

Probing dark particles indirectly at Electron-Positron Colliders

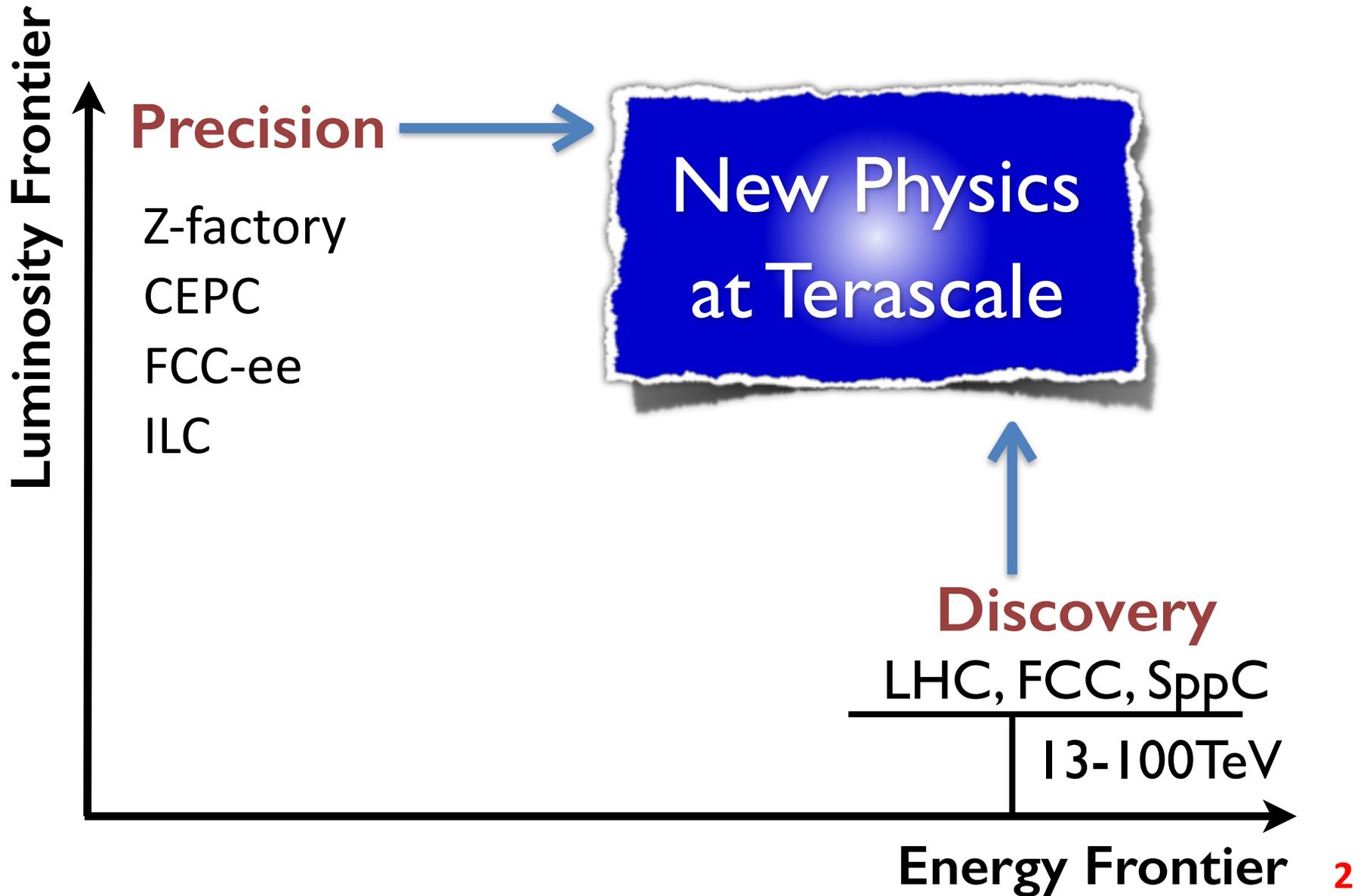
Qing-Hong Cao

Peking University

In collaboration with Yang Li, Bin Yan, Ya Zhang and Zhen Zhang

Nucl. Phys. B909 (2016) 197

Two Frontiers of High Energy Physics



DISCOVERY versus PRECISION

DISCOVERY

versus

PRECISION

New Resonance

SM Parameter values

Excesses (anomaly)

Anomalous couplings

DISCOVERY

versus

PRECISION

New Resonance

SM Parameter values

Excesses (anomaly)

Anomalous couplings

Direct search

Indirect search

DISCOVERY

versus

PRECISION

New Resonance

SM Parameter values

Excesses (anomaly)

Anomalous couplings

Direct search

Indirect search

Need good precision
of SM backgrounds
(mainly QCD)

Need accurate calculation
of SM signal processes
(mainly EW)

DISCOVERY

versus

PRECISION

New Resonance

SM Parameter values

Excesses (anomaly)

Anomalous couplings

Direct search

Indirect search

Need good precision
of SM backgrounds
(mainly QCD)

Need accurate calculation
of SM signal processes
(mainly EW)

SM parameter values?
(mainly limits)

DISCOVERY

versus

PRECISION

New Resonance

SM Parameter values

Excesses (anomaly)

Anomalous couplings

Direct search

Indirect search

Need good precision
of SM backgrounds
(mainly QCD)

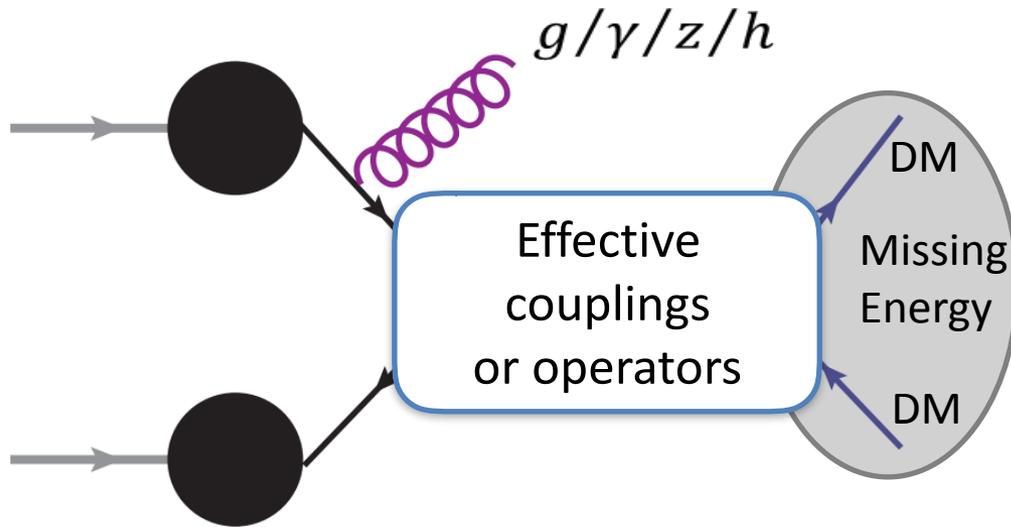
Need accurate calculation
of SM signal processes
(mainly EW)

SM parameter values?
(mainly limits)

Now

Future

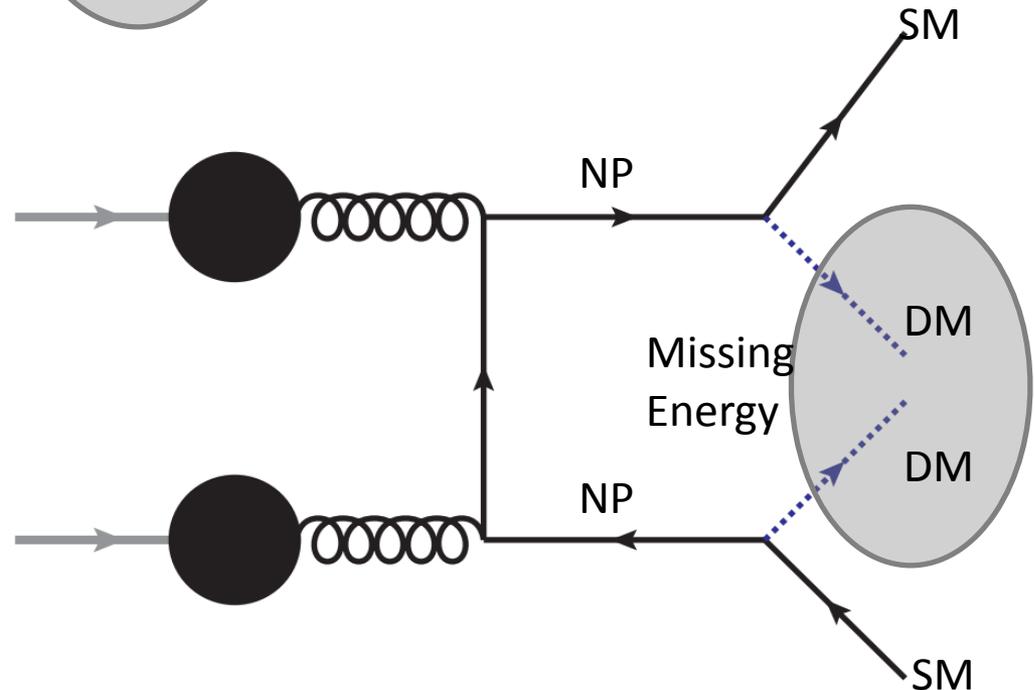
Dark Matter at the LHC



Mono-X plus MET

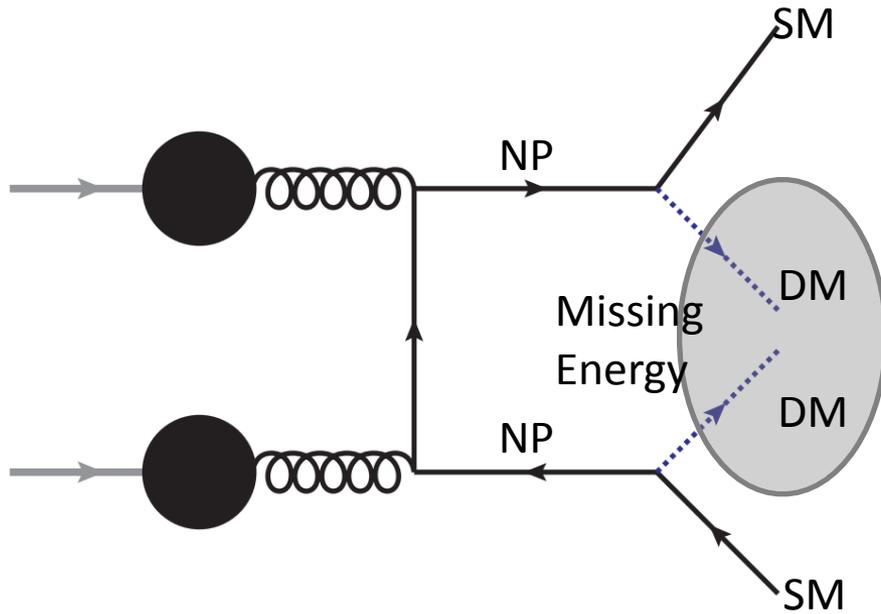
V. Khachatryan et al. (CMS), Eur. Phys. J. C75,235 (2015);
G. Aad et al. (ATLAS), Eur. Phys. J. C75,299 (2015)

SM pair plus MET



Qing Hong Cao et al. Phys. Rev. Lett.109 (2012) 152004;
Haipeng An et al. Phys. Rev. Lett.115 (2015) 181602;
Qing Hong Cao, Chuan-Ren Chen and Yandong Liu, arxiv:1512.09144

Dark Matter at the LHC



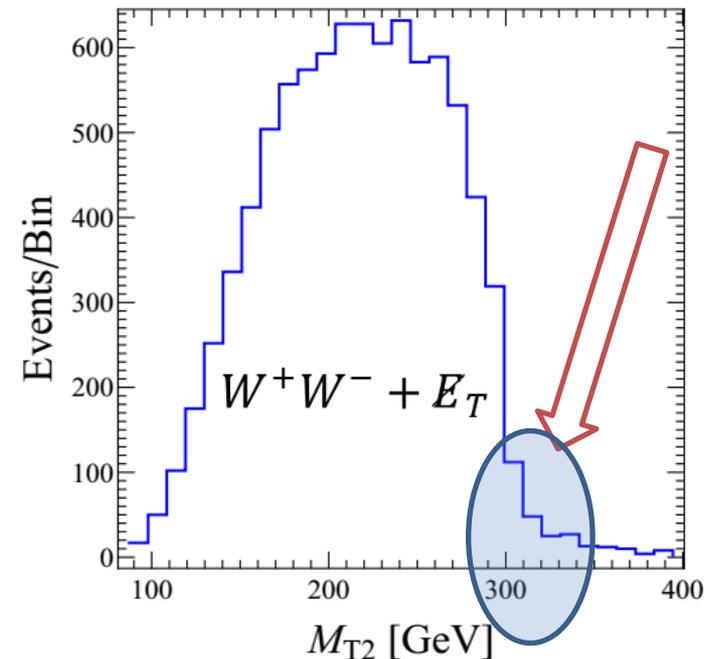
SM pair plus MET

$$M_{NP} \gg M_{DM}$$

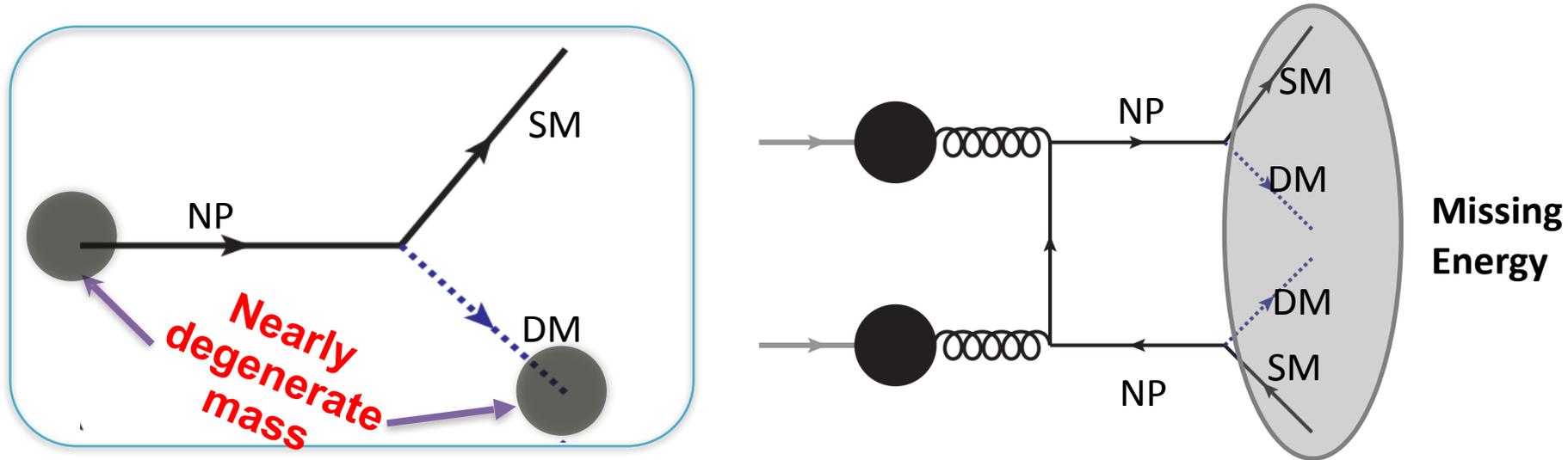
M_{T2} \longleftrightarrow mother particle's mass

$$M_{NP} = 300 \text{ GeV}, \quad M_{DM} = 0$$

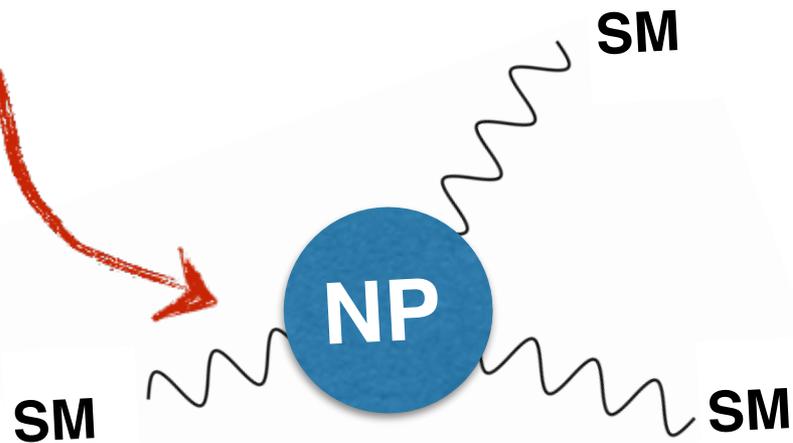
Qing-Hong Cao, Chuan-Ren Chen and Yandong Liu,
arxiv:1512.09144



Degenerate Dark Matter Model



NP affects SM processes through loop



Loop does not care about mass split at all, but demands **HIGH PRECISION** measurements

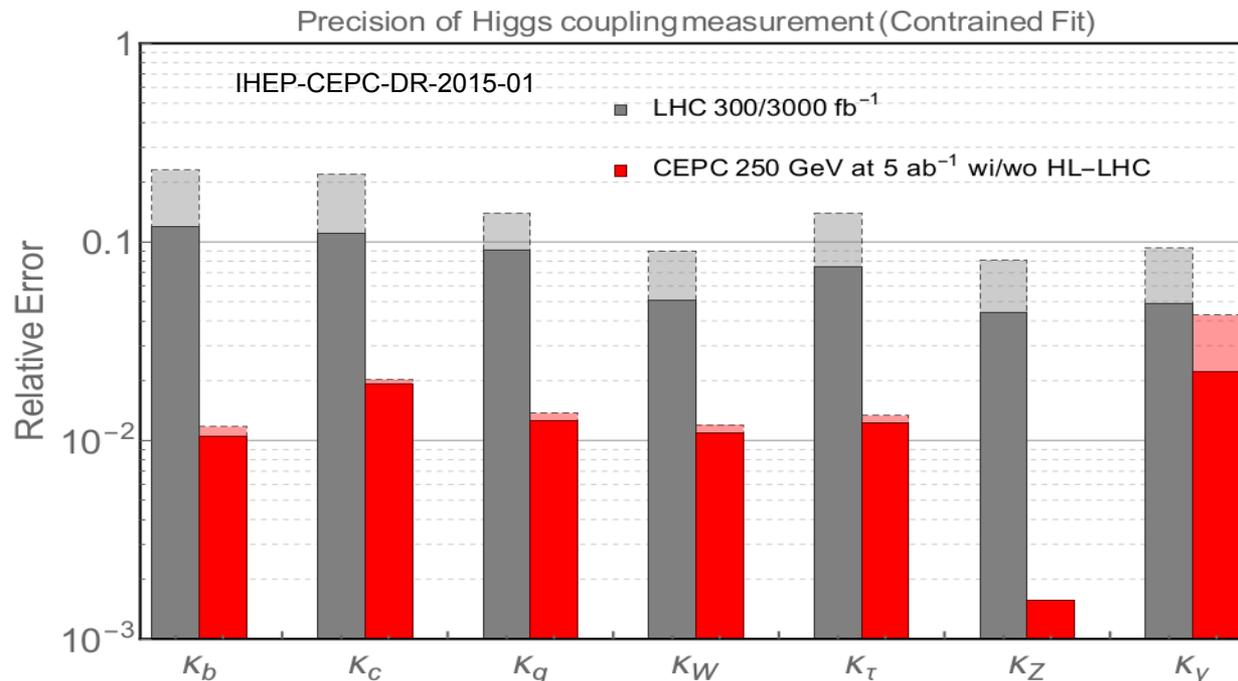
The demand for e^+e^- colliders

high luminosity

clean background

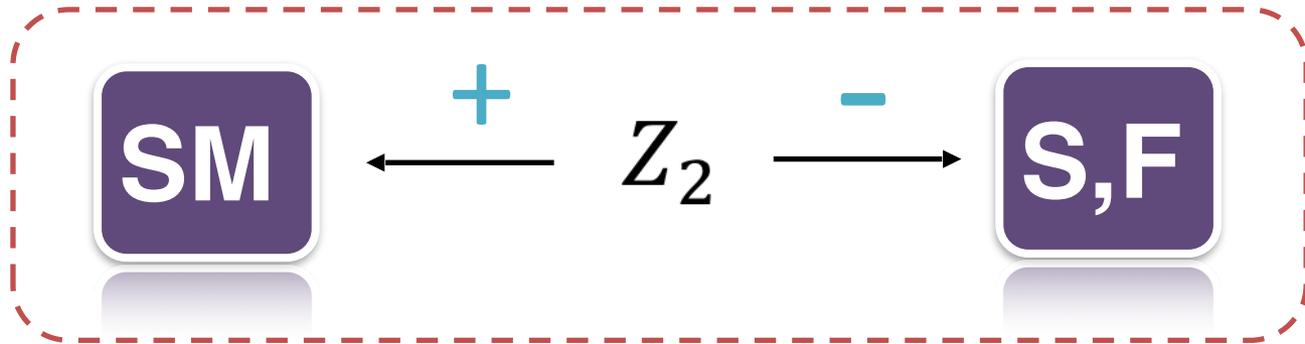
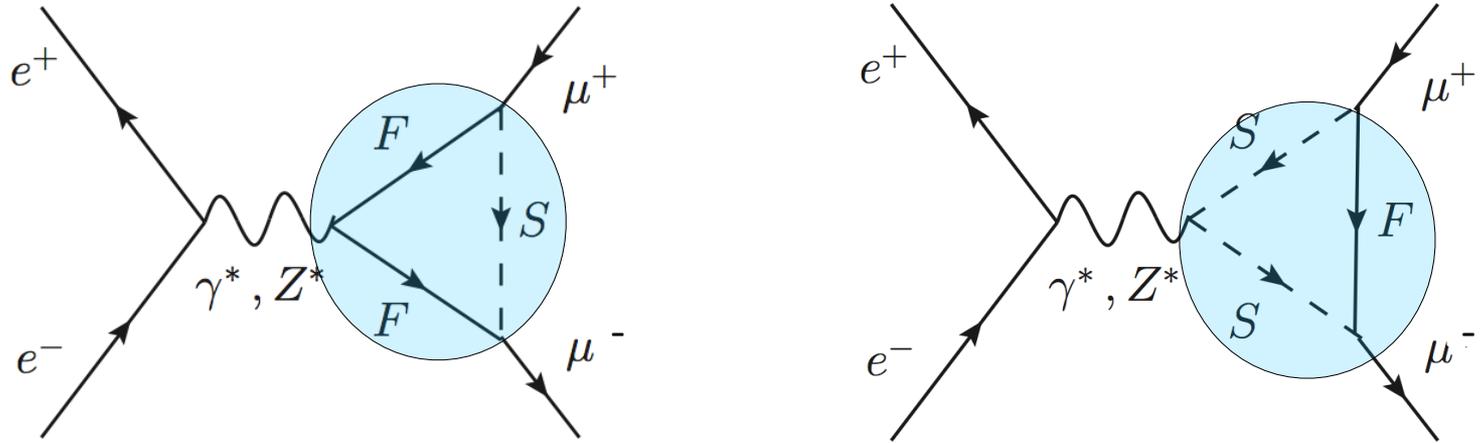
High precision

$\sim 10^{-3}$
precision



Precisions of a few percents are achievable for some of the couplings.
The CEPC can robustly improve this precision by an order of magnitude.

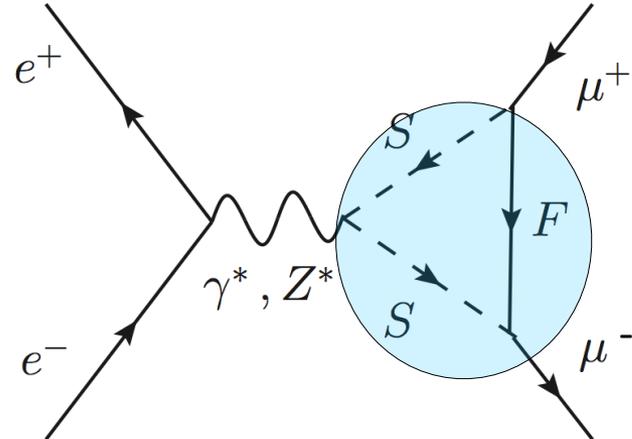
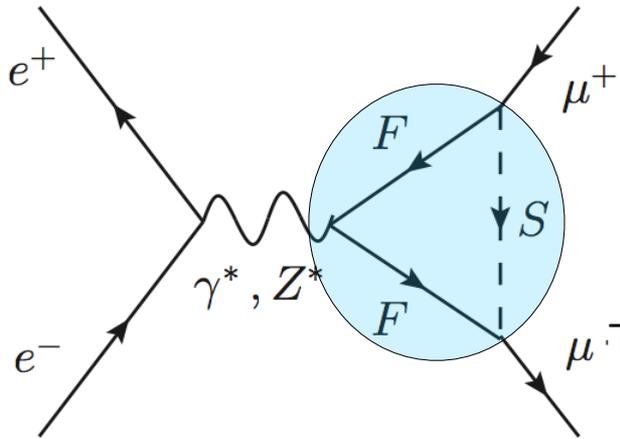
Simplified New Physics Model



Assuming NP does not talk to electron directly

$$\Delta\mathcal{L} = \bar{F}(i\not{D} - M_F)F + |D_\nu S|^2 - M_S^2 S^\dagger S - V(S, H) + \mathcal{L}_{\text{Yuk}}$$

Simplified New Physics Model



$$\Delta\mathcal{L} = \bar{F}(i\not{D} - M_F)F + |D_\mu S|^2 - M_S^2 S^\dagger S - V(S, H)$$

Left-handed scenario

$$L_{Yuk} = y C_{ijk} S^i \bar{\mu}_L^k F^j + h.c.$$

$$C_{ijk} = \langle I_{\mu L} k | I_S I_F; ij \rangle$$

$$I_S = I_F \pm 1/2$$

Right-handed scenario

$$L_{Yuk} = y C_{ij} S^i \bar{\mu}_R F^j + h.c.$$

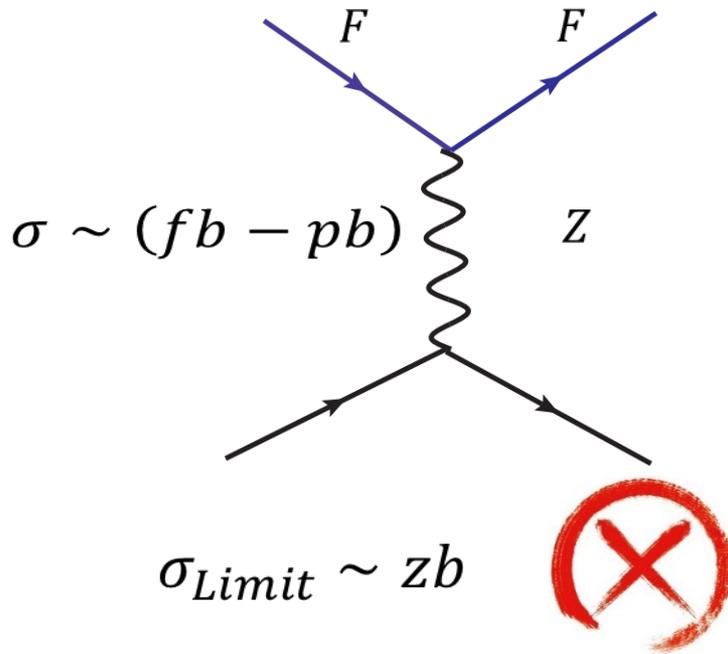
$$C_{ij} = \langle I_{\mu R} 0 | I_S I_F; ij \rangle$$

$$I_S = I_F$$

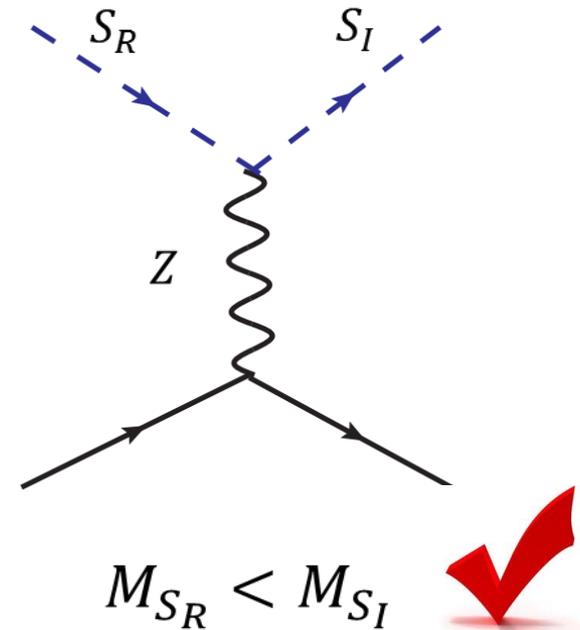
V.S.

Dark Matter Candidate

$$\Delta\mathcal{L} = \bar{F}(i\not{D} - M_F)F + |D_\mu S|^2 - M_S^2 S^\dagger S - V(S, H)$$



Dark Matter
Direct Detection



Except singlet Fermion

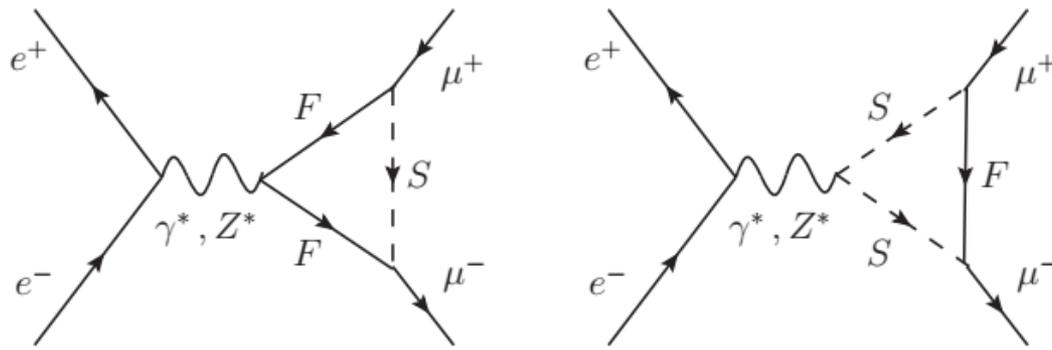
D.S. Akerib et al. (LUX) (2005), Arxiv:1512.03506

Scalar dark matter

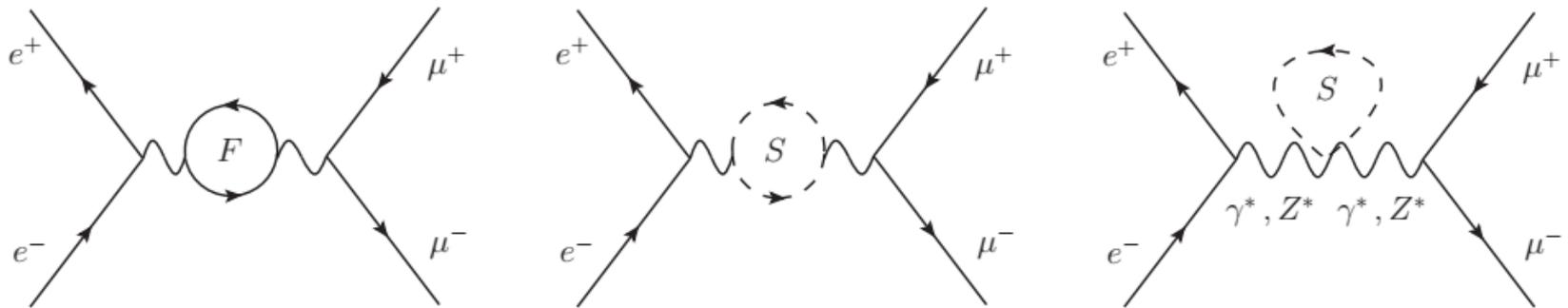
$Y_S \in \{-I_S, -I_S + 1, \dots, I_S\}$

One-Loop Correction

Yukawa interaction: $\mathcal{O}(y^2)$



Gauge interaction: $\mathcal{O}(e^2)$



Purely Yukawa Interaction

Loop induced effective coupling $V\mu^+\mu^-$

$$-ie\bar{u}(k_-)(\alpha_V\gamma^\mu + i\beta_V\sigma^{\mu\nu}q_\nu + \xi_{1,V}\gamma^\mu\gamma_5 + \xi_{2,V}q^\mu\gamma_5)v(k_+)$$

On-shell scheme

A. Stange and S. Willenbrock, Phys. Rev. D48,2054(1993)
Q.-H.Cao et al, Phys. Rev.D 79,015004(2009)

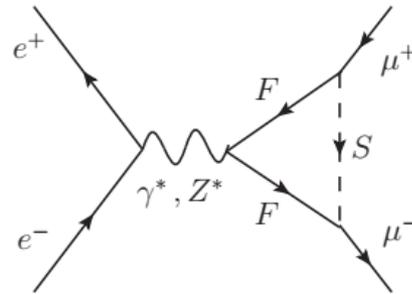
$$\alpha_V = \alpha_{V,\Delta} + \delta\alpha_V$$

$$\beta_V = \beta_{V,\Delta}$$

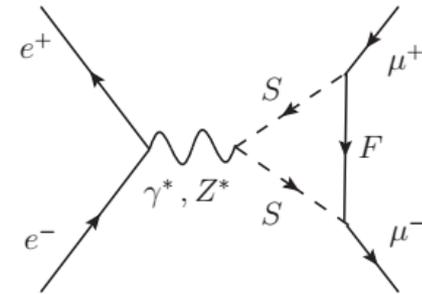
$$\xi_{1,V} = \xi_{1,V,\Delta} + \delta\xi_{1,V}$$

$$\xi_{2,V} = \xi_{2,V,\Delta}$$

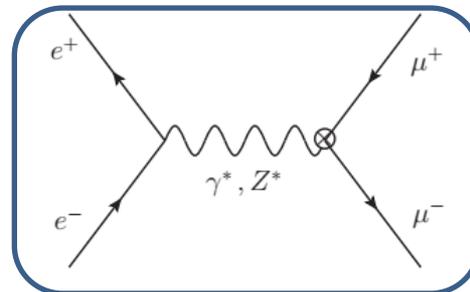
Wave function of muon



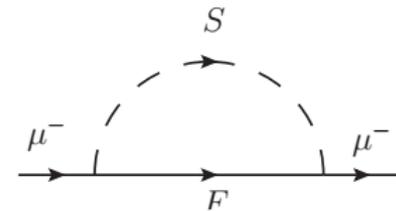
(a)



(b)

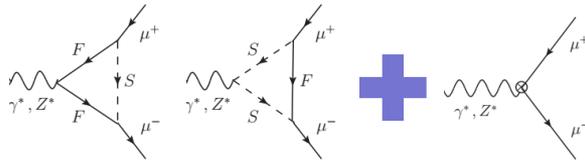


(c)



(d)

Purely Yukawa Interaction (Left-Handed Scenario)



$$= -ie\bar{u}(k_-) (\alpha_V \gamma^\mu + i\beta_V \sigma^{\mu\nu} q_\nu + \xi_{1,V} \gamma^\mu \gamma_5 + \xi_{2,V} q^\mu \gamma_5) v(k_+)$$

$$V = \gamma, Z$$

$$\alpha_V = \alpha_{V,\Delta} + \delta\alpha_V, \quad \beta_V = \beta_{V,\Delta}, \quad \xi_{1,V} = \xi_{1,V,\Delta} + \delta\xi_{1,V}, \quad \xi_{2,V} = \xi_{2,V,\Delta},$$

Stange, Willenbrock, Phys. Rev. D48, 2054 (1993).

$$J_{F3} = \sum_{ijk} C_{ik-\frac{1}{2}} T_{F,kj}^3 C_{ij-\frac{1}{2}}^*$$

B_0 and C_0 scalar functions

Dimensional Regularization
On-shell scheme

Aoki, et al, Prog. Theor. Phys. Suppl. 73, 1 (1982).
Passarino, Veltman, Nucl. Phys. B160, 151 (1979).

$$a \equiv \frac{|y|^2}{16\pi^2} [a^1 + a^2 B_0(s) + a^3 B_0(m_\mu^2) + a^4 B'_0(m_\mu^2) + a^5 C_0]. \quad \text{where } a = \alpha_V, \beta_V, \xi_{1,V}, \xi_{2,V}$$

$$\alpha_\gamma^1 = -\frac{s(2Y_S + 2J_{S3} + 1)}{4(s - 4m_\mu^2)}, \quad \alpha_\gamma^2 = -\alpha_\gamma^3 = \frac{16m_\mu^4 - 2s(6J_{S3} + 6Y_S + 7)m_\mu^2 + s^2}{4(s - 4m_\mu^2)^2},$$

$$\alpha_\gamma^4 = -\frac{m_\mu^2}{2},$$

$$\alpha_\gamma^5 = -\frac{sm_\mu^2 [s(J_{S3} + Y_S + 1) - 4M^2(2J_{S3} + 2Y_S + 1)] + sm_\mu^4(2J_{S3} + 2Y_S - 3) + M^2s^2(2J_{S3} + 2Y_S + 1) + 8m_\mu^6}{2(s - 4m_\mu^2)^2},$$

$$\beta_\gamma^1 = \frac{m_\mu(2J_{S3} + 2Y_S + 1)}{2(s - 4m_\mu^2)}, \quad \beta_\gamma^2 = -\beta_\gamma^3 = \frac{m_\mu(2m_\mu^2 + s)(2J_{S3} + 2Y_S + 1)}{2(s - 4m_\mu^2)^2},$$

$$\beta_\gamma^5 = -\frac{m_\mu(2J_{S3} + 2Y_S + 1)[m_\mu^4 + m_\mu^2(4M^2 - s) - M^2s]}{(s - 4m_\mu^2)^2},$$

$$\xi_{1,\gamma}^1 = \frac{1}{4}(2J_{S3} + 2Y_S + 1), \quad \xi_{1,\gamma}^2 = -\xi_{1,\gamma}^3 = -\frac{(4Y_S + 4J_{S3} + 2)m_\mu^2 + s}{4(s - 4m_\mu^2)},$$

$$\xi_{1,\gamma}^5 = -\frac{m_\mu^2 [s(J_{S3} + Y_S + 1) - m_\mu^2(2J_{S3} + 2Y_S + 1)] - M^2(s - 4m_\mu^2)(2J_{S3} + 2Y_S + 1)}{2(s - 4m_\mu^2)},$$

$$\alpha_Z^1 = -\frac{[4c_W^2 J_{S3} - 2s_W^2(2Y_S + 1) + 1]s}{8c_W s_W (s - 4m_\mu^2)},$$

$$\alpha_Z^2 = -\alpha_Z^3 = \frac{2m_\mu^2 s [2s_W^2(6J_{S3} + 6Y_S + 7) - 12J_{S3} - 7] + (2s_W^2 - 1)(16m_\mu^4 + s^2)}{8c_W s_W (s - 4m_\mu^2)^2},$$

$$\sim (a^2 + a^3) \frac{1}{\epsilon} \quad \epsilon = \frac{4-n}{2}$$

$\alpha_V, \beta_V, \xi_{1,V}, \xi_{2,V}$
are UV finite

Purely Yukawa Interaction (Left-Handed Scenario)

When M is Large

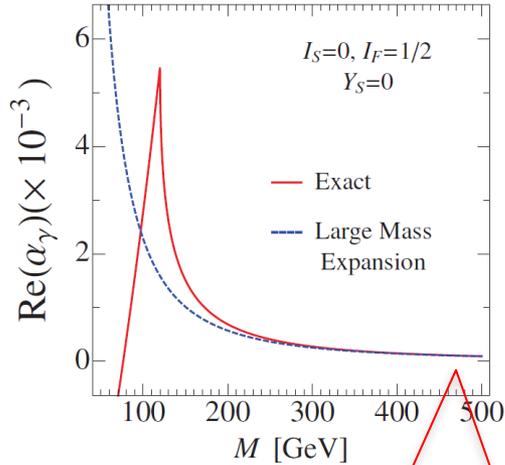


$$B_0(p^2; M^2, M^2) = B_0(0; M^2, M^2) + \frac{p^2}{6M^2} + \frac{p^4}{60M^4} + \mathcal{O}(M^{-6}),$$

$$C_0(p_1^2, p_2^2, p_3^2; M^2, M^2, M^2) = -\frac{1}{2M^2} - \frac{p_1^2 + p_2^2 + p_3^2}{24M^4} + \mathcal{O}(M^{-6}).$$



$$\begin{aligned} \alpha_\gamma &= +\frac{|y|^2}{768\pi^2} \frac{s}{M^2} (2J_{S3} + 2Y_S + 3) & \alpha_Z &= +\frac{|y|^2}{768\pi^2 c_W s_W} \frac{s}{M^2} \left(2c_W^2 J_{S3} - 2s_W^2 Y_S - 3s_W^2 + \frac{3}{2} + \frac{m_\mu^2}{s} \right), \\ \beta_\gamma &= +\frac{|y|^2}{768\pi^2} \frac{m_\mu}{M^2} (4J_{S3} + 4Y_S + 2) & \beta_Z &= +\frac{|y|^2}{768\pi^2 c_W s_W} \frac{m_\mu}{M^2} (4c_W^2 J_{S3} - 4s_W^2 Y_S - 2s_W^2 + 1), \\ \xi_{1,\gamma} &= -\frac{|y|^2}{768\pi^2} \frac{s}{M^2} (2J_{S3} + 2Y_S + 3) & \xi_{1,Z} &= -\frac{|y|^2}{768\pi^2 c_W s_W} \frac{s}{M^2} \left(2c_W^2 J_{S3} - 2s_W^2 Y_S - 3s_W^2 + \frac{3}{2} - \frac{m_\mu^2}{s} \right), \\ \xi_{2,\gamma} &= +\frac{|y|^2}{768\pi^2} \frac{m_\mu}{M^2} (4J_{S3} + 4Y_S + 6) & \xi_{2,Z} &= +\frac{|y|^2}{768\pi^2 c_W s_W} \frac{m_\mu}{M^2} (4c_W^2 J_{S3} - 4s_W^2 Y_S - 6s_W^2 + 3). \end{aligned}$$



Decoupling

$$\xi_{1,\gamma} \gamma^\nu \gamma_5 + \xi_{2,\gamma} q^\nu \gamma_5 = \xi_{1,\gamma} (\gamma^\nu - 2m_\mu/s q^\nu) \gamma_5$$

by $U(1)_{EM}$ symmetry

Anapole Moment

$$(\alpha_\gamma \gamma^\nu + i \beta_\gamma q_\rho \sigma^{\nu\rho} + \xi_{1,\gamma} \gamma^\nu \gamma_5 + \xi_{2,\gamma} q^\nu \gamma_5) \rightarrow 0$$

in the Thomson limit,

i.e., $q^\mu \rightarrow 0$

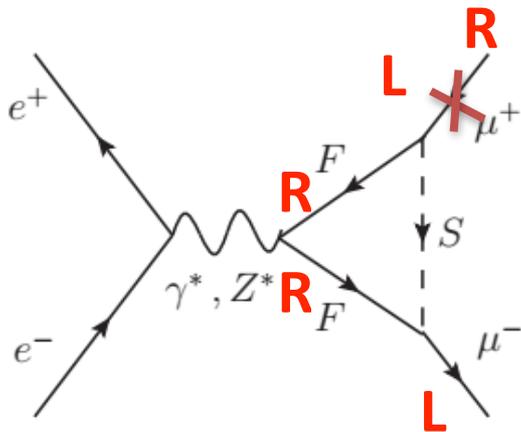
Left-Handed Scenario (Large Mass Expansion)

$$\alpha_\gamma = + \frac{|y|^2}{768\pi^2} \frac{s}{M^2} (2J_{S3} + 2Y_S + 3) \quad -ie\bar{u}(k_-)(\alpha_V\gamma^\mu + i\beta_V\sigma^{\mu\nu}q_\nu)v(k_+)$$

$$\beta_\gamma = + \frac{|y|^2}{768\pi^2} \frac{m_\mu}{M^2} (4J_{S3} + 4Y_S + 2) \quad -ie\bar{u}(k_-)(\xi_{1,V}\gamma^\mu\gamma_5 + \xi_{2,V}q^\mu\gamma_5)v(k_+)$$

$$\xi_{1,\gamma} = - \frac{|y|^2}{768\pi^2} \frac{s}{M^2} (2J_{S3} + 2Y_S + 3) \quad J_{S3} = \begin{cases} \frac{1}{3}I_S, & \text{for } I_F = I_S + \frac{1}{2}, \\ -\frac{1}{3}(I_S + 1), & \text{for } I_F = I_S - \frac{1}{2}. \end{cases}$$

$$\xi_{2,\gamma} = + \frac{|y|^2}{768\pi^2} \frac{m_\mu}{M^2} (4J_{S3} + 4Y_S + 6)$$



$$\alpha_Z = + \frac{|y|^2}{768\pi^2 c_W s_W} \frac{s}{M^2} \left(2c_W^2 J_{S3} - 2s_W^2 Y_S - 3s_W^2 + \frac{3}{2} + \frac{m_\mu^2}{s} \right),$$

$$\beta_Z = + \frac{|y|^2}{768\pi^2 c_W s_W} \frac{m_\mu}{M^2} \left(4c_W^2 J_{S3} - 4s_W^2 Y_S - 2s_W^2 + 1 \right),$$

$$\xi_{1,Z} = - \frac{|y|^2}{768\pi^2 c_W s_W} \frac{s}{M^2} \left(2c_W^2 J_{S3} - 2s_W^2 Y_S - 3s_W^2 + \frac{3}{2} - \frac{m_\mu^2}{s} \right),$$

$$\xi_{2,Z} = + \frac{|y|^2}{768\pi^2 c_W s_W} \frac{m_\mu}{M^2} \left(4c_W^2 J_{S3} - 4s_W^2 Y_S - 6s_W^2 + 3 \right).$$

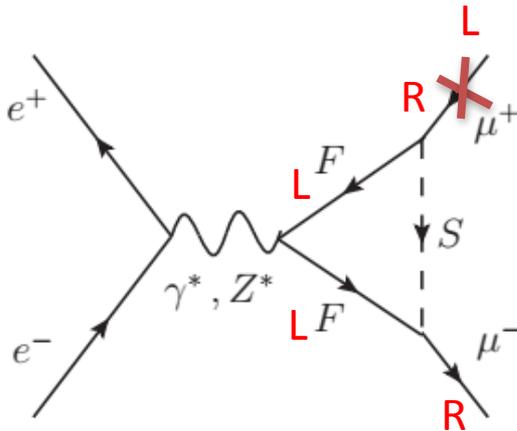
Right-Handed Scenario (Large Mass Expansion)

$$\alpha_\gamma = +\frac{|y|^2}{768\pi^2} \frac{s}{M^2} (2Y_S + 3), \quad \alpha_Z = +\frac{|y|^2}{768\pi^2 c_W s_W} \frac{s}{M^2} \left(-2s_W^2 Y_S - 3s_W^2 - \frac{m_\mu^2}{s} \right)$$

$$\beta_\gamma = +\frac{|y|^2}{768\pi^2} \frac{m_\mu}{M^2} (4Y_S + 2), \quad \beta_Z = +\frac{|y|^2}{768\pi^2 c_W s_W} \frac{m_\mu}{M^2} (-4s_W^2 Y_S - 2s_W^2),$$

$$\xi_{1,\gamma} = +\frac{|y|^2}{768\pi^2} \frac{s}{M^2} (2Y_S + 3), \quad \xi_{1,Z} = +\frac{|y|^2}{768\pi^2 c_W s_W} \frac{s}{M^2} \left(-2s_W^2 Y_S - 3s_W^2 + \frac{m_\mu^2}{s} \right)$$

$$\xi_{2,\gamma} = -\frac{|y|^2}{768\pi^2} \frac{m_\mu}{M^2} (4Y_S + 6), \quad \xi_{2,Z} = -\frac{|y|^2}{768\pi^2 c_W s_W} \frac{m_\mu}{M^2} (-4s_W^2 Y_S - 6s_W^2).$$



$$-ie\bar{u}(k_-)(\alpha_V\gamma^\mu + i\beta_V\sigma^{\mu\nu}q_\nu)v(k_+)$$

$$-ie\bar{u}(k_-)(\xi_{1,V}\gamma^\mu\gamma_5 + \xi_{2,V}q^\mu\gamma_5)v(k_+)$$

Purely Gauge Interaction

Loop induced effective coupling $V\mu^+\mu^-$

$$-ie\bar{u}(k_-)(\alpha_V\gamma^\mu + i\beta_V\sigma^{\mu\nu}q_\nu + \xi_{1,V}\gamma^\mu\gamma_5 + \xi_{2,V}q^\mu\gamma_5)v(k_+)$$

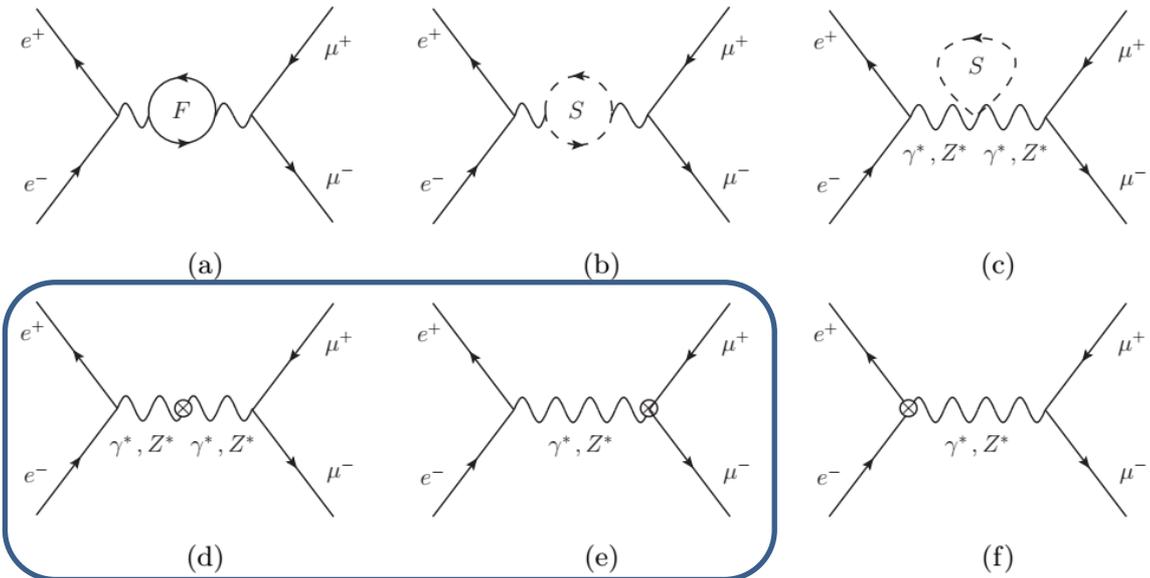
On-shell scheme

$$\alpha_V = \alpha_{V,0} + \delta\alpha_V$$

$$\beta_V = \beta_{V,0}$$

$$\xi_{1,V} = \xi_{1,V,0} + \delta\xi_{1,V}$$

$$\xi_{2,V} = \xi_{2,V,0} + \delta\xi_{2,V}$$



Wave function of gauge boson

Purely Gauge Interaction

$$-ie\bar{u}(k_-)(\alpha_V\gamma^\mu + i\beta_V\sigma^{\mu\nu}q_\nu + \xi_{1,V}\gamma^\mu\gamma_5 + \xi_{2,V}q^\mu\gamma_5)v(k_+)$$

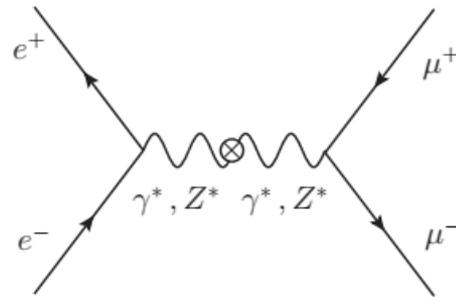
On-shell scheme

$$\alpha_V = \alpha_{V,0} + \delta\alpha_V$$

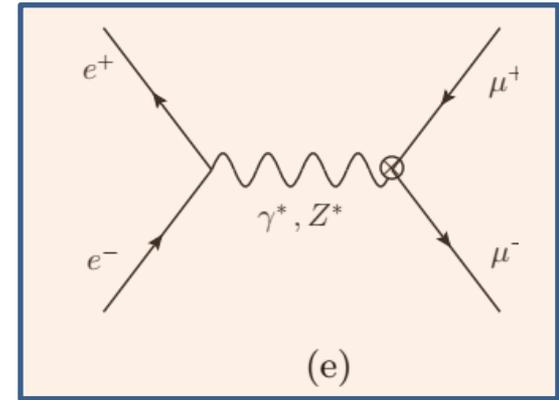
$$\beta_V = \beta_{V,0}$$

$$\xi_{1,V} = \xi_{1,V,0} + \delta\xi_{1,V}$$

$$\xi_{2,V} = \xi_{2,V,0} + \delta\xi_{2,V}$$



(d)



(e)

A. Denner, Fortsch.Phys.41,307(1993),0709.1075

$$\delta\alpha_\gamma = C_v^\gamma + \left(-\frac{g_v}{s - m_Z^2} C_{AZ} + \frac{Q}{s} C_{AA} \right)$$

$$\delta\xi_{1,\gamma} = C_a^\gamma + \left(-\frac{g_a}{s - m_Z^2} C_{AZ} \right)$$

$$\delta\xi_{2,\gamma} = 0 + \left(-\frac{g_a 2m_\mu}{s - m_Z^2} C'_{AZ} \right)$$

$$\delta\alpha_Z = C_v^Z + \left(-\frac{g_v}{s - m_Z^2} C_{ZZ} + \frac{Q}{s} C_{AZ} \right)$$

$$\delta\xi_{1,Z} = C_a^Z + \left(-\frac{g_a}{s - m_Z^2} C_{ZZ} \right)$$

$$\delta\xi_{2,Z} = 0 + \left(-\frac{g_a 2m_\mu}{s - m_Z^2} C'_{ZZ} \right)$$

Numerical Results

SM tree-level

► $\Delta\sigma(\equiv \sigma - \sigma_0) \sim \text{SM-NP interference}$

$$\frac{d\Delta\sigma}{dt} = \frac{\pi\alpha_{\text{EM}}^2}{s^2} \sum_{ij} \left[\mathcal{A}_{ij}^L \left(g_i^\gamma + g_L g_i \frac{s}{s - m_Z^2} \right) \left(F_{\gamma,j}^L + F_{Z,j}^L \frac{s}{s - m_Z^2} \right) + (L \rightarrow R) \right].$$

$$F_{\gamma,j}^L = 2\Re \{ \alpha_\gamma + C_L^\gamma, \xi_{1,\gamma}, m_\mu \beta_\gamma \},$$

$$F_{Z,j}^L = 2\Re \{ g_L \alpha_Z + C_L^Z g_v, g_L \xi_{1,Z} + C_L^Z g_a, g_L m_\mu \beta_Z \},$$

$$F_{\gamma,j}^R = 2\Re \{ \alpha_\gamma + C_R^\gamma, \xi_{1,\gamma}, m_\mu \beta_\gamma \},$$

$$F_{Z,j}^R = 2\Re \{ g_R \alpha_Z + C_R^Z g_v, g_R \xi_{1,Z} + C_R^Z g_a, g_R m_\mu \beta_Z \}.$$

► Input parameters – G_μ scheme:

$$G_\mu = 1.1663787 \times 10^{-5} \text{ GeV}^{-2},$$

$$\alpha_{\text{EM}}(0) = 1/137.035999139,$$

$$m_Z = 91.1876 \text{ GeV},$$

$$m_W = 80.385 \text{ GeV},$$

$$m_\mu = 105.6583745 \text{ MeV},$$

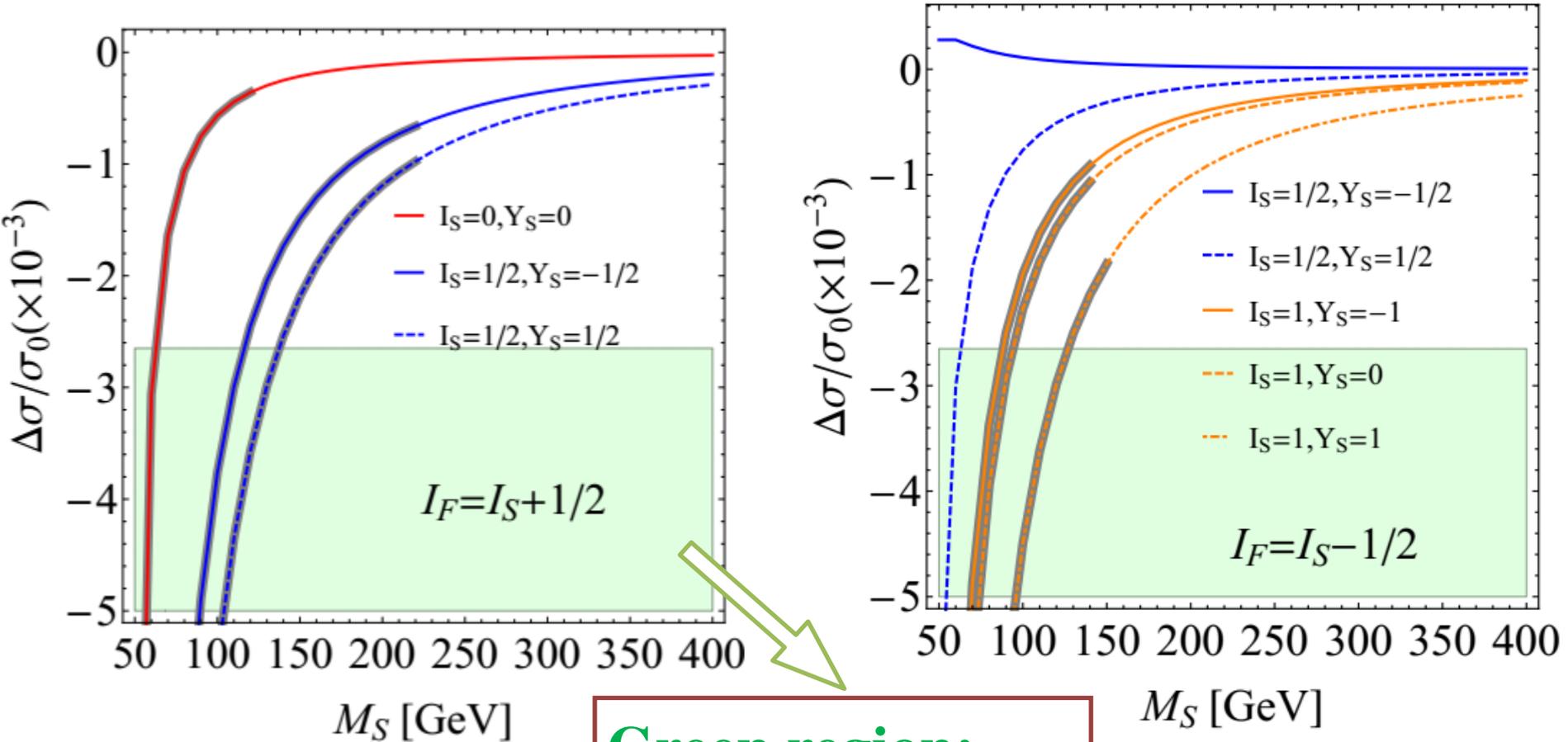
$$c_W = m_W/m_Z$$

P. J. Mohr, D. B. Newell, and B. N. Taylor (2015)

► **LoopTools** for the evaluation of the scalar functions.

LEP-I Constraint (Left-handed Scenario)

$\sqrt{s} = 91 \text{ GeV} \quad \gamma=1$



Green region:
Excluded by Z-peak
data at LEP-I

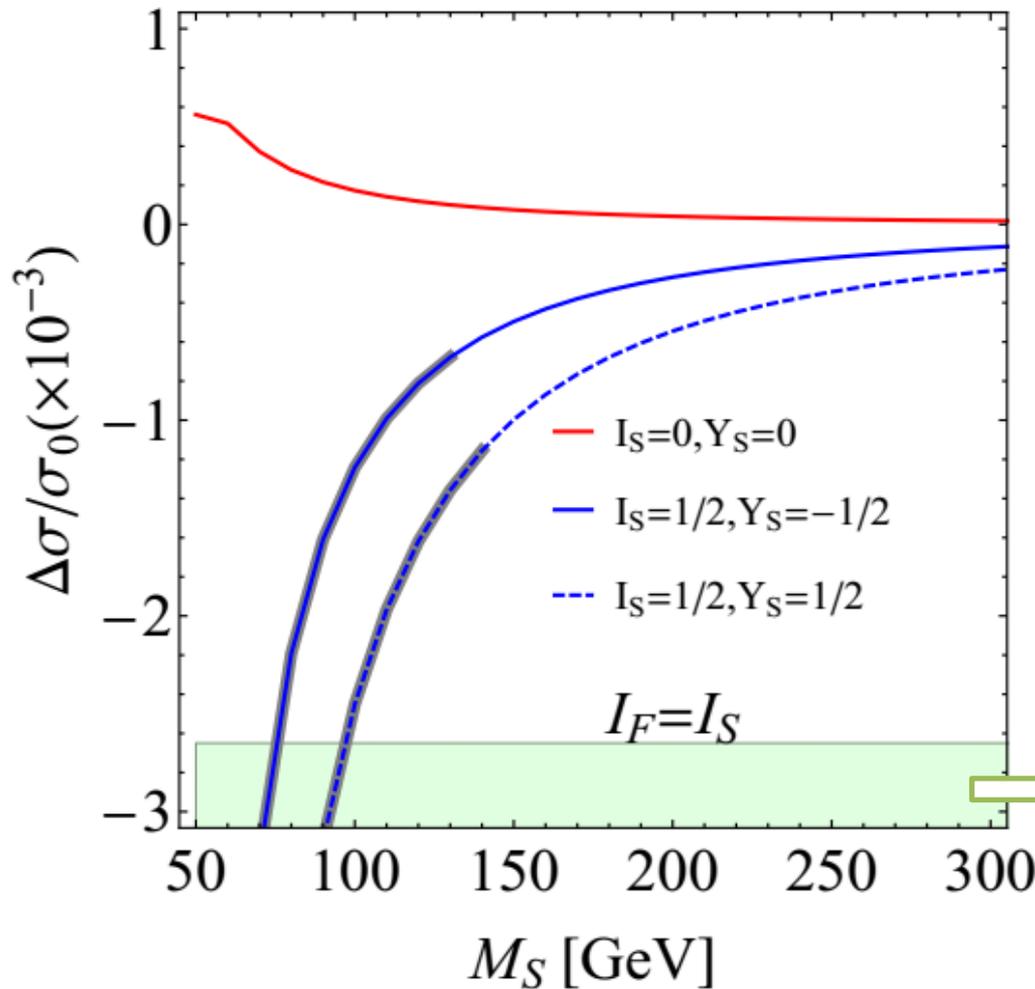
Gray shaded bands:
 excluded by Mono-jet + MET data

M_S [GeV]
 Statistical error: 0.5%
 Systematic error: 0.09%
 Physics Reports 427
 (2006)257-454

LEP-I constraint (Right-handed Scenario)

$$\sqrt{s} = 91 \text{ GeV}$$

$$y=1$$



Statistical error: 0.5%

Systematic error: 0.09%

Physics Reports 427
(2006)257-454

Green region :
Excluded by Z-peak
data at LEP-I

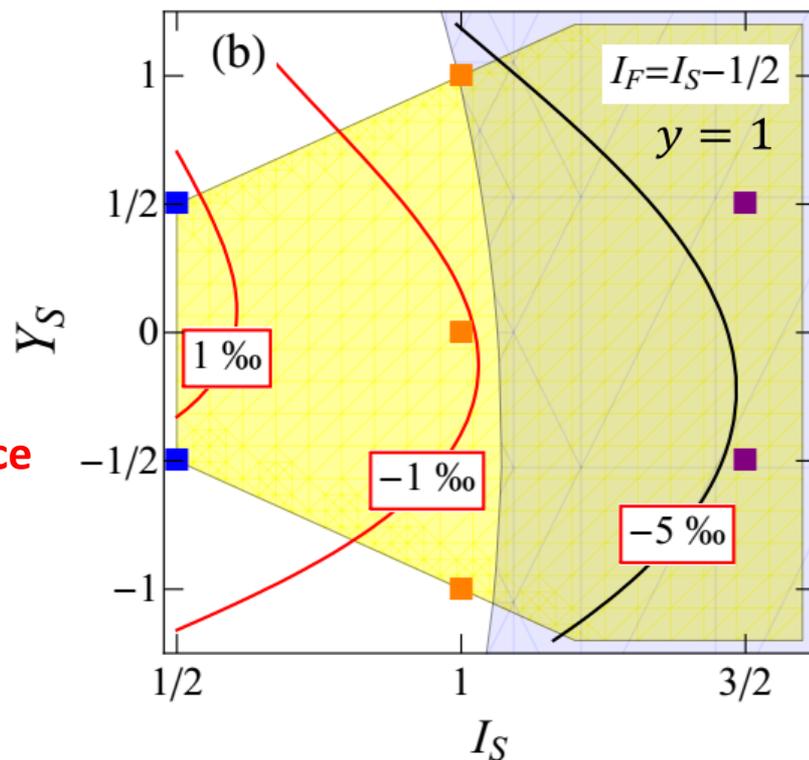
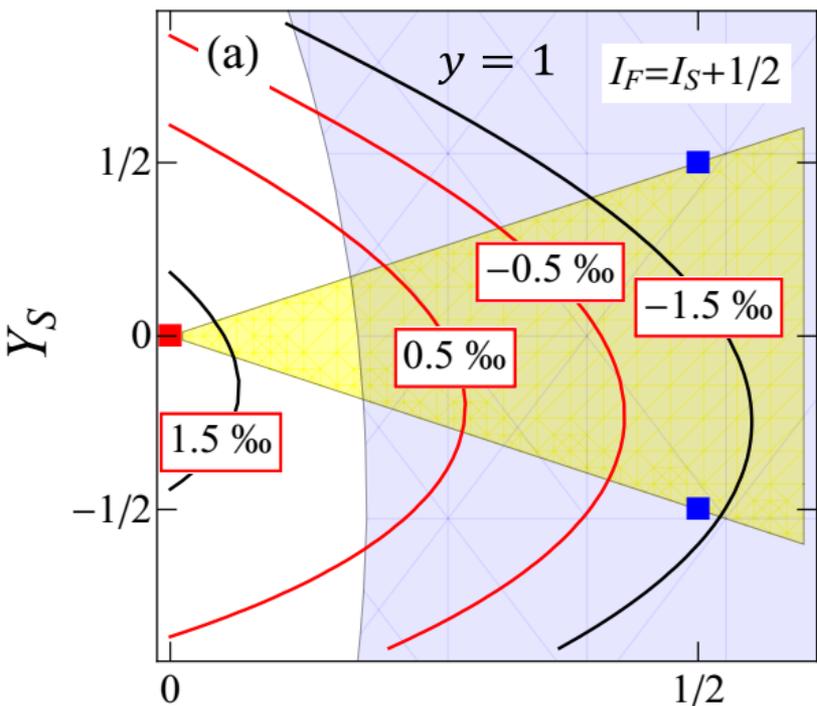
Gray shaded bands:

excluded by Mono-jet + MET data

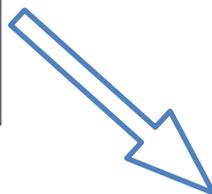
Left-Handed Scenario ($E_{\text{cm}}=240\text{GeV}$)

$M = 150 \text{ GeV}$

$\Delta\sigma/\sigma_0$ contours



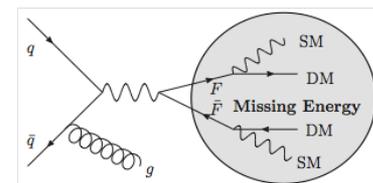
SM-NP interference



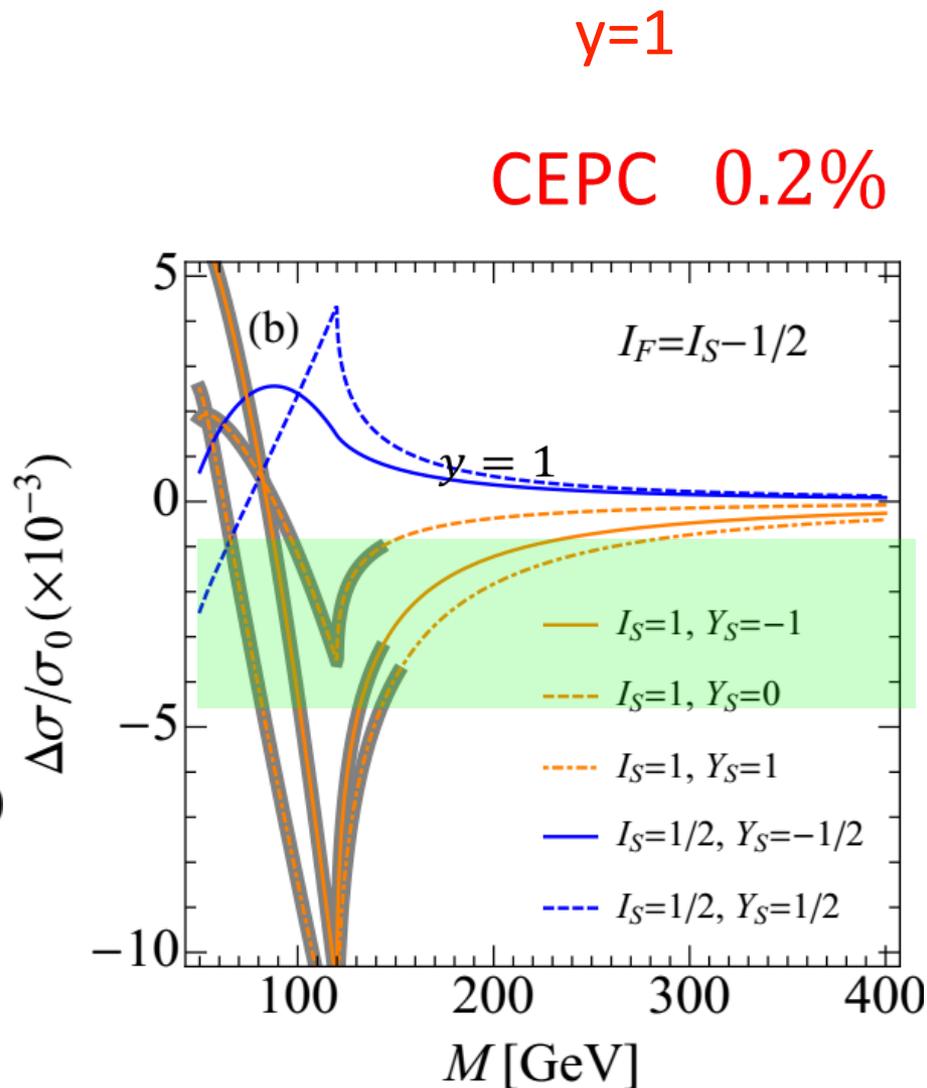
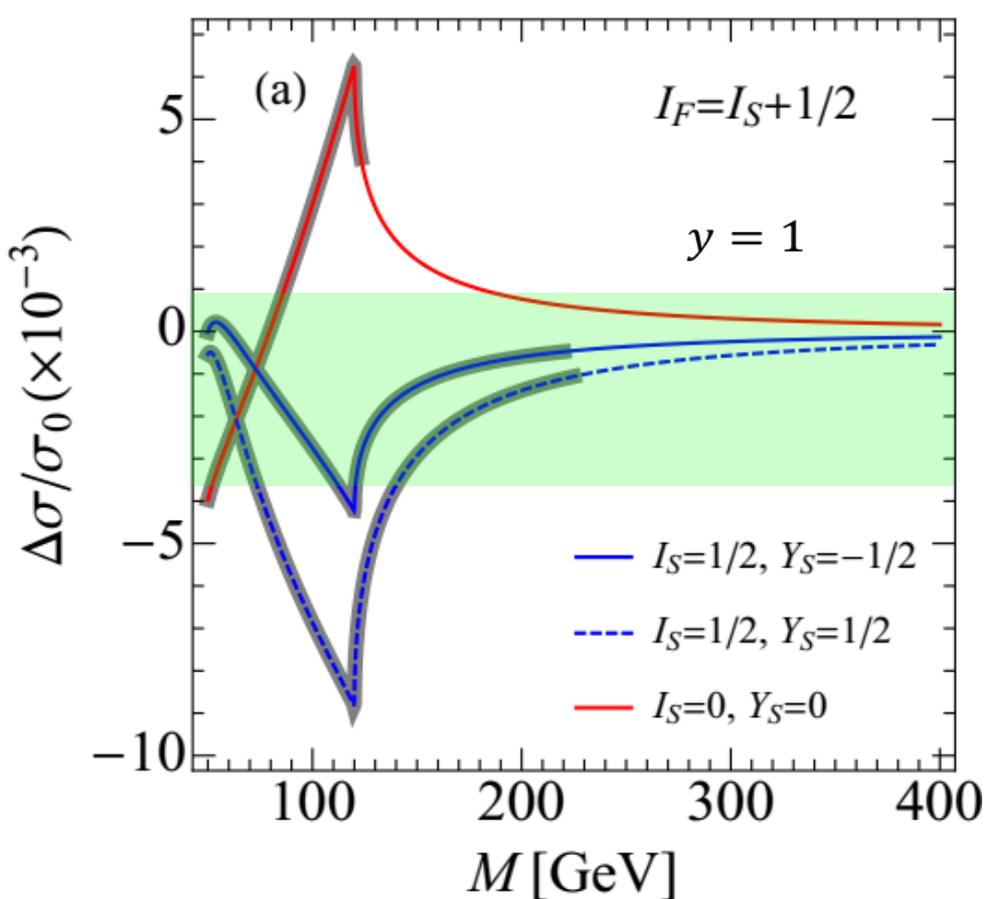
Yellow region: I_S
Representations
with DM candidates

Blue region :
**Higher Reprs excluded by
mono-jet + MET data**

$$Y_S \in \{-I_S, -I_S + 1, \dots, I_S\}$$



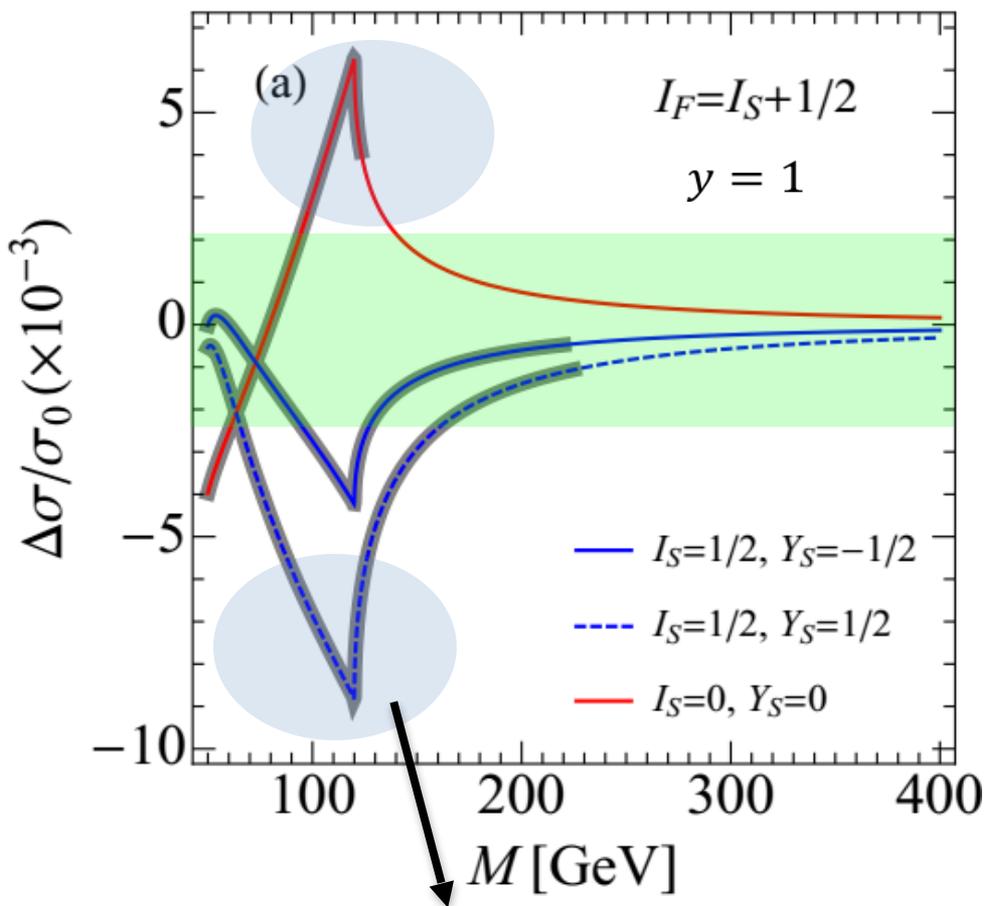
Left-Handed Scenario ($E_{\text{cm}}=240\text{GeV}$)



Gray shaded bands:

excluded by Mono-jet + MET data

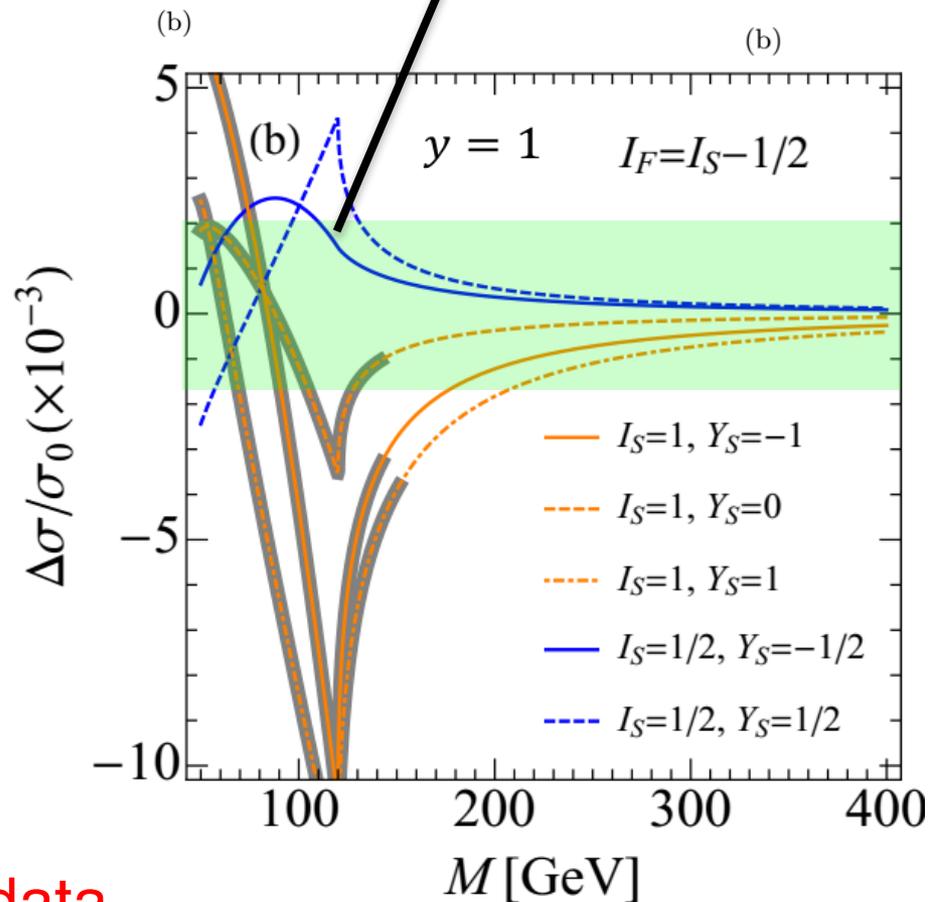
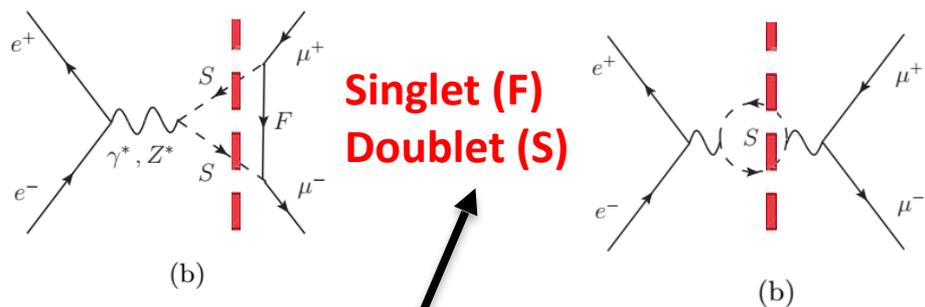
Left-Handed Scenario ($E_{\text{cm}}=240\text{GeV}$)



Threshold effect

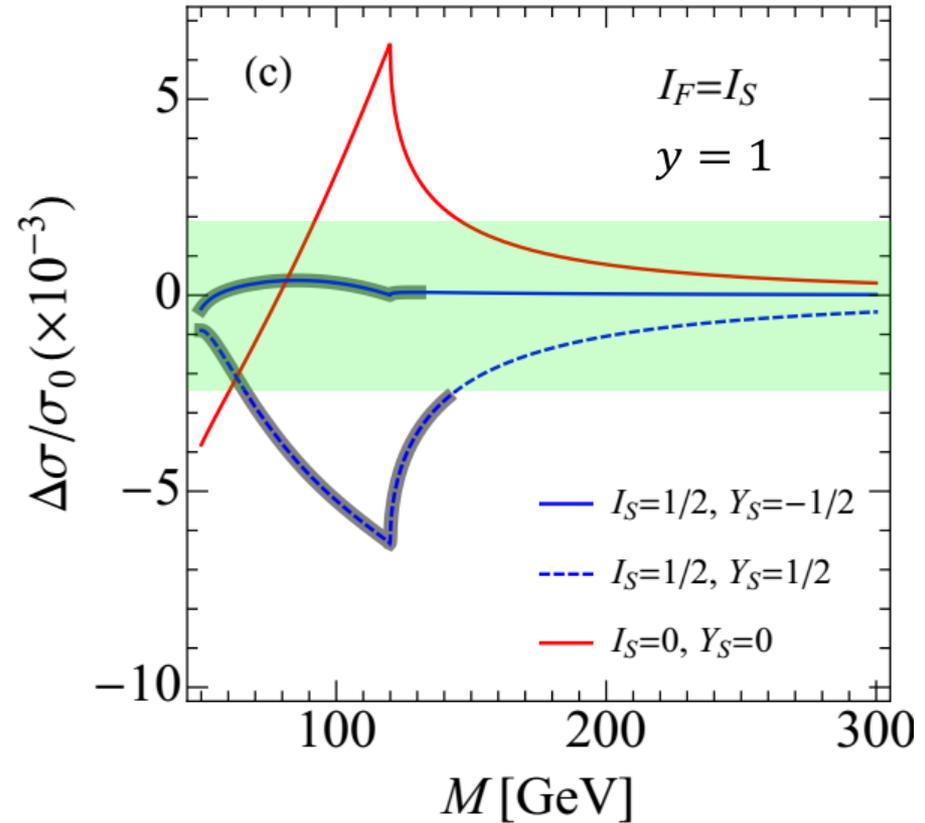
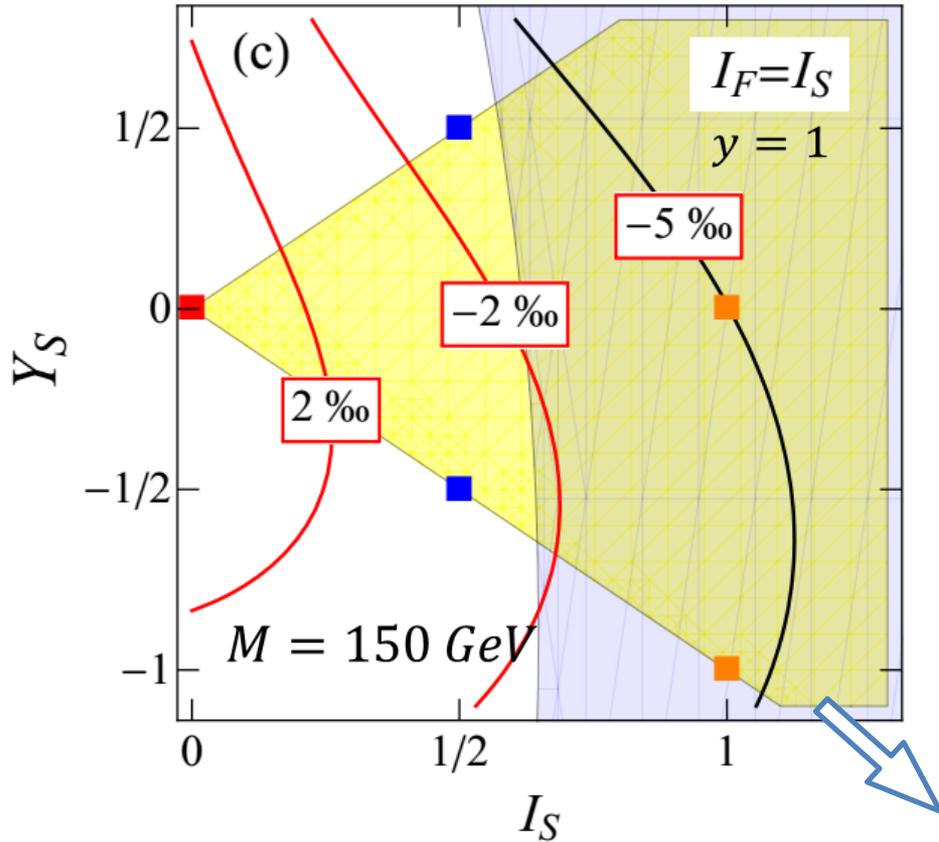
Gray shaded bands:

excluded by Mono-jet + MET data



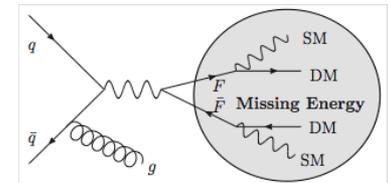
Right-Handed Scenario ($E_{\text{cm}}=240\text{GeV}$)

CEPC 0.2%



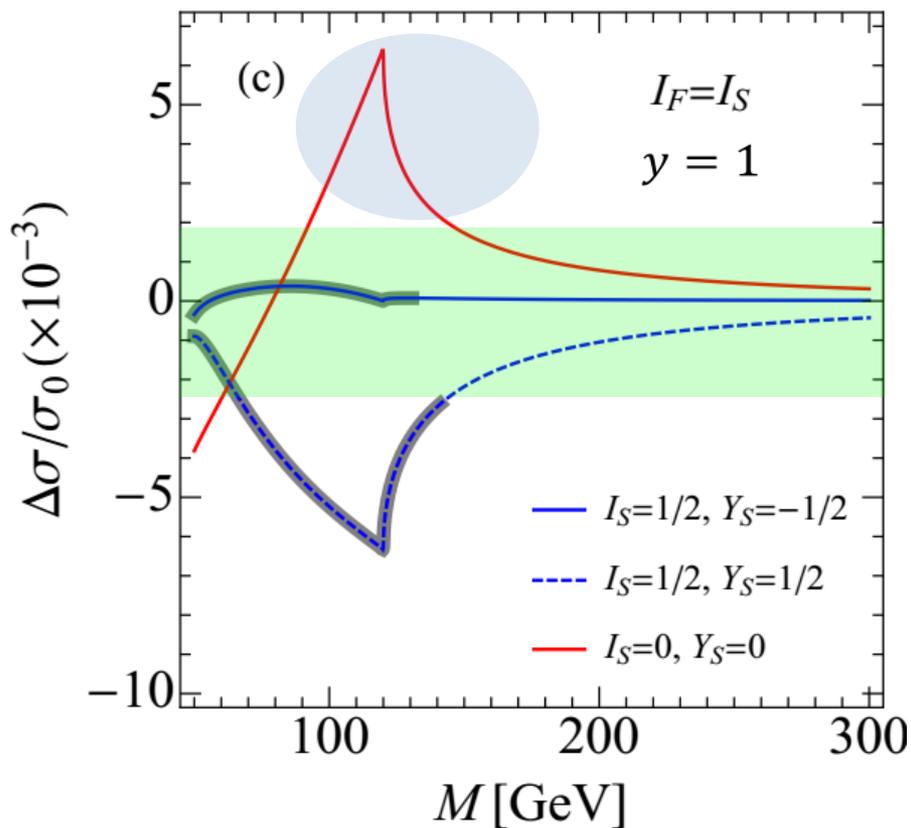
Yellow region:
Representations
with DM candidates

Blue region :
**Higher Reprs excluded by
mono-jet + MET data**

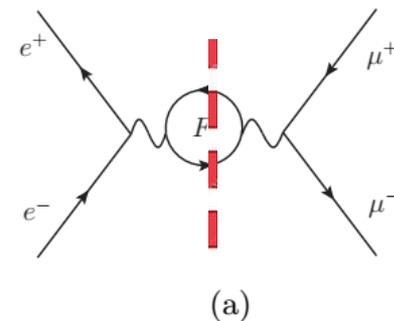
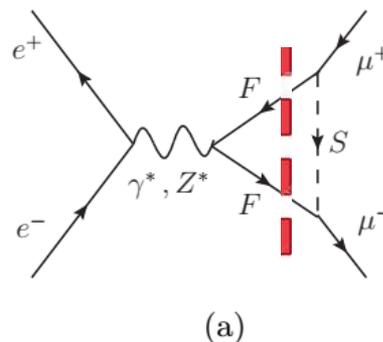


Right-Handed Scenario ($E_{\text{cm}}=240\text{GeV}$)

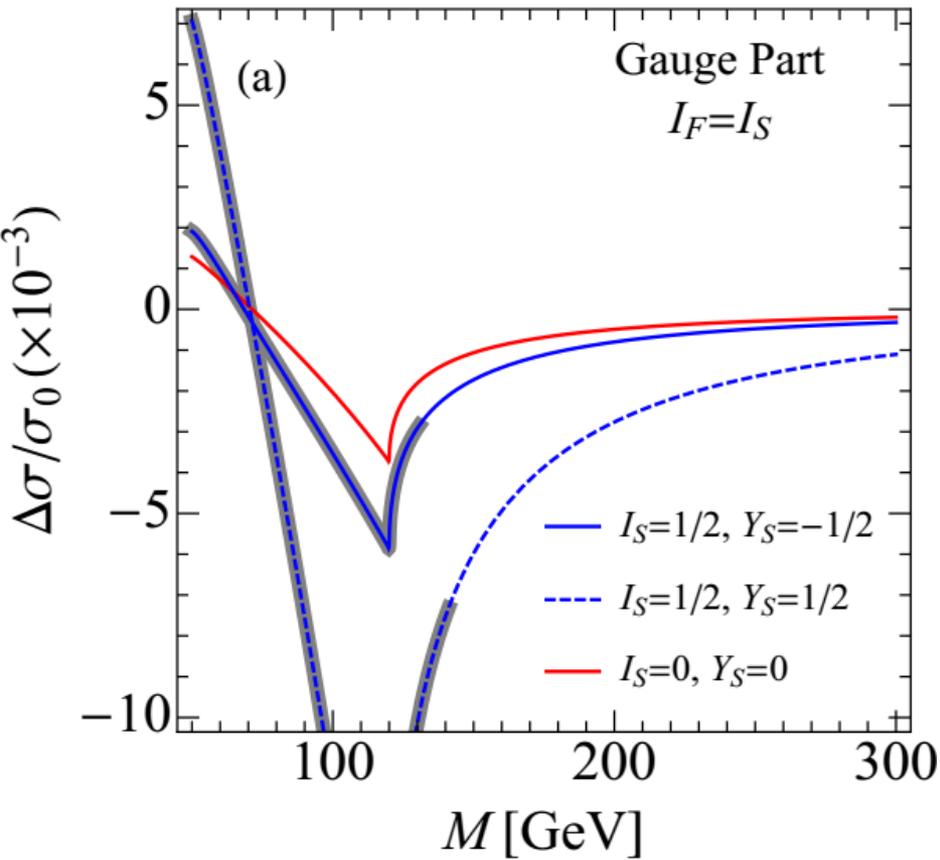
Threshold effects



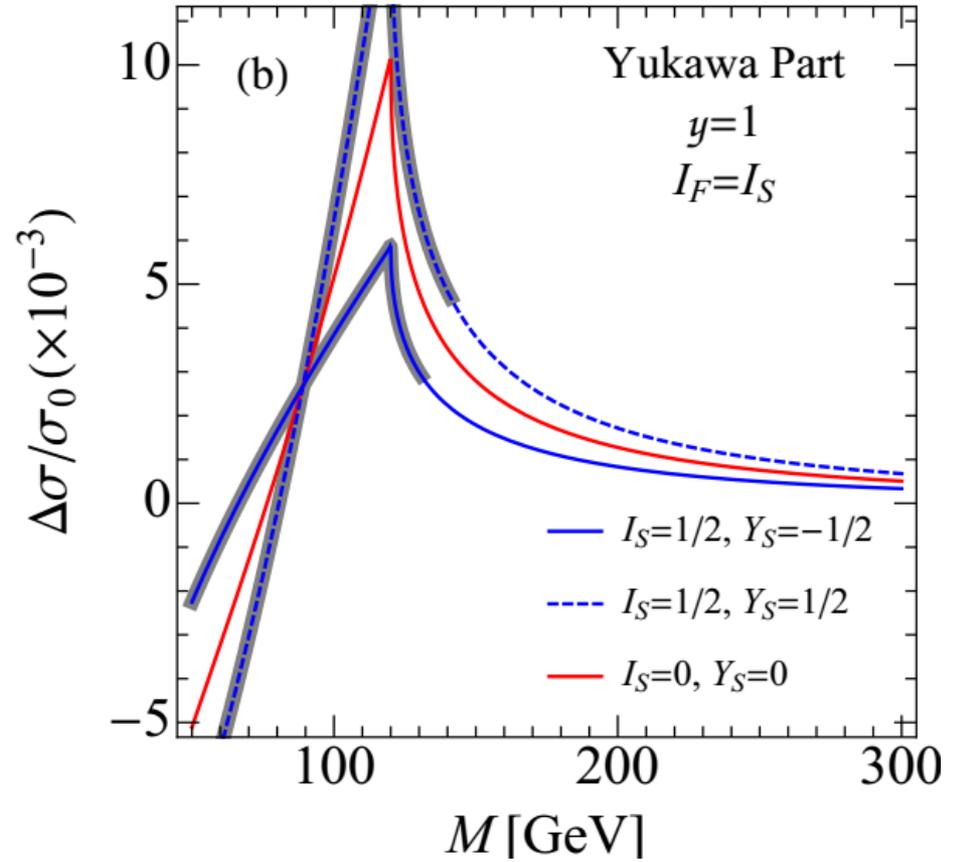
$M = 150 \text{ GeV}$



Right-Handed Scenario



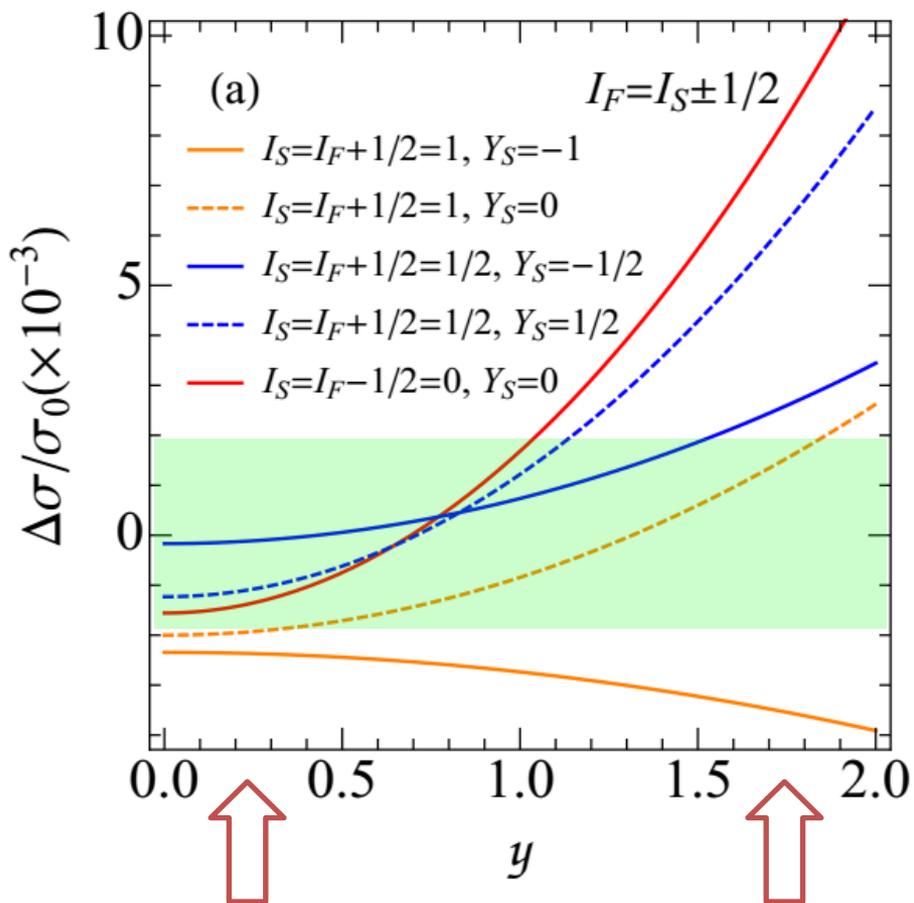
Negative



Positive

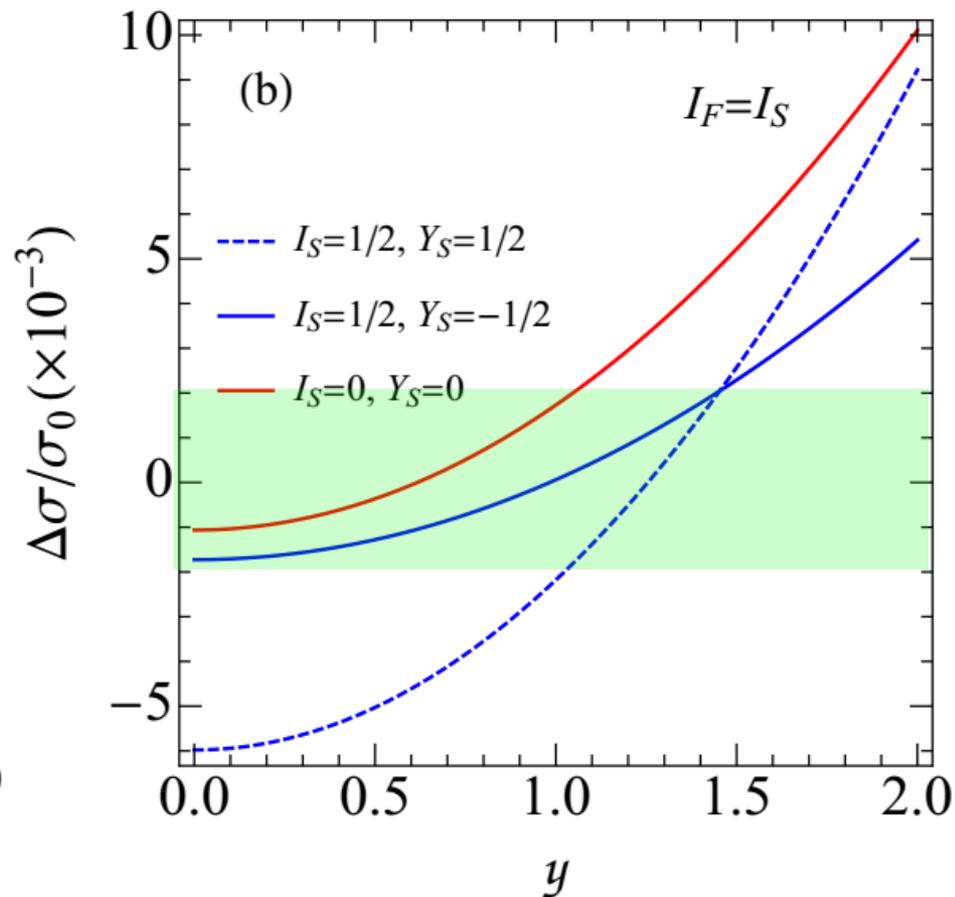
Large cancellation between gauge part and Yukawa part

Yukawa Coupling



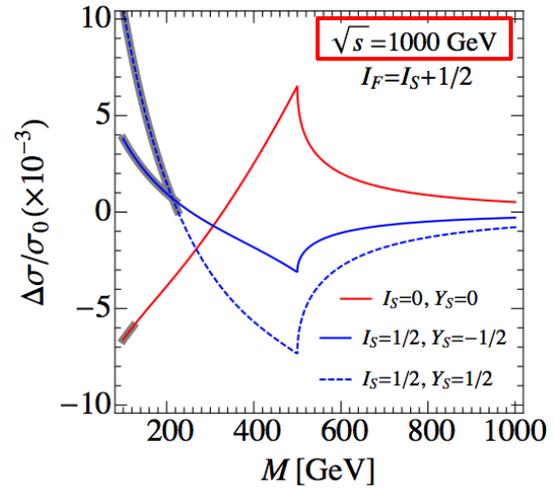
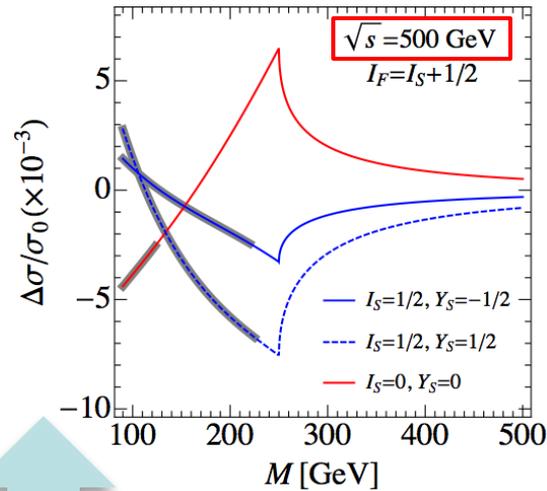
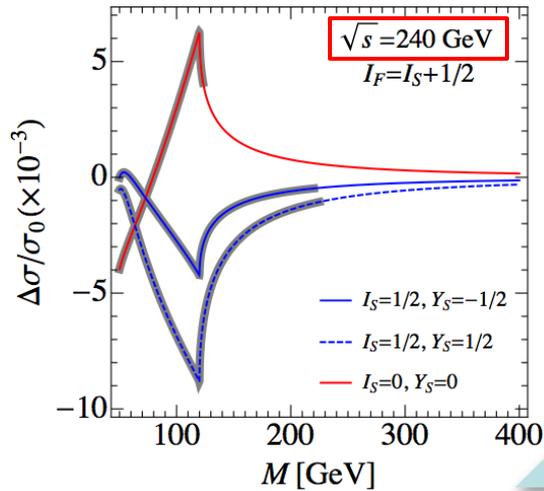
Gauge part
dominate

Yukawa part
dominate

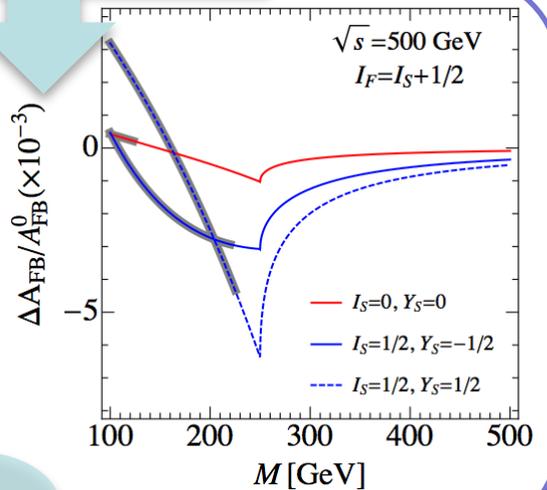
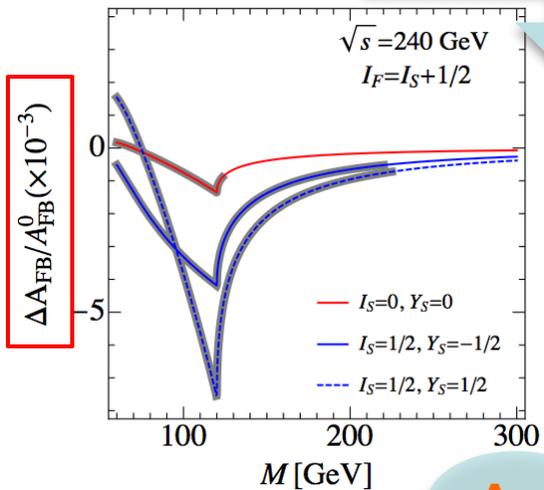


$M = 150 \text{ GeV}$

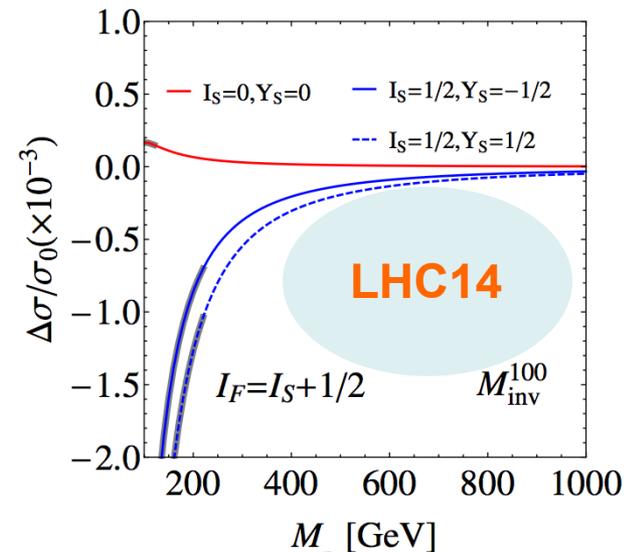
Other collision energies, A_{FB} , pp collider



e^+e^- collider

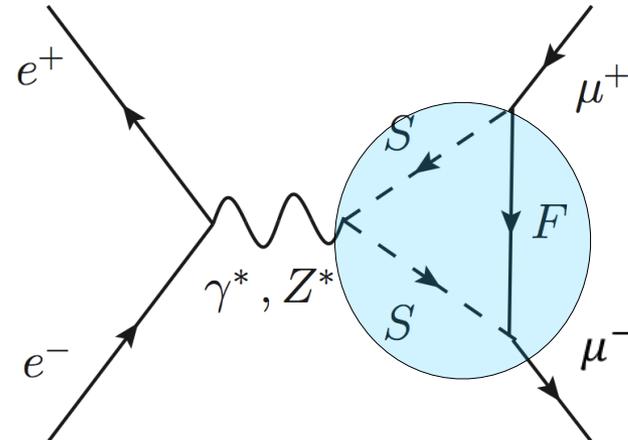
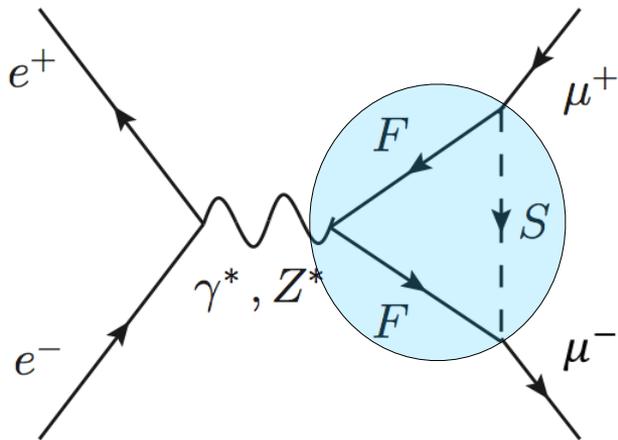
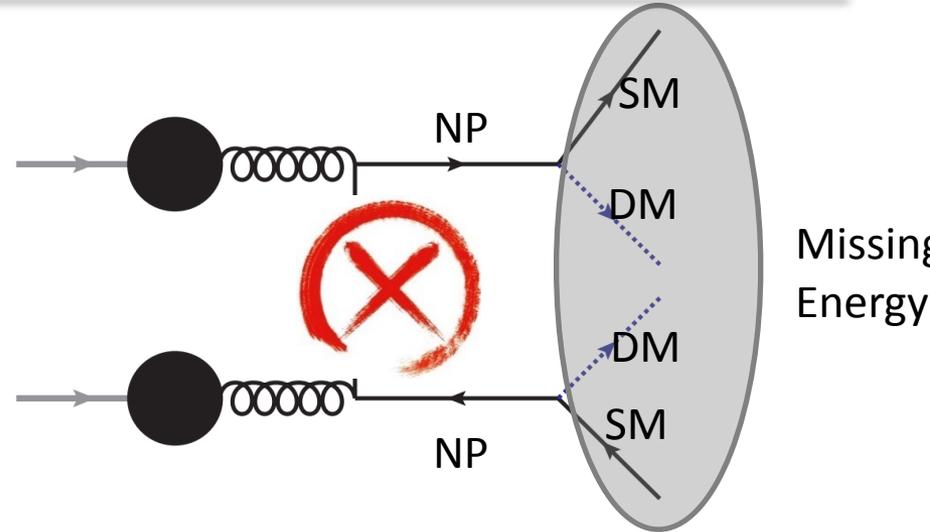
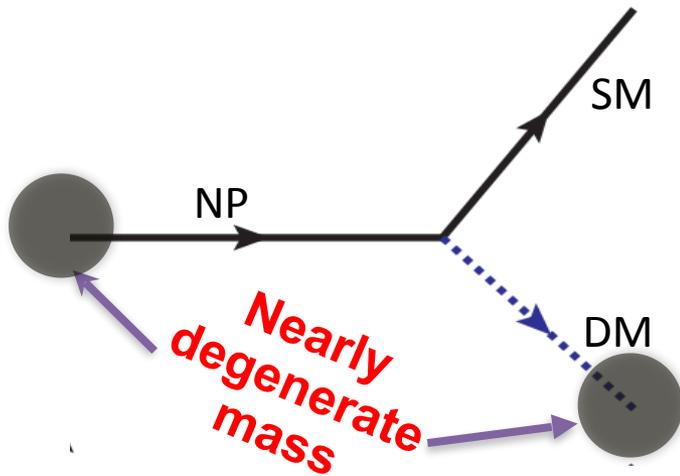


A_{FB}



Summary

Nearly degenerate DM models, which are hard to be probed directly at the LHC, can be tested at electron-positron colliders potentially.



Backup slides

$$M_{T2} \left(p_{vis}^{(1)}, p_{vis}^{(2)}, E_T; m_{inv} \right) = \min_{E_T = E_T^{(1)} + E_T^{(2)}} \left[\max \left\{ M_T \left(p_{vis}^{(1)}, E_T^{(1)}; m_{inv} \right), M_T \left(p_{vis}^{(2)}, E_T^{(2)}; m_{inv} \right) \right\} \right]$$

$$M_T \left(p_{vis}^{(i)}, E_T^{(i)}; m_{inv} \right) = \sqrt{m_{inv}^2 + m_{vis}^2 + 2 \left(E_T^{vis} E_T^{(i)} - \vec{p}_{vis}^i \vec{E}_T^i \right)}$$

$$J_{S3} = \begin{cases} \frac{1}{3} I_S, & \text{for } I_F = I_S + \frac{1}{2}, \\ -\frac{1}{3} (I_S + 1), & \text{for } I_F = I_S - \frac{1}{2}. \end{cases}$$

Input parameters

G_μ Scheme:

$$G_\mu = 1.1663787 \times 10^{-5} \text{GeV}^{-2}$$

$$\alpha_{EM}(0) = 1/137.035999138$$

$$m_Z = 91.1876 \text{GeV}$$

$$m_W = 80.385 \text{GeV}$$

$$m_\mu = 105.6583745 \text{MeV}$$

$$c_W = m_W/m_Z$$

Renormalization Constant

Yukawa interaction:

$$\begin{aligned}\delta\alpha_\gamma &= -\frac{1}{2}Q \left(\delta Z_\mu^R + \delta Z_\mu^L \right), & \delta\alpha_Z &= \frac{1}{2} \left(g_R \delta Z_\mu^R + g_L \delta Z_\mu^L \right), \\ \delta\xi_{1,\gamma} &= -\frac{1}{2}Q \left(\delta Z_\mu^R - \delta Z_\mu^L \right), & \delta\xi_{1,Z} &= \frac{1}{2} \left(g_R \delta Z_\mu^R - g_L \delta Z_\mu^L \right),\end{aligned}$$

$$\Sigma(\not{p}) = \not{p} [\Sigma_L(p^2) P_L + \Sigma_R(p^2) P_R] + m_\mu \Sigma_S(p^2),$$

$$\delta Z_\mu^{L,R} = -m_\mu^2 \frac{\partial}{\partial p^2} \Re [\Sigma_L(p^2) + \Sigma_R(p^2) + 2\Sigma_S(p^2)] \Big|_{p^2=m_\mu^2} - \Re \Sigma_{L,R}(m_\mu^2).$$

Gauge interaction:

$$C_{L/R}^\gamma = -Q \left(\frac{1}{2} \delta Z_{AA} + \delta Z_e \right) + g_{L/R} \frac{1}{2} \delta Z_{ZA}, \quad C_{L/R}^Z = g_{L/R} \left(\frac{\delta g_{L/R}}{g_{L/R}} + \frac{1}{2} \delta Z_{ZZ} \right) - Q \frac{1}{2} \delta Z_{AZ},$$

$$C_{AA} = s \delta Z_{AA}, \quad C_{AZ} = \delta Z_{ZA} (s - m_Z^2) + s \delta Z_{AZ}, \quad C_{ZZ} = \delta Z_{ZZ} (s - m_Z^2) - \delta m_Z^2,$$

$$C'_{AZ} = \frac{1}{2} (\delta Z_{AZ} + \delta Z_{ZA}), \quad C'_{ZZ} = \delta Z_{ZZ},$$

$$\delta g_L = \frac{T^3}{-c_W s_W} \left[\frac{\delta s_W (s_W^2 - c_W^2)}{c_W^2 s_W} + \delta Z_e \right] + \delta g_R, \quad \delta g_R = \frac{s_W}{c_W} Q \left[\frac{\delta s_W}{c_W^2 s_W} + \delta Z_e \right],$$

Feynman Rules

Appendix A: Feynman rules

The Feynman rules for the Yukawa couplings in Eq. 3 and Eq. 13 are displayed in Fig. 7 (a) and (b), respectively. The CG coefficients are given explicitly as follows,

$$C_{ij-\frac{1}{2}} = \begin{cases} (-1)^{I_S-i} \sqrt{\frac{I_S+i+1}{2I_S^2+3I_S+1}} \delta_{i+j,-\frac{1}{2}}, & \text{for } I_F = I_S + \frac{1}{2}, \\ (-1)^{I_S-i-1} \sqrt{\frac{I_S-i}{I_S(2I_S+1)}} \delta_{i+j,-\frac{1}{2}}, & \text{for } I_F = I_S - \frac{1}{2}, \end{cases} \quad (\text{A1})$$

and

$$C_{ij} = (-1)^{I_S-i} \frac{1}{\sqrt{2I_S+1}} \delta_{i+j,0} \quad . \quad (\text{A2})$$

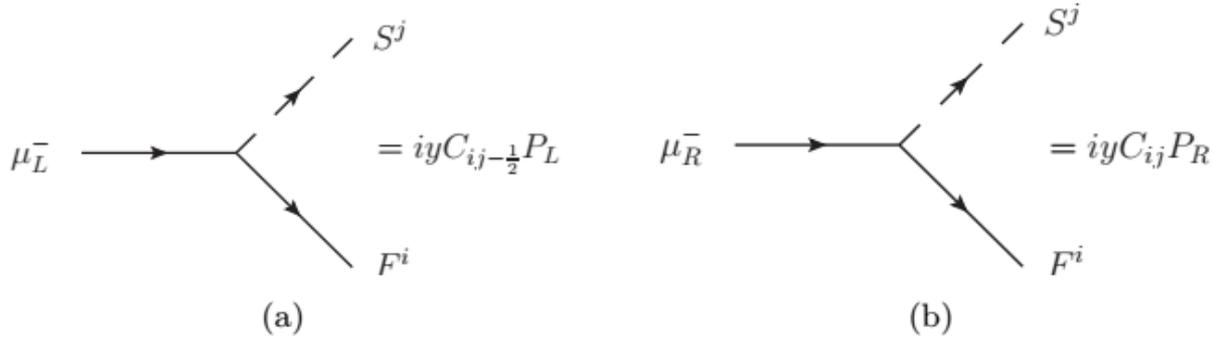


FIG. 7: Feynman rules of the $S\bar{\mu}_L F$ (a) and $S\bar{\mu}_R F$ (b) couplings in Eq. 3 and 13 respectively.