Determination of the total absorption peak in an electromagnetic calorimeter

—— Calorimeter Function

Yiming Zhang, Zhe Wang

Tsinghua University, Beijing, China

Jia-Hua Cheng

Institute of Physics, National Chiao-Tung University, Hsinchu

CPS/HEP 2016

Electromagnetic calorimeter

An irreplaceable tool in particle physics experiment

- Useful information from energy measurement
- Still widely used in many experiments for new discoveries
- First step of the energy analysis: energy calibration with peaks

Finite size and insensitive material

- Leakage and resolution bring the energy uncertainty
- Many functions used for spectra fit
  -- Gaussian, exponentially modified Gaussian, Crystal Ball function...
- Some low energy EM calorimeter need high energy accuracy, such as Dayabay and JUNO

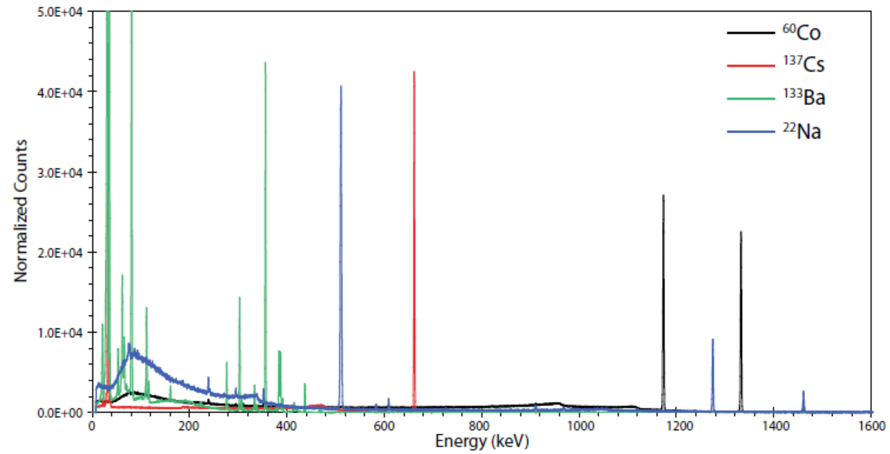
——How to find the peaks in the spectra accurately?

How to find the peaks

Good resolution——HPGe spectra

- Outstanding spikes
- Easily fit by

  Gaussian + localized background

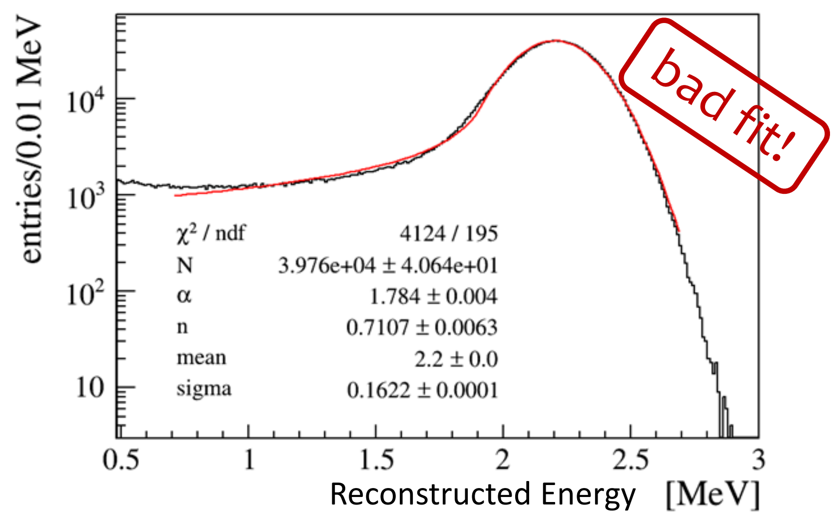


Not good resolution and leakage——Liquid Scintillator spectra

Gamma spectrum from neutron captured on hydrogen @ Dayabay

- leakage and relatively not good resolution

Crystal Ball function fit? (a trial here) ⟶



Bad fit for the peak and tail The Crystal Ball function tail is **non-physical**

We need some better choice!

New function to find the peak

A semi-empirical function based on physics and MC——**Calorimeter function**

Nucl. Instrum. Methods A827 (2016) 165-170

$$f_{\text{cal.}} = f_{\text{deposited energy}} \otimes f_{\text{resolution}}$$

$$f_{\text{deposited energy}} = \alpha f_{\text{peak}} + (1-\alpha) f_{\text{tail}} \longrightarrow$$

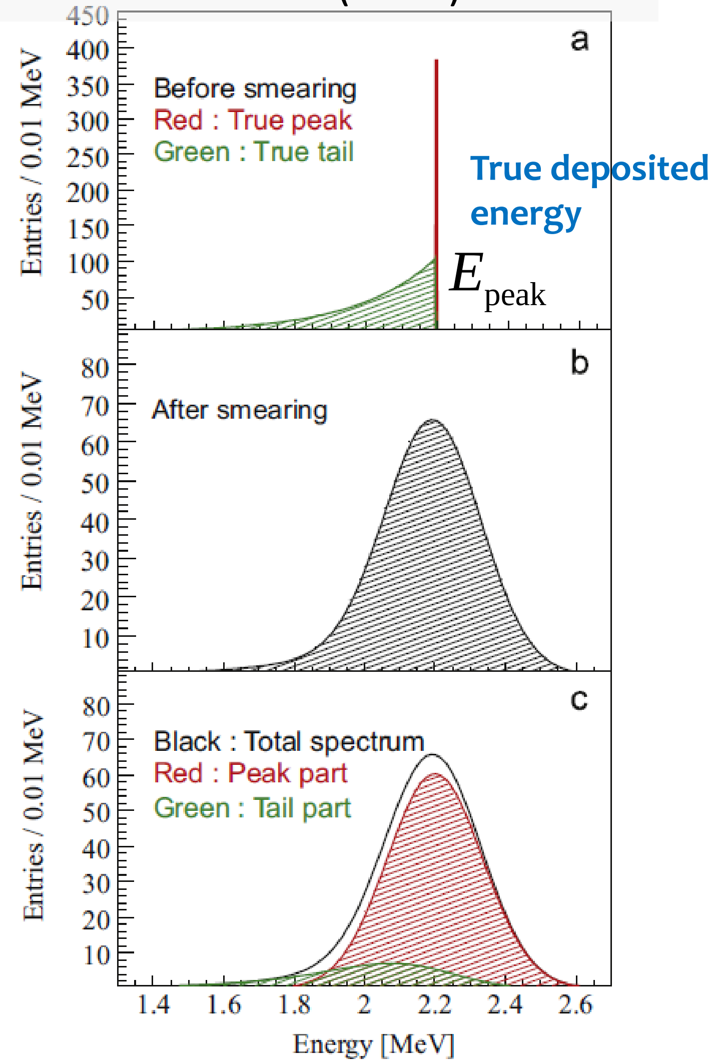
$$f_{\text{peak}} = \delta(E_{\text{peak}})$$

eg.

$$f_{\text{tail}} = \beta f_{\text{exp}} + (1-\beta) f_{\text{const}}$$

$$f_{\text{tail}} = \beta f_{\text{exp1}} + (1-\beta) f_{\text{exp2}}$$

(Cut off at E_peak)



Smearing: adding the resolution effect

$$f_{\text{resolution}} = Gaus(E, \sigma)$$

A) $\sigma = const$ Used in following slides

B) $\sigma = c\sqrt{E}$ Details in backup

New function to find the peak

A semi-empirical function based on physics and MC

$$f_{\text{cal.}} = f_{\text{deposited energy}} \otimes f_{\text{resolution}}$$

$$f_{\text{deposited energy}} = \alpha f_{\text{peak}} + (1-\alpha) f_{\text{tail}}$$

$$f_{\text{peak}} = \delta(E_{\text{peak}}) \qquad f_{\text{tail}} = \beta f_{\text{exp1}} + (1-\beta) f_{\text{exp2}} \quad (0 < E < E_{\text{peak}})$$

Case A: fixed resolution

$$f_{\text{peak}} \otimes f_{\text{resolution}} = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(E-E_{\text{peak}})^2}{2\sigma^2}}$$

$$f_{\text{exp}} \otimes f_{\text{resolution}} = \frac{\lambda}{2} e^{\frac{\sigma^2\lambda^2+2\lambda E}{2}} [\operatorname{erf}(\frac{E_{\text{peak}} - E - \sigma^2\lambda}{\sqrt{2}\sigma}) - \operatorname{erf}(\frac{-E-\sigma^2\lambda}{\sqrt{2}\sigma})]$$

$$f_{\text{const}} \otimes f_{\text{resolution}} = \frac{1}{2E_{\text{peak}}} [\operatorname{erf}(\frac{E_{\text{peak}} - E}{\sqrt{2}\sigma}) - \operatorname{erf}(\frac{-E}{\sqrt{2}\sigma})]$$

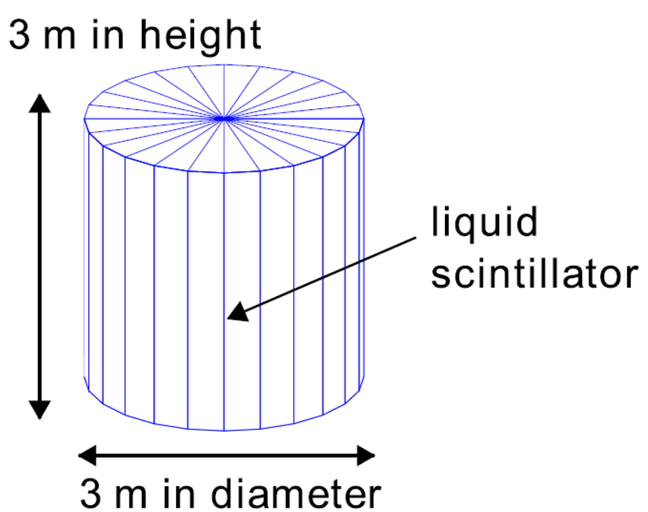
$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$$

Precision good enough for many spectra
Performance checked with MC

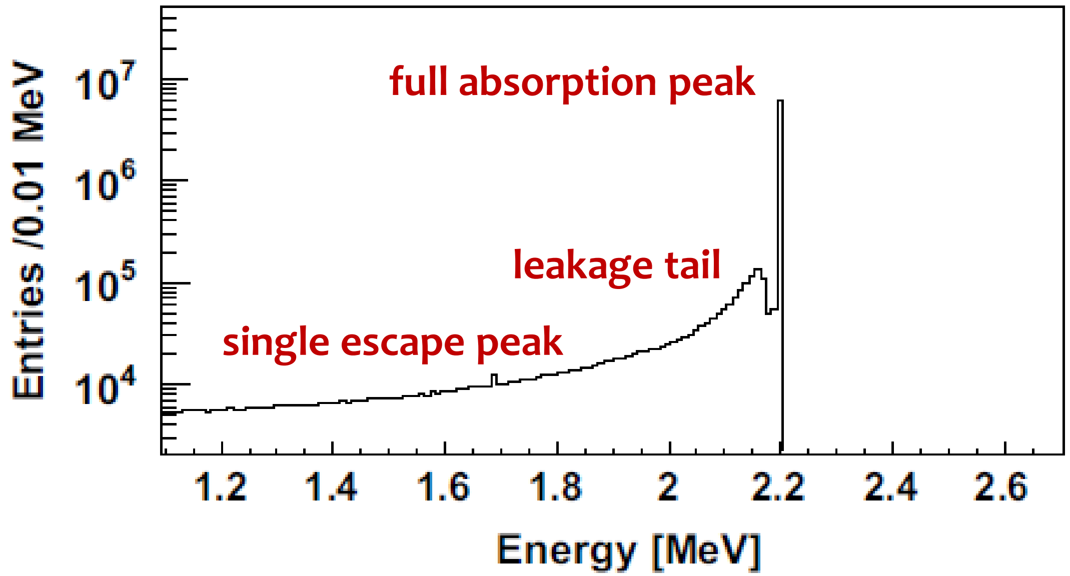
New function performance

GEANT4 MC to check the performance of the new function

A) 2.2 MeV gamma uniformly distributed in a liquid scintillator cylinder
similar to nH events at Dayabay GdLS tank with resolution as 6%



Detector setup



Spectrum of true deposited energy

- The tail in true deposited energy could be well described by exp1 + exp2 function
- Smear the deposited energy to get measured energy (avoiding electronics response impact)

New function performance

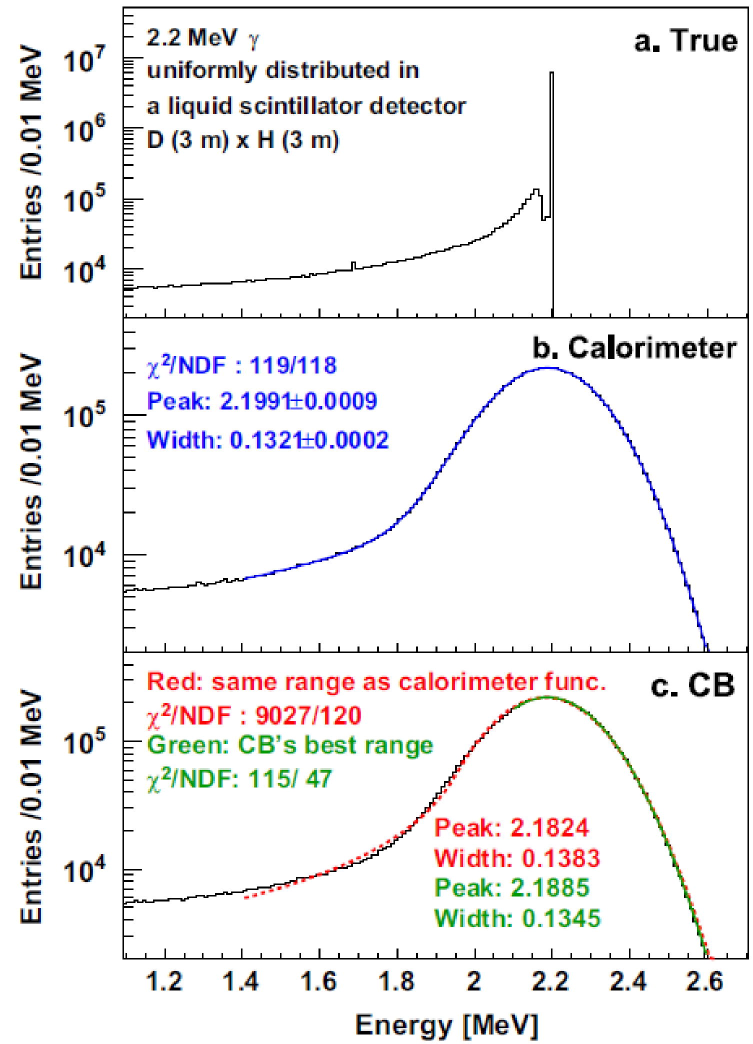
A) 2.2 MeV gamma uniformly distributed in a liquid scintillator cylinder

Comparison between Calorimeter function and the Crystal Ball function

$$\text{Accuracy} \equiv \frac{\text{fit result} - \text{true value}}{\text{true value}} \times 100\%$$

	Calorimeter	Crystal Ball	
Fit range	Best fit	Same range	Best fit
Peak accu.	-0.042%	-0.80%	-0.53%
Resolution accu.	0.044%	4.8%	1.9%
Peak area accu.	-0.59%	22%	19%
χ^2/NDF	119/118	9027/120	115/47

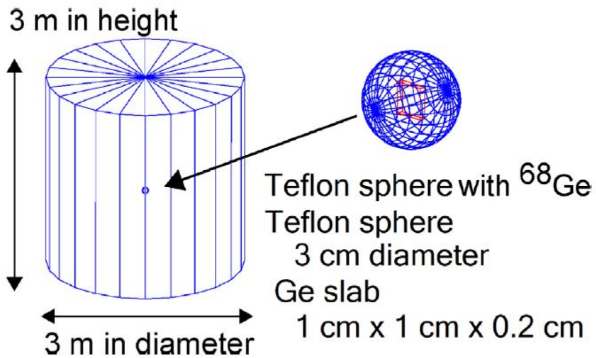
Much better result of Calorimeter function than Crystal Ball function



New function performance

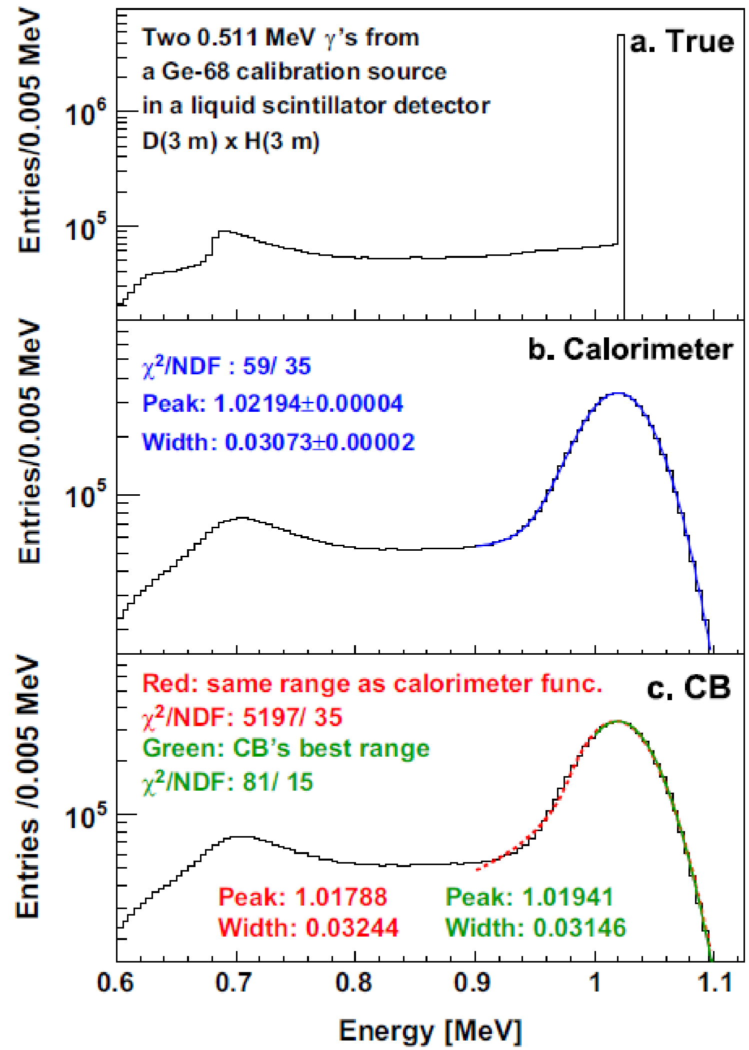
B) ^{68}Ge source in a liquid scintillator cylinder center

Detector setup



Detector resolution set to be 3% according to JUNO

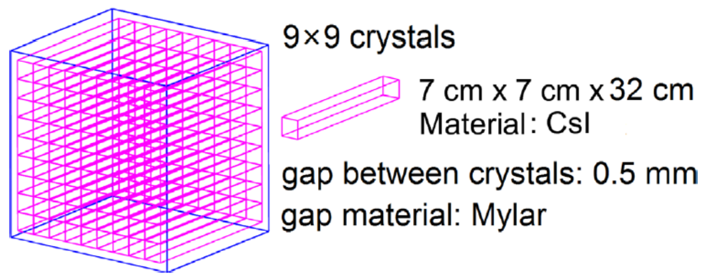
	Calorimeter	Crystal Ball	
Fit range	Best fit	Same range	Best fit
Peak accu.	0.005%	-0.40%	-0.25%
Resolution accu.	0.22%	5.8%	2.6%
Peak area accu.	-0.095%	18%	14%
χ^2/NDF	59/35	5197/35	81/15



New function performance

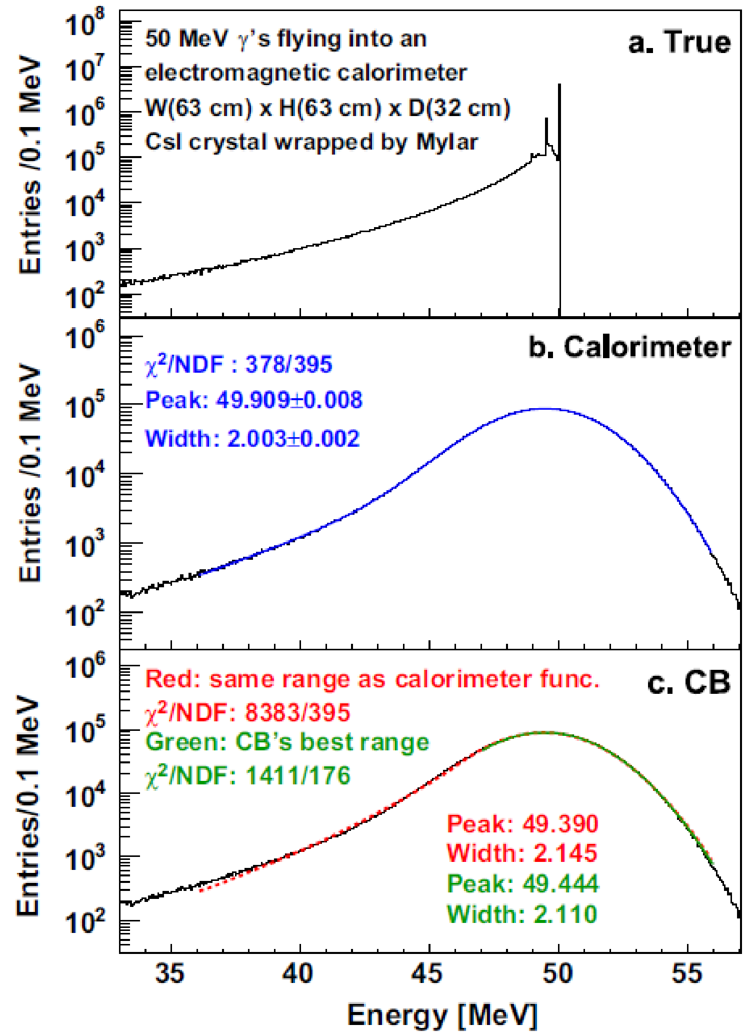
c) 50 MeV gamma into a CsI crystal array

Detector setup



Detector resolution set to be 4% according to BESIII

	Calorimeter	**Crystal Ball**	
Fit range	Best fit	Same range	Best fit
Peak accu.	-0.18%	-1.2%	-1.1%
Resolution accu.	0.15%	7.3%	5.5%
Peak area accu.	17%	128%	124%
χ^2/NDF	378/395	8383/395	1411/176



New function performance

Summary table for three EM calorimeter MC

		Calorimeter function	Crystal Ball function	
	Fit range	Own best fit	Same as Calorimeter	Own best fit
2.2 MeV γ in liquid scintillator cylinder	Peak accuracy	-0.042%	-0.80%	-0.53%
	Resolution accuracy	0.044%	4.8%	1.9%
	Peak area accuracy	-0.59%	22%	19%
	$\chi2/NDF$	119/118	9027/120	115/47
^{68}Ge source in liquid scintillator cylinder	Peak accuracy	0.005%	-0.40%	-0.25%
	Resolution accuracy	0.22%	5.8%	2.6%
	Peak area accuracy	-0.095%	18%	14%
	$\chi2/NDF$	59/35	5197/35	81/15
50 MeV γ into a CsI crystal array	Peak accuracy	-0.18%	-1.2%	-1.1%
	Resolution accuracy	0.15%	7.3%	5.5%
	Peak area accuracy	17%	128%	124%
	$\chi2/NDF$	378/395	8383/395	1411/176

Summary

We introduced a new function to describe the peak shape in electromagnetic calorimeter——Calorimeter function

- Based on the true deposited energy with resolution effect
- Various forms of tail shape in true deposited energy
  - typically exp + const, exp1 + exp2 …
- Fixed resolution and energy dependent resolution both considered

We check the performance of the Calorimeter function in three kinds of typical EM calorimeters and compared it with the Crystal Ball function

- In all cases, the Calorimeter function could fit the full peak, resolution and peak area well
- It is distinctly much better than the Crystal Ball function

The Calorimeter function has been used in the Dayabay data analysis and should be applicable in other EM calorimeters

Nucl. Instrum. Methods A827 (2016) 165-170

Thank you!

Backup

New function to find the peak

A semi-empirical function based on physics and MC

$$f_{\text{cal.}} = f_{\text{deposited energy}} \otimes f_{\text{resolution}}$$

$$f_{\text{deposited energy}} = \alpha f_{\text{peak}} + (1-\alpha) f_{\text{tail}}$$

$$f_{\text{peak}} = \delta(E_{\text{peak}}) \qquad f_{\text{tail}} = \beta f_{\exp 1} + (1-\beta) f_{\exp 2} \quad (0 < E < E_{peak})$$

Case B: energy dependent resolution

$$f_{\text{peak}} \otimes f_{\text{resolution}} = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(E-E_{\text{peak}})^2}{2\sigma^2}}$$

$$f_{\exp} \otimes f_{\text{resolution}} = \int_0^{E_{\text{peak}}} \lambda e^{\lambda E'} \frac{1}{c\sqrt{2\pi E'}} e^{-\frac{(E'-E)^2}{2c^2E'}}$$

$$= \frac{\lambda}{2A} e^{\frac{1-A}{c^2}E} [\text{erfc}(\frac{E-AE_{\text{peak}}}{\sqrt{2c^2E_{\text{peak}}}}) - e^{\frac{2A}{c^2}E} \text{erfc}(\frac{E+AE_{\text{peak}}}{\sqrt{2c^2E_{\text{peak}}}})]$$

$$\text{erfc}(x) \equiv 1 - \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt \qquad A \equiv \sqrt{1-2c^2\lambda} \quad (\lambda < \frac{1}{2c^2})$$