Two-loop corrections to hadronic B decays in QCD factorization approach

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based on **0911.3655**, **1507.03700**, **1606.02888** and **work in progress**

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Outline

B physics:

- What is B physics: the productions and decays of hadrons containing a b-quark;

$$B_u\ (u\bar{b}),\quad B_d\ (d\bar{b}),\quad B_s\ (s\bar{b}),\quad B_c\ (c\bar{b}),\quad \Lambda_b\ (udb),\quad \Upsilon(nS),\quad \cdots$$

- Motivation of B physics:
  - to measure the SM flavour-related parameters, to test the CKM mechanism of CP violation, to search for/constrain on NP signals beyond the SM;

    $\hookrightarrow$ complementary to EWP tests @(LEP, Tevatron) and direct NP searches @(LHC)

  - to further understand how quarks and gluons are confined into hadrons through strong interactions, i.e., the non-perturbative aspects of QCD;

    $\hookrightarrow$ operator product expansion, QCD effective field theories, factorization theorems

  - to probe the hadronic structure (i.e., the distribution amplitudes) of b hadrons as well as of their decay products;

    $\hookrightarrow$ important theoretical and phenomenological inputs for other processes

Classification of B decays:

- At the quark level: B-hadron weak decays are mediated by flavour-changing weak charged-current $J^\mu_{\rm CC}$ coupled to the W-boson;

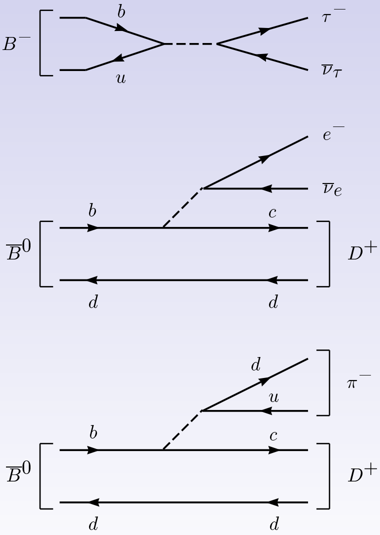
$$\mathcal{L}_{\rm CC} = -\frac{g}{\sqrt{2}} J^\mu_{\rm CC} W^\dagger_\mu + \text{h.c.}$$

$$J^\mu_{\rm CC} = (\bar{\nu}_e, \bar{\nu}_\mu, \bar{\nu}_\tau)\,\gamma^\mu \begin{pmatrix} e_{\rm L} \\ \mu_{\rm L} \\ \tau_{\rm L} \end{pmatrix}$$

$$+\,(\bar{u}_{\rm L}, \bar{c}_{\rm L}, \bar{t}_{\rm L})\,\gamma^\mu\, V_{\rm CKM} \begin{pmatrix} d_{\rm L} \\ s_{\rm L} \\ b_{\rm L} \end{pmatrix}$$

- Three different classes: depending on the different final states, B-hadron weak decays can be divided into three classes:

leptonic, semi-leptonic, non-leptonic



Simple quark-line diagrams

Non-leptonic B decays:

- Play a crucial role in testing the CKM mechanism and in quantifying the CP violation:

  - α: from time-dep. CP asym. in $B \to \pi\pi$, $\pi\rho$ and $\rho\rho$ decays;

    $(90.4^{+2.0}_{-1.0})^\circ$

  - β: from $B \to J/\psi K_S$ and other charmonium modes;

    $(22.62^{+0.44}_{-0.42})^\circ$

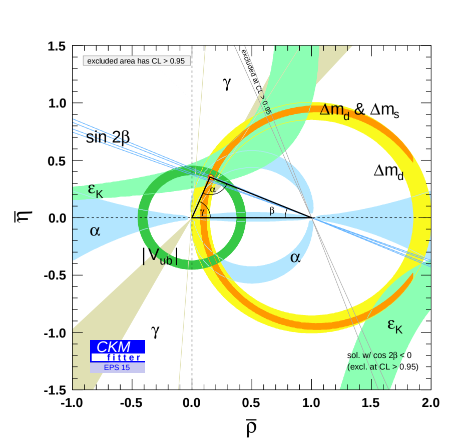
  - γ: from $B \to DK$, $B \to K\pi\pi$, $B \to KKK$ decays;

    $(67.01^{+0.88}_{-1.99})^\circ$

  - β_s: from $B_s \to J/\psi\phi$ and $B_s \to \phi\phi$ decays, $\cdots$;

    $(0.01882^{+0.00036}_{-0.00042})\,rad$

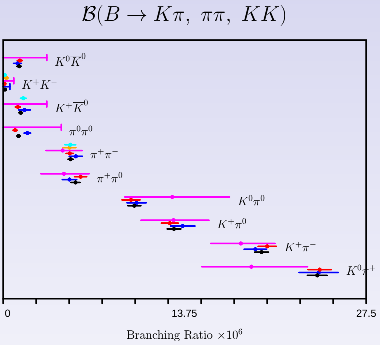
$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$

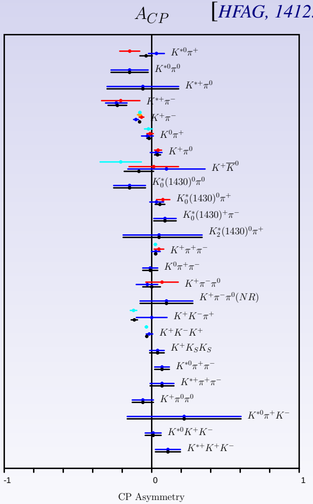


taken from CKMfitter group as of Summer 2015.

Status of exp. data on non-leptonic B decays:

- Thanks to BaBar, Belle, Tevatron and LHCb, more and more precise data available now for many non-leptonic B decays; [*HFAG, 1412.7515*]





Some of the data are now much more precise than the theoretical predictions!

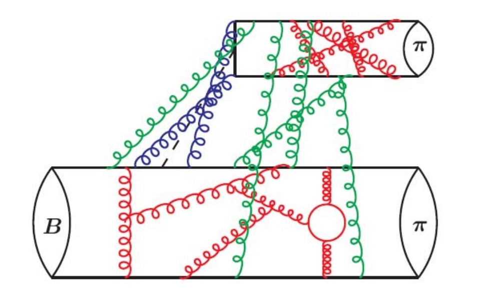
- To catch up with the precise exp. measurements, it is now very necessary and urgent to improve the theoretical calculation!

Difficulties in non-leptonic B decays:

- For a hadronic decay, theoretically very difficult: initial- and final-states are all hadrons; in the real world, quarks are confined inside hadrons through the exchange of soft gluons;

  $\hookrightarrow$the simplicity of weak interactions overshadowed by the complexity of strong interactions!

  $B \to \pi\pi$ decay:



- Hadronic B decay is a multi-scale problem with highly hierarchical interaction scales:

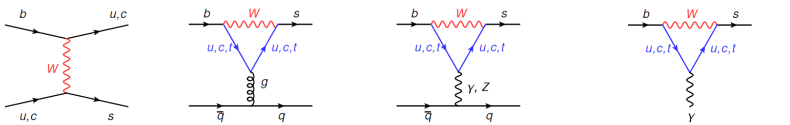
EW interaction scale	$\gg$	ext. mom'a in B rest frame	$\gg$	QCD-bound state effects
$m_W \sim 80$ GeV $m_Z \sim 91$ GeV	$\gg$	$m_b \sim 5$ GeV	$\gg$	$\Lambda_{\rm QCD} \sim 1$ GeV

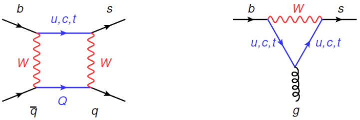
Effective weak Hamiltonion for non-leptonic B decays:

- The starting point $\mathcal{H}_{\rm eff}$: integrating out the heavy degrees of freedom $(m_W, m_Z, m_t \gg m_b)$, containing physics above $\mu \sim m_b$; [*BBL basis: Buras, Buchalla, Lautenbacher '96*; *CMM basis: Chetyrkin, Misiak, Münz '98*]

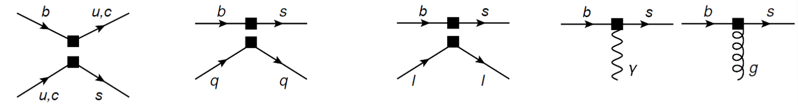
$$\mathcal{L}_{\rm eff} \sim G_F\, V_{\rm CKM} \times \Big[\sum_{p=u,c} \sum_{i=1,2} C_i\, \mathcal{O}_i^p + \sum_{3,\dots,6} C_i\, \mathcal{O}_i + \sum_{7,\dots,10} C_i\, \mathcal{O}_i + \sum_{7\gamma,8g} C_i\, \mathcal{O}_i \Big]$$

charged current | QCD-penguin | EW-penguin | electro- & chromo-mgn





- $\mathcal{Q}_i$: local operators;
- C_i: pert. calculable;



Weak effective Hamiltonian for non-leptonic B decays:

- Three steps to obtain $\mathcal{H}_{\text{eff}} \propto \sum C_i Q_i$: [*BBL basis: Buras, Buchalla, Lautenbacher '96*;

  - Calculation of matching coefficients c_i in fixed-order perturbation theory:

$$C_i(m_W) = c_i^{(0)} + \frac{\alpha_s}{4\pi} c_i^{(1)} + \ldots$$

⟵ SM! + New Physics?

  - Perturbative calculation of anomalous dimensions γ_{ij} of operators in H_{eff}

$$\gamma_{ij} = \gamma_{ij}^{(0)} + \frac{\alpha_s}{4\pi} \gamma_{ij}^{(1)} + \ldots$$

⟵ QCD (+QED)

  - Use renormalization group to sum large logarithms $\ln \frac{m_b}{m_W}$:

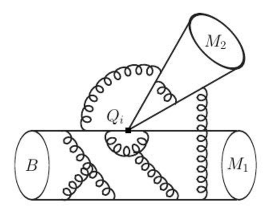
$$C_i(m_W) \to C_i(m_b) = \left[\frac{\alpha_s(m_b)}{\alpha_s(m_W)}\right]^{-\gamma_{ij}^{(0)}/2\beta_0} C_j(m_W) + \ldots$$

⟵ RGE

- Wilson coefficients C_i: perturbatively calculable; the NNLO program now complete;

  - ▷ 2-loop/3-loop matching calculations at the initial scale; [Bobeth, Misiak, Urban 99; Misiak, Steinhauser 04]
  - ▷ 3-loop/4-loop anomalous dimension matrices for running; [Gorbahn, Haisch 04; Gorbahn, Haisch, Misiak 05; Czakon, Haisch, Misiak 06]

Calculation of the hadronic matrix elements of Q_i:



- Hadronic matrix elements $\langle M_1 M_2 | Q_i | \bar{B} \rangle$: $\hookrightarrow$ *a quite difficult, multi-scale, strong-interaction problem!*
- Effective theories/Factorization theorem/Approximate symmetries: express $\langle M_1 M_2 | Q_i | \bar{B} \rangle$ in terms of (few) universal non-perturbative hadronic quantities;
- To match the exp. precision, we need to try to improve the calculation of $\langle Q_i \rangle$!
  - Dynamical approaches based on factorization theorems: PQCD, QCDF, SCET, $\cdots$

    [Keum, Li, Sanda, Lü, Yang '00;
    Beneke, Buchalla, Neubert, Sachrajda, '00;
    Bauer, Flemming, Pirjol, Stewart, '01; Beneke, Chapovsky, Diehl, Feldmann, '02]
  - Symmetries of QCD: Isospin, U-Spin, V-Spin, and flavour SU(3) symmetries, $\cdots$ [Zeppenfeld, '81;
    London, Gronau, Rosner, Chiang, Cheng *et al.*]
  - Some other methods based on QCD: Lattice QCD, QCD sum rule, Dyson-Schwinger equations, $\cdots$

    [HPQCD, RBC/UKQCD, Fermilab Lattice and MILC;
    [Khodjamirian, '00; Khodjamirian, Mannel, Melic, '03; Huang *et al*, '04;
    Ivanov, Korner, Kovalenko and Roberts, '07]

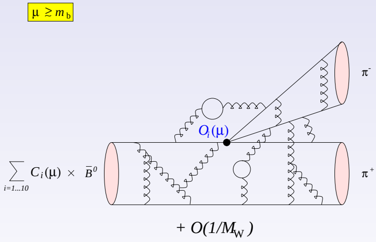
Hadronic matrix elements in QCDF approach:

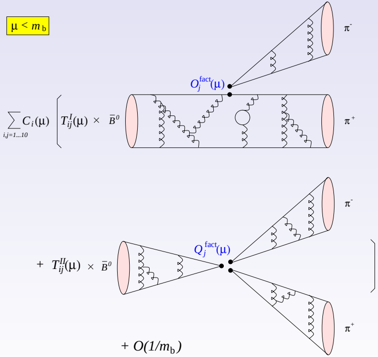
- In the heavy-quark limit, $\langle M_1 M_2 | Q_i | \bar{B} \rangle$ obeys the factorization formula: [BBNS'99-'04]

$$\langle M_1 M_2 | Q_i | \bar{B} \rangle \simeq m_B^2 F_+^{BM_1}(0) f_{M_2} \int du \; T_i^I(u) \, \phi_{M_2}(u) + (M_1 \leftrightarrow M_2)$$

$$+ f_B f_{M_1} f_{M_2} \int d\omega dv du \; T_i^{II}(\omega, v, u) \, \phi_B(\omega) \, \phi_{M_1}(v) \, \phi_{M_2}(u)$$

$$+ \mathcal{O}(1/m_b)$$

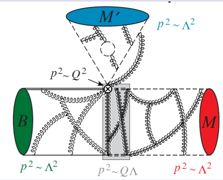




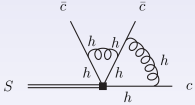
- A systematic framework to all orders in α_s, but limited accuracy by $1/m_b$ corrections.

Factorization formula from the SCET point of view:

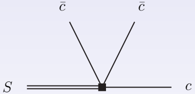
- Soft-collinear effective theory (SCET): an EFT designed to describe processes involving energetic hadrons/jets; [Bauer, Flemming, Pirjol, Stewart, '01; Beneke, Chapovsky, Diehl, Feldmann, '02]
- In a two-body charmless $B \to MM'$ decays: relevant degrees of freedom including
  - low-virtuality modes:
    - ⋆ HQET fields: $p - m_b v \sim$ $\mathcal{O}(\Lambda)$
    - ⋆ soft spectators in B meson: $p_s^\mu \sim \Lambda \ll m_b, \quad p_s^2 \sim$ $\mathcal{O}(\Lambda^2)$
    - ⋆ collinear quarks and gluons in pion: $E_c \sim m_b, \quad p_c^2 \sim$ $\mathcal{O}(\Lambda^2)$
  - high-virtuality modes:
    - ⋆ hard modes: $(\text{heavy quark} + \text{collinear})^2 \sim$ $\mathcal{O}(m_b^2)$
    - ⋆ hard-collinear modes: $(\text{soft} + \text{collinear})^2 \sim$ $\mathcal{O}(m_b\Lambda)$



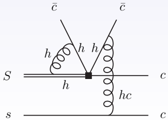
- For T^I: one-step matching, QCD $\to$ $\mathrm{SCET_I}(hc, c, s)$



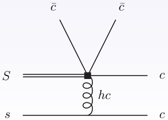
$\xrightarrow{\mu \simeq \mu_h}$ T_i^I



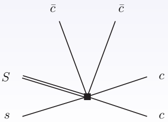
- For T^{II}: two-step matching, QCD $\to$ $\mathrm{SCET_I}(hc, c, s)$ $\to$ $\mathrm{SCET_{II}}(c, s)$



$\xrightarrow{\mu \simeq \mu_h}$ H_i

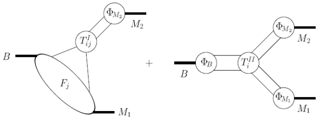


$\xrightarrow{\mu \simeq \mu_{hc}}$ $H_i \otimes J$



Factorization formula from the SCET point of view:

- In SCET formalism, factorization is accomplished by showing that the various types of fields with differing kinematics do not couple at the level of the $\mathcal{L}_{\text{tot}} = \mathcal{L}_n + \mathcal{L}_{\bar{n}} + \mathcal{L}_s$; [Bauer, Flemming, Pirjol, Stewart, '01; Beneke, Chapovsky, Diehl, Feldmann, '02; Becher, Broggio, Ferroglia '14]
  - Light-cone kinematics; - Large and small collinear spinor components; - Multi-pole expansion and field re-difinitions;
- Factorization formula for $B \to M_1 M_2$ in SCET formalism:



Form factor term + Spectator scattering

$$T, C, P^{c,u}, \ldots \quad \sim \quad \langle M_1 M_2 | C_i O_i | \bar{B} \rangle_{\mathcal{L}_{\text{eff}}} = \sum_{\text{terms}} C(\mu_h) \times \Big\{ F_{B\to M_1} \times \underbrace{T^{\text{I}}(\mu_h, \mu_s)}_{1+\alpha_s+\ldots} \star f_{M_2} \Phi_{M_2}(\mu_s)$$

$$+ f_B \Phi_B(\mu_s) \star \Big[\underbrace{T^{\text{II}}(\mu_h, \mu_I)}_{1+\ldots} \star \underbrace{J^{\text{II}}(\mu_I, \mu_s)}_{\alpha_s+\ldots} \Big] \star f_{M_1} \Phi_{M_1}(\mu_s) \star f_{M_2} \Phi_{M_2}(\mu_s) \Big\}$$

$$+ 1/m_b\text{-suppressed terms}$$

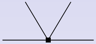
- SCET: field-theoretical basis for QCDF, theoretical basis of Feynman diagrammatic QCD factorization; $\hookrightarrow$ SCET factorization is exactly the same as QCDF; [Beneke '15]

Perturbative calculation of the hard kernels $T^{I,II}$:

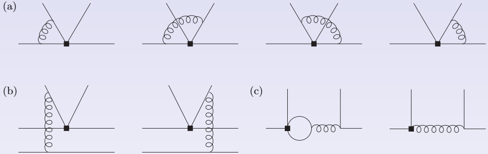
- T^I and T^{II}: perturbatively calculable order by order in α_s;

  vertex corrections: $T^I = 1 + \mathcal{O}(\alpha_s) + \cdots$;   spectator scattering: $T^{II} = \mathcal{O}(\alpha_s) + \cdots$.

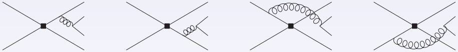
  - up to NLO in α_s, the relevant Feynman diagrams include:



  - hard and IR contributions are separated properly, thus validating the soft-collinear factorization at 1-loop level;



  - strong phases from final-state interactions $\sim \mathcal{O}(\alpha_s), \mathcal{O}(1/m_b)$;
  - annihilation topologies and higher Fock states give power-suppressed contributions;



- Main conclusions: in the heavy-quark limit, all "non-factorizable" diagrams are dominated by hard gluons and can be calculated as expansion in $\alpha_s(m_b)$; Soft gluons are suppressed as $\Lambda_{\rm QCD}/m_b$; $\hookrightarrow$ "colour transparency argument"   [Bjorken, '89]

QCDF/SCET analyses of $B \to M_1 M_2$ at NLO:

- Analyses of complete sets of final states:
  - $B \to PP, PV$: [Beneke, Neubert, hep-ph/0308039; Cheng, Chua, 0909.5229, 0910.5237;]
  - $B \to VV$: [Beneke, Rohrer, Yang, hep-ph/0612290; Cheng, Yang, 0805.0329; Cheng, Chua, 0909.5229, 0910.5237;]
  - $B \to AP, AV, AA$: [Cheng, Yang, 0709.0137, 0805.0329;]
  - $B \to SP, SV$: [Cheng, Chua, Yang, hep-ph/0508104, 0705.3079; Cheng, Chua, Yang, Zhang, 1303.4403;]
  - $B \to TP, TV$: [Cheng, Yang, 1010.3309;]
- Well-established successes based on the NLO hard-scattering functions!

- Successes of QCDF/SCET:
  - Colour-allowed tree-dominated and penguin-dominated Brs are usually quantitatively OK;
  - Dynamical explanation of intricate patterns of penguin interference seen in PP, PV, VP and VV modes:

$$PP \sim a_4 + r_\chi a_6, \quad PV \sim a_4 \approx \frac{PP}{3}$$

$$VP \sim a_4 - r_\chi a_6 \sim -PV$$

$$VV \sim a_4 \sim PV$$

  - Qualitative explanation of polarization puzzle in $B \to VV$ decays, due to the large weak annihilation;
  - Strong phases start at $\mathcal{O}(\alpha_s)$, dynamical explanation of smallness of direct CP asymmetries;

Some issues in QCDF/SCET:

- Some issues in QCDF/SCET:
  - factorization of power corrections is generally broken, due to the appearance of end-point divergence;
  - could not account for some data, such as the large $\mathrm{Br}(B \to \pi^0\pi^0)$, the unmatched CP asymmetries in $B \to \pi K$ decays, $\cdots$;
  - how important is the higher-order perturbative corrections? Factorization theorem is still established?
  - what is the correct theory for power corrections? Can never exclude large sizeable power corrections theoretically!
- Motivation for nontrivial NNLO perturbative calculation:
  - conceptual aspect: check if factorization theorem still held at the NNLO?
  - phenomenologically: strong phases first start at $\mathcal{O}(\alpha_s)$, NNLO is only the NLO to them; quite relevant for precise direct CP prediction;
  - exp. data driven: α_2 seems to be too small, and the $A_{CP}(\pi K)$ puzzle; Does NNLO short-distance prediction tend toward the right direction?

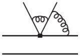
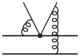
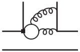
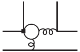
Status of perturbative calculation of the hard kernels $T^{I,II}$:

- To ascertain the SD contribution: need a reliable $\mathcal{O}(\alpha_s^2)$ hard-scattering kernels;

Two hard-scattering kernels for each operator insertion

$$\langle M_1 M_2 | Q_i | B \rangle \simeq F^{BM_1} T_i^I \otimes \phi_{M_2} + T_i^{II} \otimes \phi_B \otimes \phi_{M_1} \otimes \phi_{M_2}$$

and two classes of topological amplitudes

Status	2-loop vertex corrections (T_i^I)	1-loop spectator scattering (T_i^{II})
Trees	 [GB 07, 09] [Beneke, Huber, Li 09]	 [Beneke, Jäger 05] [Kivel 06] [Pilipp 07]
Penguins	 [GB, Beneke, Huber, Li 15 + in progress]	 [Beneke, Jäger 06] [Jain, Rothstein, Stewart 07]

- For the spectator-scattering kernel T_i^{II}: already completed, both for tree and penguin;

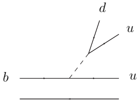
  [*Beneke, Jäger '05; Kivel '06; Pilipp '07; Jain, Rothstein, Stewart '07*]

- For the form-factor term T_i^I: already known for tree, for penguin now only with $Q_{1,2}^p$;

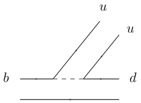
  [*Bell '07-'09; Beneke, Huber, Li '09; Bell, Beneke, Huber, Li '15 and work in progress*]

Typical topological amplitudes for $B \to M_1 M_2$:

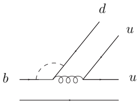
- For a non-leptonic B decay, three topological amplitudes are mostly relevant:



colour-allowed tree α_1



colour-suppressed tree α_2



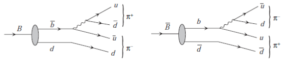
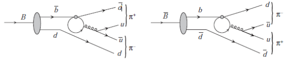
QCD penguins α_4

- Due to CKM unitarity, the amplitude for a $\bar{B} \to \bar{f}$ decay can always be written as:

$$\bar{A}(\bar{B} \to \bar{f}) = \lambda_u^{(D)} \left[T + \ldots\right] + \lambda_c^{(D)} \left[P_c + \ldots\right]$$

  - $\lambda_u^{(D)}$ part: $b \to u\bar{u}D$ transition, dominated by tree amplitudes $T = \alpha_{1,2}(M_1M_2)$;
  - $\lambda_c^{(D)}$ part: $b \to Dq\bar{q}$ transition, dominated by penguin amplitudes $P_c = \alpha_4^c(M_1M_2)$;

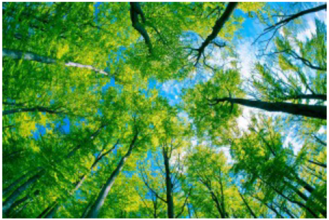
- For a specific decay mode, if both T and P_c involved, then direct CP asymmetry:

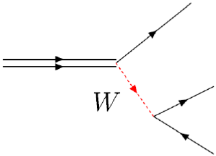
  - Tree amplitude: 
  - Penguin amplitude: 

$$\mathcal{A}(\bar{B}^0 \to \pi^+\pi^-) = i\frac{G_F}{\sqrt{2}} m_B^2 F_0^{B\to\pi}(0) f_\pi$$

$$\left[\lambda_u a_1 + \lambda_p(a_4^p + a_{10}^p) + \lambda_p r_\chi^\pi(a_6^p + a_8^p)\right]$$

Tree-dominated decay modes:





- Two-loop vertex corrections to T^I:

G. Bell, *"NNLO vertex corrections in charmless hadronic B decays: Imaginary part,"* Nucl. Phys. B **795** (2008) 1 [arXiv:0705.3127 [hep-ph]].

G. Bell, *"NNLO vertex corrections in charmless hadronic B decays: Real part,"* Nucl. Phys. B **822** (2009) 172 [arXiv:0902.1915 [hep-ph]].

G. Bell and V. Pilipp, *"$B \to \pi^- \pi^0 / \rho^- \rho^0$ to NNLO in QCD factorization,"* Phys. Rev. D **80** (2009) 054024 [arXiv:0907.1016 [hep-ph]].

M. Beneke, T. Huber and X. Q. Li, *"NNLO vertex corrections to non-leptonic B decays: Tree amplitudes,"* Nucl. Phys. B **832** (2010) 109 [arXiv:0911.3655 [hep-ph]].

- CMM operator basis in full QCD:

$$Q_1^p = \bar{p}\gamma^\mu P_L T^A b \;\; \bar{D}\gamma_\mu P_L T^A p,$$

$$Q_2^p = \bar{p}\gamma^\mu P_L b \;\; \bar{D}\gamma_\mu P_L p,$$

$$Q_3 = \bar{D}\gamma^\mu P_L b \;\; \sum_q \bar{q}\gamma_\mu q,$$

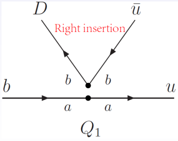
$$Q_4 = \bar{D}\gamma^\mu P_L T^A b \;\; \sum_q \bar{q}\gamma_\mu T^A q,$$

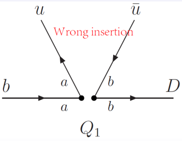
$$Q_5 = \bar{D}\gamma^\mu\gamma^\nu\gamma^\rho P_L b \;\; \sum_q \bar{q}\gamma_\mu\gamma_\nu\gamma_\rho q,$$

$$Q_6 = \bar{D}\gamma^\mu\gamma^\nu\gamma^\rho P_L T^A b \;\; \sum_q \bar{q}\gamma_\mu\gamma_\nu\gamma_\rho T^A q,$$

$$Q_{8g} = \frac{-g_s}{32\pi^2}\, m_b\, \bar{D}\sigma_{\mu\nu}(1+\gamma_5)G^{\mu\nu} b.$$

$+$ 8 evanescent operators in QCD





- Nonlocal SCET operator basis for RI:

$$O_1 = \left[\bar{\chi}\,\frac{\not{n}_-}{2} P_L \chi\right]\left[\bar{\xi}\,\not{n}_+ P_L h_v\right],$$

$$O_2 = \left[\bar{\chi}\,\frac{\not{n}_-}{2} P_L \gamma_\perp^\alpha\gamma_\perp^\beta \chi\right]\left[\bar{\xi}\,\not{n}_+ P_L \gamma^\perp_\beta\gamma^\perp_\alpha h_v\right],$$

$$O_3 = \left[\bar{\chi}\,\frac{\not{n}_-}{2} P_L \gamma_\perp^\alpha\gamma_\perp^\beta\gamma_\perp^\gamma\gamma_\perp^\delta \chi\right]\left[\bar{\xi}\,\not{n}_+ P_L \gamma^\perp_\delta\gamma^\perp_\gamma\gamma^\perp_\beta\gamma^\perp_\alpha h_v\right]$$

- Nonlocal SCET operator basis for WI:

$$\tilde{O}_1 = \left[\bar{\xi}\,\gamma_\perp^\alpha P_L \chi\right]\left[\bar{\chi} P_R \gamma^\perp_\alpha h_v\right],$$

$$\tilde{O}_2 = \left[\bar{\xi}\,\gamma_\perp^\alpha\gamma_\perp^\beta\gamma_\perp^\gamma P_L \chi\right]\left[\bar{\chi} P_L \gamma^\perp_\alpha\gamma^\perp_\gamma\gamma^\perp_\beta h_v\right],$$

$$\tilde{O}_3 = \left[\bar{\xi}\,\gamma_\perp^\alpha\gamma_\perp^\beta\gamma_\perp^\gamma\gamma_\perp^\delta\gamma_\perp^\epsilon P_L \chi\right]\left[\bar{\chi} P_R \gamma^\perp_\alpha\gamma^\perp_\epsilon\gamma^\perp_\delta\gamma^\perp_\gamma\gamma^\perp_\beta h_v\right]$$

- 8 evanescent operators in QCD: although vanish in 4-dim., but needed to complete the operator basis under renormalization! [Gorbahn, Haisch 04; Gorbahn, Haisch, Misiak 05]

Matching calculation from QCD onto SCET$_{\rm I}$: I

- To extract the hard kernels from matching, construct the factorized QCD operator:

$$O_{\rm QCD} \equiv [\bar{q}\,\frac{\not{n}_-}{2}(1-\gamma_5)q]\,[\bar{q}\,\not{n}_+(1-\gamma_5)b] = C_{FF}\,C_{\bar{q}q}\,O_1$$

$C_{\bar{q}q}$ and C_{FF}: matching coefficients for QCD currents to SCET currents; [Beneke, Huber, Li, '08]

$\Longrightarrow \langle O_{\rm QCD}\rangle$ is the product of a light-meson LCDA and the *full QCD* heavy-to-light transition form factor.

- For "right insertion":

$$\langle Q_i\rangle = T_i\,\langle O_{\rm QCD}\rangle + \sum_{a>1} H_{ia}\langle O_a\rangle$$

- For "wrong insertion":

$$\langle Q_i\rangle = \widetilde{T}_i\,\langle O_{\rm QCD}\rangle + \tilde{H}_{i1}\langle \tilde{O}_1 - O_1\rangle + \sum_{a>1} \tilde{H}_{ia}\langle \tilde{O}_a\rangle$$

- On the QCD side, we have:

$$\langle Q_i\rangle = \Big\{ A_{ia}^{(0)} + \frac{\alpha_s}{4\pi}\Big[A_{ia}^{(1)} + Z_{ext}^{(1)}\,A_{ia}^{(0)} + Z_{ij}^{(1)}A_{ja}^{(0)}\Big] + \Big(\frac{\alpha_s}{4\pi}\Big)^2\Big[A_{ia}^{(2)} + Z_{ij}^{(1)}A_{ja}^{(1)} + Z_{ij}^{(2)}A_{ja}^{(0)} + Z_{ext}^{(1)}\,A_{ia}^{(1)} + Z_{ext}^{(2)}\,A_{ia}^{(0)} + Z_{ext}^{(1)}\,Z_{ij}^{(1)}A_{ja}^{(0)} + Z_{\alpha}^{(1)}A_{ia}^{(1)} + (-i)\,\delta m^{(1)}\,A_{ia}'^{(1)}\Big] + \mathcal{O}(\alpha_s^3)\Big\}\,\langle O_a\rangle^{(0)}$$

- On the SCET side, we have:

$$\langle O_a\rangle = \Big\{ \delta_{ab} + \frac{\hat{\alpha}_s}{4\pi}\Big[M_{ab}^{(1)} + Y_{ext}^{(1)}\,\delta_{ab} + Y_{ab}^{(1)}\Big] + \Big(\frac{\hat{\alpha}_s}{4\pi}\Big)^2\Big[M_{ab}^{(2)} + Y_{ac}^{(1)}M_{cb}^{(1)} + Y_{ab}^{(2)} + Y_{ext}^{(1)}\,M_{ab}^{(1)} + Y_{ext}^{(2)}\,\delta_{ab} + Y_{ext}^{(1)}\,Y_{ab}^{(1)} + \hat{Z}_{\alpha}^{(1)}M_{ab}^{(1)}\Big] + \mathcal{O}(\alpha_s^3)\Big\}\,\langle O_b\rangle^{(0)}$$

Matching calculation from QCD to SCET$_{\mathrm{I}}$: II

- Final result for RI at 1- and 2-loop level:

$$
\begin{aligned}
T_i^{(1)} &= A_{i1}^{(1),nf} + Z_{ij}^{(1)}\, A_{j1}^{(0)}\,, \\
T_i^{(2)} &= A_{i1}^{(2),nf} + Z_{ij}^{(1)}\, A_{j1}^{(1)} + Z_{ij}^{(2)}\, A_{j1}^{(0)} + Z_{\alpha}^{(1)}\, A_{i1}^{(1),nf} + (-i)\,\delta_m^{(1)}\, A_{i1}'^{(1),nf} \\
&\quad + T_i^{(1)}\left[-C_{FF}^{(1)} - Y_{11} + Z_{ext}^{(1)}\right] - \sum_{b>1} H_{ib}^{(1)}\, Y_{b1}^{(1)}\,.
\end{aligned}
$$

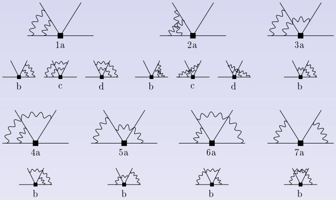
- Final result for WI at 1- and 2-loop level:

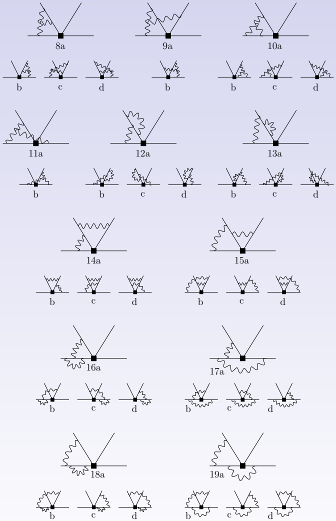
$$
\begin{aligned}
\widetilde{T}_i^{(1)} &= \widetilde{A}_{i1}^{(1),nf} + Z_{ij}^{(1)}\, \widetilde{A}_{j1}^{(0)} + \widetilde{A}_{i1}^{(1),f} - A^{(1),f}\, \widetilde{A}_{i1}^{(0)} - [\widetilde{Y}_{11}^{(1)} - Y_{11}^{(1)}]\, \widetilde{A}_{i1}^{(0)} \\
\widetilde{T}_i^{(2)} &= \widetilde{A}_{i1}^{(2),nf} + Z_{ij}^{(1)}\, \widetilde{A}_{j1}^{(1)} + Z_{ij}^{(2)}\, \widetilde{A}_{j1}^{(0)} + Z_{\alpha}^{(1)}\, \widetilde{A}_{i1}^{(1),nf} \\
&\quad + (-i)\,\delta_m^{(1)}\, \widetilde{A}_{i1}'^{(1),nf} + Z_{ext}^{(1)}\,[\widetilde{A}_{i1}^{(1),nf} + Z_{ij}^{(1)}\, \widetilde{A}_{j1}^{(0)}] \\
&\quad - \widetilde{T}_i^{(1)}\,[C_{FF}^{(1)} + \widetilde{Y}_{11}^{(1)}] - \sum_{b>1} \widetilde{H}_{ib}^{(1)}\, \widetilde{Y}_{b1}^{(1)} \\
&\quad + [\widetilde{A}_{i1}^{(2),f} - A^{(2),f}\, \widetilde{A}_{i1}^{(0)}] + (-i)\,\delta_m^{(1)}\,[\widetilde{A}_{i1}'^{(1),f} - A'^{(1),f}\, \widetilde{A}_{i1}^{(0)}] \\
&\quad + (Z_{\alpha}^{(1)} + Z_{ext}^{(1)} + \xi_{45}^{(1)})[\widetilde{A}_{i1}^{(1),f} - A^{(1),f}\, \widetilde{A}_{i1}^{(0)}] \\
&\quad - C_{FF}^{(1)}\, \widetilde{A}_{i1}^{(0)}\,[\widetilde{Y}_{11}^{(1)} - Y_{11}^{(1)}] - [\widetilde{Y}_{11}^{(2)} - Y_{11}^{(2)}]\, \widetilde{A}_{i1}^{(0)}
\end{aligned}
$$

- For $Q_{5,6}$ insertion: evanescent operator contributions are already non-zero at tree-level;

Two-loop Feynman diagrams for tree amplitudes:

- The two-loop non-factorizable diagrams: [*Beneke,Buchalla,Neubert,Sachrajda 00*]





  - totally 62 “non-factorizable” diagrams;
  - vacuum polarization insertions in gluon propagators;
  - the one-loop counter-term insertions;

- For the tree amplitudes α_1 and α_2, now complete and have been cross-checked; [*G.Bell 07, 09; M. Beneke, T. Huber, X. Q. Li 09*]

Multi-loop calculations in a nutshell: I

- Adopt the DR scheme with $D = 4 - 2\epsilon$, to regulate both the UV and IR div.; at two-loop order, UV and IR poles appear up to $1/\epsilon^2$ and $1/\epsilon^4$, respectively.

- Basis strategy and procedure:

  - perform the general tensor reduction via Passarino-Veltman ansatz, $\Longrightarrow$thousands of scalar integrals, [*Passarino,Veltman '79*]

  - reduce them to Master Integrals via Laporta algorithm based on IBP identities $\Longrightarrow$totally 42 MIs, [*Tkachov '81; Chetyrkin,Tkachov '81; Laporta '01; Anastasiou,Lazopoulos '04*]

  - calculate these MIs, very challenging as we need analytical results.

- Techniques used to calculate MIs (developed very rapidly in recent years):

  - standard Feynman/Schwinger parameterisation, only for very simpler MIs;

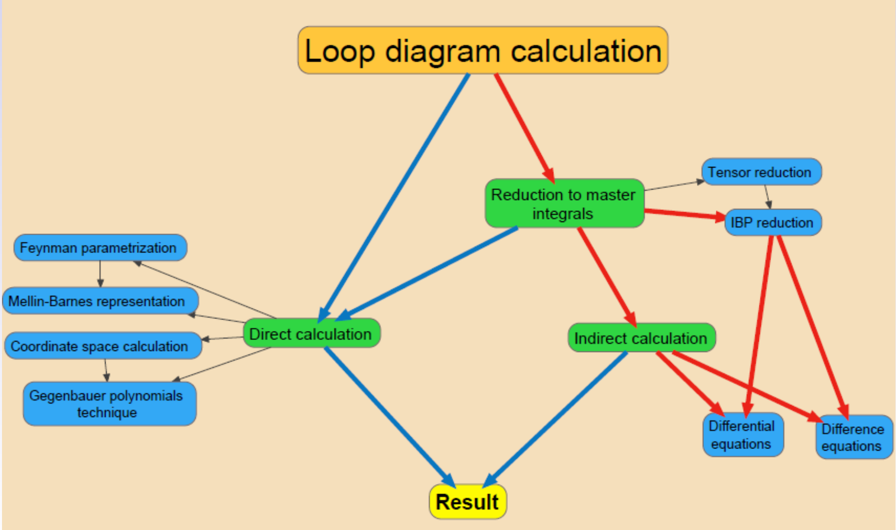
  - method of differential equations; [*Kotikov '91; Remiddi '97; Henn '13*]

  - Mellin-Barnes techniques; [*Smirnov '99; Tausk '99*]

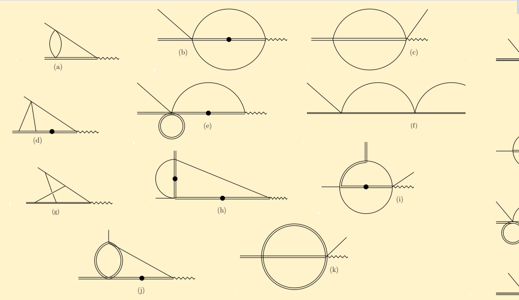
  - method of sector decomposition, for numerical check! [*Binoth, Heinrich 00*]

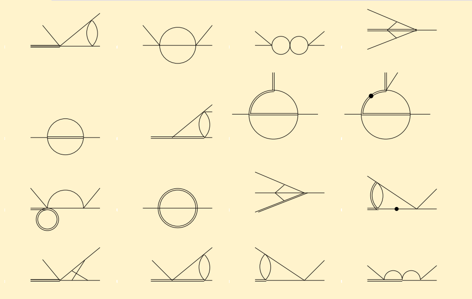
Multi-loop calculations in a nutshell:

- General precedure of the multi-loop calculations: [*R.N. Lee talk at ACAT 2013*]



List of the resulted Master Integrals:





- The double lines are massive, while the single lines massless;
- The dot on lines denotes the squared propagator;
- LHS MIs have been cross-checked in inclusive $B \to X_u l\nu$ calculations;
  [*Bell '08; Bonciani, Ferroglia '08; Asatrian, Greub, Pecjak '08; Beneke, Huber, Li '08*]
- RHS MIs are needed for charmless hadronic B decays and have also been cross-checked;
  [*Bell '07, '09; Beneke, Huber, Li '09*]
- Two-loop crossed six-line vertex MI with two massive lines: derived from a three-fold Mellin-Barnes representation;
  [*Huber '09*]

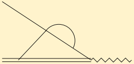
Illustration of the calculation techniques: I

- IBP IDs: for the two-loop case, there are eight IDs per scalar integral;

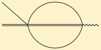
$$\int \frac{d^D k}{(2\pi)^D} \int \frac{d^D l}{(2\pi)^D} \frac{\partial}{\partial a^\mu} \left[b^\mu f(k,l,p_i) \right] = 0 \,; \quad a^\mu = k^\mu,\, l^\mu \,; \quad b^\mu = k^\mu,\, l^\mu,\, p_i^\mu$$

- Solve systems of these equations via Laporta algorithm;

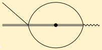
$\Longrightarrow$a scalar integral can be expressed as a linear combination of some MIs:



$$= \frac{(8-3D)(7uD-8D-24u+28)}{3(D-4)^2\, m_b^4\, u^3}$$



$$- \frac{2[u^2(D-4)+(16D-56)(1-u)]}{3(D-4)^2\, m_b^2\, u^3}$$



- Differential equations: [*Kotikov '91; Remiddi '97; Henn '13*]

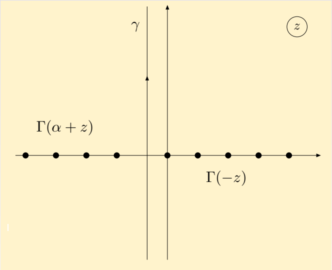
$$\frac{\partial}{\partial u} \mathrm{MI}_i(u) = f(u,\epsilon)\, \mathrm{MI}_i(u) + \sum_{j\neq i} g_j(u,\epsilon)\, \mathrm{MI}_j(u)$$

- needs results from Laporta reduction. - $\mathrm{MI}_j(u)$ are known simpler MIs.
- should fix the boundary condition by some other methods.
- choose "optimal" basis of MIs to get simple iterated integrations in each order in ϵ-expansion.

Illustration of the calculation techniques: II

- Mellin-Barnes representation: makes Feynman parameter integrals simpler; [*Smirnov '99; Tausk '99*]

$$\frac{1}{(A_1 + A_2)^{\alpha}} = \oint_{\gamma} \frac{dz}{2\pi i} A_1^z A_2^{-\alpha-z} \frac{\Gamma(-z)\,\Gamma(\alpha+z)}{\Gamma(\alpha)}$$



  - has been partially automated;
  - can be used as a numerical cross-check of our analytic calculation; [*Czakon '05; Gluza, Kajda, Riemann '07*]

- Special functions frequently used:
  - HPL up to weight 4 with argument u or $1-u$;
  - Generalized polylogarithms Li_2, Li_3, Li_4 with argument u, $1-u$, $\frac{u}{1-u}$;
  - Hypergeometric function pFq, needs perform ϵ-expansion; [*Maitre,Huber'05,'07*]

Numerical results for α_1 and α_2 at NNLO:

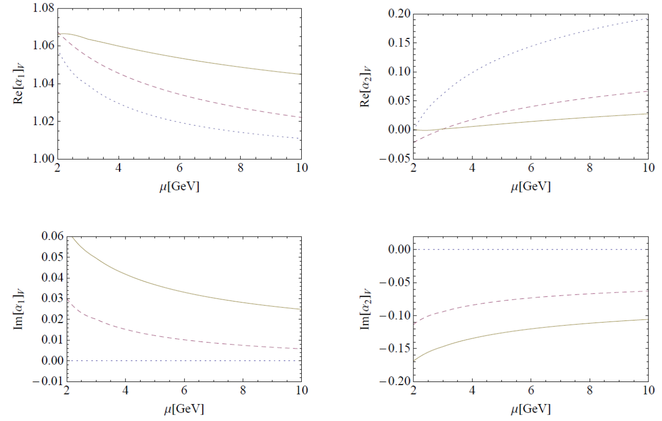
- Numerical results for colour-allowed α_1 and colour-suppress α_2 at NNLO: [*Beneke, Huber, Li, arXiv:0911.3655 [hep-ph]; Bell, arXiv:0902.1915 [hep-ph]; Bell and Pilipp, arXiv:0907.1016 [hep-ph]*]

$$\begin{aligned}
\alpha_1(\pi\pi) &= 1.009 + [0.023 + 0.010\, i]_{\rm NLO} + [0.026 + 0.028\, i]_{\rm NNLO} \\
&\quad - \left[\frac{r_{\rm sp}}{0.445}\right] \left\{[0.014]_{\rm LOsp} + [0.034 + 0.027 i\,]_{\rm NLOsp} + [0.008]_{\rm tw3}\right\} \\
&= 1.000^{+0.029}_{-0.069} + (0.011^{+0.023}_{-0.050}) i
\end{aligned}$$

$$\begin{aligned}
\alpha_2(\pi\pi) &= 0.220 - [0.179 + 0.077\, i]_{\rm NLO} - [0.031 + 0.050\, i]_{\rm NNLO} \\
&\quad + \left[\frac{r_{\rm sp}}{0.445}\right] \left\{[0.114]_{\rm LOsp} + [0.049 + 0.051 i\,]_{\rm NLOsp} + [0.067]_{\rm tw3}\right\} \\
&= 0.240^{+0.217}_{-0.125} + (-0.077^{+0.115}_{-0.078}) i
\end{aligned}$$

- individual NNLO corrections significant, but cancelled between vertex and spectator;
- precise prediction for α_1, but larger hadronic uncertainties for α_2 from $r_{\rm sp} = \frac{9 f_\pi \hat{f}_B}{m_b f_+^{B\pi}(0) \lambda_B}$;
- *The NNLO contributions have only a marginal effect on tree-dominated B decays*.

Dependence of $\alpha_{1,2}$ on the hard scale μ_h, only vertex part!



- dotted line: LO result
- dashed line: NLO result
- solid line: NNLO result

- The real parts on the scale dependence substantially reduced!
- The imaginary parts less pronounced, since it is just a first-order effect!
- Sizeable correction to imaginary part (phases), but cancellation between vertex and spectator-scattering!

Factorization test with the semi-leptonic data:

$$R_\pi \equiv \frac{\Gamma(B^- \to \pi^- \pi^0)}{d\Gamma(\bar{B}^0 \to \pi^+ l^- \bar{\nu})/dq^2\big|_{q^2=0}} = 3\pi^2 f_\pi^2 \, |V_{ud}|^2 \, |\alpha_1(\pi\pi) + \alpha_2(\pi\pi)|^2$$

- From exclusive semi-leptonic data (HFAG 2014):

  $$[\,|\alpha_1(\pi\pi) + \alpha_2(\pi\pi)|\,]_{\text{exp}} = 1.27 \pm 0.04$$

- Prediction with $\lambda_B = 0.35$ GeV:

  $$|\alpha_1(\pi\pi) + \alpha_2(\pi\pi)| = 1.24^{+0.16}_{-0.10}$$

- Good agreements observed, and hence supporting QCD factorization!

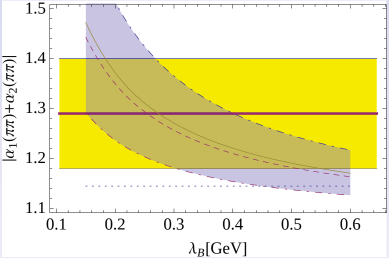


Figure from BHL2009 with obsolete data (yellow band)

$$[\,|\alpha_1(\pi\pi) + \alpha_2(\pi\pi)|\,]_{\text{exp}} = 1.29 \pm 0.11$$

- Colour-suppressed tree amplitude α_2 can be large only if it also has a large relative phase!

- It is interesting to extend to the other final state, $R_\rho = 1.75^{+0.37}_{-0.24}$ $(2.08^{+0.50}_{-0.46})$;

Branching ratios for tree-dominated decays:

	Theory I		Theory II		Experiment
$B^- \to \pi^-\pi^0$	$5.43^{+0.06}_{-0.06}{}^{+1.45}_{-0.84}$	$(\star)$	$5.82^{+0.07}_{-0.06}{}^{+1.42}_{-1.35}$	$(\star)$	$5.59^{+0.41}_{-0.40}$
$\bar{B}^0_d \to \pi^+\pi^-$	$7.37^{+0.86}_{-0.69}{}^{+1.22}_{-0.97}$	$(\star)$	$5.70^{+0.70}_{-0.55}{}^{+1.16}_{-0.97}$	$(\star)$	5.16 ± 0.22
$\bar{B}^0_d \to \pi^0\pi^0$	$0.33^{+0.11}_{-0.08}{}^{+0.42}_{-0.17}$		$0.63^{+0.12}_{-0.10}{}^{+0.64}_{-0.42}$		1.55 ± 0.19
			BELLE CKM 14:		0.90 ± 0.16
$B^- \to \pi^-\rho^0$	$8.68^{+0.42}_{-0.41}{}^{+2.71}_{-1.56}$	$(\star\star)$	$9.84^{+0.41}_{-0.40}{}^{+2.54}_{-2.52}$	$(\star\star)$	$8.3^{+1.2}_{-1.3}$
$B^- \to \pi^0\rho^-$	$12.38^{+0.90}_{-0.77}{}^{+2.18}_{-1.41}$	$(\star)$	$12.13^{+0.85}_{-0.73}{}^{+2.23}_{-2.17}$	$(\star)$	$10.9^{+1.4}_{-1.5}$
$\bar{B}^0 \to \pi^+\rho^-$	$17.80^{+0.62}_{-0.56}{}^{+1.76}_{-2.10}$	$(\star)$	$13.76^{+0.49}_{-0.44}{}^{+1.77}_{-2.18}$	$(\star)$	15.7 ± 1.8
$\bar{B}^0 \to \pi^-\rho^+$	$10.28^{+0.39}_{-0.39}{}^{+1.37}_{-1.42}$	$(\star\star)$	$8.14^{+0.34}_{-0.33}{}^{+1.35}_{-1.49}$	$(\star\star)$	7.3 ± 1.2
$\bar{B}^0 \to \pi^\pm\rho^\mp$	$28.08^{+0.27}_{-0.19}{}^{+3.82}_{-3.50}$	$(\dagger)$	$21.90^{+0.20}_{-0.12}{}^{+3.06}_{-3.55}$	$(\dagger)$	23.0 ± 2.3
$\bar{B}^0 \to \pi^0\rho^0$	$0.52^{+0.04}_{-0.03}{}^{+1.11}_{-0.43}$		$1.49^{+0.07}_{-0.07}{}^{+1.77}_{-1.29}$		2.0 ± 0.5
$B^- \to \rho^-_L\rho^0_L$	$18.42^{+0.23}_{-0.21}{}^{+3.92}_{-2.55}$	$(\star\star)$	$19.06^{+0.24}_{-0.22}{}^{+4.59}_{-4.22}$	$(\star\star)$	$22.8^{+1.8}_{-1.9}$
$\bar{B}^0_d \to \rho^+_L\rho^-_L$	$25.98^{+0.85}_{-0.77}{}^{+2.93}_{-3.43}$	$(\star\star)$	$20.66^{+0.68}_{-0.62}{}^{+2.99}_{-3.75}$	$(\star\star)$	$23.7^{+3.1}_{-3.2}$
$\bar{B}^0_d \to \rho^0_L\rho^0_L$	$0.39^{+0.03}_{-0.03}{}^{+0.83}_{-0.36}$		$1.05^{+0.05}_{-0.04}{}^{+1.62}_{-1.04}$		$0.55^{+0.22}_{-0.24}$

Theory I: $f_+^{B\pi}(0) = 0.25 \pm 0.05$, $A_0^{B\rho}(0) = 0.30 \pm 0.05$, $\lambda_B(1\,\text{GeV}) = 0.35 \pm 0.15$ GeV

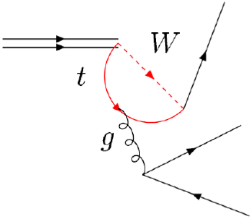
Theory II: $f_+^{B\pi}(0) = 0.23 \pm 0.03$, $A_0^{B\rho}(0) = 0.28 \pm 0.03$, $\lambda_B(1\,\text{GeV}) = 0.20^{+0.05}_{-0.00}$ GeV

- First error from γ and V_{cb}; V_{ub} uncertainty not included;
- Second error from hadronic inputs; form-factor uncertainty not included for marked modes;
- Theory II: small λ_B and form-factor hypothesis are more favoured;

Summary for tree-dominated B decays:

- NNLO corrections individually sizeable, but ultimately not large due to cancellation between vertex and spectator;
- Colour-allowed modes well described by factorization, less the purely colour-suppressed ones;
- NNLO corrections are end of the road at leading power; No indication of further large radiative corrections;
- Size of the spectator-scattering contributions in QCDF determined by $\lambda_B^{-1}(\mu) = \int \frac{d\omega}{\omega} \phi_B(\omega; \mu)$:
  - the current $B \to \pi\pi$ and $\pi\rho$ data prefers smaller $\lambda_B \sim 200$ MeV, compared to QCD sum rule estimate $\lambda_B(1 \text{ GeV}) \sim 350 - 500$ MeV; [*Braun, Ivanov, Korchemsky '03*]
  - λ_B can be measured in $B \to \gamma\ell\nu$ decays: $\Gamma(B \to \gamma\ell\nu) \propto 1/\lambda_B^2$, NLO+$1/m_b$ corrections; [*Beneke, Rohrwild '11; Braun, Khodjamirian '12*]
  - weak constraint from BaBar '09 data: $\lambda_B > 115$ MeV; Belle '15 data: $\lambda_B > 217$ MeV [*Belle, 1504.05831*]
  - much progress in our theoretical understanding of $\phi_B(\omega; \mu)$; [*Bell, Feldmann, Wang, Yip, '13; Braun, Manashov '14; Feldmann, Lange, Wang, '14*]
- Other attempts for enhanced colour-suppressed tree amplitude:
  - $1/m_b$ power correction as a "nuisance parameter": $a_2 \to a_2 \left(1 + \rho_c e^{i\phi_c}\right)$; [*Cheng, Chua, '09*]
  - introduce the Glauber gluon effects in spectator amplitudes; [*Li, Mishima, arXiv:1407.7647 [hep-ph]*]
  - the renormalization scale for spectator interactions is much lower after applying the principle of maximum conformality, $Q_1^H \simeq 0.75 - 0.90$ GeV; [*Qiao, Zhu, Wu, Brodsky, arXiv:1408.1158 [hep-ph]*]

Penguin-dominated decay modes:



- Two-loop vertex corrections:

G. Bell, M. Beneke, T. Huber and X. Q. Li, *"Two-loop current – current operator contribution to the non-leptonic QCD penguin amplitude,"* Phys. Lett. B **750** (2015) 348 [arXiv:1507.03700 [hep-ph]].

C. S. Kim and Y. W. Yoon, *"Order α_s^2 magnetic penguin correction for B decay to light mesons,"* JHEP **1111** (2011) 003 [arXiv:1107.1601 [hep-ph]].

Motivation for NNLO corrections to penguin amplitudes:

- Many decay channels are penguin-dominated, very sensitive to penguin amplitudes α_4^p;

$$\mathcal{A}_{B^-\to\pi^-\bar{K}^0} = \lambda_c^{(s)}[P_c - \frac{1}{3}P_c^{C,EW}] + \lambda_u^{(s)}[P_u - \frac{1}{3}P_u^{C,EW}]$$

$$\sqrt{2}\,\mathcal{A}_{B^-\to\pi^0K^-} = \lambda_c^{(s)}[P_c + P_c^{EW} + \frac{2}{3}P_c^{C,EW}] + \lambda_u^{(s)}[T + C + P_u + P_u^{EW} + \frac{2}{3}P_u^{C,EW}]$$

$$\mathcal{A}_{\bar{B}^0\to\pi^+K^-} = \lambda_c^{(s)}[P_c + \frac{2}{3}P_c^{C,EW}] + \lambda_u^{(s)}[T + P_u + \frac{2}{3}P_u^{C,EW}]$$

$$\sqrt{2}\,\mathcal{A}_{\bar{B}^0\to\pi^0\bar{K}^0} = \lambda_c^{(s)}[-P_c + P_c^{EW} + \frac{1}{3}P_c^{C,EW}] + \lambda_u^{(s)}[C - P_u + P_u^{EW} + \frac{1}{3}P_u^{C,EW}]$$

Mode	Br $[10^{-6}]$	A_{CP}	S_{CP}
$B^+\to\pi^+K^0$	$23.79^{+0.75}_{-0.75}$	-0.015 ± 0.019	
$B^+\to\pi^0K^+$	$12.94^{+0.52}_{-0.51}$	0.040 ± 0.021	
$B^0\to\pi^-K^+$	$19.57^{+0.53}_{-0.52}$	-0.082 ± 0.006	
$B^0\to\pi^0K^0$	$9.93^{+0.49}_{-0.49}$	-0.01 ± 0.10	0.57 ± 0.17

- Due to CKM unitarity, the amplitude for a $\bar{B}\to\bar{f}$ decay can always be written as:

$$\bar{A}(\bar{B}\to\bar{f}) = \lambda_u^{(D)}\,[T+\ldots] + \lambda_c^{(D)}\,[P_c+\ldots]$$

- To predict direct CP asymmetries, need calculate both T and P_c to a high precision level;

The dominant contribution to a_4^p: I

- The leading penguin amplitudes including the $\mathcal{O}(\alpha_s^2)$ spectator terms: [*Beneke, Jäger '06*]

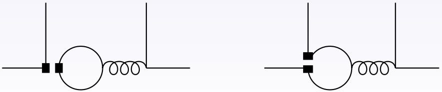
$$a_4^u(\pi\pi) = -0.029 - [0.002 + 0.001i]_{\rm V} + [0.003 - 0.013i]_{\rm P} + [??+??i]_{\rm NNLO}$$
$$+ \left[\frac{r_{\rm sp}}{0.485}\right]\left\{[0.001]_{\rm LO} + [0.001 + 0.001i]_{\rm HV} - [0.000 + 0.001i]_{\rm HP} + [0.001]_{\rm tw3}\right\}$$

$$a_4^c(\pi\pi) = -0.029 - [0.002 + 0.001i]_{\rm V} - [0.001 + 0.007i]_{\rm P} + [??+??i]_{\rm NNLO}$$
$$+ \left[\frac{r_{\rm sp}}{0.485}\right]\left\{[0.001]_{\rm LO} + [0.001 + 0.001i]_{\rm HV} + [0.000 - 0.000i]_{\rm HP} + [0.001]_{\rm tw3}\right\}$$

- Although $\mathcal{O}(\alpha_s^2)$ spectator effect on $\alpha_{1,2}$ significant, but small on a_4^p due to numerical cancellation $\Rightarrow$ *how about the 2-loop vertex corrections to a_4^p? significant or marginal?*

- The NNLO correction to a_4^p comes mainly from penguin contractions with $Q_{1,2}$ insertions:

$$\begin{pmatrix} C_1 & C_2 & C_3 & C_4 \\ -0.285 & 1.010 & -0.006 & -0.087 \\ C_5 & C_6 & C_7^{\rm eff} & C_8^{\rm eff} \\ 0.0004 & 0.001 & -0.302 & -0.164 \end{pmatrix}$$



Matching from QCD to SCET$_{\mathrm{I}}$:

- The CMM operator basis in full QCD:

$$Q_1^p = (\bar{p}_L \gamma^\mu T^A b_L)\,(\bar{D}_L \gamma_\mu T^A p_L), \qquad Q_2^p = (\bar{p}_L \gamma^\mu b_L)\,(\bar{D}_L \gamma_\mu p_L),$$

$$+\ \text{QCD penguin operators}$$

$$+\ \text{evanescent operators}$$

- The nonlocal operator basis in SCET:

$$O_1 = \bar{\chi}\,\frac{\not{n}_-}{2}(1-\gamma_5)\chi\;\bar{\xi}\,\not{n}_+(1-\gamma_5)h_v\,,$$

$$\tilde{O}_n = \bar{\xi}\,\gamma_\perp^\alpha \gamma_\perp^{\mu_1} \gamma_\perp^{\mu_2} \ldots \gamma_\perp^{\mu_{2n-2}} \chi\;\bar{\chi}(1+\gamma_5)\gamma_{\perp\alpha}\gamma_{\perp\mu_{2n-2}}\gamma_{\perp\mu_{2n-3}}\cdots\gamma_{\perp\mu_1} h_v,$$

$\tilde{O}_1 - O_1/2$ is another evanescent operator

- The master formulae at LO, NLO, and NNLO read, respectively,

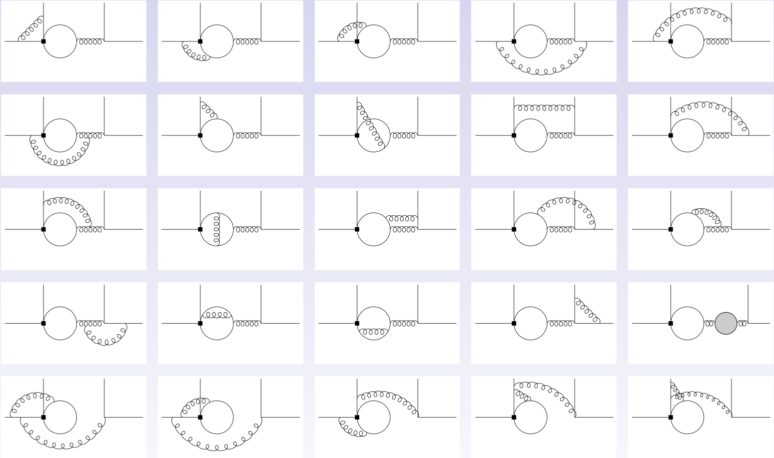
$$\widetilde{T}_i^{(0)} = \widetilde{A}_{i1}^{(0)}\,, \qquad \widetilde{T}_i^{(1)} = \widetilde{A}_{i1}^{(1)\mathrm{nf}} + Z_{ij}^{(1)}\,\widetilde{A}_{j1}^{(0)} + \ldots\,,$$

$$\widetilde{T}_i^{(2)} = \widetilde{A}_{i1}^{(2)\mathrm{nf}} + Z_{ij}^{(1)}\,\widetilde{A}_{j1}^{(1)} + Z_{ij}^{(2)}\,\widetilde{A}_{j1}^{(0)} + Z_\alpha^{(1)}\,\widetilde{A}_{i1}^{(1)\mathrm{nf}} + (-i)\,\delta m^{(1)}\,\widetilde{A}_{i1}^{\prime(1)\mathrm{nf}}$$

$$+ Z_{ext}^{(1)}\,\big[\widetilde{A}_{i1}^{(1)\mathrm{nf}} + Z_{ij}^{(1)}\,\widetilde{A}_{j1}^{(0)}\big] - \widetilde{T}_i^{(1)}\big[C_{FF}^{(1)} + \widetilde{Y}_{11}^{(1)}\big] + \ldots$$

NNLO penguin amplitudes with $Q_{1,2}^p$ insertion:

- On the QCD side, relevant Feynman diagrams with $Q_{1,2}^p$ insertion ($\sim$ 70 diagrams):



- The quark in the fermion loop can either be massless (for $p = u$) or massive (for $p = c$);
- In the massive case, a genuine two-loop, two-scale problem involved; also $s_c = m_c^2/m_b^2$;

Calculate the MIs in a canonical basis:

- For the massive charm-type insertions, 29 new MIs found and computed based on the DE approach in a canonical basis; [*Bell, Huber '14*]

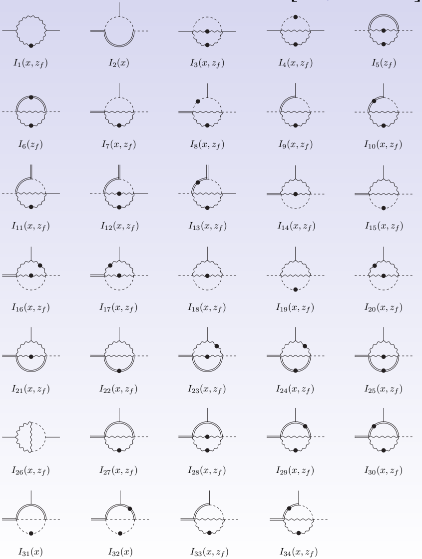
- Choose an "optimal" basis of MIs, so that the DEs decouple order-by-order in ϵ expansion, and the dependence of MIs on the kinematic variables is factorised from that on the ϵ: [*Henn, 1304.1806*]

$$\frac{\partial}{\partial x_m}\vec{M}(\epsilon, x_n) = \epsilon\, A_m(x_n)\, \vec{M}(\epsilon, x_n)$$

- The above simplified form of DEs trivial to solve in terms of iterated integrals; [*Bell, Huber '14*]

- Together with boundary conditions, analytic results of the MIs obtained in terms of generalised HPLs (or Goncharov polylogarithms); [*Maitre, 0703052*]

- The analytic results make it much easier to handel the threshold at $\bar{u}m_b^2 = 4m_c^2$ and the convolution integral $\int_0^1 du T^I(u)\phi(u)$; [*Bell, Beneke, Huber, Li '15*]



Numerical result for a_4^p with $Q_{1,2}^p$ insertion:

- For $Q_{1,2}^u$ insertion:

$$a_4^u(\pi\bar{K})/10^{-2} = -2.87 - [0.09 + 0.09i]_{\mathrm{V}_1} + [0.49 - 1.32i]_{\mathrm{P}_1} - [0.32 + 0.71i]_{\mathrm{P}_2}$$
$$+ \left[\frac{r_{\mathrm{sp}}}{0.434}\right] \left\{[0.13]_{\mathrm{LO}} + [0.14 + 0.12i]_{\mathrm{HV}} - [0.01 - 0.05i]_{\mathrm{HP}} + [0.07]_{\mathrm{tw3}}\right\}$$
$$= (-2.46^{+0.49}_{-0.24}) + (-1.94^{+0.32}_{-0.20})i$$

$\sim 15\%$ correction to real part, $\sim 40\%$ to imaginary part for $a_4^u(\pi\bar{K})$

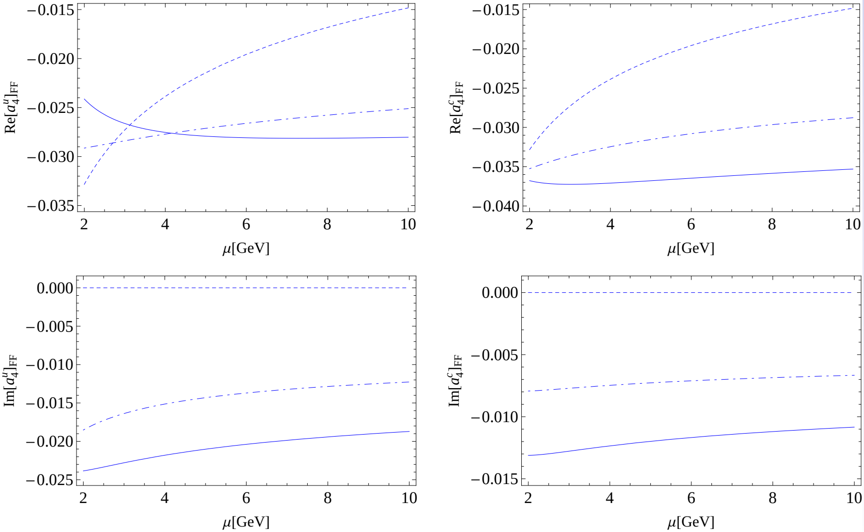
- For $Q_{1,2}^c$ insertion:

$$a_4^c(\pi\bar{K})/10^{-2} = -2.87 - [0.09 + 0.09i]_{\mathrm{V}_1} + [0.05 - 0.62i]_{\mathrm{P}_1} - [0.77 + 0.50i]_{\mathrm{P}_2}$$
$$+ \left[\frac{r_{\mathrm{sp}}}{0.434}\right] \left\{[0.13]_{\mathrm{LO}} + [0.14 + 0.12i]_{\mathrm{HV}} + [0.01 + 0.03i]_{\mathrm{HP}} + [0.07]_{\mathrm{tw3}}\right\}$$
$$= (-3.34^{+0.43}_{-0.27}) + (-1.05^{+0.45}_{-0.36})i$$

$\sim 25\%$ correction to real part, $\sim 50\%$ to imaginary part for $a_4^c(\pi\bar{K})$

Numerical result for a_4^p with $Q_{1,2}^p$ insertion: III

- With the NNLO corrections included, the scale dependence of a_4^u and a_4^c reduced:



form-factor term only! red-dashed: LO; red-line: NLO; qquad blue-line: NNLO

The full QCD penguin amplitude in QCDF:

- In QCDF, the full QCD penguin amplitude is defined as: [*Beneke, Neubert '03*]

$$\hat{\alpha}_4^p(M_1M_2) = a_4^p(M_1M_2) \pm r_\chi^{M_2} a_6^p(M_1M_2) + \beta_3^p(M_1M_2)$$

  - a_4^p: the only leading-power contribution, with its real part being of order -0.03;
  - $r_\chi^{M_2} a_6^p(M_1M_2)$: the power-suppressed scalar penguin amplitude; very small when $M_2 = V$, but larger than $a_4^p(M_1M_2)$ for $M_2 = P$ due to the "chiral enhancement" factor r_χ^P;
  - β_3^p: $1/m_b$-suppressed annihilation contribution; can only be estimated based on a two-parameter model,

$$\int_0^1 \frac{dx}{x} \to X_A = (1 + \varrho_A\, e^{i\phi_A}) \ln \frac{m_B}{\Lambda_h}, \text{ with } \Lambda_h = 500 \text{ MeV};$$

  - the interference between $a_4^p(M_1M_2)$ and $a_6^p(M_1M_2)$ is constructive for PP, but destructive for VP;

- The impact of a correction to a_4^p is always diluted by the other power-suppressed terms;
- When $M_2 = V$, the computation of a_4^p ascertains the SD contribution, and hence the direct CP asymmetry, but there is an uncertain annihilation contribution of similar size;
- When $M_2 = P$, there is another NNLO SD contribution from a_6^p, difficult though not impossible to calculate, since it is power-suppressed;

Direct CP asymmetries in $B \to \pi K$ decays:

f	NLO	NNLO	NNLO + LD	Exp
$\pi^- \bar{K}^0$	$0.71^{+0.13}_{-0.14}{}^{+0.21}_{-0.19}$	$0.77^{+0.14}_{-0.15}{}^{+0.23}_{-0.22}$	$0.10^{+0.02}_{-0.02}{}^{+1.24}_{-0.27}$	-1.7 ± 1.6
$\pi^0 K^-$	$9.42^{+1.77}_{-1.76}{}^{+1.87}_{-1.88}$	$10.18^{+1.91}_{-1.90}{}^{+2.03}_{-2.62}$	$-1.17^{+0.22}_{-0.22}{}^{+20.00}_{-6.62}$	4.0 ± 2.1
$\pi^+ K^-$	$7.25^{+1.36}_{-1.36}{}^{+2.13}_{-2.58}$	$8.08^{+1.52}_{-1.51}{}^{+2.52}_{-2.65}$	$-3.23^{+0.61}_{-0.61}{}^{+19.17}_{-3.36}$	-8.2 ± 0.6
$\pi^0 \bar{K}^0$	$-4.27^{+0.83}_{-0.77}{}^{+1.48}_{-2.23}$	$-4.33^{+0.84}_{-0.78}{}^{+3.29}_{-2.32}$	$-1.41^{+0.27}_{-0.25}{}^{+5.54}_{-6.10}$	1 ± 10
$\delta(\pi \bar{K})$	$2.17^{+0.40}_{-0.40}{}^{+1.39}_{-0.74}$	$2.10^{+0.39}_{-0.39}{}^{+1.40}_{-2.86}$	$2.07^{+0.39}_{-0.39}{}^{+2.76}_{-4.55}$	12.2 ± 2.2
$\Delta(\pi \bar{K})$	$-1.15^{+0.21}_{-0.22}{}^{+0.55}_{-0.84}$	$-0.88^{+0.16}_{-0.17}{}^{+1.31}_{-0.91}$	$-0.48^{+0.09}_{-0.09}{}^{+1.09}_{-1.15}$	-14 ± 11

$$\delta(\pi \bar{K}) = A_{\rm CP}(\pi^0 K^-) - A_{\rm CP}(\pi^+ K^-)$$

$$\Delta(\pi \bar{K}) = A_{\rm CP}(\pi^+ K^-) + \frac{\Gamma_{\pi^- \bar{K}^0}}{\Gamma_{\pi^+ K^-}} A_{\rm CP}(\pi^- \bar{K}^0) - \frac{2\Gamma_{\pi^0 K^-}}{\Gamma_{\pi^+ K^-}} A_{\rm CP}(\pi^0 K^-) - \frac{2\Gamma_{\pi^0 \bar{K}^0}}{\Gamma_{\pi^+ K^-}} A_{\rm CP}(\pi^0 \bar{K}^0)$$

- "NLO" and "NNLO": only perturbatively calculable SD contribution included;
- "NNLO+LD": power-suppressed spectator and annihilation terms included back;
- For πK, the NNLO change is minor, since a_4^c only part of the SD penguin amplitude;
- NNLO correction does not help resolving the observed πK CP asymmetry puzzle;

Direct CP asymmetries in $B \to \pi K^*$ and ρK decays:

f	NLO	NNLO	NNLO + LD	Exp
$\pi^- \bar{K}^{*0}$	$1.36\,^{+0.25}_{-0.26}\,^{+0.60}_{-0.47}$	$1.49\,^{+0.27}_{-0.29}\,^{+0.69}_{-0.56}$	$0.27\,^{+0.05}_{-0.05}\,^{+3.18}_{-0.67}$	-3.8 ± 4.2
$\pi^0 K^{*-}$	$13.85\,^{+2.40}_{-2.70}\,^{+5.84}_{-5.86}$	$18.16\,^{+3.11}_{-3.52}\,^{+7.79}_{-10.57}$	$-15.81\,^{+3.01}_{-2.83}\,^{+69.35}_{-15.39}$	-6 ± 24
$\pi^+ K^{*-}$	$11.18\,^{+2.00}_{-2.15}\,^{+9.75}_{-10.62}$	$19.70\,^{+3.37}_{-3.80}\,^{+10.54}_{-11.42}$	$-23.07\,^{+4.35}_{-4.05}\,^{+86.20}_{-20.64}$	-23 ± 6
$\pi^0 \bar{K}^{*0}$	$-17.23\,^{+3.33}_{-3.00}\,^{+7.59}_{-12.57}$	$-15.11\,^{+2.93}_{-2.65}\,^{+12.34}_{-10.64}$	$2.16\,^{+0.39}_{-0.42}\,^{+17.53}_{-36.80}$	-15 ± 13
$\delta(\pi \bar{K}^*)$	$2.68\,^{+0.72}_{-0.67}\,^{+5.44}_{-4.30}$	$-1.54\,^{+0.45}_{-0.58}\,^{+4.60}_{-9.19}$	$7.26\,^{+1.21}_{-1.34}\,^{+12.78}_{-20.65}$	17 ± 25
$\Delta(\pi \bar{K}^*)$	$-7.18\,^{+1.38}_{-1.28}\,^{+3.38}_{-5.35}$	$-3.45\,^{+0.67}_{-0.59}\,^{+9.48}_{-4.95}$	$-1.02\,^{+0.19}_{-0.18}\,^{+4.32}_{-7.86}$	-5 ± 45
$\rho^- \bar{K}^0$	$0.38\,^{+0.07}_{-0.07}\,^{+0.16}_{-0.27}$	$0.22\,^{+0.04}_{-0.04}\,^{+0.19}_{-0.17}$	$0.30\,^{+0.06}_{-0.06}\,^{+2.28}_{-2.39}$	-12 ± 17
$\rho^0 K^-$	$-19.31\,^{+3.42}_{-3.61}\,^{+13.95}_{-8.96}$	$-4.17\,^{+0.75}_{-0.80}\,^{+19.26}_{-19.52}$	$43.73\,^{+7.07}_{-7.62}\,^{+44.00}_{-137.77}$	37 ± 11
$\rho^+ K^-$	$-5.13\,^{+0.95}_{-0.97}\,^{+6.38}_{-4.02}$	$1.50\,^{+0.29}_{-0.27}\,^{+8.69}_{-10.36}$	$25.93\,^{+4.43}_{-4.90}\,^{+25.40}_{-75.63}$	20 ± 11
$\rho^0 \bar{K}^0$	$8.63\,^{+1.59}_{-1.65}\,^{+2.31}_{-1.69}$	$8.99\,^{+1.66}_{-1.71}\,^{+3.60}_{-7.44}$	$-0.42\,^{+0.08}_{-0.08}\,^{+19.49}_{-8.78}$	6 ± 20
$\delta(\rho \bar{K})$	$-14.17\,^{+2.80}_{-2.96}\,^{+7.98}_{-5.39}$	$-5.67\,^{+0.96}_{-1.01}\,^{+10.86}_{-9.79}$	$17.80\,^{+3.15}_{-3.01}\,^{+19.51}_{-62.44}$	17 ± 16
$\Delta(\rho \bar{K})$	$-8.75\,^{+1.62}_{-1.66}\,^{+4.78}_{-6.48}$	$-10.84\,^{+1.98}_{-2.09}\,^{+11.67}_{-9.09}$	$-2.43\,^{+0.46}_{-0.42}\,^{+4.60}_{-19.43}$	-37 ± 37

- For πK^* a_6^c is small; for ρK cancellation between a_4^c and a_6^c;

$\hookrightarrow$ large modification for $\pi^0 K^{*-}$, $\pi^+ K^{*-}$, $\rho^0 K^-$, $\rho^+ K^-$, less for the others;

The penguin-to-tree ratio in QCDF: I

- The magnitude of the penguin-to-tree ratio can be extracted from data, and provides a crucial test of the QCDF approach: [*Beneke, Neubert '03*]

$$\left|\frac{\hat{\alpha}_4^c(\pi\bar{K})}{\alpha_1(\pi\pi)+\alpha_2(\pi\pi)}\right| = \left|\frac{V_{ub}}{V_{cb}}\right| \frac{f_\pi}{f_K} \left[\frac{\Gamma_{\pi^-\bar{K}^0}}{2\Gamma_{\pi^-\pi^0}}\right]^{1/2}$$

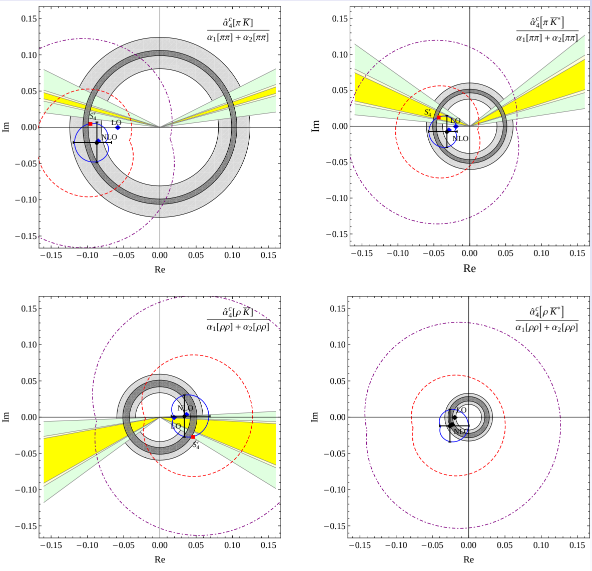
- The relative strong phase of the ratio can be probed by considering: [*Beneke, Neubert '03*]

$$-\sin\psi + \frac{\mathrm{Im}\,\mathcal{R}}{\mathrm{Re}\,\mathcal{R}}\cos\psi = \frac{1}{2\sin\gamma\,\mathrm{Re}\,\mathcal{R}} \left|\frac{V_{cs}}{V_{us}}\right| \frac{f_\pi}{f_K} \frac{\Gamma_{\pi^+K^-}}{\sqrt{2\Gamma_{\pi^-\pi^0}\Gamma_{\pi^-\bar{K}^0}}} A_{\mathrm{CP}}(\pi^+K^-)$$

  - ψ: the phase of the amplitude ratio;
  - $\mathcal{R} = \frac{\alpha_1(\pi\bar{K})+\hat{\alpha}_4^u(\pi\bar{K})}{\alpha_1(\pi\pi)+\alpha_2(\pi\pi)}$

- For $\pi\bar{K}$, $\pi\bar{K}^*$, normalized to the $\pi\pi$ final state, thus free of $f_+^{B\to\pi}$ uncertainty;
- For $\rho\bar{K}$, $\rho_L\bar{K}_L^*$, normalized to the $\rho_L\rho_L$ final state, thus free of $A_0^{B\to\rho}$ uncertainty;
- Together with the exp. data, these two equations provide useful information on the ratio;

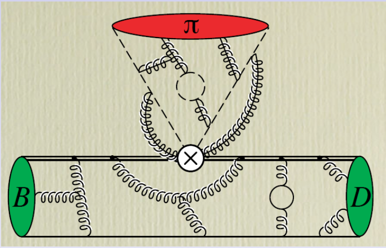
The penguin-to-tree ratio in QCDF: II

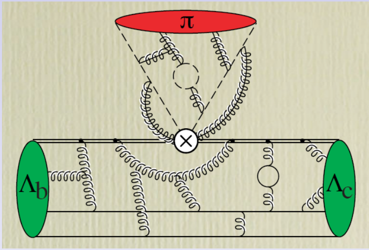


- Different penguin magnitudes of *PP* vs. *PV*, *VP* and *VV* clearly reflected in the data as predicted in QCDF;
- Nearly circular contours: varying $\phi_A \in (0, 2\pi)$ for fixed $\varrho_A = 1, 2, 3$;
- S_4': favoured parameter set, $\varrho_A = 1$ and $\phi_A = -55^\circ$ (PP), $\phi_A = -45^\circ$ (PV), $\phi_A = -50^\circ$ (VP); almost universal for different modes!
- Despite sizable NNLO correction to a_4^c, difference between NNLO and NLO small due to “dilution” and partial cancellation in amplitude ratio;
- An annihilation of 0.02 to 0.03 required, except for the longitudinal *VV*;

- Only the πK CP asymmetry now requires a value larger than $\varrho_A = 1$ for a perfect fit;

The NNLO vertex correction to $\bar{B}_{(s)} \to D_{(s)}^{(*)+} L^-$:





- Two-loop vertex corrections to T^I:

M. Beneke, G. Buchalla, M. Neubert and C. T. Sachrajda, *"QCD factorization for exclusive, nonleptonic B meson decays: General arguments and the case of heavy light final states,"* Nucl. Phys. B **591** (2000) 313 [hep-ph/0006124].

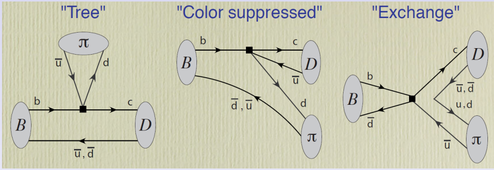
T. Huber and S. Kränkl, *"Two-loop master integrals for non-leptonic heavy-to-heavy decays,"* JHEP **1504** (2015) 140 [arXiv:1503.00735 [hep-ph]].

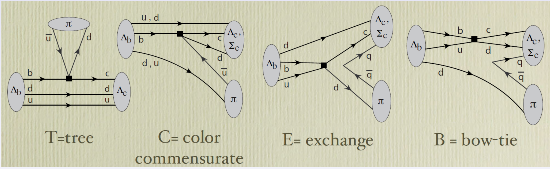
T. Huber, S. Kränkl and Xin-Qiang Li, *"Two-body non-leptonic heavy-to-heavy decays at NNLO in QCD factorization,"* arXiv:1606.02888 [hep-ph].

Features for $\bar{B}_{(s)} \to D_{(s)}^{(*)+} L^-$:

- Only colour-allowed tree amplitude, no colour-suppressed tree nor penguin contributions;
- Only vertex kernels to NF, spectator-scattering and weak annihilation are power-suppressed;

  [*Beneke, Buchalla, Neubert, Sachrajda, '00; Bauer, Pirjol, Stewart, '01; Leibovich, Ligeti, Stewart, Wise, '03*]





- Factorization theorem is established in these class-I decays;

  [*Beneke, Buchalla, Neubert, Sachrajda, '00; Bauer, Pirjol, Stewart, '01; Leibovich, Ligeti, Stewart, Wise, '03*]

- Motivation for NNLO: NLO results colour-suppressed alongside with small WC; At NNLO colour suppression gets lifted and large WC re-enters, comparable in size to NLO;

  $\hookrightarrow$ how about the NNLO corrections?

Factorization formula for $\bar{B}_{(s)} \to D_{(s)}^{(*)+} L^-$:

- In the heavy-quark limit, the decay amplitude for $\bar{B}^0 \to D^+\pi^-$ is given by: [*BBNS, '00*]

$$\langle D^+\pi^-|Q_i|\bar{B}^0\rangle = \sum_j F_j^{B\to D}(m_\pi^2)\int_0^1 du\, T_{ij}^I(u)\phi_L(u)$$

- Demonstration of factorization based on Feynman diagrams at two-loop order: [*BBNS, '00*]

$$F_{B\to D}^{(0)} \cdot T^{(0)} * \Phi_\pi^{(0)} = A^{(0)}$$

$$F_{B\to D}^{(0)} \cdot T^{(1)} * \Phi_\pi^{(0)} = A^{(1)} - F_{B\to D}^{(1)} \cdot T^{(0)} * \Phi_\pi^{(0)} - F_{B\to D}^{(0)} \cdot T^{(0)} * \Phi_\pi^{(1)}$$

$$F_{B\to D}^{(0)} \cdot T^{(2)} * \Phi_\pi^{(0)} = A^{(2)} - F_{B\to D}^{(0)} \cdot T^{(1)} * \Phi_\pi^{(1)} - F_{B\to D}^{(1)} \cdot T^{(1)} * \Phi_\pi^{(0)}$$
$$- F_{B\to D}^{(2)} \cdot T^{(0)} * \Phi_\pi^{(0)} - F_{B\to D}^{(0)} \cdot T^{(0)} * \Phi_\pi^{(2)} - F_{B\to D}^{(1)} \cdot T^{(0)} * \Phi_\pi^{(1)}$$

- Proof within SCET: factorization $\Leftrightarrow$ separation of scales and decoupling $\Leftrightarrow$ $Q_i = Q_c \times Q_s$ at the Langrangian level $\mathcal{L} = \mathcal{L}_c^0 + \mathcal{L}_s^0$; [*Bauer, Pirjol, Stewart, '01*]

$$\langle D\pi|(\bar{c}b)(\bar{u}d)|B\rangle = N\,\xi(v\cdot v')\int_0^1 dx\, T(x,\mu)\,\phi_\pi(x,\mu)$$

Universal functions:

$$\langle D^{(*)}|O_s|B\rangle = \xi(v\cdot v')$$
$$\langle \pi|O_c(x)|0\rangle = f_\pi \phi_\pi(x)$$

Calculate T, $\alpha_s(Q)$

$Q = E_\pi, m_b, m_c$

corrections will be $\Lambda/m_c \sim 30\%$

Explicit calculation of NNLO vertex corrections to T^I:

- Matching from QCD to SCET: complicated by massive charm quark!

$$\langle Q_i \rangle = T_i \langle O^{\rm QCD} \rangle + T_i' \langle O'^{\rm QCD} \rangle + \sum_{a>1} \left[H_{ia} \langle O_a \rangle + H_{ia}' \langle O_a' \rangle \right]$$

$$Q^{(\prime)\rm QCD} = \left[\bar{q} \frac{\not{n}_-}{2} (1-\gamma_5) q \right] \left[\bar{c} \not{n}_+ (1 \mp \gamma_5) b \right] = C_{\bar{q}q} C_{FF}^{\rm D} O_1^{(\prime)} + C_{\bar{q}q} C_{FF}^{\rm ND} O_1'^{()}$$

- The SCET operator basis: two different chiral structures due to massive charm!

$$O_1^{(\prime)} = \bar{\chi} \frac{\not{n}_-}{2} (1-\gamma_5) \chi \quad \bar{h}_{v'} \not{n}_+ (1 \mp \gamma_5) h_v$$

$$O_2^{(\prime)} = \bar{\chi} \frac{\not{n}_-}{2} (1-\gamma_5) \gamma_\perp^\alpha \gamma_\perp^\beta \chi \quad \bar{h}_{v'} \not{n}_+ (1 \mp \gamma_5) \gamma_{\perp,\beta} \gamma_{\perp,\alpha} h_v$$

$$O_3^{(\prime)} = \bar{\chi} \frac{\not{n}_-}{2} (1-\gamma_5) \gamma_\perp^\alpha \gamma_\perp^\beta \gamma_\perp^\gamma \gamma_\perp^\delta \chi \quad \bar{h}_{v'} \not{n}_+ (1 \mp \gamma_5) \gamma_{\perp,\delta} \gamma_{\perp,\gamma} \gamma_{\perp,\beta} \gamma_{\perp,\alpha} h_v$$

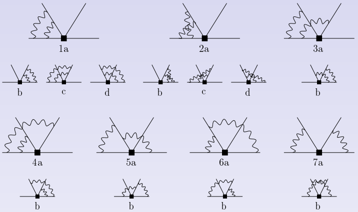
- Final master formulas for the scattering kernels:

$$T_i^{(0)} = A_{i1}^{(0)}, \qquad T_i^{(1)} = A_{i1}^{(1)nf} + Z_{ij}^{(1)} A_{j1}^{(0)}$$

$$T_i^{(2)} = A_{i1}^{(2)nf} + Z_{ij}^{(1)} A_{j1}^{(1)} + Z_{ij}^{(2)} A_{j1}^{(0)} + Z_\alpha^{(1)} A_{i1}^{(1)nf} - \hat{T}_i^{(1)} \left[C_{FF}^{\rm D(1)} + Y_{11}^{(1)} - Z_{ext}^{(1)} \right]$$
$$- C_{FF}^{\rm ND(1)} \hat{T}_i'^{(1)} + (-i)\delta m_b^{(1)} A_{i1}^{*(1)nf} + (-i)\delta m_c^{(1)} A_{i1}^{**(1)nf} - \sum_{b\neq 1} H_{ib}^{(1)} Y_{b1}^{(1)}$$

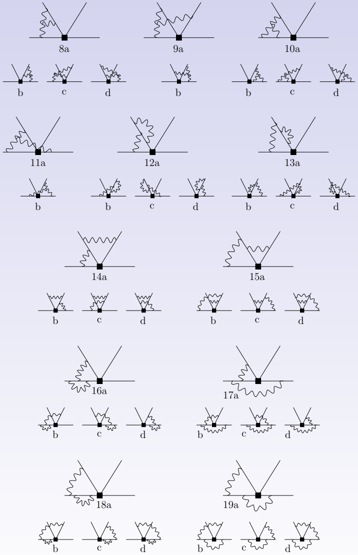
Explicit calculation of NNLO vertex corrections to T^I:

- Two-loop non-factorizable Feynman diagrams contributing to $A_{i1}^{(2)nf}$: [*BBNS '01*]



  - about 70 two-loop diagrams;
  - Laporta reduction based on IBP;
  - 23 new MIs and solved using DEs in a canonical basis;

- Both UV and IR div. cancelled analytically, thus factorization established!



Predictions for $a_1(D^{(*)+}L^-)$:

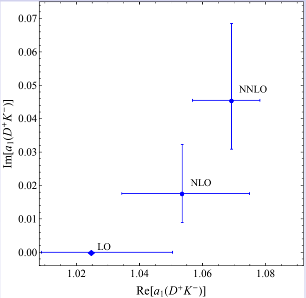
- Numerical results for $a_1(D^+K^-)$:

$$a_1(D^+K^-) = 1.025 + [0.029 + 0.018i]_{\text{NLO}} + [0.016 + 0.028i]_{\text{NNLO}}$$

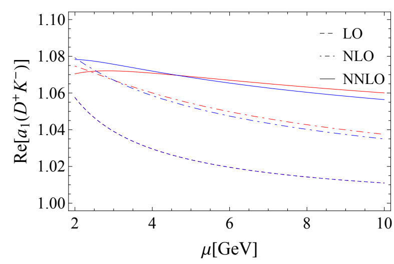
$$= (1.069^{+0.009}_{-0.012}) + (0.046^{+0.023}_{-0.015})i$$

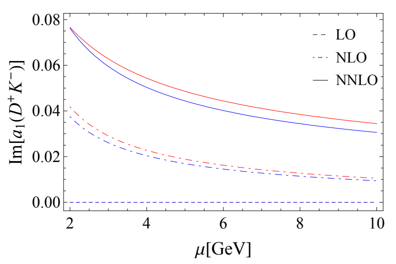
$\sim 2\%$ correction to real part, $\sim 60\%$ to imaginary part.

both the NLO and NNLO contribute constructively to the LO result.



- Dependence on μ and quark-mass scheme: pole (in blue) and $\overline{\text{MS}}$ (red) for m_b and m_c;





Test of factorization in class-I decays:

- Particularly clean and direct method: [*Bjorken, '89; Neubert and Stech, '97*]

$$R_L^{(*)} \equiv \frac{\Gamma(\bar{B}_d \to D^{(*)+}L^-)}{d\Gamma(\bar{B}_d \to D^{(*)+}\ell^-\bar{\nu}_\ell)/dq^2\,|_{q^2=m_L^2}} = 6\pi^2\,|V_{ij}|^2 f_L^2\,|a_1(D^{(*)+}L^-)|^2\,X_L^{(*)}$$

$X_V = X_V^* = 1$ for a vector or axial-vector meson, for a pseudoscalar $X_L^{(*)}$ deviates from 1 below the percent level;

$\lvert a_1(D^{(*)+}L^-)\rvert$	LO	NLO	NNLO	Exp.
$\lvert a_1(D^+\pi^-)\rvert$	1.025	$1.054^{+0.022}_{-0.020}$	$1.073^{+0.012}_{-0.014}$	0.89 ± 0.05
$\lvert a_1(D^{*+}\pi^-)\rvert$	1.025	$1.052^{+0.020}_{-0.018}$	$1.071^{+0.013}_{-0.014}$	0.96 ± 0.03
$\lvert a_1(D^+\rho^-)\rvert$	1.025	$1.054^{+0.022}_{-0.019}$	$1.072^{+0.012}_{-0.014}$	0.91 ± 0.08
$\lvert a_1(D^{*+}\rho^-)\rvert$	1.025	$1.052^{+0.020}_{-0.018}$	$1.071^{+0.013}_{-0.014}$	0.86 ± 0.06
$\lvert a_1(D^+K^-)\rvert$	1.025	$1.054^{+0.022}_{-0.019}$	$1.070^{+0.010}_{-0.013}$	0.87 ± 0.06
$\lvert a_1(D^{*+}K^-)\rvert$	1.025	$1.052^{+0.020}_{-0.018}$	$1.069^{+0.010}_{-0.013}$	0.97 ± 0.04
$\lvert a_1(D^+K^{*-})\rvert$	1.025	$1.054^{+0.022}_{-0.019}$	$1.070^{+0.010}_{-0.013}$	0.99 ± 0.09
$\lvert a_1(D^+a_1^-)\rvert$	1.025	$1.054^{+0.022}_{-0.019}$	$1.072^{+0.012}_{-0.014}$	0.76 ± 0.19

Test of factorization and SU(3)symmetry:

- Ratios of $\bar{B}_{d,s} \to D_{s,d}^{(*)+} L^-$ decay rates: [*Neubert, Stech, '97; Fleischer, Serra, Tuning, '04, '12*]

$$\mathcal{A}(\bar{B}_d^0 \to D^{(*)+}\pi^-) = \text{Tree} + \text{W-exchange} \quad \mathcal{A}(\bar{B}_d^0 \to D^{(*)+}K^-) = \text{Tree}$$

useful to gain information on W-exchange contribution, as well as to test factorization;

Ratios	LO	NLO	NNLO	Exp.
$\frac{\mathrm{Br}(\bar{B}_d \to D^+\rho^-)}{\mathrm{Br}(\bar{B}_d \to D^+\pi^-)}$	2.654	$2.653^{+0.163}_{-0.158}$	$2.653^{+0.163}_{-0.158}$	2.80 ± 0.47
$\frac{\mathrm{Br}(\bar{B}_d \to D^+K^{*-})}{\mathrm{Br}(\bar{B}_d \to D^{*+}K^-)}$	2.019	$2.026^{+0.404}_{-0.358}$	$2.023^{+0.403}_{-0.358}$	2.103 ± 0.363
$\frac{\mathrm{Br}(\bar{B}_d \to D^+K^-)}{\mathrm{Br}(\bar{B}_d \to D^+\pi^-)}$	0.077	$0.077^{+0.002}_{-0.002}$	$0.077^{+0.002}_{-0.002}$	0.074 ± 0.009
$\frac{\mathrm{Br}(\bar{B}_d \to D^{*+}K^-)}{\mathrm{Br}(\bar{B}_d \to D^{*+}\pi^-)}$	0.075	$0.075^{+0.002}_{-0.002}$	$0.075^{+0.002}_{-0.002}$	0.078 ± 0.007
$\frac{\mathrm{Br}(\bar{B}_s \to D_s^+\pi^-)}{\mathrm{Br}(\bar{B}_d \to D^+K^-)}$	14.67	$14.67^{+1.34}_{-1.28}$	$14.67^{+1.34}_{-1.28}$	15.43 ± 2.02
$\frac{\mathrm{Br}(\bar{B}_s \to D_s^+\pi^-)}{\mathrm{Br}(\bar{B}_d \to D^+\pi^-)}$	1.120	$1.120^{+0.109}_{-0.104}$	$1.120^{+0.109}_{-0.104}$	1.134 ± 0.102

Showing both a small impact of the W-exchange topology and of nonfac. SU(3)-breaking effects!

With LQCD for $B_{(s)} \to D_{(s)}$ FFs, the last two allow precise measurement of f_s/f_d! [*Fleischer, Serra, Tuning, '12*]

Predictions for $\Lambda_b \to \Lambda_c^+ L^-$ decays:

- At the LHC, Λ_b production constitutes $\sim 20\%$ of b-hadrons; [*LHCb, arXiv:1111.2357*]
- Due to $S = \frac{1}{2}$, its decays complementary to B-meson decays, a new testing ground for different QCD models and factorization assumptions used in B-meson case.

Decay mode	LO	NLO	NNLO	Exp.
$\Lambda_b \to \Lambda_c^+ \pi^-$	2.60	$2.75\,^{+0.53}_{-0.53}$	$2.85\,^{+0.54}_{-0.54}$	$4.30\,^{+0.36}_{-0.35}$
$\bar{B}_d \to D^+ \pi^-$	3.58	$3.79\,^{+0.44}_{-0.42}$	$3.93\,^{+0.43}_{-0.42}$	2.68 ± 0.13
$\Lambda_b \to \Lambda_c^+ K^-$	2.02	$2.14\,^{+0.40}_{-0.39}$	$2.21\,^{+0.40}_{-0.40}$	3.42 ± 0.33
$\bar{B}_d \to D^+ K^-$	2.74	$2.90\,^{+0.33}_{-0.31}$	$3.01\,^{+0.32}_{-0.31}$	1.97 ± 0.21
$\frac{\mathrm{Br}(\Lambda_b \to \Lambda_c^+ \mu^- \bar{\nu})}{\mathrm{Br}(\Lambda_b \to \Lambda_c^+ \pi^-)}$	18.88	$17.87\,^{+2.31}_{-2.33}$	$17.25\,^{+2.19}_{-2.18}$	$16.6\,^{+4.1}_{-4.7}$
$\frac{\mathrm{Br}(\Lambda_b \to \Lambda_c^+ K^-)}{\mathrm{Br}(\Lambda_b \to \Lambda_c^+ \pi^-)}$ (%)	7.77	$7.77\,^{+0.19}_{-0.18}$	$7.77\,^{+0.19}_{-0.18}$	7.31 ± 0.23
$\frac{\mathrm{Br}(\Lambda_b \to \Lambda_c^+ \pi^-)}{\mathrm{Br}(\bar{B}_d \to D^+ \pi^-)}$	0.73	$0.73\,^{+0.16}_{-0.15}$	$0.73\,^{+0.16}_{-0.15}$	3.3 ± 1.2

For mesonic decays, larger than data; but for baryonic decays, lower than data, and NNLO has a right directions!

Conclusion and outlook

- NNLO calculation for hadronic B decays at leading power in QCDF (almostly) complete:
  - two-loop vertex corrections to tree amplitudes $a_{1,2}$ now complete;
  - two-loop corrections with $Q^p_{1,2}$ insertion to a^p_4 now complete;
  - two-loop corrections from $Q_{3,4,5,6}$ and Q_{8g} operators in progress;
- For $a_{1,2}$: NNLO corrections individually sizeable, but cancelled between vertex and spectator; a_1-dominated modes well described by QCDF, but less for a_2-dominated modes;
- For a^p_4 with $Q^p_{1,2}$ insertion: NNLO corrections sizeable to the SD part of direct CP, but their effect tempered by power-suppressed a^p_6 (chirally-enhanced) and β^p_3 (annihilation) terms;
- The NNLO correction does not help resolving the large $\mathrm{Br}(\pi^0\pi^0)$ and the πK CP asymmetry puzzle, nor does it render the poorly known annihilation terms redundant;
- $\Lambda_b \to \Lambda_c^+ L^-$ decays provide another testing ground for different QCD models and factorization assumptions used in B-meson case;

谢　谢　大　家!