QCD and Monte-Carlo Event Generators

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U.S. DEPARTMENT OF ENERGY
Office of Science

SLAC NATIONAL ACCELERATOR LABORATORY

Suggested reading

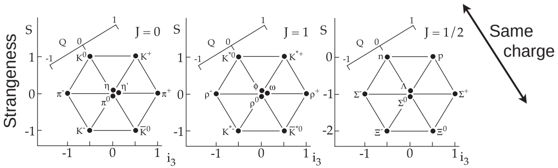
- L. Dixon, F. Petriello (Editors)
  Journeys Through the Precision Frontier
  Proceedings of TASI 2014, Worldscientific, 2015
- R. K. Ellis, W. J. Stirling, B. R. Webber
  QCD and Collider Physics
  Cambridge University Press, 2003
- R. D. Field
  Applications of Perturbative QCD
  Addison-Wesley, 1995
- A. Buckley et al.
  General-Purpose Event Generators for LHC Physics
  Phys. Rept. 504 (2011) 145
- T. Sjöstrand, S. Mrenna, P. Z. Skands
  PYTHIA 6.4 Physics and Manual
  JHEP 05 (2006) 026

Outline of lectures

- Recap and brief history of QCD
- The structure of Monte Carlo events
- Short recap of MC methods
- Fixed-order calculations
- Resummation and parton showers
- Matching NLO calculations and parton showers
- Merging matched computations with parton showers
- Non-factorizable terms and secondary interactions
- Hadronization & Particle decays

Quark model

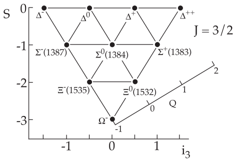
- 1960's → large zoo of hadrons dicovered, systematization needed
- Gell-Mann, Ne'eman '62 → "eight-fold way", prediction of Ω



- Use SU(3) of flavor (isospin & strangeness) to classify hadrons
- Describes structure of low-lying multiplets very well

Basic idea of QCD

- Decuplets contains state with three identical quarks



Isospin third component

- Corresponds to all symmetric state $\Delta^{++} = |u^\uparrow u^\uparrow u^\uparrow\rangle$
  $\rightarrow$ forbidden by Fermi statistics
- Rescue by postulating new degree of freedom $\rightarrow$ "color"

$$\Delta^{++} = N \sum \varepsilon_{ijk} |u_i^\uparrow u_j^\uparrow u_k^\uparrow\rangle$$

- Color unobserved experimentally $\rightarrow$ SU(3) invariance of Lagrangian

Representations

- Lie algebra of SU(3) is $[T^a, T^b] = if^{abc}T_c$
- Fundamental representation usually in terms of $T^a = \lambda^a/2$ with Gell-Mann matrices

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \quad \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad \lambda_8 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

- $\lambda_a^\dagger = \lambda_a$, $\mathrm{Tr}(\lambda^a) = 0$, $\mathrm{Tr}(\lambda^a\lambda^b) = 2\delta^{ab}$
- Fierz identity

$$\lambda^a_{ij}\lambda^a_{kl} = 2\left(\delta_{il}\delta_{jk} - \frac{1}{3}\delta_{ij}\delta_{kl}\right)$$

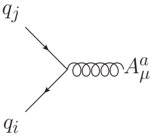
- Adjoint representation: $F^a_{bc} = -if_{abc}$

QCD Lagrangian

- Behaviour of quark field under gauge transformation

$$|u\rangle = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} \rightarrow |u'\rangle = \exp\left\{ig_s\alpha_a T^a\right\}\begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}$$

- Covariant derivative $D_\mu = \delta_\mu + ig_s t^a A^a_\mu$ with $A_\mu \rightarrow$gauge fields
  $\rightarrow$ quark kinetic and quark-gluon interaction term: $\mathcal{L} = \bar{\psi}(i\not{D} - m)\psi$



$$= \quad g_s \bar{q}_j \gamma^\mu T^a_{ji} q_i A^a_\mu$$

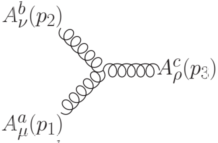
- Gluon kinetic and self-interaction term from QCD analog $\mathcal{L} = \frac{1}{4}F^{\mu\nu}F_{\mu\nu}$

$$F^a_{\mu\nu} = \delta_\mu A^a_\nu - \delta_\nu A^a_\nu - g_s f_{abc} A^a_\mu A^b_\nu$$

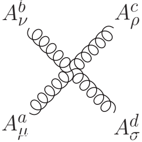
$\rightarrow$ 3- and 4-gluon interaction terms

QCD Lagrangian

- Non-abelian structure of QCD manifests itself in 3- and 4-gluon vertices



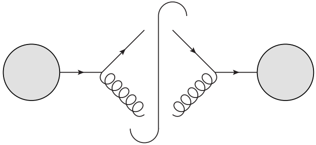
$$= g_s f_{abc}\left[\; g_{\mu\nu}(p_1 - p_2)_\rho + g_{\nu\rho}(p_2 - p_3)_\mu + g_{\rho\mu}(p_3 - p_1)_\nu \right]$$



$$= -ig_s^2\left[\; f_{abe}f_{ecd}(g_{\mu\rho}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\rho}) + f_{ade}f_{ecb}(g_{\mu\rho}g_{\nu\sigma} - g_{\mu\nu}g_{\rho\sigma}) + f_{ace}f_{ebd}(g_{\mu\sigma}g_{\nu\rho} - g_{\mu\nu}g_{\rho\sigma})\right]$$

External quark splitting

- Consider process $q \to qg$ attached to some other diagrams



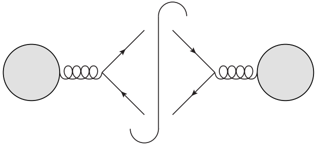
- Proportional to $g_s^2 T^a_{ij} T^a_{jk} = 4\pi\alpha_s C_F \delta_{ik}$, where

$$C_F = \frac{N_C^2 - 1}{2N_C}$$

- Identical to QED case except for color factor C_F

External gluon splitting

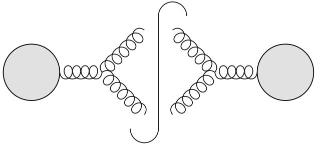
- Consider process $g \to q\bar{q}$ attached to some other diagrams



- Proportional to $g_s^2 T^a_{ij} T^b_{ji} = 4\pi\alpha_s T_R \delta^{ab}$, where

  $T_R = \frac{1}{2}$

- Consider process $g \to q\bar{q}$ attached to some other diagrams



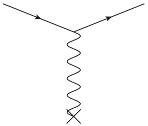
- Proportional to $g_s^2 f^{abc} f^{bcd} = 4\pi\alpha_s C_A \delta^{ad}$, where

  $C_A = 3$

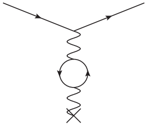
- $\to$ Gluons couple stronger to gluons than to quarks

Screening in QED

- At low energy, QED potential looks like $V(R) = -\frac{\alpha}{R}$



- When $R \lesssim 1/m_e$, vacuum polarization effects set in

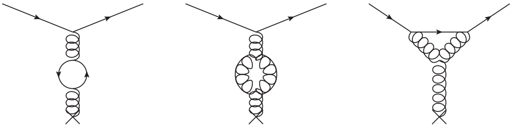


  Potential changes to

$$V(R) = -\frac{\alpha}{R}\left[1 + \frac{2\alpha}{3\pi}\log\frac{1}{m_e R} + \mathcal{O}(\alpha^2)\right] = \frac{\bar{\alpha}(R)}{R}$$

  where $\bar{\alpha}(R) \rightarrow$ effective (running) coupling
- Understand effective coupling in analogy to solid state physics:
  - In insulators, charge screened by polarization of atoms
  - In QED, charge screened by newly created e^+e pairs

- In QCD, gauge bosons carry color $\rightarrow$ screening & anti-screening



- Sum of contributions defines running of strong coupling at 1-loop

$$\mu^2 \frac{\partial \alpha_s(\mu^2)}{\partial \mu^2} = \beta(\alpha_s) \quad \text{where} \quad \beta(\alpha_s) = -\alpha_s \sum_{n=0} \frac{\alpha_s}{4\pi} \beta_n$$

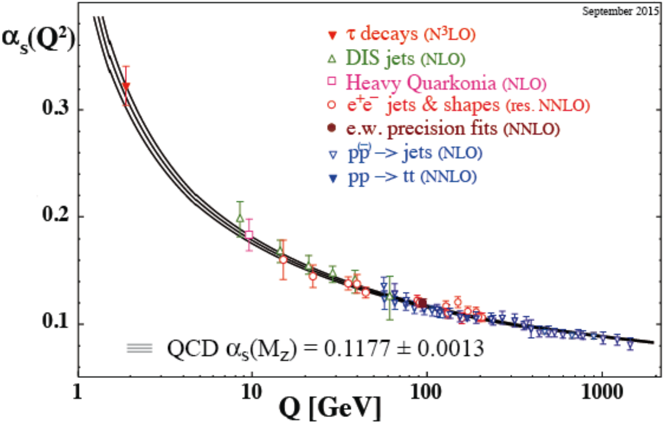
- Coefficients known up to four loops

$$\beta_0 = \frac{11}{3} N_c - \frac{4}{3} T_R n_f$$
$$\beta_1 = \frac{17}{12} C_A^2 - \left(\frac{5}{6} C_A + \frac{1}{2} C_F \right) T_R n_f$$
$$\ldots$$

- Unless $n_f \geq 17$, 1-loop beta function is negative $\rightarrow$ confinement

Running of the strong coupling

[Bethke] Proc. HP α_s (2015)



QCD in e^+e^- annihilation

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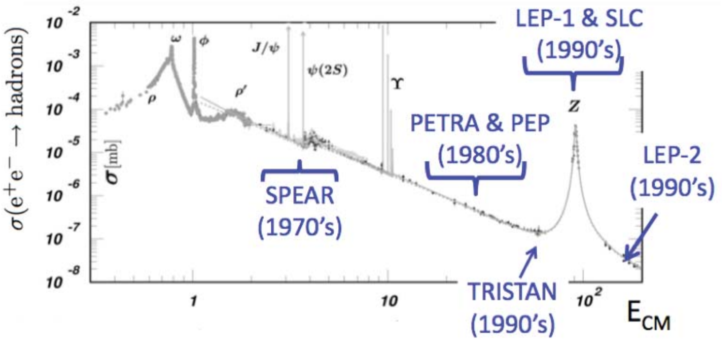
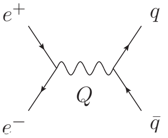
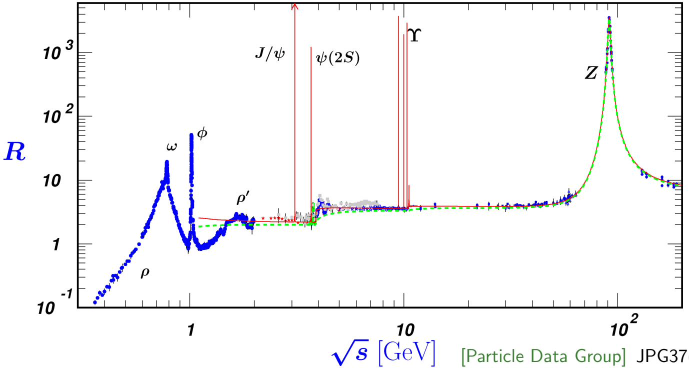


- SPEAR (SLAC): Discovery of quark jets
- PETRA (DESY) & PEP (SLAC): First high energy (>10 GeV) jets
  Discovery of gluon jets (PETRA) & pioneering QCD studies
- LEP (CERN) & SLC (SLAC): Large energies $\to$ more reliable
  QCD calculations, smaller hadronization uncertainties
  Large data samples $\to$ precision tests of QCD

Basic process for e$^+$e$^-$ →hadrons



- Prediction for $e^+e^- \to q\bar{q}$ at leading perturbative order differs from $e^+e^- \to \mu^+\mu^-$ only by quark charges
- Define ratio $R = \dfrac{\sigma_{e^+e^- \to \text{hadrons}}}{\sigma_{e^+e^- \to \mu^+\mu^-}} \xrightarrow{\text{LO}} \sum_i e_{q,i}^2$

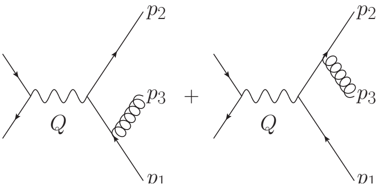


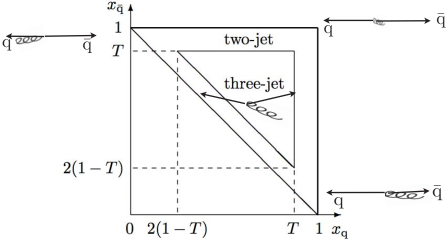
[Particle Data Group] JPG37(2010)075021

- Kinematic variables $x_i = \frac{2p_i \cdot Q}{Q^2}$

  $\to x_i < 1, \quad x_1 + x_2 + x_3 = 2$
- Differential cross section

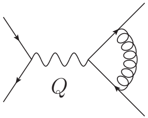
  $$\frac{d^2\sigma}{dx_1 dx_2} = \sigma_0 \frac{\alpha_s}{\pi} C_F \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$
- Divergent as
  - $x_1 \to 1$ $(p_3 \parallel p_1)$
  - $x_2 \to 1$ $(p_3 \parallel p_2)$
  - $(x_1, x_2) \to (1,1)$ $(x_3 \to 0)$





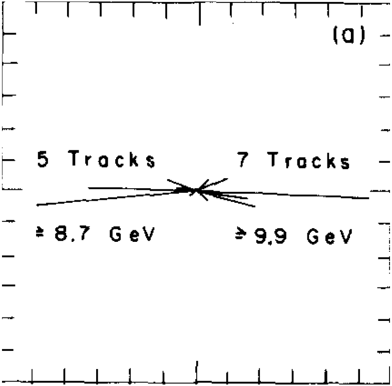
- Divergences canceled by virtual correction
  Total correction to $e^+e^- \to$ hadrons:

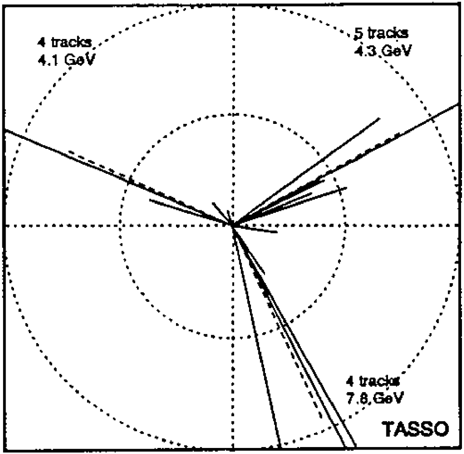
  $$\sigma^{\text{NLO}} = \sigma_0 \left(1 + \frac{3}{4} C_F \frac{\alpha_s}{\pi}\right)$$



Discovery of the gluon

[TASSO] PLB86(1979)243 & Proc. Neutrino '79, Vol.1, p.113

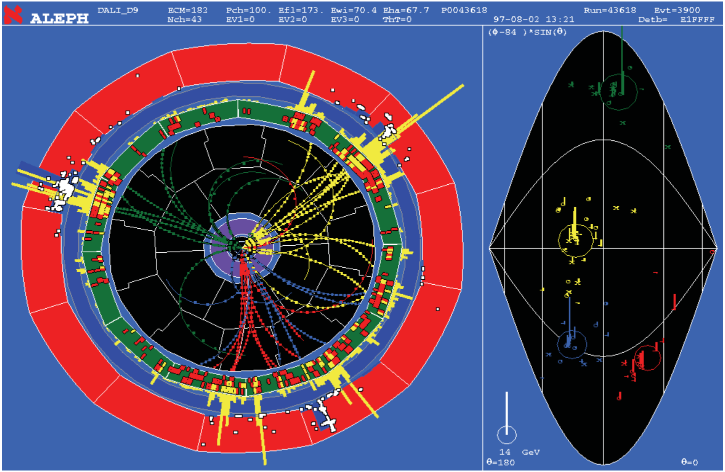


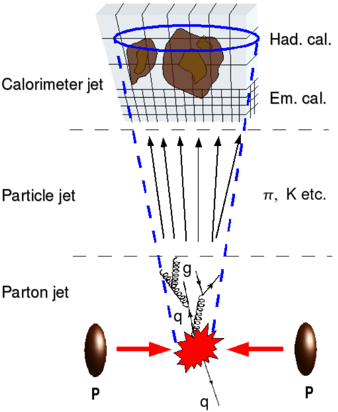


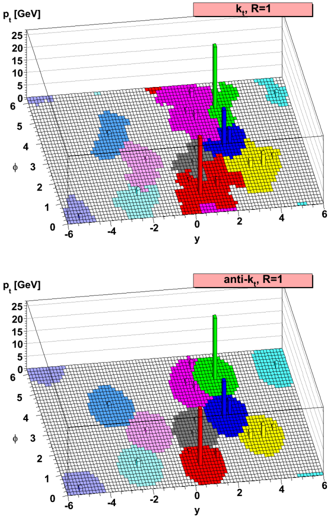
- Gluon discovery at the PETRA collider at DESY
- Typical three-jet event (right) vs. two-jet event (left)

High-energy colliders and jets

[ALEPH]







- Identify hadronic activity in experiment with partonic activity in pQCD theory
- See lecture by Matteo Cacciari

[Cacciari,Salam,Soyez] arXiv:0802.1189

Event shape variables

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- Shape variables characterize event as a whole
- Thrust (introduced 1978 at PETRA)

  $$T = \max_{\vec{n}} \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{\sum_j |\vec{p}_j|}$$

  - $T \to 1$ – back-to-back event
  - $T \to 1/2$ – spherically symmetric event

  Vector for which maximum is obtained $\to$ thrust axis $\vec{n}_T$
- Thrust major/minor

  $$T_{\mathrm{maj}} = \max_{\vec{n}_{\mathrm{maj}} \perp \vec{n}_T} \frac{\sum_i |\vec{p}_i \cdot \vec{n}_{\mathrm{maj}}|}{\sum_j |\vec{p}_j|}$$

  $$T_{\mathrm{min}} = \frac{\sum_i |\vec{p}_i \cdot (\vec{n}_T \times \vec{n}_{\mathrm{maj}})|}{\sum_j |\vec{p}_j|}$$
- Jet broadening

  $$B_i = \frac{\sum_{k \in H_i} |\vec{p}_k \times \vec{n}_T|}{2 \sum_j |\vec{p}_j|}$$

  Computed for two hemispheres w.r.t. $\vec{n}_T$, then
  - $B_W = \max(B_1, B_2)$ – Wide jet broadening
  - $B_N = \min(B_1, B_2)$ – Narrow jet broadening

Event shape variables

- Jet mass

$$M_i^2 = \frac{1}{E_{cm}^2}\left(\sum_{k\in H_i} p_k\right)^2$$

  Computed for two hemispheres w.r.t. $\vec{n}_T$, then
  - $\rho = \max(M_1^2, M_2^2)$ – Heavy jet mass
  - $M_L = \min(M_1^2, M_2^2)$ – Light jet mass
- Quadratic momentum tensor

$$M^{\alpha\beta} = \frac{\sum_i p_i^\alpha p_i^\beta}{\sum_j |\vec{p}_j|^2}\,, \qquad \alpha,\beta = 1,2,3$$

  Eigenvalues $\lambda_1 \geq \lambda_2 \geq \lambda_3$ used to define
  - $S = \frac{3}{2}(\lambda_2 + \lambda_3)$ – Sphericity
  - $A = \frac{3}{2}\lambda_3$ – Aplanarity
  - $P = \lambda_2 - \lambda_3$ – Planarity
- C-Parameter

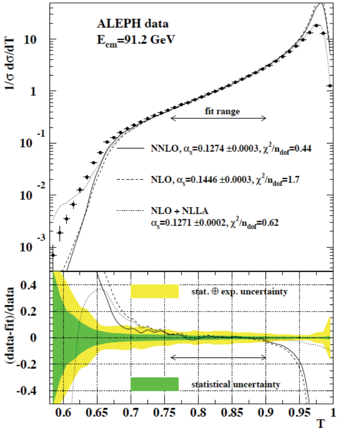
  Linearized momentum tensor

$$\Theta^{\alpha\beta} = \frac{1}{\sum_j |\vec{p}_j|}\sum_i \frac{p_i^\alpha p_i^\beta}{|\vec{p}_i|}\,,$$

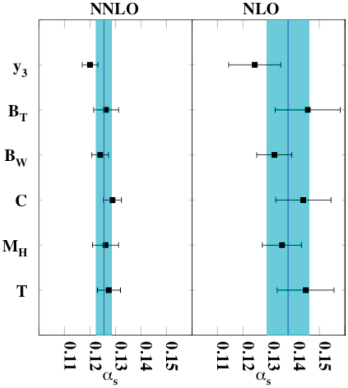
  Eigenvalues λ_i define $C = 3(\lambda_1\lambda_2 + \lambda_2\lambda_3 + \lambda_3\lambda_1)$

Application of event shape variables

- Discovery of quark and gluon jets – Sphericity & Oblateness
- Measurement of strong coupling constant – T, C, B, ρ, Durham jet rates

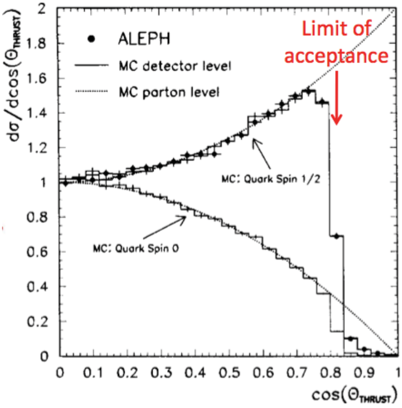


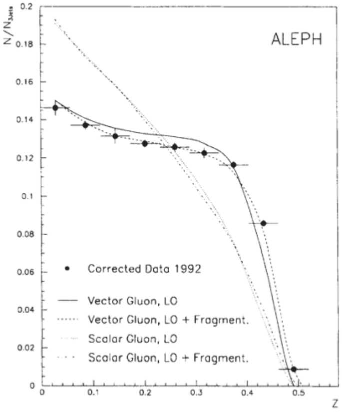
[Dissertori et al.] arXiv:0906.3436



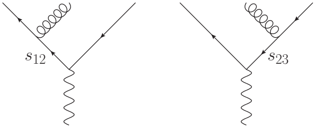
Application of event shape variables

- Measurement of quark (and gluon) spin – Thrust axis
- Measurement of triple-gluon vertex – BZ angle





Parton evolution



- Consider $e^+e^- \to 3$ partons

$$\frac{1}{\sigma_{2\to2}}\frac{\mathrm{d}\sigma_{2\to3}}{\mathrm{d}\cos\theta\mathrm{d}z} \sim C_F\frac{\alpha_s}{2\pi}\frac{2}{\sin^2\theta}\frac{1+(1-z)^2}{z}$$

  θ - angle of gluon emission
  z - fractional energy of gluon

- Divergent in
  - Collinear limit: $\theta \to 0, \pi$
  - Soft limit: $z \to 0$
- Separate into two independent jets

$$\frac{2\mathrm{d}\cos\theta}{\sin^2\theta} = \frac{\mathrm{d}\cos\theta}{1-\cos\theta} + \frac{\mathrm{d}\cos\theta}{1+\cos\theta} = \frac{\mathrm{d}\cos\theta}{1-\cos\theta} + \frac{\mathrm{d}\cos\bar{\theta}}{1-\cos\bar{\theta}} \approx \frac{\mathrm{d}\theta^2}{\theta^2} + \frac{\mathrm{d}\bar{\theta}^2}{\bar{\theta}^2}$$

- Independent evolution with θ

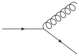
$$\mathrm{d}\sigma_3 \sim \sigma_2 \sum_{\text{jets}} C_F\frac{\alpha_s}{2\pi}\frac{\mathrm{d}\theta^2}{\theta^2}\mathrm{d}z\,\frac{1+(1-z)^2}{z}$$

Parton evolution

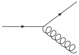
- Same equation for any variable with same limiting behavior
  - Transverse momentum $k_T^2 = z^2(1-z)^2\theta^2 E^2$
  - Virtuality $t = z(1-z)\theta^2 E^2$
- Call this the "evolution variable"

$$\frac{\mathrm{d}\theta^2}{\theta^2} = \frac{\mathrm{d}k_T^2}{k_T^2} = \frac{\mathrm{d}t}{t} \qquad \leftrightarrow \qquad \text{collinear divergence}$$

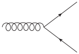
- Absorb z-dependence into flavor-dependent splitting kernel $P_{ab}(z)$



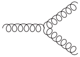
$$= C_F \frac{1+z^2}{1-z}$$



$$= C_F \frac{1+(1-z)^2}{z}$$



$$= T_R \left[z^2 + (1-z)^2 \right]$$



$$= C_A \left[\frac{z}{1-z} + \frac{1-z}{z} + z(1-z) \right]$$

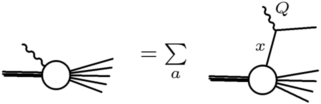
- DGLAP evolution equation emerges, but so far computation in pQCD

$$\mathrm{d}\sigma_{n+1} \sim \sigma_n \sum_{\text{jets}} \frac{\mathrm{d}t}{t} \mathrm{d}z \, \frac{\alpha_s}{2\pi} P_{ab}(z)$$

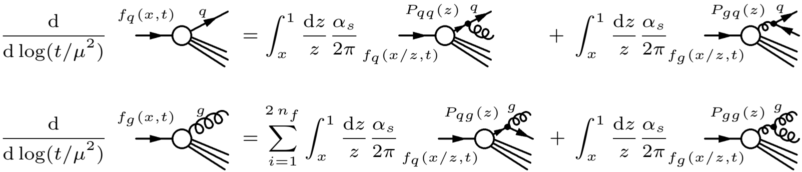
The DGLAP equation

- Assume cross section factorizes into perturbative & non-perturbative piece

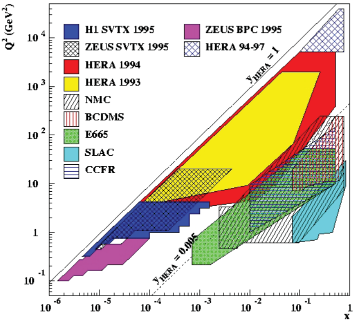
$$\sigma = \sum_{a=q,g} \int \mathrm{d}x\, f_a(x,Q^2)\hat{\sigma}_a(Q^2)$$

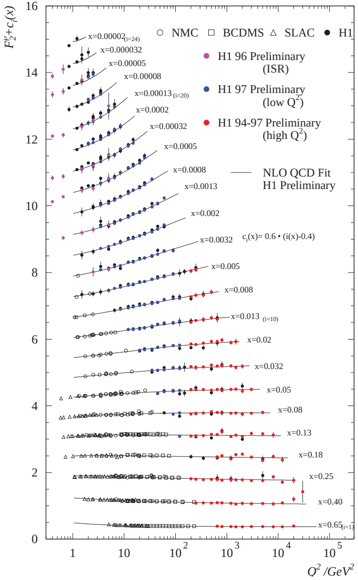


- Evolution from previous slide turns into evolution equation for $f_a(x,Q^2)$
- $f_a(x,Q^2)$ cannot be predicted as function of x
  but dependence on Q^2 can be computed order-by-order in pQCD
- DGLAP equation



PDF measurements





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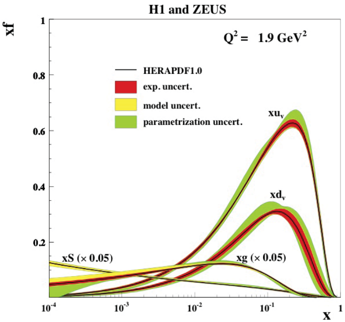
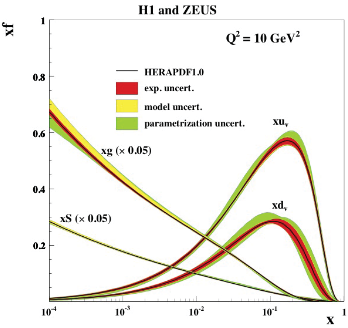
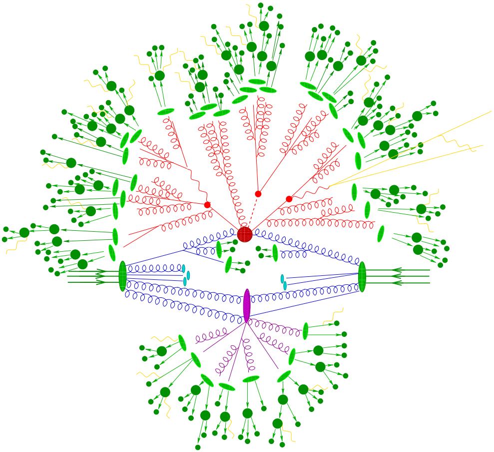




- Hard interaction
- QCD evolution
- Secondary hard interactions
- Hadronization
- Hadron decays
- Higher-order QED corrections



General-purpose event generators for LHC physics

SLAC

[Buckley et al.] arXiv:1101.2599

Herwig

- Originated in coherent shower studies → angular ordered PS
- Front-runner in development of MC@NLO and POWHEG
- Simple in-house ME generator & spin-correlated decay chains
- Original framework for cluster fragmentation

Pythia

- Originated in hadronization studies → Lund string
- Leading in development of multiple interaction models
- Pragmatic attitude to ME generation → external tools
- Extensive PS development and earliest ME$\oplus$PS matching

Sherpa

- Started with PS generator APACIC++ & ME generator AMEGIC++
- Current MPI model and hadronization pragmatic add-ons
- Leading in development of automated ME$\oplus$PS merging
- Automated framework for NLO calculations and MC@NLO

Analysis and tuning tools

Rivet [Buckley et al.] arXiv:0103.0694

- LHC-successor to HZTool
  Collection of exp. data & matching analysis routines
- Spirit: "Right MC describes everything at the same time"

Professor [Buckley et al.] arXiv:0907.2973

- Tuning in multi-dimensional parameter space of MC
- Generate event samples at random parameter points
  Analyze them with Rivet
  Parametrize observables
  Minimize χ^2 and cross-check

Tune comparisons

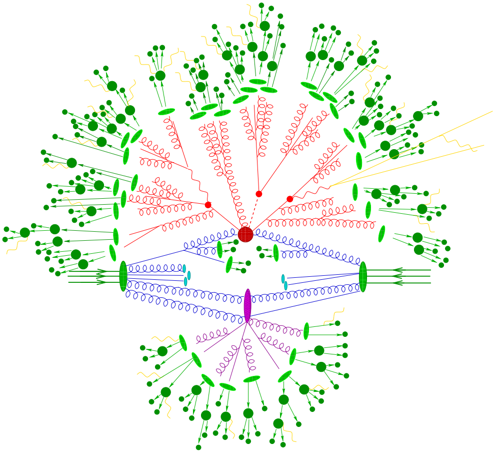
Deviation metrics per gen/tune and observable group:

Gen	Tune	UE	Dijets	Multijets	Jet shapes	W and Z	Fragmentation	B frag
AlpGen	HERWIG6	—	1.83	5.36	2.48	0.91	—	—
	PYTHIA6-AMBT1	—	1.55	2.80	0.61	0.53	—	—
	PYTHIA6-D6T	—	1.38	2.67	2.31	1.67	—	—
	PYTHIA6-P2010	—	1.09	2.65	2.03	1.48	—	—
	PYTHIA6-P2011	—	1.12	2.60	0.48	0.24	—	—
	PYTHIA6-Z2	—	1.48	2.63	0.55	0.48	—	—
	PYTHIA6-profQ2	—	1.16	2.65	1.43	1.29	—	—
HERWIG	AUET2-CTEQ6L1	0.43	0.55	0.77	0.35	0.58	22.80	2.38
	AUET2-LOxx	0.25	0.71	0.60	0.39	0.88	22.13	2.29
Herwig++	2.5.1-UE-EE-3-CTEQ6L1	0.27	0.87	0.78	0.51	0.98	10.58	1.32
	2.5.1-UE-EE-3-MRSTLOxx	0.23	1.05	0.78	0.50	0.65	10.58	1.32
PYTHIA6	AMBT1	0.39	1.20	0.54	0.77	0.27	0.93	1.65
	AUET2B-CTEQ6L1	0.16	0.92	0.44	0.59	0.74	0.67	1.29
	AUET2B-LOxx	0.13	1.33	0.55	0.58	1.15	0.67	1.30
	D6T	0.58	0.79	0.50	0.56	1.25	0.36	2.63
	DW	0.81	0.78	0.61	0.56	1.33	0.36	2.63
	P2010	0.30	0.93	0.82	1.07	0.30	0.44	1.75
	P2011	0.12	0.89	0.67	1.02	0.53	0.43	2.13
	ProfQ2	0.51	0.67	0.81	0.51	0.64	0.30	1.65
	Z2	0.18	0.94	0.73	0.80	0.30	0.95	2.78
Pythia8	4C	0.30	0.97	0.93	0.50	0.90	0.38	1.12
Sherpa	1.3.1	0.68	0.47	0.34	0.71	0.36	0.75	2.48

[LH'11 SM WG] arXiv:1203.6803 [hep-ph]

The structure of MC events

- Hard interaction
- QCD evolution
- Secondary hard interactions
- Hadronization
- Hadron decays
- Higher-order QED corrections



Amplitude generation

- **Textbook**: Use completeness relations to square amplitudes
  sum/average over external states (helicity and color)
  Computational effort grows quadratically with number of diagrams
- **Real life**: Amplitudes are complex numbers
  first compute them, then add and square
  Effort grows linearly with number of diagrams

- Applies to dynamical degrees of freedom only
  - Consider helicity: Polarizations depend on momenta
    need to recompute for each phase-space point
  - Consider color: Mostly summed over at low multiplicity
    independent of other d.o.f. $\rightarrow$ no need to recompute

Helicity

[Dixon] hep-ph/9601359
[Dittmaier] hep-ph/9805445

- Weyl-van-der-Waerden spinors for helicity states $+/-$

$$\chi_+(p) = \begin{pmatrix} \sqrt{p^+} \\ \sqrt{p^-}\, e^{i\phi_p} \end{pmatrix} \qquad \chi_-(p) = \begin{pmatrix} \sqrt{p^-}\, e^{i\phi_p} \\ -\sqrt{p^+} \end{pmatrix} \qquad \begin{aligned} p^\pm &= p^0 \pm p^3 \\ p_\perp &= p^1 + ip^2 \end{aligned}$$

  Basic building blocks for all amplitudes
  $+, -, \perp$ directions define "spinor gauge"

- Massive Dirac spinors in terms of WvdW spinors

$$u_+(p,m) = \frac{1}{\sqrt{2\,\bar{p}}} \begin{pmatrix} \sqrt{p_0 - \bar{p}}\; \chi_+(\hat{p}) \\ \sqrt{p_0 + \bar{p}}\; \chi_+(\hat{p}) \end{pmatrix} \qquad \bar{p} = \mathrm{sgn}(p_0)\, |\vec{p}|$$

$$u_-(p,m) = \frac{1}{\sqrt{2\,\bar{p}}} \begin{pmatrix} \sqrt{p_0 + \bar{p}}\; \chi_-(\hat{p}) \\ \sqrt{p_0 - \bar{p}}\; \chi_-(\hat{p}) \end{pmatrix} \qquad \hat{p} = (\bar{p}, \vec{p})$$

- γ^5 conveniently defined in Weyl representation

$$\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} -\sigma^0 & 0 \\ 0 & \sigma^0 \end{pmatrix}$$

  Projection operator $P_{R,L} = P_\pm = (1 \pm \gamma^5)/2$ identifies
  lower/upper component of Dirac spinors as right-/left-handed

Helicity

- Massless polarizations constructed from $u_\pm(p)$ and $u_\pm(k)$ with external light-like gauge vector k

$$\varepsilon^\mu_\pm(p,k) \;=\; \pm\frac{\bar{u}_\mp(k)\gamma^\mu u_\mp(p)}{\sqrt{2}\,\bar{u}_\mp(k)u_\pm(p)}\;.$$

  Defines light-like axial gauge

- For massive particles decompose momentum p using k

$$b = p - \kappa k \qquad \kappa = \frac{p^2}{2pk} \qquad \Rightarrow \qquad b^2 = 0$$

  Transverse polarizations as in massless case ($p \to b$) plus longitudinal

$$\varepsilon^\mu_0(p,k) = \frac{1}{m}\left(\,\bar{u}_-(b)\gamma^\mu u_-(b) - \kappa\,\bar{u}_-(k)\gamma^\mu u_-(k)\,\right)$$

- Vertices & propagators have simpler structure

- Building blocks for Standard model complete!

Color

[Maltoni,Stelzer,Willenbrock] hep-ph/0209271
[Duhr,SH,Maltoni] hep-ph/0607057

- QCD amplitudes can be stripped of color factors
- Fundamental representation for n-gluons

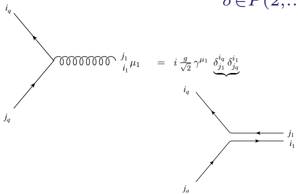
$$\mathcal{A}_n(p_1,\ldots,p_n) = \sum_{\vec{\sigma}\in P(2,\ldots,n)} \mathrm{Tr}(\lambda^{a_1}\lambda^{a_{\sigma_2}}\ldots\lambda^{a_{\sigma_n}})\, A(p_1,p_{\sigma_2},\ldots,p_{\sigma_n})$$

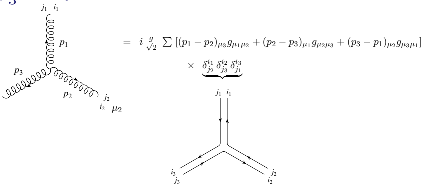
- Adjoint representation for n-gluons

$$\mathcal{A}_n(p_1,\ldots,p_n) = \sum_{\vec{\sigma}\in P(2,\ldots,n-1)} \left[F^{a_{\sigma_2}}\ldots F^{a_{\sigma_{n-1}}}\right]^{a_1}_{a_n} A(p_1,p_{\sigma_2},\ldots,p_{\sigma_{n-1}},p_n)$$

- Color-flow representation for n-gluons

$$\mathcal{A}_n(p_1,\ldots,p_n) = \sum_{\vec{\sigma}\in P(2,\ldots,n)} \delta^{i_1}_{j_{\sigma_2}}\,\delta^{i_{\sigma_2}}_{j_{\sigma_3}}\ldots\delta^{i_{\sigma_n}}_{j_1}\, A(p_1,p_{\sigma_2},\ldots,p_{\sigma_n})$$





Color

- We can sample colors just like we sample momenta
- Assign one in (r, g, b) / $(\bar{r}, \bar{g}, \bar{b})$ to each external (anti-)quark & gluon
- Average number of partial amplitudes is then smallest in color-flow basis

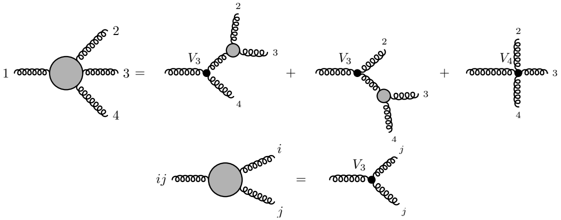
	Average # of partials		
n	Gell-Mann	Color-flow	Adjoint
4	4.83	1.28	1.15
5	15.2	1.83	1.52
6	56.5	3.21	2.55
7	251	6.80	5.53
8	1280	17.0	15.8
9	7440	48.7	56.4
10	47800	158	243

	Time [s/10^4pt]	
n	CO	CD
4	1.20	1.04
5	3.78	2.69
6	14.2	7.19
7	58.5	23.7
8	276	82.1
9	1450	270
10	7960	864

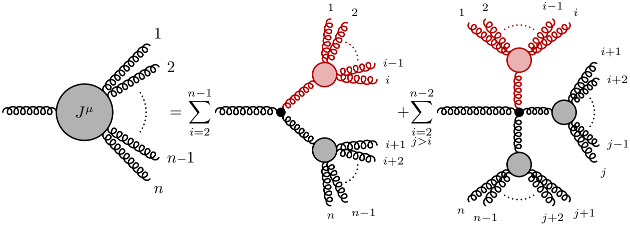
- Computational effort reduced further by not stripping amplitudes of color factors
- Evaluate dynamically at each vertex $\rightarrow$ straightforward computer algorithm
- Color dressing (CD) vs. color ordering (CO)

Amplitude construction

Example: Diagrams for
$g(1)g(2) \to g(3)g(4)$



[Berends,Giele] NPB306(1988)759



Example: Currents for
$g(1)g(2) \to g(3)g(4)$

Step 1	$J_1 = \varepsilon(1)$	$J_2 = \varepsilon(2)$	$J_3 = \varepsilon(3)$	$J_4 = \varepsilon(4)$
Step 2	J_{12}	J_{13}	J_{23}	
Step 3	J_{123}			
Step 4	$A(1,2,3,4) = J_4^* \, J_{123}$			

Phase space

[James] CERN-68-15
[Byckling,Kajantie] NPB9(1969)568

- Need to evaluate in a process-independent way

$$\mathrm{d}\Phi_n(p_a,p_b;p_1,..,p_n) = \left[\prod_{i=1}^{n}\frac{\mathrm{d}^3p_i}{(2\pi)^3\,2E_i}\right]\delta^4\left(p_a+p_b-\sum_{i=1}^{n}p_n\right)$$

- Use factorization properties of phase-space integral

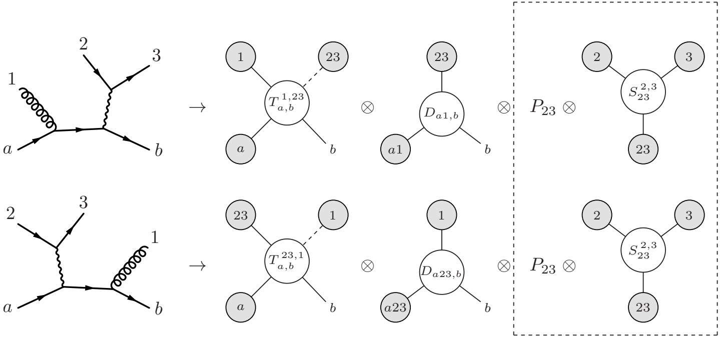
$$\begin{aligned}\mathrm{d}\Phi_n(p_a,p_b;p_1,..,p_n) &= \mathrm{d}\Phi_{n-m+1}(p_a,p_b;p_{1m},p_{m+1},..,p_n)\\ &\times\frac{\mathrm{d}s_{1m}}{2\pi}\,\mathrm{d}\Phi_m(p_{1m};p_1,..,p_m)\end{aligned}$$

- Apply repeatedly until only 2-particle phase spaces remain

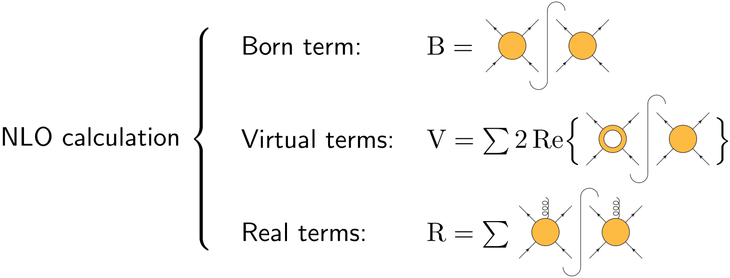
$$\mathrm{d}\Phi_2 = \frac{\lambda(s_{ij},m_i^2,m_j^2)}{16\pi^2\,2s_{ij}}\mathrm{d}\cos\theta_i\mathrm{d}\phi_i$$

$\lambda^2(a,b,c) = (a-b-c)^2-4bc$ - Källen function

Phase space



- Construct one integrator per diagram and combine into multi-channel
- Intuitive notion of pole structure, multi-channel determines balance
- Factorial growth with number of diagrams can be tamed by recursion



- UV divergences in V removed by renormalization procedure
- V and R both still infrared divergent
- IR divergences cancel between V and R (KLN theorem)
- Exploit this fact to construct finite integrand for MC

Toy model for NLO

[Frixione,Webber] hep-ph/0204244

- Assume system of charged particles
  which radiates "photons" of fractional energy x.
- Predicting infrared-safe observables O
  amounts to computing expectation values

$$\langle O\rangle = \lim_{\varepsilon\to 0}\int_0^1 \mathrm{d}x\, x^{-2\varepsilon} O(x)\left[\left(\frac{\mathrm{d}\sigma}{\mathrm{d}x}\right)_{\mathrm{B}} + \left(\frac{\mathrm{d}\sigma}{\mathrm{d}x}\right)_{\mathrm{V}} + \left(\frac{\mathrm{d}\sigma}{\mathrm{d}x}\right)_{\mathrm{R}}\right]$$

- Born, virtual and real-emission contributions given by

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}x}\right)_{\mathrm{B,V,R}} = \quad \mathrm{B}\,\delta(x), \qquad \left(\mathrm{V}_f + \frac{\mathrm{B}C}{2\varepsilon}\right)\delta(x), \qquad \frac{\mathrm{R}(x)}{x}$$

  Real-emission behaves as $\lim_{x\to 0}\mathrm{R}(x) = \mathrm{B}C$

  Virtual correction $\begin{cases} \mathrm{V}_f & - \quad \text{finite piece} \\ \mathrm{B}C/2\varepsilon & - \quad \text{singular piece} \end{cases}$

  Implicit: All higher-order terms proportional to coupling α

Subtraction in the toy model

- Perform NLO calculation in subtraction method

$$\langle O\rangle_R = \mathrm{B}C\, O(0) \int_0^1 \mathrm{d}x \frac{x^{-2\varepsilon}}{x} + \int_0^1 \mathrm{d}x\, \frac{\mathrm{R}(x)\, O(x) - \mathrm{B}C\, O(0)}{x^{1+2\varepsilon}}$$

- Second integral non-singular → set $\varepsilon = 0$

$$\langle O\rangle_R = -\frac{\mathrm{B}C}{2\varepsilon}\, O(0) + \int_0^1 \mathrm{d}x\, \frac{\mathrm{R}(x)\, O(x) - \mathrm{B}C\, O(0)}{x}$$

- Combine everything with Born and virtual correction

$$\langle O\rangle = \Big(\mathrm{B} + \mathrm{V}_f\Big)\, O(0) + \int_0^1 \frac{\mathrm{d}x}{x} \Big[\mathrm{R}(x)\, O(x) - \mathrm{B}C\, O(0)\Big]$$

  Both terms separately finite

- Rewrite for further reference

$$\langle O\rangle = \Big(\mathrm{B} + \mathrm{V} + \mathrm{I}\Big)\, O(0) + \int_0^1 \frac{\mathrm{d}x}{x} \Big[\mathrm{R}(x)\, O(x) - \mathrm{S}\, O(0)\Big]$$

  $\mathrm{I} = -\mathrm{B}C/2\varepsilon$ → Integrated subtraction term
  $\mathrm{S} = \mathrm{B}C$ → Real subtraction term

QCD subtraction

- QCD subtraction a little more cumbersome due to spin and colour correlations in R
- Basic features surviving from toy model are phase-space mapping and subtraction terms as products of Born times splitting operator
- Commonly used techniques
  - Dipole method

    [Catani,Seymour] NPB485(1997)291

    [Catani,Dittmaier,Seymour,Trocsanyi] NPB627(2002)189
  - FKS method

    [Frixione,Kunszt,Signer] NPB467(1996)399

Matrix element generators

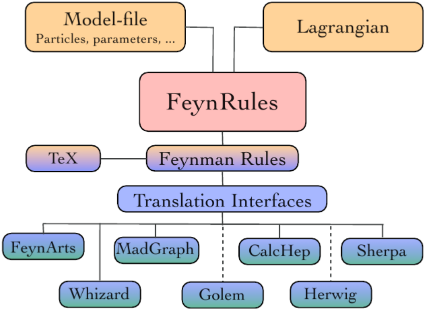
- Commonly used ME generators

	Built-in models	$2 \to$	$\|M_n\|^2$	$d\Phi_n$	NLO
ALPGEN	SM	8	recursive	Multi	-
AMEGIC	SM,MSSM,ADD	6	diagrams	Multi	sub
Comix	SM	8	recursive	Multi	sub
CompHEP	SM,MSSM	4	textbook	Single	-
HELAC	SM	8	recursive	Multi	sub+loop
MadEvent	SM,MSSM,UED	6	diagrams	Multi	sub+loop
Whizard	SM,MSSM,LH	8	recursive	Multi	sub

New physics models

[Christensen,Duhr] arXiv:0806.4194

- Most ME generators suited for any physics model, but implementing Feynman rules tedious and error-prone
- Automated by FeynRules package
- Extracts vertices from Lagrangian based on minimal information about particle content
- Writes generator-specific output permitting easy cross-checks



SLAC

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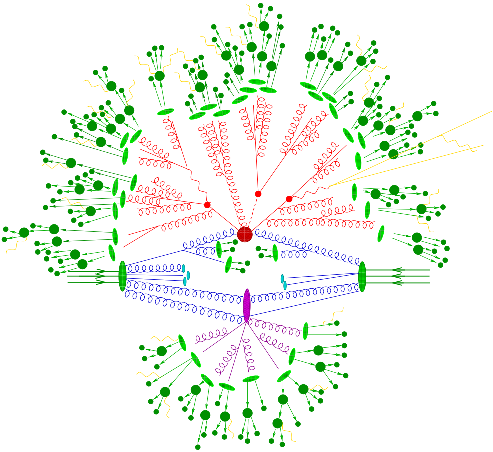
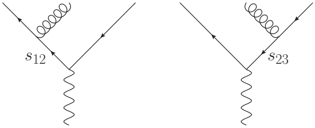


Parton evolution

- Consider $e^+e^- \to 3$ partons

$$\frac{1}{\sigma_{2\to 2}}\frac{\mathrm{d}\sigma_{2\to 3}}{\mathrm{d}\cos\theta \mathrm{d}z} \sim C_F \frac{\alpha_s}{2\pi}\frac{2}{\sin^2\theta}\frac{1+(1-z)^2}{z}$$



θ - angle of gluon emission

z - fractional energy of gluon

- Divergent in
  - Collinear limit: $\theta \to 0, \pi$
  - Soft limit: $z \to 0$
- Separate into two independent jets

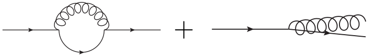
$$\frac{2\mathrm{d}\cos\theta}{\sin^2\theta} = \frac{\mathrm{d}\cos\theta}{1-\cos\theta} + \frac{\mathrm{d}\cos\theta}{1+\cos\theta} = \frac{\mathrm{d}\cos\theta}{1-\cos\theta} + \frac{\mathrm{d}\cos\bar{\theta}}{1-\cos\bar{\theta}} \approx \frac{\mathrm{d}\theta^2}{\theta^2} + \frac{\mathrm{d}\bar{\theta}^2}{\bar{\theta}^2}$$

- Independent jet evolution

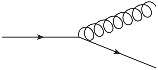
$$\mathrm{d}\sigma_3 \sim \sigma_2 \sum_{\text{jets}} C_F \frac{\alpha_s}{2\pi}\frac{\mathrm{d}\theta^2}{\theta^2}\mathrm{d}z\,\frac{1+(1-z)^2}{z} \to \sigma_2 \sum_{\text{jets}} \frac{\mathrm{d}t}{t}\mathrm{d}z\,\frac{\alpha_s}{2\pi}P_{ab}(z)$$

The Sudakov form factor

- Collinear partons not separately resolvable
- Introduce finite resolution criterion, e.g. $t > t_c$



- virtual+unresolved = finite



- resolved = finite

- Unitarity / Probability conservation $\to$ resolved + unresolved = 1
- Must implement no-emission probability (Poisson statistics)

$$\mathrm{d}\mathcal{P}_{\mathrm{emit}}(t) = \frac{\mathrm{d}t}{t} \int \mathrm{d}z \, \frac{\alpha_s}{2\pi} P_{ab}(z) \quad \to \quad \mathcal{P}_{\mathrm{no}}(t,t') = \exp\left\{ -\int_t^{t'} \frac{\mathrm{d}\bar{t}}{\bar{t}} \int \mathrm{d}z \, \frac{\alpha_s}{2\pi} P_{ab}(z) \right\}$$

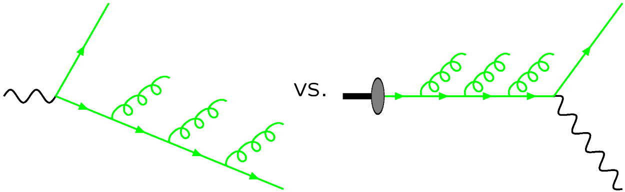
- Call $\Delta(t,t') := \mathcal{P}_{\mathrm{no}}(t,t')$ the Sudakov form factor
- Total probability for parton produced at t' to radiate at t is

$$\mathrm{d}\mathcal{P}(t) = \mathrm{d}\mathcal{P}_{\mathrm{emit}}(t)\mathcal{P}_{\mathrm{no}}(t,t') = \mathrm{d}t \, \frac{\mathrm{d}\Delta(t,t')}{\mathrm{d}t}$$

Initial-state evolution

[Sjöstrand] PLB175(1985)321

- Iteration leads to tree-like approximation of higher-order configuration
- Slight difference between final-state and initial-state evolution



- Initial-state emission probability must account for probability to resolve (different) parton at larger x

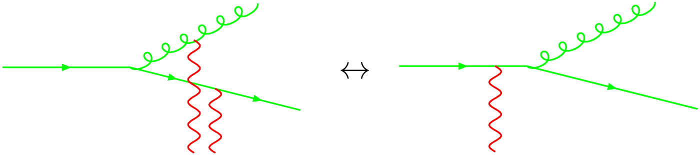
$$\mathrm{d}\mathcal{P}_{\mathrm{emit}}(x,t) = \frac{\mathrm{d}t}{t}\int\frac{\mathrm{d}z}{z}\,\frac{\alpha_s}{2\pi}P_{ab}(z)\,\frac{f_b(x/z,t)}{f_a(x,t)}$$

- Hard to implement in forward evolution (increasing t)
- Standard method is to evolve backward in initial state

Color coherence and angular ordering

[Marchesini,Webber] NPB310(1988)461

- Gluons with large wavelength not capable of resolving charges of emitting color dipole individually



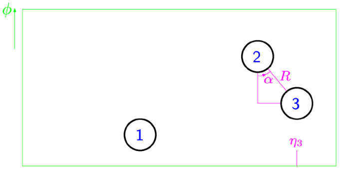
- Emission occurs with combined charge of mother parton instead
- Net effect is destructive interference outside cone with opening angle defined by emitting color dipole
- Can be implemented directly by angular ordering variable or additional ordering criterion in parton showers

Color coherence and angular ordering

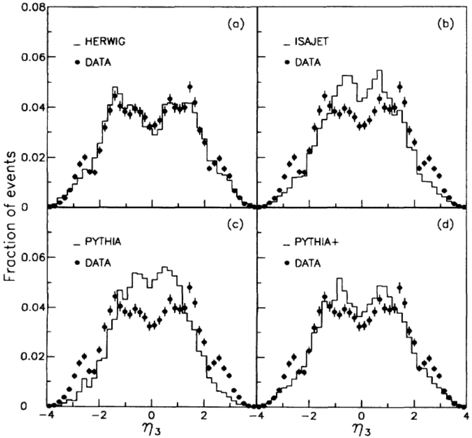
[CDF] PRD50(1994)5562

- Color coherence observed experimentally in 3-jet events

- Purely virtuality ordered PS's produce too much radiation in central region
- Angular ordered / angular vetoed PS's ok

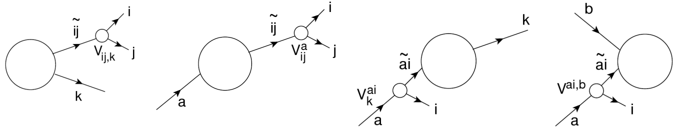


$E_{T1} > 110$ GeV, $E_{T3} > 10$ GeV.



Dipole showers

- In parton showers, the collinear/soft limit is never reached
  But who absorbs recoil when a splitting parton goes off mass-shell?
- No answer in DGLAP evolution equations $\leftrightarrow$ collinear limit
  Ambiguity introduces large uncertainties, especially at large t
- Natural solution provided by $2 \to 3$ splittings
  Spectator kinematics enters splitting probability
- Basic concept of dipole showers

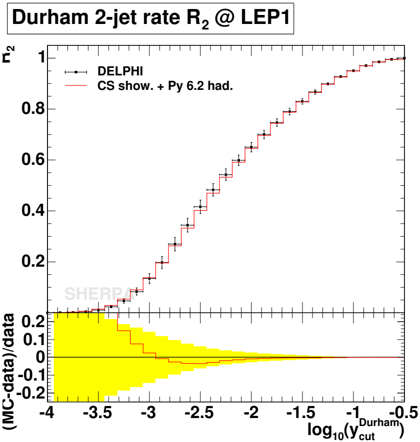


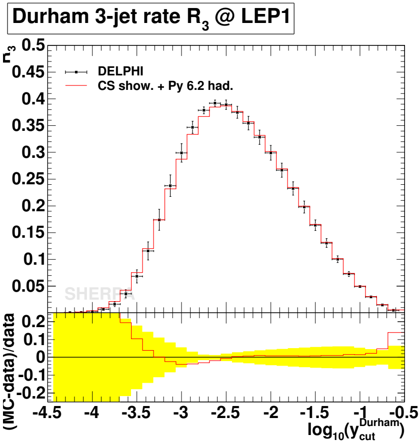
final-final final-initial initial-final initial-initial

- Publicly available generators

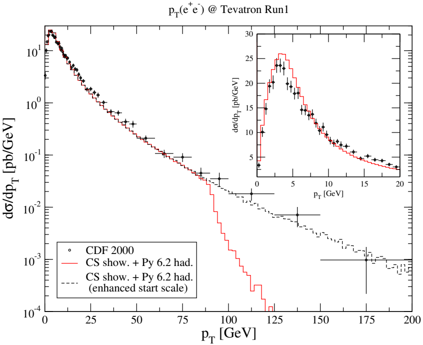
	Evolution variable	Splitting variable	Coherence
Ariadne	dipole-$k_\perp^2$	Rapidity	Antenna
Herwig	$E^2\theta^2$	Energy fraction	AO
Herwig++	$(t - m^2)/z(1-z)$	LC mom fraction	AO/Dipole
Pythia $<$6.4	t	Energy fraction	Enforced
Pythia $\geq$6.4	$k_\perp^2$	LC mom fraction	Enforced
Sherpa $<$1.2	t	Energy fraction	Enforced
Sherpa $\geq$1.2	$k_\perp^2$	LC mom fraction	Dipole
Vincia	variable	variable	Antenna

Effects of the parton shower





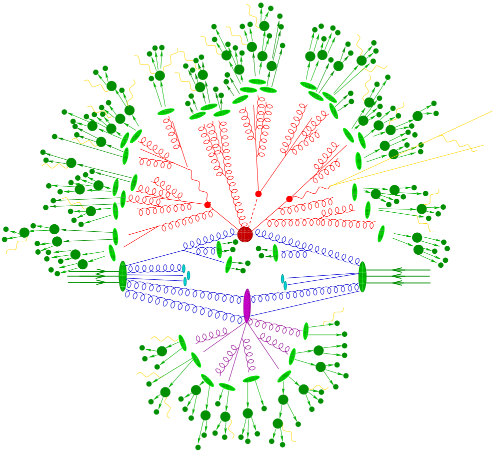
Effects of the parton shower



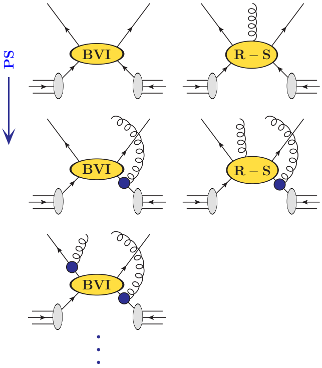
- Example: Drell-Yan lepton pair production at Tevatron
- If ME computed at leading order, then parton shower is only source of transverse momentum

The structure of MC events

- Hard interaction
- QCD evolution
- Secondary hard interactions
- Hadronization
- Hadron decays
- Higher-order QED corrections



NLO−PS matching



Modified subtraction

SLAC

[Frixione,Webber] hep-ph/0204244

- Revisit toy model for NLO

$$\langle O\rangle = \Big(\mathrm{B}+\mathrm{V}+\mathrm{I}\Big)\,O(0) + \int_0^1 \frac{\mathrm{d}x}{x}\Big[\mathrm{R}(x)\,O(x) - \mathrm{S}\,O(0)\Big]$$

- In parton showers, any number of “photons” can be emitted
- Emission probability controlled by Sudakov form factor

$$\Delta(x_1,x_2) = \exp\left\{-\int_{x_1}^{x_2}\frac{\mathrm{d}x}{x}\,\mathrm{K}(x)\right\}$$

Evolution kernel behaves as $\lim_{x\to 0}\mathrm{K}(x) = \lim_{x\to 0}\mathrm{R}(x)/\mathrm{B} = C$

- Define generating functional of PS $\to$ $\mathcal{F}^{(n)}_{\mathrm{MC}}(x) \leftrightarrow$ PS starting from n emissions at x
- $\mathcal{F}^{(n)}_{\mathrm{MC}}(x,O)$ now replaces observable O Naively:

  $O(0) \Leftrightarrow$ start MC with 0 emissions $\to \mathcal{F}^{(0)}_{\mathrm{MC}}(O)$

  $O(x) \Leftrightarrow$ start MC with 1 emission $\to \mathcal{F}^{(1)}_{\mathrm{MC}}(x,O)$

Modified subtraction

- Combined generating functional would be

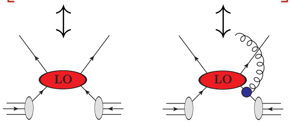
$$\left[\left(\mathrm{B}+\mathrm{V}+\mathrm{I}\right)-\int_0^1\frac{\mathrm{d}x}{x}\mathrm{S}\right]\mathcal{F}^{(0)}_{\mathrm{MC}}(O)+\int_0^1\frac{\mathrm{d}x}{x}\,\mathrm{R}(x)\,\mathcal{F}^{(1)}_{\mathrm{MC}}(x,O)$$

- This is wrong because

$$\mathcal{F}^{(0)}_{\mathrm{MC}}(O)=\Delta(x_c,1)\,O(0)+\int_{x_c}^1\frac{\mathrm{d}x}{x}\mathrm{K}(x)\Delta(x,1)\,O(x)+\ldots$$

- So $\mathrm{B}\,\mathcal{F}^{(0)}_{\mathrm{MC}}$ generates an $\mathcal{O}(\alpha)$ term that spoils NLO accuracy

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}x}\right)_{\mathrm{MC}}O(x)=\mathrm{B}\left[-\frac{\mathrm{K}(x)}{x}\,O(0)+\frac{\mathrm{K}(x)}{x}\,O(x)\right]$$



Modified subtraction

- The proper MC@NLO is obtained by subtracting this $\mathcal{O}(\alpha)$ contribution

$$\mathcal{F}_{\mathrm{MC@NLO}}(O) = \underbrace{\left[\Big(\mathrm{B}+\mathrm{V}+\mathrm{I}\Big) + \int_0^1 \frac{\mathrm{d}x}{x}\Big(\mathrm{BK}(x)-\mathrm{S}\Big)\right]}_{\text{NLO-weighted Born cross section}} \mathcal{F}^{(0)}_{\mathrm{MC}}(O)$$

$$+ \int_0^1 \frac{\mathrm{d}x}{x} \underbrace{\Big[\mathrm{R}(x) - \mathrm{BK}(x)\Big]}_{\text{modified subtraction}} \mathcal{F}^{(1)}_{\mathrm{MC}}(x, O)$$

- Like at fixed order, both terms are separately finite
- We call events from the first term $\mathbb{S}$-events ($\mathbb{S}$tandard) and events from the second term $\mathbb{H}$-events ($\mathbb{H}$ard)
- For further reference, define $\mathrm{D}^{(\mathrm{K})}(x) := \mathrm{BK}(x)$ as well as

$$\bar{\mathrm{B}}^{(\mathrm{K})} = \Big(\mathrm{B}+\mathrm{V}+\mathrm{I}\Big) + \int_0^1 \frac{\mathrm{d}x}{x}\Big(\mathrm{D}^{(\mathrm{K})}(x) - \mathrm{S}\Big)\,, \quad \mathrm{H}^{(\mathrm{K})}(x) = \mathrm{R}(x) - \mathrm{D}^{(\mathrm{K})}(x)$$

$\rightarrow$ compact notation

$$\mathcal{F}_{\mathrm{MC@NLO}}(O) = \bar{\mathrm{B}}^{(\mathrm{K})}\,\mathcal{F}^{(0)}_{\mathrm{MC}}(O) + \int_0^1 \frac{\mathrm{d}x}{x}\,\mathrm{H}^{(\mathrm{K})}(x)\,\mathcal{F}^{(1)}_{\mathrm{MC}}(x, O)$$

[Frixione,Webber] hep-ph/0204244

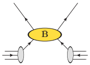
- Apply toy model to QCD, but include $1/x$-terms in coefficient functions
  Also need to sum over all flavor contributions at real-emission level

$$\bar{\mathrm{B}}_n^{(\mathrm{K})}(\Phi_n) = \Big(\mathrm{B}_n(\Phi_n) + \tilde{\mathrm{V}}_n(\Phi_n) + \mathrm{I}_n(\Phi_n)\Big) + \int \mathrm{d}\Phi_1 \Big(\mathrm{D}_n^{(\mathrm{K})}(\Phi_{n+1}) - \mathrm{S}_n(\Phi_{n+1})\Big)$$

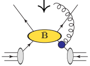
$$\mathrm{H}_n^{(\mathrm{K})}(\Phi_{n+1}) = \mathrm{R}_n(\Phi_{n+1}) - \mathrm{D}_n^{(\mathrm{K})}(\Phi_{n+1})$$

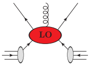
- Full differential event rate up to first emission

$$\mathrm{d}\sigma_{\mathrm{MC@NLO}} = \mathrm{d}\Phi_n \bar{\mathrm{B}}_n^{(\mathrm{K})}(\Phi_n) \bigg[\Delta_n^{(\mathrm{PS})}(t_c, \mu_Q^2) \quad \leftrightarrow$$



$$+ \int_{t_c}^{\mu_Q^2} \mathrm{d}\Phi_1 \mathrm{K}_n(\Phi_1) \Delta_n^{(\mathrm{PS})}(t(\Phi_1), \mu_Q^2) \bigg] + \mathrm{d}\Phi_n \int \mathrm{d}\Phi_1 \, \mathrm{H}_n^{(\mathrm{K})}(\Phi_{n+1})$$





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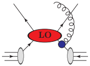
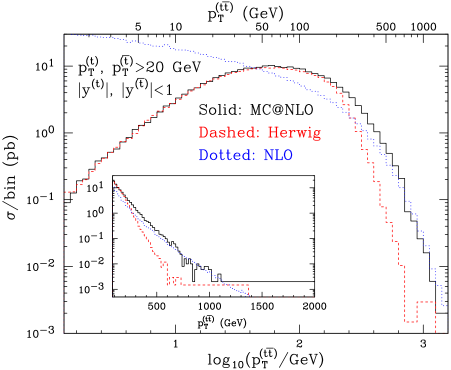


MC@NLO features

[Nason,Webber] arXiv:1202.1251

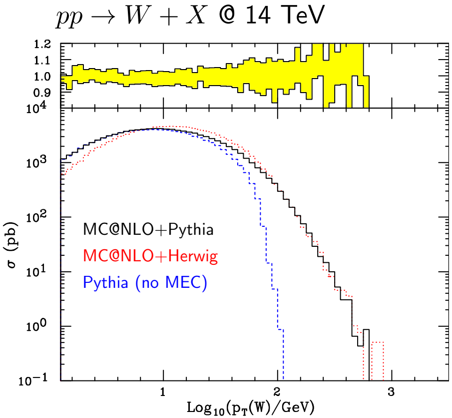
$pp \to t\bar{t} + X$ @ 14 TeV



- MC@NLO interpolates smoothly between real-emission ME and PS

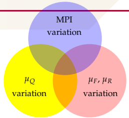
MC@NLO features

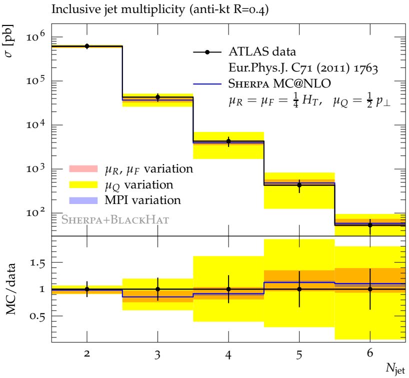
[Torrielli,Frixione] arXiv:1002.4293

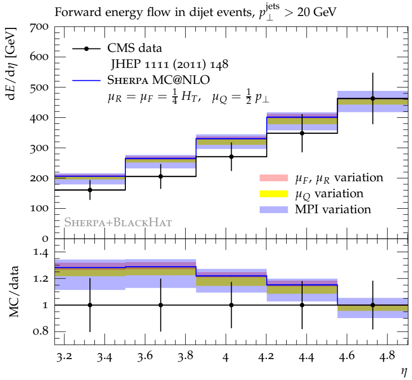


- MC@NLO with different PS agree at high $p_T \leftrightarrow$ NLO
- Differences at low p_T due to differences in PS

Which uncertainties are important?

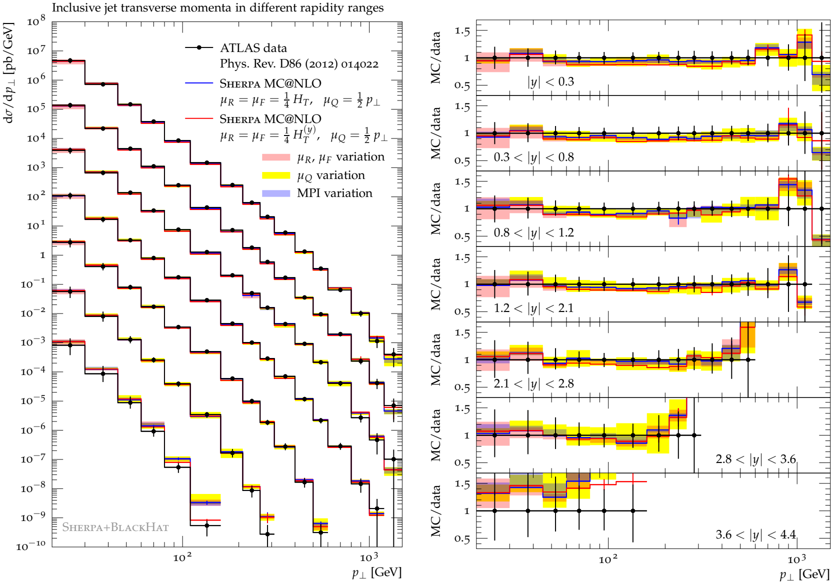




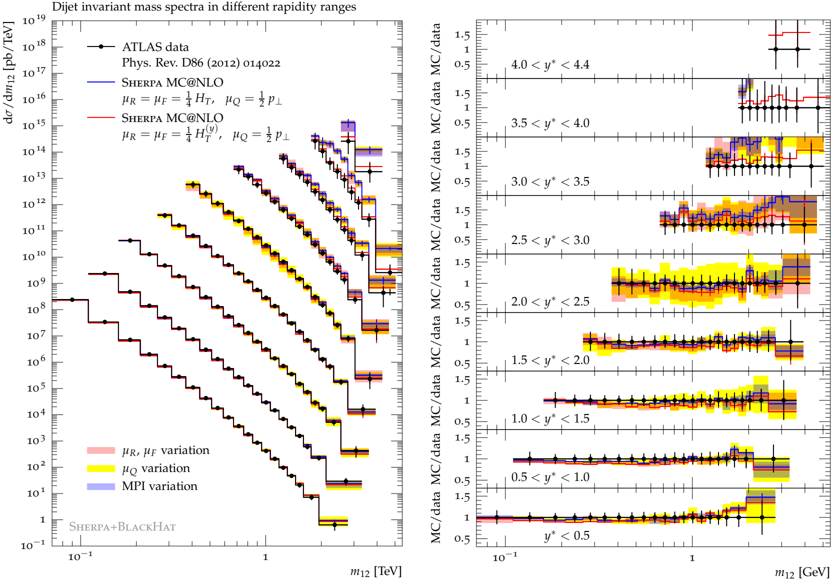


- Jet multiplicity $\rightarrow$ uncertainty due to choice of μ_Q^2
- Forward energy flow $\rightarrow$ major uncertainty from underlying event

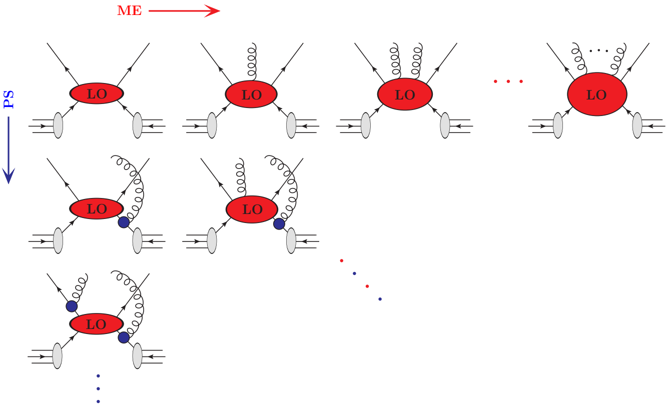
Inclusive jet production at the LHC



Inclusive jet production at the LHC

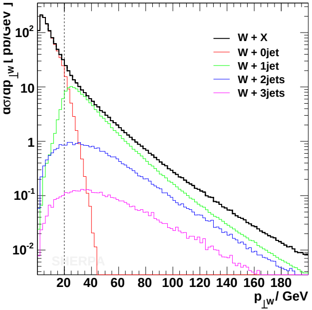


Matrix elements & parton showers



Basic idea of merging

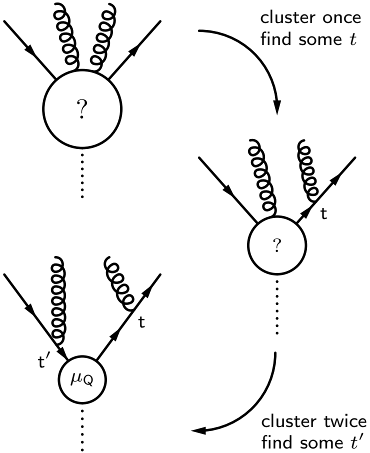
- Separate phase space into "hard" and "soft" region
- Matrix elements populate hard domain
- Parton shower populates soft domain
- Need criterion to define "hard" & "soft" $\to$ jet measure Q and corresponding cut, $Q_{\rm cut}$



Parton-shower histories

[André,Sjöstrand] hep-ph/9708390

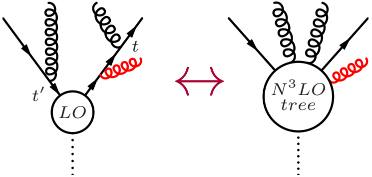
- Start with some "core" process for example $e^+e^- \to q\bar{q}$
- This process is considered inclusive It sets the resummation scale μ_Q^2
- Higher-multiplicity ME can be reduced to core by clustering
- If we want to match ME & PS the correct clustering algorithm suggests itself
  - Identify most likely splitting according to PS emission probability
  - Combine partons into mother according to PS kinematics
  - Continue until core process



SLAC

Efficient scheme to compute Sudakov suppression: Pseudo-showers

- Start PS from core process
- Evolve until predefined branching
  $\leftrightarrow$ truncated parton shower
- Emissions that would produce
  additional hard jets
  lead to event rejection (veto)



This corresponds to computing a Sudakov form factor given by

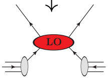
$$\Delta_n^{(\mathrm{PS})}(t,\mu_Q^2;>Q_{\mathrm{cut}}) = \exp\left\{-\int_t^{\mu_Q^2} \mathrm{d}\Phi_1\, K_n(\Phi_1)\,\Theta(Q-Q_{cut})\right\}$$

MEPS merging in MC@NLO notation

- Observable O to $\mathcal{O}(\alpha_s)$ given by

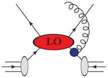
$$\langle O \rangle = \int \mathrm{d}\Phi_B \mathrm{B}(\Phi_B) \Bigg[\Delta^{(\mathrm{K})}(t_c, \mu_Q^2)\, O(\Phi_B)$$

$\updownarrow$



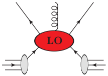
$$+ \int_{t_c}^{\mu_Q^2} \mathrm{d}\Phi_1 \mathrm{K}(\Phi_1)\, \Delta^{(\mathrm{K})}(t(\Phi_1), \mu_Q^2)\, \Theta(Q_{\mathrm{cut}} - Q)\, O(\Phi_R) \Bigg]$$

$\updownarrow$



$$+ \int \mathrm{d}\Phi_R \mathrm{R}(\Phi_R)\, \Delta^{(\mathrm{K})}(t(\Phi_R), \mu_Q^2; > Q_{\mathrm{cut}})\, \Theta(Q - Q_{\mathrm{cut}})\, O(\Phi_R)$$

$\updownarrow$



- Jet veto in PS
- Jet cut on $n+1$-parton final state

CKKW-L and METS merging

Algorithms with

- Exact correspondence between clustering & PS evolution
- Sudakov form factors as defined in parton shower

[Lönnblad] hep-ph/0112284
[Lönnblad,Prestel] arXiv:1109.4829

CKKW-L (Pythia)

- Truncated showers generate suppression, but no emissions
- Jet criterion dynamically redefined during PS evolution

[SH,Krauss,Schumann,Siegert] arXiv:0903.1219
[Hamilton,Richardson,Tully] arXiv:0905.3072

METS (Herwig, Sherpa)

- Truncated parton showers generate emissions and suppression
- Accounts for mismatch between jet criterion and evolution variable

CKKW and MLM merging

Algorithms with

- Approximate correspondence between clustering & PS evolution
- Approximate Sudakov form factors

[Mangano,Moretti,Pittau] hep-ph/0108069
[Mangano,Moretti,Piccinini,Treccani] hep-ph/0611129

MLM

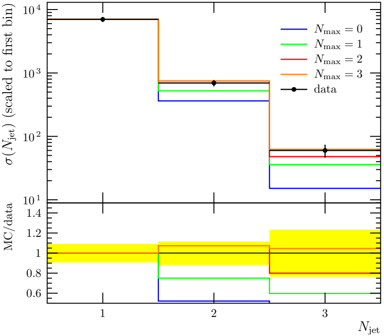
- No truncated parton showers, evolution starts at highest scale
- Sudakov suppression achieved by jet matching

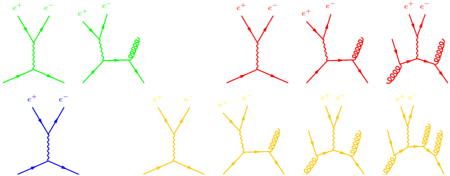
[Catani,Krauss,Kuhn,Webber] hep-ph/0109231
[Krauss] hep-ph/0205283

CKKW

- No truncated parton showers, evolution starts at modified scales
- Sudakov suppression by reweighting with NLL-approximated Sudakov

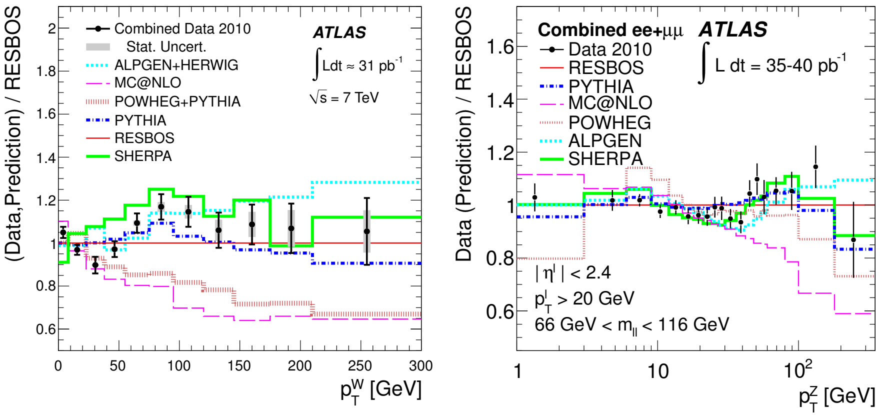
Effects of merging - Z+jets at the Tevatron





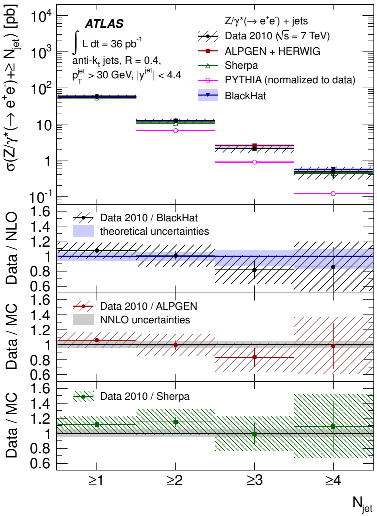
- MC predictions for exclusive n-jet rates match data well as long as corresponding final states are described by matrix elements

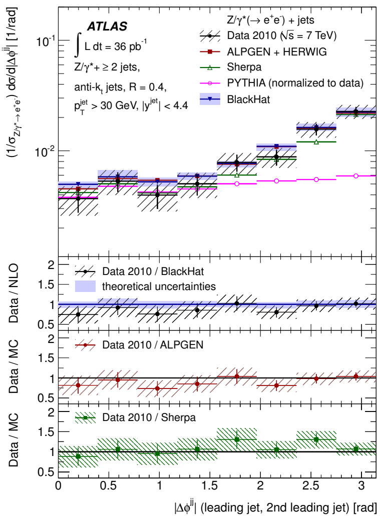
[ATLAS] arXiv:1108.6308 arXiv:1107.2381



- Comparison of vector boson transverse momentum spectra

[ATLAS] arXiv:1111.2690

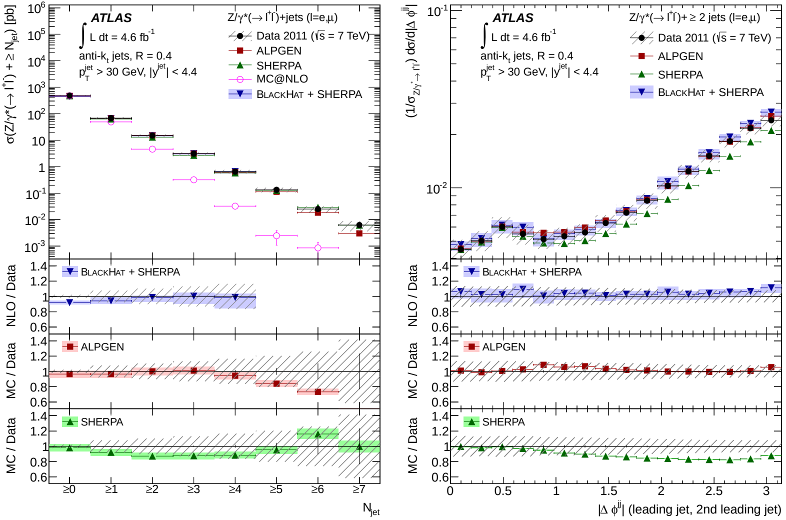




- Good agreement with both ALPGEN (MLM) and Sherpa
- PS alone fails for $n_{\text{jet}} \geq 2$

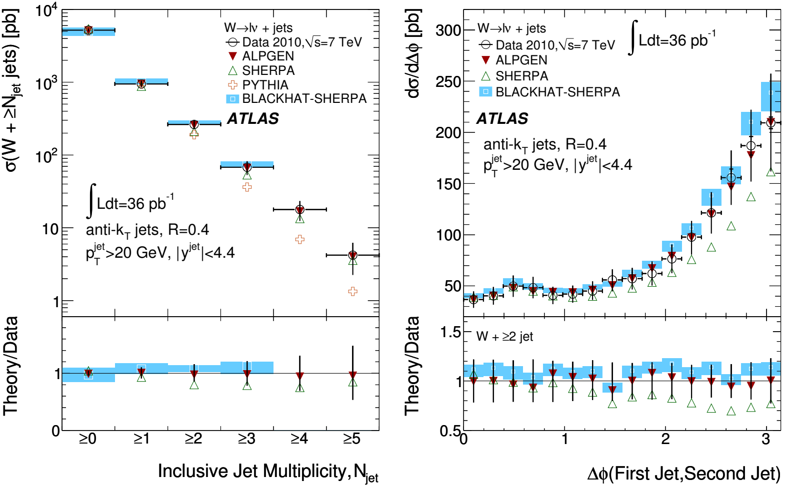
Z+jets

[ATLAS] arXiv:1304.7098



- Good agreement with ALPGEN (MLM), not so good with Sherpa
- MC@NLO alone fails for $n_{\text{jet}} > 2$

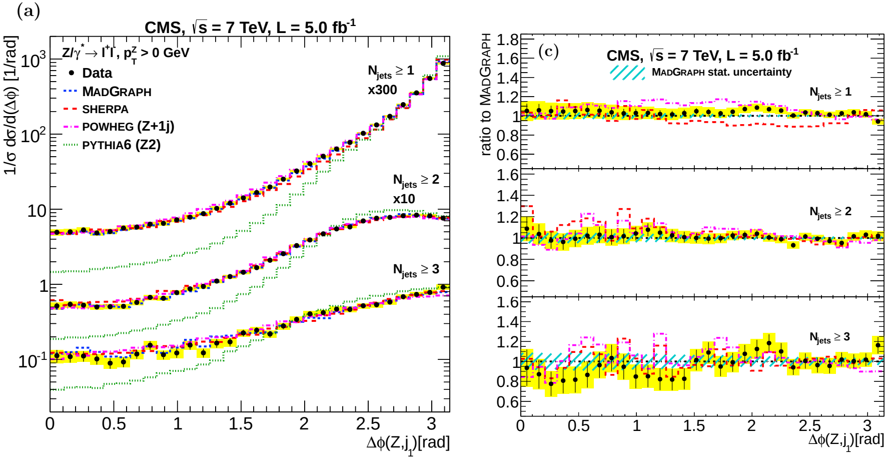
[ATLAS] arXiv:1201.1267



- Good agreement with ALPGEN (MLM), not so good with Sherpa

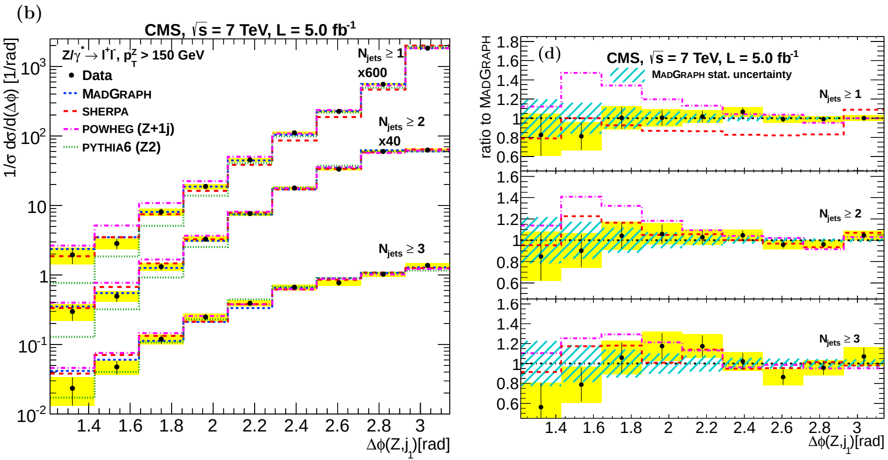
Z+jets

[CMS] arXiv:1301.1646



- Azimuthal separation between Z-boson and leading jet
- Inclusive event sample

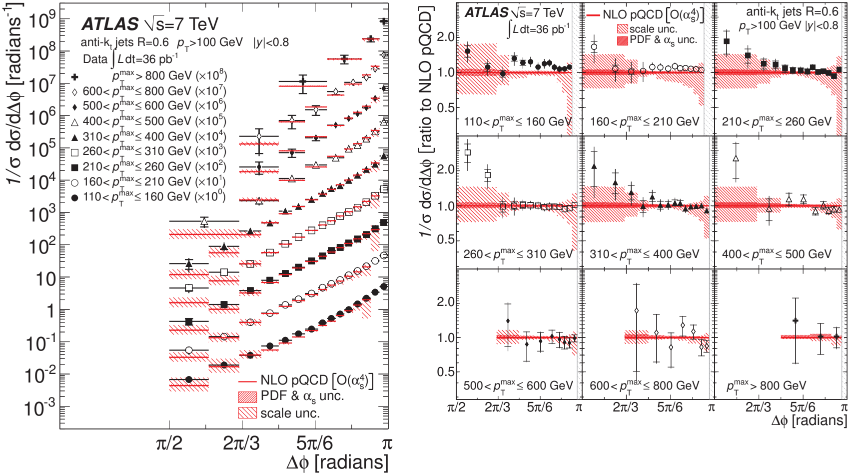
[CMS] arXiv:1301.1646



- Azimuthal separation between Z-boson and leading jet
- Boosted event sample

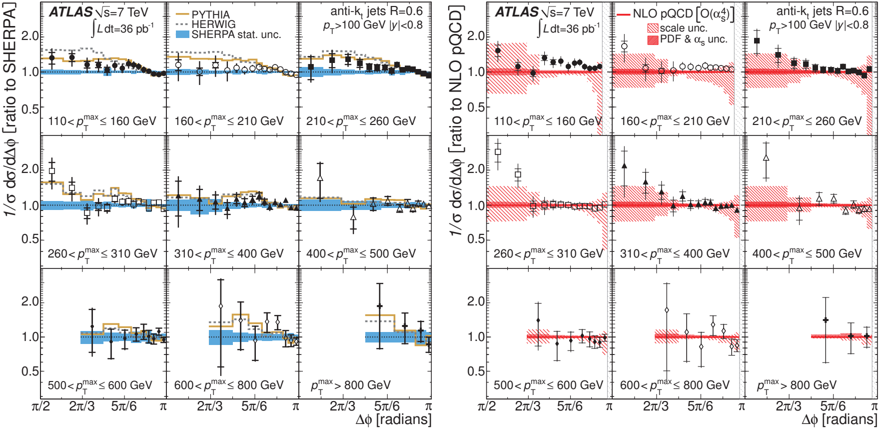
Pure jets

[ATLAS] arXiv:1102.2696



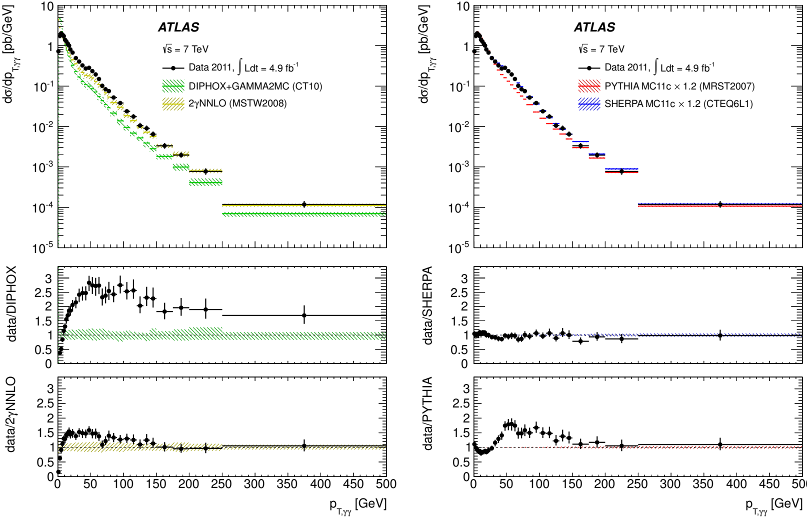
Pure jets

[ATLAS] arXiv:1102.2696



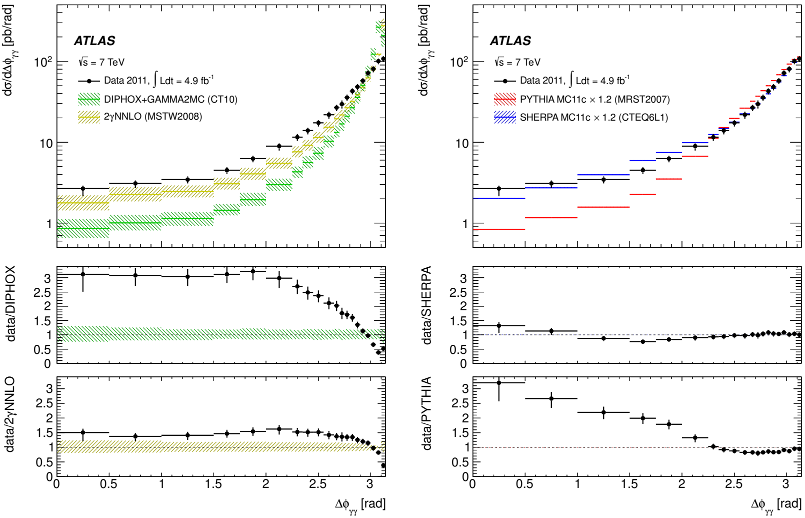
Diphotons

[ATLAS] arXiv:1211.1913

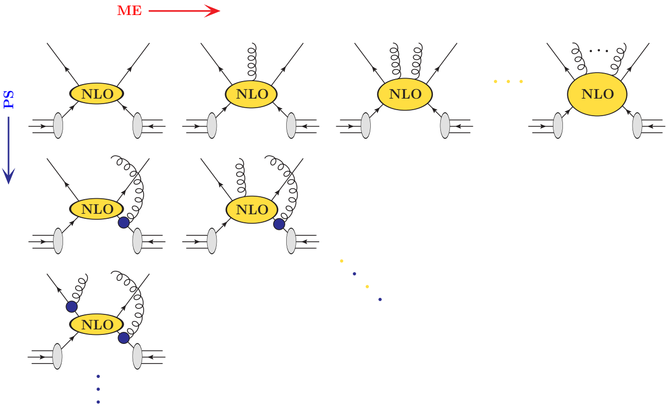


Diphotons

[ATLAS] arXiv:1211.1913



Merging NLO-matched calculations (MEPS@NLO)



Constructing MEPS@NLO

[Lavesson,Lönnblad,Prestel] arXiv:0811.2912 arXiv:1211.7278
[Gehrmann,Krauss,Schönherr,Siegert,SH] arXiv:1207.5031 arXiv:1207.5030
[Frederix,Frixione] arXiv:1209.6215

- MEPS for 0+1-jet in MC@NLO notation

$$\langle O\rangle = \int \mathrm{d}\Phi_B \mathrm{B}(\Phi_B)\Big[\Delta^{(\mathrm{K})}(t_c)\,O(\Phi_B) + \int_{t_c} \mathrm{d}\Phi_1 \mathrm{K}(\Phi_1)\,\Delta^{(\mathrm{K})}(t)\,\Theta(Q_{\mathrm{cut}}-Q)\,O(\Phi_R)\Big]$$
$$+ \int \mathrm{d}\Phi_R\,\mathrm{R}(\Phi_R)\,\Delta^{(\mathrm{K})}(t(\Phi_R);>Q_{\mathrm{cut}})\,\Theta(Q-Q_{\mathrm{cut}})\,O(\Phi_R)$$

- Reorder by parton multiplicity k, change notation $\mathrm{R}_k \to \mathrm{B}_{k+1}$
- Analyze exclusive contribution from k hard partons only $(t_0 = \mu_Q^2)$

$$\langle O\rangle_k^{\mathrm{excl}} = \int \mathrm{d}\Phi_k\,\mathrm{B}_k \prod_{i=0}^{k-1} \Delta_i^{(\mathrm{K})}(t_{i+1},t_i;>Q_{\mathrm{cut}})\,\Theta(Q_k-Q_{\mathrm{cut}})$$
$$\times \Big[\Delta_k^{(\mathrm{K})}(t_c,t_k)\,O_k + \int_{t_c}^{t_k} \mathrm{d}\Phi_1\,\mathrm{K}_k\,\Delta_k^{(\mathrm{K})}(t_{k+1},t_k)\,\Theta(Q_{\mathrm{cut}}-Q_{k+1})\,O_{k+1}\Big]$$

Constructing MEPS@NLO

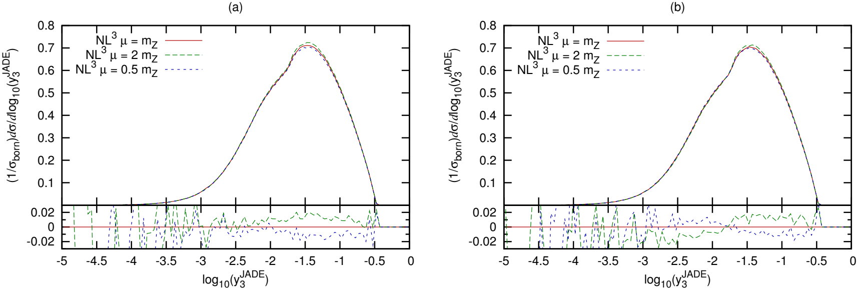
SLAC

- Analyze exclusive contribution from k hard partons

$$
\begin{aligned}
\langle O\rangle_k^{\rm excl} \;=\; &\int {\rm d}\Phi_k\,\bar{\rm B}_k^{\rm (D)}\,\prod_{i=0}^{k-1}\Delta_i^{\rm (K)}(t_{i+1},t_i;>Q_{\rm cut})\,\Theta(Q_k-Q_{\rm cut})\\
&\times\left(1+\frac{{\rm B}_k}{\bar{\rm B}_k^{\rm (D)}}\sum_{i=0}^{k-1}\int_{t_{i+1}}^{t_i}{\rm d}\Phi_1{\rm K}_i\,\Theta(Q_i-Q_{\rm cut})\right)\\
&\times\left[\Delta_k^{\rm (D)}(t_c,t_k)\,O_k\;+\int_{t_c}^{t_k}{\rm d}\Phi_1\;{\rm K}_k\,\Delta_k^{\rm (D)}(t_{k+1},t_k)\,\Theta(Q_{\rm cut}-Q_{k+1})\,O_{k+1}\right]\\
&+\int {\rm d}\Phi_{k+1}\,{\rm H}_k^{\rm (D)}\,\Delta_k^{\rm (K)}(t_k;>Q_{\rm cut})\,\Theta(Q_k-Q_{\rm cut})\,\Theta(Q_{\rm cut}-Q_{k+1})\,O_{k+1}
\end{aligned}
$$

- PS evolution kernels $\to$ dipole terms
- Born matrix element $\to$ NLO-weighted Born
- Add hard remainder function
- Subtract $\mathcal{O}(\alpha_s)$ terms contained in truncated PS

[Lavesson,Lönnblad] arXiv:0811.2912

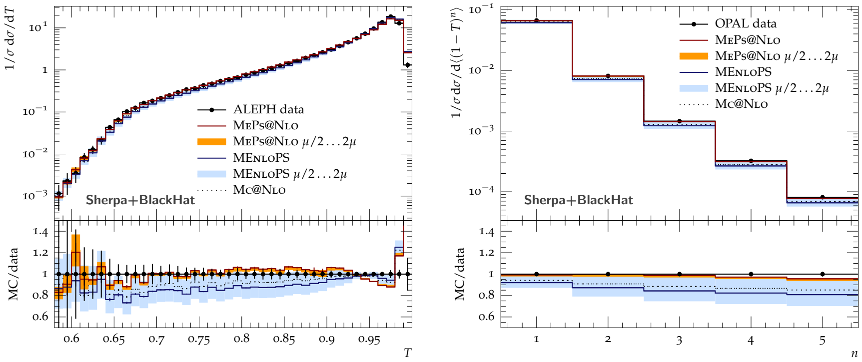


- Scale variations around 2%
- Agreement between 1- and 2-loop
  but no further reduction of uncertainty

$e^+e^- \to$hadrons at LEP

SLAC

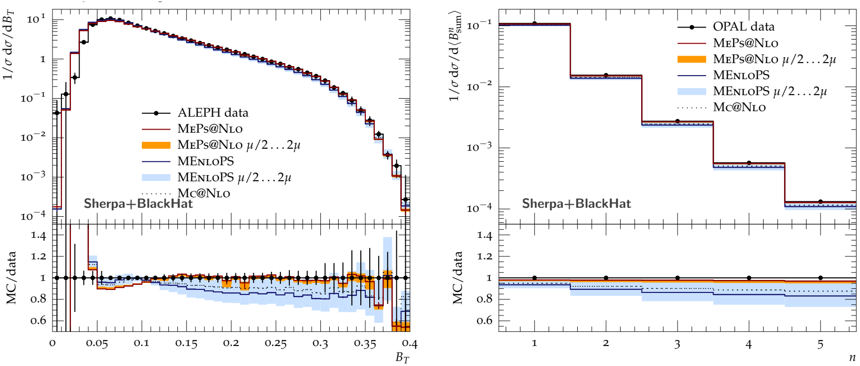
[Gehrmann,Krauss,Schönherr,Siegert,SH] arXiv:1207.5031



- Thrust & its moments
- MEPS@NLO with 2,3&4 jet PL at NLO plus 5&6 jet PL at LO vs MENLOPS with up to 6 jets at LO

$e^+e^-\to$hadrons at LEP

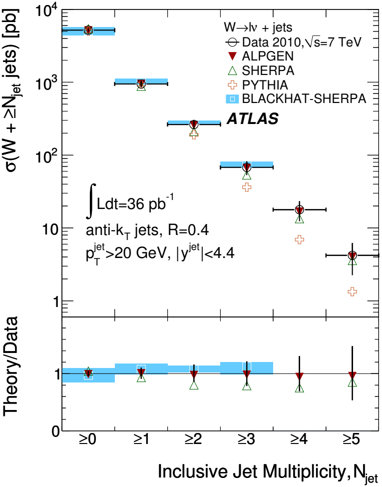
[Gehrmann,Krauss,Schönherr,Siegert,SH] arXiv:1207.5031

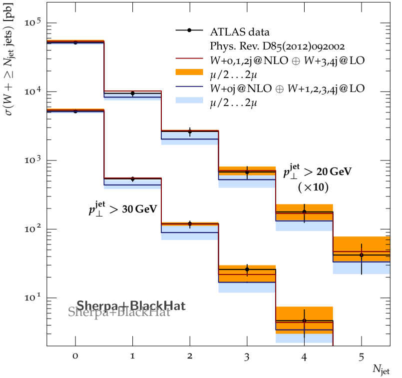


- Total jet broadening & its moments
- MEPS@NLO with 2,3&4 jet PL at NLO plus 5&6 jet PL at LO vs MENLOPS with up to 6 jets at LO

W+jets production at the LHC

[ATLAS] arXiv:1201.1276
[SH,Krauss,Schönherr,Siegert] arXiv:1207.5030





- MEPS@NLO with 0,1&2 jet PL at NLO plus 3&4 jet PL at LO
- MENLOPS with up to 4 jets at LO

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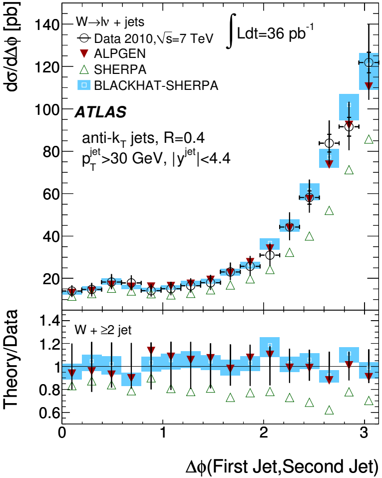
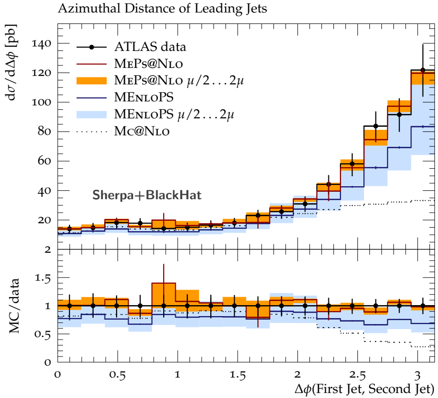
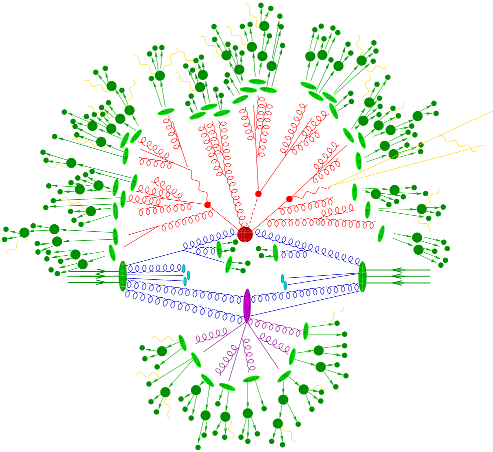




- MEPS@NLO with 0,1&2 jet PL at NLO plus 3&4 jet PL at LO
- MENLOPS with up to 4 jets at LO

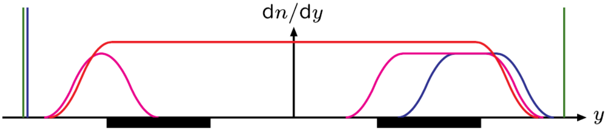
The structure of MC events

- Hard interaction
- QCD evolution
- Secondary hard interactions
- Hadronization
- Hadron decays
- Higher-order QED corrections

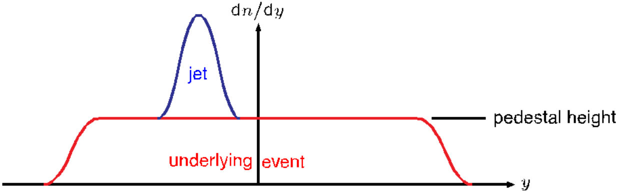


- Soft inclusive collision

$$\sigma_{tot} = \sigma_{\text{elastic}} + \sigma_{\text{single diffractive}} + \sigma_{\text{double diffractive}} + \sigma_{\text{non-diffractive}}$$

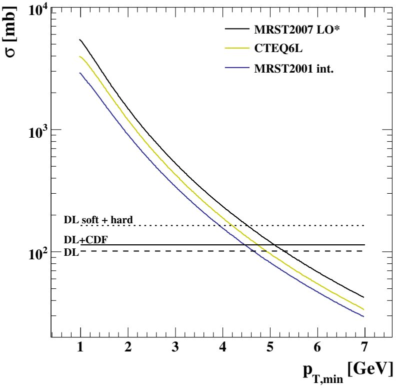


- Underlying event



Modeling the pedestal

[Sjöstrand,Zijl] PRD36(1987)2019

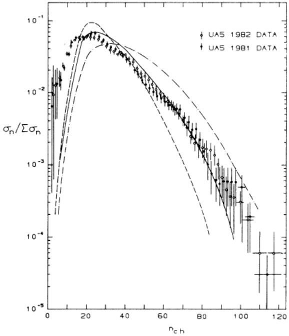


- Partonic cross sections diverge roughly like $\mathrm{d}p_T^2/p_T^4$
- Total cross section at LHC exceeded for $p_T \approx$ 2-5 GeV
- Interpretation as possibility for multiple hard scatters with

$$\langle n \rangle = \frac{\sigma_{\text{hard}}}{\sigma_{\text{non-diffractive}}}$$

- Main free parameter is $p_{T,\mathrm{min}}$
  Determines size of σ_{hard}

Modeling the pedestal

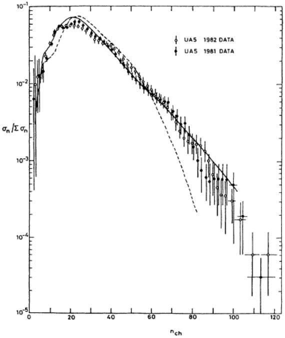


- Despite MPI wrong charged multi distribution
  Impact parameter dependent model needed
- Various hadron shape models in b-space
  (Exponential, Gaussian, double Gaussian)

$$\langle n \rangle = \frac{\sigma_{\text{hard}}}{\sigma_{\text{non-diffractive}}}$$

$$\downarrow$$

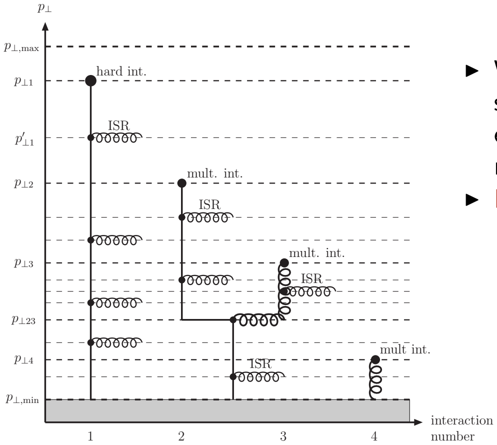
$$\langle \tilde{n}(b) \rangle = f_c f(b) \frac{\sigma_{\text{hard}}}{\sigma_{\text{non-diffractive}}}$$



- Hardness of the collision determines overlap
  Collisions with large overlap in turn
  have more secondary interactions

Combination with the parton shower

[Sjöstrand,Skands] hep-ph/0408302

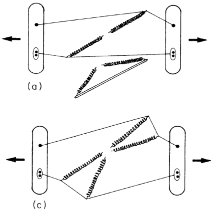


- When attaching IS shower to secondary scattering can ask at each point whether emission or new interaction is more likely
- New evolution equation

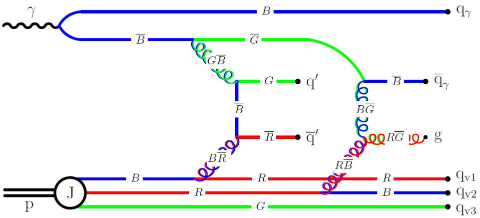
$$\frac{d\mathcal{P}}{dp_T} = \left(\frac{d\mathcal{P}_{\mathrm{MI}}}{dp_T} + \frac{d\mathcal{P}_{\mathrm{ISR}}}{dp_T}\right)$$
$$\times \exp\left\{-\int_{p_T} dp'_T \left(\frac{d\mathcal{P}_{\mathrm{MI}}}{dp'_T} + \frac{d\mathcal{P}_{\mathrm{ISR}}}{dp'_T}\right)\right\}$$

Color connections and beam remnants

[Sjöstrand,Skands] hep-ph/0402078



- Secondary scatterings need to be color-connected to something
- Simplest model would decouple them from proton remnants
- Next-to-simplest model would put all scatters on one color string



- New models embed scatters into existing color topology
- Three different options for string drawing
  - At random
  - Rapidity ordered
  - String length optimized

A model for minimum bias collisions

[Butterworth,Forshaw,Seymour] hep-ph/9601371
[Borozan,Seymour] hep-ph/0207283

- Assume parton distribution within beam hadron is

$$\frac{\mathrm{d}n_a(x,\mathbf{b})}{\mathrm{d}^2\mathbf{b}\mathrm{d}x} = f_a(x)\,G(\mathbf{b})$$

- Use electromagnetic form factor

$$G(\mathbf{b}) = \int \frac{\mathrm{d}^2\mathbf{k}}{(2\pi)^2}\,\frac{\exp(\mathbf{k}\cdot\mathbf{b})}{(1+\mathbf{k^2}/\mu^\mathbf{2})^\mathbf{2}}$$

- EM measurements indicate $\mu_P = 0.71$ GeV
  μ is however left free in model $\rightarrow$ tuning
- Continue model below $p_{T,\mathrm{min}}$ with same b-space parametrization
  but cross section as Gaussian in p_T $\rightarrow$ inclusive non-diffractive events

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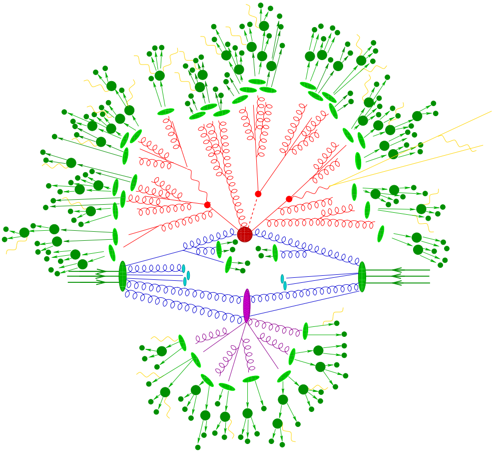
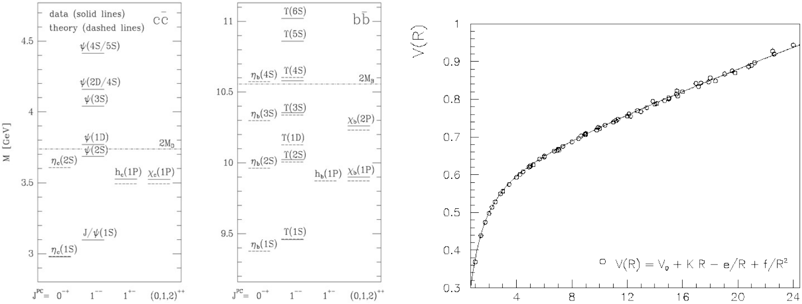


The inter-quark potential



- Measure QCD potential from quarkonia masses
- Or compute using lattice QCD
- Approximately linear potential ↔ QCD flux tube

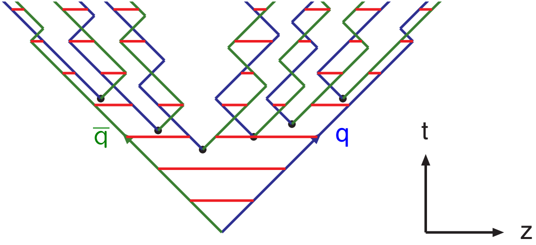
The Lund string model

[Andersson,Gustafson,Ingelman,Sjöstrand] PR97(1983)31

- Start with example $e^+e^- \to q\bar{q}$
- QCD flux tube with constant energy per unit rapidity $\leftrightarrow$ 
- New $q\bar{q}$-pairs created by tunneling (κ - string tension)

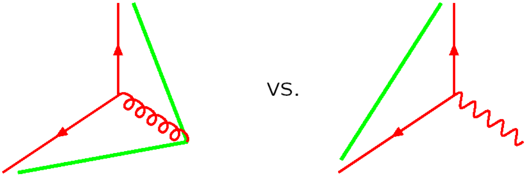
$$\frac{\mathrm{d}\mathcal{P}}{\mathrm{d}x\mathrm{d}t} = \exp\left\{-\frac{\pi^2 m_q^2}{\kappa}\right\}$$

- Expanding string breaks into hadrons, then yo-yo modes
- Baryons modeled as quark-diquark pairs



The Lund string model

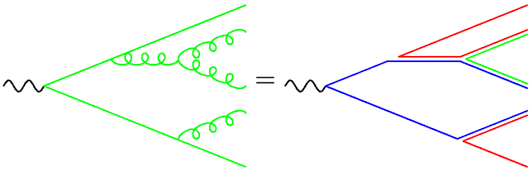
- String model very well motivated, but many parameters
- But also gives genuine prediction of "string effect"
- Gluons are kinks on string
  String accelerated in direction of gluon
- Infrared safe matching to parton showers
  Gluons with $k_T \lesssim 1/\kappa$ irrelevant



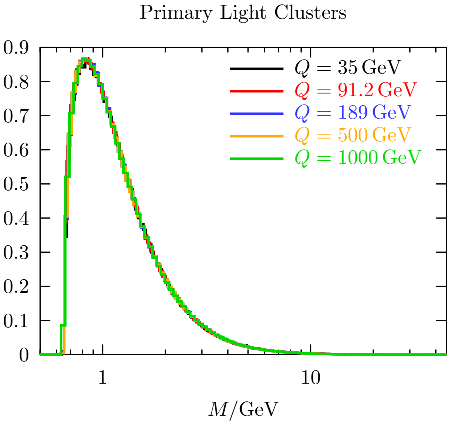
The cluster model

[Webber] NPB238(1984)492

- Underlying idea: Preconfinement
- Follow color structure of parton showers: color singlets end up close in phase space
- Mass of color singlets peaked at low scales ($\approx t_c$)



The cluster model



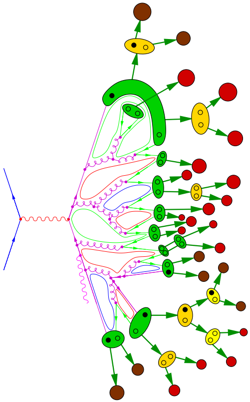
- Mass spectrum of primordial clusters independent of cm energy

Naïve model

- Split gluons into $q\bar{q}$-pairs
- Color-adjacent pairs form primordial clusters
- Clusters decay into hadrons according to phase space $\rightarrow$ baryon & heavy quark production suppressed

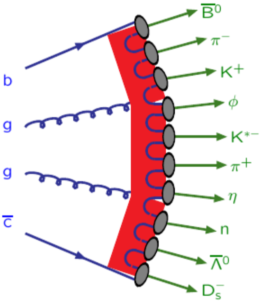
Improved model

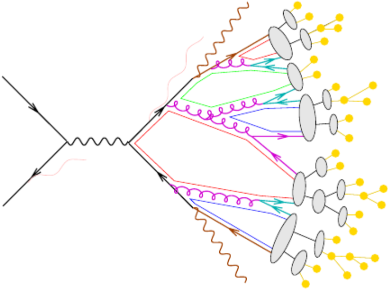
- Heavy clusters decay into lighter ones
- Three options: C$\rightarrow$CC, C$\rightarrow$CH & C$\rightarrow$HH
- Leading particle effects



String vs Cluster

[T.Sjöstrand, Durham'09]





program	PYTHIA	HERWIG
model	string	cluster
energy–momentum picture	powerful	simple
	predictive	unpredictive
parameters	few	many
flavour composition	messy	simple
	unpredictive	in-between
parameters	many	few

"There ain't no such thing as a parameter-free *good* description"

The structure of MC events

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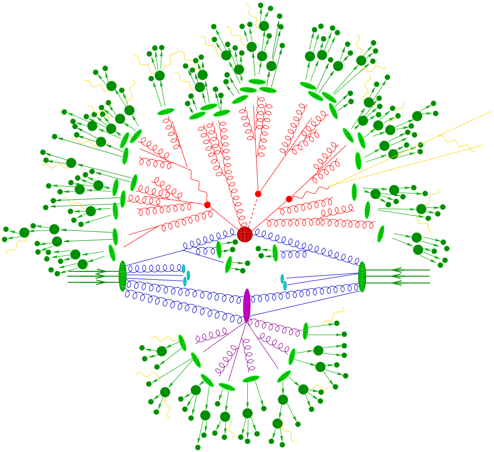
- Higher-order QED corrections

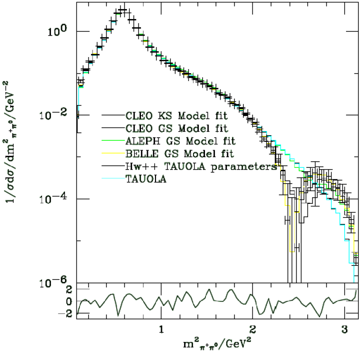


Secondary particle decays

- String and clusters decay to some stable hadrons
  but main outcome are unstable resonances
- These decay further according to the PDG decay tables
- Many hadron decays according to phase space
  but also a large variety of form factors known
- Not all branching ratios known precisely
  plus many BR's in PDG tables do not add up to one
- Significant effect on hadronization yields,
  hadronization corrections to event shapes, etc.

Secondary particle decays & photon radiation

- Previous generations of generators relied on external decay packages Tauola (τ-decays) & EvtGen (B-decays)
- New generation programs Herwig++ & Sherpa contain at least as complete a description
- Spin correlations and B-mixing built in
- No interfacing issues



- Previous generations of generators relied on external package Photos to simulate QED radiation
- New generation programs Herwig++ & Sherpa have simulation of QED radiation built in

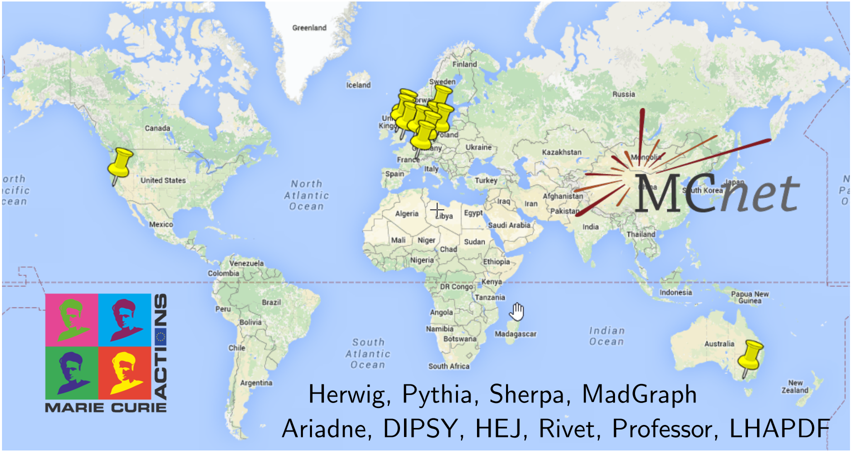
Summary

- Parton showers resum logarithmic corrections and generate exclusive radiation pattern
- Virtual corrections approximated by unitarity, can be included in full using MC@NLO
- POWHEG eliminates negative weight events in NLO matched simulations
- Large uncertainties associated with choice of resummed part of real correction

Summary

- ME+PS merging includes higher multiplicity matrix elements at tree-level
- NL^{3}SP, MEPS@NLO & FxFx allow to merge NLO-matched simulations of anything+jets
- UNLOPS unitarises entire simulation
- Underlying event typically simulated by MPI can be combined with PS in common evolution
- Two models (string & cluster) for parton to hadron fragmentation

MCnet phase 3 funded by European Commission



7 Countries, 12 Institutes, $\sim$ 100 Scientists

Apply now to work with us!

Join us for MC4BSM '17 at SLAC!

Try your generator of choice during the tutorial!

Stating the problem

- Want to compute expectation values of observables

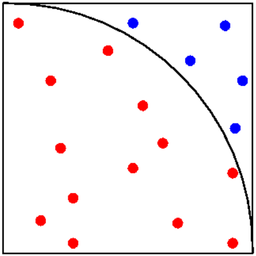
$$\langle O\rangle = \sum_n \int \mathrm{d}\Phi_n \, P(\Phi_n) \, O(\Phi_n)$$

  Φ_n - Point in n-particle phase-space
  $P(\Phi_n)$ - Probability to produce Φ_n
  $O(\Phi_n)$ - Value of observable at Φ_n

- Problem #1: Computing $P(\Phi_n)$
- Problem #2: Performing the integral

- At LO and NLO problem #2 is harder to solve
  This is where MC event generators come in

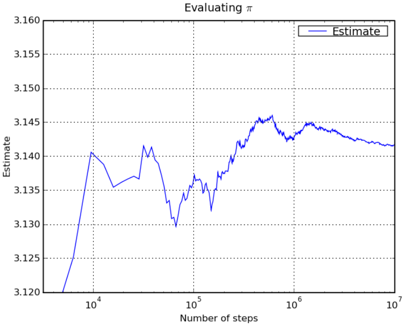
The hit-or-miss method



$$\frac{\text{Hits}}{\text{Misses} + \text{Hits}} \rightarrow \frac{\pi}{4}$$

Throw random points (x,y),
with x, y in [0,1]

For hits: $(x^2 + y^2) < r^2 = 1$



Importance sampling

- In many cases we can approximate the integral of $f(x)$ with some known function $g(x)$ such that primitive $G(x)$ is known
- This amounts to a variable transformation

$$I = \int_a^b \mathrm{d}x\, g(x)\, \frac{f(x)}{g(x)} = \int_{G(a)}^{G(b)} \mathrm{d}G(x)\, w(x) \quad \text{where} \quad w(x) = \frac{f(x)}{g(x)}$$

- Integral and error estimate are

$$I = \big[G(b) - G(a)\big]\langle w\rangle \qquad \sigma = \big[G(b) - G(a)\big]\sqrt{\frac{\langle w^2\rangle - \langle w\rangle^2}{N-1}}$$

  N - Number of MC events (points)
- MC error scales as $1/\sqrt{N}$ independent of number of dimensions!
- Note that I is independent of $g(x)$, but σ is not $\rightarrow$ suitable choice of $g(x)$ can be used to minimize error

Selection from a known distribution

- Random number generators produce uniform pseudo-random numbers in $[0, 1]$
- Assume we want points following the distribution $g(x)$ with known primitive $G(x)$ instead
- Probability of producing point in $[x, x + \mathrm{d}x]$ is $g(x)\,\mathrm{d}x$
- Can generate x according to

$$\int_a^x \mathrm{d}x'\, g(x') = R \int_a^b \mathrm{d}x'\, g(x')$$

where R is a uniform random number in $[0, 1]$

$$x = G^{-1}\Big[G(a) + R\big(G(b) - G(a)\big)\Big]$$

Selection from a Poisson distribution

- Assume nuclear decay process described by $g(x)$
- Nucleus can decay only if it has not decayed already
  Must account for survival probability $\leftrightarrow$ Poisson distribution

$$\mathcal{G}(x) \;=\; g(x)\Delta(x,b) \qquad \text{where} \qquad \Delta(x,b) \;=\; \exp\left\{-\int_x^b \mathrm{d}x'\, g(x')\right\}$$

- If $G(x)$ is known, then we also know the integral of $\mathcal{G}(x)$

$$\int_x^b \mathrm{d}x' \mathcal{G}(x') \;=\; \int_x^b \mathrm{d}x' \,\frac{\mathrm{d}\Delta(x',b)}{\mathrm{d}x'} \;=\; 1-\Delta(x,b)$$

- Can generate events by requiring $1-\Delta(x,b) = 1-R$

$$x = G^{-1}\Big[G(b) + \log R\Big]$$

The veto algorithm

SLAC

- Veto algorithm $\leftrightarrow$ Hit-or-miss method for Poisson distributions
  - Generate event according to $\mathcal{G}(x)$
  - Accept with $w(x) = f(x)/g(x)$
  - If rejected, continue starting from x
- Probability for immediate acceptance

$$\frac{f(x)}{g(x)}\,g(x)\exp\left\{-\int_x^b \mathrm{d}x'\,g(x')\right\}$$

- Probability for acceptance after one rejection

$$\frac{f(x)}{g(x)}\,g(x)\int_x^b \mathrm{d}x_1 \exp\left\{-\int_x^{x_1} \mathrm{d}x'\,g(x')\right\}\left(1-\frac{f(x_1)}{g(x_1)}\right)g(x_1)\exp\left\{-\int_{x_1}^b \mathrm{d}x'\,g(x')\right\}$$

- For n intermediate rejections we obtain n nested integrals $\int_x^b \int_{x_1}^b \ldots \int_{x_{n-1}}^b$
- Disentangling yields $1/n!$ and summing over all possible rejections gives

$$f(x)\exp\left\{-\int_x^b \mathrm{d}x'\,g(x')\right\}\sum_{n=0}^{\infty}\frac{1}{n!}\left[\int_x^b \mathrm{d}x'\,\left[g(x')-f(x')\right]\right]^n = f(x)\exp\left\{-\int_x^b \mathrm{d}x'\,f(x')\right\}$$

Non-standard veto algorithm

[Siegert,Schumann,SH] arXiv:0912.3501

- In MC@NLO generate emission according to

$$\Delta^{(\mathrm{S})}(t,t') = \exp\left\{ -\int_t^{t'} \mathrm{d}\Phi_1 \frac{\mathrm{S}(\Phi_B;t,z,\phi)}{\mathrm{B}(\Phi_B)} \right\}$$

- "dipole-term-correct" PS

$$f(t) \to \int \mathrm{d}z\,\mathrm{d}\phi\, J(t,z,\phi)\, \frac{\mathrm{S}(\Phi_B;t,z,\phi)}{\mathrm{B}(\Phi_B)}$$

$$g(t) \to \int \mathrm{d}z\,\mathrm{d}\phi\, J(t,z,\phi)\, \mathrm{K}(t,z,\phi)$$

- $f(t)/g(t)$ may be negative e.g. due to subleading color dipoles
  $\to$ probabilistic interpretation not possible
- Split weight into MC and analytic part using auxiliary function $h(t)$

$$\frac{f(t)}{h(t)}\, g(t)\, \exp\left\{ -\int_t^{t_1} \mathrm{d}\bar{t}\, g(\bar{t}) \right\} \prod_{i=1}^{n} \left[\int_{t_{i-1}}^{t_{n+1}} \mathrm{d}t_i \left(1 - \frac{f(t_i)}{h(t_i)} \right) g(t_i)\, \exp\left\{ -\int_{t_i}^{t_{i+1}} \mathrm{d}\bar{t}\, g(\bar{t}) \right\} \right]$$

$$w(t,t_1,\ldots,t_n) = \frac{h(t)}{g(t)} \prod_{i=1}^{n} \frac{h(t_i)}{g(t_i)} \frac{g(t_i)-f(t_i)}{h(t_i)-f(t_i)}$$

- $h(t)$ can be chosen freely, as long as $\mathrm{sgn}(h) = \mathrm{sgn}(f)$, $|f| < |h|$

The MC algorithm for parton showers

- Start with set of n partons at scale t', which evolve collectively
  Sudakov form factors factorize, schematically

$$\Delta(t,t') = \prod_{i=1}^{n} \Delta_i(t,t') \qquad\qquad \Delta_i(t,t') = \prod_{j=q,g} \Delta_{i\to j}(t,t')$$

- Use veto algorithm to find new scale t where branching occurs
  - Generate t using overestimate $\alpha_s^{\max} P_{ab}^{\max}(z)$
  - Determine "winner" parton i and select new flavor j
  - Select splitting variable according to overestimate
  - Accept point with weight $\alpha_s(k_T^2) P_{ab}(z) / \alpha_s^{\max} P_{ab}^{\max}(z)$
- Construct splitting kinematics and update color flow
- Continue until $t < t_c$

Recap: Basics of NLO+PS matching

- Leading-order calculation for observable O

$$\langle O\rangle = \int \mathrm{d}\Phi_B\, \mathrm{B}(\Phi_B)\, O(\Phi_B)$$

- NLO calculation for same observable

$$\langle O\rangle = \int \mathrm{d}\Phi_B \left\{\mathrm{B}(\Phi_B) + \tilde{V}(\Phi_B)\right\} O(\Phi_B) + \int \mathrm{d}\Phi_R\, \mathrm{R}(\Phi_R)\, O(\Phi_R)$$

- Parton-shower result until first emission

$$\langle O\rangle = \int \mathrm{d}\Phi_B\, \mathrm{B}(\Phi_B) \left[\Delta^{(\mathrm{K})}(t_c)\, O(\Phi_B) + \int_{t_c} \mathrm{d}\Phi_1\, \mathrm{K}(\Phi_1)\, \Delta^{(\mathrm{K})}(t(\Phi_1))\, O(\Phi_R)\right]$$

$$\overset{\mathcal{O}(\alpha_s)}{\to} \int \mathrm{d}\Phi_B\, \mathrm{B}(\Phi_B) \left\{1 - \int_{t_c} \mathrm{d}\Phi_1 \mathrm{K}(\Phi_1)\right\} O(\Phi_B) + \int_{t_c} \mathrm{d}\Phi_B \mathrm{d}\Phi_1\, \mathrm{B}(\Phi_B)\, \mathrm{K}(\Phi_1)\, O(\Phi_R)$$

Phase space: $\mathrm{d}\Phi_1 = \mathrm{d}t\, \mathrm{d}z\, \mathrm{d}\phi\, J(t,z,\phi)$

Splitting functions: $\mathrm{K}(t,z) \to \alpha_s/(2\pi t) \sum \mathrm{P}(z)\, \Theta(\mu_Q^2 - t)$

Sudakov factors: $\Delta^{(\mathrm{K})}(t) = \exp\left\{ - \int_t \mathrm{d}\Phi_1 \mathrm{K}(\Phi_1)\right\}$

NLO+PS matching - Dealing with color and spin

- Subtract $\mathcal{O}(\alpha_s)$ PS terms from NLO result

$$\int \mathrm{d}\Phi_B \left\{\mathrm{B}(\Phi_B) + \tilde{\mathrm{V}}(\Phi_B) + \mathrm{B}(\Phi_B)\int \mathrm{d}\Phi_1 \mathrm{K}(\Phi_1)\right\} \ldots$$

$$+ \int \mathrm{d}\Phi_R \left\{\mathrm{R}(\Phi_R) - \mathrm{B}(\Phi_B)\,\mathrm{K}(\Phi_1)\right\} \ldots$$

- In DLL approximation both terms finite $\rightarrow$
  MC events in two categories, $\mathbb{S}$tandard and $\mathbb{H}$ard

$$\mathbb{S} \;\rightarrow\; \bar{\mathrm{B}}^{(\mathrm{K})}(\Phi_B) = \mathrm{B}(\Phi_B) + \tilde{\mathrm{V}}(\Phi_B) + \mathrm{B}(\Phi_B)\int \mathrm{d}\Phi_1 \mathrm{K}(\Phi_1)$$

$$\mathbb{H} \;\rightarrow\; \mathrm{H}^{(\mathrm{K})} = \mathrm{R}(\Phi_R) - \mathrm{B}(\Phi_B)\mathrm{K}(\Phi_1)$$

- Color & spin correlations $\rightarrow$ NLO subtraction needed
  $1/N_c$ corrections can be faded out in soft region by smoothing function

$$\bar{\mathrm{B}}^{(\mathrm{K})}(\Phi_B) = \mathrm{B}(\Phi_B) + \tilde{\mathrm{V}}(\Phi_B) + \mathrm{I}(\Phi_B) + \int \mathrm{d}\Phi_1 \left[\mathrm{S}(\Phi_R) - \mathrm{B}(\Phi_B)\,\mathrm{K}(\Phi_1)\right] f(\Phi_1)$$

$$\mathrm{H}^{(\mathrm{K})}(\Phi_R) = \left[\mathrm{R}(\Phi_R) - \mathrm{B}(\Phi_B)\,\mathrm{K}(\Phi_1)\right] f(\Phi_1)$$

NLO+PS matching - Dealing with color and spin

Method 1 [Frixione,Webber] hep-ph/0204244

- $f(\Phi_1) \to 0$ in soft-gluon limit
- Full NLO only in hard / collinear region
  Missing subleading color terms in soft domain
- Only affects unresolved gluons $\to$ no need to correct

Method 2 [Krauss,Schönherr,Siegert,SH] arXiv:1111.1220

- Replace $\mathrm{B}(\Phi_B)\mathrm{K}(\Phi_1) \to \mathrm{S}(\Phi_R)$, i.e. include color & spin correlations
- May lead to non-probabilistic $\Delta^{(\mathrm{S})}(t)$
  Requires modification of veto algorithm
- Exact cancellation of all divergences without additional smoothing
  Equivalent to one-step full color parton shower algorithm

MC@NLO

[Frixione,Webber] hep-ph/0204244

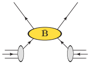
- Add parton shower, described by generating functional $\mathcal{F}_{\rm MC}$

$$\langle O\rangle = \int {\rm d}\Phi_B\, \bar{\rm B}^{\rm (K)}(\Phi_B)\, \mathcal{F}^{(0)}_{\rm MC}(\mu_Q^2, O) + \int {\rm d}\Phi_R\, {\rm H}^{\rm (K)}(\Phi_R)\, \mathcal{F}^{(1)}_{\rm MC}(t(\Phi_R), O)$$

  Probability conservation $\leftrightarrow$ $\mathcal{F}_{\rm MC}(t,1) = 1$

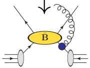
- Expansion of matched result until first emission

$$\langle O\rangle = \int {\rm d}\Phi_B \bar{\rm B}^{\rm (K)}(\Phi_B)\Big[\Delta^{\rm (K)}(t_c)\, O(\Phi_B) \quad \leftrightarrow$$

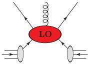


$$+ \int_{t_c} {\rm d}\Phi_1 {\rm K}(\Phi_1)\Delta^{\rm (K)}(t(\Phi_1))\, O(\Phi_r)\Big] + \int {\rm d}\Phi_R\, {\rm H}^{\rm (K)}(\Phi_{n+1})\, O(\Phi_R)$$

$\updownarrow$



$\updownarrow$



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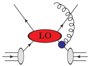


- Parametrically $\mathcal{O}(\alpha_s)$ correct
- Preserves logarithmic accuracy of PS

POWHEG

[Nason] hep-ph/0409146

- Aim of the method: Eliminate negative weights from MC@NLO
- Replace $\mathrm{BK} \to \mathrm{R} \Rightarrow$ no $\mathbb{H}$-events $\quad\Rightarrow\quad \bar{\mathrm{B}}^{(\mathrm{R})}$ positive in physical region
- Expectation value of observable is

$$\langle O\rangle = \int \mathrm{d}\Phi_B \bar{\mathrm{B}}^{(\mathrm{R})}(\Phi_B)\Bigg[\Delta^{(\mathrm{R})}(t_c, s_{\mathrm{had}})\, O(\Phi_B) + \int_{t_c}^{s_{\mathrm{had}}} \mathrm{d}\Phi_1 \frac{\mathrm{R}(\Phi_R)}{\mathrm{B}(\Phi_B)} \Delta^{(\mathrm{R})}(t(\Phi_1), s_{\mathrm{had}})\, O(\Phi_R)\Bigg]$$

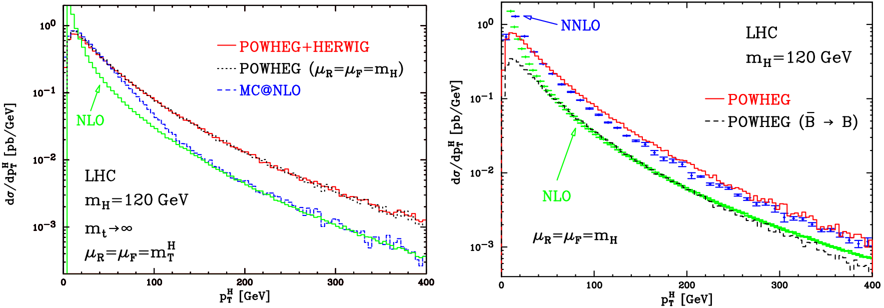
- μ_Q^2 has changed to hadronic centre-of-mass energy squared, s_{had}, as full phase space for real-emission correction, R, must be covered
- Absence of $\mathbb{H}$-events leads to enhancement of high-p_T region by

  $$K = \frac{\bar{\mathrm{B}}}{\mathrm{B}} = 1 + \mathcal{O}(\alpha_s)$$

  Formally beyond NLO, but sizeable corrections in practice

POWHEG features

[Alioli,Nason,Oleari,Re] arXiv:0812.0578



- Large enhancement at high $p_{T,h}$
- Can be traced back to large NLO correction
- Fortunately, NNLO correction is also large $\to \sim$ agreement

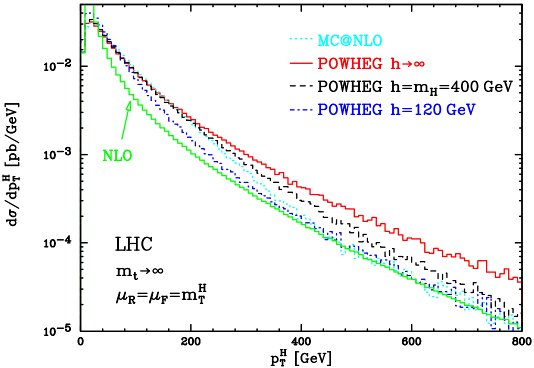
Improved POWHEG

- To avoid problems in high-p_T region, split real-emission ME into singular and finite parts as $\mathrm{R} = \mathrm{R}^s + \mathrm{R}^f$
- Treat singular piece in $\mathbb{S}$-events and finite piece in $\mathbb{H}$-events Similar to MC@NLO with redefined PS evolution kernels
- Differential event rate up to first emission

$$\langle O \rangle = \int \mathrm{d}\Phi_B \bar{\mathrm{B}}^{(\mathrm{R^s})}(\Phi_B) \bigg[\Delta^{(\mathrm{R^s})}(t_c, s_{\mathrm{had}})\, O(\Phi_B) + \int_{t_c}^{s_{\mathrm{had}}} \mathrm{d}\Phi_1 \frac{\mathrm{R}^s(\Phi_R)}{\mathrm{B}(\Phi_B)} \Delta^{(\mathrm{R^s})}(t(\Phi_1), s_{\mathrm{had}})\, O(\Phi_R) \bigg] + \int \mathrm{d}\Phi_R\, \mathrm{R}_n^f(\Phi_R)$$

POWHEG features

[Alioli,Nason,Oleari,Re] arXiv:0812.0578



- Singular real-emission part here defined as

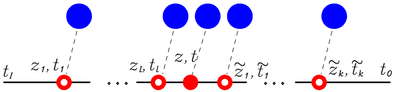
$$\mathrm{R}^s = \mathrm{R}\,\frac{h^2}{p_T^2 + h^2}$$

- Can "tune" NNLO contribution by varying free parameter h

Truncated parton showers

- Usually want to match hardest emission to matrix elements
- Thus need to identify hardness with evolution parameter
- What if evolution parameter of parton shower differs?

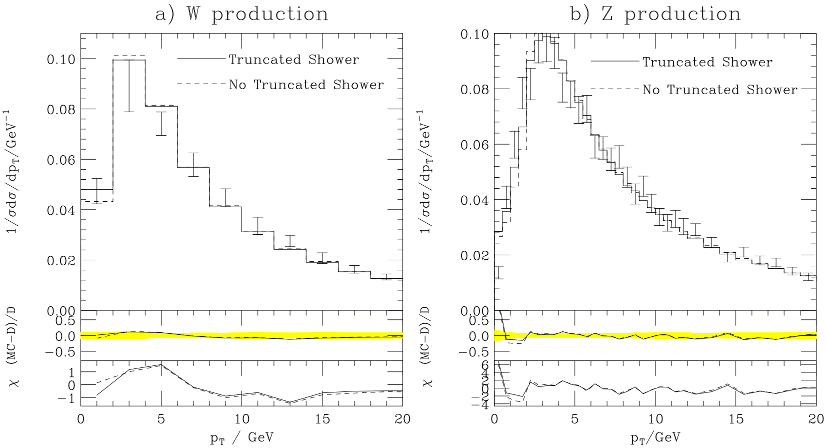
[Nason] hep-ph/0409146



- Must implement additional emissions to account for missing Sudakov form factor "above" predefined branching
- Must also veto on emissions harder than matched branching
- Truncated, vetoed parton shower

Effect of truncated showers on matching

[Richardson, Tully] arXiv:0806.0290



- Small effect, but also insensitive observable
- In general effects not small, see next lecture

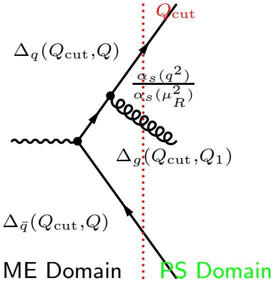
Review: CKKW merging

[Catani,Krauss,Kuhn,Webber] hep-ph/0109231
[Krauss] hep-ph/0205283

- Select ME according to $\sigma(> Q_{\rm cut})$
- Construct PS history
- Weight each vertex with $\alpha_s(q^2)/\alpha_s(\mu_R^2)$
- Weight parton of type i from Q_j to Q_k by

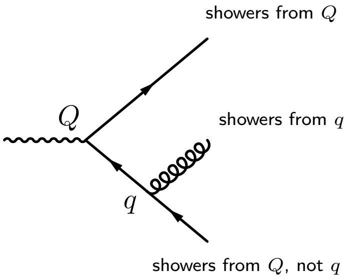
  $\Delta_i(Q_{\rm cut}, Q_j)/\Delta_i(Q_{\rm cut}, Q_k)$

  $\rightarrow$ Sudakov suppression to NLL accuracy
- Veto parton shower if emission has $Q > Q_{\rm cut}$



Problems with CKKW

- Convenient choices made
  - Clustering with Durham k_T-algorithm
  - Analytic Sudakov form factors
  - No truncated showers
    Instead redefined starting scales
- Correct to NLL, but
  exact correspondence with PS is lost
- Problems due to modified colour flow
  and kinematics



MLM merging

[Mangano,Moretti,Pittau] hep-ph/0108069
[Mangano,Moretti,Piccinini,Treccani] hep-ph/0611129

- Define parton-level jets using cone algorithm

$$R_{ij}^2 = (\eta_i - \eta_j)^2 + (\phi_i - \phi_j)^2$$
$$E_{T,i} > E_{T,\min}, \qquad R_{ij} > R_{\min}$$

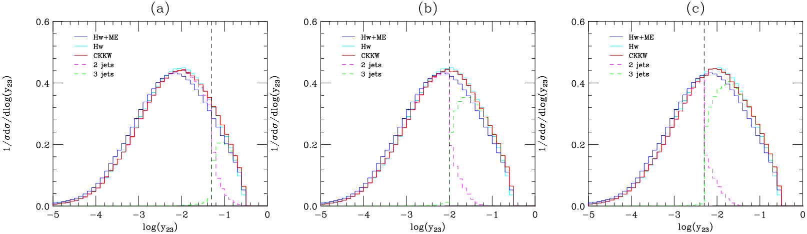
- Generate parton showers from n-jet events
  No Sudakov suppression at this point
- Form new jets on showered final state
- Reject if number of jets increased
  or jets not "matched" to partons in R_{ij}
- Sudakov suppression achieved by jet matching

Effect of truncated showers in $e^+e^- \to$ hadrons

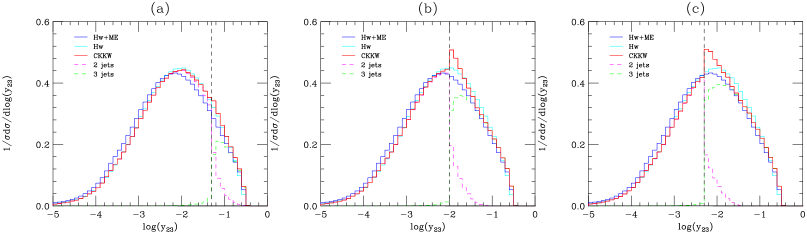
SLAC

[Hamilton, Richardson, Tully] arXiv:0905.3072

truncated shower on

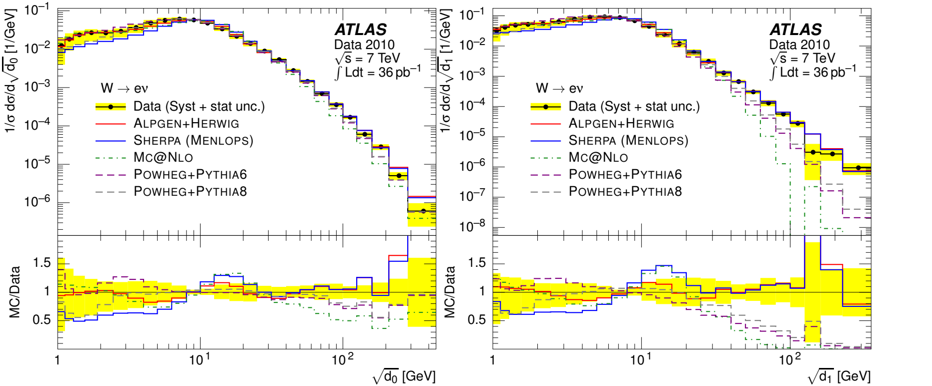


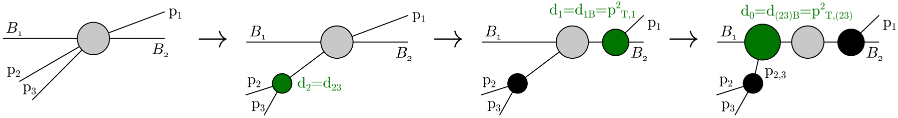
truncated shower off



K_T splitting scales in W+jets

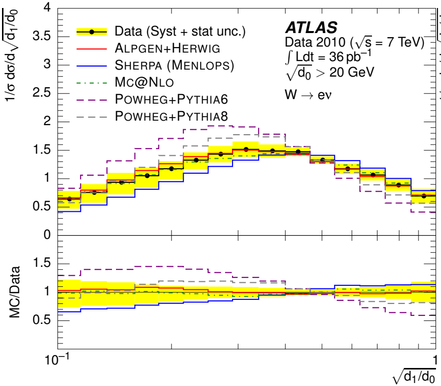
[ATLAS] 1302.1415

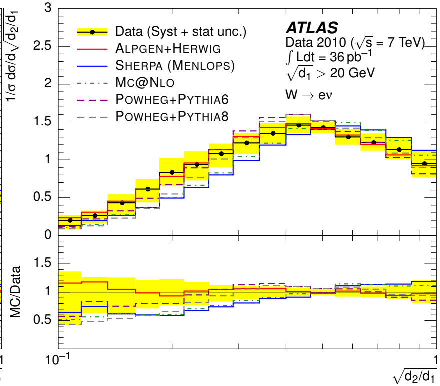


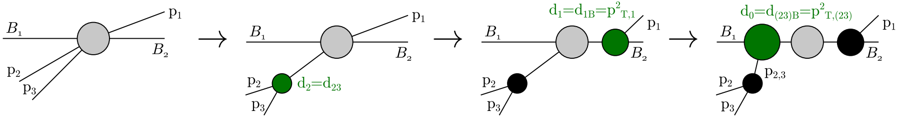


K_T splitting scales in W+jets

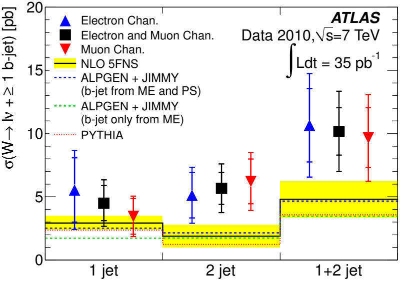
[ATLAS] 1302.1415







[ATLAS] arXiv:1109.1470



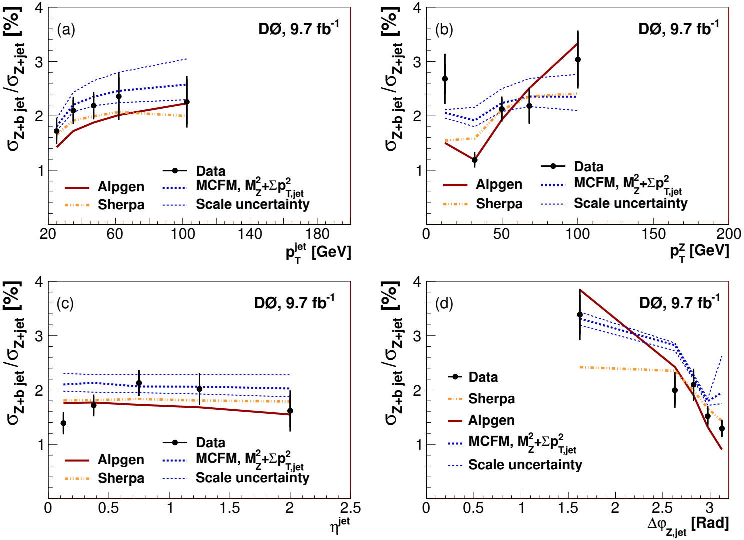
Z+b cross section

[ATLAS] arXiv:1109.1403

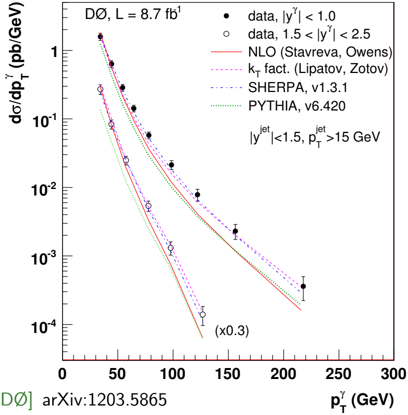
Experiment	$3.55^{+0.82}_{-0.74}(\text{stat})^{+0.73}_{-0.55}(\text{syst}) \pm 0.12(\text{lumi})$ pb
MCFM	3.88 ± 0.58 pb
ALPGEN	2.23 ± 0.01 (stat only) pb
SHERPA	3.29 ± 0.04 (stat only) pb

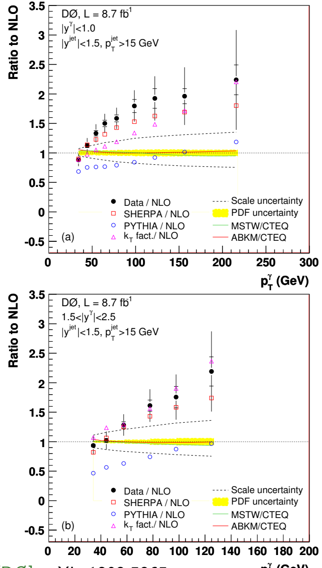
V+heavy flavor

[DØ] arXiv:1301.2233



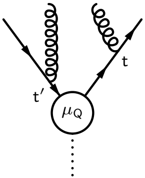
DØ analysis of γ+bjet production





[DØ] arXiv:1203.5865

MEPS@NLO from a different perspective



- Define compound evolution kernel

$$\tilde{\mathrm{D}}_k(\Phi_{k+1}) = \mathrm{D}_k(\Phi_{k+1})\,\Theta(t_k - t_{k+1}) + \mathrm{B}_k(\Phi_k)\sum_{i=n}^{k-1}\mathrm{K}_i(\Phi_i)\,\Theta(t_i - t_{k+1})\Theta(t_{k+1} - t_{i+1})$$

- Extend MC@NLO modified subtraction

$$\tilde{\mathrm{B}}_k^{(\mathrm{D})}(\Phi_k) = \Big[\mathrm{B}_k(\Phi_k) + \tilde{\mathrm{V}}_k(\Phi_k) + \mathrm{I}_k(\Phi_k)\Big] + \int \mathrm{d}\Phi_1\Big[\tilde{\mathrm{D}}_k(\Phi_{k+1}) - \mathrm{S}_k(\Phi_{k+1})\Big]$$

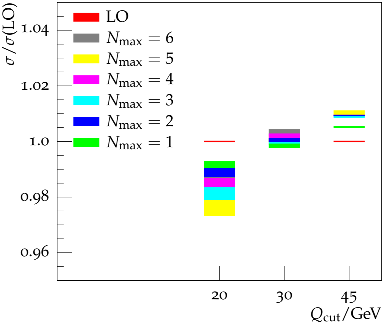
$$\tilde{\mathrm{H}}_k^{(\mathrm{D})}(\Phi_{k+1}) = \mathrm{R}_k(\Phi_{k+1}) - \tilde{\mathrm{D}}_k(\Phi_{k+1})$$

- Differential event rate for exclusive $n+k$-jet events

$$
\begin{aligned}
\langle O\rangle_k^{\rm excl} \;=&\; \int {\rm d}\Phi_k\, \tilde{\rm B}_k^{\rm (D)}\, \Theta(Q_k - Q_{\rm cut}) \\
&\times \left[\tilde\Delta_k^{\rm (D)}(t_c,\mu_Q^2)\, O_k + \int_{t_c}^{\mu_Q^2} {\rm d}\Phi_1\, \frac{\tilde{\rm D}_k^{\rm (D)}}{{\rm B}_k}\, \tilde\Delta_k^{\rm (D)}(t,\mu_Q^2)\, \Theta(Q_{\rm cut} - Q_{k+1})\, O_{k+1} \right] \\
&\quad + \int {\rm d}\Phi_{k+1}\, \tilde{\rm H}_k^{\rm (D)}\, \tilde\Delta_k^{\rm (K)}(t_{k+1},\mu_Q^2; > Q_{\rm cut})\, \Theta(Q_{\rm cut} - Q_{k+1})
\end{aligned}
$$

- Structurally equivalent to MENLOPS!
- Truncated PS contributes at $\mathcal{O}(\alpha_s)$

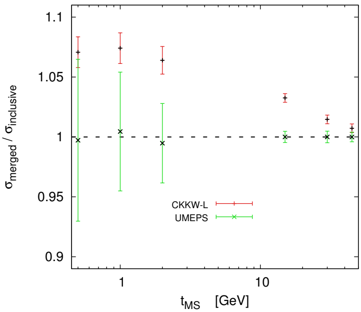
Effects of merging - Z+jets at the Tevatron



- MEPS effectively replaces splitting kernels of the parton shower with ratios of LO matrix elements for the emission terms
- We have not corrected the Sudakov form factors, hence there is a mismatch between emission- and no-emission probability
- The inclusive cross section changes, but corrections are small

Unitarized ME+PS merging

[Lönnblad,Prestel] JHEP02(2013)094



- Unitarity condition of PS:

$$1 = \Delta^{(K)}(t_c) + \int_{t_c} d\Phi_1 \, K(\Phi_1) \, \Delta^{(K)}(t)$$

- CKKW-like merging violates PS unitarity as ME ratio replaces splitting kernels in emission terms, but not in Sudakovs

$$K(\Phi_1) \to \frac{R(\Phi_1, \Phi_B)}{B(\Phi_B)}$$

- Mismatch removed by explicit subtraction

$$1 = \underbrace{\left\{ \Delta^{(K)}(t_c) + \int_{t_c} d\Phi_1 \left[K(\Phi_1) - \frac{R(\Phi_1, \Phi_B)}{B(\Phi_B)} \right] \Theta(Q - Q_{cut}) \, \Delta^{(K)}(t) \right\}}_{\text{unresolved emission / virtual correction}}$$

$$+ \underbrace{\int_{t_c} d\Phi_1 \left[K(\Phi_1)\Theta(Q_{cut} - Q) + \frac{R(\Phi_1, \Phi_B)}{B(\Phi_B)} \Theta(Q - Q_{cut}) \right] \Delta^{(K)}(t)}_{\text{resolved emission}}$$

Lessons from DIS @ HERA

[Carli,Gehrmann,SH] arXiv:0912.3715

Simulation often too focused on resonant contributions

Need be inclusive to describe DIS, low-mass Drell-Yan or photon / diphoton production