

# $B_c \rightarrow B_{sJ}$ form factors and $B_c$ decays into $B_{sJ}$ in covariant light-front approach

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# Outline

- Introduction
- $B_c \rightarrow B_{sJ}$  form factors
  - Definition
  - Covariant light-front approach
  - Formulation
  - Numerical fit
- Search for  $B_{sJ}$ 
  - Semileptonic  $B_c$  decays
  - Nonleptonic  $B_c$  decays
- Summary

## ✓ Introduction

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# Introduction

- Discovery of hadron exotics such as  $X(3872)$ ,  $Z_c(3900)$   
→ Mass region of heavy quarkonium
- Quarkonium-like states usually compose of a pair of heavy constituents  
→ We can study heavy-light hadrons first

# Introduction

- Many charm-strange hadrons have been observed  
→ such as  $D_s(2317)$ ,  $D_s(2460)$ ,  $D_{s1}(2536)$
- Heavy quark symmetry:  $c \rightarrow b$
- But only a few bottom-strange mesons are observed  
→ Use  $B_c$  decays to study the  $B_{sJ}$  spectrum
- $B_{sJ} \in \{B_s, B_s^*, B_{s0}, B_{s1}, B'_{s1}, B_{s2}\}$

# Introduction

$B_{sJ}$	$^{2S+1}L_J$	type
$B_s$	$^1S_0$	pseudoscalar
$B_s^*$	$^3S_1$	vector
$B_{s0}$	$^3P_0$	scalar
$B_{s1}$	$^3P_1$	axis
$B'_{s1}$	$^1P_1$	axis
$B_{s2}$	$^3P_2$	tensor

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# $B_c \rightarrow B_{sJ}$ form factors: Definition

Form factors of  $B_c \rightarrow B_{sJ}(V)$ :

$$\langle V(P'', \varepsilon'') | V_\mu | B_c(P') \rangle = -\frac{1}{m_{B_c} + m_V} \epsilon_{\mu\nu\alpha\beta} \varepsilon''^{\nu*\nu} P^\alpha q^\beta V^{B_c}(q^2)$$

$$\begin{aligned} \langle V(P'', \varepsilon'') | A_\mu | B_c(P') \rangle &= 2im_V \frac{\varepsilon''^*\cdot q}{q^2} q_\mu A_0^{B_c}(q^2) + i(m_{B_c} + m_V) A_1^{B_c}(q^2) \left[ \varepsilon''^* - \frac{\varepsilon''^*\cdot q}{q^2} q_\mu \right] \\ &\quad - i \frac{\varepsilon''^*\cdot P}{m_{B_c} + m_V} A_2^{B_c}(q^2) \left[ P_\mu - \frac{m_{B_c}^2 - m_V^2}{q^2} q_\mu \right] \end{aligned}$$

### $B_c \rightarrow B_s$ form factors: Definition

## Form factors of $B_c \rightarrow B_{sJ}(T)$ :

$$\langle T(P'', \varepsilon'') | V_\mu | B_c(P') \rangle = -\frac{2V^{B_c T}(q^2)}{m_{B_c} + m_T} \epsilon^{\mu\nu\rho\sigma} (\varepsilon_T^*)_\nu (P')_\rho (P'')_\sigma,$$

$$\langle T(P'', \varepsilon'') | A_\mu | B_c(P') \rangle = 2im_T \frac{\varepsilon_T^* \cdot q}{q^2} q_\mu A_0^{B_c T}(q^2) + i(m_{B_c} + m_T) A_1^{B_c T}(q^2) \left[ \varepsilon_{T\mu}^* - \frac{\varepsilon_T^* \cdot q}{q^2} q_\mu \right] \\ - i \frac{\varepsilon_T^* \cdot P}{m_{B_c} + m_T} A_2^{B_c T}(q^2) \left[ P_\mu - \frac{m_{B_c}^2 - m_T^2}{q^2} q_\mu \right]$$

The spin-2 polarization tensor can be constructed using spin-1 polarization vector  $\varepsilon$ :

$$\varepsilon''_{\mu\nu}(P'', \pm 2) = \varepsilon_\mu(\pm)\varepsilon_\nu(\pm), \quad \varepsilon''_{\mu\nu}(P'', \pm 1) = \frac{1}{\sqrt{2}}[\varepsilon_\mu(\pm)\varepsilon_\nu(0) + \varepsilon_\nu(\pm)\varepsilon_\mu(0)],$$

$$\varepsilon''_{\mu\nu}(P'', 0) = \frac{1}{\sqrt{6}} [\varepsilon_\mu(+)\varepsilon_\nu(-) + \varepsilon_\nu(+)\varepsilon_\mu(-)] + \sqrt{\frac{2}{3}} \varepsilon_\mu(0)\varepsilon_\nu(0).$$

# Covariant light-front approach → Light-cone variables

Light-front decomposition of the mesons' momentum is

$$P = (P^-, P^+, P_\perp)$$

$$P^\pm = P^0 \pm P^3, P^2 = P^+ P^- - P_\perp^2$$

The quark and antiquark inside the meson have momenta  $p_1$  and  $p_2$ . Expressed by the internal variables  $(x_i, p_\perp)$  as:

$$p_{1,2}^+ = x_{1,2} P^+, \quad p_{1,2\perp} = x_{1,2} P_\perp \pm p_\perp$$

# Covariant light-front approach → Meson bound state

A meson bound state with the total momentum  $P$  and spin  $J$  can be expressed as:

$$|M(P^{2S+1}L_J, J_z)\rangle = \int \{d^3 p_1\} \{d^3 p_2\} 2(2\pi)^3 \delta^3(\tilde{P} - \tilde{p}_1 - \tilde{p}_2) \cdot \sum_{\lambda_1 \lambda_2} \Psi_{LS}^{JJ_z}(\tilde{p}_1, \tilde{p}_2, \lambda_1, \lambda_2) |q_1(p_1, \lambda_1) \bar{q}_2(p_2, \lambda_2)\rangle$$

$$\Psi_{LS}^{JJ_z} = \frac{1}{\sqrt{N_c}} \langle L, S; L_z, S_z | L, S; J, J_z \rangle R_{\lambda_1 \lambda_2}^{SS_z}(x, p_\perp) \varphi(x, p_\perp)$$

$R_{\lambda_1 \lambda_2}^{SS_z}(x, p_\perp)$  constructs a state of definite spin  $(S, S_z)$  out of light helicity  $(\lambda_1, \lambda_2)$  eigenstates. It has covariant form:

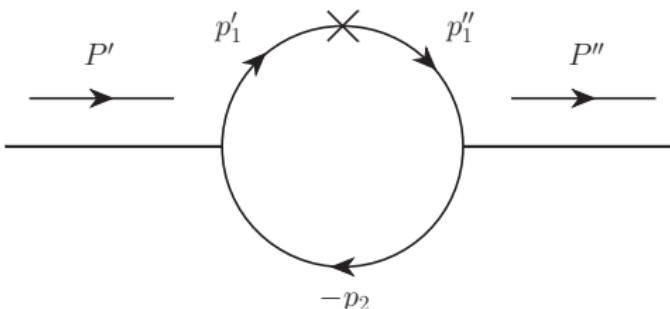
$$R_{\lambda_1 \lambda_2}^{SS_z}(x, p_\perp) = \frac{1}{\sqrt{2}\tilde{M}_0} \bar{u}(p_1, \lambda_1) \Gamma v(p_2, \lambda_2)$$

# Covariant light-front approach → Effective vertex

By light-front approach we can get the meson-quark-antiquark vertices for various mesons.

$M(^{2S+1}L_J)$	$i\Gamma'_M$
pseudoscalar ( ${}^1S_0$ )	$H'_P \gamma_5$
scalar ( ${}^3P_0$ )	$H'_S$
vector ( ${}^3S_1$ )	$iH'_V [\gamma_\mu - \frac{1}{W'_V} (p'_1 - p_2)_\mu]$
axial ( ${}^3P_1$ )	$iH'_{3A} [\gamma_\mu + \frac{1}{W'_{3A}} (p'_1 - p_2)_\mu] \gamma_5$
axial ( ${}^1P_1$ )	$iH'_{1A} [\frac{1}{W'_{1A}} (p'_1 - p_2)_\mu] \gamma_5$
tensor ( ${}^3P_2$ )	$i\frac{1}{2} H'_T [\gamma_\mu - \frac{1}{W'_T} (p'_1 - p_2)_\mu] (p'_1 - p_2)_\nu$

# Derivation of form factors



Hadronic matrix element with a pseudoscalar outgoing meson:

$$\langle P(P') | V_\mu | P(P') \rangle \equiv \mathcal{B}_\mu^{PP},$$

With the use of meson-quark-antiquark vertices, it's straightforward to obtain:

$$\mathcal{B}_\mu^{PP} = i^3 \frac{N_c}{(2\pi)^4} \int d^4 p'_1 \frac{H'_P H''_P}{N'_1 N''_1 N_2} S_{V\mu}^{PP}$$

$$S_{V\mu}^{PP} = \text{Tr}[\gamma_5(\not{p}'_1 + m'_1) \gamma_\mu (\not{p}'_1 + m'_1) \gamma_5 (\not{p}_2 - m_2)],$$

$$N'_1{}^{(\prime)} = p'_1{}^{(\prime)2} - m'_1{}^{(\prime)2} + i\epsilon, \quad N_2 = p_2^2 - m_2^2 + i\epsilon$$

# Derivation of form factors

In the  $q^+ = 0$  frame. The  $p_1'^-$  integration picks up the residue  
 $p_2 = \hat{p}_2 = [(p_{2\perp}^2 + m_2^2)/p_2^+, p_2^+, p_{2\perp}]$  and leads to:

$$\begin{aligned} N_1^{(\prime\prime)} &\rightarrow \hat{N}_1^{(\prime\prime)} = x_1(M'^{(\prime\prime)2} - M_0'^{(\prime\prime)2}), \\ H_P^{(\prime\prime)} &\rightarrow h_P^{(\prime\prime)}, \end{aligned}$$

$$\int \frac{d^4 p'_1}{N'_1 N''_1 N_2} H'_P H''_P S^{PP} \rightarrow -i\pi \int \frac{dx_2 d^2 p'_{\perp}}{x_2 \hat{N}'_1 \hat{N}''_1} h'_P h''_P \hat{S}^{PP}$$

# Derivation of form factors → Analytical results

Finally we obtain the form factor  $f_+(q^2)$  and  $f_-(q^2)$  and then  $F_1(q^2)$  and  $F_0(q^2)$ :

$$F_1^{PP}(q^2) = f_+(q^2), \quad F_0^{PP}(q^2) = f_+(q^2) + \frac{q^2}{q \cdot P} f_-(q^2)$$

Where:

$$\begin{aligned} f_+(q^2) = & \frac{N_c}{16\pi^3} \int dx_2 d^2 p'_\perp \frac{h'_P h''_P}{x_2 \hat{N}'_1 \hat{N}''_1} \left[ x_1(M'_0{}^2 + M''_0{}^2) + x_2 q^2 - x_2(m'_1 - m''_1)^2 \right. \\ & \left. - x_1(m'_1 - m_2)^2 - x_1(m''_1 - m_2)^2 \right] \end{aligned}$$

# Numerical results → Fit method

In the  $q^+ = 0$  frame,

$$q^2 = -q_\perp^2 < 0$$

To access the  $q^2$  distribution to  $q^2 > 0$ , the following fit formula is used to fit the exact form factor expressions:

$$F(q^2) = \frac{F(0)}{1 - \frac{q^2}{m_{\text{fit}}^2} + \delta(\frac{q^2}{m_{\text{fit}}^2})^2}$$

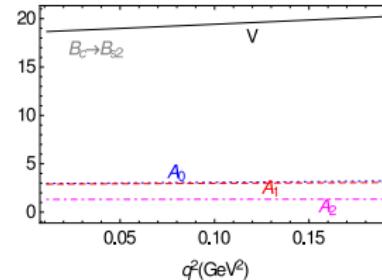
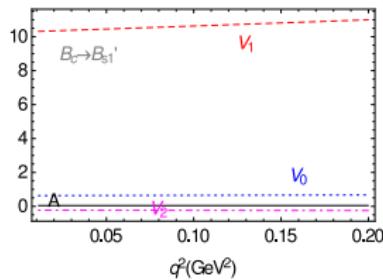
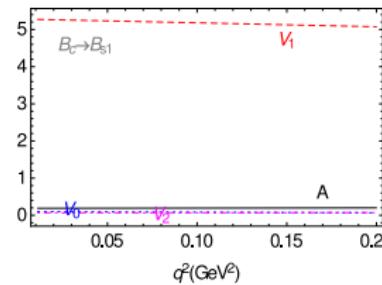
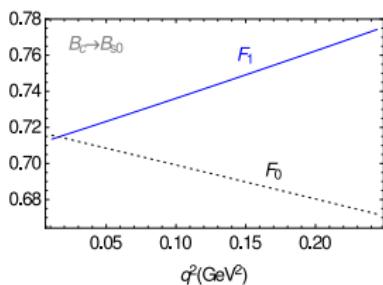
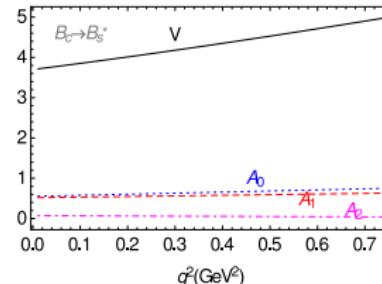
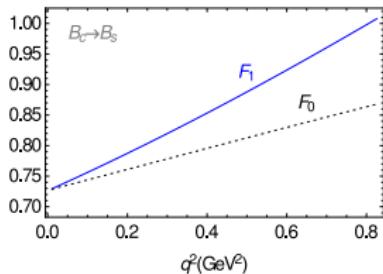
i.e. their  $q^2$ -dependence are obtained by the extrapolation.

# Numerical results → $(B_c \rightarrow B_s J/\psi)$

$B_c \rightarrow B_s, B_s^*, B_{s0}$  form factors in the light-front quark model:

$F$	$F(0)$	$m_{fit}$	$\delta$	$F$	$F(0)$	$m_{fit}$	$\delta$
$F_1^{BcBs}$	0.73	1.57	0.49	$F_0^{BcBs}$	0.73	2.07	0.82
$V^{BcBs^*}$	3.70	1.57	0.48	$A_0^{BcBs^*}$	0.55	1.49	0.61
$A_1^{BcBs^*}$	0.52	1.90	0.56	$A_2^{BcBs^*}$	0.07	1.04	0.37
$F_1^{BcBs_0^*}$	0.71	1.69	0.48	$F_0^{BcBs_0^*}$	0.72	1.98	1.43

# Numerical results → Figures

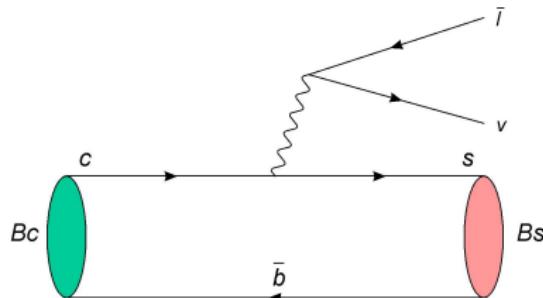


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# Semileptonic $B_c$ decays $\rightarrow$ Helicity amplitude

Decay process of  $B_c \rightarrow B_s J \bar{\ell} \nu$ :



For  $B_c \rightarrow B_s J \bar{\ell} \nu$ , divide the decay amplitude into hadronic part and leptonic part:

$$i\mathcal{M} = -i \frac{G_F}{\sqrt{2}} V_{CKM} H^\mu L^\nu g_{\mu\nu}$$

$$H^\mu = \langle M | V^\mu - A^\mu | B_c(P') \rangle \quad \rightarrow \quad \text{Form factors}$$

$$L^\nu = \langle \bar{\ell} \nu | \bar{\nu} \gamma^\nu (1 - \gamma_5) | 0 \rangle \quad \rightarrow \quad \text{Straightforward calculation}$$

# Semileptonic $B_c$ decays $\rightarrow$ Decay width

Then it's straightforward to obtain the partial decay width:

$$\frac{d\Gamma(B_c \rightarrow P \bar{l} \nu)}{dq^2} = (1 - \hat{m}_l^2)^2 \frac{\sqrt{\lambda(m_{B_c}^2, m_P^2, q^2)} G_F^2 |V_{CKM}|^2}{384 m_{B_c}^3 \pi^3} \left\{ (\hat{m}_l^2 + 2) \right. \\ \times \left. \lambda(m_{B_c}^2, m_P^2, q^2) F_1^2(q^2) + 3\hat{m}_l^2(m_{B_c}^2 - m_P^2)^2 F_0^2(q^2) \right\}$$

where we define:

$$\lambda(m_{B_c}^2, m_l^2, q^2) = (m_{B_c}^2 + m_l^2 - q^2)^2 - 4m_{B_c}^2 m_l^2$$

$$\hat{m}_l = m_l / \sqrt{q^2}$$

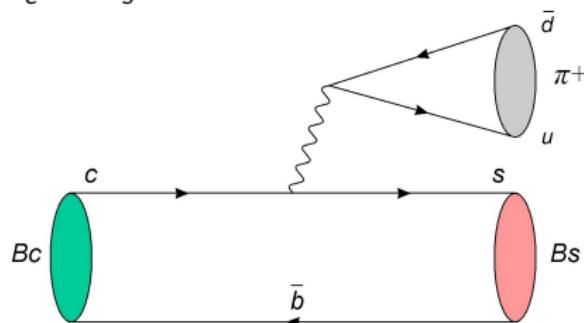
# Semileptonic $B_c$ decays → Numerical results

Branching ratios for  $B_c \rightarrow B_s J \bar{\ell} \nu (\ell = e, \mu)$ :

$\ell = e$	$\mathcal{B}_{\text{total}}$	$\mathcal{B}_L/\mathcal{B}_T$	$\ell = \mu$	$\mathcal{B}_{\text{total}}$	$\mathcal{B}_L/\mathcal{B}_T$
$B_c \rightarrow B_s \ell \nu$	$1.51 \times 10^{-2}$	--	$B_c \rightarrow B_s \ell \nu$	$1.43 \times 10^{-2}$	--
$B_c \rightarrow B_s^* \ell \nu$	$1.96 \times 10^{-2}$	1.13	$B_c \rightarrow B_s^* \ell \nu$	$1.83 \times 10^{-2}$	1.10
$B_c \rightarrow B_{s0} \ell \nu$	$6.58 \times 10^{-4}$	--	$B_c \rightarrow B_{s0} \ell \nu$	$5.23 \times 10^{-4}$	--
$B_c \rightarrow B_{s1} \ell \nu$	$8.31 \times 10^{-5}$	0.57	$B_c \rightarrow B_{s1} \ell \nu$	$6.33 \times 10^{-5}$	0.52
$B_c \rightarrow B'_{s1} \ell \nu$	$5.38 \times 10^{-4}$	2.38	$B_c \rightarrow B'_{s1} \ell \nu$	$3.98 \times 10^{-4}$	2.09
$B_c \rightarrow B_{s2} \ell \nu$	$2.98 \times 10^{-5}$	2.29	$B_c \rightarrow B_{s2} \ell \nu$	$1.97 \times 10^{-5}$	1.97

# Nonleptonic $B_c$ decays → Decay amplitude

Decay process of  $B_c \rightarrow B_s \pi^+$ :



$\pi^+$  decay constant is defined as:

$$\langle \pi^+(p) | \bar{u} \gamma_\mu \gamma_5 d | 0 \rangle = -if_\pi p_\mu$$

Thus the amplitude becomes:

$$i\mathcal{M} = \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} a_1 f_\pi q_\mu \langle B_s | \bar{s}_\alpha \gamma^\mu (1 - \gamma^5) c_\alpha | B_c^+ \rangle$$

Where  $a_1 = C_1/3 + C_2$  and  $C_1, C_2$  are Wilson coefficients.

# Nonleptonic $B_c$ decays → Decay amplitude

Some  $B_c \rightarrow B_{sJ}\pi$  decay amplitudes:

$$i\mathcal{M}(B_c^+ \rightarrow B_s\pi^+) = N m_{B_c}^2 (1 - r_{B_s}^2) F_0^{B_c B_s}(m_\pi^2)$$

$$i\mathcal{M}(B_c^+ \rightarrow B_s^*\pi^+) = (-i)N \sqrt{\lambda(m_{B_c}^2, m_{B_s^*}^2, m_\pi^2)} A_0^{B_c B_s^*}(m_\pi^2)$$

$$i\mathcal{M}(B_c^+ \rightarrow B_{s0}\pi^+) = (-i)N m_{B_c}^2 (1 - r_{B_{s0}}^2) F_0^{B_c B_{s0}}(m_\pi^2)$$

Where  $N = G_F/\sqrt{2} V_{cs}^* V_{ud} a_1 f_\pi$ . The partial decay width for  $B_c \rightarrow B_{sJ}\pi$  is:

$$\Gamma = \frac{|\vec{p}_1|}{8\pi m_{B_c}^2} |\mathcal{M}|^2$$

with  $|\vec{p}_1|$  being the magnitude of three-momentum of  $B_{sJ}$ .

# Nonleptonic $B_c$ decays → Numerical results

Numerical results for  $B_c \rightarrow B_{sJ}\pi$  branching ratios turn out to be as follows:

$$\mathcal{B}(B_c^+ \rightarrow B_s\pi^+) = 4.1\%$$

$$\mathcal{B}(B_c^+ \rightarrow B_s^*\pi^+) = 2.0\%$$

$$\mathcal{B}(B_c^+ \rightarrow B_{s0}\pi^+) = 0.68\%$$

$$\mathcal{B}(B_c^+ \rightarrow B_{s1}\pi^+) = 0.0082\%$$

$$\mathcal{B}(B_c^+ \rightarrow B'_{s1}\pi^+) = 0.36\%$$

$$\mathcal{B}(B_c^+ \rightarrow B_{s2}\pi^+) = 0.023\%$$

Zhi Yang, Xing-Gang Wu, Gu Chen, Qi-Li Liao, Jia-Wei Zhang,  
[arXiv:1112.5169](https://arxiv.org/abs/1112.5169)

$Z - peak : 10^5 - 10^7 B_c$  events

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# Summary

- Covariant LFQM
- Form factors from LFQM
  - Numerical fit and extrapolating to  $q^2 > 0$
  - Stable dependence on  $q^2$
- Semileptonic  $B_c$  decays
  - Branching ratios of  $B_c \rightarrow B_{sJ}(e^+/\mu^+)\nu$
  - To be examined by experiments.
- Nonleptonic  $B_c$  decays
  - Branching ratios of  $B_c \rightarrow B_{sJ}\pi^+$
  - To be examined by experiments.

**Thank you for your attention!**