中微子的mu-tau对称性

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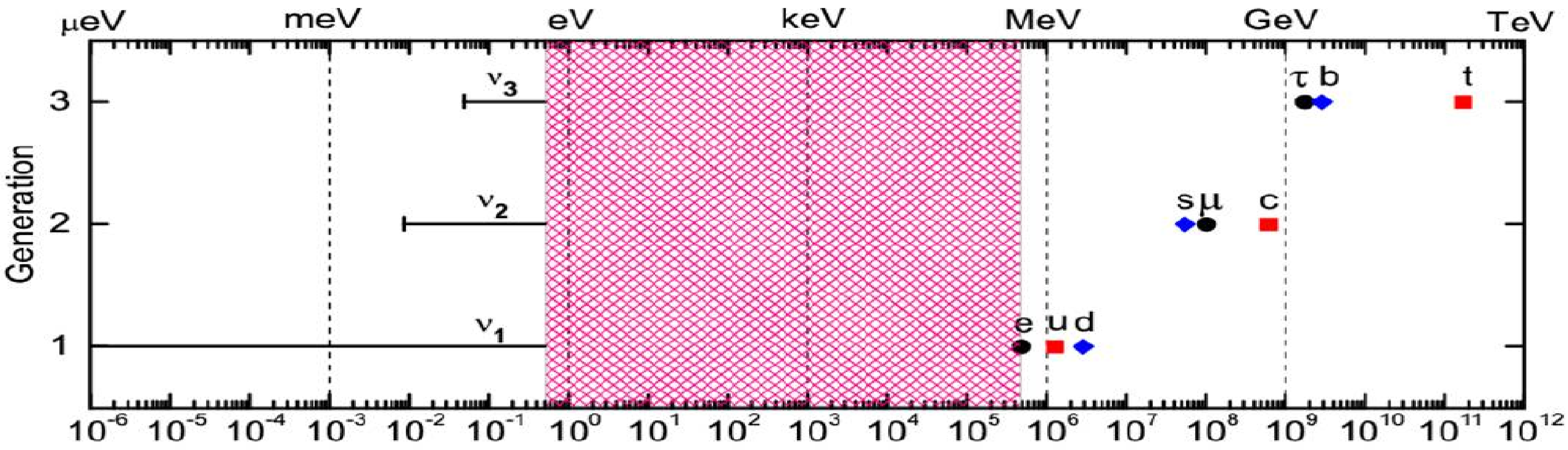
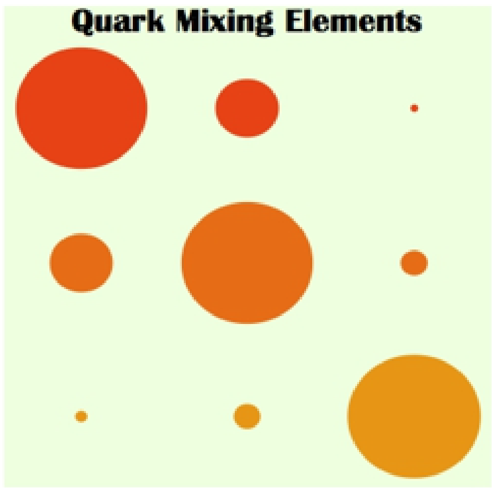
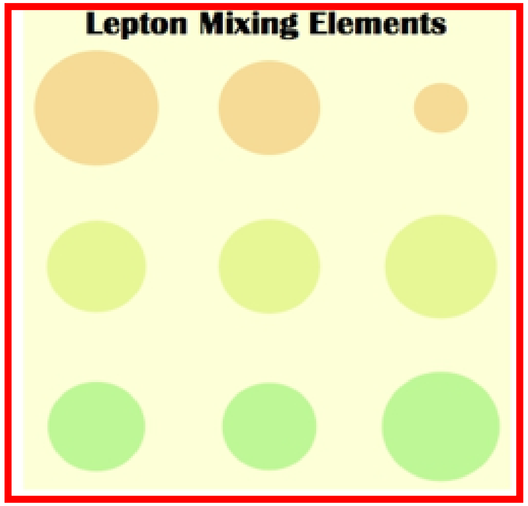
A review of mu-tau flavor symmetry in neutrino physics







Vissani 2014

味道问题

- 质量等级?
- 混合模式?
- CPV来源?

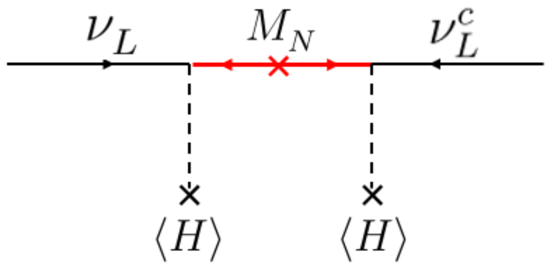
- 中微子混合与mu-tau对称性
- mu-tau对称性的破坏及其来源
- 总结与展望

中微子混合

中微子振荡现象表明中微子具有非零质量及味道混合

seesaw机制可自然解释中微子质量之轻并预言其为Majorana粒子 Minkowski 1977; Yanagida 1979…

$$\overline{\nu_L}\langle H\rangle N_R + M_N \overline{N_R^c} N_R \Longrightarrow \frac{\langle H\rangle}{M_N}\overline{\nu_L}\langle H\rangle \nu_L^c$$



弱作用本征态

$$\overline{l_L} M_l l_R \qquad \overline{\nu_L} M_\nu \nu_L^c$$

$$\frac{1}{\sqrt{2}}\overline{l_L}\gamma^\mu \nu_L W_\mu^-$$

质量本征态

$$\overline{l'_L} V_l^\dagger M_l M_l^\dagger V_l l'_L \qquad \overline{\nu'_L} V_\nu^\dagger M_\nu V_\nu^* \nu_L'^c$$

$$\frac{1}{\sqrt{2}}\overline{l'_L}\gamma^\mu \boxed{V_l^\dagger V_\nu}\, \nu'_L W_\mu^-$$

$$U = \begin{pmatrix} c_{12} & s_{12} & \\ -s_{12} & c_{12} & \\ & & 1 \end{pmatrix} \begin{pmatrix} c_{13} & & s_{13}e^{-i\delta} \\ & 1 & \\ -s_{13}e^{i\delta} & & c_{13} \end{pmatrix} \begin{pmatrix} 1 & & \\ & c_{23} & s_{23} \\ & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} e^{i\rho} & & \\ & e^{i\sigma} & \\ & & 1 \end{pmatrix}$$

Majorana相位

实验结果

太阳、大气、反应堆与加速器为多种中微子(反中微子)振荡实验提供了中微子源

Parameter	Best-fit	1σ range	3σ range
$\sin^2\theta_{12}$	0.308	0.291–0.325	0.259–0.359
$\sin\theta_{13}$	0.153	0.147–0.159	0.133–0.172
θ_{23}	41.4	40.0–43.3	37.8–52.3
δ/π	1.39	1.12–1.77	0.00–2.00

- θ_{13} 相对于0而言较大
- θ_{23} 对π/4的可能偏离
- $\delta \sim 3\pi/2$的初步迹象

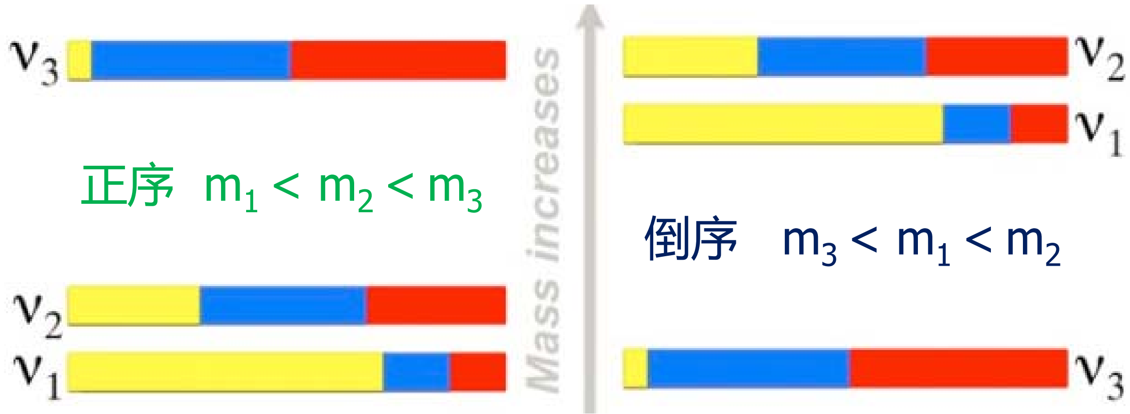
Capozzi et al 2014

振荡现象只对质量平方差敏感

$$\Delta m^2_{21} \simeq 7.54 \times 10^{-5}\ \mathrm{eV}^2 \qquad 1$$

$$|\Delta m^2_{31}| \simeq 2.47 \times 10^{-3}\ \mathrm{eV}^2 \qquad 33$$

$$\boxed{\Delta m^2_{ij} = m^2_i - m^2_j}$$



宇宙学测量限制中微子质量和 $m_1 + m_2 + m_3 < 0.23$ eV Planck 2013

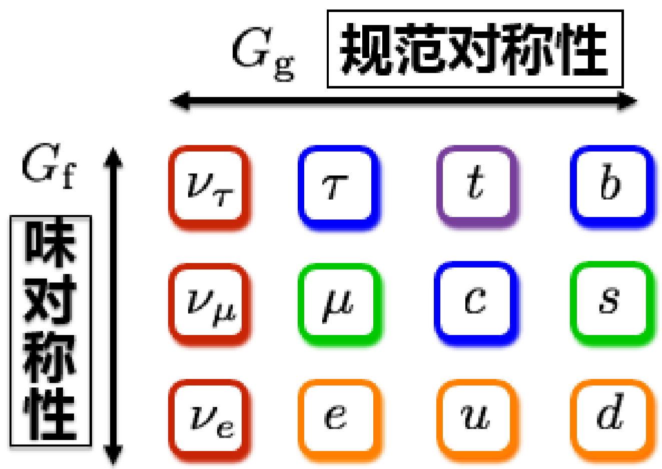
味道对称性

$$\sin\theta_{12} = 1/\sqrt{3} \quad \theta_{23} = \pi/4 \quad \theta_{13} = 0$$

$$U_{\mathrm{TB}} = \frac{1}{\sqrt{6}}\begin{pmatrix} 2 & \sqrt{2} & 0 \\ -1 & \sqrt{2} & -\sqrt{3} \\ -1 & \sqrt{2} & \sqrt{3} \end{pmatrix}$$

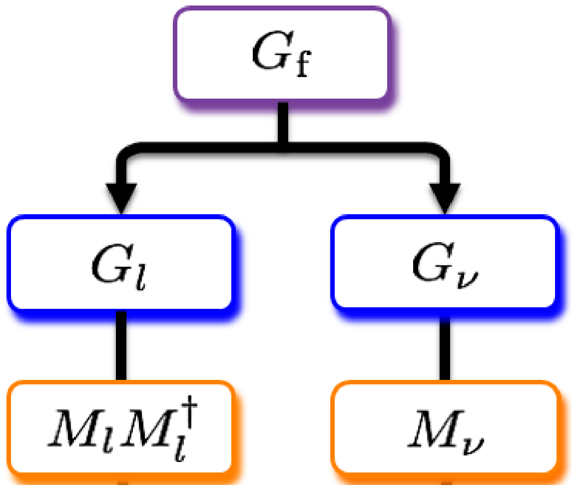
TB混合模式 Harrison, Perkins & Scott; Xing 2002

$$M_l = \begin{pmatrix} m_e & & \\ & m_\mu & \\ & & m_\tau \end{pmatrix} \qquad M_\nu = \begin{pmatrix} C+D-B & B & B \\ B & C & D \\ B & D & C \end{pmatrix}$$



Li Caichang's talk

King & Luhn 2013



mu-tau对称性

$$|U_{\mu i}| = |U_{\mu i}| \ \ (i=1,2,3) \iff \begin{cases} \theta_{23} = \pi/4 \ , \ \ \theta_{13} = 0 \ \ \text{or} \\ \theta_{23} = \pi/4 \ , \ \ \delta = \pm\pi/2 \end{cases}$$

交换对称性 Fukuyama & Nishiura 1997

$$\nu_e \to \nu_e, \quad \nu_\mu \to \nu_\tau, \quad \nu_\tau \to \nu_\mu$$

$$M_{e\mu} = M_{e\tau} \quad \& \quad M_{\mu\mu} = M_{\tau\tau}$$

$$M_\nu = \begin{pmatrix} A & B & B \\ B & C & D \\ B & D & C \end{pmatrix}$$

反射对称性 Harrison & Scott 2002

$$\nu_e \to \nu_e^c \ , \quad \nu_\mu \to \nu_\tau^c \ , \quad \nu_\tau \to \nu_\mu^c$$

$$M_{e\mu} = M_{e\tau}^* \ , \quad M_{\mu\mu} = M_{\tau\tau}^*, \quad M_{ee}, M_{\mu\tau} \ \text{Real}$$

$\theta_{23}=\pi/2$, $\delta=\pm\pi/2$, Majorana相位取平庸值

$$M_\nu = \begin{pmatrix} |A| & B & B^* \\ B & C & |D| \\ B^* & |D| & C^* \end{pmatrix}$$

近似mu-tau对称性

$$M_{e\mu} = M_{e\tau} \quad \& \quad M_{\mu\mu} = M_{\tau\tau} \quad \Longrightarrow \quad \epsilon_1 = \frac{M_{e\mu} - M_{e\tau}}{M_{e\mu} + M_{e\tau}} \quad \& \quad \epsilon_2 = \frac{M_{\mu\mu} - M_{\tau\tau}}{M_{\mu\mu} + M_{\tau\tau}}$$

当对称性破坏参数为小量时（如$|\varepsilon_{1,2}|<0.2$）mu-tau对称性近似成立

具有近似mu-tau对称性的中微子质量矩阵可借助$\varepsilon_{1,2}$表达为如下形式

$$M_\nu = \begin{pmatrix} A & B & B \\ B & C & D \\ B & D & C \end{pmatrix} + \begin{pmatrix} \delta_{ee} & \delta_{e\mu} & \delta_{e\tau} \\ \delta_{e\mu} & \delta_{\mu\mu} & \delta_{\mu\tau} \\ \delta_{e\tau} & \delta_{\mu\tau} & \delta_{\tau\tau} \end{pmatrix} \Longrightarrow \begin{pmatrix} A' & B'(1+\epsilon_1) & B'(1-\epsilon_1) \\ B'(1+\epsilon_1) & C'(1+\epsilon_2) & D' \\ B'(1-\epsilon_1) & D' & C'(1-\epsilon_2) \end{pmatrix}$$

$\Delta\theta_{23} = \theta_{23} - \pi/4$与$\theta_{13}$ 的大小为$\varepsilon_{1,2}$所控制 并依赖于中微子质量谱及Majorana相位 Grimus et al 2004

$$\begin{aligned} \Delta\theta_{23} = \mathrm{Re} \; \Big\{ & (2\Delta m^2_{31})^{-1} \left[2m_{12}c_{12}s_{12}\left(\overline{m}^*_1\epsilon_1 + m_3\epsilon^*_1\right) + (m_{22}+m_3)s^2_{12}\left(\overline{m}^*_1\epsilon_2 + m_3\epsilon^*_2\right)\right] \\ - \; & (2\Delta m^2_{32})^{-1} \left[2m_{12}c_{12}s_{12}\left(\overline{m}^*_2\epsilon_1 + m_3\epsilon^*_1\right) - (m_{22}+m_3)c^2_{12}\left(\overline{m}^*_2\epsilon_2 + m_3\epsilon^*_2\right)\right] \Big\} \end{aligned}$$

$$\begin{aligned} \theta_{13}e^{-i\delta} = & (2\Delta m^2_{31})^{-1} \left[2m_3m_{12}c^2_{12}\epsilon_1 + 2\overline{m}_1m^*_{12}c^2_{12}\epsilon^*_1 + m_3(m_{22}+m_3)c_{12}s_{12}\epsilon_2 + \overline{m}_1(m^*_{22}+m_3)c_{12}s_{12}\epsilon^*_2\right] \\ + & (2\Delta m^2_{32})^{-1} \left[2m_3m_{12}s^2_{12}\epsilon_1 + 2\overline{m}_2m^*_{12}s^2_{12}\epsilon^*_1 - m_3(m_{22}+m_3)c_{12}s_{12}\epsilon_2 - \overline{m}_2(m^*_{22}+m_3)c_{12}s_{12}\epsilon^*_2\right] \end{aligned}$$

CP守恒 $\varepsilon_{1,2}$ 为实数 ρ & σ = 0 或 $\pi/2$

$m_1 < m_2 \ll m_3$

$$\theta_{13} \sim \frac{1}{2}\sqrt{\frac{\Delta m^2_{21}}{\Delta m^2_{31}}}\, c_{12}s_{12}\,(2\epsilon_1 - \epsilon_2) \simeq 0.04\,(2\epsilon_1 - \epsilon_2)$$

$m_1 \simeq m_2 \gg m_3$

$\rho=\sigma=0$:

$$\theta_{13} \sim \frac{1}{4}\frac{\Delta m^2_{21}}{\Delta m^2_{31}} c_{12}s_{12}\,(2\epsilon_1 - \epsilon_2) \simeq 0.004\,(2\epsilon_1 - \epsilon_2)$$

$\rho=0,\ \sigma=\pi/2$:

$$\theta_{13} \sim \frac{1}{2}\cos 2\theta_{12}\sin 2\theta_{12}\,(2\epsilon_1 - \epsilon_2) \simeq 0.18\,(2\epsilon_1 - \epsilon_2)$$

$m_1 \simeq m_2 \simeq m_3$

$\rho=\sigma=0$:

$$\theta_{13} \sim \frac{2m_1^2}{\Delta m^2_{31}}\frac{\Delta m^2_{21}}{\Delta m^2_{31}} c_{12}s_{12}\epsilon_2 < 0.1\epsilon_2 \quad \text{for} \quad m_1 < 0.1 \ \text{eV}$$

$\rho=0,\ \sigma=\pi/2$:

$$\boxed{\begin{aligned}\theta_{13} &\sim \frac{2m_1^2}{\Delta m^2_{31}} c_{12}s_{12}\left(2c^2_{12}\epsilon_1 + s^2_{12}\epsilon_2\right)\\ \Delta\theta_{23} &\sim \frac{2m_1^2}{\Delta m^2_{31}} s^2_{12}\left(2c^2_{12}\epsilon_1 + s^2_{12}\epsilon_2\right)\end{aligned}}$$

$$\boxed{|\Delta\theta_{23}| \sim \theta_{13}s_{12}/c_{12} \simeq 6^\circ}$$

当中微子质量近简并且(ρ, σ)=(0, $\pi/2$)时 足够小的$\varepsilon_{1,2}$便可产生现实的θ_{13}

此时$\Delta\theta_{23}$比较大(若考虑CP破坏, 则可在$\Delta\theta_{23}$很小的情况下得到所需的θ_{13})

具有等级结构的M_ν

在非主导的矩阵元中 相较于矩阵元本身 对称性破坏的项未必是小量

领头阶: 主导的矩阵元遵守mu-tau对称性, 非主导的矩阵元为零

$$M_\nu = m_0 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix} \Longrightarrow \begin{array}{l} m_1 = m_2 = 0 \quad \& \quad m_3 = 2m_0 \\ \theta_{12} = \theta_{13} = 0 \quad \& \quad \theta_{23} = \pi/4 \end{array}$$

次领头阶贡献破坏mu-tau对称性 Mohapatra 2004

$$M_\nu = m_0 \begin{pmatrix} d\epsilon & c\epsilon & b\epsilon \\ c\epsilon & 1 + a\epsilon & -1 \\ b\epsilon & -1 & 1+\epsilon \end{pmatrix} \Longrightarrow \begin{array}{l} m_{1,2} \simeq \frac{1}{4}\epsilon m_0 \left(2d + a + 1 \mp \sqrt{(2d-a-1)^2 + 8(b+c)^2}\right) \\ \theta_{13} \simeq \frac{1}{2\sqrt{2}}(b-c)\,\epsilon\,, \quad \Delta\theta_{23} \simeq \frac{1}{4}(a-1)\,\epsilon \end{array}$$

θ_{13}的"小"与中微子质量等级$m_1<m_2<<m_3$关联 $\boxed{\epsilon \sim \sqrt{\Delta m^2_{21}/\Delta m^2_{31}} \simeq 0.17}$

重整化跑动引起的对称性破坏

味道对称性通常在很高能标处引入, 在将其与低能实验结果比较时需考虑重整化跑动效应

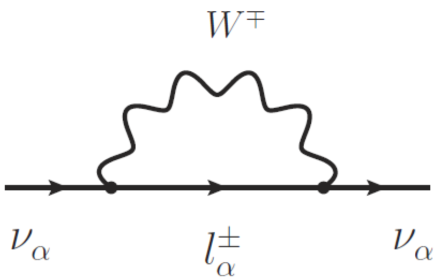
m_μ与m_τ 的不同会导致mu-tau对称性的破坏　Ellis & Lola; Chankowski, Krolikowski & Pokorski 1999

$$M'_\nu = I_\alpha \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1-\Delta_\tau \end{pmatrix} M_\nu \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1-\Delta_\tau \end{pmatrix} \Longrightarrow \begin{pmatrix} A' & B'\left(1+\frac{1}{2}\Delta_\tau\right) & B'\left(1-\frac{1}{2}\Delta_\tau\right) \\ B'\left(1+\frac{1}{2}\Delta_\tau\right) & C'\left(1+\Delta_\tau\right) & D' \\ B'\left(1-\frac{1}{2}\Delta_\tau\right) & D' & C'\left(1-\Delta_\tau\right) \end{pmatrix}$$

MSSM

$$\Delta_\tau = \int_{\Lambda_{\rm EW}}^{\Lambda_{\rm FS}} y_\tau^2 {\rm d}t \quad \text{with} \quad y_\tau^2 \simeq \tan^2\beta \frac{m_\tau^2}{v^2}$$

0.002 to 0.044 for 10<tanβ<50



在一般情形下的结果中取$\varepsilon_2=2\varepsilon_1=2\Delta_\tau$ 即可得到重整化跑动产生的 θ_{13} 及 $\Delta\theta_{23}$

$$\theta_{13} \simeq c_{12}s_{12}\frac{m_3\left|\overline{m}_1-\overline{m}_2\right|}{\Delta m_{31}^2}\Delta_\tau \,, \quad \Delta\theta_{23} \simeq \frac{\left|\overline{m}_1+m_3\right|^2 s_{12}^2 + \left|\overline{m}_2+m_3\right|^2 c_{12}^2}{2\Delta m_{31}^2}\Delta_\tau$$

在中微子质量小于0.1 eV的前提下，tanβ需大于50重整化跑动效应才可产生实际的θ_{13}

因此重整化跑动效应不足以作为mu-tau对称性破坏的唯一来源　zhang & zhou 2016

带电轻子部分的贡献

$\boxed{U = V_l^\dagger V_\nu}$ M_l 非对角时 V_l 也将对中微子混合有所贡献

$\theta^\nu_{23} = \pi/4 \;\; \& \;\; \theta^\nu_{13} = 0, \;\; \text{while} \;\; \theta^l_{23}, \;\; \theta^l_{13} \;\; \text{and} \;\; \theta^l_{12} \;\; \text{are small}$ Antusch & King 2005

$$\tilde{s}_{12} \simeq \tilde{s}^\nu_{12} - \tilde{\theta}^l_{12} c^\nu_{12} c^\nu_{23} + \tilde{\theta}^l_{13} c^\nu_{12} \tilde{s}^{\nu *}_{23} \; , \quad \tilde{s}_{13} \simeq -\tilde{\theta}^l_{13} c^\nu_{23} - \tilde{\theta}^l_{12} \tilde{s}^\nu_{23} \; , \quad \tilde{s}_{23} \simeq \tilde{s}^\nu_{23} - \tilde{\theta}^l_{23} c^\nu_{23}$$

$\text{CKM-like } V_l, \text{ i.e. } \theta^l_{12} \simeq \theta_C \gg \theta^l_{13}, \theta^l_{23} \Longrightarrow \boxed{\theta_{13} \simeq \theta_C/\sqrt{2}}$ 0.15 vs $0.22/\sqrt{2}$

在大统一模型框架下联系夸克与轻子两部分 (特别是M_d与M_l)

需选择合适的Higgs场以产生实际的费米子质量谱及 $\theta^l_{12} \simeq \theta_C$

$$M_d = \begin{pmatrix} 0 & b \\ c & a \end{pmatrix} \quad M_l = \begin{pmatrix} 0 & c_c c \\ c_b b & c_a a \end{pmatrix}$$

Antusch & Spinrath 2009

$\Downarrow$

$$M_d = \begin{pmatrix} 0 & b \\ c & a \end{pmatrix} \quad M_l = \begin{pmatrix} 0 & 6c \\ b/2 & 6a \end{pmatrix}$$

Antusch & Maurer; Marzocca, Petcov, Romanino & Spinrath 2011

Operator dimension	y_e/y_d
4	1
	-3
5	-1/2
	1
	$\pm 3/2$
	-3
	9/2
	6
	9
	-18

惰性中微子引起的对称性破坏

短基线中微子振荡反常暗示了eV惰性中微子的存在　Li Yufeng's talk

普通与惰性中微子的混合可贡献mu-tau对称性破坏　Goh, Mohapatra & Ng 2002

$$M_s = \begin{pmatrix} m_{ee} & m_{e\mu} & m_{e\mu} & m_{es} \\ m_{e\mu} & m_{\mu\mu} & m_{\mu\tau} & m_{\mu s} \\ m_{e\mu} & m_{\mu\tau} & m_{\mu\mu} & m_{\tau s} \\ m_{es} & m_{\mu s} & m_{\tau s} & m_{ss} \end{pmatrix}$$

Merle, Morisi & Winter 2014

CP守恒假设下　一般的4×4中微子质量矩阵导致6个混合角与4个质量　θ_{14}　θ_{24}　θ_{34}　m_4

3×3子矩阵所受约束条件$m_{e\mu}=m_{e\tau}$ & $m_{\mu\mu}=m_{\tau\tau}$ 有助于减少两个自由度

$$s_{34} = \sqrt{2}\frac{(m_3 - m_{22})\,\Delta\theta_{23} - m_{12}s_{13}}{m_{12}\Delta\theta_{23} + m_{11}s_{13} - m_3 s_{13}} s_{14} - s_{24}\;, \quad m_4 = \sqrt{2}\frac{m_{12}\Delta\theta_{23} + m_{11}s_{13} - m_3 s_{13}}{s_{14}\,(s_{34} - s_{24})}$$

若进一步假设3×3子矩阵能够给出TB等混合模式　新引入的物理量将受到更强的限制

Borah 1607.05556; DeV, Raj & Gautam 1607.08051

总结与展望

- mu-tau对称性需要破坏以容纳较大的θ_{13}以及可能的$\Delta\theta_{23}$
- 当中微子质量近简并且$\rho-\sigma=\pm\pi/2$时 mu-tau对称性的近似度较好
- θ_{23}, δ及中微子质量需要精确测量以判定mu-tau对称性是否足够好
- 重整化跑动效应不足以作为mu-tau对称性破坏唯一的源
- $\theta_{13} \simeq \theta_C/\sqrt{2}$ 增强了人们在大统一模型中联系夸克与轻子的动机
- 鉴于其有趣唯象后果 反射对称性在今后值得特别关注