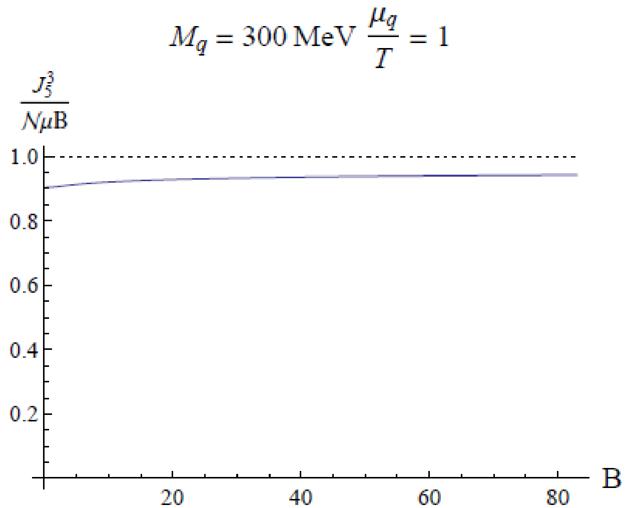
$g = \#\frac{M_q^2}{T^2} + o(M_q^2)$ when $\mu_q \ll T, B \ll T^2$

Guo, SL. 1609.XXXXX

Parameter dependence of CSE correction







M_q suppresses CSE

μ_q and B enhance CSE

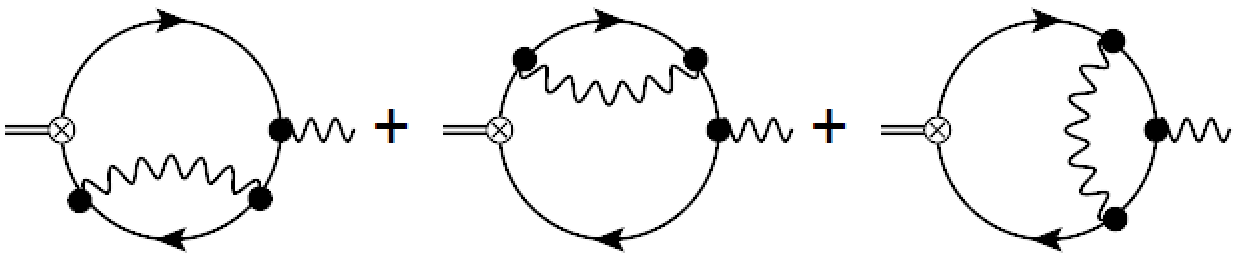
$$\Delta j_5 = -\#\frac{{M_q}^2}{T^2}\mu B + o({M_q}^2)$$

Correction numerically small

Compare to QED calculation at T=0

free theory $\qquad \mathbf{j}_5 = e\mathbf{B}\sqrt{\mu^2 - m^2}/(2\pi^2)$

perturbative correction

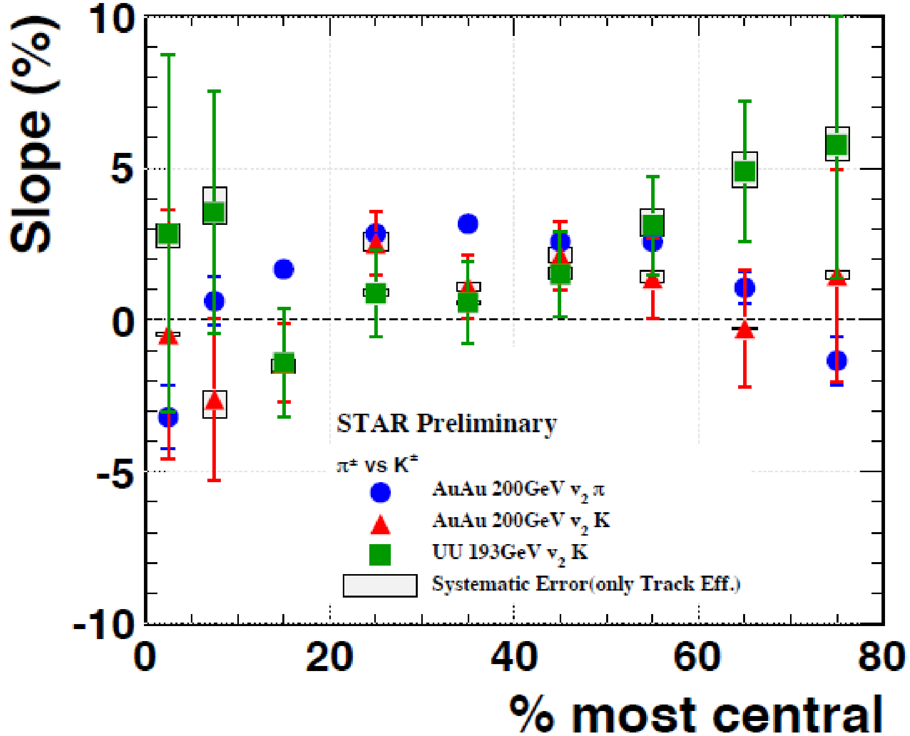


$$\langle j_5^3 \rangle_\alpha = -\frac{\alpha e B \mu}{2\pi^3}\left(\ln\frac{2\mu}{m} + \ln\frac{m_\gamma^2}{m^2} + \frac{4}{3}\right) - \frac{\alpha e B m^2}{2\pi^3 \mu}\left(\ln\frac{2^{3/2}\mu}{m_\gamma} - \frac{11}{12}\right)$$

$m_\gamma \qquad$ IR cutoff

Gorbar et al, PRD 2013

Quark mass dependence of CMW



Qiye Shou, J.Phys.Conf.Ser (2014)

Theoretical results also point to small quark mass effect

Scaling of correlators

$$\sigma \equiv \bar{\psi}\psi \qquad \sigma_5 \equiv im\bar{\psi}\gamma^5\psi \qquad n \equiv \bar{\psi}\gamma^0\psi.$$

$$G_{\mu\nu}(k) = \int d^4(x-y)e^{i\vec{k}\vec{x}}\langle O_\mu(x), O_\nu(y)\rangle$$

$$\mu, \nu = \sigma,\ n,\ \sigma_5$$

$$G_{\sigma 5 n}, G_{\sigma 5 \sigma} \sim O(kB)$$
$$G_{\sigma 5 \sigma 5} \sim O(k^2)$$

$$\mathbf{j}_5 = -C\mu\mathbf{B} + g(M_q^2, T, \mu_q, B)\mu\mathbf{B}.$$

$$g = \frac{2G_{\sigma 5n}}{ikB} \sim {M_q}^2$$

CSE correction proportional to correlator $G_{\sigma 5n}$
accessible on the lattice!

Guo, SL. 1609.XXXXX

$$g \sim {M_q}^2$$

Wigner function approach by Qun Wang et al

Physical meaning of correlators

$$G_{\sigma 5 n}, G_{\sigma 5 \sigma} \sim O(kB)$$
$$G_{\sigma 5 \sigma 5} \sim O(k^2)$$

$G_{\sigma 5 n}$ response of σ_5 to $\nabla\mu$
$G_{\sigma 5 \sigma}$ response of σ_5 to ∇m
$G_{n \sigma 5}$ response of n to $\nabla\phi$
$G_{\sigma\sigma 5}$ response of σ to $\nabla\phi$
$G_{\sigma 5 \sigma 5}$ response of σ_5 to $\nabla\phi$

$\nabla\phi$ defined as chiral shift by Gorbar et al

Guo, SL. 1609.XXXXX

k-scaling of correlators of σ_5

Lagrangian $m\bar{\psi}e^{i\phi\gamma^5}\psi.$

chiral rotation, $\psi \to e^{-i\gamma^5\phi/2}\psi$

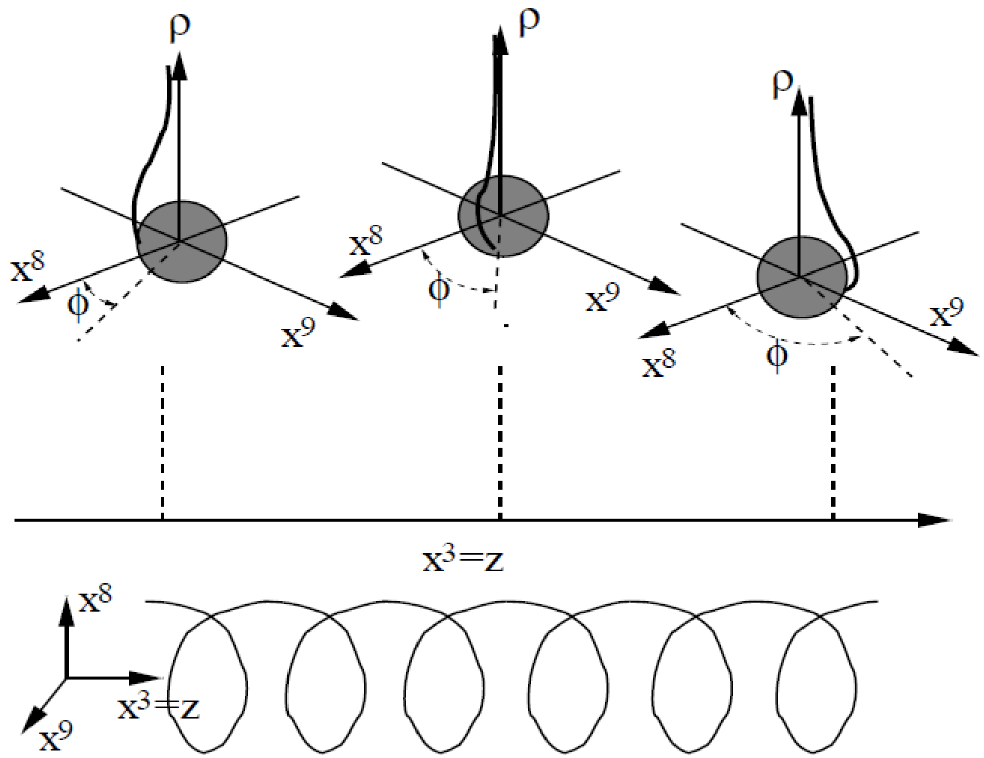
$$m\bar{\psi}e^{i\phi\gamma^5}\psi \to m\bar{\psi}\psi - \frac{\partial_\mu\phi}{2}\bar{\psi}\gamma^\mu\gamma^5\psi$$

Summary

- Quark mass enhances fluctuation of axial charge in addition to topological fluctuation
- Quark mass dissipation consistent with relaxation time approximation.
- Quark mass correction to CSE.
- k-dependence of correlators: gradient induced responses

Thank you!

Spiral phase and correction to CSE



D7 brane being a point in x8-x9 plane, spiral phase in x3 direction

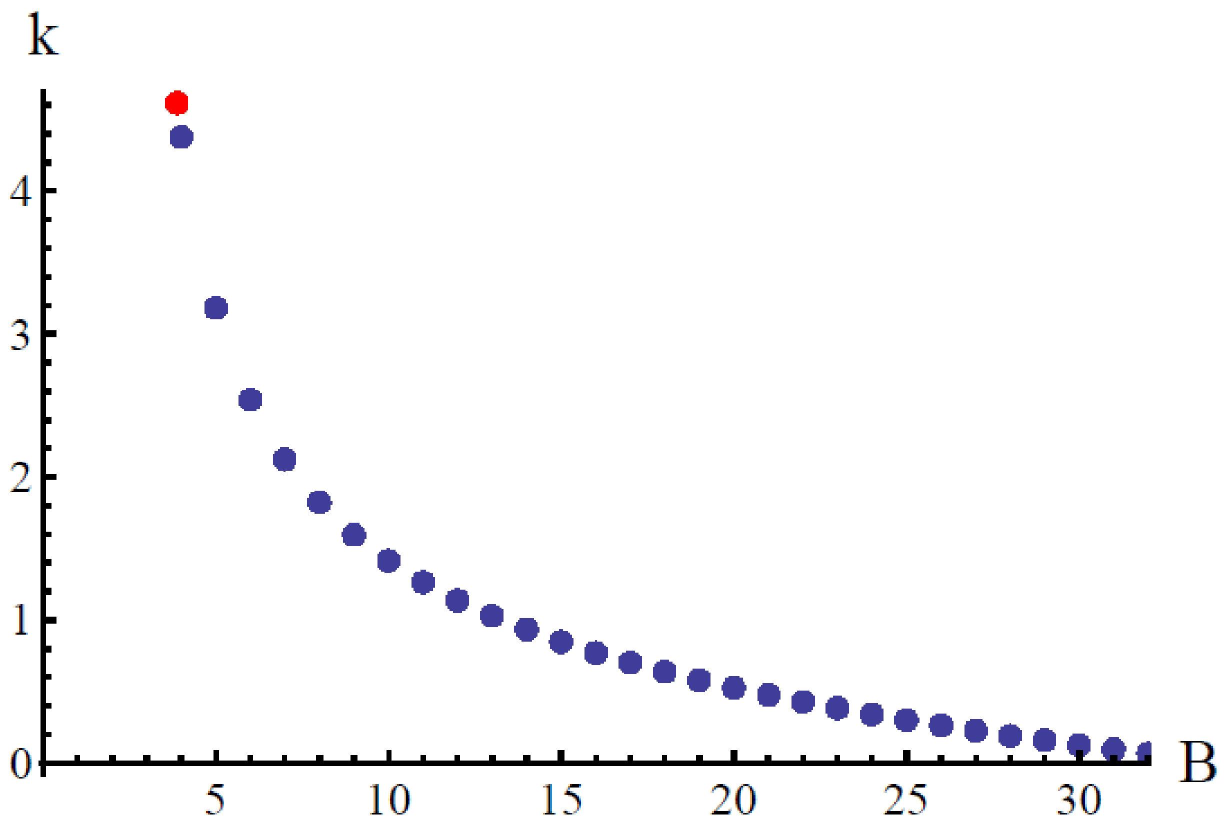
Kharzeev and Yee, PRD 2011

In spiral phase $i\bar{\psi}\gamma^5\psi \neq 0$ induces correction to CSE in massive case

In progress

Normalizable mode

$$M_q = 24.2712\,\text{MeV} \quad \frac{\mu_q}{T} = 4.92898$$



Spontaneous breaking of parity

Frameworks for axial charge dynamics

chiral kinetic theory (Berry curvature)

Son, Yamamoto, PRL (2012)
Stephanov, Yin, PRL (2012)
Pu, Gao, Wang et al, PRL (2012), (2013), PRD (2014)

Q. Wang's talk

hydrodynamics (axial charge)

Son, Surowka, PRL (2009)
Neiman, Oz, JHEP (2011)

Relativistic hydrodynamics for HIC

$$\partial_\mu T^{\mu\nu} = F^{\nu\lambda} j^V{}_\lambda$$
$$\partial_\mu j_V{}^\mu = 0$$
$$\partial_\mu j_A{}^\mu = C E^\mu B_\mu$$

Son, Surowka, PRL (2009)

with QED anomaly,
without QCD anomaly

$$j_V{}^\mu = n_V u^\mu + \nu_V{}^\mu$$
$$j_A{}^\mu = n_A u^\mu + \nu_A{}^\mu$$

talks by Jiang, Huang and Liao

Anomalous part:

CVE CME

$$\nu_V{}^\mu = C\mu_V\mu_A\omega^\mu + C\mu_A B^\mu$$
$$\nu_5{}^\mu = C/2({\mu_V}^2 + {\mu_A}^2 + \cdots)\omega^\mu + C\mu_V B^\mu$$

CSE

However, in HIC we need QCD anomaly to generate axial charge!

$$\langle N_A \rangle = 0, \langle {N_A}^2 \rangle \neq 0$$

Axial charge stochastic,
hydrodynamic noise necessary!

How hydro noise is included

Conserved charge as an example

$$\partial_\mu J^\mu = 0,$$

w/o noise

$J^0 = n$ charge density $J_k = -D\partial_k n$ diffusive current

with noise

$J^0 = n$

$$J_k = -D\partial_k n + r_k$$

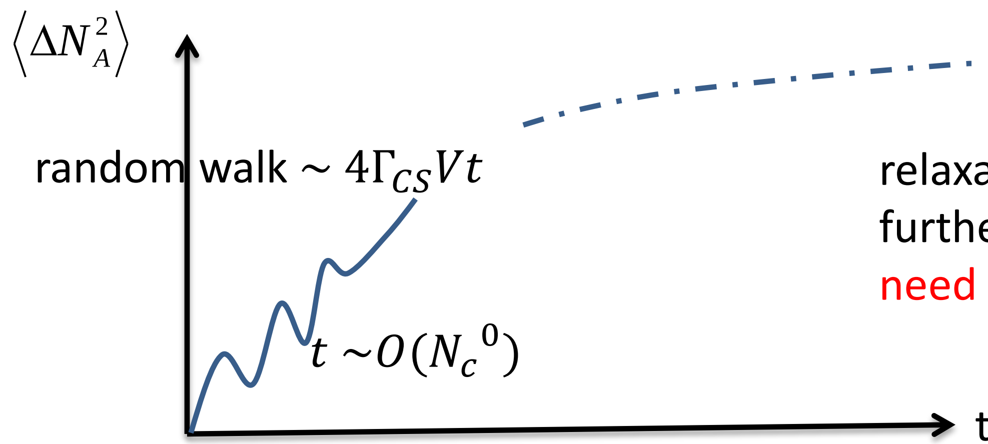
dissipation fluctuation

$$\langle r_i(\mathbf{x},t)\, r_k(\mathbf{x}',t')\rangle = C\delta_{ik}\delta(\mathbf{x}-\mathbf{x}')\delta(t-t')$$

Yan's talk

Kovtun, 1205.5040

Axial charge from topological fluctuation



$$t \sim \tau = \frac{\chi T}{2\Gamma_{CS}} \sim O(N_c)$$

relaxation: presence of n_A affects further topological charge fluctuation.
need fermion dynamics

Chern-Simon diffusion rate $\quad \Gamma_{CS} = \int d^4x \langle q(x) q(0) \rangle$

$q \sim trG\tilde{G}$ topological charge density

weak coupling extrapolation: $\Gamma_{CS} \sim 30 \alpha_s{}^4 T^4$ — Moore, Tassler, JHEP 2011

strong coupling: $\Gamma_{CS} = \alpha_s{}^2 N_c{}^2 T^4 / 16\pi$ — Son, Starinets, JHEP 2002

strong coupling w/B: $\Gamma_{CS} \sim \alpha_s{}^2 N_c{}^2 B T^2$ — Basar, Kharzeev, PRD 2012

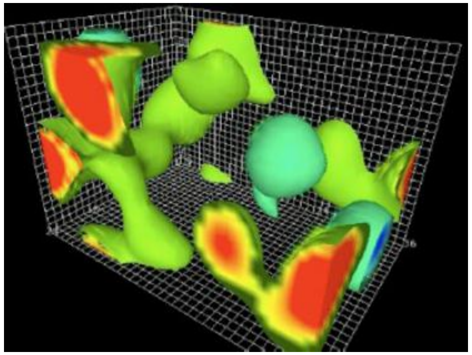
Topological fluctuation as hydro noise

Size of QGP >> fluid cell >> size of topological fluctuation

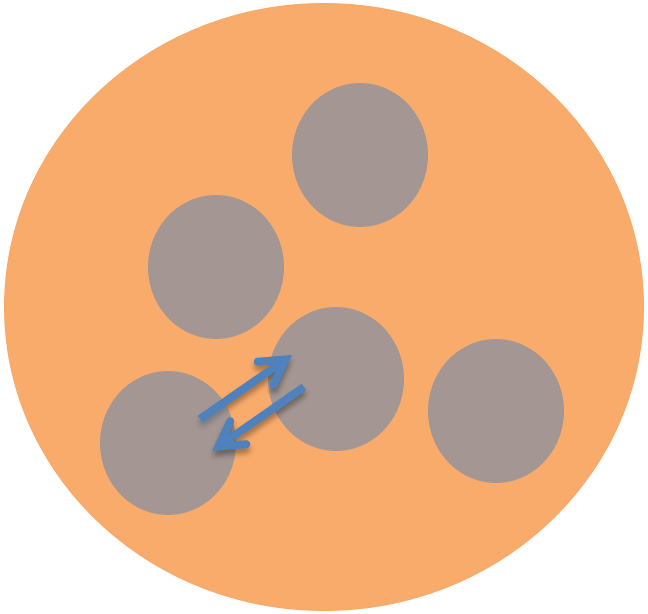
Axial charge fluctuation localized in fluid cell

Topological transition additional source of noise!

within one fluid cell



between fluid cells



The Sakai-Sugimoto model (D4/D8)

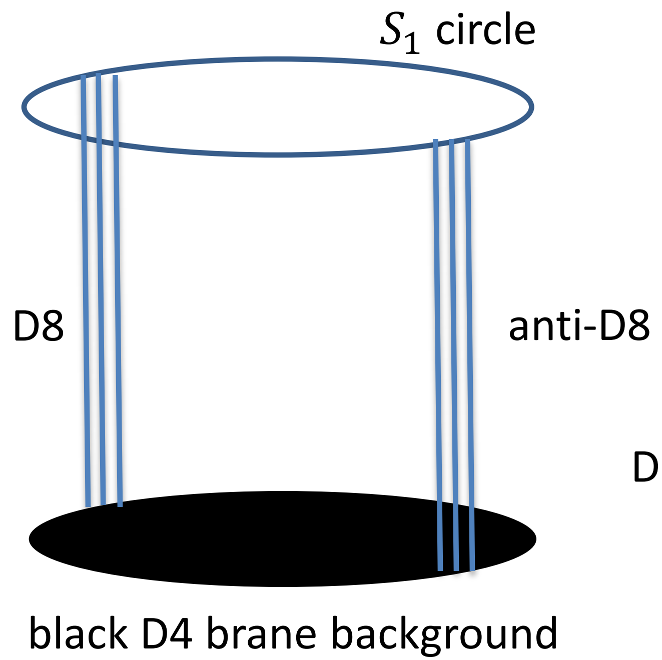
N_c D4 branes wrapped on S^1 circle + N_f D8/anti D8 branes being a point on S^1.

⇑ gluons

⇑ left/right handed quarks

mass gap $M_{KK} = \dfrac{1}{R_4}$

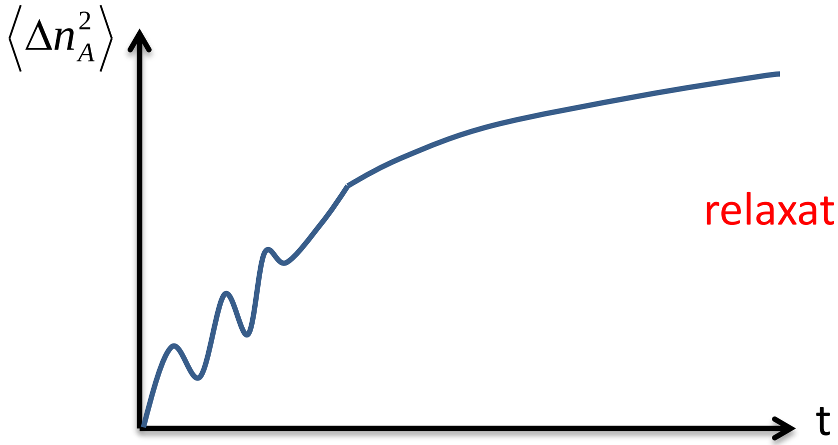
Sakai, Sugimoto, Prog.Theor.Phys, 2005



Deconfined, chiral symmetry restored

Aharony et al, Annal. Phys. 2006

Axial charge relaxation



relaxation, explicit in our model with fermions

Response of q to n_A

$$q = \frac{\Gamma_{CS}}{\chi T} n_A \quad \Longrightarrow \quad \frac{dn_A}{dt} = -2q = -\frac{2\Gamma_{CS}}{\chi T} n_A = -\frac{n_A}{\tau_{sph}}$$

χ: static susceptibility $\quad \tau_{sph} = \frac{\chi T}{2\Gamma_{CS}}$: relaxation time

consistent with early statistical argument

Also work by Akamatsu, Rothkopf, Yamamoto, JHEP 2016

Iatrakis, SL, Yin, JHEP 2015

Stochastic hydrodynamics for axial charge

Dynamical equation

$$\partial_t n_A(t,x) + \nabla \cdot j_A(t,x) = -2q(t,x)$$

Constitutive equations

$$j_A(t,x) = -D\nabla n_A(t,x) + \xi(t,x)$$

$$q(t,x) = \frac{n_A(t,x)}{2\tau_{\mathrm{sph}}} + \xi_q(t,x)$$

Non-topological fluctuation $\langle \xi_i(t,x)\xi_j(t,x')\rangle = 2\sigma T\delta_{ij}\delta(t-t')\delta^3(x-x')$

topological fluctuation $\langle \xi_q(t,x)\xi_q(t,x')\rangle = \Gamma_{\mathrm{CS}}\delta(t-t')\delta^3(x-x')$

Iatrakis, SL, Yin, JHEP 2015

Time evolution of axial charge from stochastic hydrodynamics

$$C_{nn}(t,x) \equiv \langle [n_A(t,x) - n_A(0,x)][n_A(t,0) - n_A(0,0)] \rangle$$

$$C_{nn}(t,\boldsymbol{x}) = (\chi T)\left[\underbrace{\delta^3(\boldsymbol{x})}_{\text{within cell}} - \frac{1}{(8\pi D t)^{3/2}} \underbrace{e^{-\frac{2t}{\tau_{\rm sph}}}}_{\text{across cells}} e^{-\frac{|\boldsymbol{x}|^2}{8Dt}}\right]$$

Early time $t \ll \tau_{\rm sph}$

$$C_{nn}(t,\boldsymbol{x}) \approx 4\Gamma_{\rm CS}\, t\, \delta^3(\boldsymbol{x})$$

Late time $t \gg \tau_{\rm sph}$

$$C_{nn}(t \to \infty, \boldsymbol{x}) \to (\chi T)\,\delta^3(\boldsymbol{x})$$ thermodynamic limit