Recent progress on scattering amplitude and loop-level calculation

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Content

- BCFW(hep-th/0501052.)
- Integrand Reduction (Bern, Kosower, Ossola)
- Integral Reduction(Kosower, Ita, Zhang, Larsen,Chen)
- Canonical Integral Basis(Henn, Caron-Huot, 1304.1806,1404.2922)

BCFW

Using spinor technique, $V_{a\dot{a}} = V_\mu \sigma^\mu_{a\dot{a}}$

massless states and their momenta can be written in spinor forms

$$p_{a\dot{a}} = \lambda_a \tilde{\lambda}_{\dot{a}} \qquad \epsilon^+_{a\dot{a}} = \frac{\mu_a \tilde{\lambda}_{\dot{a}}}{\langle \mu, \lambda \rangle}$$

BCFW momenta shifts

$$\hat{\lambda}_i = \lambda_i + z\lambda_j$$

$$\hat{\tilde{\lambda}}_j = \tilde{\lambda}_j - z\tilde{\lambda}_i$$

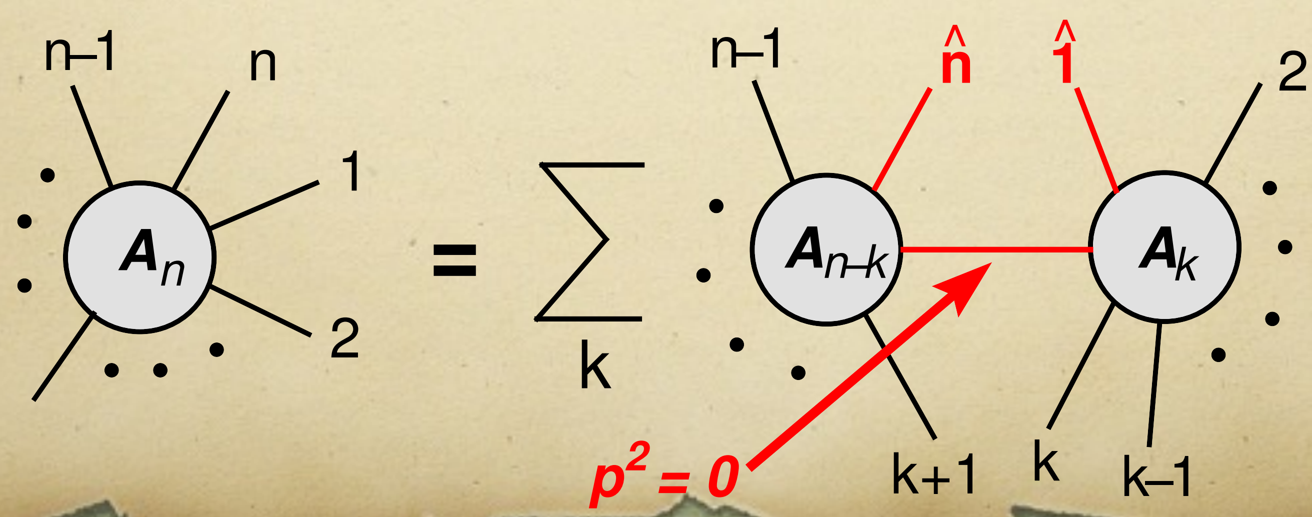
- **momentum conservation**
- **on-shell condition:** $\hat{p}_i \cdot \epsilon_i = 0, \quad \hat{p}_j \cdot \epsilon_j = 0$
- **linear transformation in spinor**

Tree-level amplitude $\hat{A}(z)$ is a rational function of z under BCFW shifts,

$$\hat{A}(z) = A^n z^n + \cdots + A^1 z + A^0 + \sum_i \frac{Res A_{a_i}}{z - a_i}$$

Amplitudes are determined by Singularity

$$\begin{cases} \text{pole positions} & : \quad \frac{1}{\hat{P}^2_{1..k}} = \frac{1}{P^2_{1..k} - z\langle\lambda_1|P_{1..k}|\tilde{\lambda}_n]} \\ \text{their Residues} & : \quad Res A_{a_i} = A_{n-k}(a_i) A_k(a_i) \end{cases}$$



Loop level Reduction

Integrand Reduction

Integrand Reduction

The general form of the loop integration

$$\int dl_1^d \cdots dl_L^d \frac{R(l_i \cdot l_j, l_i \cdot k_j, l_i \cdot \epsilon_j)}{D_1 D_2 \cdots D_m}$$

where $D_i = (l_i + k_j)^2$ is propagation

A convenience parameterization for all loop momentum

$$D_1 \cdots D_m D_{m+1}^{\perp} \cdots D_M^{\perp}$$

Example:1-loop

$$\int dl^d \frac{l \cdot k_1}{D_1 D_2 \cdots D_m}$$

$$D_1 = l^2, D_2 = (l + k_1)^2$$

$$l \cdot k_1 = \frac{D_2 - D_1}{2} - k_1^2$$

$$\frac{1}{D_1 D_3 \cdots D_m} - \frac{1}{D_2 D_3 \cdots D_m} - \frac{k_1^2}{D_1 D_2 D_3 \cdots D_m}$$

Fewer Dominators

Lower order for Numerator

m>4, the integrand can all be reduced similarly

Integrand Reduce to

$$\frac{1}{D_1 D_2 D_3 \cdots D_m} \longrightarrow \frac{1}{D_1 D_2 D_3 D_4 D_5}$$

$$\left.\begin{array}{c} \dfrac{D_5^{n_5}}{D_1 D_2 D_3 D_4} \\ \dfrac{D_4^{n_4} D_5^{n_5}}{D_1 D_2 D_3} \\ \dfrac{D_3^{n_3} D_4^{n_4} D_5^{n_5}}{D_1 D_2} \end{array}\right\} \xrightarrow{\text{Renormalization}} \begin{array}{c} \dfrac{\{1, D_5, D_5^2\}}{D_1 D_2 D_3 D_4} \\ \dfrac{\{D_4, D_5\}}{D_1 D_2 D_3} \\ \dfrac{1}{D_1 D_2} \end{array}$$

Integral Reduction

IBP Relation

Under the transition of loop momentum

$$l_i = l_i + \alpha v_j \qquad v_j^\mu \in \{l_1^\mu, l_2^\mu, \cdots, l_L^\mu, p_1^\mu, \cdots, p_\beta^\mu\}$$

$$\int dl_1^d \cdots dl_L^d \partial_{l_I^\mu} v_J^\mu \frac{C_{IJ}}{D_1 D_2 \cdots D_m} = 0$$

Integral Basis one-loop

$$0 = \int \frac{D_4}{D_1 D_2 D_3} - \frac{[(d-4)k_{14} + (d-2)k_{24}]}{(d-4)k_{12}} \frac{1}{D_1 D_3}$$

After using all IBP relations, the integral can be reduced to following basis

$$\frac{1}{D_1 D_2 D_3 D_4 D_5} \quad \frac{1}{D_1 D_2 D_3 D_4} \quad \frac{1}{D_1 D_2 D_3} \quad \frac{1}{D_1 D_2}$$

Algorithm

- F5
- Linear Algebra
- C2Z(arxiv:1511.01058)

Canonical Integral Basis

Denoting the kinematical variables by

$$s_{ij} = k_i \cdot k_j$$

The basis integral under the first differential of s_{ij}

$$\partial_{s_{ij}} I_r(\epsilon, \vec{s}) = A_{ij}(\epsilon, \vec{s}) I_r(\epsilon, \vec{s}) \qquad d = 4 - 2\epsilon$$

A conjecture: Transform the integral basis

$$I \to BI \qquad A_{ij} \to B^{-1} A_{ij} B - B^{-1}(\partial_{s_{ij}} B)$$

s.t. $\partial_{s_{ij}} I_r(\epsilon, \vec{s}) = \epsilon A_{ij}(\vec{s}) I_r(\epsilon, \vec{s})$

It has formal solution $I = P e^{\epsilon \int_C A_{ij}(\vec{s}) ds_{ij}} I(\epsilon = 0)$

One loop example

Master Integral

$$\frac{1}{D_1 D_2 D_3 D_4}$$

$$D_1 = l_1^2 \ , \quad D_2 = (l_1 - p_1)^2 \ ,$$
$$D_3 = (l_1 - p_1 - p_2)^2, D_4 = (l - p_4)^2$$

$$s_{12}\partial_{s_{12}}\begin{pmatrix} G0101 & = & 1/D_2D_4 \\ G1010 & = & 1/D_1D_3 \\ G0011 & = & 1/D_3D_4 \\ G1111 & = & 1/D_1D_2D_3D_4 \end{pmatrix}$$

$$= \begin{pmatrix} -\epsilon & 0 & 0 & 0 \\ 0 & -\epsilon & 0 & 0 \\ 0 & 0 & -\epsilon\frac{2s_{12}+s_{24}}{2(s_{12}+s_{24})} & 0 \\ * & * & * & -2\epsilon-2 \end{pmatrix}\begin{pmatrix} G0101 \\ G1010 \\ G0011 \\ G1111 \end{pmatrix}$$

By transformation $G1111 \to s_{12}^2 G1111$

$$\begin{pmatrix} -\epsilon & 0 & 0 & 0 \\ 0 & -\epsilon & 0 & 0 \\ 0 & 0 & -\epsilon\frac{2s_{12}+s_{24}}{2(s_{12}+s_{24})} & 0 \\ * & * & * & -2\epsilon \end{pmatrix}$$

- Two loop basis?
- Canonical Integral Basis: Exist or Not?
- If exist, clue for hidden algebraic structures

Thanks