High order calculation for quarkonium production

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第二届中国高精度高能物理学术研讨会

IHEP, Beijing, June 16th, 2016

Outline

I. Introduction

II. Momentum expansion

III. Loop integration reduction

IV. Summary

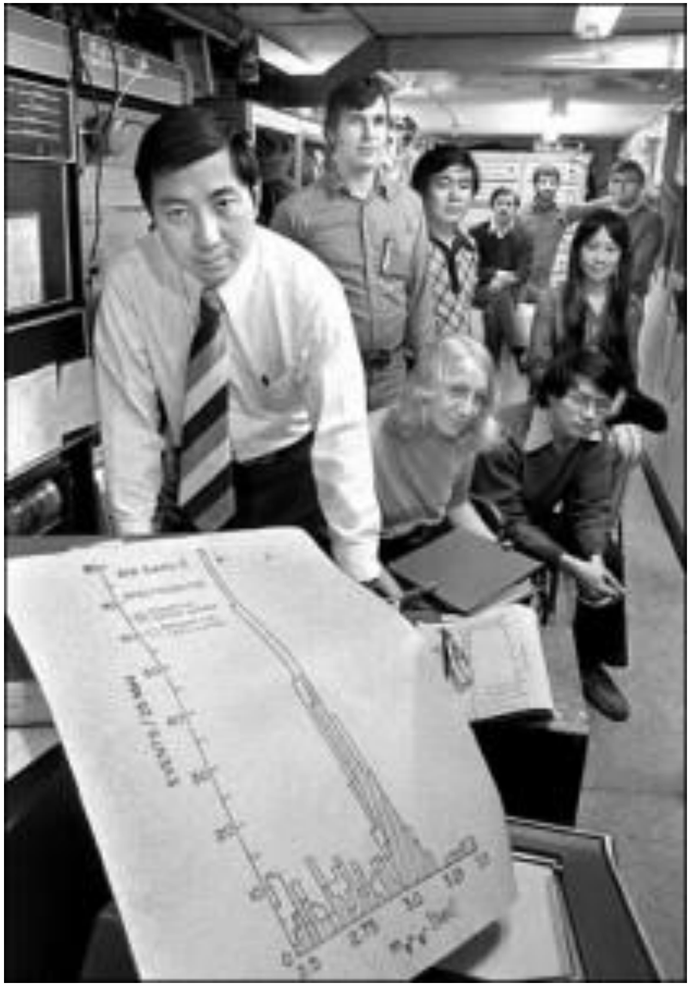
Discovery of the J/ψ: J particle

- **GIM mechanism and charm quark**

  To suppress FCNC process, Glashow–Iliopoulos–Maiani mechanism required the existence of a fourth quark

- **J particle discovered at BNL**

  - In $p + Be \rightarrow e^{+} + e^{-} + X$
  - 3.1 GeV, about three times heavier than the proton
  - With $J^{PC} = 1^{--}$

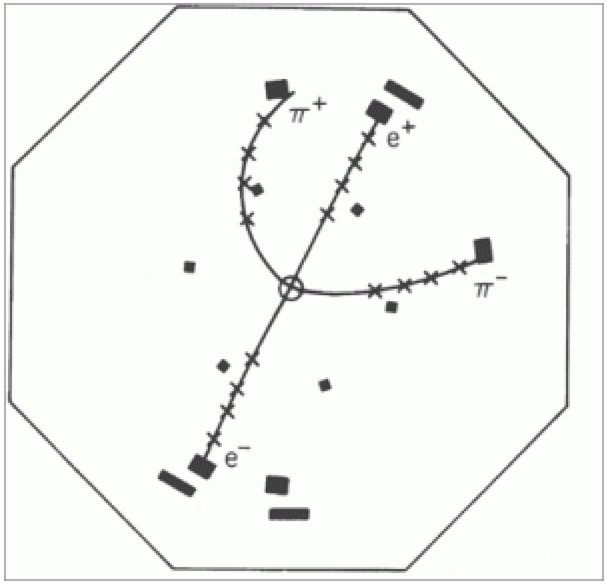


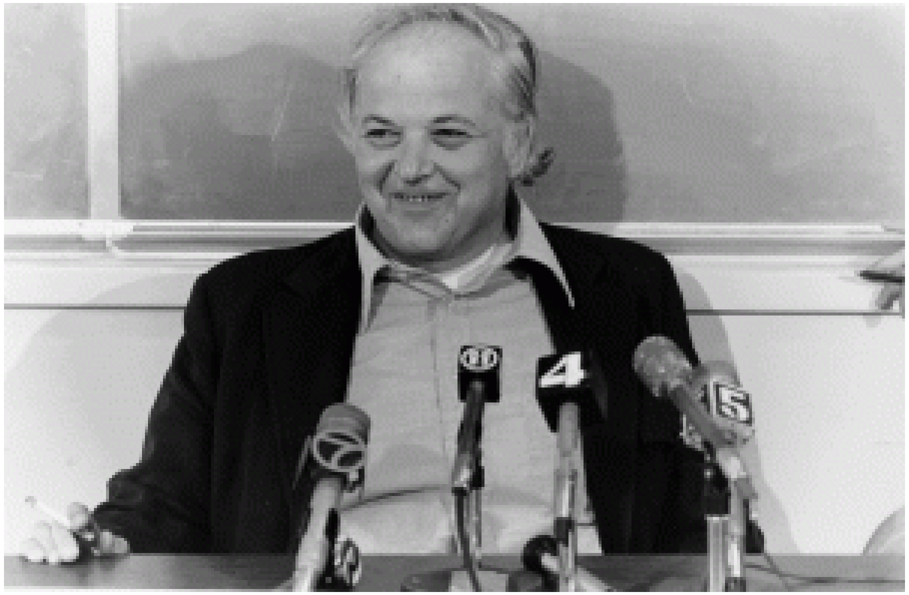
Samuel Ting and his BNL team.
Nobel Prize in 1976

Discovery of the J/ψ: ψ particle

- ψ **particle discovered at SLAC**

In $e^+ + e^- \to \pi^+ + \pi^-$



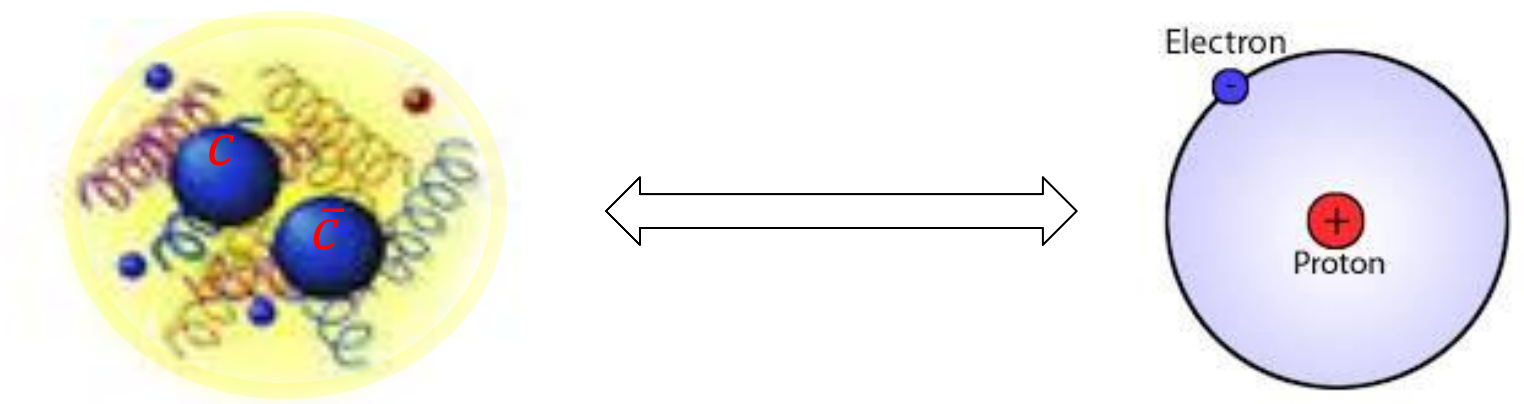


Burton Richter following the announcement of co-winning the 1976 Nobel Prize.

Heavy quarkonium

- **Bound state of $Q\bar{Q}$ pair under strong interaction**

Eg: $J/\psi\ \ \psi', \chi_{cJ}, \Upsilon(nS), \chi_{bJ}(nP) \cdots$



- ✓ **The simplest system in QCD: two-body problem**
- ✓ **“Hydrogen atom in QCD”, “an ideal laboratory in QCD”**

Velocity of heavy quarks

- **Coulomb potential between color singlet heavy quark pair:** $V(r) = -C_F \frac{\alpha_s(1/r)}{r}$
- **Virial theorem:** $mv^2 \sim V(r) \sim \frac{\alpha_s(1/r)}{r}$
- **Uncertainty principle:** $r \sim \frac{1}{mv}$
- **Velocity is determined by quark mass**

$$\alpha_s(mv) \sim mv^2\, r \sim v$$

Property

- **A non-relativistic QCD system: $v^2 \ll 1$**

  Charmonium: $\mathrm{m}{\sim}1.3\mathrm{Ge}V,\ v^2 \approx 0.3$

  Bottomonium: $m{\sim}4.5GeV,\ v^2 \approx 0.1$

- **Multiple well-separated scales :**

  Quark mass: M
  Momentum: Mv
  Energy: Mv^2

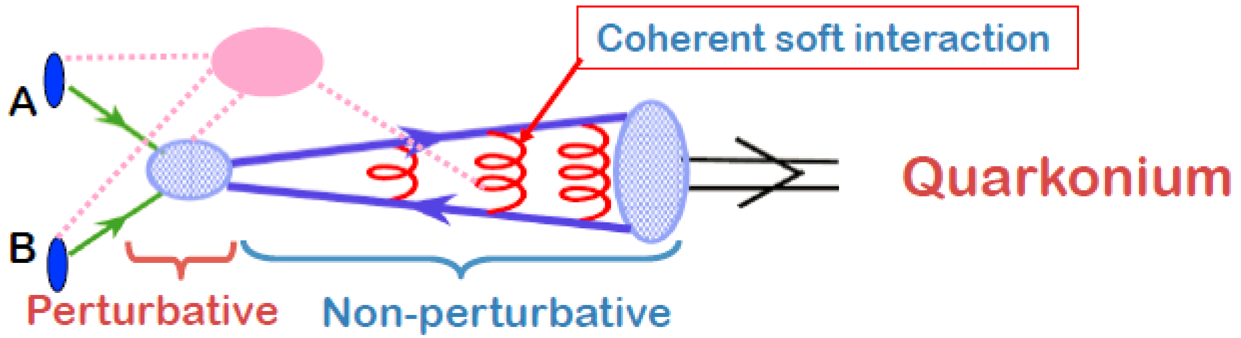
  $M \gg Mv \gg Mv^2 \sim \Lambda_{QCD}$

- **Involving both perturbative and nonperturbative physics**

- **Production: ideal to understand hadronization, to study QGP**

Space-time picture for production

- **Hadronization followed by production of an off-shell heavy quark pair**



$\Delta r \sim \frac{1}{2m}, \quad \frac{1}{m\,v}, \quad \frac{1}{m\,v^2}$

- Time scale for producing heavy quark pair: $\frac{1}{2\,m}$
- Time scale for expansion: $\frac{1}{m\,v}$
- Time scale for forming bound state: $\frac{1}{m\,v^2}$

Approximation

➢ **On-shell pair + hadronization**

$$\sigma_{AB\to H+X} = \sum_n \int_n d\Gamma_{(Q\bar{Q})_n} \left[\frac{d\hat{\sigma}(Q^2)}{d\Gamma_{(Q\bar{Q})_n}}\right] F_{(Q\bar{Q})_n\to H}(p_Q, p_{\bar{Q}}, P_H)$$

- **Corrections are at higher order in v**
- **Different assumptions/treatments on how the heavy quark pair becomes a heavy quarkonium: different factorization models**

Historical theories for quarkonium production

1. 1974 - Discovery of J/ψ, CSM and CEM

CSM: IR divergence, ψ' surplus — Einhorn, Ellis (1975), Chang (1980), Berger, Jone (1981), …

CEM: wrong for ratio — Fritzsch (1977), Halzen (1977), …

2. 1994 - NRQCD

Bodwin, Braaten, Lepage, 9407339, …

No divergence up to now, solving many puzzles

Plain NRQCD fails when $p_T \gg M$ or $p_T \ll M$, leak all order proof

3. 2014 -

High p_T: collinear factorization — Kang, Qiu, Sterman, 1109.1520; Fleming, Leibovich, Mehen, Rothstein 1207.2578; Kang, YQM, Qiu, Sterman, 1401.0923, …

Low p_T: CGC+NRQCD — Kang, YQM, Venugopalan, 1309.7337; Qiu, Sun, Xiao, Yuan, 1310.2230, …

………..: ?????

NRQCD Factorization

Bodwin, Braaten, Lepage, 9407339

➢ Factorization formula

$$d\sigma_\psi = \sum_{i,j,n} \underbrace{\int dx_1 dx_2 \, G_{i/A} G_{j/B}}_{\Lambda_{QCD}} \times \underbrace{\hat{\sigma}[ij \to c\bar{c}[n] + X]}_{m_c} \times \underbrace{\langle O_n^\psi \rangle}_{m_c v}$$

Parton distribution function

Hadronization (LDMEs)

Production of heavy quark pair

- n: quantum numbers of the pair, spectroscopic notation ${}^{2S+1}L_J^{[c]}$.
- Color, spin, orbital angular momentum, total angular momentum

Definition of hard part

- $\hat{\sigma}(c\bar{c}[n])$: **production of $c\bar{c}$ with quantum number n**
- **Expansion of relative momentum at origin**

Momenta of the pair: $p_c = p + q, p_{\bar{c}} = p - q$

$$M^{\kappa,J_z,(1,8c)}(p) = \sqrt{\frac{1}{m}} \sum_{L_z,S_z} \sum_{s,\bar{s}} \sum_{i,\bar{i}} \langle LL_z; SS_z | JJ_z \rangle \left\langle \frac{1}{2}s; \frac{1}{2}\bar{s} | SS_z \right\rangle \langle 3i; \bar{3}\bar{i} | (1,8c) \rangle$$

$$\times \begin{cases} M^F_{s\bar{s};i\bar{i}}(p,0), & \text{if } \kappa \text{ is } S\text{-wave}, \\ \epsilon^*_\beta(L_z) M^{F,\beta}_{s\bar{s};i\bar{i}}(p,0), & \text{if } \kappa \text{ is } P\text{-wave}, \end{cases}$$

$$M^{F,\beta}_{s\bar{s};i\bar{i}}(p,0) = \frac{\partial}{\partial q^\beta} M^F_{s\bar{s};i\bar{i}}(p,q)\Big|_{q=0}$$

- **Subtraction of IR divergence based factorization is needed (no discussed in the following)**

Differences between traditional LI

- **Complicated pole structure**
  - Dependence of external momentum
  - High power of propagator denominators
  - Coulomb singularies: linear divergence
  - Light cone singularities for fragmentation functions

- **No efficient numerical method available**

Momentum expansion VS loop integration

- **Loop integration:**

$$\frac{d}{dq^\alpha}\int d^4l\,\frac{1}{l^2[(l+p+q)^2-m^2][(l-p+q)^2-m^2]}$$

- **Expansion after loop integration: correct, but complicated**
- **Expansion before loop integration: correct only for** $l \gg q$

The method of region

Beneke, Smirnov, 9711391

- Find out all relevant regions of loop integration

- Doing allowed expansion in each region

  - Dimensional regularization is crucial!

Example

Ma, Qiu, Zhang, 1401.0524

- **With $\beta \ll 1$**

To perform the y integration, we introduce a parameter $\Lambda \gg \beta$ and rewrite the y integration as

$$\int_{-1}^{1} \frac{y^{2k}\mathrm{d}y}{(y^2-\beta^2-i\varepsilon)^{1+\epsilon}} = \left(\int_{-1}^{-\Lambda} + \int_{\Lambda}^{1}\right) \frac{y^{2k}\mathrm{d}y}{(y^2-\beta^2-i\varepsilon)^{1+\epsilon}} + \int_{-\Lambda}^{\Lambda} \frac{y^{2k}\mathrm{d}y}{(y^2-\beta^2-i\varepsilon)^{1+\epsilon}}. \quad \text{(A13)}$$

Since $y^2 \geq \Lambda^2 \gg \beta^2$ in the first term above, we can expand β^2 before performing the y integration and obtain

$$\frac{y^{2k}}{(y^2-\beta^2-i\varepsilon)^{1+\epsilon}} = \frac{y^{2k}}{y^{2-2\epsilon}} + (1+\epsilon)\frac{y^{2k}}{y^{4+2\epsilon}}\beta^2 + \cdots \equiv E_k(y^2)$$

Example cont.

Ma, Qiu, Zhang, 1401.0524

$$\int_{-1}^{1} \frac{y^{2k}\mathrm{d}y}{(y^2-\beta^2-i\varepsilon)^{1+\epsilon}} = \left(\int_{-1}^{-\Lambda} + \int_{\Lambda}^{1}\right) E_k(y^2)\mathrm{d}y + \int_{-\Lambda}^{\Lambda} \frac{y^{2k}\mathrm{d}y}{(y^2-\beta^2-i\varepsilon)^{1+\epsilon}}.$$

This identity can also be written as

$$\int_{-1}^{1} \frac{y^{2k}\mathrm{d}y}{(y^2-\beta^2-i\varepsilon)^{1+\epsilon}} - \int_{-1}^{1} E_k(y^2)\mathrm{d}y = \int_{-\Lambda}^{\Lambda} \frac{y^{2k}\mathrm{d}y}{(y^2-\beta^2-i\varepsilon)^{1+\epsilon}} - \int_{-\Lambda}^{\Lambda} E_k(y^2)\mathrm{d}y.$$

Example cont.

Ma, Qiu, Zhang, 1401.0524

$$\int_{-1}^{1}\frac{y^{2k}\mathrm{d}y}{(y^2-\beta^2-i\varepsilon)^{1+\epsilon}}-\int_{-1}^{1}E_k(y^2)\mathrm{d}y$$

Hard region: $|y| \gg \beta$

$$=\int_{-\infty}^{+\infty}\frac{y^{2k}\mathrm{d}y}{(y^2-\beta^2-i\varepsilon)^{1+\epsilon}}$$

$$=\beta^{2k-1-2\epsilon}\int_{-\infty}^{+\infty}\frac{y^{2k}\mathrm{d}y}{(y^2-1-i\varepsilon)^{1+\epsilon}}. \qquad (A17)$$

Soft region: $|y| \sim \beta$

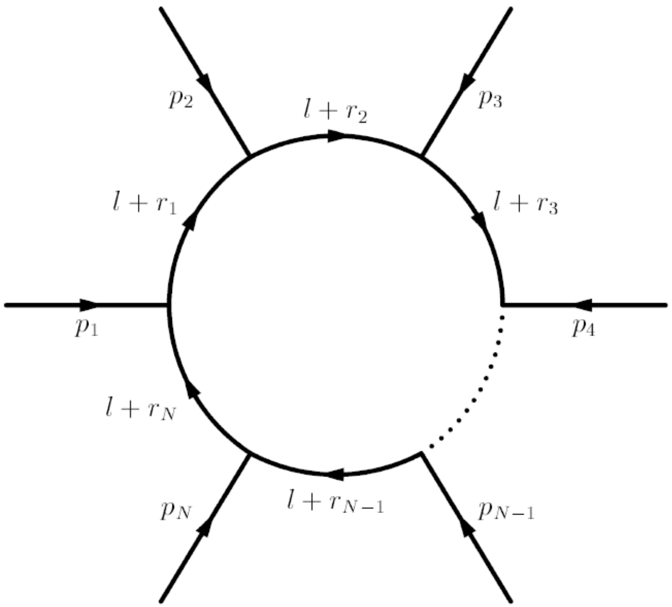
In deriving the above simplified identify, we used

$$\int_{-\infty}^{+\infty}E_k(y^2)\mathrm{d}y=\int_{-\infty}^{+\infty}\left[\frac{y^{2k}}{y^{2+2\epsilon}}+(1+\epsilon)\frac{y^{2k}}{y^{4+2\epsilon}}\beta^2+\cdots\right]\mathrm{d}y=0. \quad (A18)$$

- Hard region, soft region, potential region, usoft region
- Only hard region is needed. All other regions can be subtracted by factorization

Tensor integral to scalar integral

$$I^N_{\mu_1\cdots\mu_P}(D;\{\nu_i\}) \equiv (\mu^2)^{2-D/2}\int\frac{\mathrm{d}^D l}{(2\pi)^D}\frac{l_{\mu_1}\cdots l_{\mu_P}}{A_1^{\nu_1}A_2^{\nu_2}\cdots A_N^{\nu_N}}$$



$$I^N_{\mu_1\cdots\mu_P}(D;\{\nu_i\}) = \sum_{\substack{k,j_1,\cdots,j_N\geq 0\\ 2k+\Sigma j_i=P}}\left\{[g]^k[r_1]^{j_1}\cdots[r_N]^{j_N}\right\}_{\mu_1\cdots\mu_P}$$

$$\times\frac{(4\pi\mu^2)^{P-k}}{(-2)^k}\left[\prod_{i=1}^N\frac{\Gamma(\nu_i+j_i)}{\Gamma(\nu_i)}\right]I_0^N(D+2(P-k);\{\nu_i+j_i\})$$

Davydychev, PLB (1991)

Integrate by part

Duplancic, Nizic, 0303184

$$0 \equiv \int \frac{d^D l}{(2\pi)^D} \frac{\partial}{\partial l^\mu} \left(\frac{z_0 l^\mu + \sum_{i=1}^N z_i r_i^\mu}{A_1^{\nu_1} \cdots A_N^{\nu_N}} \right)$$

1. $\det(S_N) \neq 0, \det(R_N) \neq 0$
2. $\det(S_N) \neq 0, \det(R_N) = 0$
3. $\det(S_N) = 0, \det(R_N) \neq 0$
4. $\det(S_N) = 0, \det(R_N) = 0$

$$S_N \quad \begin{pmatrix} 0 & 1 & 1 & \cdots & 1 \\ 1 & 0 & r_{12} & \cdots & r_{1N} \\ 1 & r_{12} & 0 & \cdots & r_{2N} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & r_{1N} & r_{2N} & \cdots & 0 \end{pmatrix}$$

$$R_N \quad \begin{pmatrix} 0 & r_{12} & \cdots & r_{1N} \\ r_{12} & 0 & \cdots & r_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ r_{1N} & r_{2N} & \cdots & 0 \end{pmatrix}$$

E.g. for case 1:

$$I_0^N(D;\{\nu_k\}) = \frac{1}{4\pi\mu^2(D-1-\sum_{j=1}^N \nu_j)} \Big[C\, I_0^N(D-2;\{\nu_k\}) - \sum_{i=1}^N z_i I_0^N(D-2;\{\nu_k - \delta_{ki}\}) \Big].$$

Further reading

- **Dealing with IR divergence in real corrections**

  Phase space slicing, dipole subtraction, SecDec, …

- **Generate Feynman amplitudes**

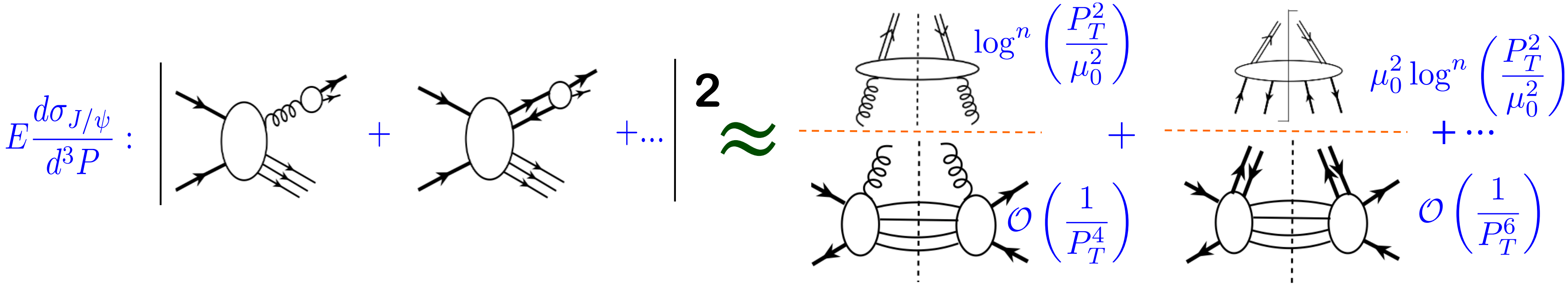
  Offshell recursion, onshell recursion, …

- **Numerical power expansion**

  Recursively construct derivative of current

Further reading cont.

- **High $p_T \gg$ m: QCD factorization up to NLP**



- **Factorization correct to all order**

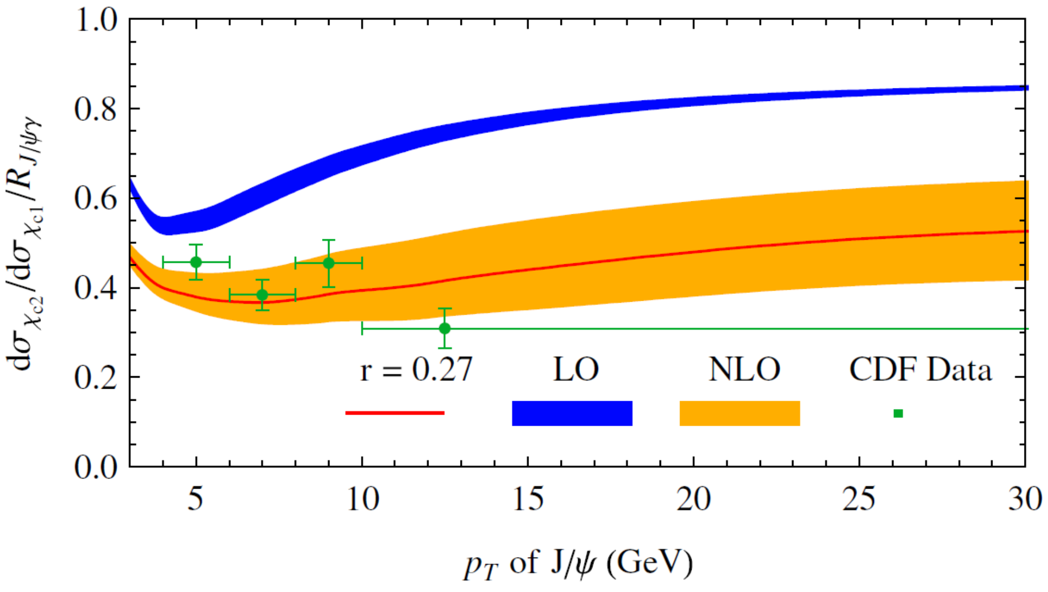
Kang, YQM, Qiu, Sterman, 1401.0923

- **Calculate hard part to high order in α_s**

To be done.

χ_{cJ}@hadron colliders

- χ_{cJ} **production:** $d\sigma_{\chi_{cJ}} \approx d\hat{\sigma}_{^3P_J^{[1]}} \langle O\left(^3P_0^{[1]}\right)\rangle + (2J+1)d\hat{\sigma}_{^3S_1^{[8]}} \langle O\left(^3S_1^{[8]}\right)\rangle$
  - $\langle O\left(^3P_0^{[1]}\right)\rangle$: can be determined by potential model
  - $\langle O\left(^3S_1^{[8]}\right)\rangle$: a number, the only free parameter, fit $d\sigma_{\chi_{c2}}/d\sigma_{\chi_{c1}}$ data

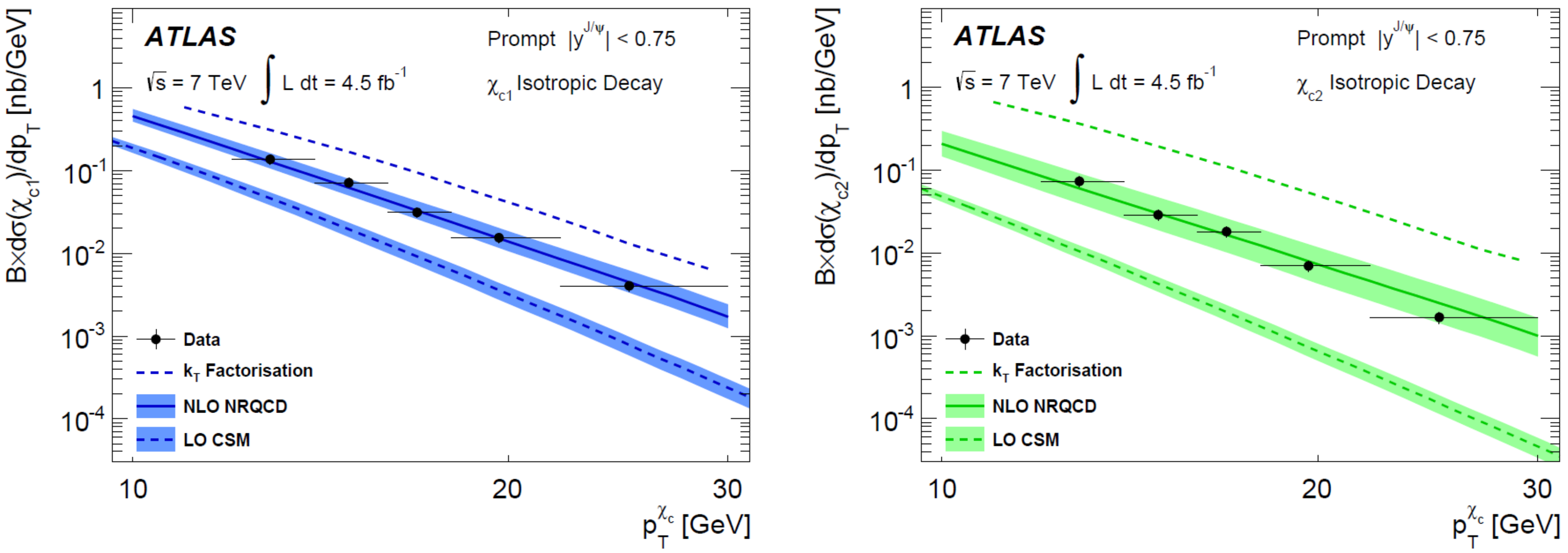


YQM, Wang, Chao, 1002.3987

Prediction

- **Comparison with new data**

ATLAS, 1404.7035



History of high order calculation: pp collision

- 0703113: Campbell, Maltoni, Tramontano

  NLO, cross section, S-wave

- 0802.3727: Gong, Wang

  NLO, polarization, S-wave

- 0806.3282: Artoisenet, Campbell, Lansberg, Maltoni, Tramontano

  $\mathrm{NNLO}^{\star}$, S-wave

- 1002.3987: YQM, Wang, Chao
- 1009.3655: YQM, Wang, Chao
- 1009.5662: Butenschöen, Kniehl

  Complete NLO (S- and P-wave), cross section

- 1201.1872: Butenschöen, Kniehl
- 1201.2675: Chao,YQM,Shao,Wang,Zhang
- 1205.6682: Gong,Wan,Wang,Zhang

  Complete NLO (S- and P-wave), with polarization

NOT fully comprehensive!!!

Summary

- **High order contributions are crucial for quarkonium production**

- **Methods discussed here can also be used for other cases**
  - Twist three/four contribution
  - Exclusive processes using light cone wave function

Thank you!