Leptogenesis and residual CP symmetry

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第十二届全国粒子物理学术会议

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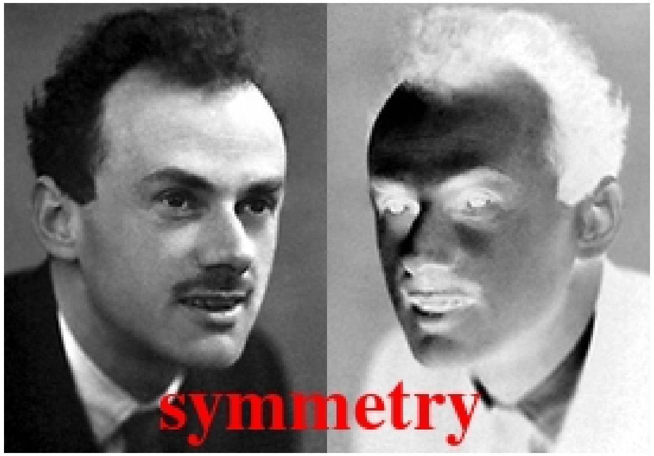
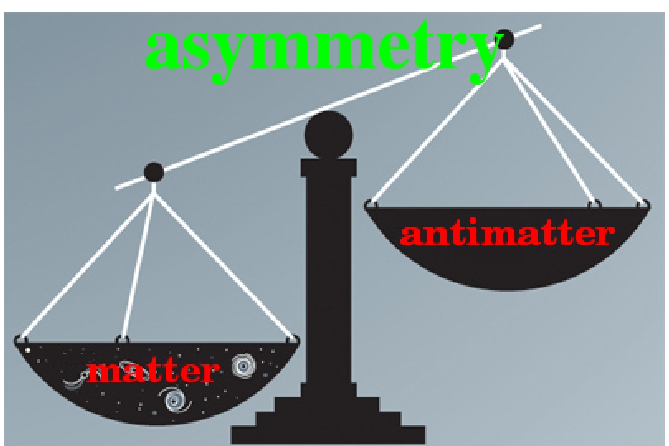
Background

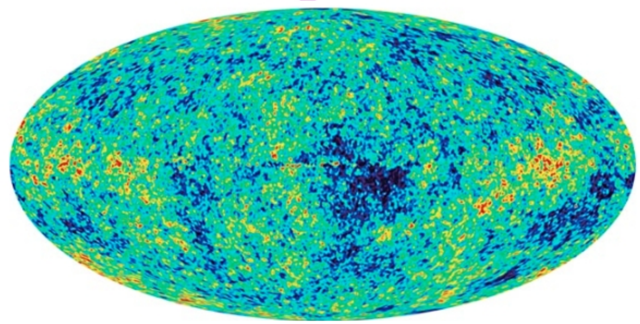


"It is quite possible that for some...stars being built up mainly of positrons and negative protons. In fact, there may be half the stars of each kind." (Dirac,1933)

matter/antimatter

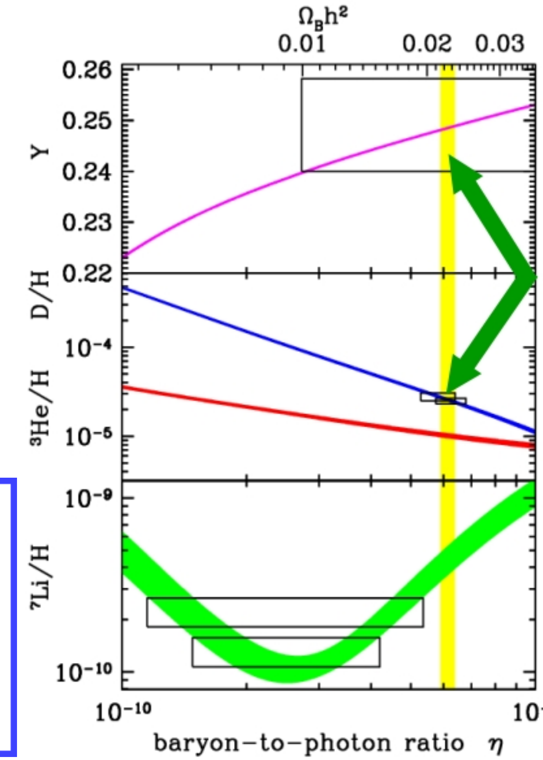


cosmic microwave background



$$Y_B^{\rm obs} = 8.66 \times 10^{-11}$$

Big Bang nucleosynthesis



$$Y_B \equiv \frac{n_B - n_{\bar{B}}}{s} : \text{ baryon asymmetry}$$

n_B : number density of baryons

s : density of entropy

Today's observations indicate that the number of baryons (protons and neutrons) in the universe is unequal to the number of antibaryons.

Leptogenesis

Fukugita, Yanagida 86

Leptogenesis is a mechanism to generate the baryon asymmetry through a lepton asymmetry

type-I seesaw

$$-\mathcal{L} = y_\alpha \bar{L}_\alpha H l_{\alpha R} + \lambda_{i\alpha} \bar{N}_{iR} \widetilde{H}^\dagger L_\alpha + \frac{1}{2} M_i \bar{N}_{iR} N_{iR}^c + h.c.$$

$\alpha = e\,,\, \mu\,,\, \tau \quad i = 1\,,2\,,3$

N_i decay $\xrightarrow[\text{out of equilibrium}]{\text{CP violation}}$ **lepton number asymmetry** $\xrightarrow[\text{sphaleron processes}]{}$ **baryon number asymmetry**

CP asymmetry

$$\epsilon_\alpha \simeq -\frac{3}{16\pi} \sum_{j\neq 1} \frac{M_1}{M_j} \frac{\mathrm{Im}\left[(\lambda\lambda^\dagger)_{1j} \lambda_{1\alpha} \lambda^*_{j\alpha}\right]}{(\lambda\lambda^\dagger)_{11}} \quad \text{when} \quad M_{2,3} \gg M_1$$

effective mass

$$\widetilde{m}_\alpha = \frac{|\lambda_{1\alpha}|^2 v^2}{M_1}$$

The baryon asymmetry $Y_B=(n_B-n_B)/s$ is given by(A. Abada, et al JHEP **0609** (2006) 010)

$$Y_B \simeq -\frac{12}{37\, g^*} \left[\epsilon_2 \eta\left(\frac{417}{589}\widetilde{m}_2\right) + \epsilon_\tau \eta\left(\frac{390}{589}\widetilde{m}_\tau\right)\right],$$

$$\epsilon_2 = \epsilon_e + \epsilon_\mu, \quad \widetilde{m}_2 = \widetilde{m}_e + \widetilde{m}_\mu,$$

$$\eta(\widetilde{m}_\alpha) \simeq \left[\left(\frac{\widetilde{m}_\alpha}{8.25\times 10^{-3}\,\mathrm{eV}}\right)^{-1} + \left(\frac{0.2\times 10^{-3}\,\mathrm{eV}}{\widetilde{m}_\alpha}\right)^{-1.16}\right]^{-1}$$

Casas-Ibarra parametrization

In order to exploit the connection between the CP violating parameters in leptogenesis and the low energy CP, a complex orthogonal matrx R is introduced as a parameterization of λ(J. A. Casas and A. Ibarra, Nucl. Phys. B 618, 171 (2001))

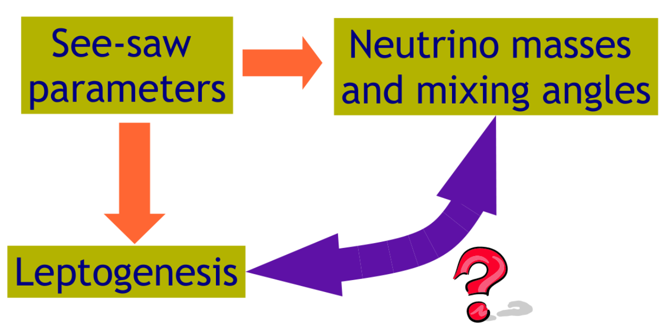
$$\boxed{R = vM^{-\frac{1}{2}}\lambda U m^{-\frac{1}{2}}\,, \qquad RR^T = R^T R = 1\,.}$$

The CP asymmetry and the effective mass reduce to

$$\epsilon_\alpha = -\frac{3M_1}{16\pi v^2}\frac{\mathrm{Im}\left(\sum_{ij}\sqrt{m_i m_j}\,m_j R_{1i}R_{1j}U^*_{\alpha i}U_{\alpha j}\right)}{\sum_j m_j |R_{1j}|^2}\,, \qquad \widetilde{m}_\alpha = \left|\sum_j m_j^{1/2} R_{1j} U^*_{\alpha j}\right|^2 .$$

Only the first row of R is relevant to CP asymmetry and effective mass.

$$\boxed{\begin{gathered}(R_{11}\,,\ R_{12}\,,\ R_{13}) \\ R_{11}^2 + R_{12}^2 + R_{13}^2 = 1\end{gathered}}$$



Leptogenesis and residual CP

We start from the type-I seesaw lagrangian

$$-\mathcal{L} = y_\alpha \bar{L}_\alpha H l_{\alpha R} + \lambda_{i\alpha} \bar{N}_{iR} \widetilde{H}^\dagger L_\alpha + \frac{1}{2} M_i \bar{N}_{iR} N^c_{iR} + h.c.$$

The effective light neutrino mass is

$$m_\nu = v^2 \lambda^T M^{-1} \lambda = U^* \mathrm{diag}(m_1, m_2, m_3) U^\dagger \,.$$

The effective neutrino mass term is invariant under the CP transformation

$$\boldsymbol{\nu_L(x) \xmapsto{CP} i X_i \gamma_0 \nu^c_L(x_P)\,.}$$

$$\boxed{X_i^T m_\nu X_i = m_\nu^*\,,}$$

$$X_i = U d_i U^T\,,$$

$$\begin{aligned} d_1 &= \mathrm{diag}(1,-1,-1)\,, \\ d_2 &= \mathrm{diag}(-1,1,-1)\,, \\ d_3 &= \mathrm{diag}(-1,-1,1)\,, \\ d_4 &= \mathrm{diag}(1,1,1)\,. \end{aligned}$$

The CP symmetry of the effective mass term may be accidental or come from high energy scale. We focus on the case two residual CP are inherited from CP symmetry in the seesaw Lagrangian.

Two CP preserved

Suppose the seesaw Lagrangian is invariant under

$$\mathrm{CP}_1: \ \nu_L \longmapsto iX_{\nu 1}\gamma_0\nu_L^c\,, \qquad N_R \longmapsto i\widehat{X}_{N1}\gamma_0 N_R^c\,,$$
$$\mathrm{CP}_2: \ \nu_L \longmapsto iX_{\nu 2}\gamma_0\nu_L^c\,, \qquad N_R \longmapsto i\widehat{X}_{N2}\gamma_0 N_R^c\,.$$

CP_1 and CP_2 give the following constraints to λ, M and the PMNS matrix U

$$\widehat{X}^\dagger_{N1}\lambda X_{\nu 1} = \lambda^*, \qquad \widehat{X}^\dagger_{N1} M \widehat{X}^*_{N1} = M^*, \qquad X_{\nu 1} U^* = U\widehat{X}_{\nu 1},$$
$$\widehat{X}^\dagger_{N2}\lambda X_{\nu 2} = \lambda^*, \qquad \widehat{X}^\dagger_{N2} M \widehat{X}^*_{N2} = M^*, \qquad X_{\nu 2} U^* = U\widehat{X}_{\nu 2}\,.$$

$$\widehat{X}_{N1}, \widehat{X}_{N2}; \widehat{X}_{\nu 1}, \widehat{X}_{\nu 2} = \mathrm{diag}\left(\pm 1, \pm 1, \pm 1\right)\,,$$

$$\widehat{X}_{N1}\widehat{X}_{N2} = P_N^T \mathrm{diag}(1,-1,-1) P_N\,,$$
$$\widehat{X}_{\nu 1}\widehat{X}_{\nu 2} = P_\nu^T \mathrm{diag}(1,-1,-1) P_\nu\,.$$

parameterization of *$X_{\nu 1}$* and *$X_{\nu 2}$*

$$X_{\nu 1} = e^{i\kappa_1}v_1 v_1^T + e^{i\kappa_2}v_2 v_2^T + e^{i\kappa_3}v_3 v_3^T\,, \qquad X_{\nu 2} = e^{i\kappa_1}v_1 v_1^T - e^{i\kappa_2}v_2 v_2^T - e^{i\kappa_3}v_3 v_3^T\,.$$

$$v_1 = \begin{pmatrix} \cos\varphi \\ \sin\varphi\cos\phi \\ \sin\varphi\sin\phi \end{pmatrix}, \ v_2 = \begin{pmatrix} \sin\varphi\cos\rho \\ -\sin\phi\sin\rho - \cos\varphi\cos\phi\cos\rho \\ \cos\phi\sin\rho - \cos\varphi\sin\phi\cos\rho \end{pmatrix}, \ v_3 = \begin{pmatrix} \sin\varphi\sin\rho \\ \sin\phi\cos\rho - \cos\varphi\cos\phi\sin\rho \\ -\cos\phi\cos\rho - \cos\varphi\sin\phi\sin\rho \end{pmatrix}$$

Constraint on PMNS matrix

The PMNS matrix can be determined by the following two equations

$$U^\dagger X_{\nu 1} U^* = \widehat{X}_{\nu 1}, \qquad U^\dagger X_{\nu 2} U^* = \widehat{X}_{\nu 2}\,.$$

$X_{\nu 1}$ and $X_{\nu 2}$ can be decomposed by the takagi factorization

$$\boxed{\begin{aligned} &X_{\nu 1} = \Sigma\Sigma^T, \qquad X_{\nu 2} = \Sigma\,\mathrm{diag}(1,-1,-1)\,\Sigma^T, \\ &\Sigma = (v_1, v_2, v_3)\,\mathrm{diag}\left(e^{i\kappa_1/2}, e^{i\kappa_2/2}, e^{i\kappa_3/2}\right)\,. \end{aligned}}$$

Then we have

$$(\widehat{X}_{\nu 1}^{-1/2} U^\dagger \Sigma)(\widehat{X}_{\nu 1}^{-1/2} U^\dagger \Sigma)^T = 1,$$

$$P_\nu(\widehat{X}_{\nu 1}^{-1/2} U^\dagger \Sigma)\mathrm{diag}(1,-1,-1) = \mathrm{diag}(1,-1,-1)P_\nu(\widehat{X}_{\nu 1}^{-1/2} U^\dagger \Sigma).$$

$\widehat{X}_{\nu 1}^{-1/2} U^\dagger \Sigma$ is unitary and orthogonal

$$P_\nu(\widehat{X}_{\nu 1}^{-1/2} U^\dagger \Sigma) = \begin{pmatrix} \pm 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix}\,.$$

Finally, the PMNS matrix can be written as

$$\boxed{U = (v_1, v_2, v_3)\,\mathrm{diag}\left(e^{i\kappa_1/2}, e^{i\kappa_2/2}, e^{i\kappa_3/2}\right) \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{pmatrix} P_\nu \widehat{X}_{\nu 1}^{-\frac{1}{2}}\,.}$$

Constraint on R

From the definition $R = vM^{-\frac{1}{2}}\lambda U m^{-\frac{1}{2}}$ we have

$$\widehat{X}_{N1} R^* \widehat{X}_{\nu 1} = R, \qquad \widehat{X}_{N2} R^* \widehat{X}_{\nu 2} = R,$$

$$P_N R P_\nu^T = \mathrm{diag}\,(1,-1,-1)\, P_N R P_\nu^T \mathrm{diag}\,(1,-1,-1)\,,$$

$$P_N R P_\nu^T = \begin{pmatrix} \times & 0 & 0 \\ 0 & \times & \times \\ 0 & \times & \times \end{pmatrix}.$$

Because *R* is a complex orthogonal matrix

$$R = P_N^T \begin{pmatrix} \pm 1 & 0 & 0 \\ 0 & \cos\eta & \sin\eta \\ 0 & -\sin\eta & \cos\eta \end{pmatrix} P_\nu\,, \quad \text{or} \quad R = P_N^T \begin{pmatrix} \pm 1 & 0 & 0 \\ 0 & \cosh\eta & i\sinh\eta \\ 0 & -i\sinh\eta & \cosh\eta \end{pmatrix} P_\nu\,.$$

Where η is a real parameter, P_N and P_ν are arbitrary permutation matrices.

one nonzero element in the 1st row of R

$$R = \begin{pmatrix} 1 \; 0 \; 0 \\ \cdots \end{pmatrix}, \qquad \epsilon_\alpha = -\frac{3M_1}{16\pi v^2} \frac{\mathrm{Im}\left(\sum_{ij} \sqrt{m_i m_j}\, m_j R_{1i} R_{1j} U_{\alpha i}^* U_{\alpha j}\right)}{\sum_j m_j |R_{1j}|^2} = -\frac{3M_1}{16\pi v^2} \frac{\mathrm{Im}\,(m_1^2 |U_{\alpha 1}|^2)}{m_1} = 0\,.$$

We consider cases there are two nonzero elements in the first row of *R*

$$C_{12} : R = \begin{pmatrix} \times \; \times \; 0 \\ \cdots \end{pmatrix}, \qquad C_{13} : R = \begin{pmatrix} \times \; 0 \; \times \\ \cdots \end{pmatrix}, \qquad C_{23} : R = \begin{pmatrix} 0 \; \times \; \times \\ \cdots \end{pmatrix}.$$

The position of zero element is determined by P_ν.

Results

In order to simplify our analysis, we introduce

$$U' \equiv U \hat{X}_{\nu 1}^{1/2}, \quad R' \equiv \hat{X}_{N1}^{1/2} R \hat{X}_{\nu 1}^{1/2}, \quad K_j \equiv (\hat{X}_{N1})_{11} (\hat{X}_{\nu 1})_{jj} .$$

The CP asymmetry and the effective mass reduce to

$$\boxed{\epsilon_\alpha = -\frac{3M_1}{16\pi v^2} W_{ab}\, I_{ab}^{\alpha} ,} \quad \boxed{\widetilde{m}_\alpha = \left| m_a^{1/2} R'_{1a} U'_{\alpha a} + m_b^{1/2} R'_{1b} U'_{\alpha b} \right|^2 ,}$$

where $W_{ab} = \dfrac{\sqrt{m_a m_b}\, R'_{1a} R'_{1b} (m_a K_a - m_b K_b)}{m_a (R'_{1a})^2 + m_b (R'_{1b})^2}, \qquad I_{ab}^{\alpha} = \mathrm{Im}\left(U'_{\alpha a} U'^{*}_{\alpha b}\right) .$

$$\boxed{\begin{aligned} I_{ab}^{e} &= \pm J_1, \\ I_{ab}^{\mu} &= \pm J_2, \\ I_{ab}^{\tau} &= \pm J_3 . \end{aligned}}$$

$$\begin{aligned} J_1 &= \frac{1}{2} \sin 2\rho \sin^2 \varphi \sin \frac{\kappa_2 - \kappa_3}{2} , \\ J_2 &= \frac{1}{8} \left[\left(2\cos 2\varphi \cos^2 \phi + 3\cos 2\phi - 1\right) \sin 2\rho - 4 \cos 2\rho \cos \varphi \sin 2\phi \right] \sin \frac{\kappa_2 - \kappa_3}{2} , \\ J_3 &= \frac{1}{8} \left[\left(2\cos 2\varphi \sin^2 \phi - 3\cos 2\phi - 1\right) \sin 2\rho + 4 \cos 2\rho \cos \varphi \sin 2\phi \right] \sin \frac{\kappa_2 - \kappa_3}{2} . \end{aligned}$$

three or four residual CP

For the case of three or four residual CP, the first row of *R* only have one nonzero element. So, all the leptogenesis CP asymmetries would be vanishing.

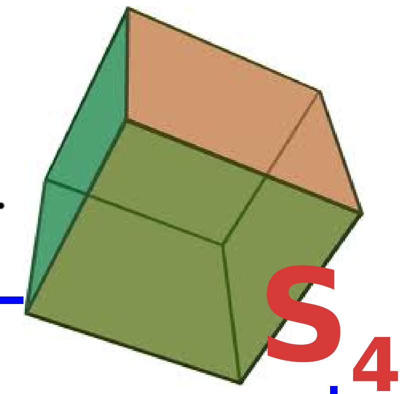
three or four residual CP

$$R = P_N^T \begin{pmatrix} \pm 1 & 0 & 0 \\ 0 & \pm 1 & 0 \\ 0 & 0 & \pm 1 \end{pmatrix} P_\nu .$$

Predictions of S_4

S_4 group can be generated by *S, T* and *U*

$$S^2 = T^3 = U^2 = (ST)^3 = (SU)^2 = (TU)^2 = (STU)^4 = 1\,.$$



generators of S_4 in the 3 representation

$$S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}, \quad T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{4i\pi/3} & 0 \\ 0 & 0 & e^{2i\pi/3} \end{pmatrix}, \quad U = -\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}.$$

Systematical and comprehensive studies have revealed that there are five possible cases which can accommodate the experimental measured values of the lepton mixing angles(G.-J. Ding et al., JHEP 1305, 084 (2013);C.-C. Li et al., Nucl. Phys. B 881, 206 (2014))

	$\mathcal{G}_l$	$(X_{\nu 1}, X_{\nu 2})$
(i)	Z_3^T	$(1, S)$
(ii)		(U, SU)
(iii)		$(1, SU)$
(iv)		(U, S)
(v)	$Z_4^{TST^2U}$	(TST^2U, T^2)

Example of S_4

case i: $(X_{\nu 1},X_{\nu 2})=(1,S)$

The CP parameters are

$$\varphi = \arccos \frac{1}{\sqrt{3}}\,, \quad \phi = \frac{\pi}{4}\,, \quad \rho = 0\,, \quad \kappa_1 = 0\,, \quad \kappa_2 = 0\,, \quad \kappa_3 = 0\,.$$

Because κ_2-κ_3=0, we can see

$$I_{13}^{e} = I_{13}^{\mu} = I_{13}^{\tau} = 0, \quad \epsilon_e = \epsilon_\mu = \epsilon_\tau = 0\,.$$

The baryon asymmetry can not be generated at the leading order.

case ii: $(X_{\nu 1},X_{\nu 2})=(U,SU)$

$$\varphi = \arccos \frac{1}{\sqrt{3}}\,, \quad \phi = \frac{\pi}{4}\,, \quad \rho = 0\,, \quad \kappa_1 = \pi\,, \quad \kappa_2 = \pi\,, \quad \kappa_3 = 0\,.$$

We obtain

$$I_{13}^{e} = 0, \quad I_{13}^{\mu} = -\frac{1}{2\sqrt{3}}, \quad I_{13}^{\tau} = \frac{1}{2\sqrt{3}}\,,$$

and

$$U' = \frac{i}{\sqrt{6}} \begin{pmatrix} 2\cos\theta & \sqrt{2} & 2\sin\theta \\ -\cos\theta + i\sqrt{3}\sin\theta & \sqrt{2} & -\sin\theta - i\sqrt{3}\cos\theta \\ -\cos\theta - i\sqrt{3}\sin\theta & \sqrt{2} & -\sin\theta + i\sqrt{3}\cos\theta \end{pmatrix}\,.$$

Example of S_4

The ratio of the baryon asymmetry Y_B to the experimental value Y_b^{obs}.

red solid line:

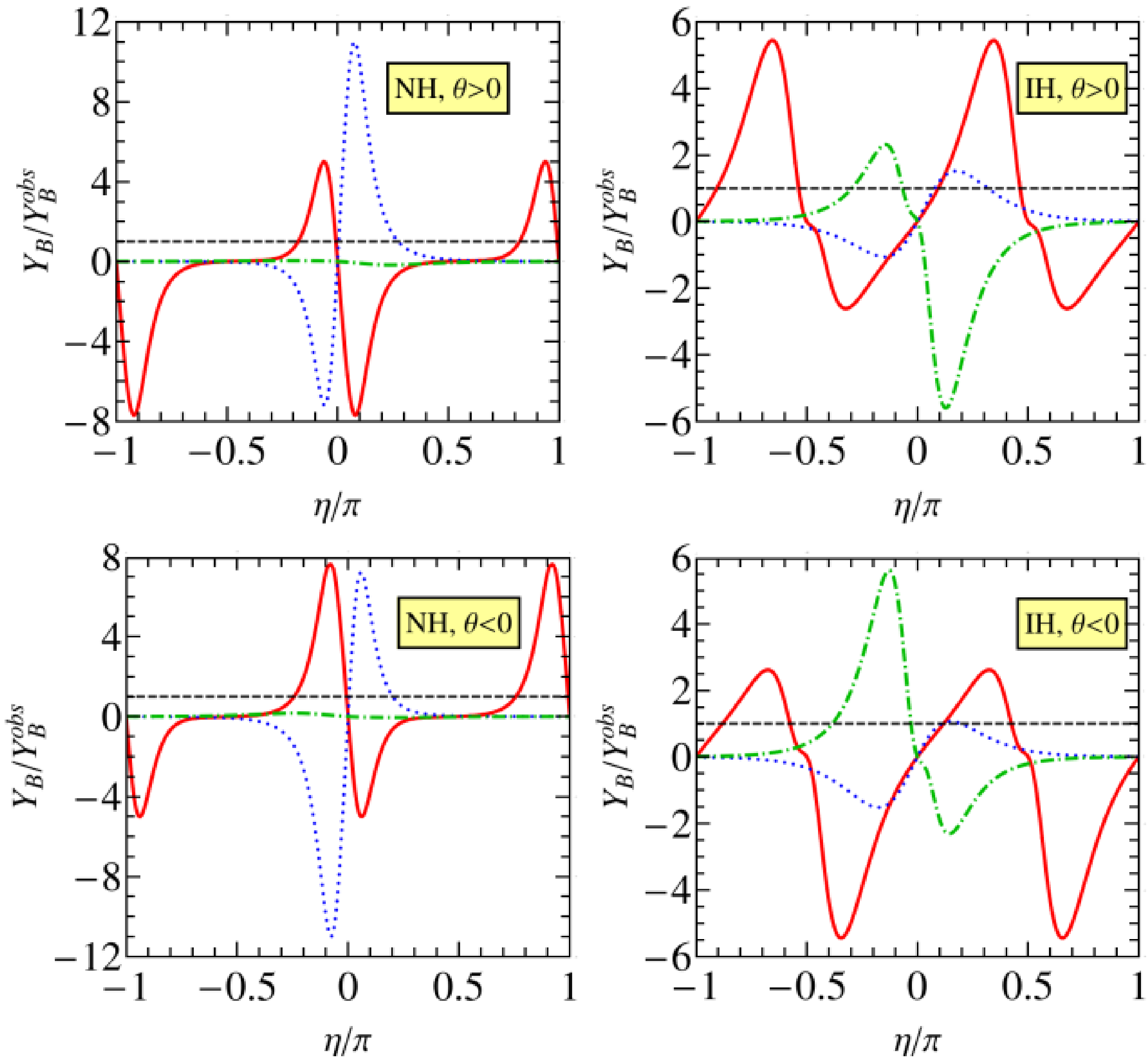
$$R' = \begin{pmatrix} \cos\eta & 0 & \sin\eta \\ & \cdots & \end{pmatrix}$$

blue dotted line:

$$R' = \begin{pmatrix} \cosh\eta & 0 & \sinh\eta \\ & \cdots & \end{pmatrix}$$

green dash-dotted line:

$$R' = \begin{pmatrix} \sinh\eta & 0 & \cosh\eta \\ & \cdots & \end{pmatrix}$$



Example of S_4

case iii: $(X_{\nu 1},X_{\nu 2})=(1,SU)$

$$\varphi = \arcsin\frac{1}{\sqrt{3}}\,, \quad \phi = \frac{5\pi}{4}\,, \quad \rho = 0\,, \quad \kappa_1 = 0\,, \quad \kappa_2 = 0\,, \quad \kappa_3 = 0\,.$$

Because κ_2-κ_3=0, the baryon asymmetry can not be generated.

case iv: $(X_{\nu 1},X_{\nu 2})=(U,S)$

$$\varphi = \arcsin\frac{1}{\sqrt{3}}\,, \quad \phi = \frac{5\pi}{4}\,, \quad \rho = 0\,, \quad \kappa_1 = \pi\,, \quad \kappa_2 = \pi\,, \quad \kappa_3 = 2\pi\,.$$

We obtain

$$I_{23}^e = 0, \quad I_{23}^\mu = \frac{1}{\sqrt{6}}, \quad I_{23}^\tau = -\frac{1}{\sqrt{6}}\,,$$

and

$$U' = \frac{i}{\sqrt{6}}\begin{pmatrix} 2 & \sqrt{2}\cos\theta & \sqrt{2}\sin\theta \\ -1 & \sqrt{2}\cos\theta + i\sqrt{3}\sin\theta & \sqrt{2}\sin\theta - i\sqrt{3}\cos\theta \\ -1 & \sqrt{2}\cos\theta - i\sqrt{3}\sin\theta & \sqrt{2}\sin\theta + i\sqrt{3}\cos\theta \end{pmatrix}\,.$$

Example of S_4

red solid line:

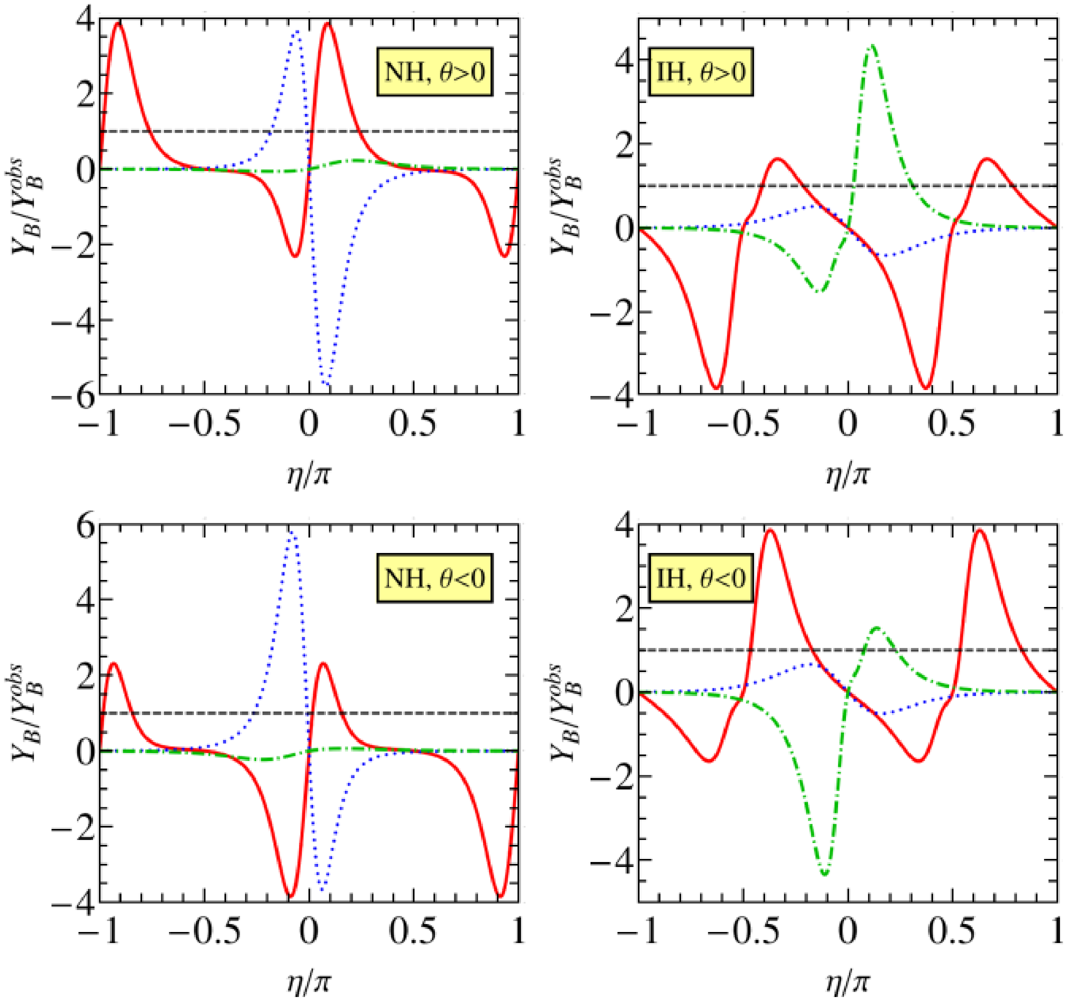
$$R' = \begin{pmatrix} 0 & \cos\eta & \sin\eta \\ \cdots & & \end{pmatrix}$$

blue dotted line:

$$R' = \begin{pmatrix} 0 & \cosh\eta & \sinh\eta \\ \cdots & & \end{pmatrix}$$

green dash-dotted line:

$$R' = \begin{pmatrix} 0 & \sinh\eta & \cosh\eta \\ \cdots & & \end{pmatrix}$$



case v: $(X_{\nu 1},X_{\nu 2})=(TST^2U,T^2)$

$$\varphi = \frac{\pi}{3}\,, \quad \phi = \arcsin\frac{1}{\sqrt{3}}\,, \quad \rho = \arccos\frac{1}{\sqrt{3}}\,, \quad \kappa_1 = 0\,, \quad \kappa_2 = 0\,, \quad \kappa_3 = 0\,.$$

κ_2-κ_3=0, the baryon asymmetry can not be generated.

Summary

- We considered a general formalism of leptogenesis in the presence of CP symmetry. In this approach, the lepton flavour mixing angle, CP phases and baryon asymmetry are strongly constrained.

- If two CP transformations are preserved. The PMNS matrix is determined in terms of a real parameter θ, the R matrix in Casas-Ibarra parametrization depends on only a single real parameter η. ϵ_α is independent of the free parameter θ. If three or four residual CP are preserved in the neutrino sector, the CP asymmetry ϵ_α would be vanishing.

- As an example, we have applied the formalism to the case of S_4 group. the correct size of the baryon asymmetry can be generated for two cases. The observed lepton mixing and the correct baryon asymmetry can be generated simultaneously.

thank you