Fluctuations of conserved charges from Lattice QCD

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1) EoS at $\mu_B>0$: arXiv:1408.6305,1412.6727,16xx.xxxx
2) Curvature of freezeout line: PRD93 (2016)1,014512

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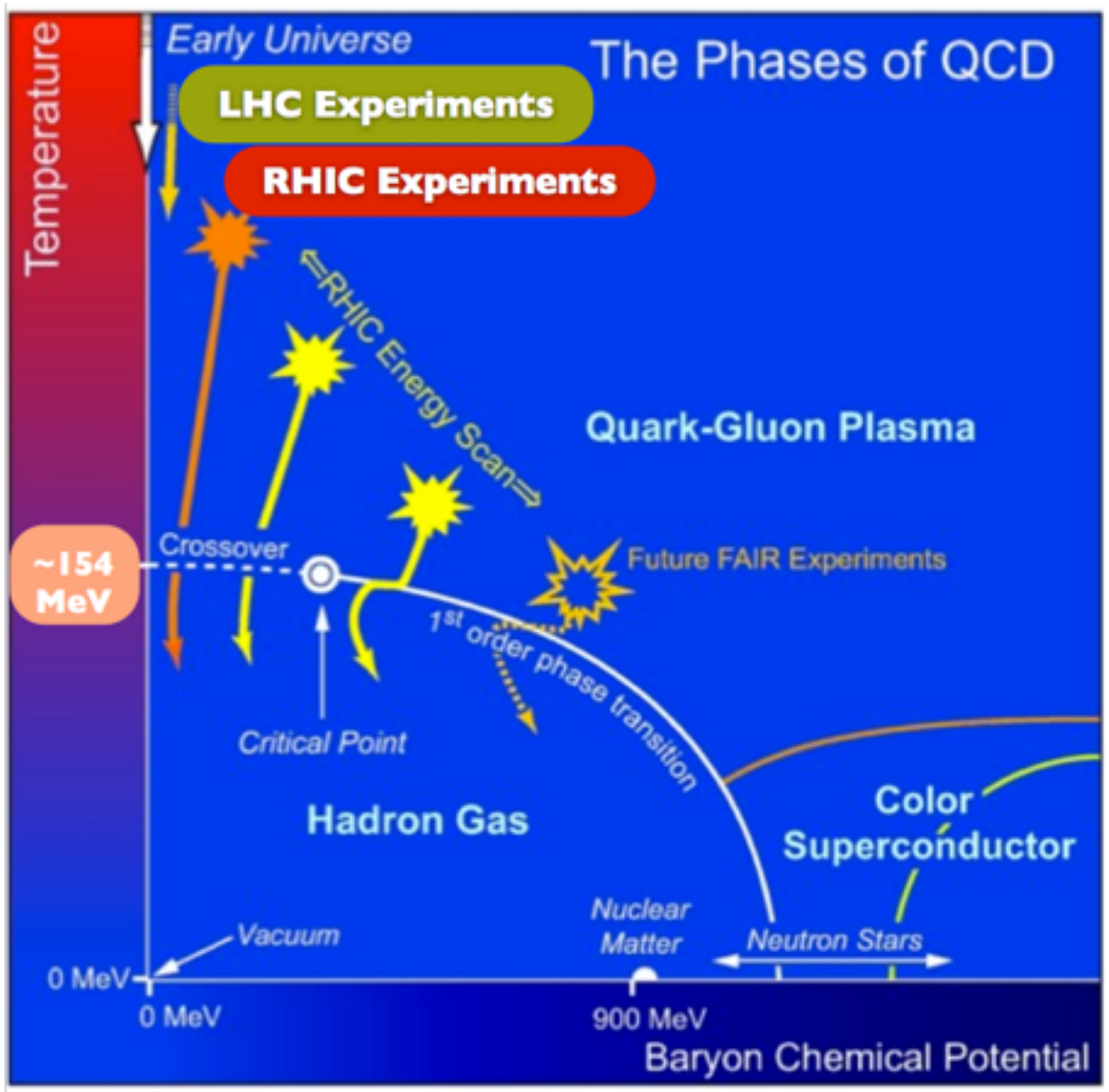
QCD Phase Diagram

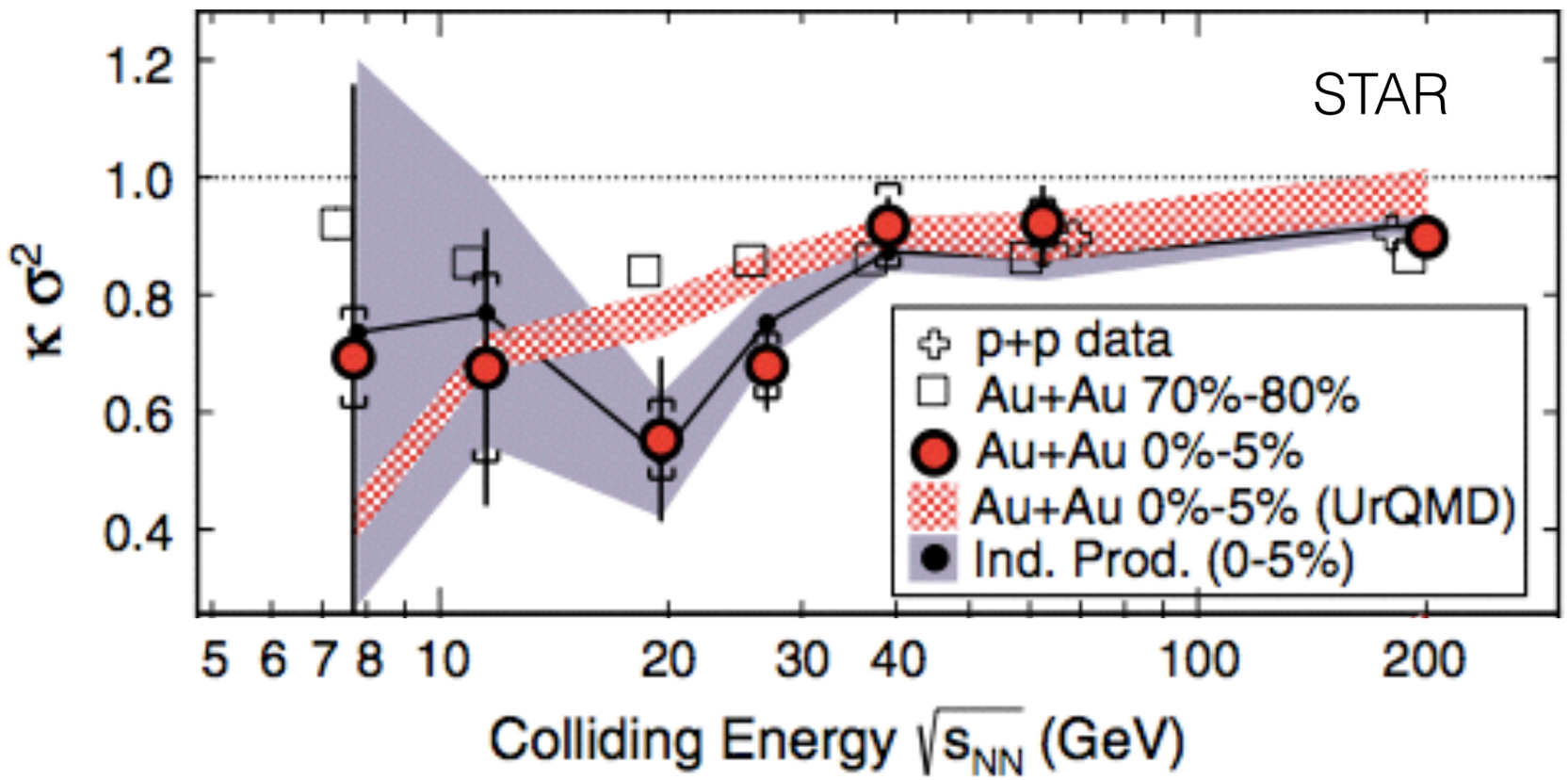


- The QCD phase structure is extensively studied by Heavy Ion Collision (HIC) Experiment
- Hadronic fluctuations and abundance are measured at freezeout
- The QCD Equation of State (EoS) is an essential input to hydrodynamic modeling of HIC

Lattice QCD can provide both EoS and fluctuations of conserved charges at small μ_B/T

Beam Energy Scan at RHIC

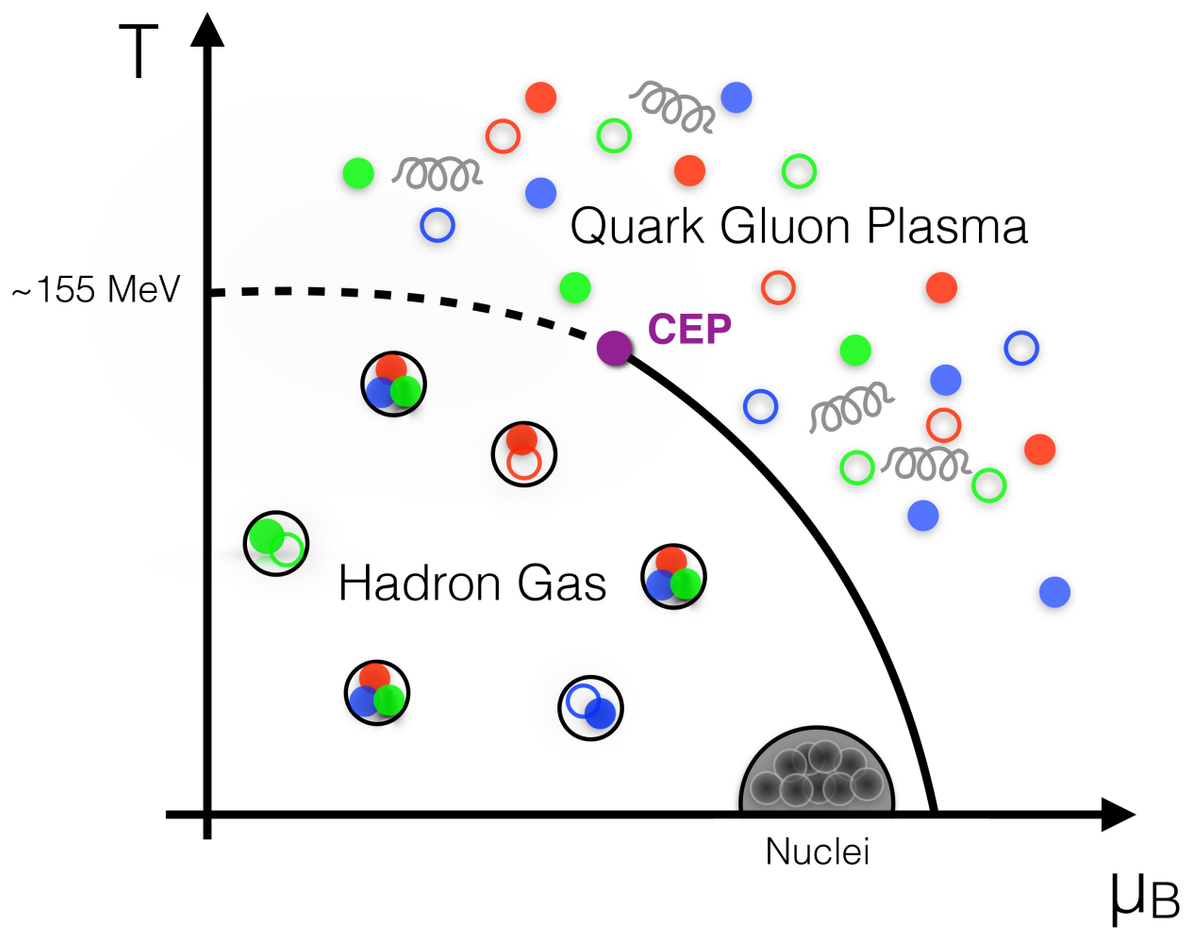
Ratio of the 4th to 2nd order proton number fluctuations



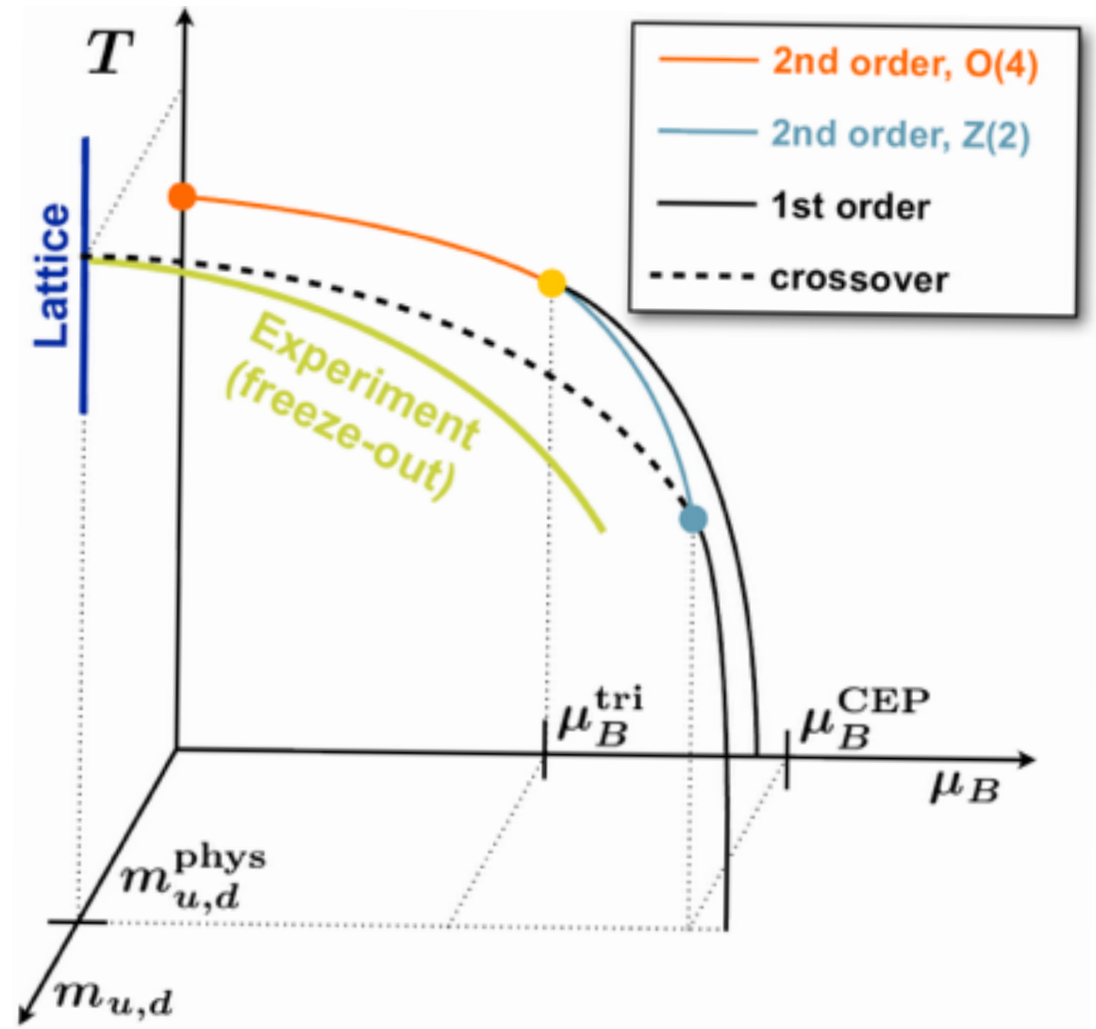
Can this non-monotonic behavior be understood in terms of the QCD thermodynamics in equilibrium?

What is the relation of this intriguing phenomenon to the critical behavior of QCD phase transition?

QCD phase diagram

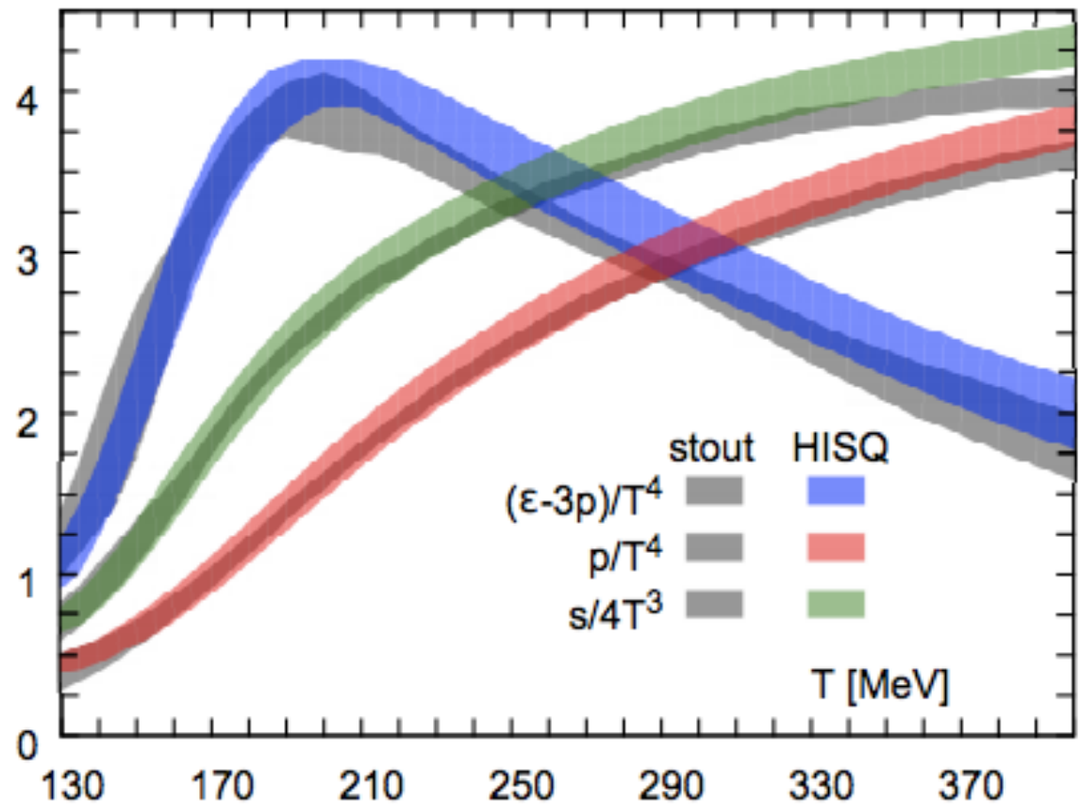


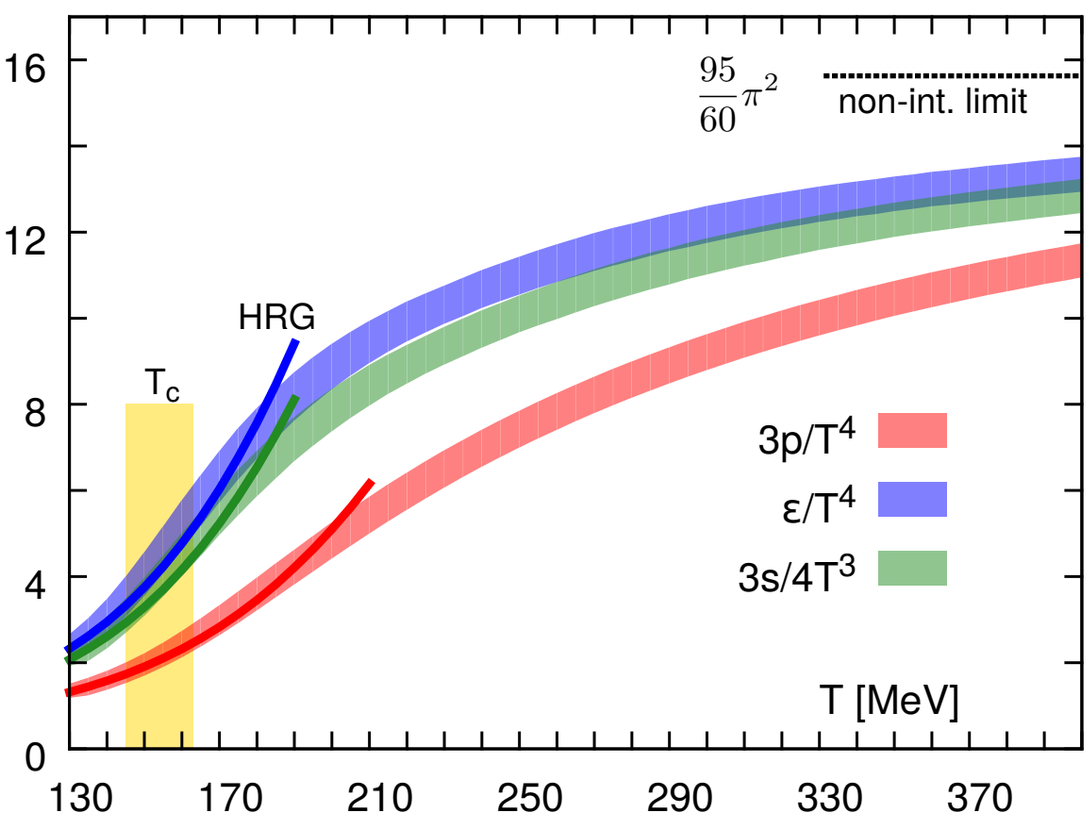
HTD, F. Karsch, S. Mukherjee, arXiv:1504.0527



- EOS at nonzero μ_B
- Freeze-out line

Equation of State at μ_B=0





- Consensus of QCD EoS obtained from two different discretization schemes HotQCD, PRD 90 (2014) 094503, Wuppertal-Budapest, Phys. Lett. B730 (2014) 99

- Parameterization of EoS for use

$$\frac{p}{T^4} = \frac{1}{2}\left(1+\tanh(c_t(\bar{t}-t_0))\right) \cdot \frac{p_{id} + a_n/\bar{t} + b_n/\bar{t}^2 + c_n/\bar{t}^3 + d_n/\bar{t}^4}{1 + a_d/\bar{t} + b_d/\bar{t}^2 + c_d/\bar{t}^3 + d_d/\bar{t}^4}$$

$\bar{t}=T/T_c$

HotQCD, PRD 90 (2014) 094503

- Smooth transition from hadronic phase to QGP phase; system is far away from the idea gas limit at ~2.7T_c

Lattice simulations at nonzero μ_B

- No direct simulation is reliable due to the infamous sign problem
- Several approaches exist: Reweighting, imaginary μ_B, complex Langevin, Lefschetz thimbles…
- Taylor Expansion Method for small values of μ_B

$$\frac{p}{T^4} = \frac{1}{VT^3} \ln \mathcal{Z}(T, V, \hat{\mu}_u, \hat{\mu}_d, \hat{\mu}_s) = \sum_{i,j,k=0}^{\infty} \frac{\chi_{ijk}^{BQS}}{i!j!k!} \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k$$

Allton et al., Phys.Rev. D66 (2002) 074507
Gavai & Gupta et al., Phys.Rev. D68 (2003) 034506

Fluctuations of conserved charges

Taylor expansion of the QCD pressure:

Allton et al., Phys.Rev. D66 (2002) 074507
Gavai & Gupta et al., Phys.Rev. D68 (2003) 034506

$$\frac{p}{T^4} = \frac{1}{VT^3} \ln \mathcal{Z}(T, V, \hat{\mu}_u, \hat{\mu}_d, \hat{\mu}_s) = \sum_{i,j,k=0}^{\infty} \frac{\chi_{ijk}^{BQS}}{i!j!k!} \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k$$

- Taylor expansion coefficients at μ=0 are computable in Lattice QCD

$$\chi_{ijk}^{BQS} \equiv \chi_{ijk}^{BQS}(T) = \frac{1}{VT^3} \frac{\partial P(T, \hat{\mu})/T^4}{\partial \hat{\mu}_B^i \partial \hat{\mu}_Q^j \partial \hat{\mu}_S^k}\Big|_{\hat{\mu}=0}$$

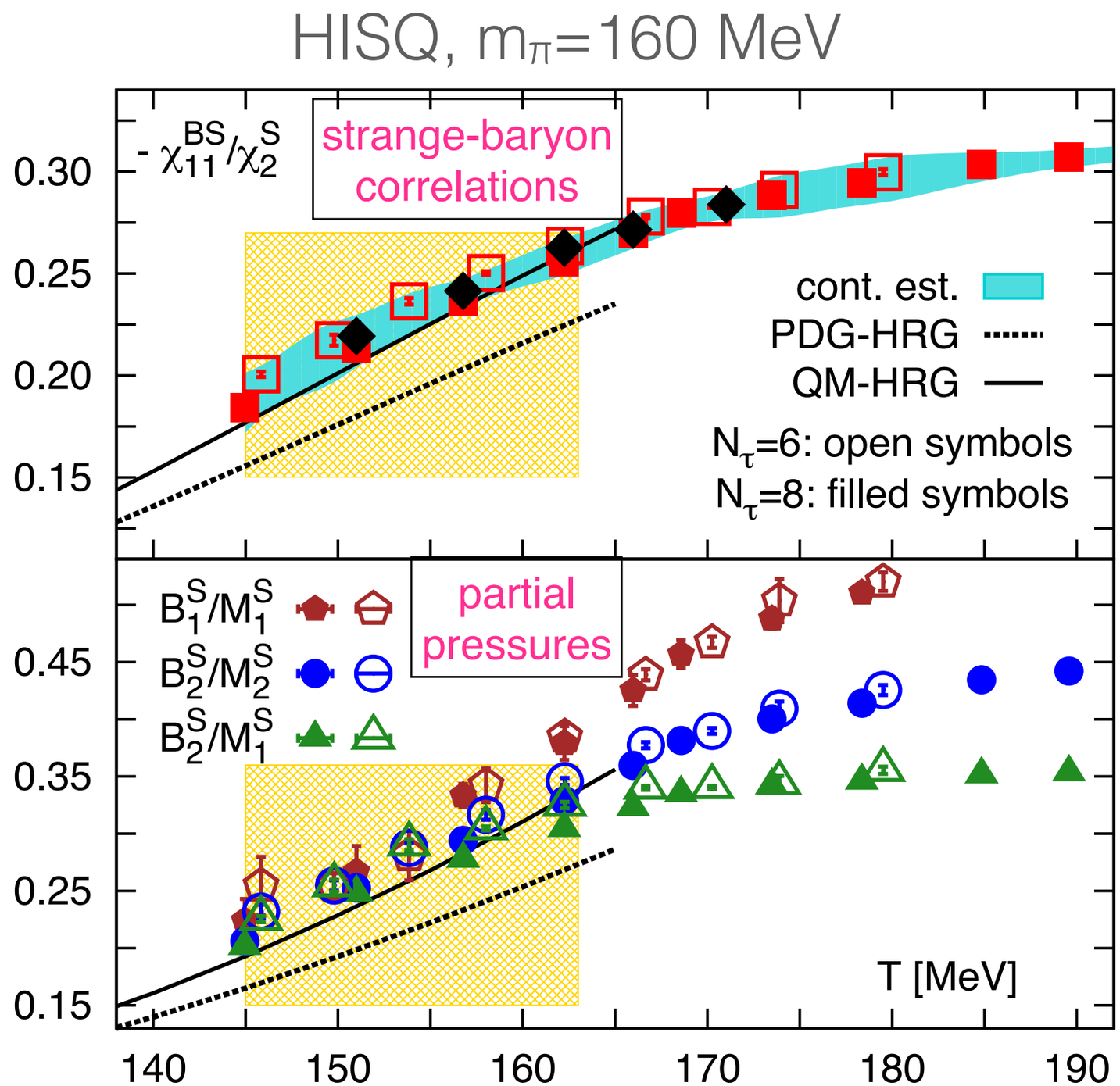
- Other quantities can be obtained using relations, e.g.

$$\frac{\epsilon - 3p}{T^4} = T \frac{\partial P/T^4}{\partial T} = \sum_{i,j,k=0}^{\infty} \frac{T\, \mathrm{d}\chi_{ijk}^{BQS}/\mathrm{d}T}{i!j!k!} \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k$$

Pressure of hadron resonance gas (HRG)

$$\frac{p}{T^4} = \sum_{m \in meson, baryon} \ln Z(T, V, \mu) \sim \exp(-m_H/T) \exp((B\mu_B + S\mu_s + Q\mu_Q)/T)$$

Indirect evidence of experimentally not yet observed strange states hinted from QCD thermodynamics



PDG-HRG: Hadron Resonance Gas model calculations with spectrum from PDG

QM-HRG: Similar as PDG-HRG but with spectrum from Quark Model

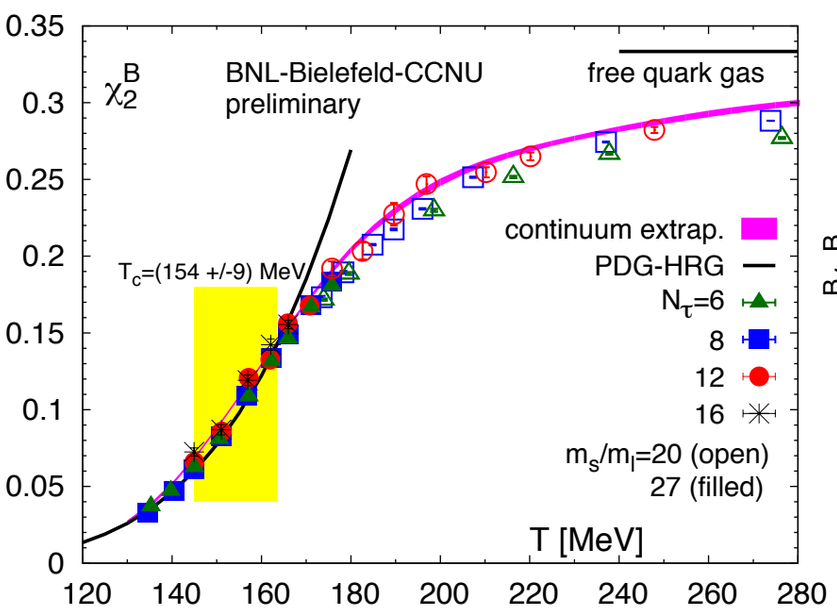
A. Bazavov et al.[Bielefeld-BNL-CCNU], Phys. Rev. Lett. 113 (2014)072001

Pressure of QCD at nonzero muB

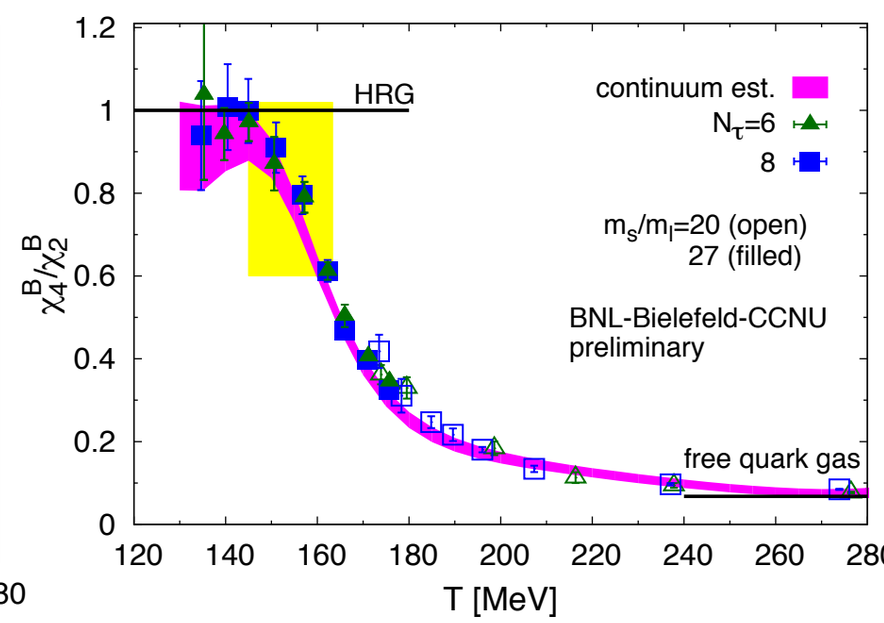
$\mu_Q=\mu_S=0$:

$$\Delta(P/T^4) = \frac{P(T,\mu_B)-P(T,0)}{T^4} = \sum_{n=1}^{\infty} \frac{\chi_{2n}^B(T)}{(2n)!}\left(\frac{\mu_B}{T}\right)^{2n}$$
$$= \frac{1}{2}\chi_2^B(T)\hat{\mu}_B^2\left(1+\frac{1}{12}\frac{\chi_4^B(T)}{\chi_2^B(T)}\hat{\mu}_B^2+\frac{1}{360}\frac{\chi_6^B(T)}{\chi_2^B(T)}\hat{\mu}_B^4+\cdots\right)$$

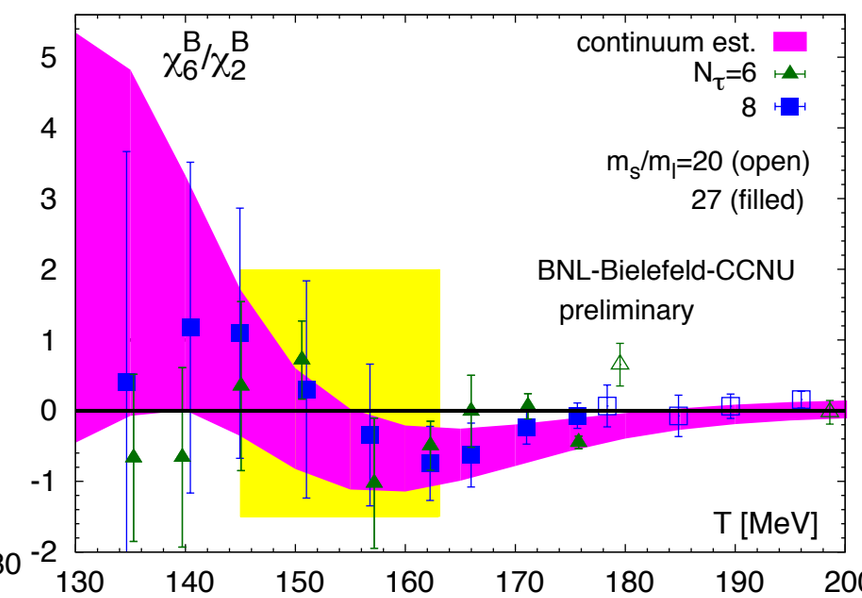
LO expansion coefficient
variance of net-baryon number distribution



NLO expansion coefficient
kurtosis * variance



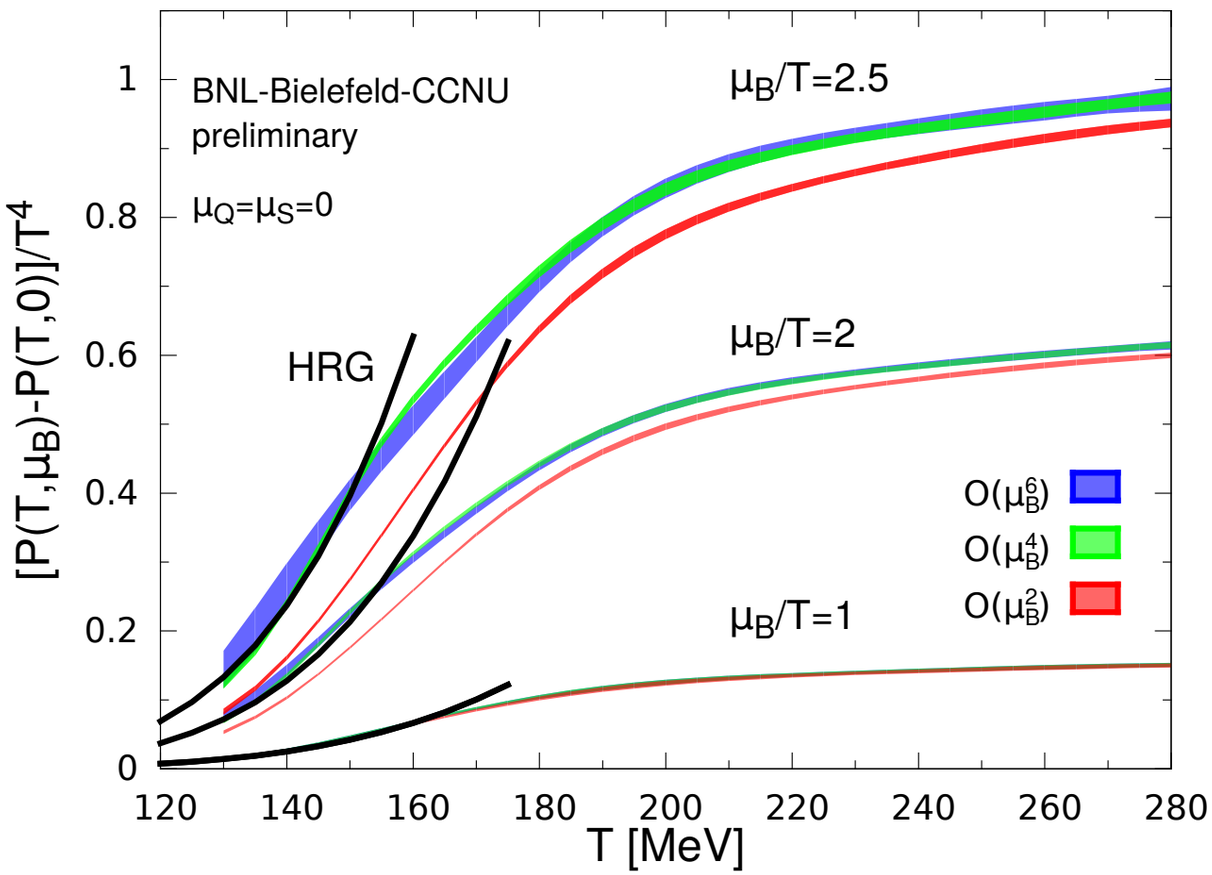
NNLO expansion coefficient

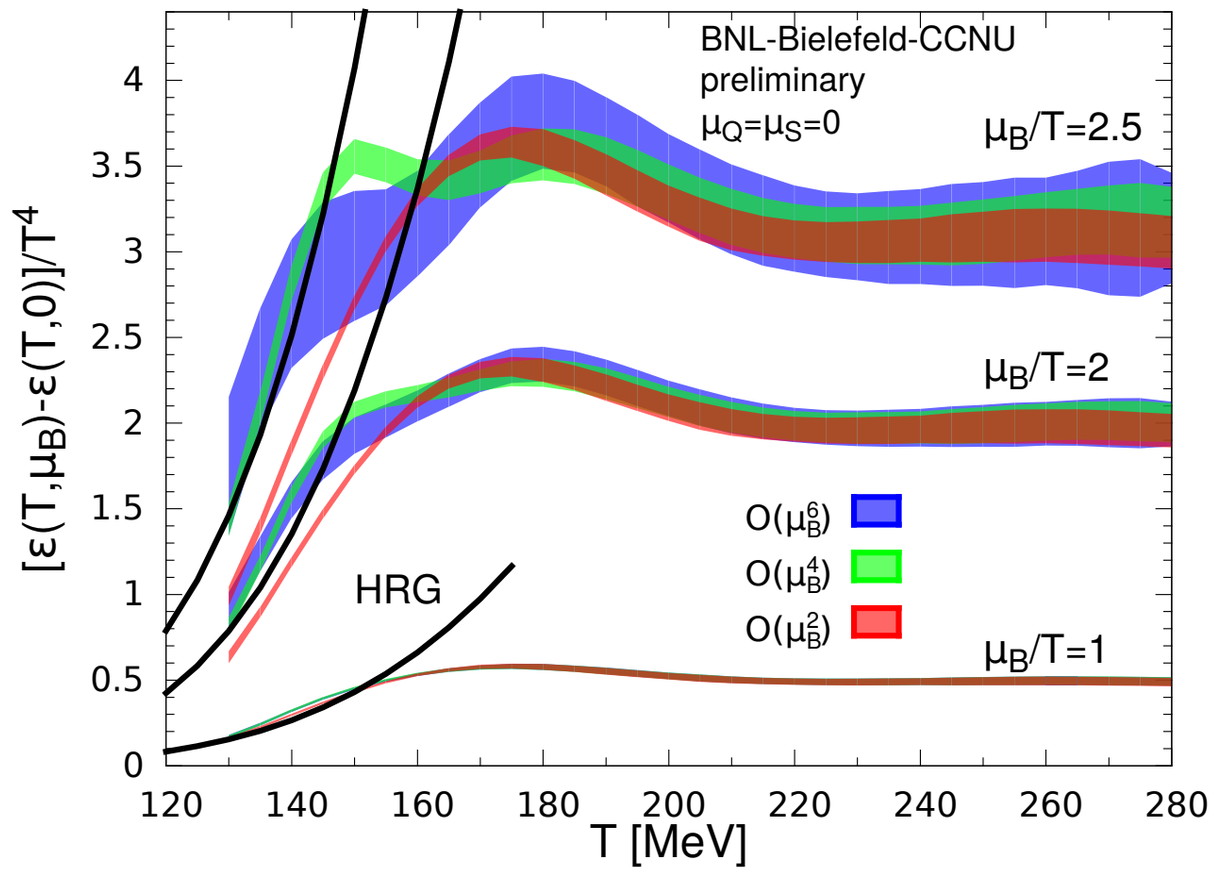


HTD, Nucl. Phys. A 931 (2014) 52-62, HTD, F. Karsch, S. Mukherjee, arXiv:1504.0527,
E. Laermann [BNL-Bielefeld-CCNU], talk given in lattice 2016

- HRG describes well on the LO expansion coefficient up to ~160 MeV while it deviates from NLO expansion coefficient ~ 40% in the crossover region
- For small muB/T the LO contribution dominates

Pressure with $\mu_Q=\mu_S=0$





E. Laermann [BNL-Bielefeld-CCNU], lattice 2016

- Leading order corrections dominate at small μ_B/T
- Higher order corrections become significant at $\mu_B/T \gtrsim 2$

Conditions meet in heavy ion collisions

- Zero net strangeness n_S=0, and n_Q/n_B=r=0.4 as in PbPb collision systems

$$\frac{n_X}{T^3} = \frac{\partial P/T^4}{\partial \hat{\mu}_X} \quad \text{, X=B,Q,S}$$

,

$$n_S = n_S^{(1)}\mu_B + n_S^{(3)}\mu_B^3 + \cdots = 0\,, \quad n_Q = n_Q^{(1)}\mu_B + n_Q^{(3)}\mu_B^3 + \cdots$$
$$n_I = n_I^{(1)}\mu_B + n_I^{(3)}\mu_B^3 + \cdots = (\frac{1}{r} - 2)n_Q$$

E.g. 1st order coefficient in n_S: $n_S^{(1)} = \chi_2^S \frac{\mu_S}{\mu_B} + \chi_{11}^{QS}\frac{\mu_Q}{\mu_B} + \chi_{11}^{BS}$

μ_S, μ_Q and μ_B are correlated

Conditions meet in heavy ion collisions

Taylor expansion of the QCD pressure:

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$\mu_Q = \mu_S = 0$:

$$\frac{p}{T^4} = \sum_{n=0} \frac{\chi_{2n}^{B}}{(2n)!} \left(\frac{\mu_B}{T}\right)^{2n}$$

strangness neutral case:

$$\frac{p}{T^4} = \sum_{n=0} \frac{\chi_{2n,\mathrm{SN}}^{B}}{(2n)!} \left(\frac{\mu_B}{T}\right)^{2n}$$

- Expand μ_Q and μ_S in terms of μ_B

$$\frac{\mu_Q}{T} = q_1 \frac{\mu_B}{T} + q_3 \left(\frac{\mu_B}{T}\right)^3 + \cdots \ , \ \frac{\mu_S}{T} = s_1 \frac{\mu_B}{T} + s_3 \left(\frac{\mu_B}{T}\right)^3 + \cdots$$

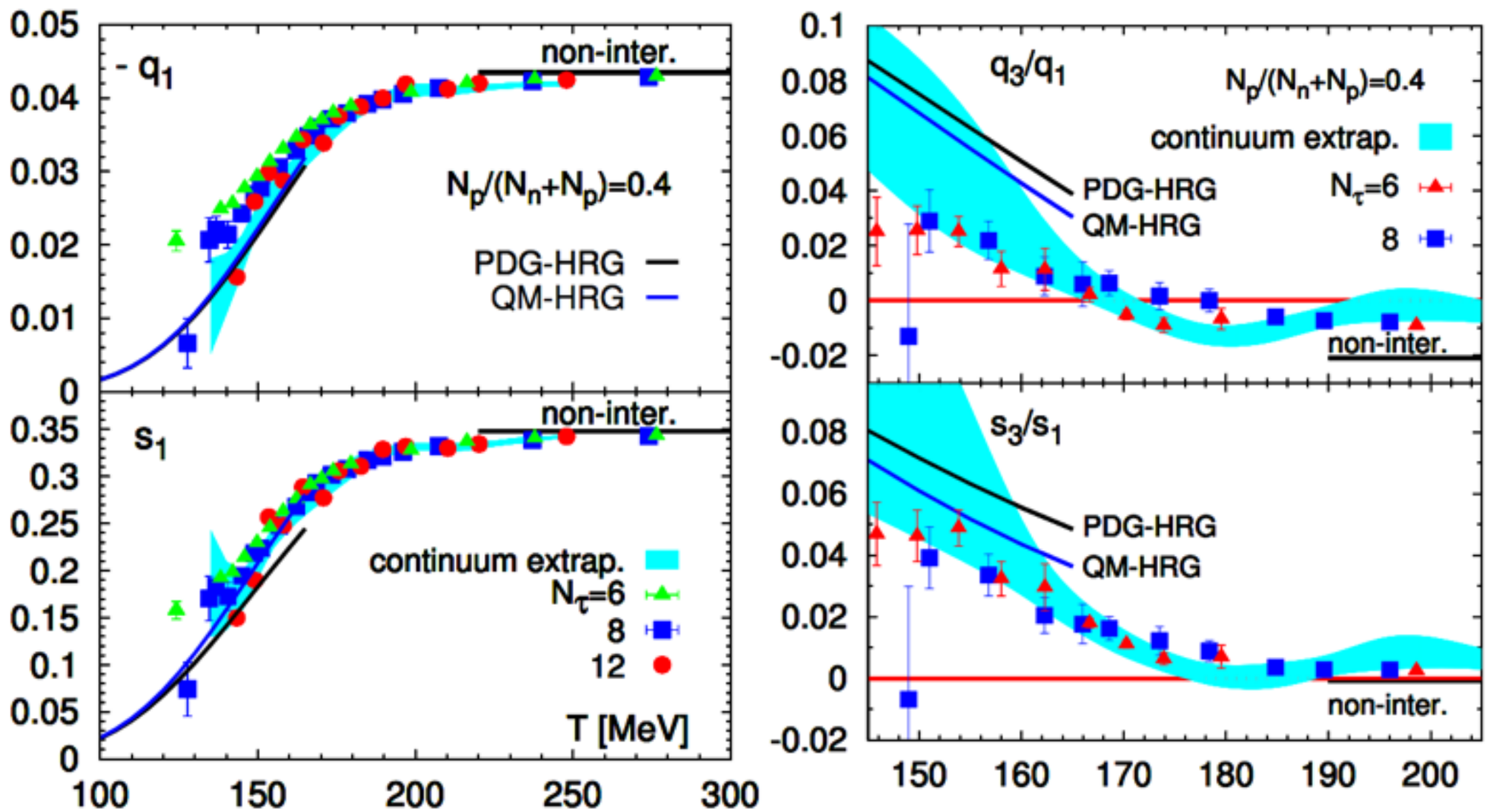
- With constrains from isospin symmetry etc., one can derive q_i and s_i order by order and then the pressure etc.

A. Bazavov, HTD et al., Phys. Rev. Lett. 109 (2012)192302

Conditions meet in heavy ion collisions

- Zero net strangeness n_S=0, and n_Q/n_B = r=0.4 as in PbPb collision systems

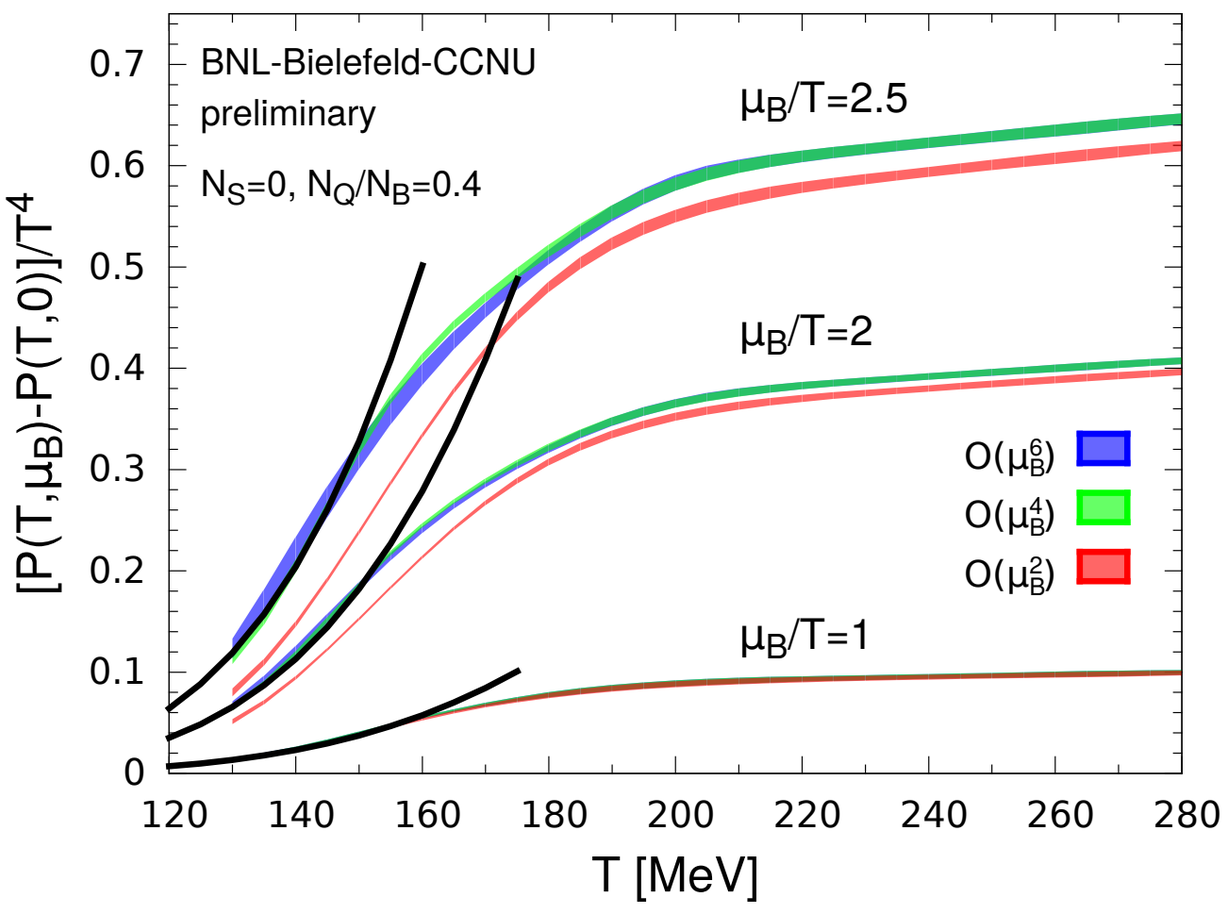
$$\frac{\mu_Q}{T} = q_1 \frac{\mu_B}{T} + q_3 \left(\frac{\mu_B}{T}\right)^3 + \cdots \ , \quad \frac{\mu_S}{T} = s_1 \frac{\mu_B}{T} + s_3 \left(\frac{\mu_B}{T}\right)^3 + \cdots$$

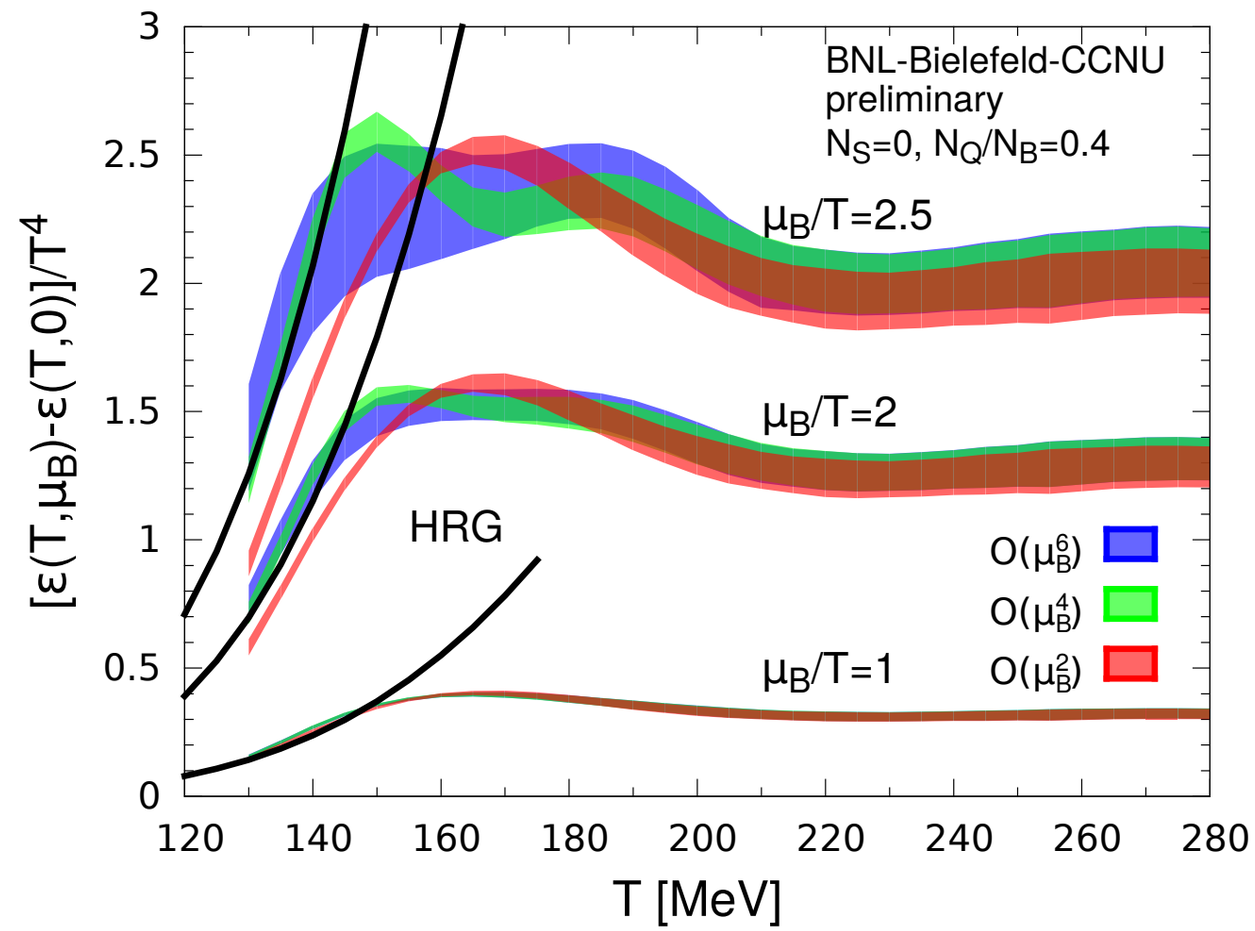


A. Bazavov, HTD et al., Phys. Rev. Lett. 109 (2012)192302

EoS in the strangeness neutral system

At LHC and RHIC: $\langle n_S \rangle = 0$, $n_Q/n_B = 0.4$:

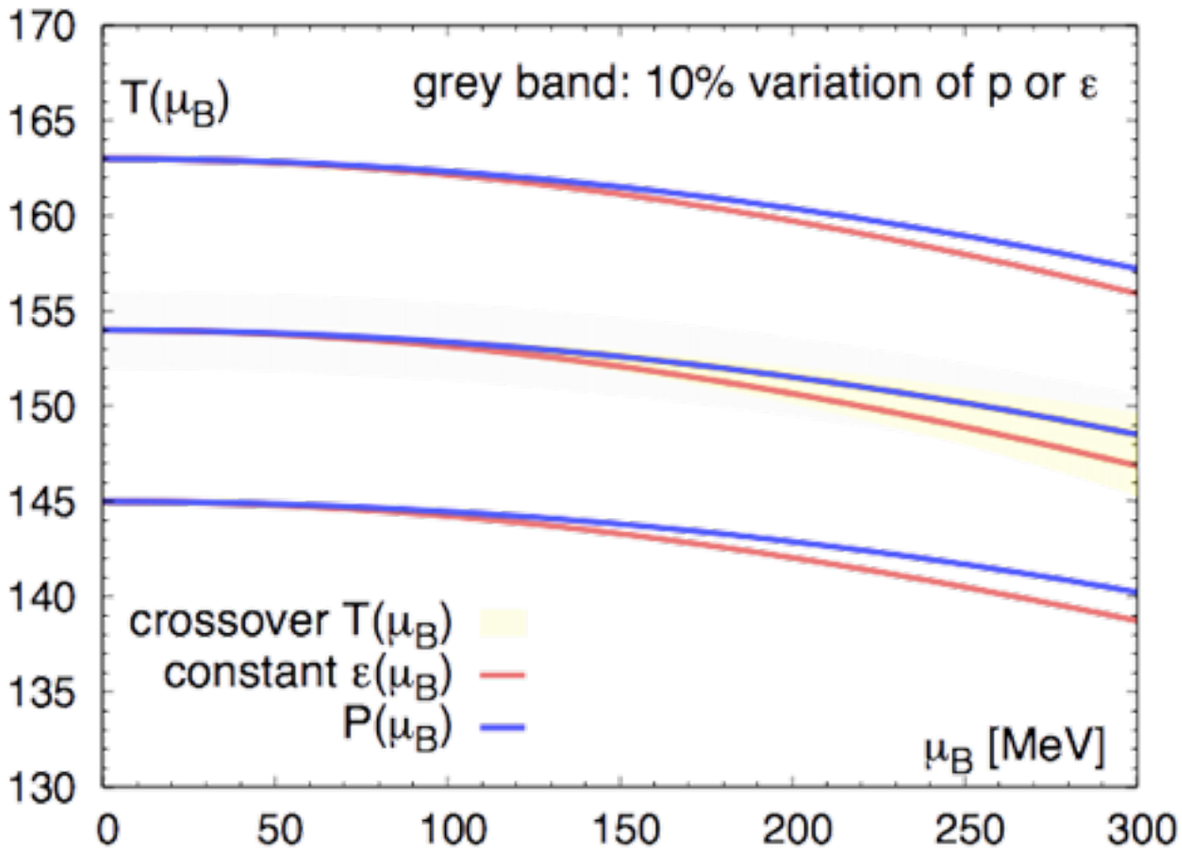


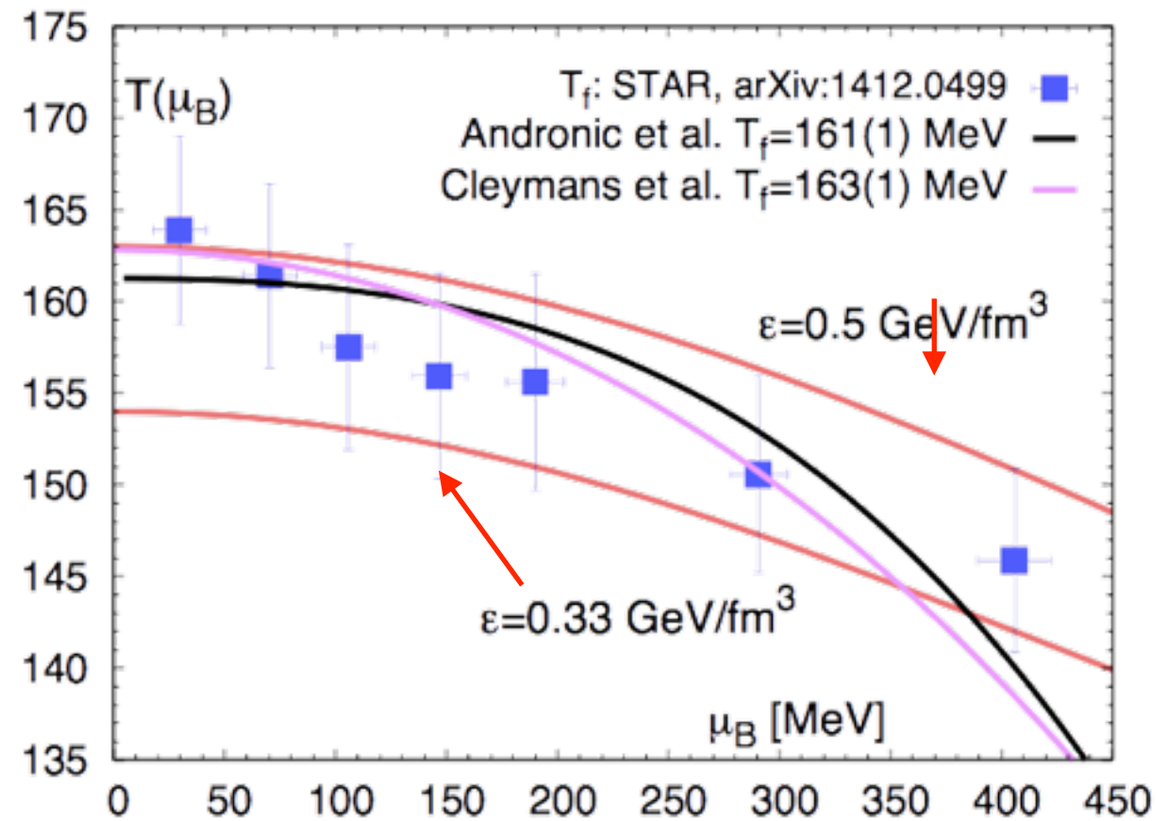


BNL-Bielefeld-CCNU, 1408.6305,1412.6727, lattice 2016

The EoS is well under control at $\mu_B/T \lesssim 2$ or $\sqrt{s_{NN}} \gtrsim 20$ GeV

Line of constant physics and freeze-out





Parameterization $T(\mu_B) = T(0)(1 - \kappa_2 \hat{\mu}_B^2 + \mathcal{O}(\hat{\mu}_B^4))$

curvature at constant pressure: $\kappa_{2,p} \approx 0.011$

curvature at constant energy: $\kappa_{2,\epsilon} \approx 0.013$

curvature of the crossover line: $\kappa_{2,c} \approx 0.006 - 0.013$

Explore the QCD phase diagram through fluctuations of conserved charges

Comparison of experimentally measured higher order cumulants of conserved charges to those from LQCD, e.g.:

$$\frac{M_Q(\sqrt{s})}{\sigma_Q^2(\sqrt{s})} = \frac{\langle N_Q \rangle}{\langle (\delta N_Q)^2 \rangle} = \frac{\chi_1^Q(T,\mu_B)}{\chi_2^Q(T,\mu_B)} = R_{12}^Q(T,\mu_B)$$

$$\frac{S_Q(\sqrt{s})\,\sigma_Q^3(\sqrt{s})}{M_Q(\sqrt{s})} = \frac{\langle (\delta N_Q)^3 \rangle}{\langle N_Q \rangle} = \frac{\chi_3^Q(T,\mu_B)}{\chi_1^Q(T,\mu_B)} = R_{31}^Q(T,\mu_B)$$

HIC

mean: M_Q

variance: σ_Q^2

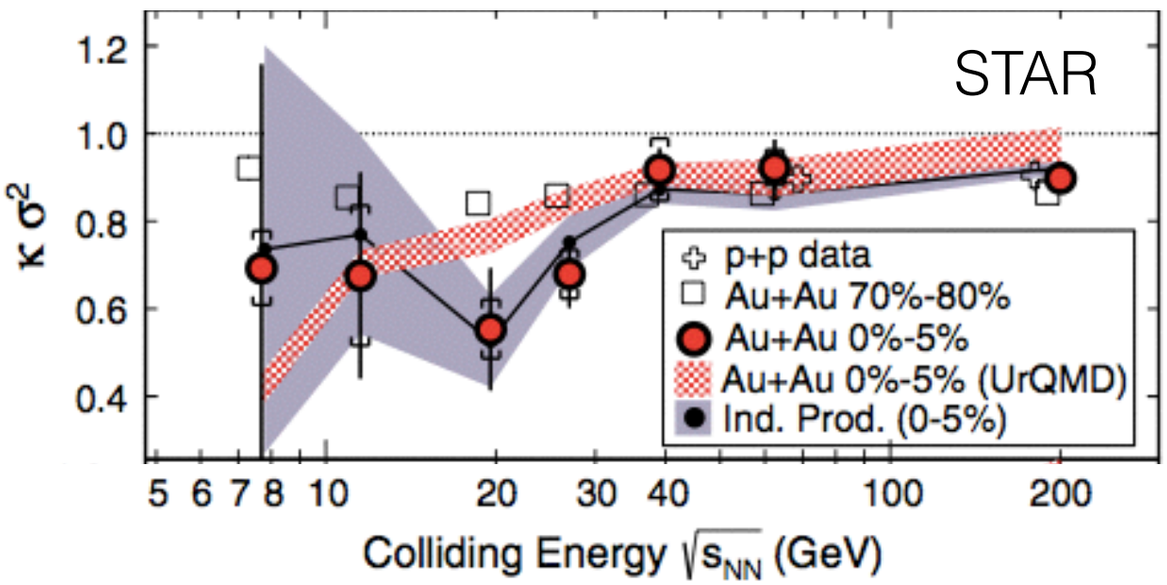
skewness: S_Q

LQCD

generalized susceptibilities

$$\chi_n^Q(T,\vec{\mu}) = \frac{1}{VT^3}\frac{\partial^n \ln Z(T,\vec{\mu})}{\partial(\mu_Q/T)^n}$$

Explore the QCD phase diagram

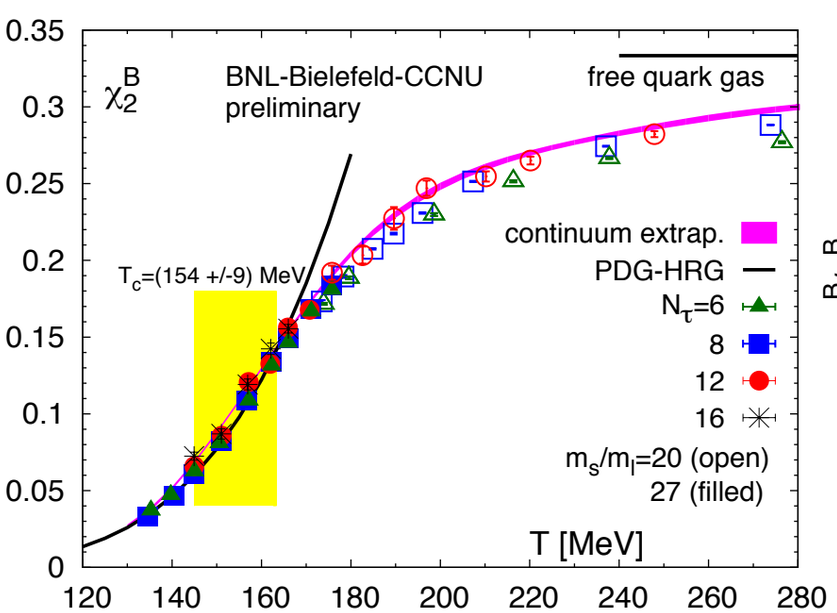


$$(\kappa\sigma^2)_B = \frac{\chi^B_{4,\mu}}{\chi^B_{2,\mu}} = \frac{\chi^B_4}{\chi^B_2}\left[1 + \left(\frac{\chi^B_6}{\chi^B_4} - \frac{\chi^B_4}{\chi^B_2}\right)\left(\frac{\mu_B}{T}\right)^2 + \cdots\right]$$

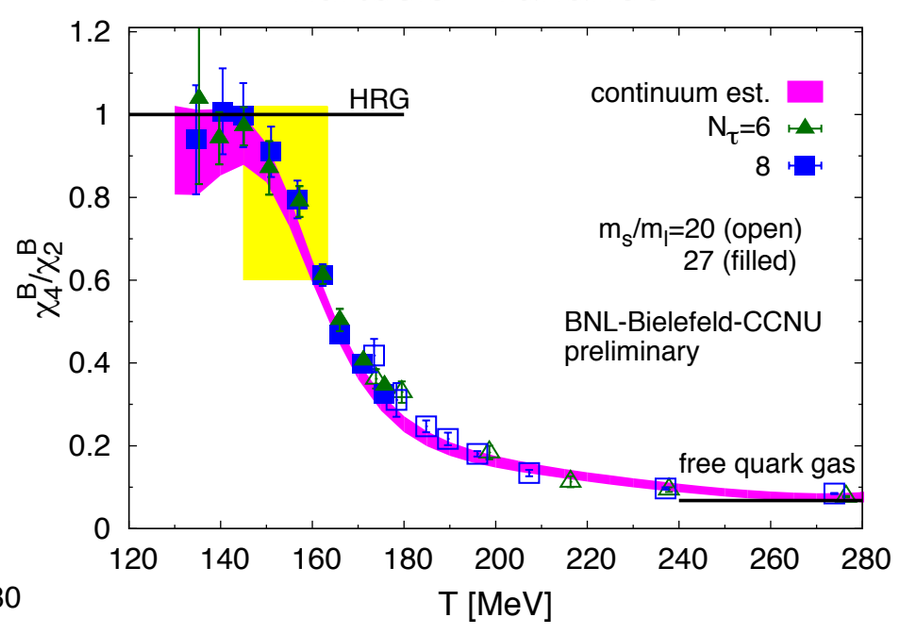
In the O(4) universality class:

$$\chi^B_6 < 0\,, \qquad T \sim T_c$$

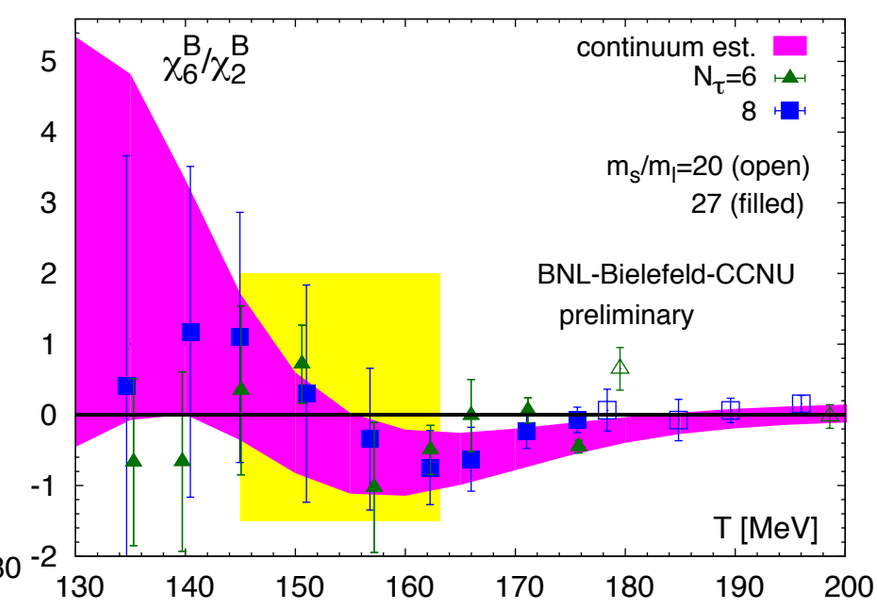
LO expansion coefficient
variance of net-baryon number distribution



NLO expansion coefficient
kurtosis * variance



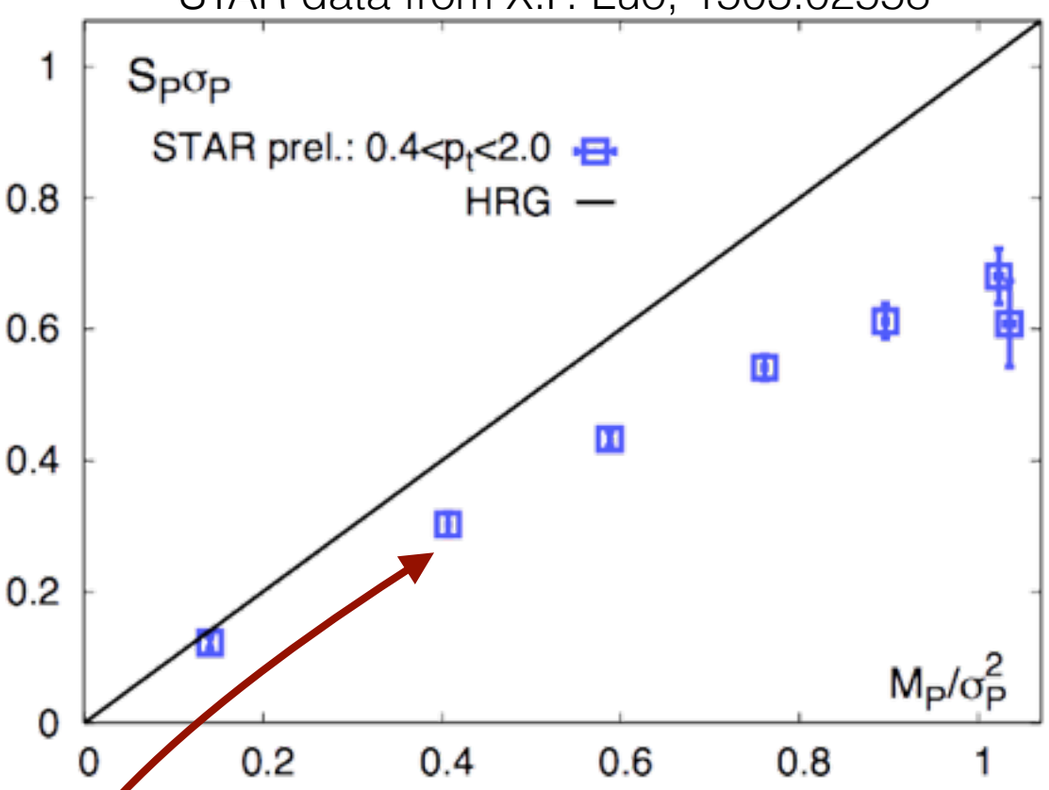
NNLO expansion coefficient



HTD, Nucl. Phys. A 931 (2014) 52-62, HTD, F. Karsch, S. Mukherjee, arXiv:1504.0527,
E. Laermann [BNL-Bielefeld-CCNU], talk given in lattice 2016

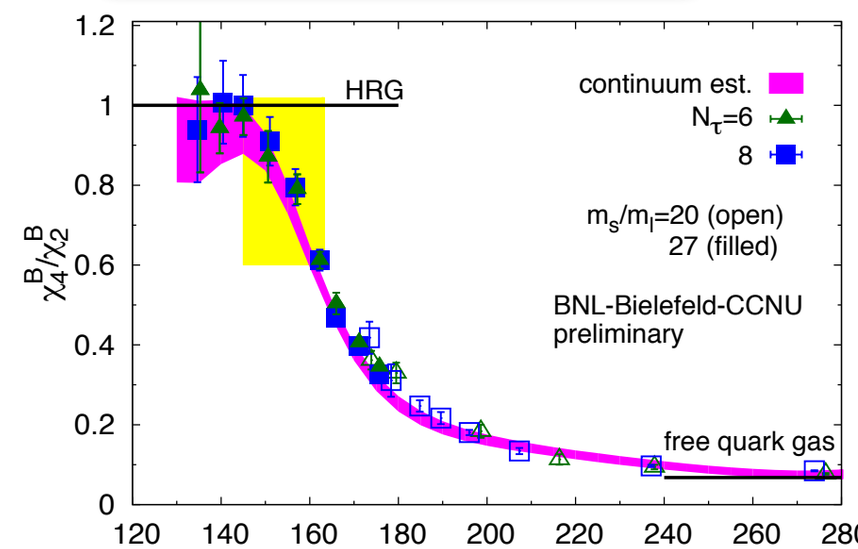
conserved charge fluctuations & freeze-out

STAR data from X.F. Luo, 1503.02558



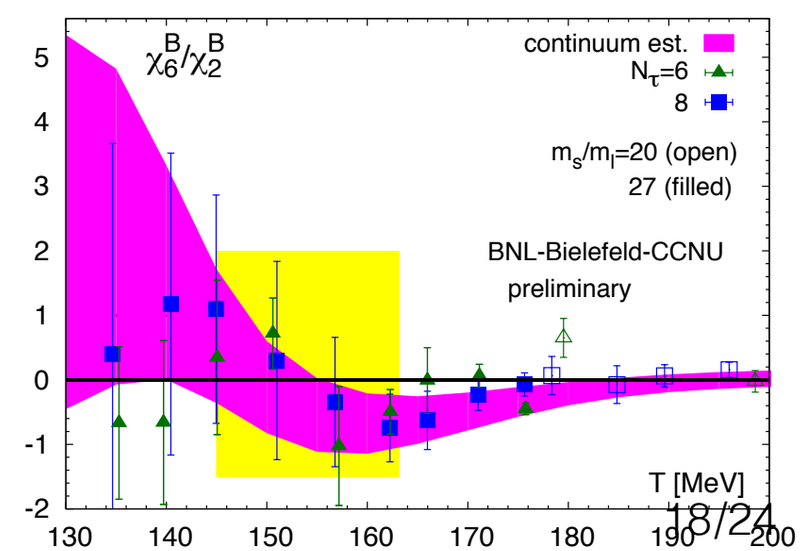
$$S_B\sigma_B = \frac{\chi^B_{3,\mu}}{\chi^B_{2,\mu}} = \frac{\mu_B}{T}\frac{\chi^B_4}{\chi^B_2}\frac{1+\frac{1}{6}\frac{\chi^B_6}{\chi^B_4}(\frac{\mu_B}{T})^2+\cdots}{1+\frac{1}{2}\frac{\chi^B_4}{\chi^B_2}(\frac{\mu_B}{T})^2+\cdots}$$

$$\frac{M_B}{\sigma_B^2} = \frac{\chi^B_{1,\mu}}{\chi^B_{2,\mu}} = \frac{\mu_B}{T}\frac{1+\frac{1}{6}\frac{\chi^B_4}{\chi^B_2}(\frac{\mu_B}{T})^2+\cdots}{1+\frac{1}{2}\frac{\chi^B_4}{\chi^B_2}(\frac{\mu_B}{T})^2+\cdots}$$



$$S_B\sigma_B = \frac{\mu_B}{T}\frac{\chi^B_4}{\chi^B_2} + \mathcal{O}((\frac{\mu_B}{T})^3) = \frac{M_B}{\sigma_B^2}\frac{\chi^B_4}{\chi^B_2} + \mathcal{O}((\frac{\mu_B}{T})^3)$$

slope is smaller than 1 as $\frac{\chi^B_4}{\chi^B_2} < 1$



Ratio on charge fluctuations on the freeze-out line

In heavy ion collisions M_s=0 and M_Q/M_B=r

ratios of mean to variance:

$$R_{12}^X(T,\mu) \equiv \frac{M_X}{\sigma_X^2} = \frac{\chi_1^X(T,\mu)}{\chi_2^X(T,\mu)} \ , \qquad X=B,Q$$

ratio of electrical charge to baryon ratio :

$$\Sigma_r^{QB} \equiv R_{12}^Q / R_{12}^B = r\,\sigma_B^2/\sigma_Q^2$$

Expand the ratio around μ_B=0:

$$\Sigma_r^{QB}(T,\hat{\mu}_B) = \Sigma_r^{QB}(T,\hat{\mu}_B=0) + \frac{1}{2!}\frac{\partial^2 \Sigma_r^{QB}(T,\hat{\mu}_B)}{\partial \hat{\mu}_B^2}\Big|_{\hat{\mu}_B=0}\hat{\mu}_B^2$$

Expand the ratio around $T_f(\mu_B)=T_f(\mu_B=0)$:

$$\Sigma_r^{QB}(T_f,\hat{\mu}_B) = \Sigma_r^{QB}(T_f=T_{f,0},\hat{\mu}_B) + \frac{\mathrm{d}\Sigma_r^{QB}(T_f,\hat{\mu}_B)}{\mathrm{d}T}\Big|_{T_{f,0}}(T_f - T_{f,0})$$

Bielefeld-BNL-CCNU, PRD 93 (2016)014512

Parameterization of freeze-out line

Parameterization of $T_f(\mu_B)$: works well in HRG models

$$T_f(\mu_B) = T_{f,0}\left(1 - \kappa_2^f\left(\mu_B/T_{f,0}\right)^2\right)$$

Cleymans et al., PRC 73(2006)034905
Andronic, Braun-Munzinger & Stachel, NPA 772(2006)167

Taylor expansion of the ratio at T= $T_f(\mu_B=0)$ and $\mu_B=0$

Bielefeld-BNL-CCNU, PRD 93 (2016)014512

$$\Sigma_r^{QB}(T,\hat{\mu}_B) = \Sigma_r^{QB}(T,\hat{\mu}_B=0) + \frac{1}{2!}\frac{\partial^2\Sigma_r^{QB}(T,\hat{\mu}_B)}{\partial\hat{\mu}_B^2}\Big|_{\hat{\mu}_B=0}\hat{\mu}_B^2$$

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Bielefeld-BNL-CCNU, PRD 93 (2016)014512

$$\Sigma_r^{QB}(T_f,\hat{\mu}_B) = \Sigma_r^{QB}(T_f,\hat{\mu}_B=0) + \frac{1}{2!}\frac{\partial^2 \Sigma_r^{QB}(T_f,\hat{\mu}_B)}{\partial \hat{\mu}_B^2}\Big|_{\hat{\mu}_B=0}\hat{\mu}_B^2$$

$$\Sigma_r^{QB}(T_f,\hat{\mu}_B) = \Sigma_r^{QB}(T_f=T_{f,0},\hat{\mu}_B) + \frac{\mathrm{d}\Sigma_r^{QB}(T_f,\hat{\mu}_B)}{\mathrm{d}T}\Big|_{T_{f,0}}(T_f - T_{f,0})$$

Parameterization of freeze-out line

Parameterization of $T_f(\mu_B)$: works well in HRG models

$$T_f(\mu_B) = T_{f,0}\left(1 - \kappa_2^f\,(\mu_B/T_{f,0})^2\right)$$

Cleymans et al., PRC 73(2006)034905
Andronic, Braun-Munzinger & Stachel, NPA 772(2006)167

Taylor expansion of the ratio at T= $T_f(\mu_B=0)$ and $\mu_B=0$

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Ratio of $(M_Q/\sigma_Q^2)/(M_B/\sigma_B^2)$ can be expressed in terms of κ_2^f:

$$\Sigma_r^{QB}(T_f,\hat{\mu}_B) = \Sigma_r^{QB}(T_{f,0},\hat{\mu}_B=0) + \left(\frac{1}{2!}\frac{\partial^2\Sigma_r^{QB}(T_f,\hat{\mu}_B)}{\partial\hat{\mu}_B^2} - \kappa_2^f\,T_{f,0}\,\frac{\mathrm{d}\Sigma_r^{QB}(T_f,\hat{\mu}_B)}{\mathrm{d}T}\right)\Big|_{T_{f,0},\hat{\mu}_B=0}\hat{\mu}_B^2$$

Experimentally accessible

LQCD computable

To be determined

Parameterization of freeze-out line

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$$\Sigma_r^{QB}(T_f,\hat\mu_B) = \Sigma_r^{QB}(T_f,\hat\mu_B=0) + \frac{1}{2!}\frac{\partial^2\Sigma_r^{QB}(T_f,\hat\mu_B)}{\partial\hat\mu_B^2}\Big|_{\hat\mu_B=0}\hat\mu_B^2$$

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Experimentally accessible

LQCD computable

To be determined

$\hat\mu_B$ above can be replaced: $R_{12}^B(T_f,\mu_B) \equiv \frac{M_B}{\sigma_B^2}(T_f,\mu_B) = \frac{\partial R_{12}^B}{\partial\hat\mu_B}\Big|_{\hat\mu_B=0}\hat\mu_B + \mathcal{O}(\hat\mu_B^3)$

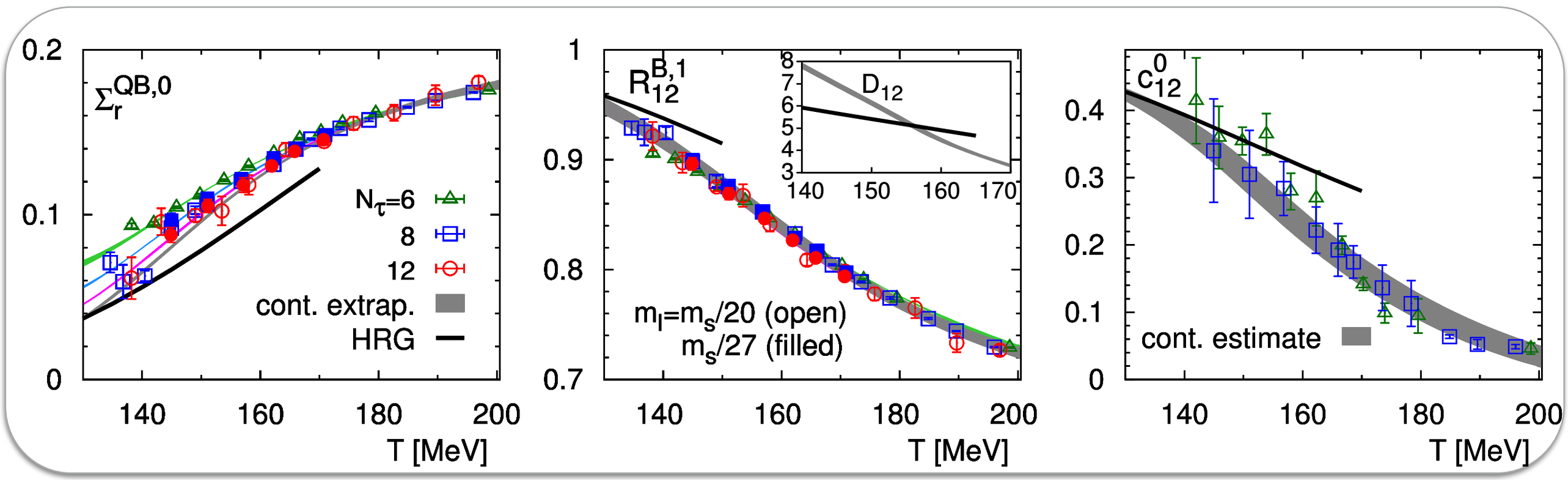
$$\frac{\partial R_{12}^B}{\partial\hat\mu_B} \equiv R_{12}^{B,1}$$

Temperature dependence of (N)LO expansion coefficients

NLO expansion of $M_Q/\sigma_Q^2/(M_B/\sigma_B^2) \equiv R_{12}^Q/R_{12}^B$:

$$\Sigma_r^{QB} = a_{12}\left[1 + \left(c_{12}^0(T_{f,0}) - \kappa_2^f D_{12}(T_{f,0})\right)\left(R_{12}^B\right)^2\right] + \mathcal{O}\left((R_{12}^B)^4\right)$$

$$a_{12} = \Sigma_r^{QB,0} \qquad D_{12}^0(T) = \left(\frac{1}{R_{12}^{B,1}}\right)^2 T\,\frac{\mathrm{d}\ln\Sigma_r^{QB,0}}{\mathrm{d}T} \qquad c_{12}^0(T) = \left(\frac{1}{R_{12}^{B,1}}\right)^2 \frac{\Sigma_r^{QB,2}}{\Sigma_r^{QB,0}}$$



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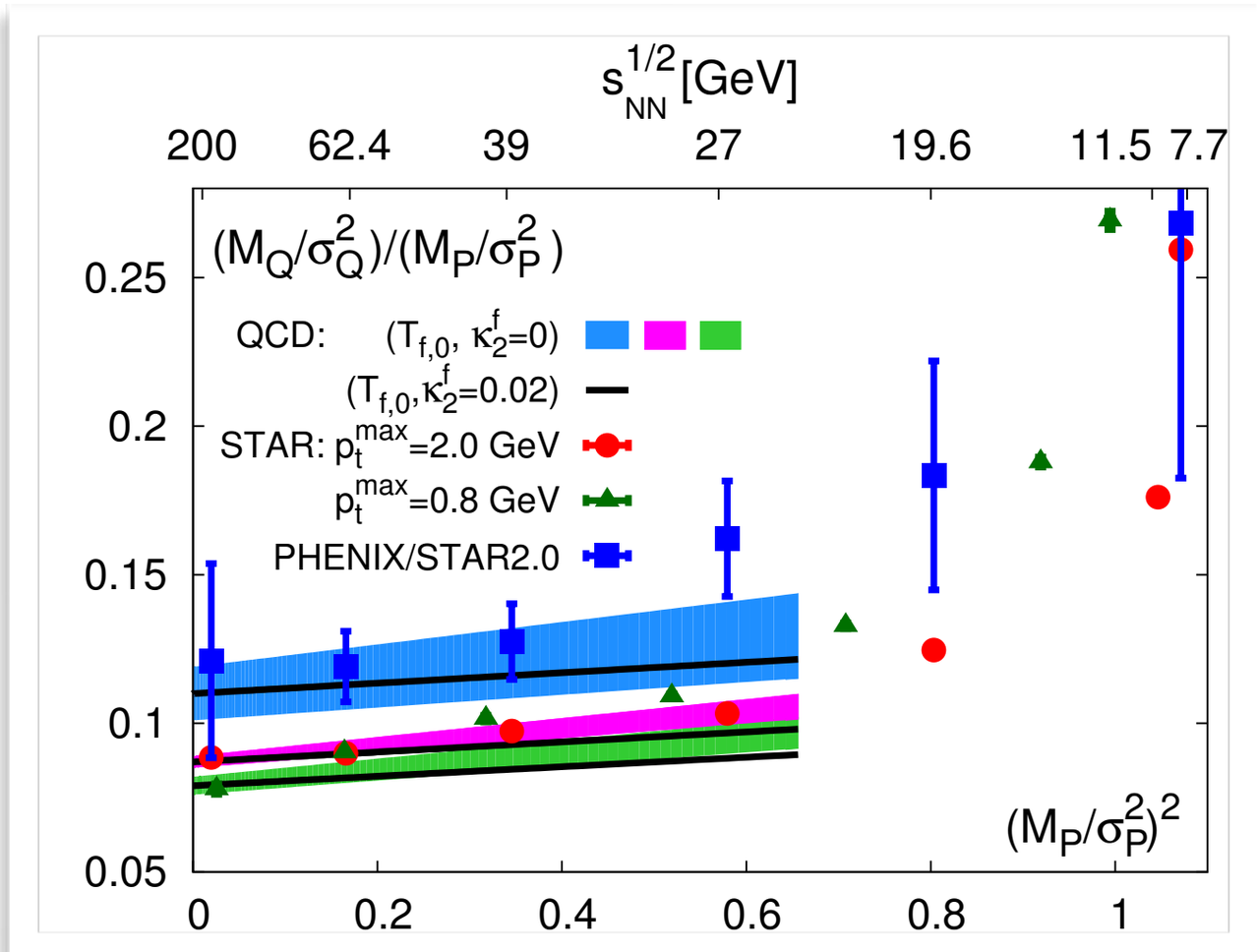
r=M_Q/M_B≃0.4 for describing AuAu or PbPb collision system

Comparison to experiment data

NLO expansion of $M_Q/\sigma_Q^2/(M_B/\sigma_B^2) \equiv R_{12}^Q/R_{12}^B$:

$$\Sigma_r^{QB} = a_{12}\left[1 + \left(c_{12}^0(T_{f,0}) - \kappa_2^f D_{12}(T_{f,0})\right)\left(R_{12}^B\right)^2\right] + \mathcal{O}\left((R_{12}^B)^4\right)$$



From HRG at the freeze-out:

$$R_{12}^P = R_{12}^B/R_{12}^{B,1}$$

Upper bound on the curvature of the freeze-out line

$$\kappa_2^f \lesssim 0.011$$

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c.f. curvature of the crossover line: $\kappa_{2,c} \approx 0.006 - 0.013$

Conclusion & Summary

- The EoS is well controlled at $\mu_B/T \lesssim 2$ or $\sqrt{s_{NN}} \gtrsim 20$ GeV

- We provided a framework that allows to determine the curvature of the freeze-out line through the direct comparison between experimental data and lattice QCD calculations of cumulant ratios

- At least for collision energy larger than 27 GeV it suggests that freeze-out happens close to the cross over & chiral phase transition line

谢谢！