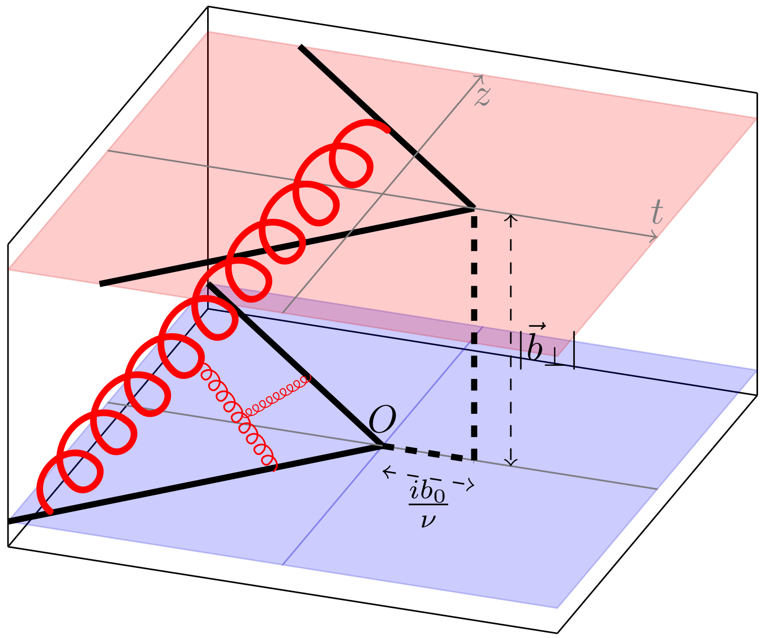
Bootstrap, symmetry, and the Higgs boson production at small transverse momentum



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and work in progress

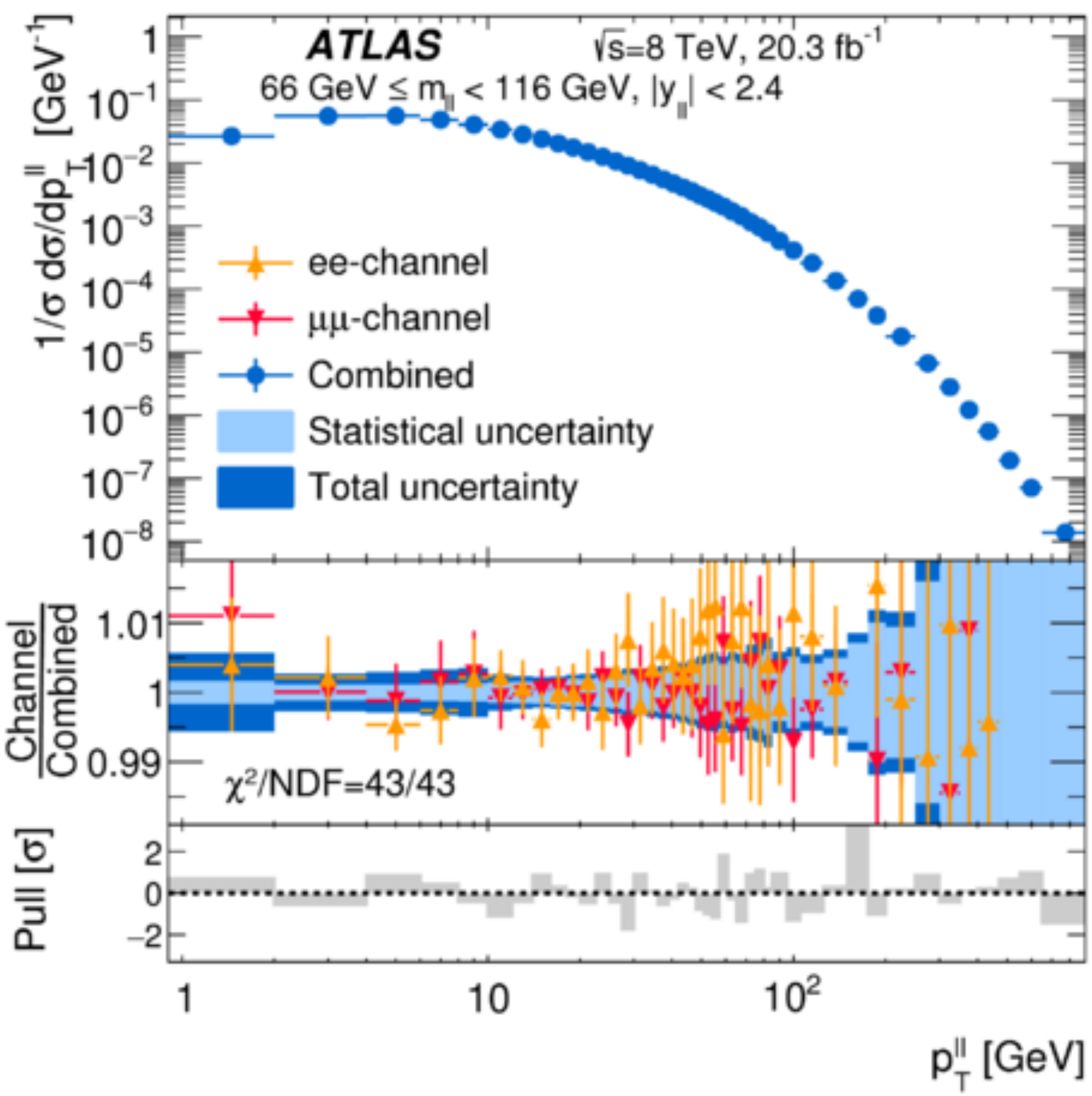
June 15, 2016

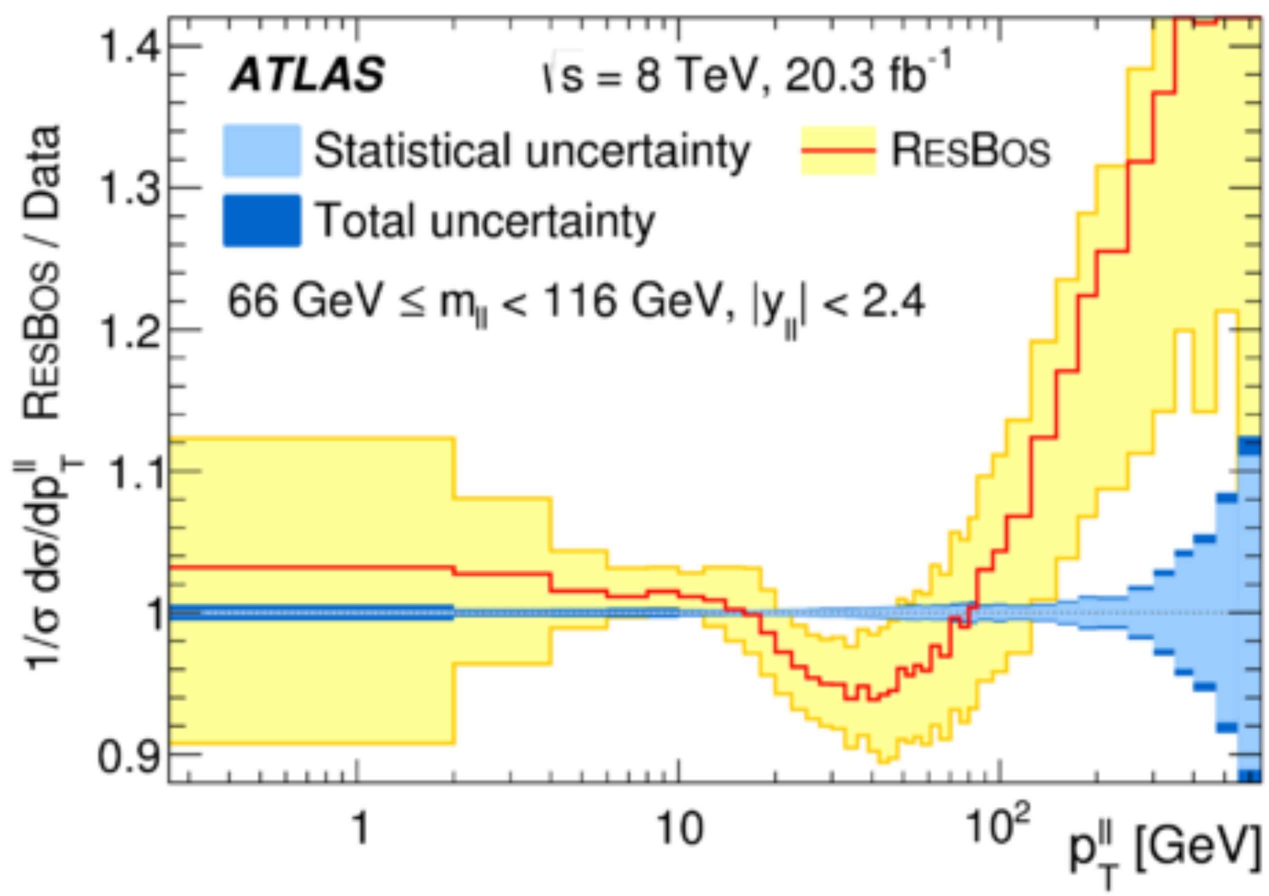
IHEP

- The main focus of this talk is the transverse-momentum spectra of Electroweak bosons (photon*, Z, h, …)
- Very clean observable, only involve electroweak particles in the final states
- One of the best studied distribution of QCD at hadron colliders

LHC physics at 1% precision

ATLAS 8 TeV arXiv:1512.02192





LHC physics at 1% precision

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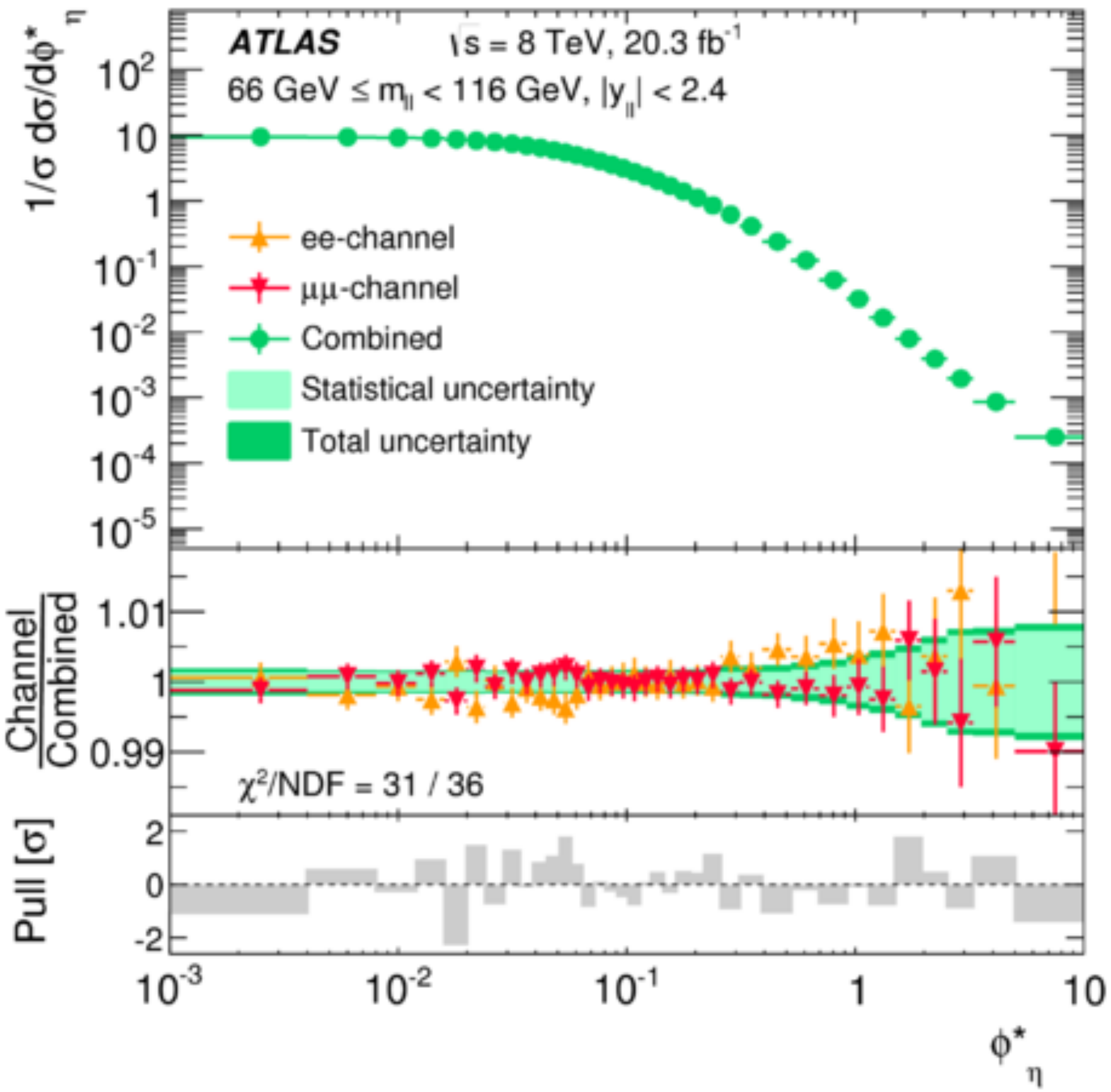
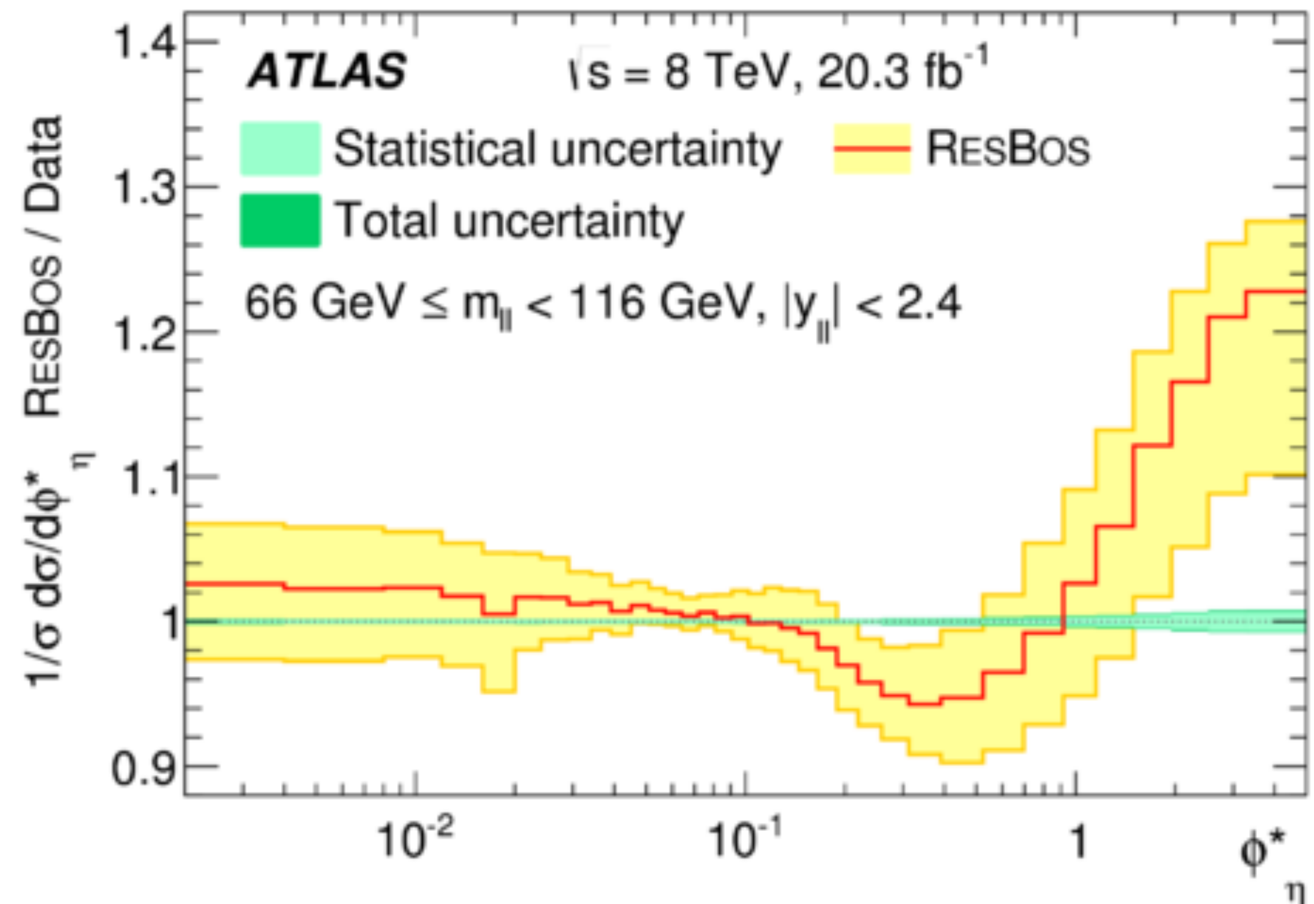
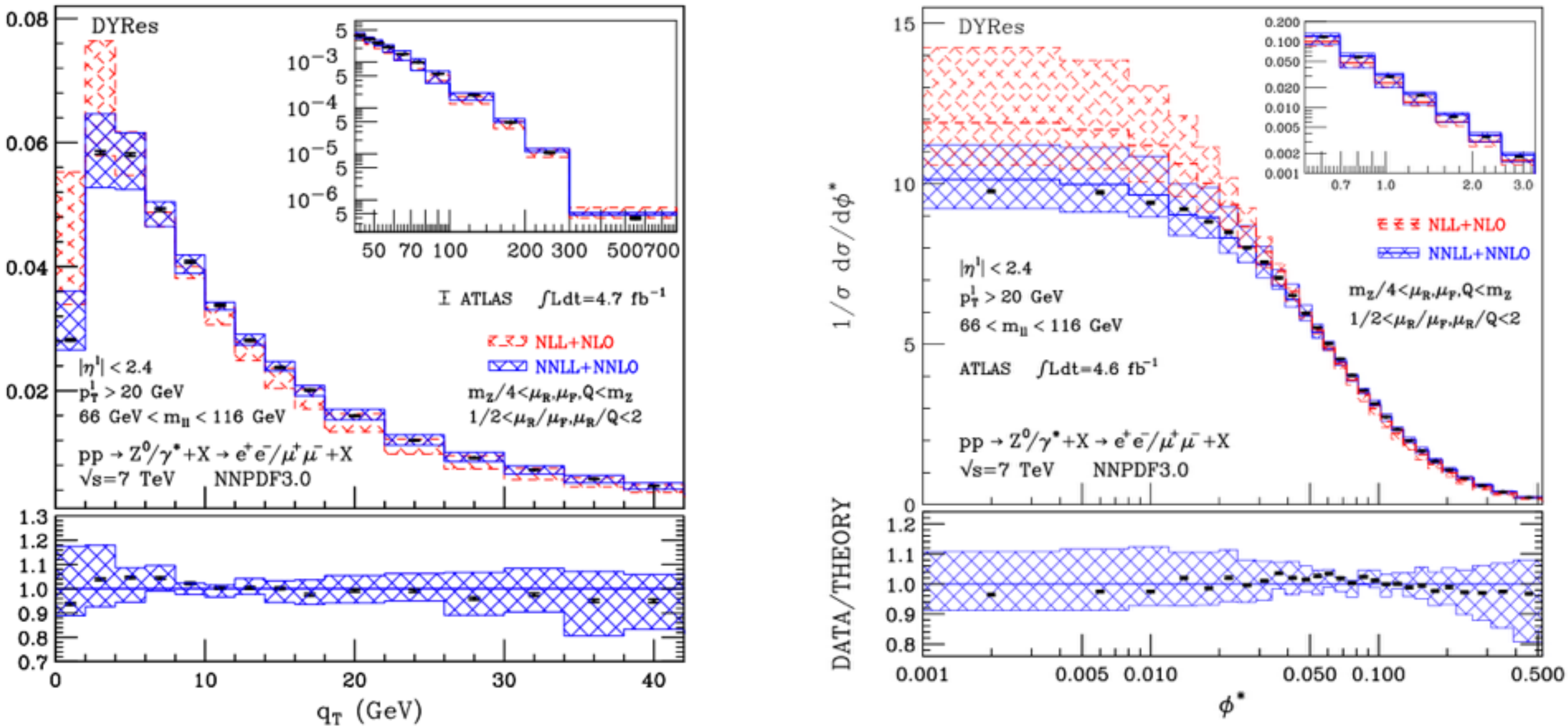




$$\phi^*_\eta = \tan\left(\frac{\pi - \Delta\phi}{2}\right) \cdot \sin(\theta^*_\eta) \qquad \cos(\theta^*_\eta) = \tanh[(\eta^- - \eta^+)/2]$$

State of the art calculation for Drell-Yan: NNLL+NNLO

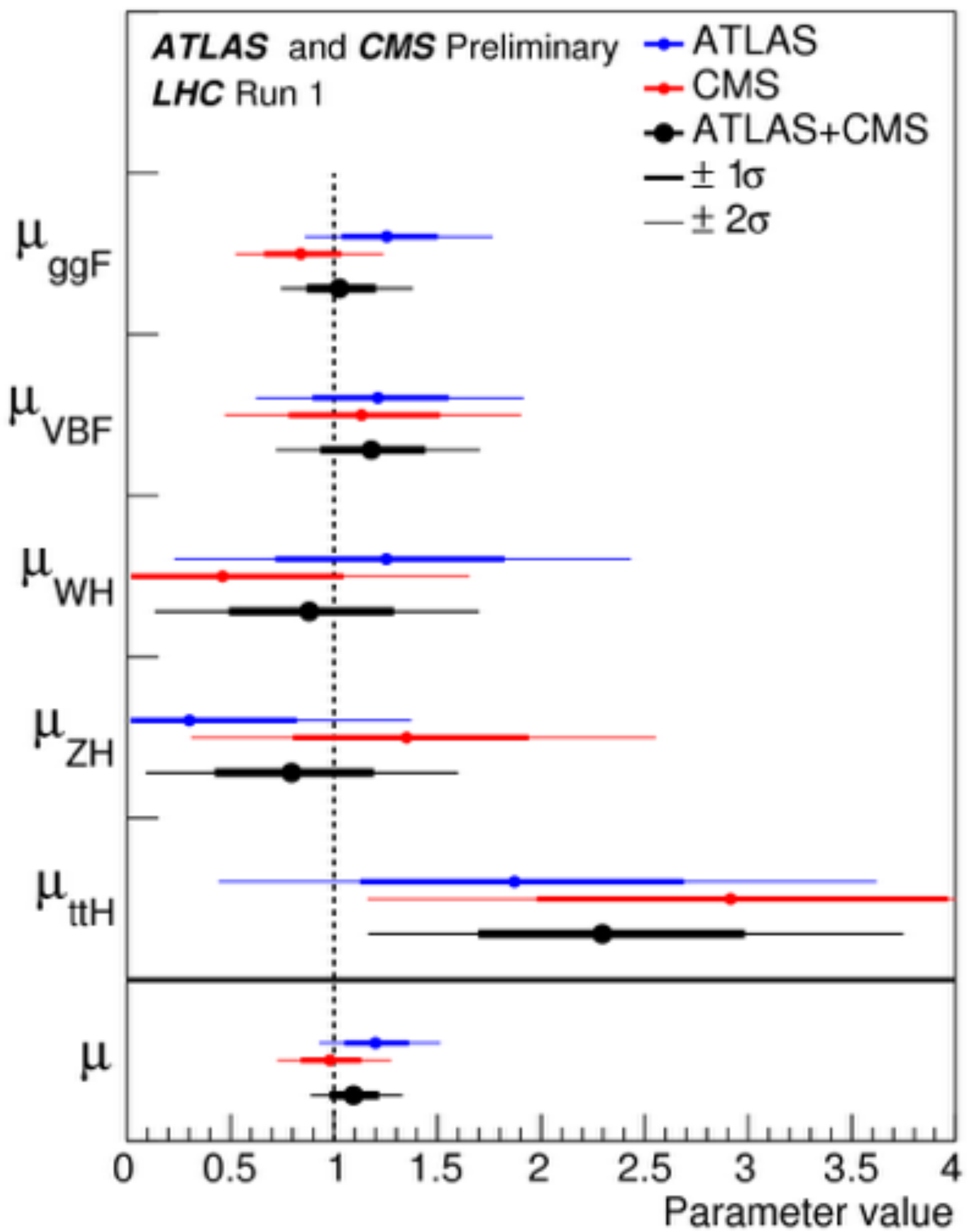
Catani et al., 1507.06937

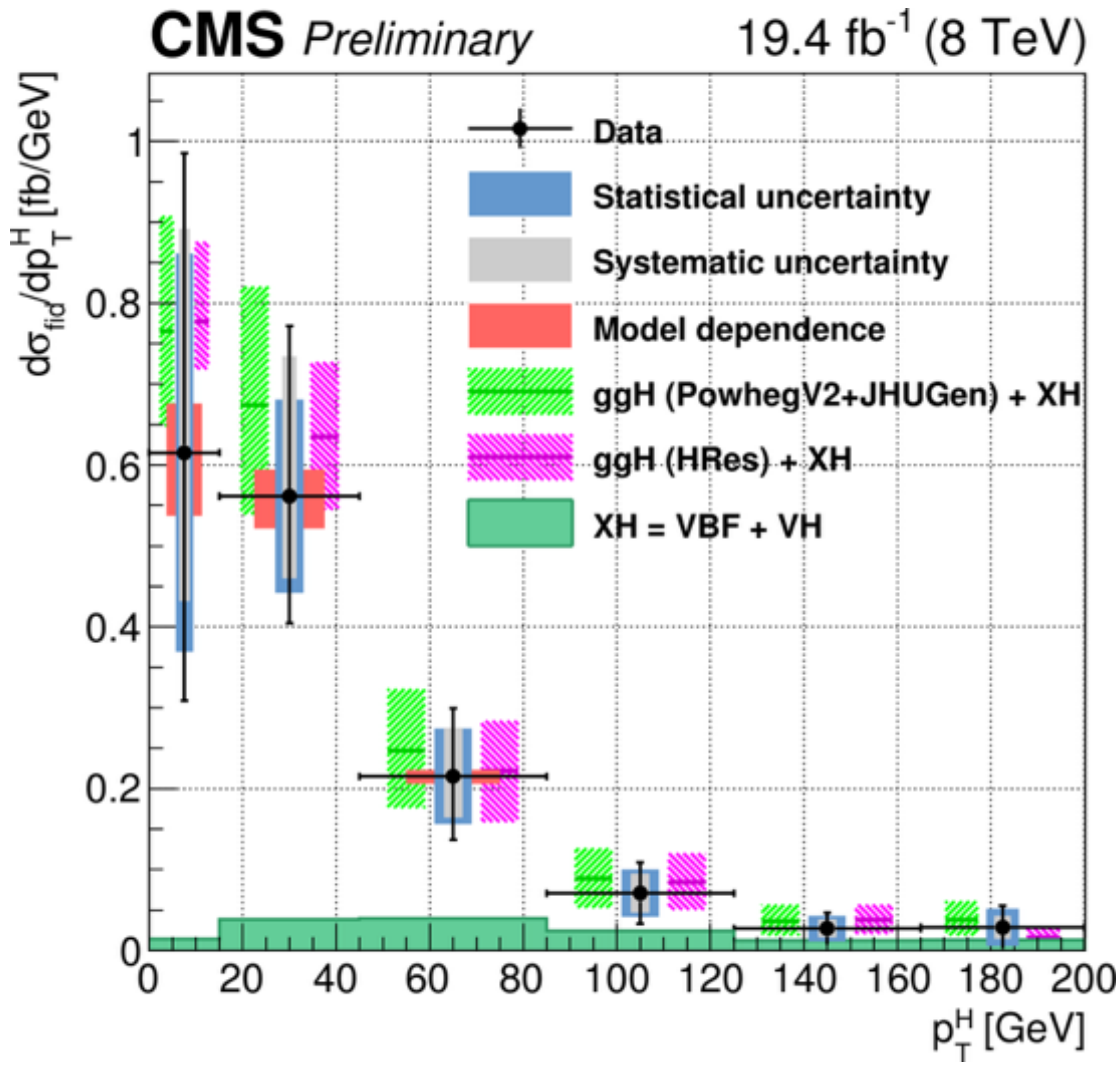


More than 20% scale theoretical uncertainties at the peak region!

Entering the era of precision Higgs physics

- **Total cross section (left) and pT distribution (right)**





Tremendous room for theoretical improvement

- At very small pT, intrinsic transverse-momentum of proton
- small pT, soft gluon resummation
- large pT, fixed-order perturbative QCD and parton shower
- Parton distribution function
- ...
- We will gain from a high precision study of pT:
  - alpha_s
  - Improved modeling of initial-state radiations for virtually all processes at the LHC
  - Precision measurement of W mass
  - Gluon's distribution inside proton
  - Improved understanding of proton's transverse structure
  - ……

QCD theory: at the dawn of N3Lx

- **Soft-virtual corrections at N3LO**

  Anastasiou, Duhr, Dulat, Furlan, Gehrmann, Herzog, Mistlberger, 2014
  Y. Li, von Manteuffel, Schabinger, HXZ, 2014

- **Resummation with 3-loop constant**

  Ahmed, Mandal, Rana, V. Ravindran, 2014
  Bonvini, Marzani, 2014
  Catani, Cieri, de Florian, Ferrera, Grazzini, 2014
  …

- **Subleading terms in threshold expansion**

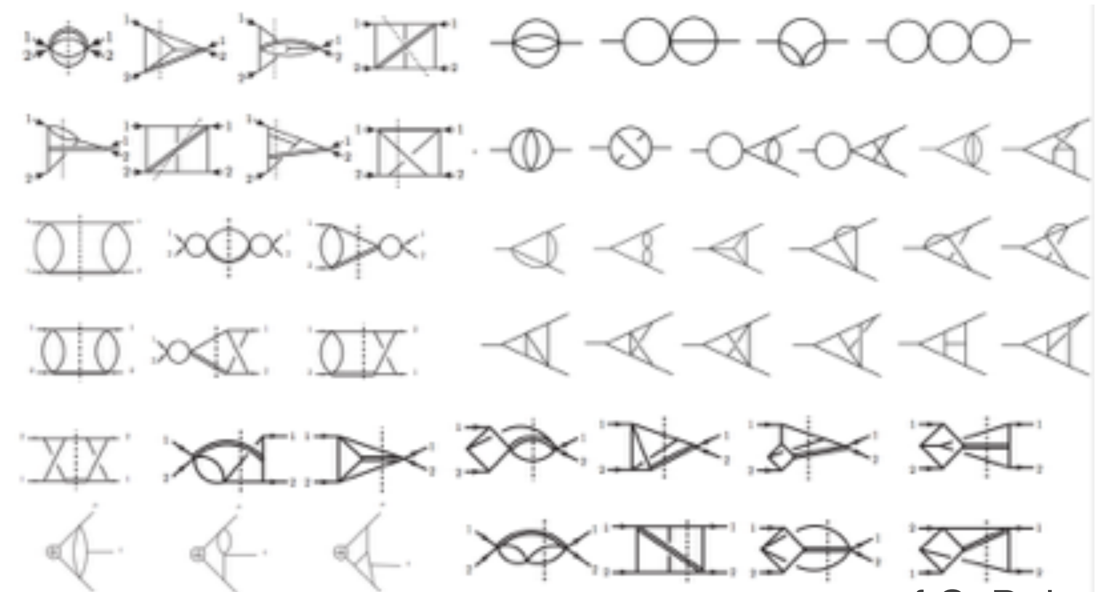
  Anastasiou, Duhr, Dulat, Furlan, Gehrmann, Herzog, Mistlberger, 2014

- **Exact N3LO cross section through powerful threshold (1-z) expansion via differential equation**

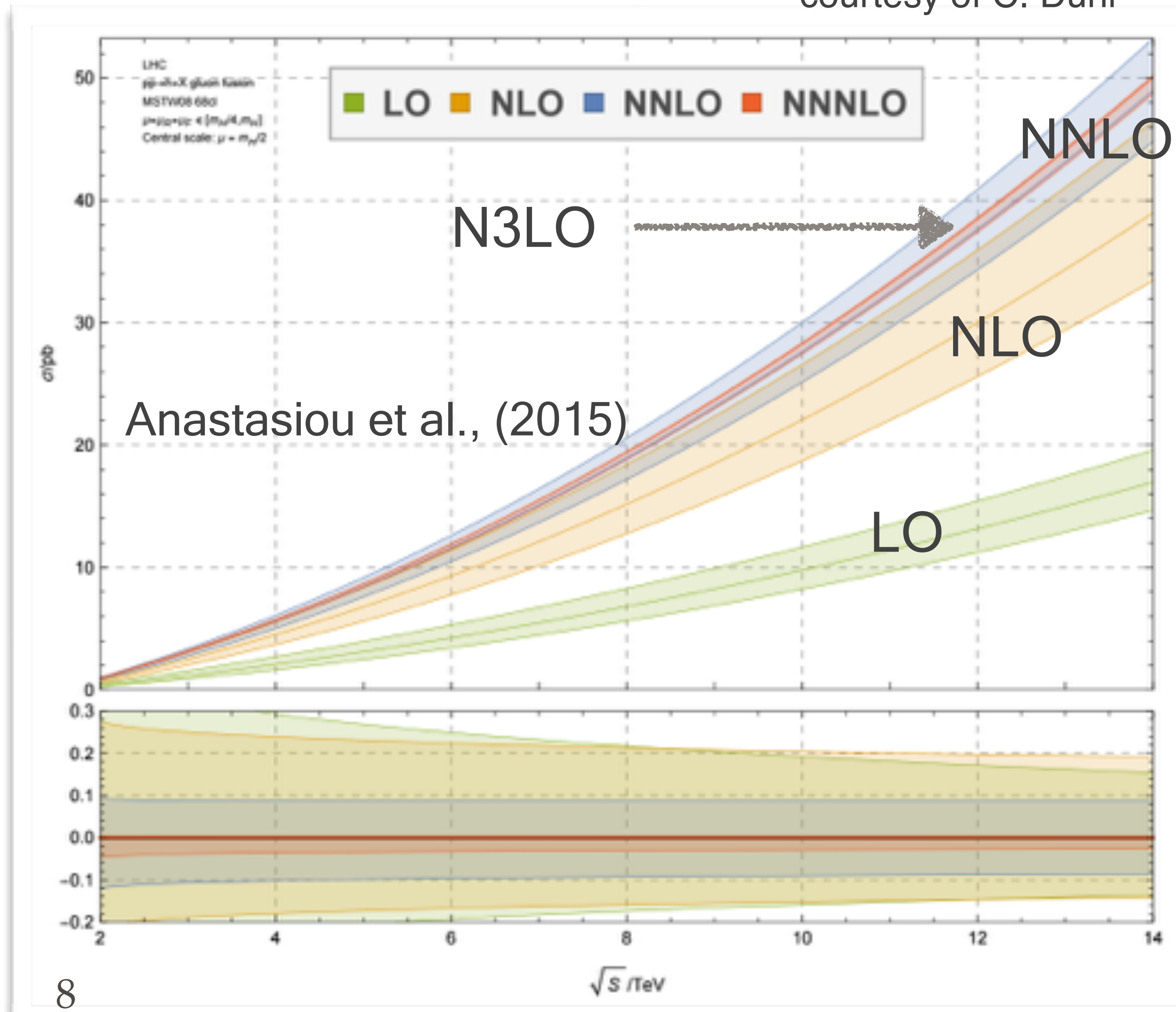
  Anastasiou, Duhr, Dulat, Mistlberger, 2015

- **Full z dependence at N3LO for quark-quark' initiated channel**

  Anzai, Hasselhuhn, Hoschele, Hoff, Kilgore, Steinhauser, Ueda, 2015

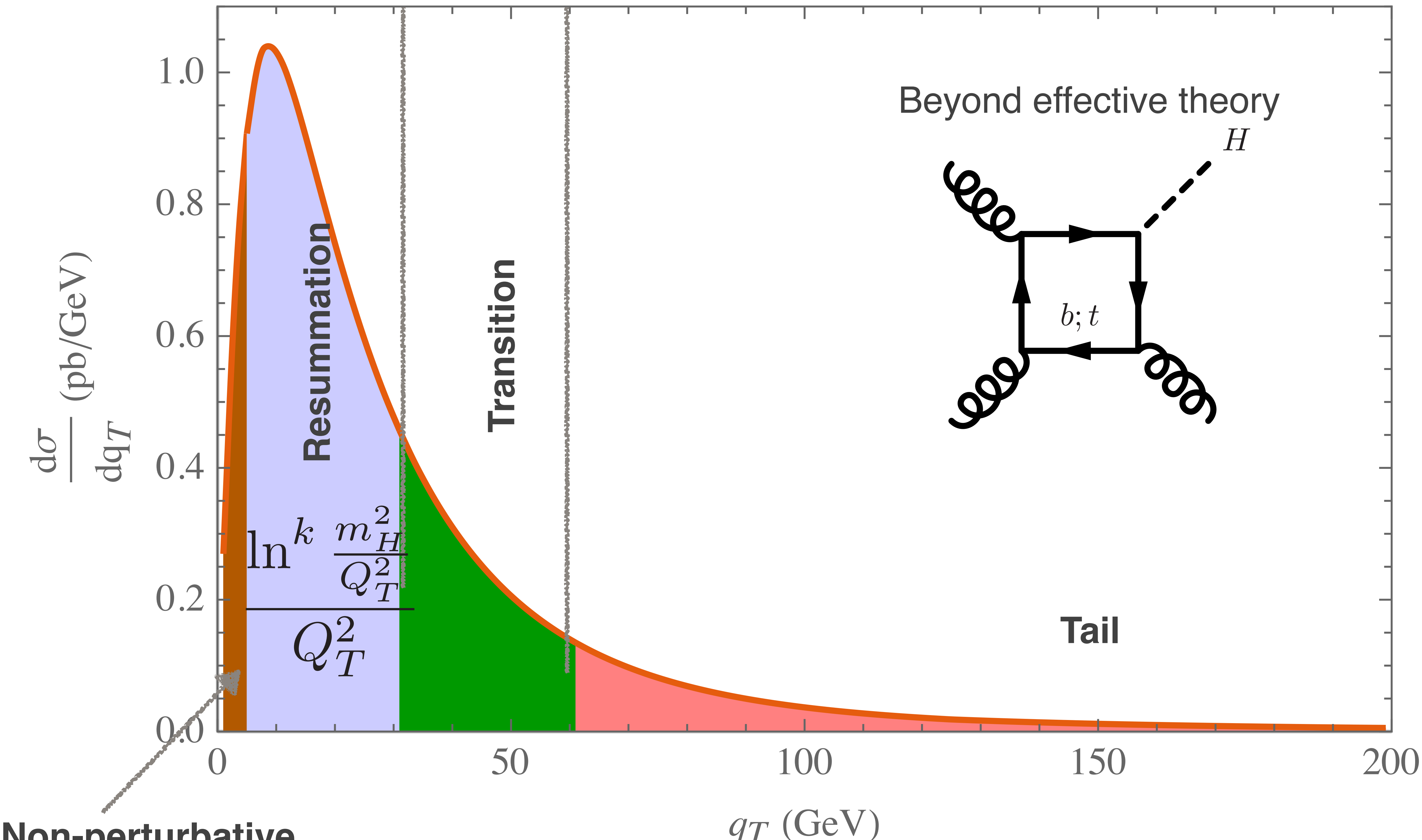


courtesy of C. Duhr



pT of Higgs: multi-scale problem

Higgs production through gluon-gluon fusion



Transverse-momentum resummation via QCD factorization

Dokshitzer, Diakonov, Troian, Parisi, Petrozio, Collins, Soper, Sterman, …

$$\frac{d\sigma}{dQ^2\,dy\,dQ_T^2} \sim \frac{4\pi^2\alpha^2}{9Q^2 s}(2\pi)^{-2}\int d^2b\, e^{iQ_T\cdot b}\sum_j e_j^2$$

PDF

$$\times \sum_a \int_{x_A}^1 \frac{d\xi_A}{\xi_A} f_{a/A}(\xi_A;1/b) \sum_b \int_{x_B}^1 \frac{d\xi_B}{\xi_B} f_{b/B}(\xi_B;1/b)$$

Sudakov factor

$$\times \exp\left\{-\int_{1/b^2}^{Q^2}\frac{d\bar{\mu}^2}{\bar{\mu}^2}\left[\ln\left(\frac{Q^2}{\bar{\mu}^2}\right)A(g(\bar{\mu})) + B(g(\bar{\mu}))\right]\right\}$$

Collinear/hard factor

$$\times C_{ja}\left(\frac{x_A}{\xi_A};g(1/b)\right)C_{jb}\left(\frac{x_B}{\xi_B};g(1/b)\right)$$

non-singular terms

$$+\frac{4\pi^2\alpha^2}{9Q^2 s}Y(Q_T;Q,x_A,x_B).$$

Collins, Soper, Sterman, 1985

Counting of the resummation order

$$\ln\sigma(b) \sim -\int_{1/b^2}^{m_H^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2}\left[\ln\left(\frac{m_H^2}{\bar{\mu}^2}\right) A[\alpha_s(\bar{\mu})] + B[\alpha_s(\bar{\mu})]\right]$$

$$= \alpha_s\left[\ln^2(b^2m_H^2) + \ln(b^2m_H^2)\right]$$

$$\alpha_s^2\left[\ln^3(b^2m_H^2) + \ln^2(b^2m_H^2) + \ln(b^2m_H^2)\right]$$

$$\alpha_s^3\left[\ln^4(b^2m_H^2) + \ln^3(b^2m_H^2) + \ln^2(b^2m_H^2) + \ln(b^2m_H^2)\right]$$

$$+\ldots$$

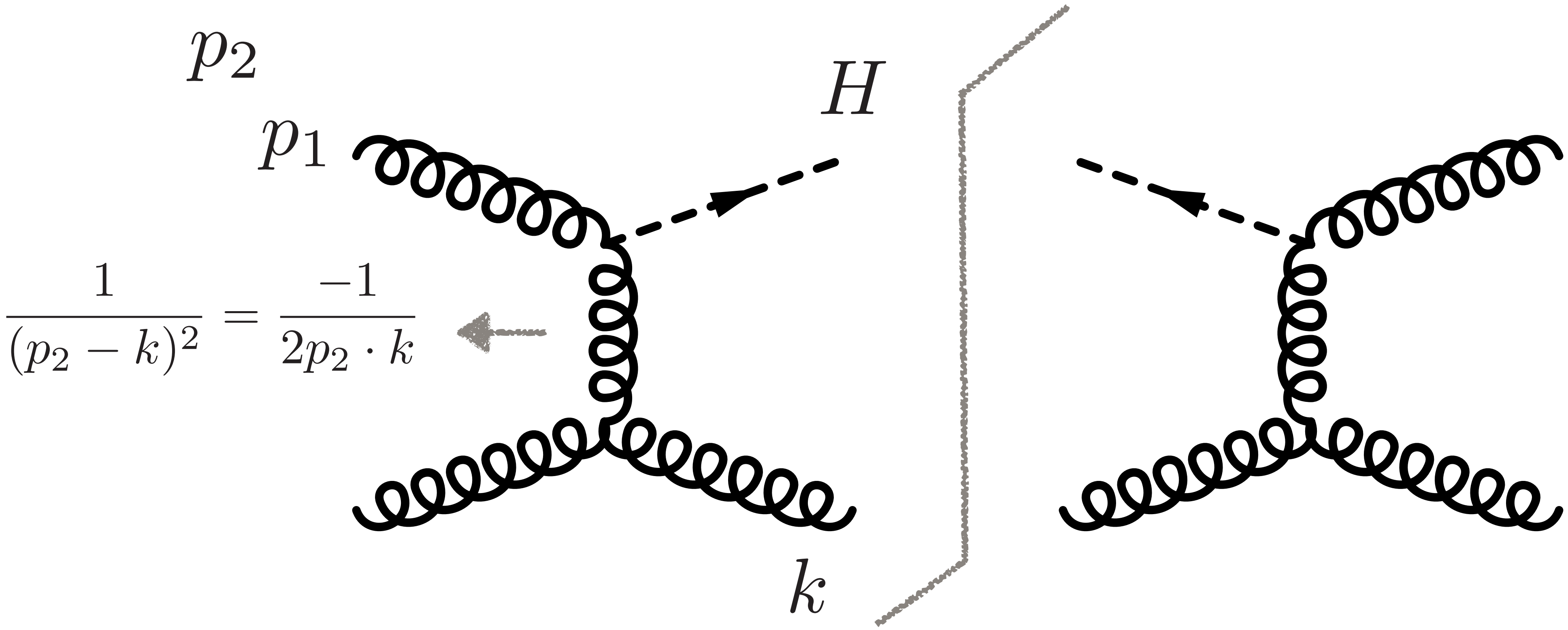
LL	NLL	NNLL	N3LL
A_1	A_2	A_3	A_4
	B_1	B_2	B_3

- **B_2 for Drell-Yan (**Davies, Webber, Stirling, 1985**); B_2 for Higgs production (**Grazzini, de Florian, 2000**)**
- **When expanded to fixed order, A_n contribute at one higher order than B_{n-1}**
- **To meet the standard of N3LL resummation we need B_3**

qT resummation in Soft-Collinear Effective Theory

- **Soft-Collinear effective theory (SCET) is an EFT of QCD. Only soft/collinear modes are kept, all the other modes are integrated out.**
- **SCET is a very convenient framework for resummation calculation.**
- **Transverse-momentum resummation in SCET:**
  - **Y. Gao, C.S. Li, J.J. Liu, 2005**
  - **Idilbi, X.D. Ji, F. Yuan, 2005**
  - **Mantry, Petriello, 2009**
  - **Becher, Neubert, 2009**
  - **Echevarria, Idilbi, Scimemi, 2011**
  - **Rapidity renormalization group: Chiu, Jain, Neill, Rothstein, 2012**

Soft/Collinear singularities in QCD



Soft singularity: k -> 0

Collinear singularity: k//p2

Soft/collinear singularity leads to large logarithms in fixed order perturbation theory

$$\alpha_s^n \frac{1}{q_T^2} \ln^m \frac{M_H^2}{q_T^2}$$

Fourier transform

$$\xrightarrow[\int d^2\vec{q}_T \exp\left[i\vec{b}\cdot\vec{q}_T\right]]{}$$

$$\alpha_s^n \ln^{m+1}(M_H^2 b^2)$$

Factorization and resummation in SCET

hard function Beam function soft function

$$\frac{1}{\sigma}\frac{d\sigma}{d^2\vec{Q}_T\,dY\,dQ^2} \sim H(\mu)\int\frac{d^2\vec{b}_\perp}{(2\pi)^2}\,e^{i\vec{b}_\perp\cdot\vec{Q}_T}\,[B\otimes B](\vec{b}_\perp,\mu,\nu)\cdot S_\perp(\vec{b}_\perp,\mu,\nu)$$

- **Cross section in SCET factorized into hard function, beam functions, and soft function**
- **Individual function contain UV and rapidity divergence. After regularization and renormalization: μ and ν dependence**

Resummation through renormalization group running

- **Separation of kinematical scale:**

$$\ln(M_H^2 b^2)$$

hard function	$\ln \frac{M_H^2}{\mu^2}$	
beam function	$\ln(b^2\mu^2)$	$\ln \frac{M_H^2}{\nu^2}$
soft function	$\ln(b^2\mu^2)$	$\ln(b^2\nu^2)$

RG equation:

$$\frac{d\ln S_\perp(\vec{b}_\perp,\mu,\nu)}{d\ln\mu^2} = \Gamma_{\rm cusp}\left[\alpha_S(\mu)\right]\ln\frac{\mu^2}{\nu^2} - \gamma_s\left[\alpha_S(\mu)\right]$$

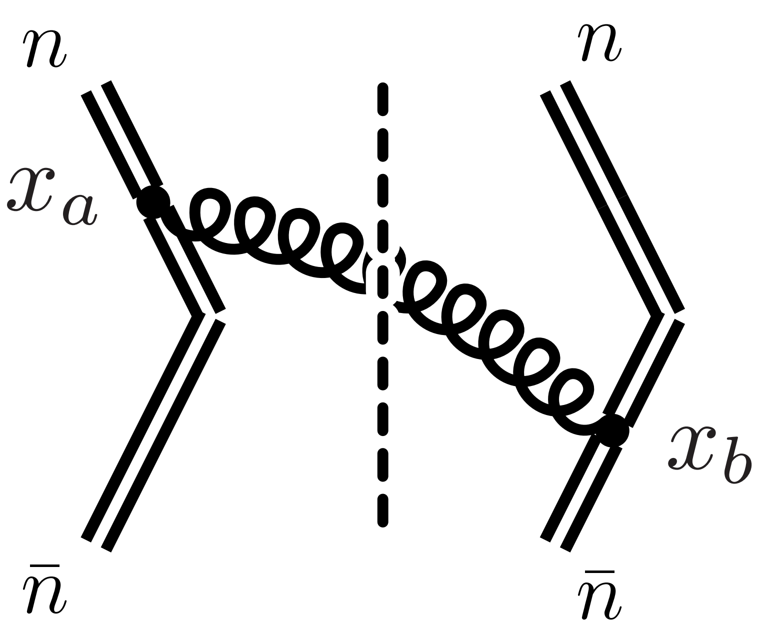
rapidity RG equation:

$$\frac{d\ln S_\perp(\vec{b}_\perp,\mu,\nu)}{d\ln\nu^2} = \int_{\mu^2}^{b_0^2/\vec{b}_\perp^2}\frac{d\bar{\mu}^2}{\bar{\mu}^2}\Gamma_{\rm cusp}\left[\alpha_S(\bar{\mu})\right] + \gamma_R\left[\alpha_S(b_0/|\vec{b}_\perp|)\right]$$

Chiu, Jain, Neill, Rothstein, 2011,2012

Origin of rapidity divergence

$$S_\perp(b) = \mathrm{Tr}\langle 0|T\left[S^\dagger_{\bar{n}} S_n(0)\right]\bar{T}\left[S^\dagger_n S_{\bar{n}}(\vec{b})\right]|0\rangle$$



$$\sim \int dx_a\, dx_b D_+(x_{ab}^2)$$

$$\sim \int_0^\infty dt_1 \int_0^\infty dt_2 \frac{1}{(t_1 t_2 + \vec{b}_\perp^2)^{1-\epsilon}}$$

change of variable $r = \frac{t_1}{t_2} \quad v = t_1 t_2$

unregulated rapidity divergence $\sim \int_0^\infty \frac{dr}{r} \int_0^\infty \frac{dv}{(v^2 + \vec{b}_\perp^2)}$

- **Several rapidity regulators have been proposed**
  - **Tilting the Wilson line off light cone:** Ji, Ma, Yuan (2004); Collins (2011)
  - **analytic regulator:** Becher, Neubert (2009); Becher, Bell (2011); **two-loop calculation:** Gehrmann, Lubbert, Yang (2012,2014)

$$\int d^dk \to \int d^dk \left(\frac{\nu}{k^+}\right)^\alpha$$

  - **delta regulator:** Echevarria, Idilbi and Scimemi (2011); **two-loop calculation:** Echevarria, Scimemi, Vladimirov (2015)

$$\frac{1}{k^+ + i\varepsilon} \to \frac{1}{k^+ + \delta}$$

  - **rapidity renormalization group:** Chiu, Jain, Neill, Rothstein (2011,2012); **two-loop calculation:** Luebbert, Oredsson, Stahlhofen (2016)

$$\int d^dk \to \int d^dk \left(\frac{\nu}{|k_z|}\right)^\eta$$

An exponential regulator for rapidity divergence and loop calculations "without" loop diagrams

Y. Li, D. Neill, HX Zhu, 1604.00392

Y. Li, HX Zhu, 1604.01404

The rapidity renormalization group formalism

Chiu, Jain, Neill, Rothstein (2012)

rapidity scale

$$\frac{1}{\sigma}\frac{d\sigma}{d^2\vec{Q}_T\, dY\, dQ^2} \sim H(\mu)\int \frac{d^2\vec{b}_\perp}{(2\pi)^2}\, e^{i\vec{b}_\perp\cdot\vec{Q}_T}\, [B\otimes B](\vec{b}_\perp,\mu,\nu)\cdot S_\perp(\vec{b}_\perp,\mu,\nu)$$

known from quark/gluon form factor

soft anomalous dimension

RG equation:

$$\frac{d\ln S_\perp(\vec{b}_\perp,\mu,\nu)}{d\ln\mu^2} = \Gamma_{\rm cusp}\left[\alpha_S(\mu)\right]\ln\frac{\mu^2}{\nu^2} - \gamma_s\left[\alpha_S(\mu)\right]$$

rapidity anomalous dim.

rapidity RG equation:

$$\frac{d\ln S_\perp(\vec{b}_\perp,\mu,\nu)}{d\ln\nu^2} = \int_{\mu^2}^{b_0^2/\vec{b}_\perp^{\,2}}\frac{d\bar{\mu}^2}{\bar{\mu}^2}\Gamma_{\rm cusp}\left[\alpha_S(\bar{\mu})\right] + \gamma_R\left[\alpha_S(b_0/|\vec{b}_\perp|)\right]$$

Evolution equation for beam function can be deduced from requiring physical cross section is independent of both RG scale and rapidity scale

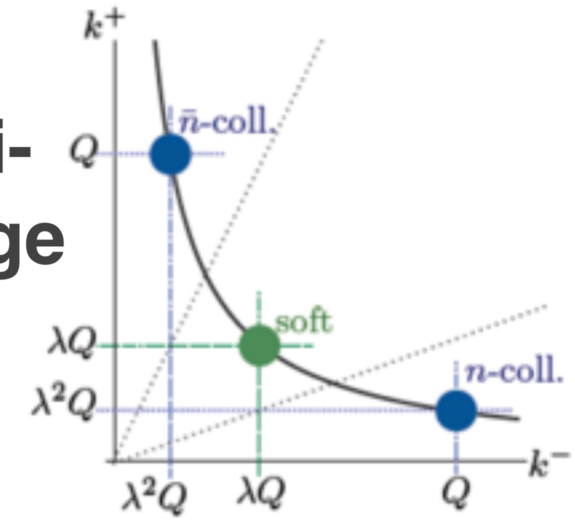
$$B_1 = \gamma_0^V - \gamma_0^R$$
$$B_2 = \gamma_1^V - \gamma_1^R + \beta_0 c_1^H$$
$$B_3 = \gamma_2^V - \gamma_2^R + \beta_1 c_1^H + 2\beta_0\left(c_2^H - \frac{1}{2}\left(c_1^H\right)^2\right)$$

γ_V **non-cusp hard anomalous dim.**

c_H **finite terms of quark/gluon form factor**

An exponential regulator for rapidity divergence

For phase space integral in each sector (soft, collinear, anti-collinear), insert an exponentially suppressing factor at large energy



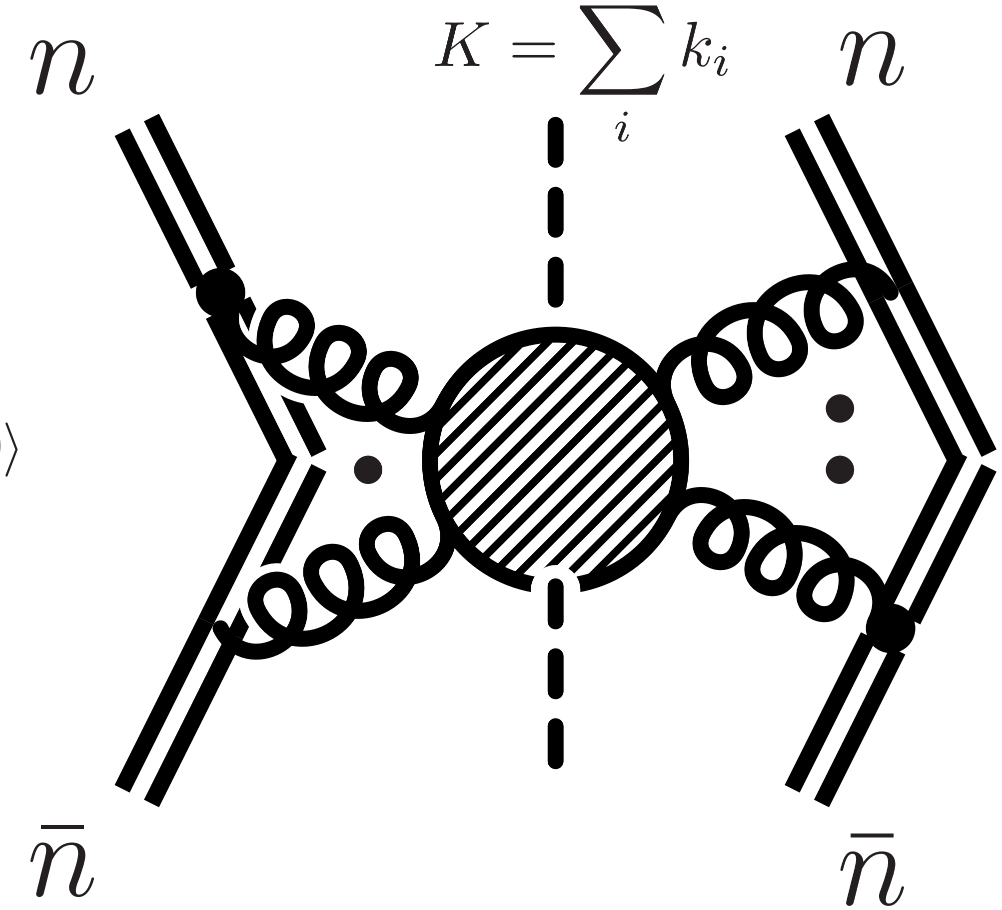
$$\int d^d K \to \lim_{\tau \to 0} \int d^d K \exp(-2 e^{\gamma_E} K^0 \tau)$$

Laplace conjugate of total energy

Take soft function as an example

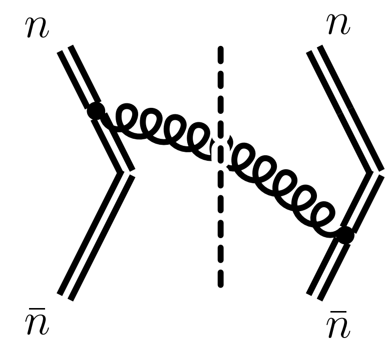
$$S(\vec{b}_\perp, \tau) = \frac{1}{C} \sum_{X_s} \mathrm{tr} \langle 0 | T\{S_{\bar{n}}^\dagger(0) S_n(0)\} \exp\left[-\mathcal{P}^0 b_0 \tau - i \vec{b}_\perp \cdot \vec{\mathcal{P}}_\perp \right] |X_s\rangle \langle X_s | \bar{T}\{S_n^\dagger(0) S_{\bar{n}}(0)\} | 0 \rangle$$

$$S_n(x) = P\mathrm{exp}\left(ig \int_{-\infty}^0 ds \, n \cdot A(x + sn) \right).$$



One-loop soft function example

non-vanishing diagram at 1 loop



$$\widetilde{S}_1(\vec{b}_\perp,\tau) = 2(4\pi)^2 C_a \left(\frac{\mu^2 e^{\gamma_E}}{4\pi}\right)^{2-d/2} \int \frac{d^dk}{(2\pi)^{d-1}}\theta(k^0)\delta(k^2) \cdot \exp\left(-2k^0\tau e^{-\gamma_E} - i\vec{b}_\perp\cdot\vec{k}_\perp\right)\frac{n\cdot\bar{n}}{k^+k^-}$$

τ is our regulator

integrating out angle, using the delta function

$$\widetilde{S}_1(\vec{b}_\perp,\tau) = 4C_a\left(\frac{b}{b_0}\mu^2\right)^{\epsilon}\int_0^\infty \frac{dk_\perp}{k_\perp^{1+\epsilon}}J_{-\epsilon}(bk_\perp)\int_0^\infty\frac{dv}{v}\exp\left[-\left(\frac{1}{\sqrt{v}}+\sqrt{v}\right)k_\perp\tau e^{-\gamma_E}\right]$$

v = exp(-2Y)

$$\widetilde{S}_1(\vec{b}_\perp,\tau) = C_a\left[\frac{4}{\epsilon^2}+\frac{4}{\epsilon}\ln(\mu^2\tau^2)+2\ln^2(\mu^2\tau^2)+4\mathrm{Li}_2\left(-\frac{\vec{b}_\perp^2}{b_0^2\tau^2}\right)+2\zeta_2\right]$$

After taking the $\tau\to 0$ **limit can identify** $\tau = \nu^{-1}$

$$S_1^\perp(\vec{b}_\perp,\nu^{-1},\mu) = C_a\left[2\ln^2\left(\frac{\vec{b}_\perp^2\mu^2}{b_0^2}\right) - 4\ln\left(\frac{\vec{b}_\perp^2\mu^2}{b_0^2}\right)\ln\left(\frac{\vec{b}_\perp^2\nu^2}{b_0^2}\right) - 2\zeta_2\right]$$

Relation to fully differential soft function

$$S(\vec{b}_\perp,\tau) = \frac{1}{C}\sum_{X_s} \mathrm{tr}\langle 0|T\{S_{\bar{n}}^\dagger(0)S_n(0)\}\exp\left[-\mathcal{P}^0 b_0\tau - i\vec{b}_\perp\cdot\vec{\mathcal{P}}_\perp\right]|X_s\rangle\langle X_s|\bar{T}\{S_n^\dagger(0)S_{\bar{n}}(0)\}|0\rangle$$

identical up to analytical continuation $\quad b^+ = b^- = ib_0\tau \quad b_0 = 2e^{-\gamma_E}$

$$S_{\mathrm{F.D.}}(b^+b^-,\vec{b}_\perp) = \frac{1}{C}\sum_{X_s} \mathrm{tr}\langle 0|T\{S_{\bar{n}}^\dagger(0)S_n(0)\}\exp\left[ib^+\mathcal{P}^-/2 + ib^-\mathcal{P}^+/2 - i\vec{b}_\perp\cdot\vec{\mathcal{P}}_\perp\right]|X_s\rangle\langle X_s|\bar{T}\{S_n^\dagger(0)S_{\bar{n}}(0)\}|0\rangle$$

Mantry, Petriello (2009)

Two-loop results for fully differential soft function available Y. Li, Mantry, Petriello (2011)

$$s^{(1)}(b,\mu) = C_F\left\{\frac{1}{2}\ln^2 L + \frac{\pi^2}{12} + \mathrm{Li}_2\left(\frac{b_\perp^2}{b^+b^-}\right)\right\},$$

$$\begin{aligned} s^{(2)}(b,\mu) = {} & C_F N_F\left\{-\frac{1}{36}\ln^3 L - \frac{5}{36}\ln^2 L - \left(\frac{7}{27} + \frac{1}{6}\mathrm{Li}_2\left(\frac{b_\perp^2}{b^+b^-}\right)\right)\ln L - \frac{41}{162} - \frac{5\pi^2}{432} + \frac{\zeta(3)}{36}\right. \\ & \left. -\frac{5}{18}\mathrm{Li}_2\left(\frac{b_\perp^2}{b^+b^-}\right) - \frac{1}{6}\mathrm{Li}_3\left(\frac{b_\perp^2}{b^+b^-}\right) + \frac{1}{6}\mathrm{S}_{1,2}\left(\frac{b_\perp^2}{b^+b^-}\right)\right\} + \\ & C_F C_A\left\{\frac{11}{72}\ln^3 L + \left(\frac{67}{72} - \frac{\pi^2}{24}\right)\ln^2 L + \left(\frac{101}{54} - \frac{7}{4}\zeta(3) + \frac{11}{12}\mathrm{Li}_2\left(\frac{b_\perp^2}{b^+b^-}\right)\right)\ln L\right. \\ & + \frac{607}{324} + \frac{67\pi^2}{864} - \frac{11}{72}\zeta(3) - \frac{\pi^4}{48} + \left(\frac{67}{36} - \frac{\pi^2}{12}\right)\mathrm{Li}_2\left(\frac{b_\perp^2}{b^+b^-}\right) + \frac{11}{12}\mathrm{Li}_3\left(\frac{b_\perp^2}{b^+b^-}\right) - \frac{1}{2}\mathrm{Li}_4\left(\frac{b_\perp^2}{b^+b^-}\right)\end{aligned}$$

13

$$\left. - \frac{11}{12}\mathrm{S}_{1,2}\left(\frac{b_\perp^2}{b^+b^-}\right) - \mathrm{S}_{1,3}\left(\frac{b_\perp^2}{b^+b^-}\right) - \frac{1}{4}\mathrm{Li}_2^2\left(\frac{b_\perp^2}{b^+b^-}\right)\right\}. \qquad (43)$$

Rewrite the result in terms of Harmonic Polylogarithms (HPLs) Remiddi, Vermaseren (1999)

HPLs are generalization of classical polylogarithms $\mathrm{Li}_n(u) = \int_0^u dt \frac{dt}{t} \mathrm{Li}_{n-1}(t)$

Defined by iterated integral:

$$H_{0,\vec{w}}(u) = \int_0^u \frac{dt}{t} H_{\vec{w}}(t) \qquad H_{1,\vec{w}}(u) = \int_0^u \frac{dt}{1-t} H_{\vec{w}}(t)$$

example: $H_{1,0,0,1}(u) \equiv H_{1,3}(u) = \int_0^u \frac{dt}{1-t} H_{0,0,1}(t)$ **transcendental weight:** $\left[H_{1,3}(u)\right] = 4$

$$S(\vec{b}_\perp, \tau, \mu) = \exp\left[\frac{\alpha_s(\mu)}{4\pi} S_1(\vec{b}_\perp, \tau, \mu) + \left(\frac{\alpha_s(\mu)}{4\pi}\right)^2 S_2(\vec{b}_\perp, \tau, \mu) + \left(\frac{\alpha_s(\mu)}{4\pi}\right)^3 S_3(\vec{b}_\perp, \tau, \mu) + \mathcal{O}(\alpha_s^4)\right]$$

Non-Abelian exponentiation. Gathered (1983); Frenkel, Taylor (1984)

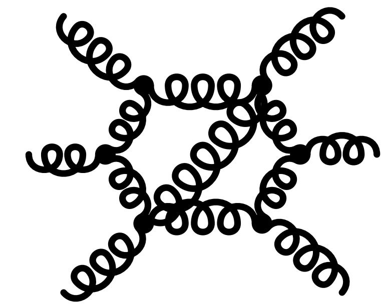
$$S_1(\vec{b}_\perp, \tau, \mu = \tau^{-1}) = c_1^s + 4C_a H_2$$

$$S_2(\vec{b}_\perp, \tau, \mu = \tau^{-1}) = c_2^s + C_A C_a \Big(-8\zeta_2 H_2 + \frac{268}{9} H_2 + \frac{44}{3} H_3 - 8H_4 - \frac{44}{3} H_{2,1} - 8H_{2,2} - 16H_{3,1} - 16H_{2,1,1}\Big) + C_a N_f \Big(-\frac{40}{9} H_2 - \frac{8}{3} H_3 + \frac{8}{3} H_{2,1}\Big)$$

$$H_{\vec{w}} \equiv H_{\vec{w}}\left(-\frac{\vec{b}_\perp^2}{b_0^2 \tau^2}\right)$$

Bootstrap program for scattering amplitude

Dixon, Drummond, Golden, Goncharov, Henn, Papathanasiou, Paulos, Spradlin, Volovich, Vergu …



Six gluon amplitudes in planar N=4 Supersymmetric Yang-Mills Theory

$$A_6^{\mathrm{MHV}}(\epsilon; s_{ij}) = A_6^{\mathrm{BDS}}(\epsilon; s_{ij}) \exp\left[R_6(u_1, u_2, u_3)\right]$$

all order ansatz **remainder function**

$$R_6^{(2)}(u_1, u_2, u_3) = \sum_{i=1}^{3}\left(L_4(x_i^+, x_i^-) - \frac{1}{2}\mathrm{Li}_4(1 - 1/u_i)\right)$$
$$- \frac{1}{8}\left(\sum_{i=1}^{3}\mathrm{Li}_2(1 - 1/u_i)\right)^2 + \frac{1}{24}J^4 + \frac{\pi^2}{12}J^2 + \frac{\pi^4}{72}$$

Goncharov, Spradlin, Vergu, Volovich (2010)

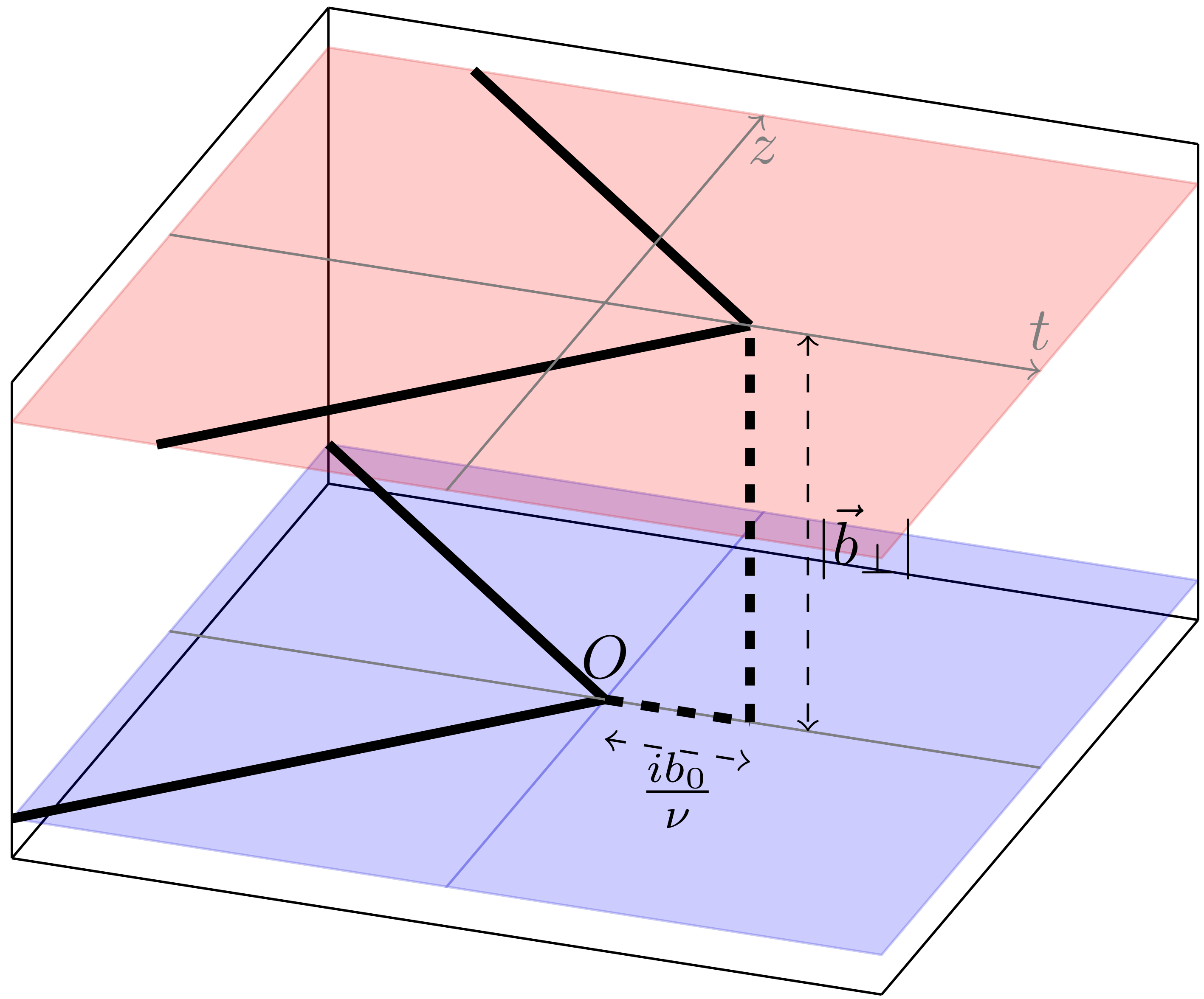
simplicity of the two-loop remainder function trigger many works on bootstrapping higher-loop remainder function without actually calculating Feynman diagrams

Identify and build up the relevant space of transcendental functions. Construct the ansatz. → **Using formal constraints (branch cut structure) and matching onto physical limit (multi-Regge, near collinear)** → **check the result**

- Usually, bootstrap is difficult for QCD: complicated rational coefficients; transcendental functions with different degrees of transcendental weight appear
- In general integrability is lost in QCD. Much less physical boundary available.
- But the fully differential soft function is special. Its exceptional simplicity makes it a good candidate for bootstrap.

Bootstrapping the fully differential soft function

$$S(\vec{b}_\perp, \tau) = \frac{1}{C} \sum_{X_s} \mathrm{tr} \langle 0 | T\{S_{\bar{n}}^\dagger(0) S_n(0)\} \exp\left[-\mathcal{P}^0 b_0 \tau - i\vec{b}_\perp \cdot \vec{\mathcal{P}}_\perp \right] |X_s\rangle \langle X_s | \bar{T}\{S_n^\dagger(0) S_{\bar{n}}(0)\} |0\rangle$$



$$\frac{d \ln S(\vec{b}_\perp, \tau, \mu)}{d \ln \mu^2} = \Gamma_{\mathrm{cusp}}[\alpha_S(\mu)] \ln(\mu^2 \tau^2) - \gamma_s[\alpha_S(\mu)]$$

will only concentrate on $S(\vec{b}_\perp, \tau, \mu = \tau^{-1})$

Ansatz for all orders fully differential soft function

- **A simple ansatz based on one and two loop "data"**

$$S_L(\vec{b}_\perp, \tau, \mu = \tau^{-1}) \doteq c_L^s + \sum_i r_i F_i(x)$$

r_i: rational number

F_i: HPLs, production of zeta value and HPL

examples of F_i: $H_{0,0,0,1}(x) \equiv H_4$ $\zeta_3 H_{0,0,1,0,1}(x) \equiv \zeta_3 H_{3,2}$

- **Weight vector draw from {0, 1}**
- **Last entry of the weight vector = 1; guaranteed by the branch cut structure of the integrals**
- **First entry: empirically observation from one and two loops**

Constraint: linearity of Log[τ] divergence

$$S(\vec{b}_\perp, \tau, \mu) = \exp\left[\frac{\alpha_s(\mu)}{4\pi} S_1(\vec{b}_\perp, \tau, \mu) + \left(\frac{\alpha_s(\mu)}{4\pi}\right)^2 S_2(\vec{b}_\perp, \tau, \mu) + \left(\frac{\alpha_s(\mu)}{4\pi}\right)^3 S_3(\vec{b}_\perp, \tau, \mu) + \mathcal{O}(\alpha_s^4)\right]$$

- **Bootstrap the soft function on the exponent**
- **At each order on the exponent, should only contain single log of rapidity divergence due to Non-Abelian Exponentiation theorem**
- **It turns out that such simple requirement already impose very strong constraint**
- **One-loop ansatz:**

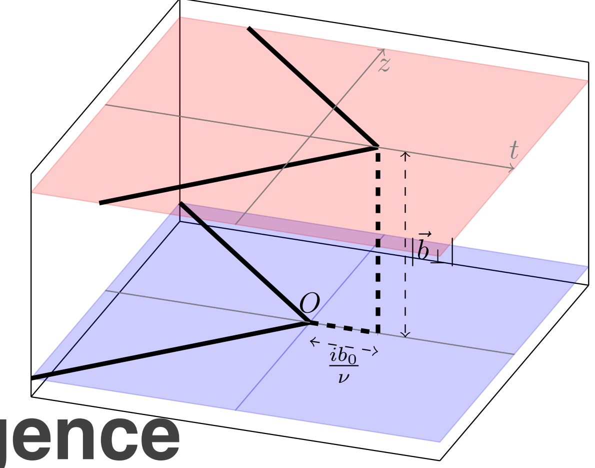
fixed by RG

$$S(\vec{b}_\perp, \tau, \mu) = C_a\left[2\ln^2(\mu^2\tau^2) + 2\zeta_2 + r_1 H_2\right]$$

Using $$\lim_{\tau\to 0} H_2(x) = \lim_{\tau\to 0} \mathrm{Li}_2\left(-\frac{\vec{b}_\perp^{\,2}}{b_0^2\tau^2}\right) = -\frac{1}{2}\ln^2\left(\frac{\vec{b}_\perp^{\,2}}{b_0^2\tau^2}\right) - \zeta_2 + \mathcal{O}(\tau)$$

$\Rightarrow$ $$r_1 = 4$$ **One-loop for free!**

Threshold expansion



- **At higher loops, the single logarithmic rapidity divergence constraint is not enough. Expansion around threshold**

$$S(\vec{b}_\perp,\tau)=\frac{1}{C}\sum_{X_s}\mathrm{tr}\langle 0|T\{S^\dagger_{\bar{n}}(0)S_n(0)\}\exp\Big[-\mathcal{P}^0 b_0\tau - i\vec{b}_\perp\cdot\vec{\mathcal{P}}_\perp\Big]|X_s\rangle\langle X_s|\bar{T}\{S^\dagger_n(0)S_{\bar{n}}(0)\}|0\rangle$$

expanding in small impact parameter

$$S(\vec{b}_\perp,\tau,\mu)=\int\frac{d^dk}{(2\pi)^d}\theta(k^0)\theta(k^2)\exp(-2k^0\tau^{-1}e^{-\gamma_E})\sum_{n=0}^{\infty}\frac{(-i\vec{b}_\perp\cdot\vec{k}_\perp)^n}{n!}\hat{S}(k,\mu)$$

fully differential soft function in momentum space

odd power vanish, only even power

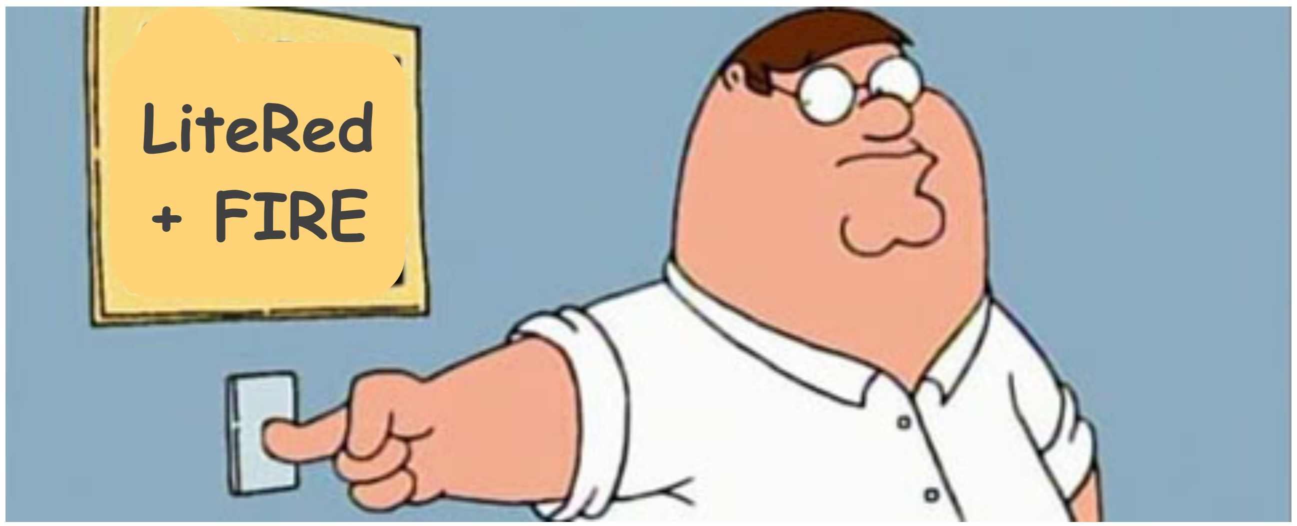
$$\frac{(-i\vec{b}_\perp\cdot\vec{k}_\perp)^{2m}}{(2m)!}=f(2m)\left(\vec{b}^{\,2}_\perp\right)^m(k^+k^- - k^2)^m$$

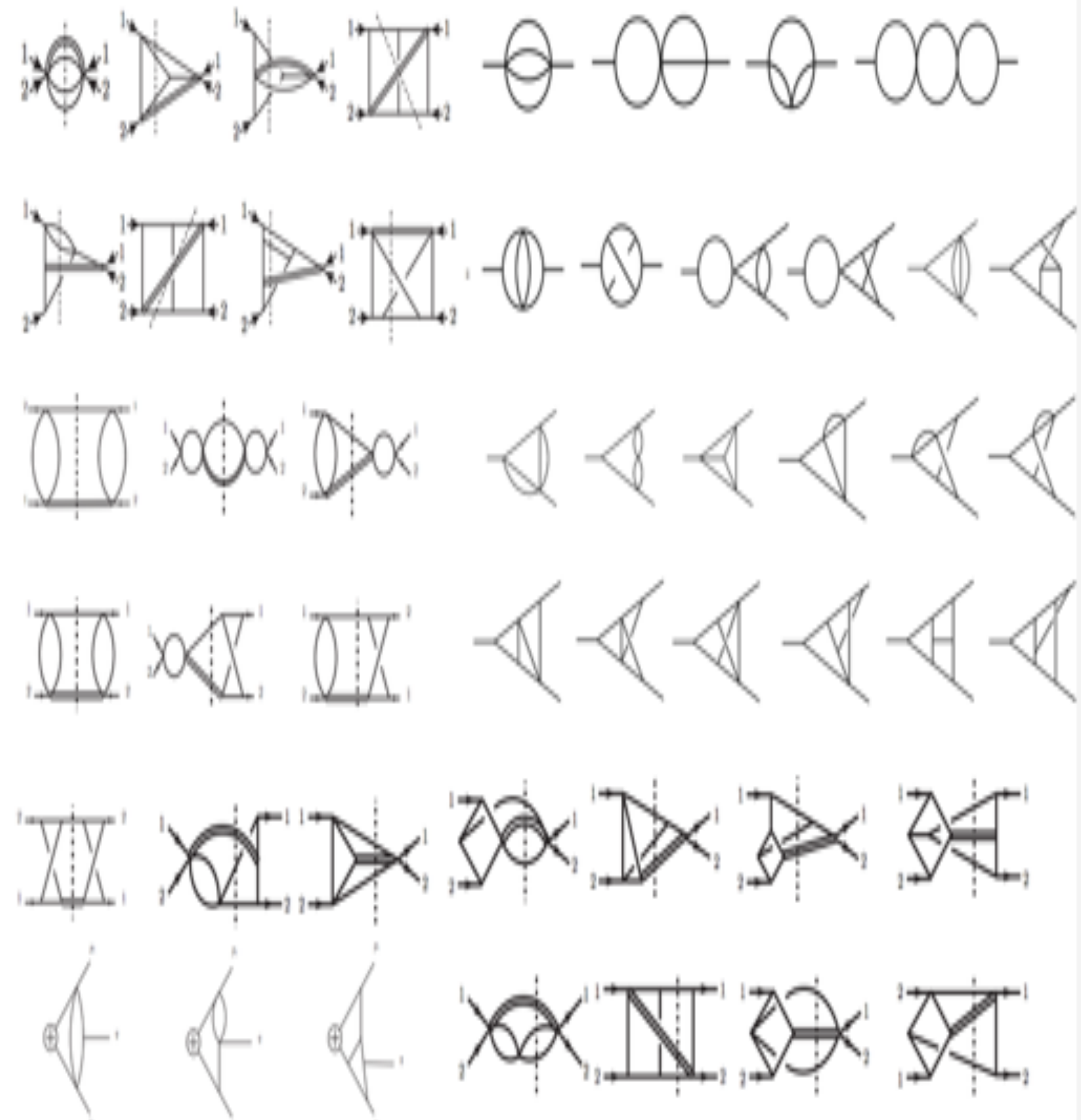
$$f(2m)=(-1)^m\frac{1\cdot 3\cdot 5\ldots(2m-1)}{d_\perp\cdot(d_\perp+2)\cdot(d_\perp+4)\ldots(d_\perp+2m-2)}$$

$$\boxed{S(\vec{b}_\perp,\tau,\mu)=\sum_{m=0}^{\infty}\frac{f(2m)}{(2m)!}(\vec{b}^{\,2}_\perp)^m\int\frac{d^dk}{(2\pi)^d}\theta(k^0)\theta(k^2)\exp\left(-\frac{2k^0}{\tau e^{\gamma_E}}\right)(k^+k^- - k^2)^m\hat{S}(k,\mu)}$$

All three-loop ingredients are known!

$$S(\vec{b}_\perp, \tau, \mu) = \sum_{m=0}^{\infty} \frac{f(2m)}{(2m)!} (\vec{b}_\perp^2)^m \int \frac{d^d k}{(2\pi)^d} \theta(k^0)\theta(k^2) \exp\left(-\frac{2k^0}{\tau e^{\gamma_E}}\right) (k^+ k^- - k^2)^m \hat{S}(k, \mu)$$





All integrals available:

- **Tree-level triple real emission**
  - Anastasiou, Duhr, Dulat, Herzog, Mistlberger (2013)
  - HXZ (2015)
- **One-loop double real emission**
  - Anastasiou, Duhr, Dulat, Furlan, Herzog, Mistlberger (2015)
  - Y. Li, von Manteuffel, Schabinger, HXZ (2014)
- **Two-loop single emission**
  - Y. Li, HXZ (2013)
  - Duhr, Gehrmann (2013)

courtesy of C. Duhr

Three ansatz for QCD:

$$S_3^{\text{ansatz}}(\vec{b}_\perp,\tau,\mu) = C_a C_A^2 \Big(r_8\zeta_2 H_2 + r_{17}\zeta_2 H_3 + r_{18}\zeta_2 H_{2,1} + r_{36}\zeta_2 H_4 + r_{37}\zeta_2 H_{3,1} + r_{38}\zeta_2 H_{2,2} + r_{39}\zeta_2 H_{2,1,1} + r_{19}\zeta_3 H_2 + r_{40}\zeta_3 H_3 + r_{41}\zeta_3 H_{2,1} + r_{42}\zeta_4 H_2 + r_1 H_2 + r_2 H_3 + r_3 H_{2,1} + r_4 H_4 + r_5 H_{3,1} + r_6 H_{2,2} + r_7 H_{2,1,1} + r_9 H_5 + r_{10} H_{4,1} + r_{11} H_{3,2} + r_{12} H_{3,1,1} + r_{13} H_{2,3} + r_{14} H_{2,2,1} + r_{15} H_{2,1,2} + r_{16} H_{2,1,1,1} + r_{20} H_6 + r_{21} H_{5,1} + r_{22} H_{4,2} + r_{23} H_{4,1,1} + r_{24} H_{3,3} + r_{25} H_{3,2,1} + r_{26} H_{3,1,2} + r_{27} H_{3,1,1,1} + r_{28} H_{2,4} + r_{29} H_{2,3,1} + r_{30} H_{2,2,2} + r_{31} H_{2,2,1,1} + r_{32} H_{2,1,3} + r_{33} H_{2,1,2,1} + r_{34} H_{2,1,1,2} + r_{35} H_{2,1,1,1,1} \Big) + \ldots$$

momentum-space fully differential soft function

$$\int \frac{d^d k}{(2\pi)^d}\theta(k^0)\theta(k^2)\exp\left(-\frac{2k^0}{\tau e^{\gamma_E}}\right)(k^+k^- - k^2)^{27}\hat{S}(k,\mu)$$

- **Number of Independent coefficients = 31**
- **Using constraint from single logarithmic rapidity divergence, independent coefficients reduced to 27**
- **Requires inserting momentum numerator (pT operator) to the power of 27! The rough estimate of computer resource consumed as function of the power N of the numerator increases factorial, N!**

Principle of maximal transcendentality

Kotikov, Lipatov, Onishchenko, Velizhanin (2002-2004)

- **This is where maximal supersymmetric Yang-Mills (N=4 SYM) can help**
- **It was observed by Kotikov et al. that the most complicated contributions (the highest transcendental-weight part) in N=4 SYM and QCD are the same for Mellin-moment of DGLAP kernel. This is called principle of maximal transcendentality**
- **In SCET threshold soft function, the same principle is also known to work** Y. Li, von Manteuffel, Scharbinger, HXZ (2014)

s.v. =

$$G^{\mathcal{N}=4}(z,0) = \mathcal{D}_0 + a\left\{\left[16\mathcal{D}_2 + 8\zeta_2\mathcal{D}_0\right]C_A\right\} + a^2\left\{\left[128\mathcal{D}_4 - 160\zeta_2\mathcal{D}_2 + 312\zeta_3\mathcal{D}_1 - 2\zeta_4\mathcal{D}_0\right]C_A^2\right\}$$
$$+ a^3\left\{\left[512\mathcal{D}_6 - 3584\zeta_2\mathcal{D}_4 + 11584\zeta_3\mathcal{D}_3 - 4928\zeta_4\mathcal{D}_2\right.\right.$$
$$\left.\left. + \left(-\frac{23200\zeta_2\zeta_3}{3} + 11904\zeta_5\right)\mathcal{D}_1 + \left(\frac{13216\zeta_3^2}{3} - \frac{8012\zeta_6}{3}\right)\mathcal{D}_0\right]C_A^3\right\} \tag{6.3}$$

= leading transcendental part of QCD

$$\mathcal{D}_0 = \delta(1-z)$$
$$\mathcal{D}_i = \left[\frac{\ln^{i-1}(1-z)}{1-z}\right]_+$$

Tackling the N=4 part first

Ansatz for three-loop N=4 SYM. Only 16 “independent” coefficients.

$$S_3^{\mathcal{N}=4}(\vec{b}_\perp,\tau,\mu) = C_A^3\Big(r_{36}\zeta_2 H_4 + r_{37}\zeta_2 H_{3,1} + r_{38}\zeta_2 H_{2,2} + r_{39}\zeta_2 H_{2,1,1} + r_{40}\zeta_3 H_3 + r_{41}\zeta_3 H_{2,1} + r_{42}\zeta_4 H_2 + r_{20} H_6 + r_{21} H_{5,1} + r_{22} H_{4,2} + r_{23} H_{4,1,1} + r_{24} H_{3,3} + r_{25} H_{3,2,1} + r_{26} H_{3,1,2} + r_{27} H_{3,1,1,1} + r_{28} H_{2,4} + r_{29} H_{2,3,1} + r_{30} H_{2,2,2} + r_{31} H_{2,2,1,1} + r_{32} H_{2,1,3} + r_{33} H_{2,1,2,1} + r_{34} H_{2,1,1,2} + r_{35} H_{2,1,1,1,1}\Big)$$

All unknown determined by threshold expansion up to 11 power. Three-loop results:

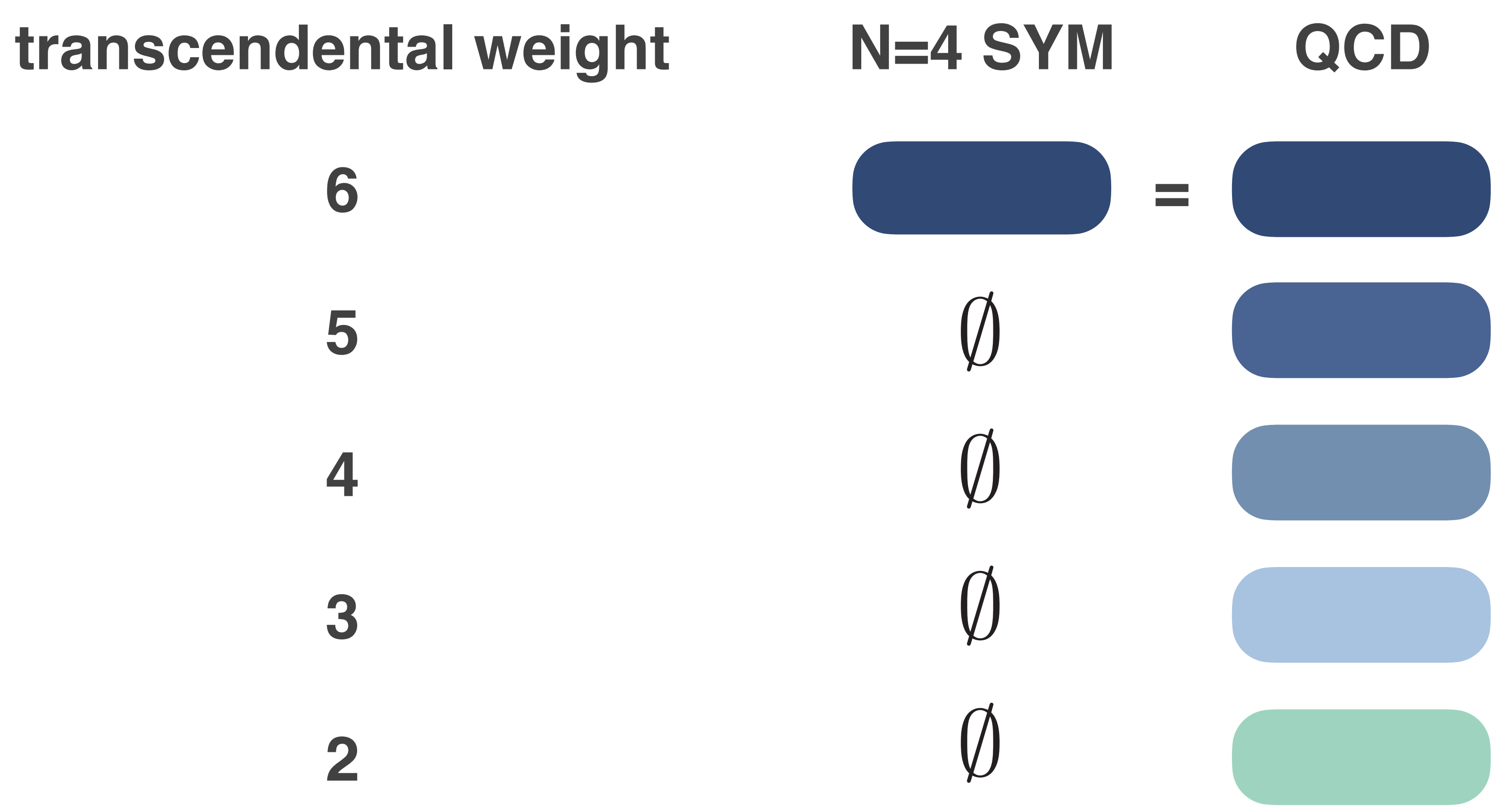
$$S_3^{\mathcal{N}=4}(\vec{b}_\perp,\tau,\mu = \tau^{-1}) = c_3^{s,\mathcal{N}=4} + N_c^3\Big(16\zeta_2 H_4 + 48\zeta_2 H_{2,2} + 64\zeta_2 H_{3,1} + 96\zeta_2 H_{2,1,1} + 120\zeta_4 H_2 + 48 H_6 + 24 H_{2,4} + 40 H_{3,3} + 72 H_{4,2} + 128 H_{5,1} + 16 H_{2,1,3} + 56 H_{2,2,2} + 80 H_{2,3,1} + 80 H_{3,1,2} + 144 H_{3,2,1} + 224 H_{4,1,1} + 64 H_{2,1,1,2} + 96 H_{2,1,2,1} + 160 H_{2,2,1,1} + 256 H_{3,1,1,1} + 192 H_{2,1,1,1,1}\Big)$$

One and two loops

$$S_1^{\mathcal{N}=4}(\tau,\vec{b}_\perp,\mu=\tau^-1) = c_1^{s,\mathcal{N}=4} + 4N_c H_2$$
$$S_2^{\mathcal{N}=4}(\tau,\vec{b}_\perp,\mu=\tau^-1) = c_2^{s,\mathcal{N}=4} + N_c^2\Big(-8\zeta_2 H_2 - 8H_4 - 8H_{2,2} - 16H_{3,1} - 16H_{2,1,1}\Big)$$

Interesting alternating overall sign.

QCD = ([N=4]) + (QCD - [N=4])



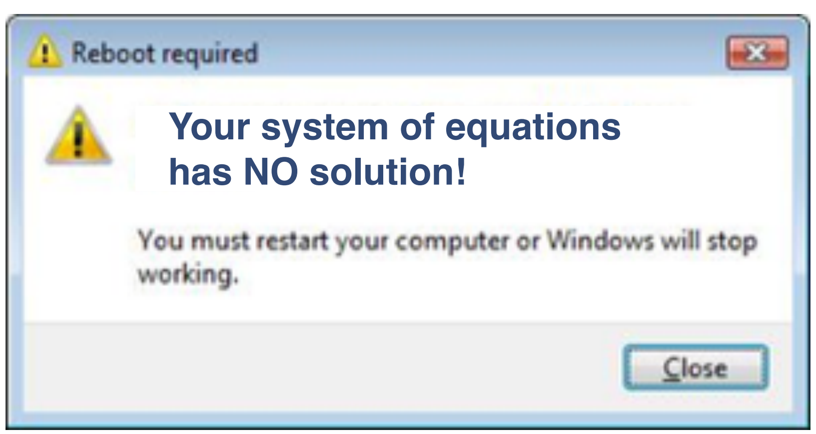
By doing a maximal supersymmetric decomposition (transcendental weight decomposition), the number of "independent" coefficients to be fixed reduced to 15, which is manageable by our computer.

$$S_3^{\text{ansatz}}(\vec{b}_\perp,\tau,\mu) = C_a C_A^2 \Big(r_8\zeta_2 H_2 + r_{17}\zeta_2 H_3 + r_{18}\zeta_2 H_{2,1} + r_{36}\zeta_2 H_4 + r_{37}\zeta_2 H_{3,1} + r_{38}\zeta_2 H_{2,2} + r_{39}\zeta_2 H_{2,1,1} + r_{19}\zeta_3 H_2 + r_{40}\zeta_3 H_3 + r_{41}\zeta_3 H_{2,1} + r_{42}\zeta_4 H_2 + r_1 H_2 + r_2 H_3 + r_3 H_{2,1} + r_4 H_4 + r_5 H_{3,1} + r_6 H_{2,2} + r_7 H_{2,1,1} + r_9 H_5 + r_{10} H_{4,1} + r_{11} H_{3,2} + r_{12} H_{3,1,1} + r_{13} H_{2,3} + r_{14} H_{2,2,1} + r_{15} H_{2,1,2} + r_{16} H_{2,1,1,1} + r_{20} H_6 + r_{21} H_{5,1} + r_{22} H_{4,2} + r_{23} H_{4,1,1} + r_{24} H_{3,3} + r_{25} H_{3,2,1} + r_{26} H_{3,1,2} + r_{27} H_{3,1,1,1} + r_{28} H_{2,4} + r_{29} H_{2,3,1} + r_{30} H_{2,2,2} + r_{31} H_{2,2,1,1} + r_{32} H_{2,1,3} + r_{33} H_{2,1,2,1} + r_{34} H_{2,1,1,2} + r_{35} H_{2,1,1,1,1} \Big) + \ldots$$

Compare with threshold expansion to fixed unknown coefficients. Amounts to solving system of linear equations.

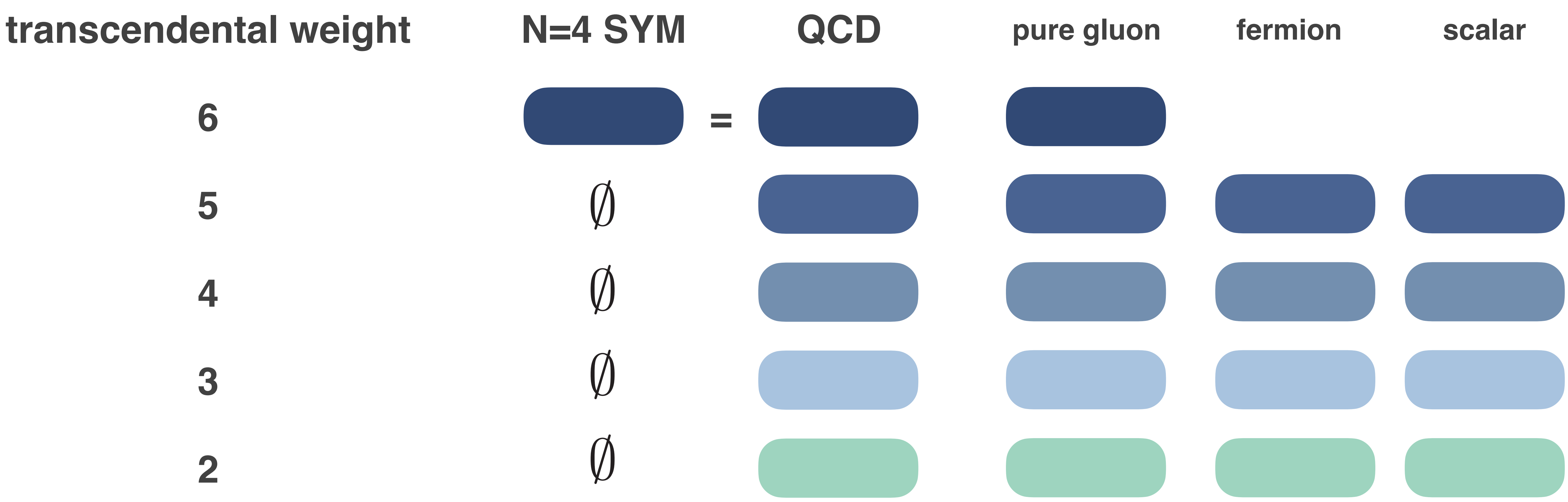
$$S_3^{QCD}(\vec{b}_\perp,\tau,\mu=\tau^{-1}) = C_a C_A^2 \Big[x\left(-\frac{1192\zeta_2}{9} - 176\zeta_3 + 120\zeta_4 + \frac{43330}{81}\right) + x^2\left(\frac{86\zeta_2}{9} - 44\zeta_3 + 30\zeta_4 - \frac{1754}{81}\right) + x^3\left(\frac{1304\zeta_2}{81} - \frac{176\zeta_3}{9} + \frac{40\zeta_4}{3} - \frac{35647}{1458}\right) + x^4\left(\frac{2039\zeta_2}{144} - 11\zeta_3 + \frac{15\zeta_4}{2} - \frac{290297}{20736}\right) + x^5\left(\frac{66149\zeta_2}{5625} - \frac{176\zeta_3}{25} + \frac{24\zeta_4}{5} - \frac{276151309}{40500000}\right) + x^6\left(\frac{26399\zeta_2}{2700} - \frac{44\zeta_3}{9} + \frac{10\zeta_4}{3} - \frac{46359709}{19440000}\right) + \mathcal{O}(x^7) \Big]$$

- **But the equations have no solution !**



- **The problem could potentially due to the incompleteness our ansatz**

QCD = ([N=4]) + (QCD - [N=4])

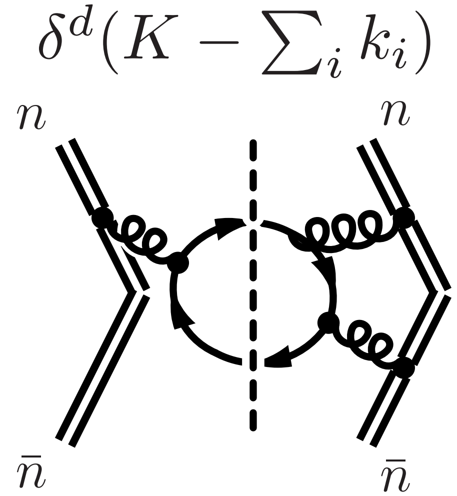


[N=4 SYM] = 1 gluon + 4 majorana fermion + 3 complex scalar

To resolve the problem, we decided to calculate the less complicated part, the part that at least involve one fermion loop using Feynman diagrams.

Direct calculation will also real what function is missing in our ansatz.

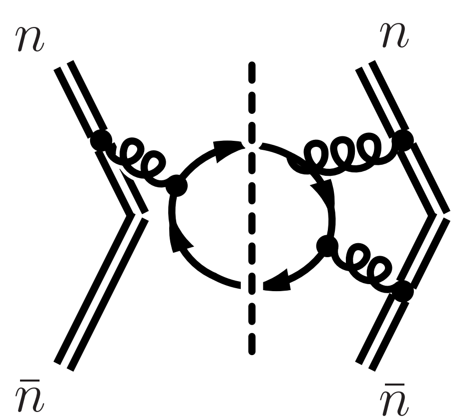
representative three-loop fermionic contribution



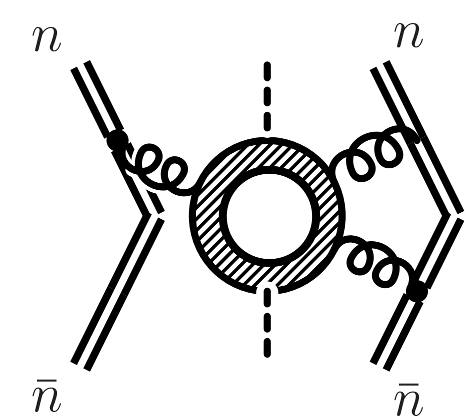
+ many other diagrams

- Systematic way to compute these contribution based on differential equation for Feynman integral Kotikov; Gehrmann, Remiddi; …

$\frac{d}{dt}$



$= \sum_i g(t)$



$_i$

HXZ (2015)

$$t = \frac{K\cdot n\, K\cdot \bar{n}}{K^2}$$

- Indeed we found functions that are not captured by our original ansatz
- We get the full results either by direct calculation (with the known highest transcendental part), or by bootstrap with the improved ansatz

Full three-loop soft function

$$S^{\rm F.D.}_{3,\mathcal{N}=4}\Big|_{\mu=\nu} = c^s_{3,\mathcal{N}=4} + N_c^3\Big(16\zeta_2 H_4 + 48\zeta_2 H_{2,2} + 64\zeta_2 H_{3,1} + 96\zeta_2 H_{2,1,1} + 120\zeta_4 H_2 + 48 H_6 + 24 H_{2,4} + 40 H_{3,3} + 72 H_{4,2} + 128 H_{5,1} + 16 H_{2,1,3} + 56 H_{2,2,2} + 80 H_{2,3,1} + 80 H_{3,1,2} + 144 H_{3,2,1} + 224 H_{4,1,1} + 64 H_{2,1,1,2} + 96 H_{2,1,2,1} + 160 H_{2,2,1,1} + 256 H_{3,1,1,1} + 192 H_{2,1,1,1,1}\Big)$$

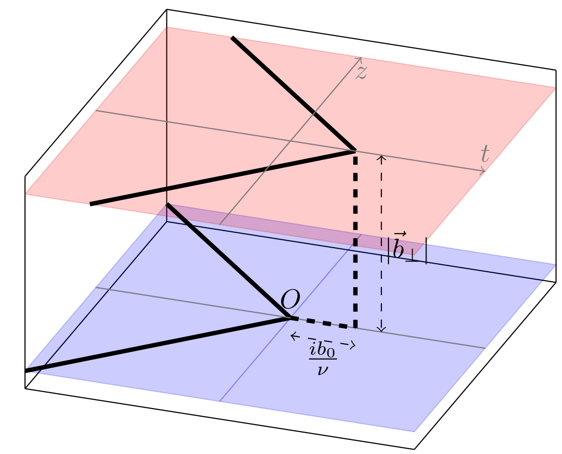
N=4 part

$$S^{\rm F.D.}_{3}\Big|_{\mu=\nu} = c^s_3 + \frac{C_a C_A^2}{N_c^3}\left(S^{\rm F.D.}_{3,\mathcal{N}=4}(x)\Big|_{\mu=\nu} - c^s_{3,\mathcal{N}=4}\right) + C_a C_A^2\Big[-\frac{1072}{9}\zeta_2 H_2 - 176\zeta_3 H_2 - \frac{88}{3}\zeta_2 H_3 + 88\zeta_2 H_{2,1} + \frac{30790}{81}H_2 + \frac{7120}{27}H_3 - \frac{104}{9}H_4 - \frac{440}{3}H_5 - \frac{8}{3}\left(H_{1,1} - \frac{H_{1,1}}{x}\right) - \frac{7120}{27}H_{2,1} - \frac{1072}{9}H_{2,2} - \frac{88}{3}H_{2,3} - \frac{3112}{9}H_{3,1} - 88 H_{3,2} - \frac{352}{3}H_{4,1} - \frac{392}{3}H_{2,1,1} + \frac{88}{3}H_{2,1,2} + \frac{352}{3}H_{2,2,1} + \frac{352}{3}H_{3,1,1} + 352 H_{2,1,1,1}\Big] + C_a C_A n_f\Big[\frac{160}{9}\zeta_2 H_2 + \frac{16}{3}\zeta_2 H_3 - 16\zeta_2 H_{2,1} - \frac{7988}{81}H_2 - \frac{2312}{27}H_3 - \frac{64}{3}H_4 + \frac{80}{3}H_5 + \frac{8}{3}\left(H_{1,1} - \frac{H_{1,1}}{x}\right) + \frac{2312}{27}H_{2,1} + \frac{160}{9}H_{2,2} + \frac{16}{3}H_{2,3} + \frac{224}{3}H_{3,1} + 16 H_{3,2} + \frac{64}{3}H_{4,1} - \frac{32}{9}H_{2,1,1} - \frac{16}{3}H_{2,1,2} - \frac{64}{3}H_{2,2,1} - \frac{64}{3}H_{3,1,1} - 64 H_{2,1,1,1}\Big] + C_a n_f^2\left(\frac{400}{81}H_2 + \frac{160}{27}H_3 + \frac{32}{9}H_4 - \frac{160}{27}H_{2,1} - \frac{32}{9}H_{3,1} + \frac{32}{9}H_{2,1,1}\right) + C_a C_F n_f\left(32\zeta_3 H_2 - \frac{110}{3}H_2 - 8H_3 + 8H_{2,1}\right) \quad (8)$$

QCD RESULT

Rapidity anomalous dimension @ 3 loop

$$S_\perp(\vec{b}_\perp,\mu,\nu) = \lim_{\tau\to 0} S(\vec{b}_\perp,\tau,\mu)\Big|_{\tau=\nu^{-1}}$$



$$S(\vec{b}_\perp,\tau) = \frac{1}{C}\sum_{X_s} \mathrm{tr}\langle 0|T\{S_{\bar{n}}^\dagger(0)S_n(0)\}\exp\left[-\mathcal{P}^0 b_0\tau - i\vec{b}_\perp\cdot\vec{\mathcal{P}}_\perp\right]|X_s\rangle\langle X_s|\bar{T}\{S_n^\dagger(0)S_{\bar{n}}(0)\}|0\rangle$$

The rapidity renormalization group

$$\frac{d\ln S_\perp(\vec{b}_\perp,\mu,\nu)}{d\ln\nu^2} = \int_{\mu^2}^{b_0^2/\vec{b}_\perp^2}\frac{d\bar{\mu}^2}{\bar{\mu}^2}\Gamma_{\mathrm{cusp}}\left[\alpha_S(\bar{\mu})\right] + \gamma_R\left[\alpha_S(b_0/|\vec{b}_\perp|)\right]$$

$$\gamma_0^R = 0$$

$$\gamma_1^R = C_a C_A\left(28\zeta_3 - \frac{808}{27}\right) + \frac{112 C_a n_f}{27}$$

$$\gamma_2^R = C_a C_A^2\left(-\frac{176}{3}\zeta_3\zeta_2 + \frac{6392\zeta_2}{81} + \frac{12328\zeta_3}{27} + 44\zeta_4 - 192\zeta_5 - \frac{297029}{729}\right)$$
$$+ C_a C_A n_f\left(-\frac{824\zeta_2}{81} - \frac{904\zeta_3}{27} + 8\zeta_4 + \frac{62626}{729}\right) + c\beta_0$$
$$+ C_a n_f^2\left(-\frac{32\zeta_3}{9} - \frac{1856}{729}\right) + C_a C_F N_f\left(-\frac{304\zeta_3}{9} - 16\zeta_4 + \frac{1711}{27}\right)$$

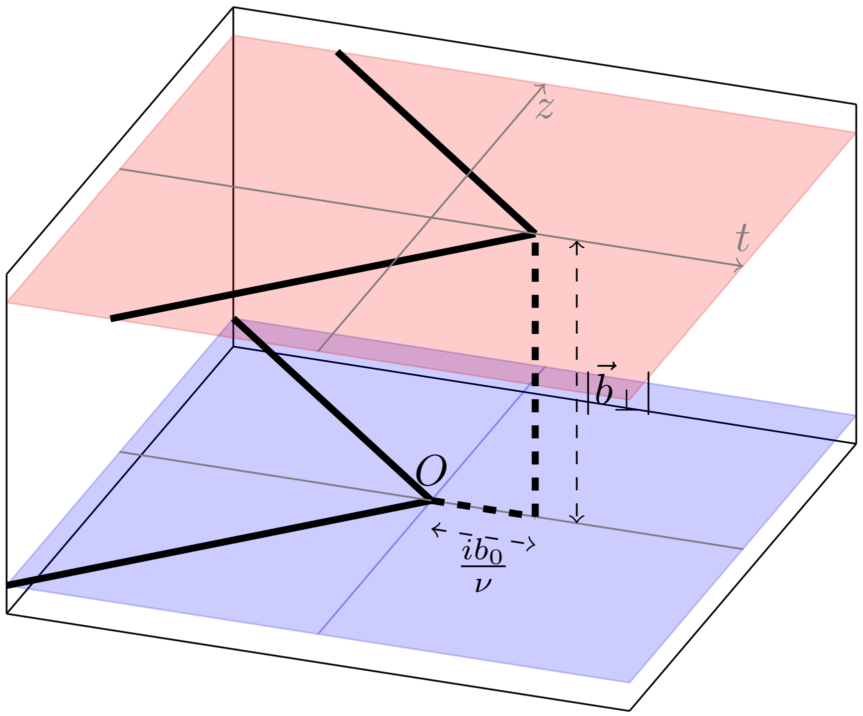
one and two loops known. Direct calculation:
Luebbert, Oredsson, Stahlhofen (2016)
also extractable from:

- Davies, Webber, Stirling (1985)
- Grazzini, de Florian (2000)
- Gehrmann, Lubbert, Yang (2012,2014)
- Echevarria, Scimemi, Vladimirov (2015)

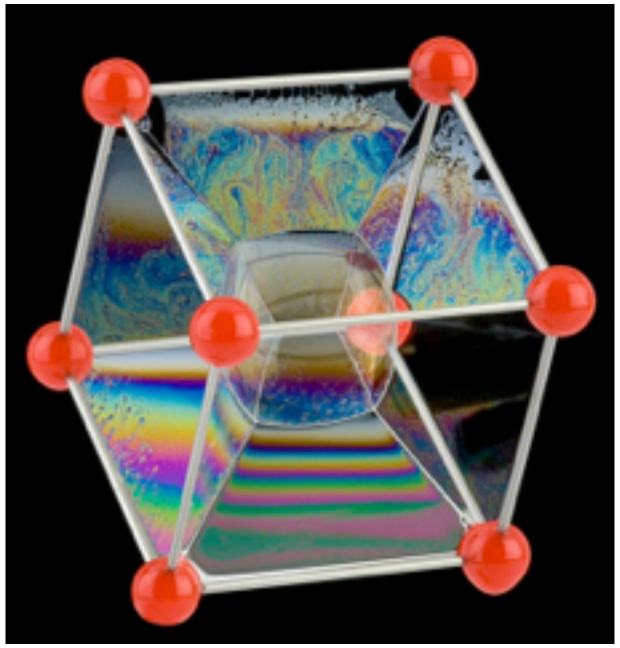
New three loop results!

Non-perturbative rapidity anomalous dimension?

$$\gamma_R = \lim_{\nu \to \infty} \nu^2 \frac{d}{d\nu^2}$$



- **At strong coupling not calculable from QCD, but feasible in planar N=4 SYM**
- **Amounts to a "soap bubble" calculation**



The application of three-loop rapidity anomalous dimension to Higgs production at small pT

work in progress

Small pT cross section for Higgs production

- **There are many different ways to perform pT resummation for Higgs production. We follow** Neill, Rothstein, Vaidya (2015)

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}^2\vec{Q}_T} = \int x_a \int x_b \, \delta\Big(x_a x_b - \frac{m_H^2}{S}\Big)\sigma_0 \int \frac{\mathrm{d}^2\vec{b}}{(2\pi)^2} e^{i\vec{b}\cdot\vec{Q}_T} W\big(x_a, x_b, m_H, \vec{b}, \mu, \nu\big) + \frac{\mathrm{d}^2\sigma}{\mathrm{d}^2\vec{Q}_T}\bigg|_{\mathrm{n.s.}}$$

$$W(x_a, x_b, m_H, \vec{b}, \mu, \nu) = \Big|C_V\big(m_t, m_H, \mu\big)\Big|^2 S(\vec{b}, \mu, \nu) B^{\alpha\beta}_{g/N_1}(x_a, Q, \vec{b}, \mu, \nu) B^{\alpha\beta}_{g/N_2}(x_b, Q, \vec{b}, \mu, \nu)$$

$$C_V(m_t, m_H, \mu) = C_V(m_t, m_H, \mu_H) \exp\left[\frac{1}{2}\int_{\mu_H^2}^{\mu^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2}\left(\Gamma_{\mathrm{cusp}}\left[\alpha_s(\bar{\mu})\right] \ln\frac{M_H^2}{\bar{\mu}^2} + \gamma^V\left[\alpha_s(\bar{\mu})\right]\right)\right]$$

$$\begin{aligned} S(\vec{b}_\perp, \mu, \nu) = S(\vec{b}_\perp, \mu_s, \nu_s) \exp\bigg[& \int_{\mu_s^2}^{\mu^2} \frac{\mathrm{d}\bar{\mu}^2}{\bar{\mu}^2}\Big(\Gamma_{\mathrm{cusp}}\left[\alpha_s(\bar{\mu})\right] \ln\frac{b^2\bar{\mu}^2}{b_0^2} + \gamma_s\left[\alpha_s(\bar{\mu})\right]\Big) \\ & + \ln\frac{\nu^2}{\nu_s^2}\Big(\int_{b_0^2/b^2}^{\mu^2} \frac{\mathrm{d}\bar{\mu}^2}{\bar{\mu}^2}\Gamma_{\mathrm{cusp}}\left[\alpha_s(\bar{\mu})\right] + \gamma_R\left[\alpha_s(b_0/b)\right]\Big)\bigg] \end{aligned}$$

The beam function (with exponential regulator)

$$B^{\alpha\beta}_{g/N}(x,\vec{b},Q,\mu,\nu) = \frac{g_\perp^{\alpha\beta}}{d-2} B_{g/N}(x,b,Q,\mu,\nu) + \left(\frac{g_\perp^{\alpha\beta}}{d-2} + \frac{b^\alpha b^\beta}{b^2}\right) B'_{g/N}(x,b,Q,\mu,\nu)$$

$$B_{g/N}(x,b,Q,\mu,\nu) = \sum_j \int_x^1 \frac{\mathrm{d}z}{z} I_{gj}(z,b,Q,\mu,\nu) f_{j/N}(x/z,\mu) + \ldots$$

- **Need two-loop results for I_{gj}. Three-loop scale dependent part from iterated solution of differential (Splitting func. @3loop.** Moch, Vogt, Vermaseren (2004)**)**
- **Two ways to obtain I_{gj}:**
  - **direct calculation** L. L. Yang, HXZ (in preparation)
  - **or using that $S_\perp B \otimes B$ is independent of rapidity regulator** (Gehrmann, Luebbert, Yang (2012,2014)

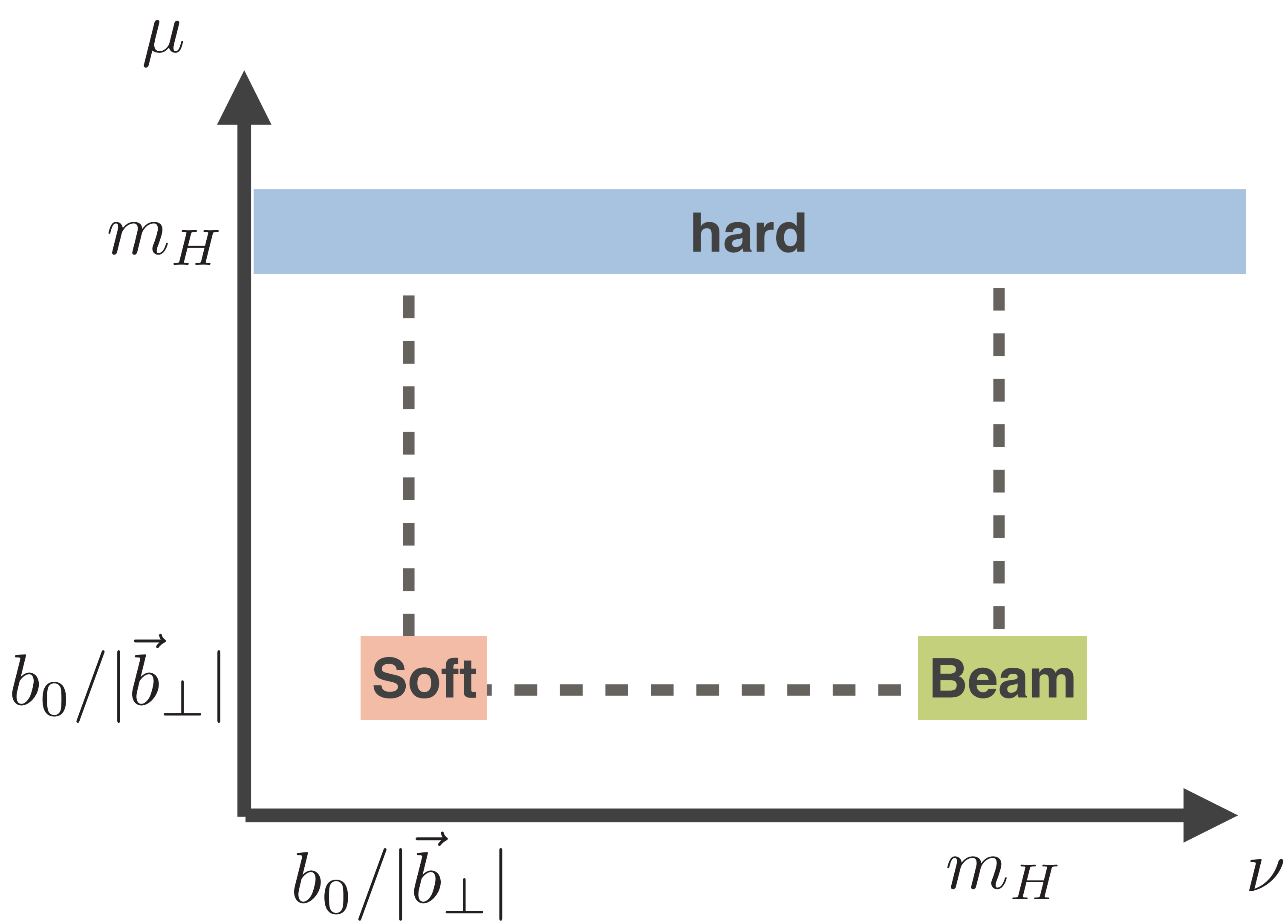
$$\widetilde{I}^{(1)}_{qq} = \left(-2\ln^2\frac{\vec{b}_\perp^2\mu^2}{b_0^2} + 2\ln\frac{\vec{b}_\perp^2\mu^2}{b_0^2}\ln\frac{\mu^2}{\nu^2} + 4\ln\frac{\vec{b}_\perp^2\mu^2}{b_0^2}\ln\frac{\mu}{Q} + 3\ln\frac{\vec{b}_\perp^2\mu^2}{b_0^2} - 2\zeta_2\right) C_F\delta(1-z)$$
$$- \ln\frac{\vec{b}_\perp^2\mu^2}{b_0^2} P^{(1)}_{qq}(z) + 2C_F(1-z)$$

Zero-bin subtraction crucial. Example: quark beam func. before and after zero-bin sub.

zero-bin sub.

$$I^{(1)}_{qq} = \left(+2\ln\frac{\vec{b}_\perp^2\mu^2}{b_0^2}\ln\frac{\nu^2}{Q^2} + 3\ln\frac{\vec{b}_\perp^2\mu^2}{b_0^2}\right) C_F\delta(1-z)$$
$$- \ln\frac{\vec{b}_\perp^2\mu^2}{b_0^2} P^{(1)}_{qq}(z) + 2C_F(1-z)$$

The hierarchy of scales

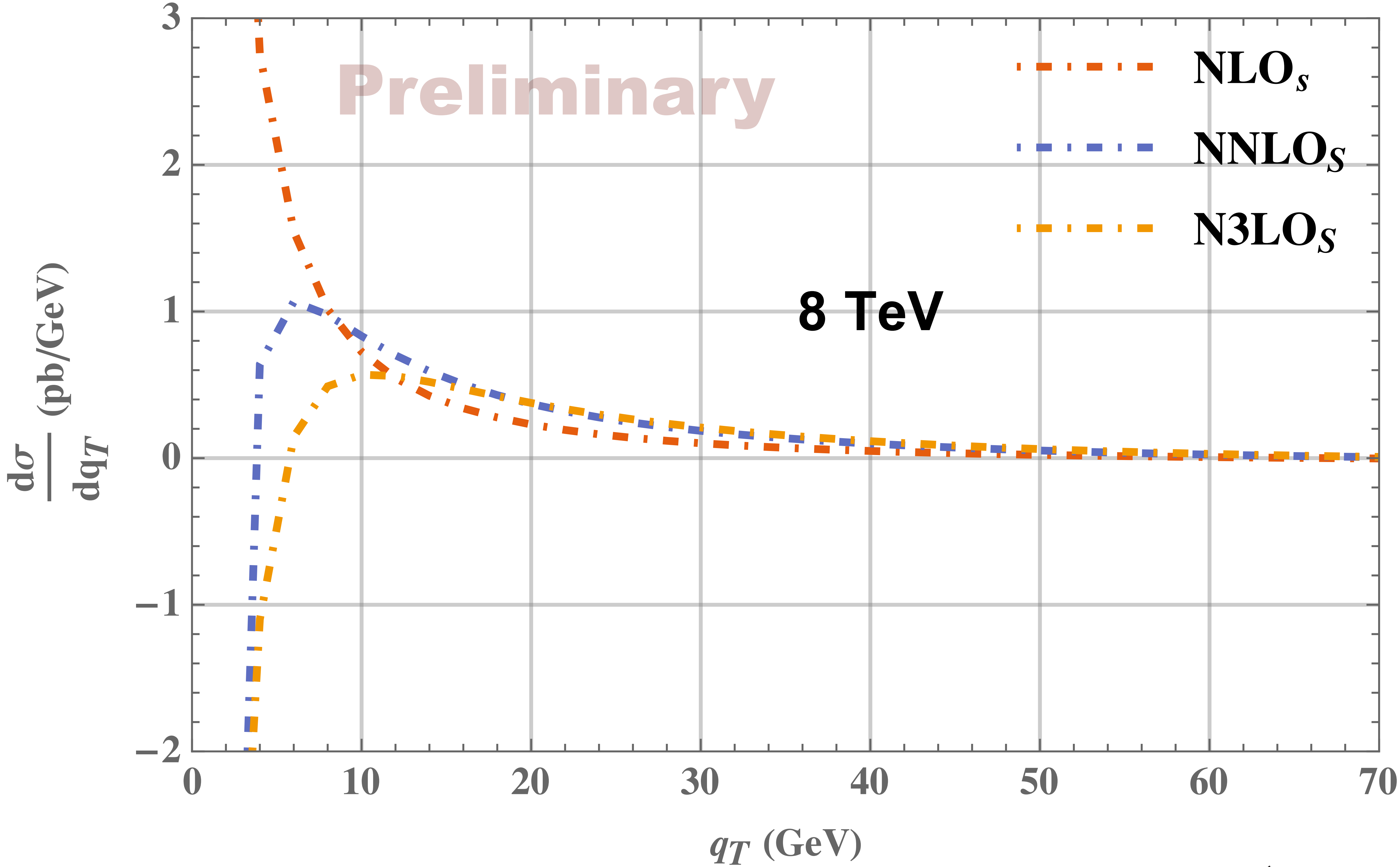


$$\mu_H \in [\frac{1}{2}M_H, 2M_H]$$

$$\mu_B \in [\frac{1}{2b}, 2/b] \qquad \nu_B \in [\frac{1}{2}M_H, 2M_H]$$

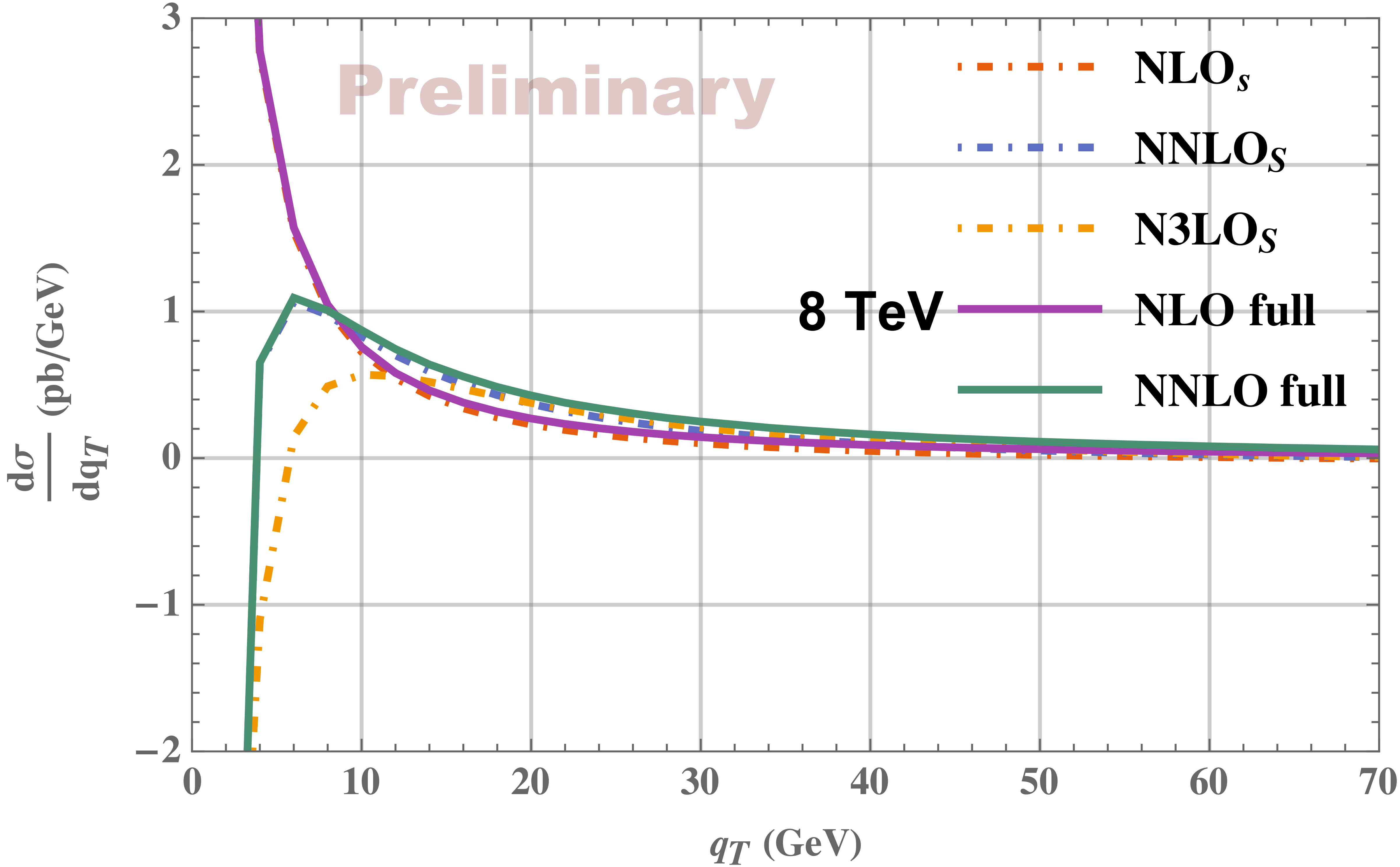
$$\mu_S \in [\frac{1}{2b}, 2/b] \qquad \nu_S \in [\frac{1}{2b}, 2/b]$$

The singular terms through to N3LO



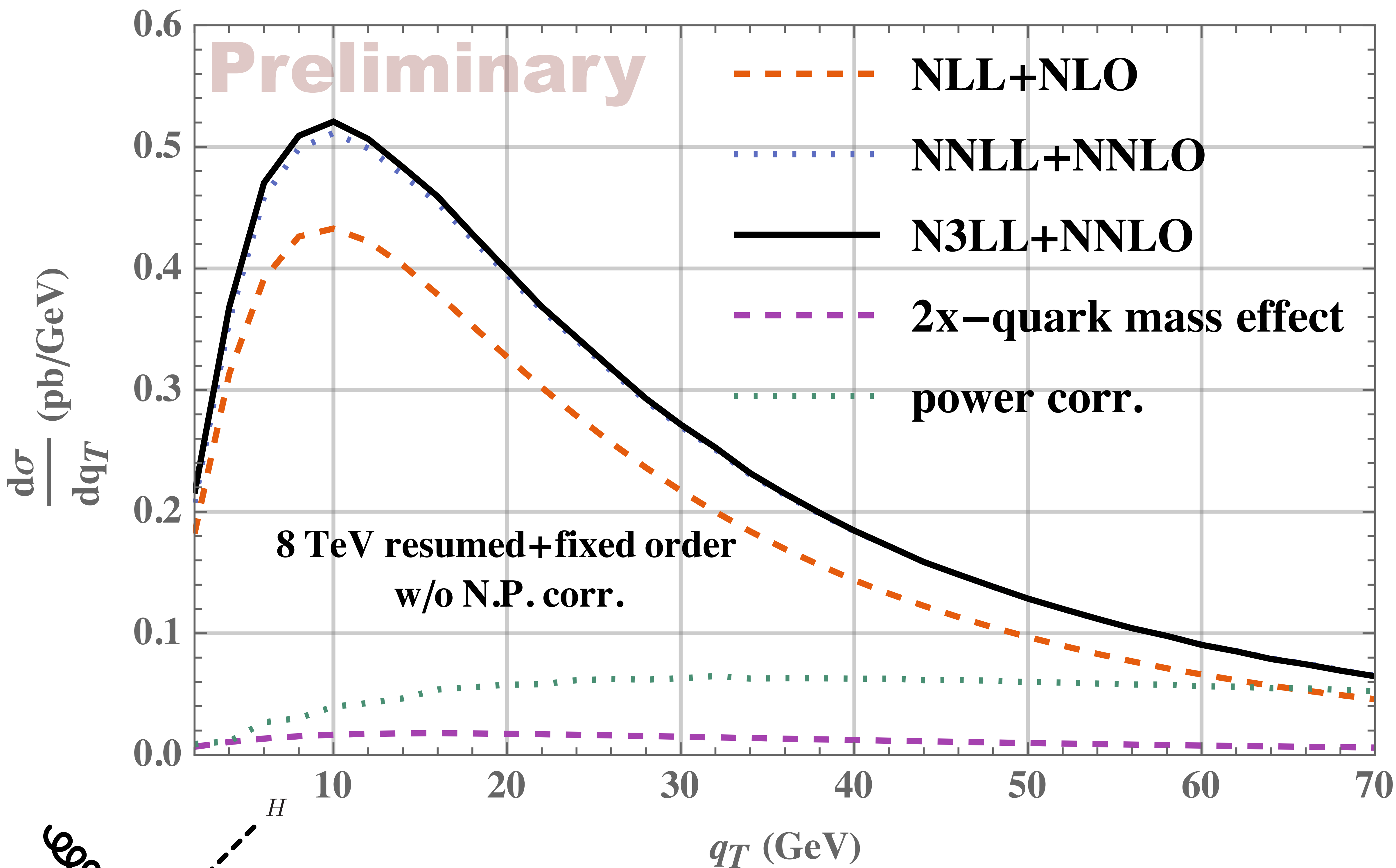
Three-loop results fully capture the singular limit of qT->0 down to $\alpha_s^3 \frac{1}{q_T}$

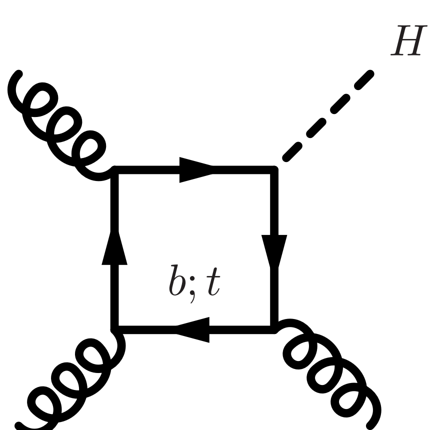
The singular terms through to N3LO



at small pT singular term hard to distinguish from full QCD.

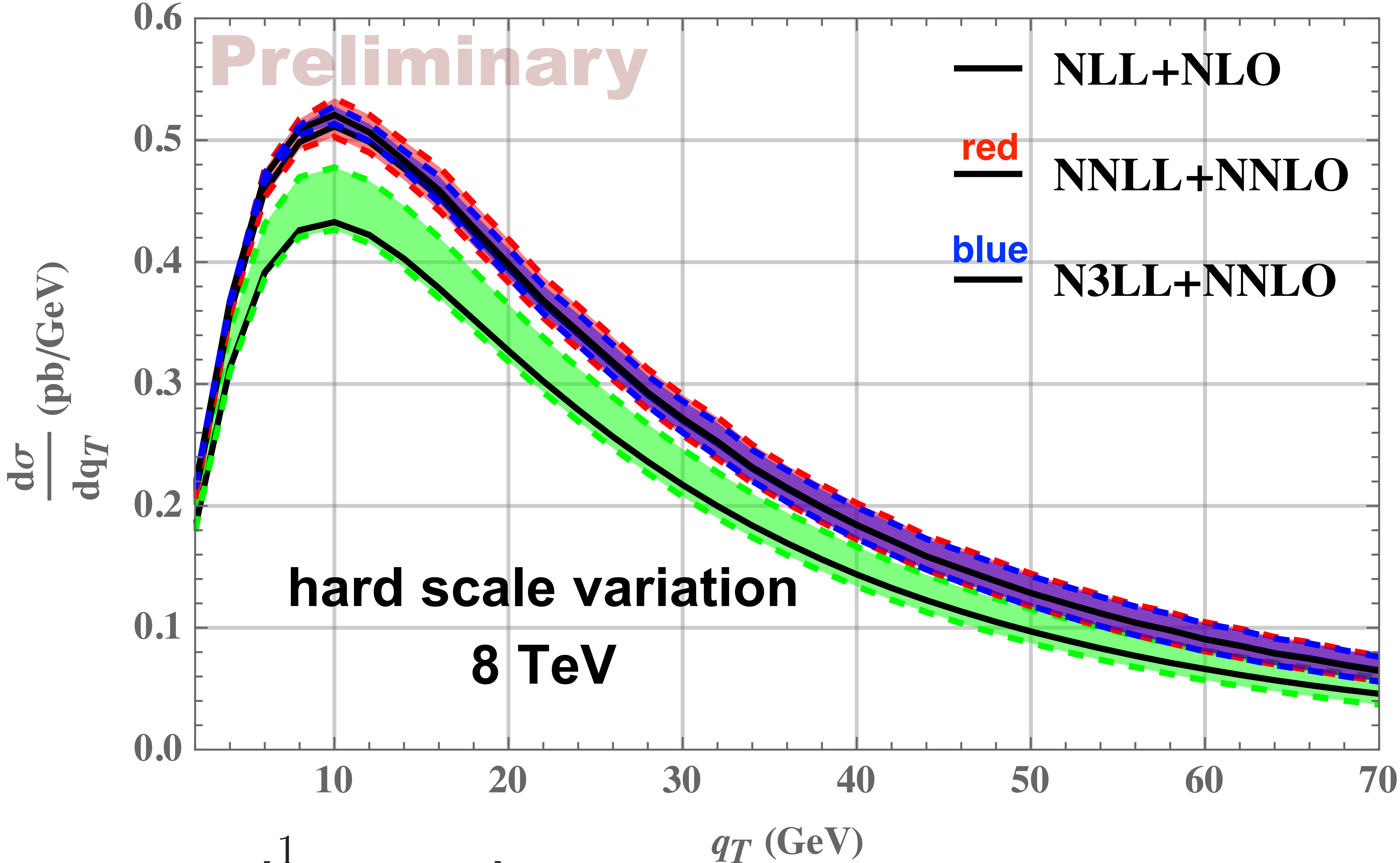
N3LL resumed prediction





w/o profile scale

Hard scale variation



$\mu_H \in [\frac{1}{2}M_H, 2M_H]$

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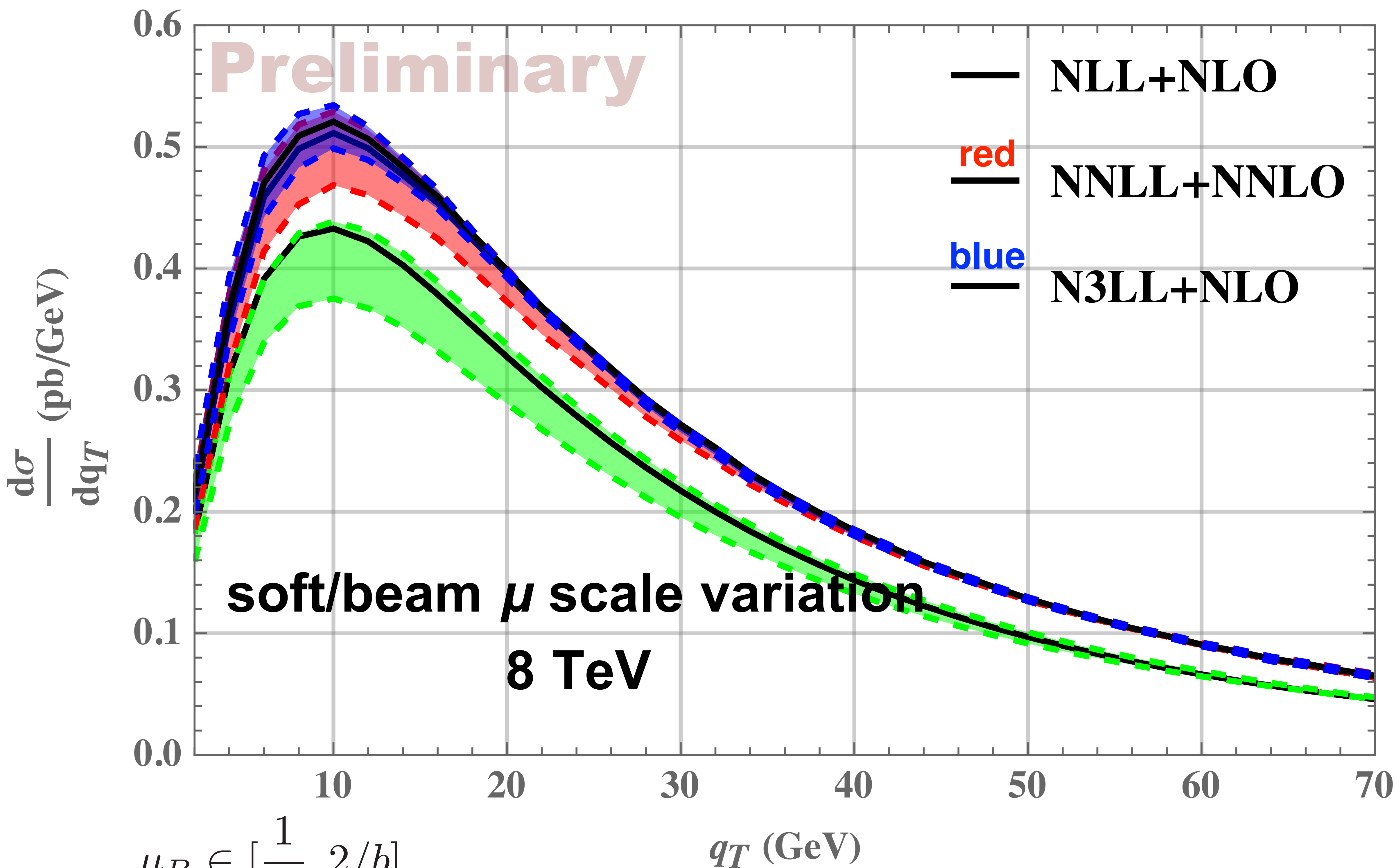
Soft and beam μ variation



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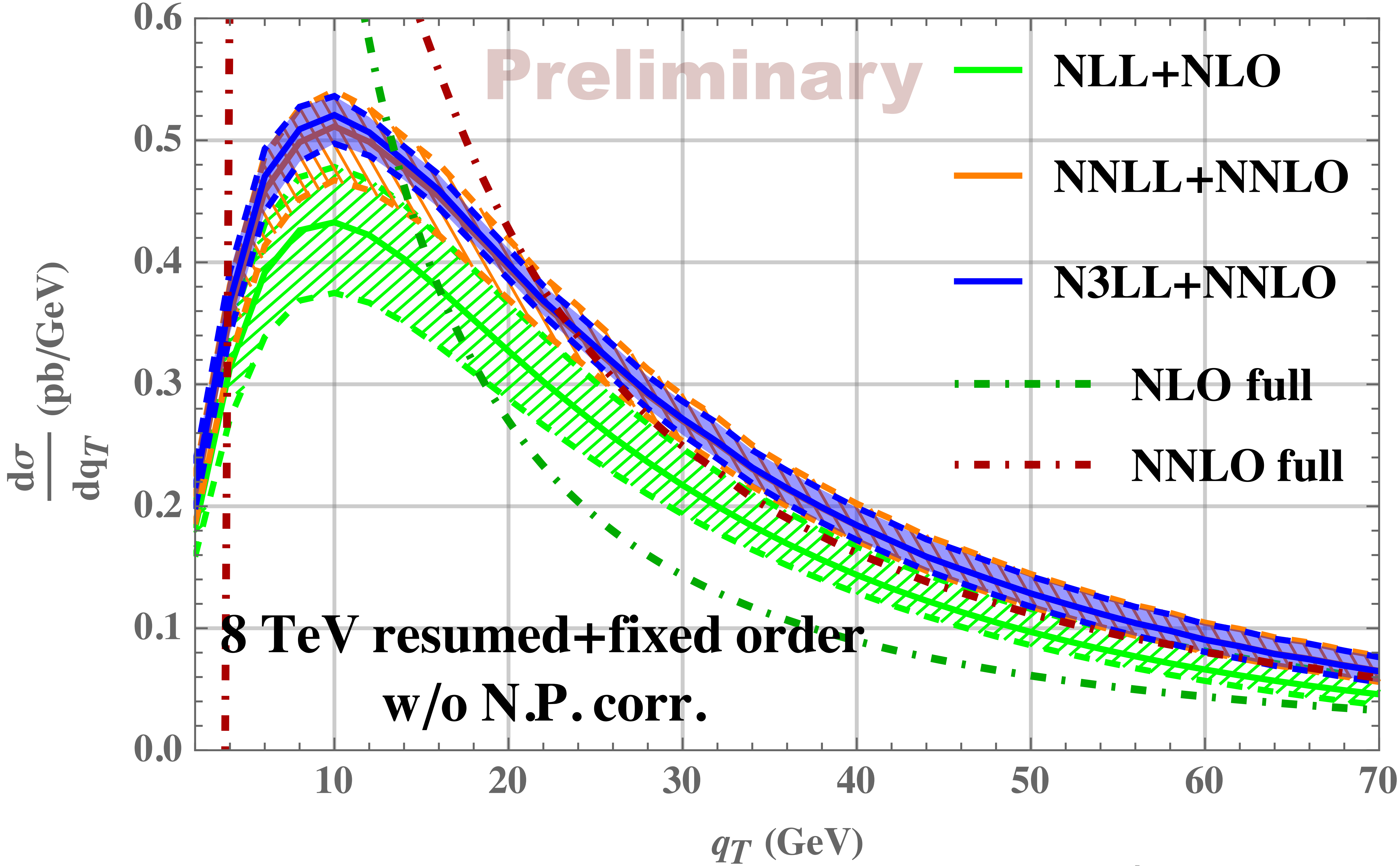
$\mu_B \in [\frac{1}{2b}, 2/b]$

$\mu_S \in [\frac{1}{2b}, 2/b]$

Higgs pT at 8 TeV



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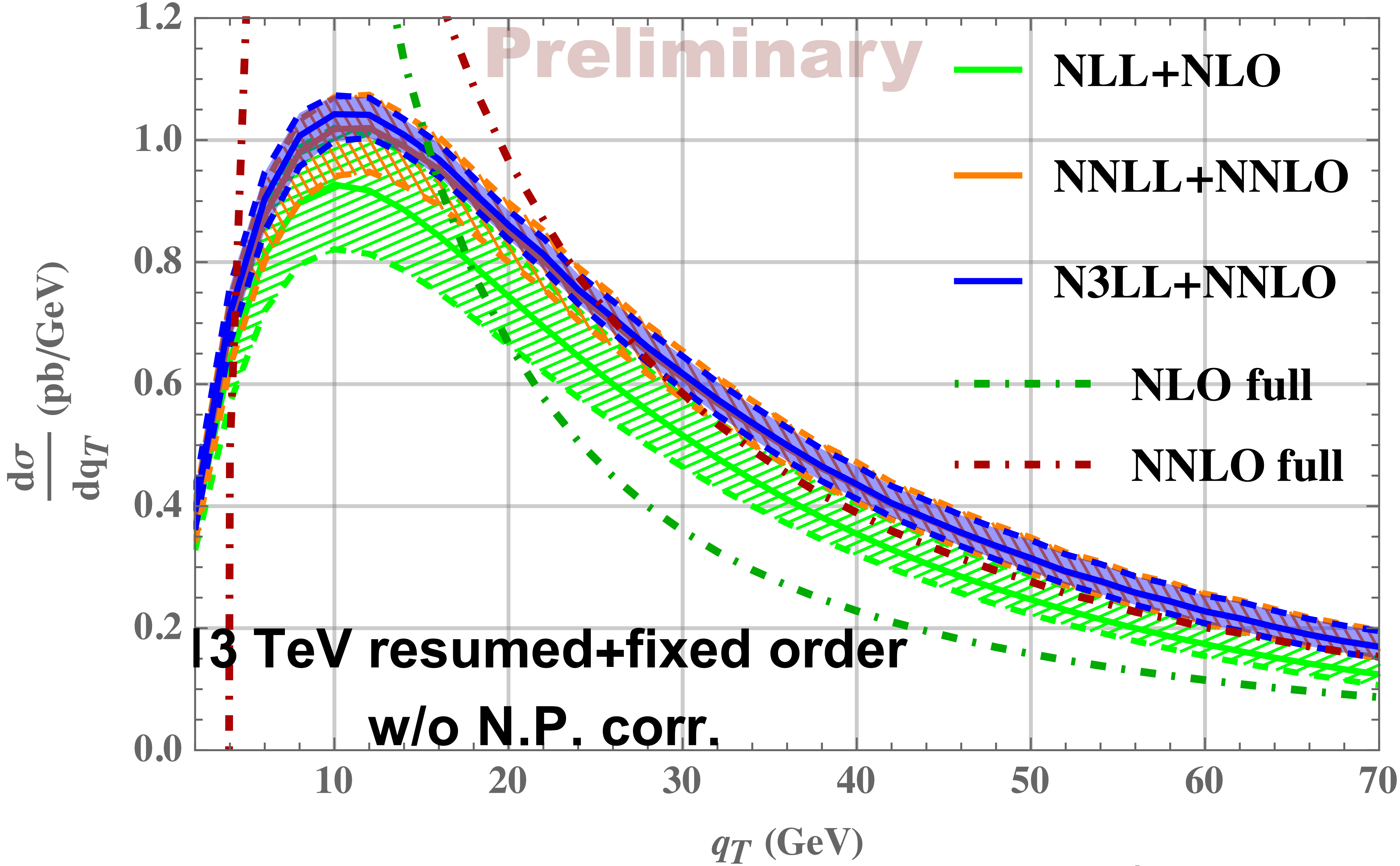
combined hard, beam and soft scale variation

Higgs pT at 13 TeV



w/o profile scale

combined hard, beam and soft scale variation

Summary

- Anomalous dimension for rapidity divergence in transverse-momentum resummation calculated to three loops for the first time, with indispensable input from
  - The rapidity renormalization group formalism from Soft-Collinear Effective Theory
  - The exponential regulator and the “data” for two-loop fully differential soft function
  - Ideas from scattering amplitude community (bootstrap program, N=4 SYM)
  - Three-loop threshold integrals calculated in the past years
- Resummation for differential observable (Higgs pT) at hadron collider now extended to Next-to-Next-to-Next-to Leading Log level