Neutrino Physics

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(IHEP, Beijing)

E. Witten (2000): for neutrino masses, the considerations have always been qualitative, and, despite some interesting attempts, there has never been a convincing quantitative model of the neutrino masses.

Part A: Neutrinos: from SM to NP

Part B: Origin of neutrino masses

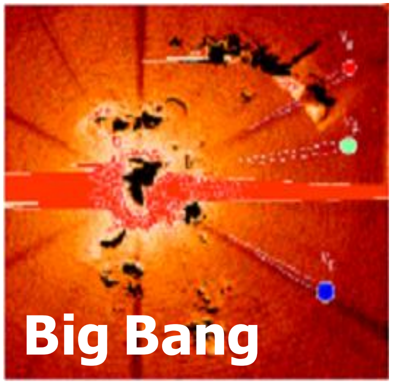
Part C: Flavor mixing and behind

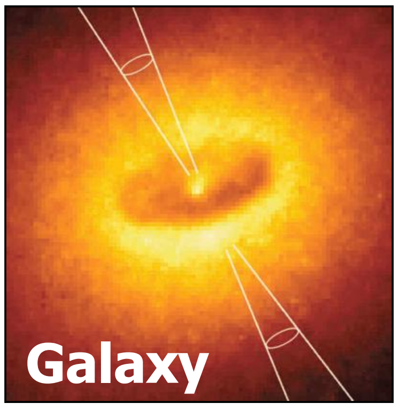
Part D: Summary and an outlook

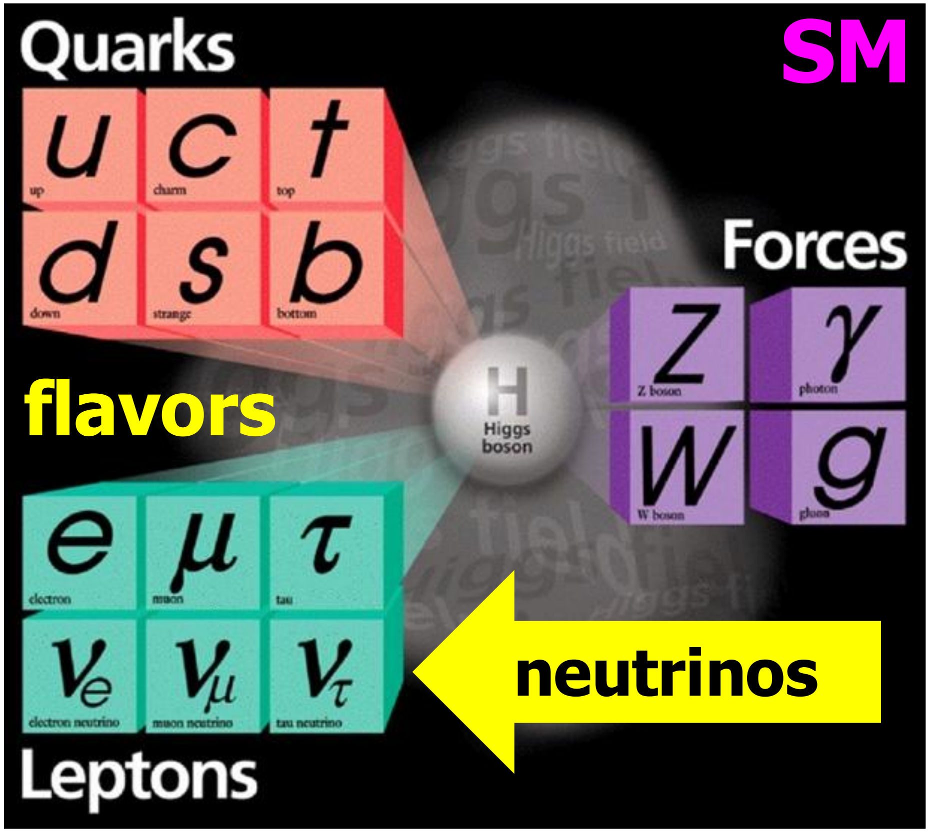
ν ν ν

@Lecture at 3rd iSTEP Summer School, Tsinghua University, 11/7/16

Neutrinos: sooooooo special?







Properties:

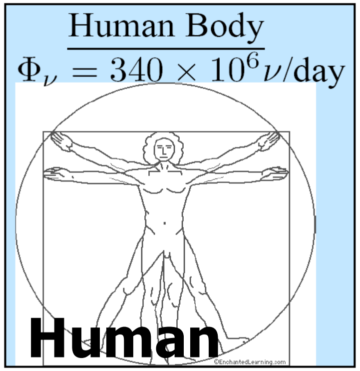
charge = 0

spin = ½

mass = 0

speed = c

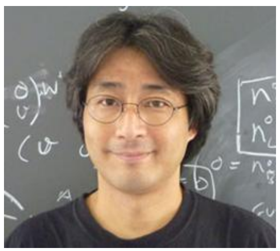
Left-handed



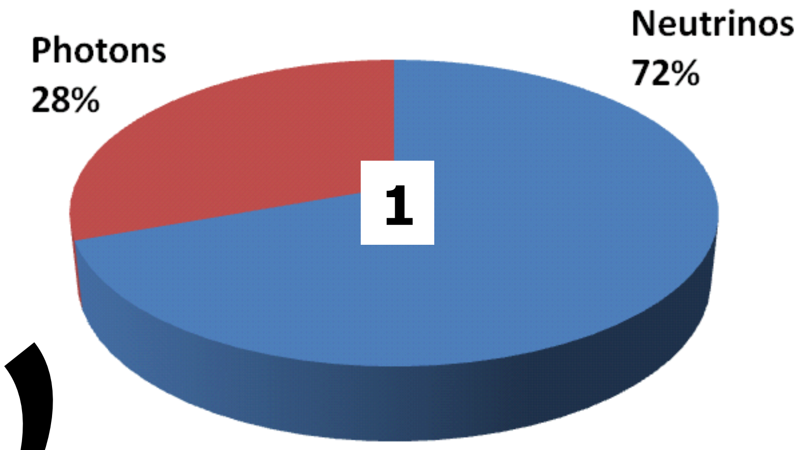
Neutrinos:

witness and participant in evolution of the Universe!

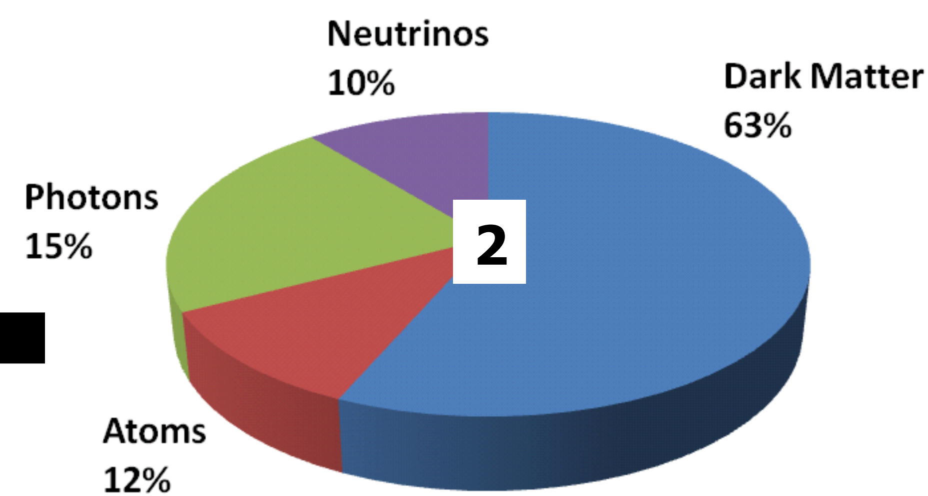
H. Murayama:
neutrinos may be our mother



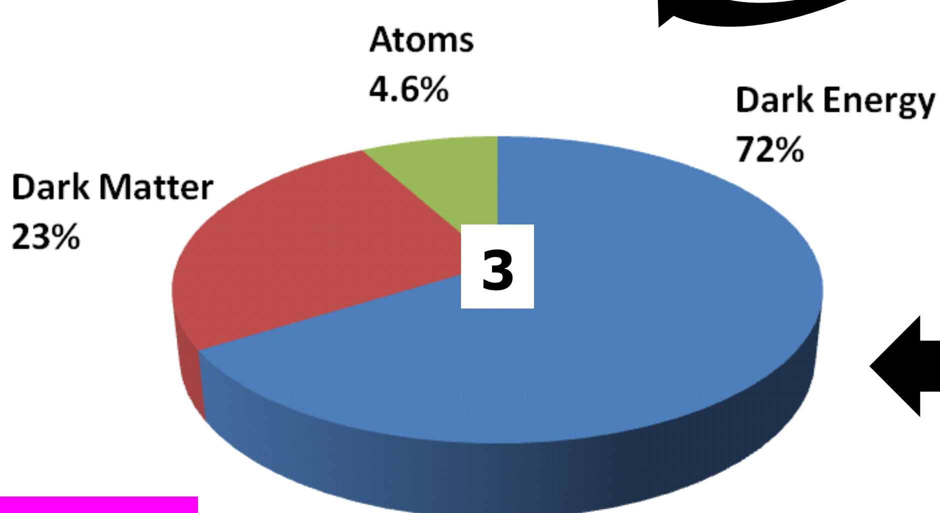
ν ν ν



neutrino decoupling
t = 1 second



photon decoupling
t = 380 000 years



≤ 1%

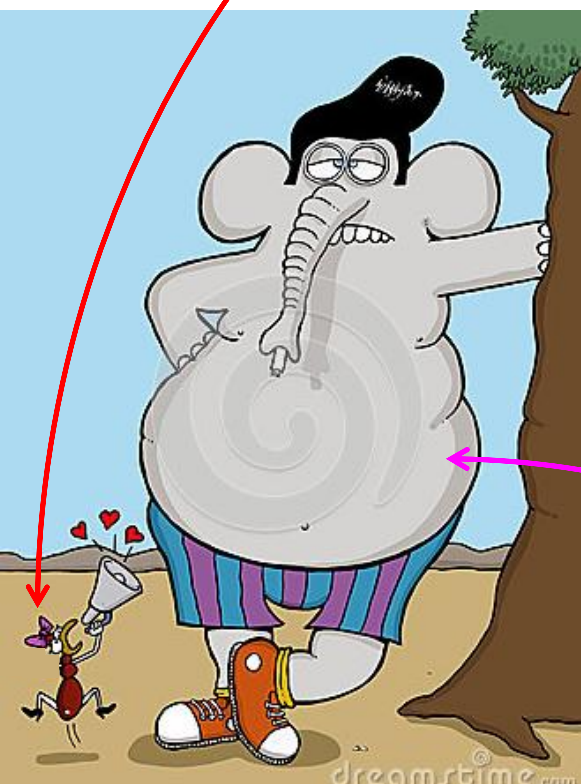
Today
t = 13.7 billion years

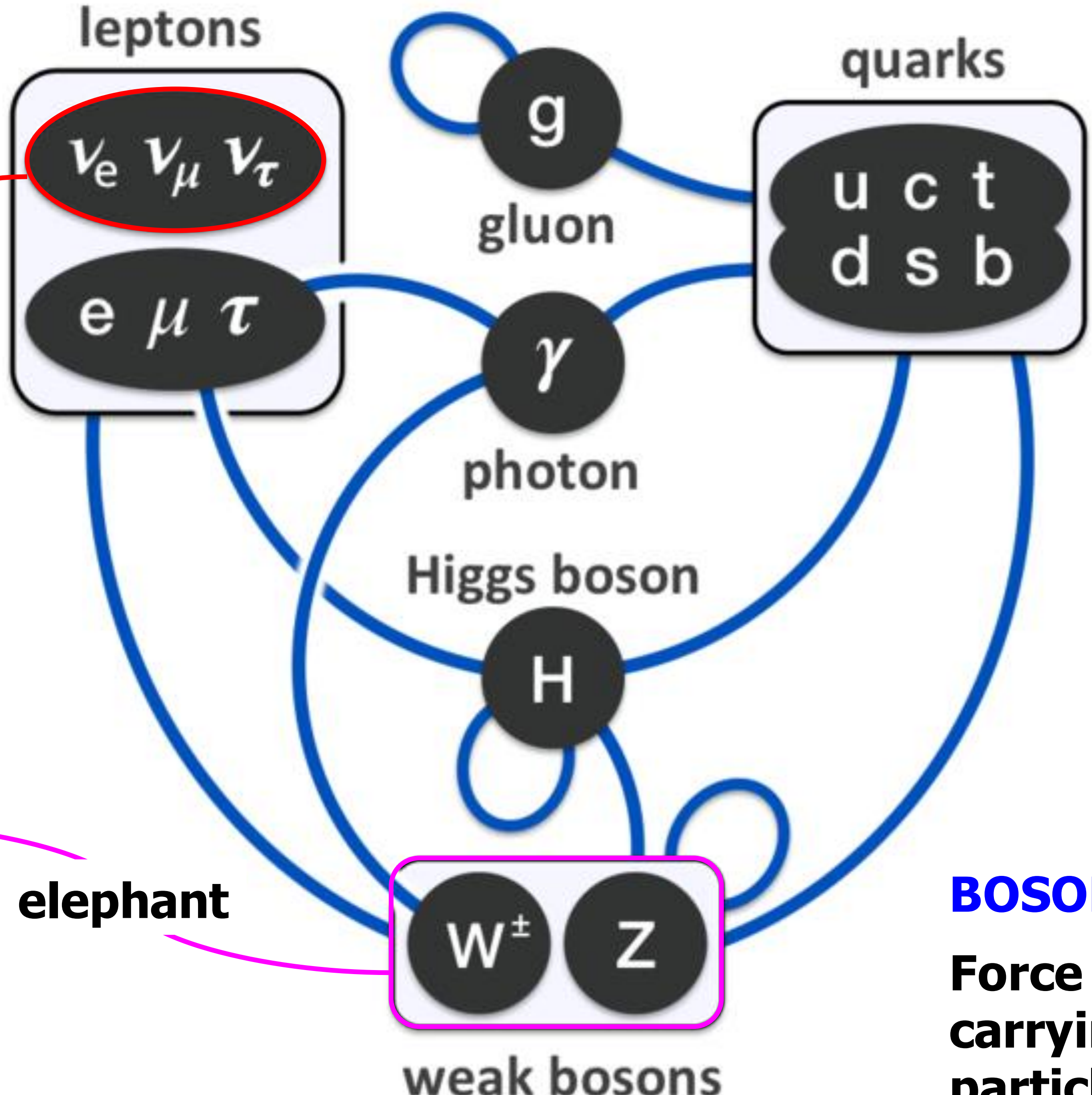
SM particle content

FERMIONS

Matter building block particles

ant





elephant

BOSONS

Force carrying particles

Electroweak Lagrangian

The standard electroweak model's Lagrangian can be written as

$$\boxed{\mathcal{L} = \mathcal{L}_{\mathrm{G}} + \mathcal{L}_{\mathrm{H}} + \mathcal{L}_{\mathrm{F}} + \mathcal{L}_{\mathrm{Y}}}$$

Sheldon Lee Glashow
Prize share: 1/3

Abdus Salam
Prize share: 1/3

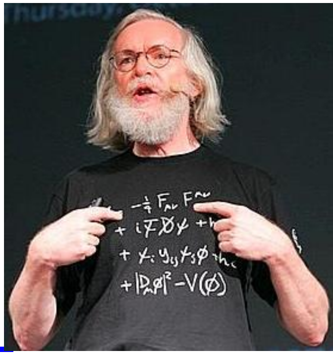
Steven Weinberg
Prize share: 1/3

$$\mathcal{L}_{\mathrm{G}} = -\frac{1}{4}\left(W^{i\mu\nu}W^{i}_{\mu\nu} + B^{\mu\nu}B_{\mu\nu}\right) \ ,$$

$$\mathcal{L}_{\mathrm{H}} = \left(D^{\mu}H\right)^{\dagger}\left(D_{\mu}H\right) - \mu^{2}H^{\dagger}H - \lambda\left(H^{\dagger}H\right)^{2} \ ,$$

$$\mathcal{L}_{\mathrm{F}} = \overline{Q_{\mathrm{L}}}\mathrm{i}\not{D}Q_{\mathrm{L}} + \overline{\ell_{\mathrm{L}}}\mathrm{i}\not{D}\ell_{\mathrm{L}} + \overline{U_{\mathrm{R}}}\mathrm{i}\not{\partial}'U_{\mathrm{R}} + \overline{D_{\mathrm{R}}}\mathrm{i}\not{\partial}'D_{\mathrm{R}} + \overline{E_{\mathrm{R}}}\mathrm{i}\not{\partial}'E_{\mathrm{R}} \ ,$$

$$\mathcal{L}_{\mathrm{Y}} = -\overline{Q_{\mathrm{L}}}Y_{\mathrm{u}}\tilde{H}U_{\mathrm{R}} - \overline{Q_{\mathrm{L}}}Y_{\mathrm{d}}HD_{\mathrm{R}} - \overline{\ell_{\mathrm{L}}}Y_{l}HE_{\mathrm{R}} + \mathrm{h.c.} \ ,$$



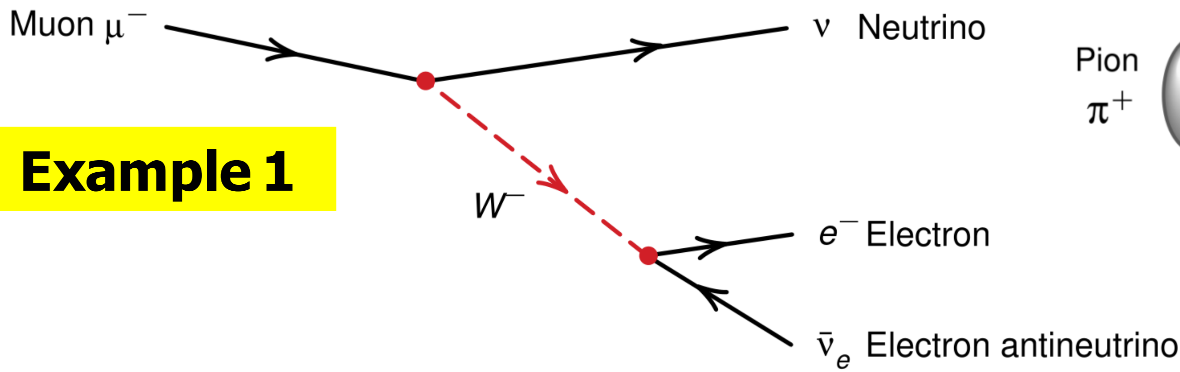
After electroweak symmetry breaking, we are left with weak neutral- and charged-current neutrino interactions:

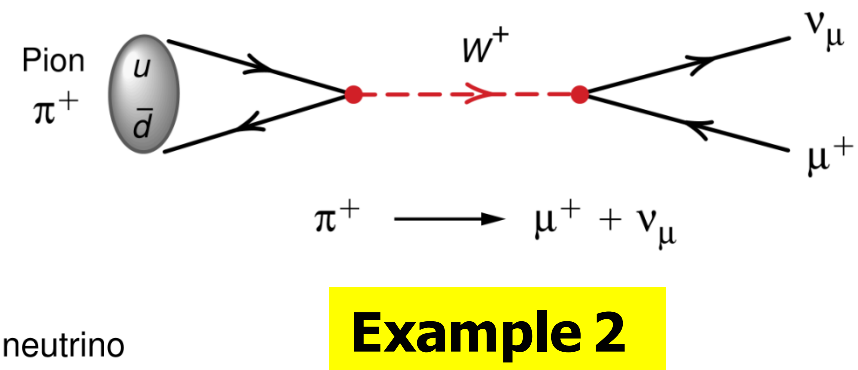
$$-\mathcal{L}_{\mathrm{cc}} = \frac{g}{2\sqrt{2}}\sum_{\alpha}\left[\overline{\alpha}\,\gamma^{\mu}\left(1-\gamma_{5}\right)\nu_{\alpha}W^{-}_{\mu} + \mathrm{h.c.}\right]$$

$$-\mathcal{L}_{\mathrm{nc}} = \frac{g}{4\cos\theta_{\mathrm{w}}}\sum_{\alpha}\left[\overline{\nu_{\alpha}}\,\gamma^{\mu}\left(1-\gamma_{5}\right)\nu_{\alpha}\right]Z_{\mu}$$

Massive neutrinos obey the same NC or CC interactions

Lepton (flavor) number (1)



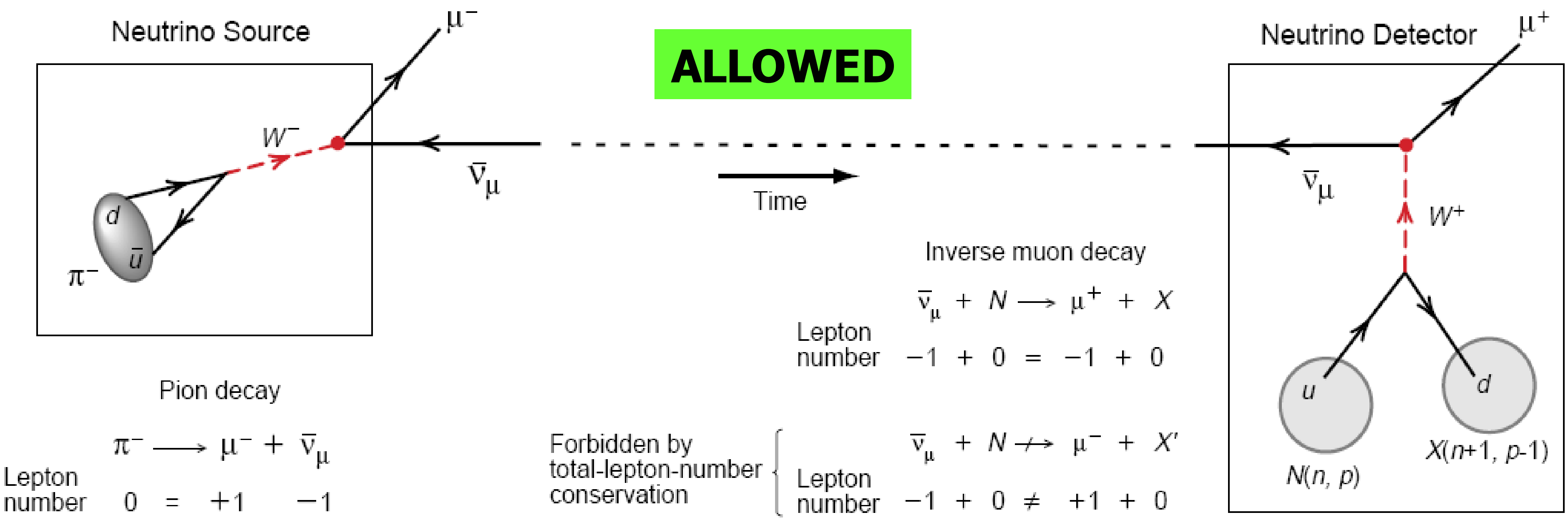


Edward Witten (opening talk at Neutrino 2000) ——"Using the fields of the SM, it is impossible at the classical level to violate the baryon and lepton number symmetries by renormalizable interactions."

	e^-	ν_e	e^+	$\overline{\nu}_e$	μ^-	ν_μ	μ^+	$\overline{\nu}_\mu$	τ^-	ν_τ	τ^+	$\overline{\nu}_\tau$
L	$+1$	$+1$	-1	-1	$+1$	$+1$	-1	-1	$+1$	$+1$	-1	-1
L_e	$+1$	$+1$	-1	-1	0	0	0	0	0	0	0	0
L_μ	0	0	0	0	$+1$	$+1$	-1	-1	0	0	0	0
L_τ	0	0	0	0	0	0	0	0	$+1$	$+1$	-1	-1

E. Witten

Lepton (flavor) number (2)



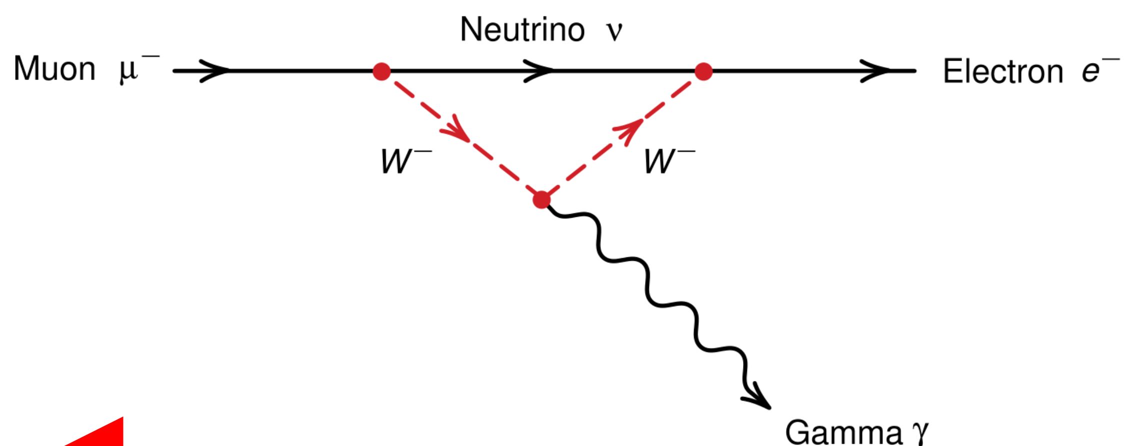
$$\mu^+ \to e^+ + \gamma$$

$$\mu^+ \to e^+ + e^- + e^+$$

$$\mu^- + N(n, p) \to e^- + N(n, p)$$

$$\mu^- + N(n, p) \to e^+ + N(n + 2, p - 2)$$

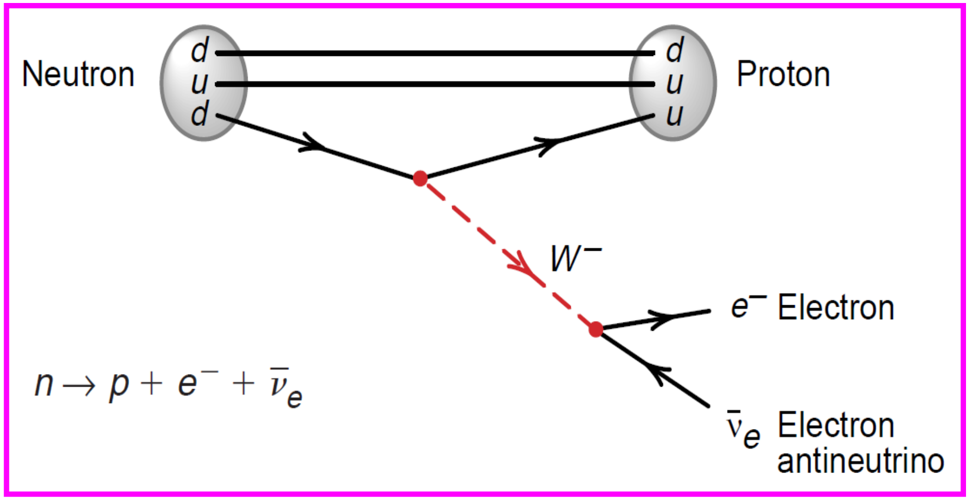
$$\mu^+ \to e^+ + \bar{\nu}_e + \nu_\mu$$



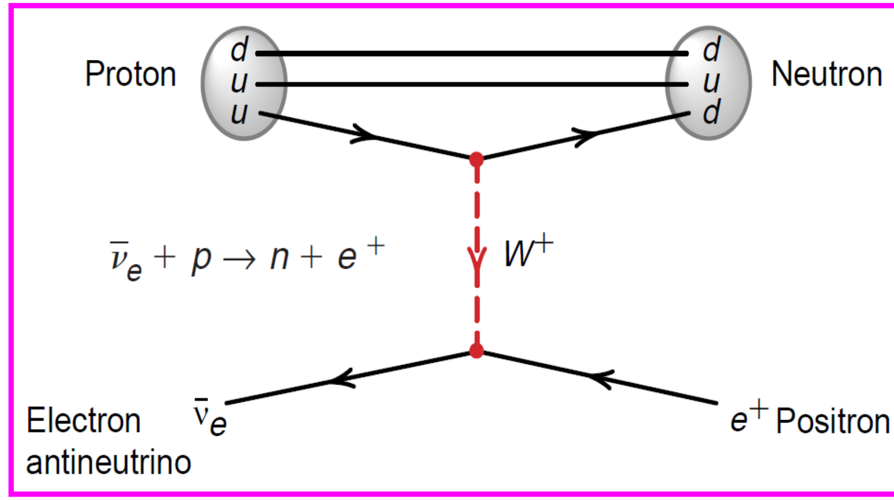
FORBIDDEN

Why the sun shines?

The beta decay

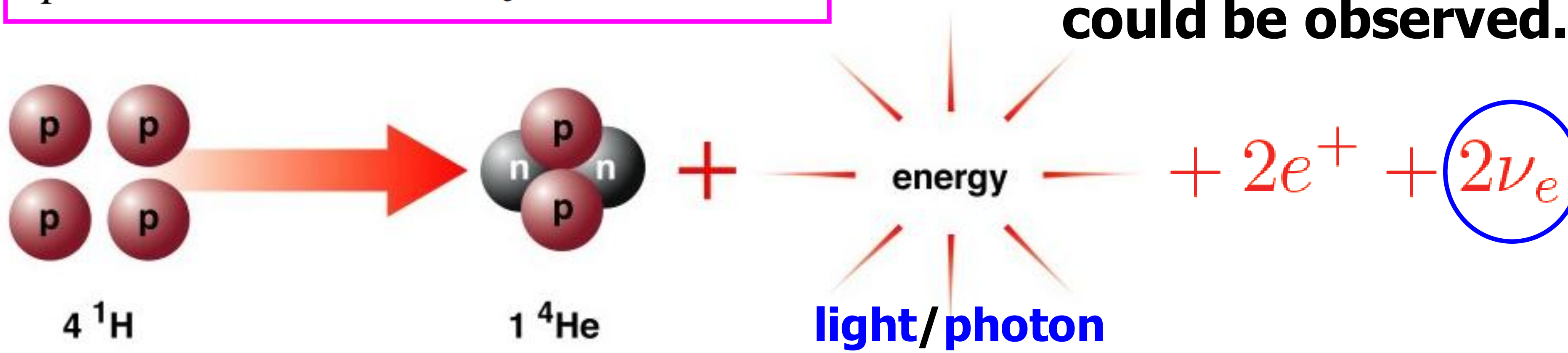


The inverse beta decay



$$4p \rightarrow \, ^4\mathrm{He} + 2e^+ + 2\nu_e + 26.73\ \mathrm{MeV}$$

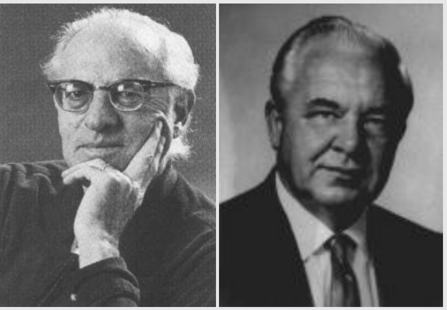
Only the neutrinos could be observed.



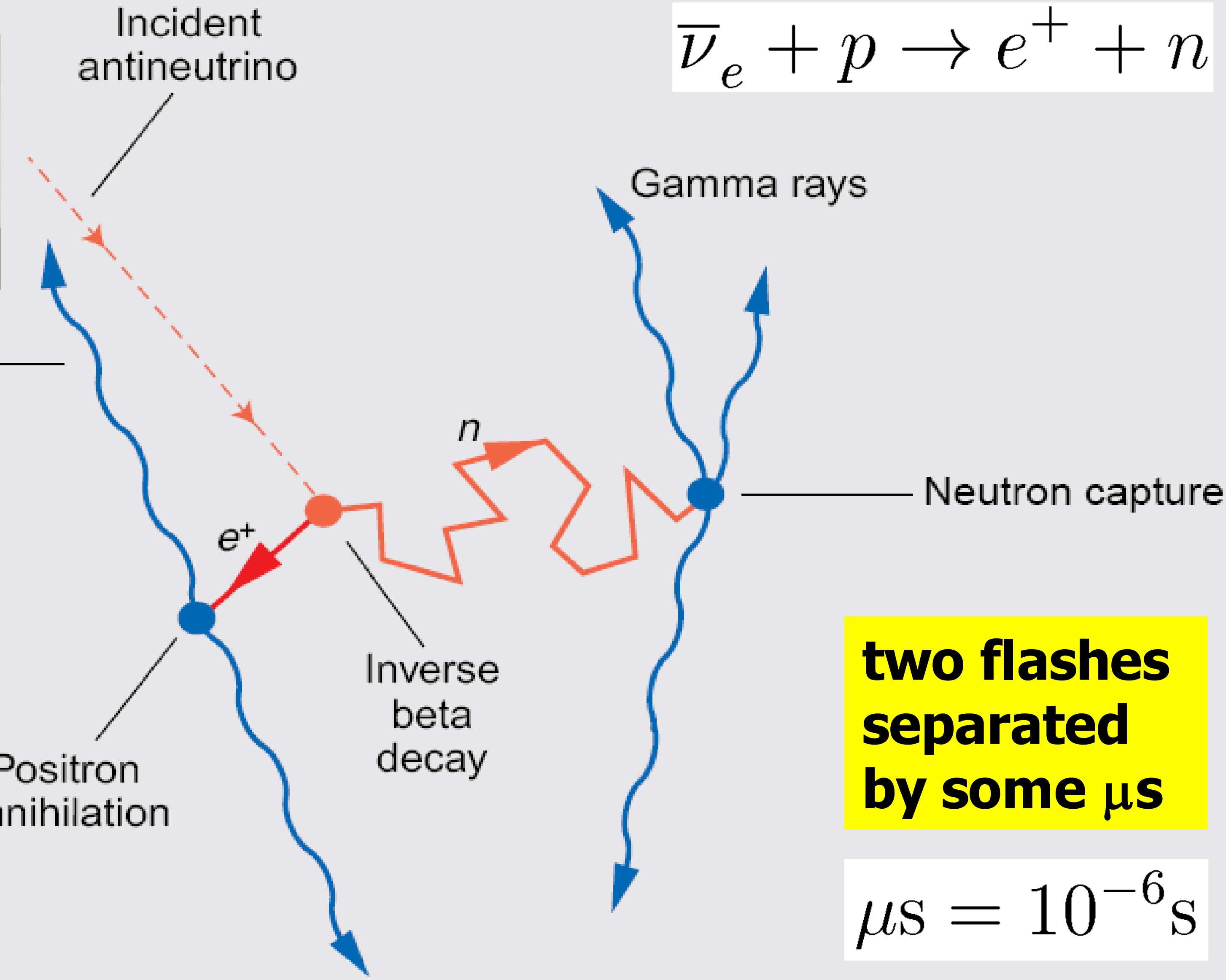
Hans Bethe (1939), George Gamow & Mario Schoenberg (1940, 1941)

First detection of neutrinos

F. Reines & C. Cowan first detected the reactor antineutrinos in 1956:



$$\overline{\nu}_e + p \rightarrow e^+ + n$$



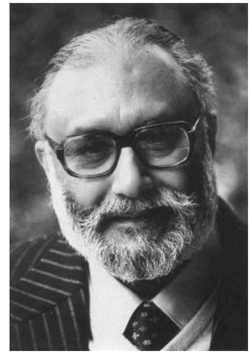
two flashes separated by some μs

$$\mu s = 10^{-6} s$$

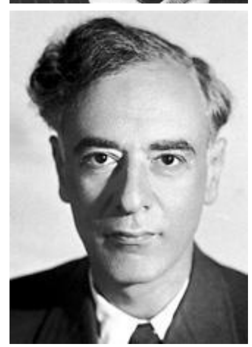
Neutrinos in 1957

The neutrino should have no mass: 2-component ν theory

★ **Abdus Salam**
received 15/11/1956,
Nuovo Cim. 5 (1957) 299



★ **Lev Landau**
received 9/1/1957,
Nucl. Phys. 3 (1957) 127



★ **T.D. Lee, C.N. Yang**
received 10/1/1957,
Phys. Rev. 105 (1957) 1671

John Ward wrote to Salam:
So many congratulations and fond hopes for at least one-third of a Nobel Prize.

——— Norman Bombey in "Abdus Salam: How to Win the Nobel Prize", Preprint arXiv:1109.1972 (9/2011).

Bruno Pontecorvo challenged the massless ν theory in 1957

Pontecorvo's idea

★ Theory of the Symmetry of Electrons and Positrons

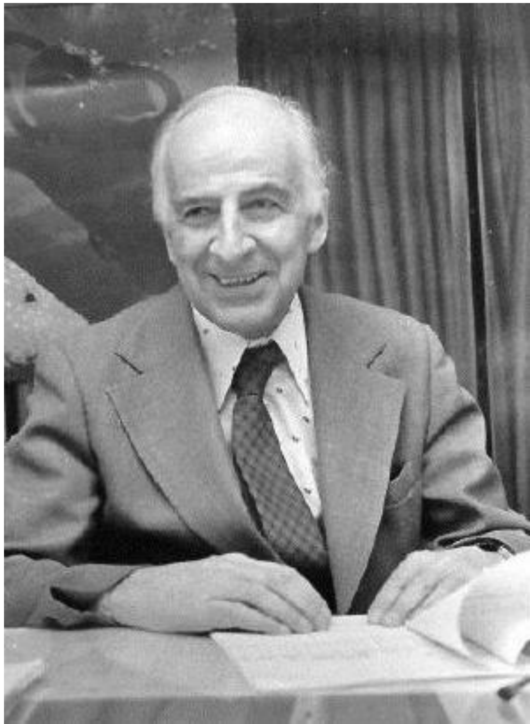
Ettore Majorana

Nuovo Cim. 14 (1937) 171

Are massive neutrinos and antineutrinos identical or different —— a fundamental puzzling question in particle physics.

★ Mesonium and Anti-mesonium

Bruno Pontecorvo



Zh. Eksp. Teor. Fiz. 33 (1957) 549
Sov. Phys. JETP 6 (1957) 429

If the two-component neutrino theory turned out to be incorrect and if the conservation law of neutrino charge didn't apply, then neutrino -antineutrino transitions would in principle be possible to take place in vacuum.

Original idea of ν-mixing

Progress of Theoretical Physics, Vol. 28, No. 5, November 1962

The paper on μ-neutrino discovery was received by PRL on 15/6/1962

Remarks on the Unified Model of Elementary Particles

Ziro MAKI, Masami NAKAGAWA and Shoichi SAKATA

$$\boxed{P(\nu_\alpha \to \nu_\alpha) = 1 - \sin^2 2\theta \sin^2\left(1.27\frac{\Delta m^2 L}{E}\right)}$$

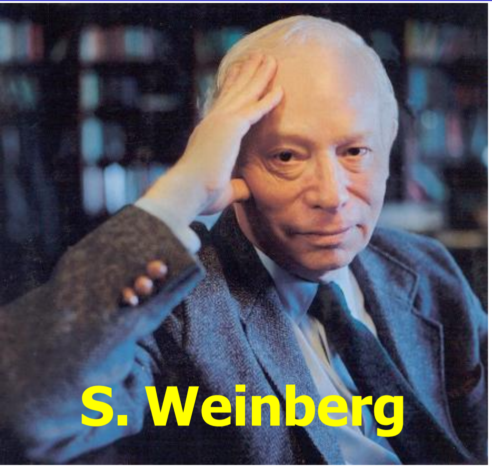
(Received June 25, 1962)

A particle mixture theory of neutrino is proposed assuming the existence of two kinds of neutrinos. Based on the neutrino-mixture theory, a possible unified model of elementary particles is constructed by generalizing the Sakata-Nagoya model.*) Our scheme gives a

$$\boxed{\begin{aligned}\nu_e &= \nu_1 \cos\delta - \nu_2 \sin\delta,\\ \nu_\mu &= \nu_1 \sin\delta + \nu_2 \cos\delta.\end{aligned}}$$

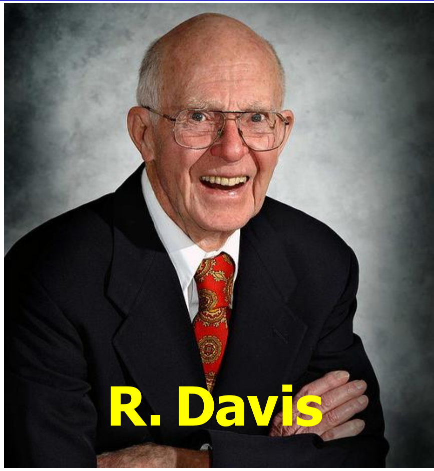
Bruno Pontecorvo formulated neutrino oscillation in 1968.

Neutrino oscillations?



Massless neutrinos: a natural assumption when the SM was formulated in **1967**, while the **solar ν anomaly** was observed in **1968**, just one year later!

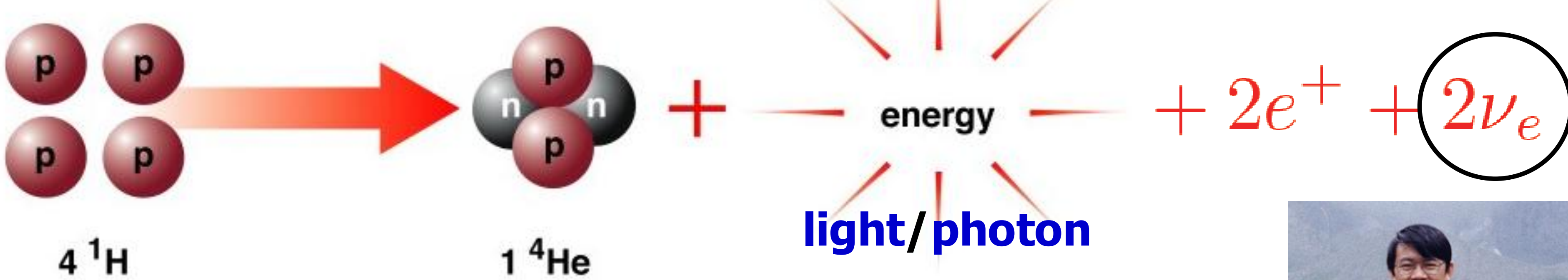
neutrino oscillation ↔ masses

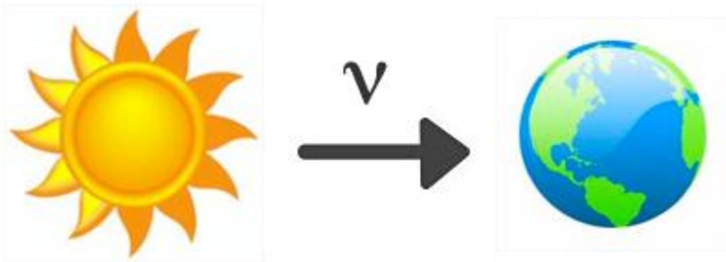


$$\boxed{4p \rightarrow {}^4\mathrm{He} + 2\mathrm{e}^+ + 2\nu_e + 26.73\ \mathrm{MeV}}$$

你相信吗？

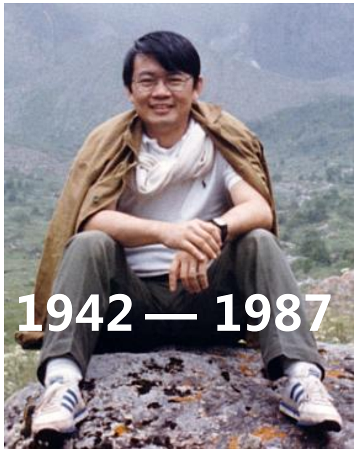
neutrino





陈华森 (1985)
重水探测器

$$\begin{aligned}
\phi_{CC} &= \phi(\nu_e) \\
\phi_{ES} &= \phi(\nu_e) + 0.1559\phi(\nu_{\mu\tau}) \\
\phi_{NC} &= \phi(\nu_e) + \phi(\nu_{\mu\tau})
\end{aligned}$$



Oscillations in 1998

Y. Suzuki
June 4

Solar ν's

Atmospheric ν's

T. Kajita
June 5

"Modest" Conclusions

(1) Flux: $\phi^{8B} = 2.44 \pm 0.05(\text{stat.})^{+0.09}_{-0.07}(\text{syst.}) \times 10^6/\text{cm}^2/\text{s}$
(0.368 for BP95, 0.474 for BP98)

(2) No seasonal variations.

(3) $(D-N)/(D+N) = -0.023 \pm 0.020(\text{stat}) \pm 0.014(\text{syst})$
no difference:
excluded regions
extended into "small angle sol
No core enhancement found.

(4) Day-Night + E-shape analysis.
(a) "No oscillation" is disfavoured @ 1~5% C.L.
(b) L.A. solution is disfavoured @ 1~5% C.L.
(c) V.O. regions are favoured (than MSW regions) @ 95% C.L.
(MSW is ok for 99% C.L.)

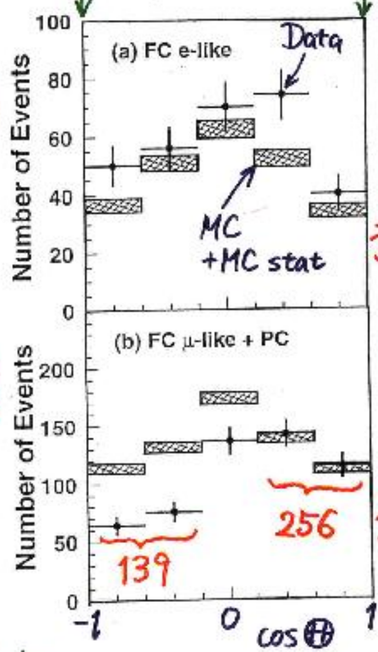
Super-K

Neutrino98
TAKAYAMA

Yes, two large mixing angles!
H. Fritzsch and Z.Z. Xing 1996:

$$\theta_{12} \simeq 42^\circ$$
$$\theta_{13} \simeq 4^\circ$$
$$\theta_{23} \simeq 52^\circ$$
$$\delta \simeq \pm 90^\circ$$

Zenith angle dependence
(Multi-GeV)



e
$\chi^2(\text{shape}) = 2.8/4\,\text{dof}$
$\frac{\text{Up}}{\text{Down}} = 0.93^{+0.13}_{-0.12}$

μ
$\chi^2(\text{shape}) = 30/4\,\text{dof}$
$\frac{\text{Up}}{\text{Down}} = 0.54^{+0.06}_{-0.05}$
(6.2σ!!)

※ Up/Down syst. error for μ-like

Prediction (flux calculation ...... ≲ 1%
1km rock above SK .... 1.5%) 1.8%

Data (Energy calib. for ↑↓ 0.7%
Non ν Background ..... < 2%) 2.1%

Bill Clinton's comments

REMARKS BY THE PRESIDENT AT MIT 1998 COMMENCEMENT
June 5, 1998

Just yesterday in Japan, physicists announced a discovery that tiny neutrinos have mass. Now, that may not mean much to most Americans, but it may change our most fundamental theories -- from the nature of the smallest subatomic particles to how the universe itself works, and indeed how it expands.

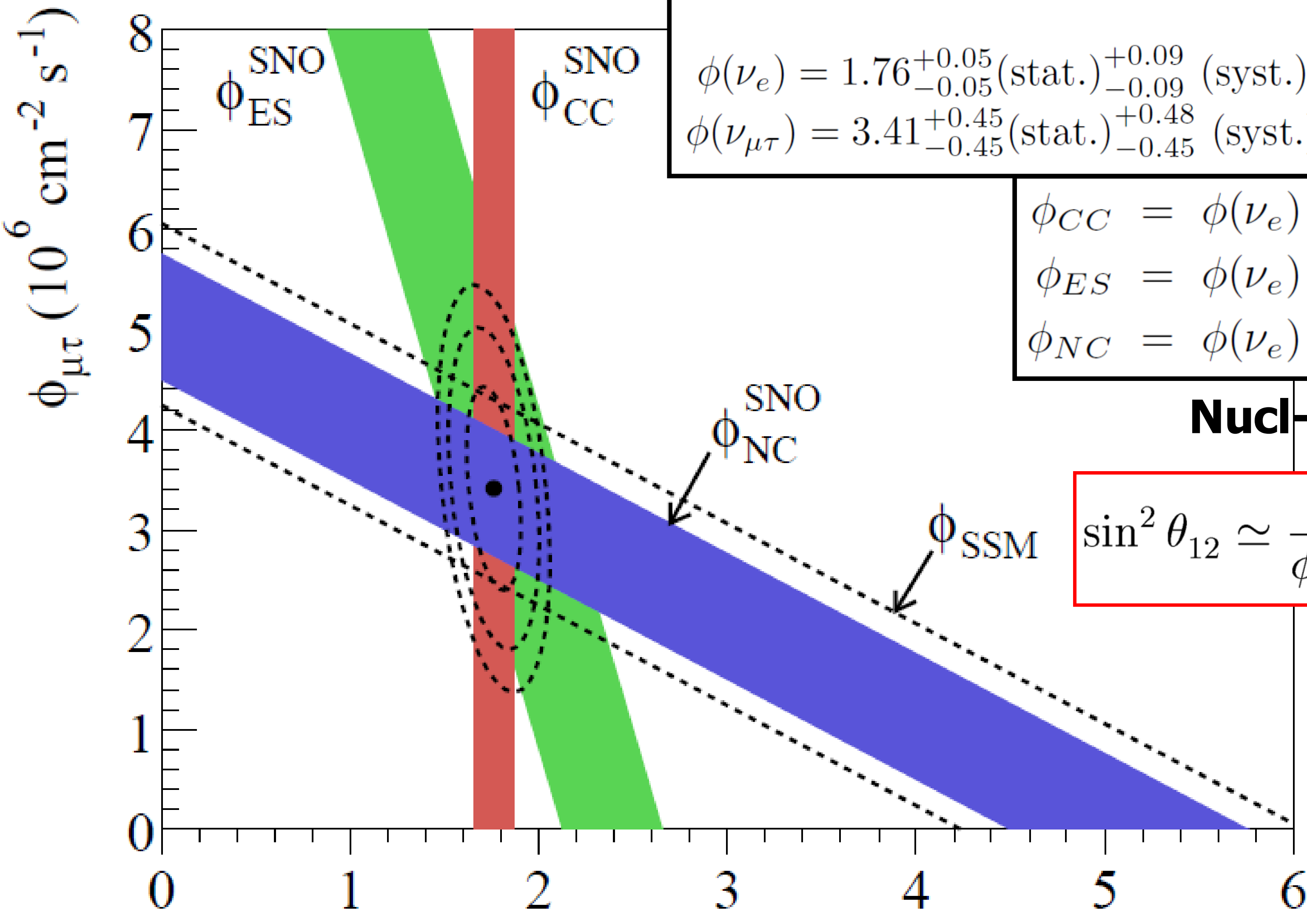
This discovery was made, in Japan, yes, but it had the support of the investment of the U.S. Department of Energy. This discovery calls into question the decision made in Washington a couple of years ago to disband the super-conducting supercollider, and it reaffirms the importance of the work now being done at the Fermi National Acceleration Facility in Illinois.

The larger issue is that these kinds of findings have implications that are not limited to the laboratory. They affect the whole of society --- not only our economy, but our very view of life, our understanding of our relations with others, and our place in time….

SNO in 2001

$$\phi_{CC} = 1.76^{+0.06}_{-0.05}(\text{stat.})^{+0.09}_{-0.09}\ (\text{syst.}) \times 10^6\ \text{cm}^{-2}\ \text{s}^{-1}$$
$$\phi_{ES} = 2.39^{+0.24}_{-0.23}(\text{stat.})^{+0.12}_{-0.12}\ (\text{syst.}) \times 10^6\ \text{cm}^{-2}\ \text{s}^{-1}$$
$$\phi_{NC} = 5.09^{+0.44}_{-0.43}(\text{stat.})^{+0.46}_{-0.43}\ (\text{syst.}) \times 10^6\ \text{cm}^{-2}\ \text{s}^{-1}$$

$$\phi(\nu_e) = 1.76^{+0.05}_{-0.05}(\text{stat.})^{+0.09}_{-0.09}\ (\text{syst.}) \times 10^6\ \text{cm}^{-2}\ \text{s}^{-1}$$
$$\phi(\nu_{\mu\tau}) = 3.41^{+0.45}_{-0.45}(\text{stat.})^{+0.48}_{-0.45}\ (\text{syst.}) \times 10^6\ \text{cm}^{-2}\ \text{s}^{-1}$$

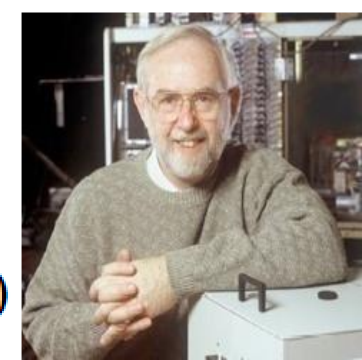


$$\begin{aligned} \phi_{CC} &= \phi(\nu_e) \\ \phi_{ES} &= \phi(\nu_e) + 0.1559\phi(\nu_{\mu\tau}) \\ \phi_{NC} &= \phi(\nu_e) + \phi(\nu_{\mu\tau}) \end{aligned}$$

Nucl-ex/0610020

$$\boxed{\sin^2\theta_{12} \simeq \frac{\phi_e}{\phi_e + \phi_{\mu\tau}} \simeq 34\%}$$

A. McDonald



Solar electron neutrinos convert to muon and tau neutrinos!

NP2015 + BP2016

2016 Breakthrough Prize in Fundamental Physics (3MUS$)


Kam-Biu Luk and the Daya Bay Collaboration


Yifang Wang and the Daya Bay Collaboration


Koichiro Nishikawa and the K2K and T2K Collaboration


Atsuto Suzuki and the KamLAND Collaboration


Arthur B. McDonald and the SNO Collaboration


Takaaki Kajita and the Super K Collaboration


Yoichiro Suzuki and the Super K Collaboration

- **Kam-Biu Luk**: the Daya Bay Collaboration
- **Yifang Wang**: the Daya Bay Collaboration
- **Koichiro Nishikawa**: the K2K/T2K Collaboration
- **Atsuto Suzuki**: the KamLAND Collaboration
- **Arthur B. McDonald**: the SNO Collaboration
- **Takaaki Kajita**: the Super K Collaboration
- **Yoichiro Suzuki**: the Super K Collaboration

2015 Nobel Prize in Physics

Global fit of current data

F. Capozzi et al (2014) —— the standard parametrization:

Parameter	Best fit	1σ range	2σ range	3σ range
Normal neutrino mass ordering ($m_1 < m_2 < m_3$)				
$\Delta m^2_{21}/10^{-5}\ \mathrm{eV}^2$	7.54	7.32 — 7.80	7.15 — 8.00	6.99 — 8.18
$\Delta m^2_{31}/10^{-3}\ \mathrm{eV}^2$	2.47	2.41 — 2.53	2.34 — 2.59	2.26 — 2.65
$\sin^2\theta_{12}/10^{-1}$	3.08	2.91 — 3.25	2.75 — 3.42	2.59 — 3.59
$\sin^2\theta_{13}/10^{-2}$	2.34	2.15 — 2.54	1.95 — 2.74	1.76 — 2.95
$\sin^2\theta_{23}/10^{-1}$	4.37	4.14 — 4.70	3.93 — 5.52	3.74 — 6.26
$\delta/180^\circ$	1.39	1.12 — 1.77	0.00 — 0.16 $\oplus$ 0.86 — 2.00	0.00 — 2.00
Inverted neutrino mass ordering ($m_3 < m_1 < m_2$)				
$\Delta m^2_{21}/10^{-5}\ \mathrm{eV}^2$	7.54	7.32 — 7.80	7.15 — 8.00	6.99 — 8.18
$\Delta m^2_{13}/10^{-3}\ \mathrm{eV}^2$	2.42	2.36 — 2.48	2.29 — 2.54	2.22 — 2.60
$\sin^2\theta_{12}/10^{-1}$	3.08	2.91 — 3.25	2.75 — 3.42	2.59 — 3.59
$\sin^2\theta_{13}/10^{-2}$	2.40	2.18 — 2.59	1.98 — 2.79	1.78 — 2.98
$\sin^2\theta_{23}/10^{-1}$	4.55	4.24 — 5.94	4.00 — 6.20	3.80 — 6.41
$\delta/180^\circ$	1.31	0.98 — 1.60	0.00 — 0.02 $\oplus$ 0.70 — 2.00	0.00 — 2.00

The PMNS matrix looks so different from the CKM matrix!

Flavor puzzles (1)

$$-\mathcal{L}_{cc} = \frac{g}{\sqrt{2}} \left[\overline{(u \;\; c \;\; t)_{\rm L}} \, \gamma^\mu \, V \begin{pmatrix} d \\ s \\ b \end{pmatrix}_{\rm L} W_\mu^+ + \overline{(e \;\; \mu \;\; \tau)_{\rm L}} \, \gamma^\mu \, U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_{\rm L} W_\mu^- \right] + \text{h.c.}$$

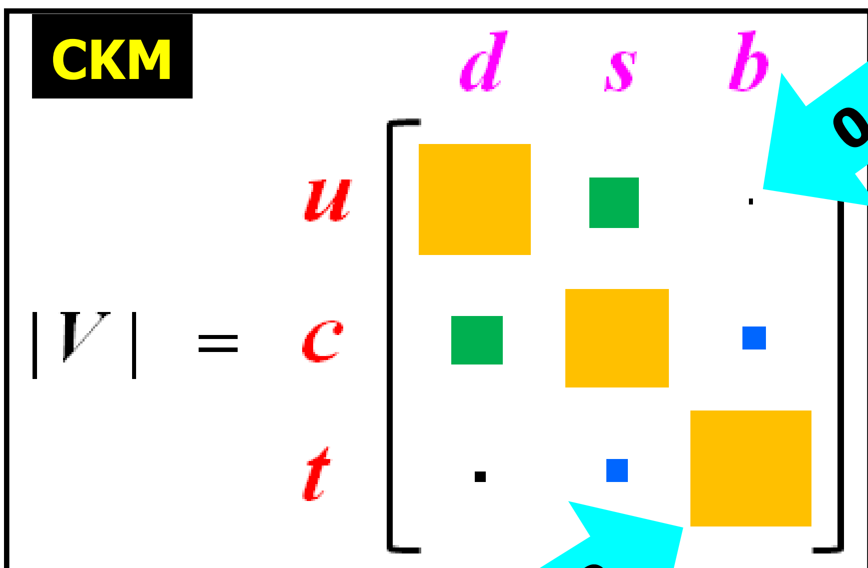
CKM **PMNS**

Wilkinson's lecture

Quark mixing: hierarchy!

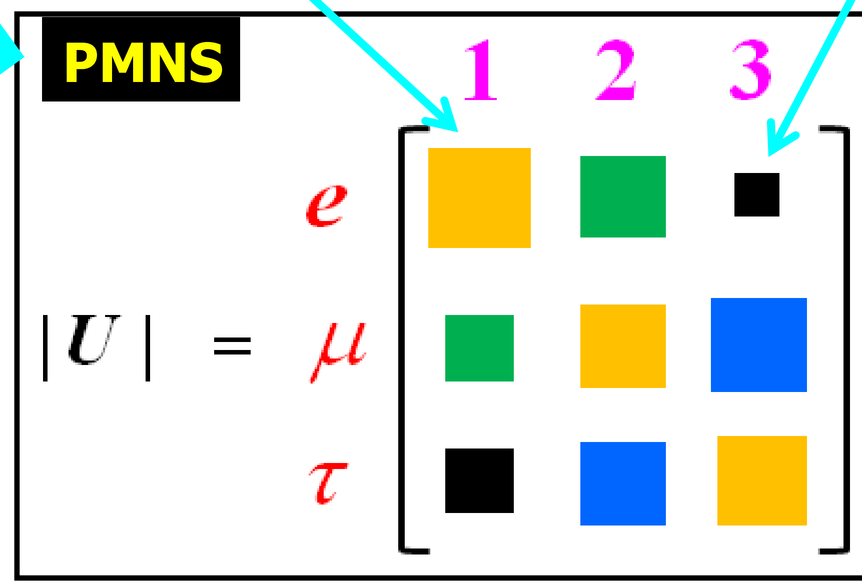
~ 0.8 **~0.15**

4/6 parameters



0.004

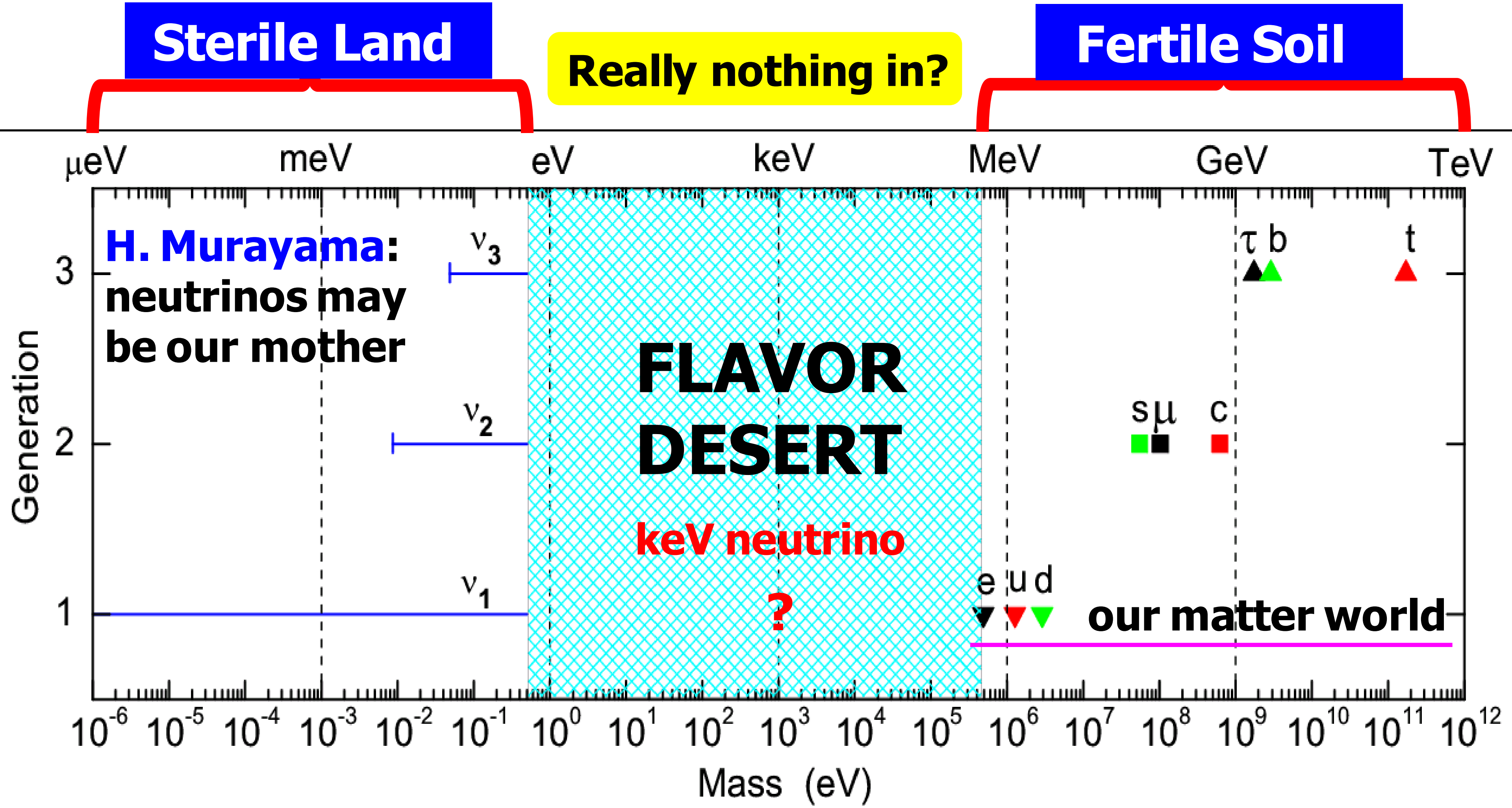
0.999



4 parameters

Lepton mixing: anarchy?

Flavor puzzles (2)



Gauge Hierarchy & Desert Puzzles / Flavor Hierarchy & Desert Puzzles

Implications of electron mass < u quark mass < d quark mass on ....

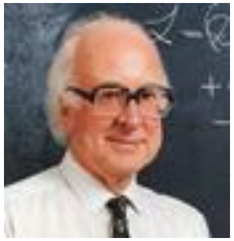
Part B

What is mass?

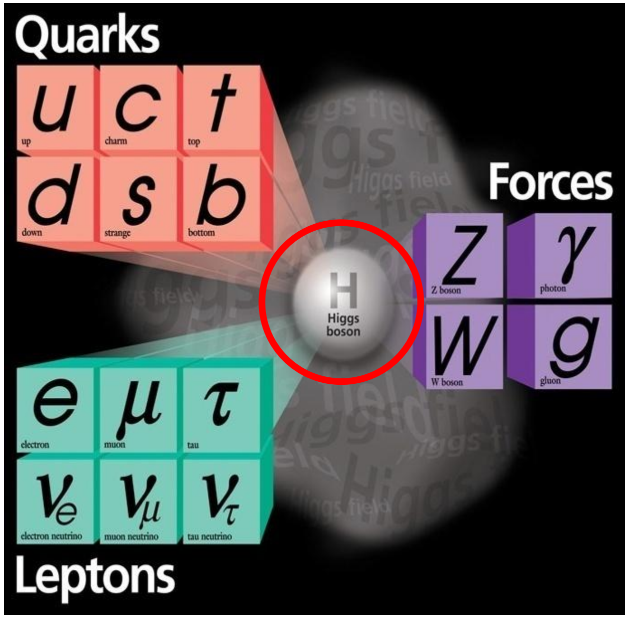
Mass is the inertial energy of a particle existing at rest.

- A **massless** particle has no way to exist at rest. It must always move at the speed of light.
- A **massive** fermion (lepton or quark) must exist in both the left- and right-handed states.

The **Brout-Englert-Higgs** mechanism is responsible for the origin of W / Z and fermion masses in the SM.



$$L_{\mathrm{SM}} = L(f,G) + \underline{L(f,H)} + \underline{L(G,H)} + L(G) - V(H)$$



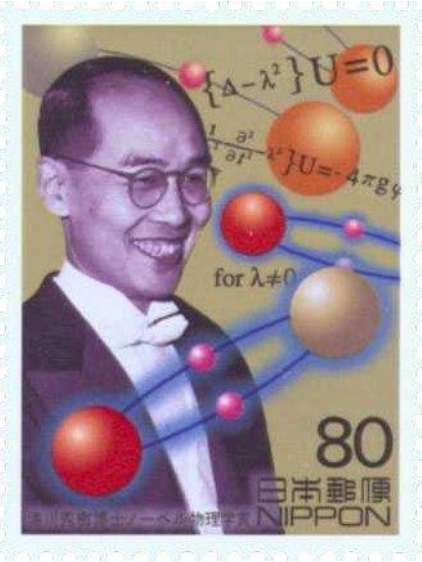
All the **bosons** were discovered in **Europe**, and most of the **fermions** were discovered in **America**.

Higgs: Yukawa interaction

force	strength	range	mediator	mass
strong	1	$10^{-15}\,\mathrm{m}$	gluon/π	$\sim 10^{2}\,\mathrm{MeV}$
EM	$1/137$	∞	photon	$=0$
weak	10^{-12}	$10^{-18}\,\mathrm{m}$	W/Z/H	$\sim 10^{2}\,\mathrm{GeV}$
gravitation	6×10^{-39}	∞	graviton	$=0$

Yukawa relation for the mediator's mass ***M*** and the force's range ***R***:

$$\boxed{M \approx \frac{200\mathrm{MeV}\times 10^{-15}\,\mathrm{m}}{R}}$$



$$\boxed{L_{\mathrm{SM}} = \underline{L(f,G)+L(f,H)}+L(G,H)+L(G)-V(H)}$$

Fermion masses, flavor mixing, CP violation

In the SM

All ν's are massless because the model's simple structure:

---- SU(2)×U(1) gauge symmetry and Lorentz invariance:

Fundamentals of a quantum field theory

---- Economical particle content:

No right-handed neutrino; only a single Higgs doublet

---- Mandatory renormalizability:

No dimension ≥ 5 operator (*B*-*L* conserved in the SM)

Neutrinos are massless in the SM: Natural or not?

YES: the neutrinos are all tooooooooo light and apparentlyleft-handed;
NO: no fundamental symmetry/conservation law to forbid ν's masses.

Possible way out: 1) the particle content can be enlarged; 2) the renormalizability can be abandoned.

Beyond the SM (1)

Way 1: to relax the requirement of **renormalizability** (S. Weinberg 79)

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm SM} + \frac{\mathcal{L}_{\rm d=5}}{\Lambda} + \frac{\mathcal{L}_{\rm d=6}}{\Lambda^2} + \cdots$$

Given the standard-model fields, the **lowest-dimension operators** that violate **lepton** and **baryon** numbers at the tree level are

$$\frac{1}{M} HHLL$$

neutrino mass

Seesaw: $m_{1,2,3} \sim \langle H \rangle^2 / M$

$$m_{1,2,3} < 1\,\mathrm{eV} \;\Rightarrow\; M > 10^{13}\,\mathrm{GeV}$$

$$\frac{1}{M^2} QQQL$$

proton decay

Example : $p \to \pi^0 + e^+$

$$\tau_p > 10^{33}\ \mathrm{years} \;\Rightarrow\; M > 10^{15}\,\mathrm{GeV}$$

Neutrino masses and **proton decays** at the **intensity frontier** offer new windows onto physics at super-high energy scales.

Beyond the SM (2)

Way 2: to add **3 right-handed** neutrinos and demand the *L* symmetry.

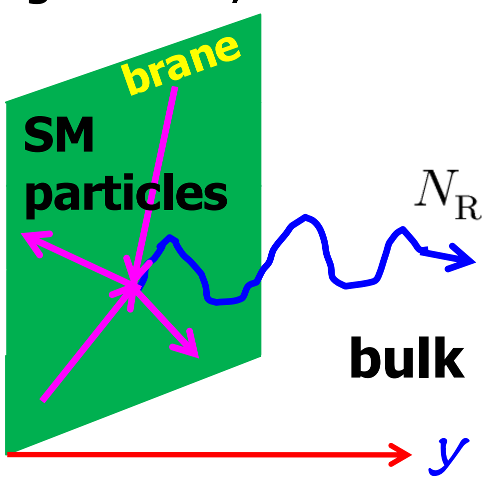
$$-\mathcal{L}_{\rm lepton} = \overline{l_{\rm L}} Y_l H E_{\rm R} + \overline{l_{\rm L}} Y_\nu \tilde{H} N_{\rm R} + {\rm h.c.}$$

$$M_l = Y_l v/\sqrt{2}\,, \quad M_\nu = Y_\nu v/\sqrt{2}$$

But, such a pure **Dirac** mass term and lepton number conservation are not convincing, because non-perturbative quantum effects break both *L* and *B* symmetries and only preserve *B* − *L* (G. 't Hooft, 1976).

The flavor hierarchy puzzle: $y_i/y_e = m_i/m_e \lesssim 0.5~{\rm eV}/0.5~{\rm MeV} \sim 10^{-6}$

A very speculative way out: the smallness of **Dirac** masses is ascribed to the assumption that *N*_R have access to an extra spatial dimension (Dienes, Dudas, Gherghetta 98; Arkani-Hamed, Dimopoulos, Dvali, March-Russell 98) :



The wavefunction of *N*_R spreads out over the extra dimension *y*, giving rise to a suppressed Yukawa interaction at *y* = 0.

$$\left[\overline{l_{\rm L}} Y_\nu \tilde{H} N_{\rm R}\right]_{y=0} \sim \boxed{\frac{1}{\sqrt{L}}} \left[\overline{l_{\rm L}} Y_\nu \tilde{H} N_{\rm R}\right]_{y=L}$$

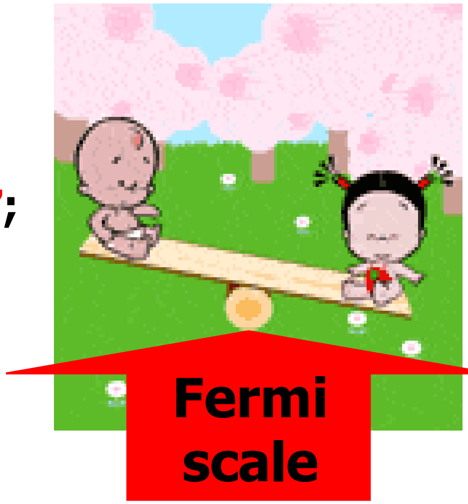
(e.g., King 08)

$$\Lambda_{\rm String}/\Lambda_{\rm Planck} \sim 10^{-12}$$

Beyond the SM (3)

Way 3: add **new heavy degrees of freedom** and allow the *L* violation.

Seesaw—A Footnote Idea:
H. Fritzsch, M. Gell-Mann,
P. Minkowski, PLB 59 (1975) 256



Type (1): SM + 3 right-handed neutrinos (Minkowski 77; Yanagida 79; Glashow 79; Gell-Mann, Ramond, Slanski 79; Mohapatra, Senjanovic 80)

$$-\mathcal{L}_{\rm lepton} = \overline{l_{\rm L}} Y_l H E_{\rm R} + \overline{l_{\rm L}} Y_\nu \tilde{H} N_{\rm R} + \frac{1}{2} \overline{N^c_{\rm R}} M_{\rm R} N_{\rm R} + {\rm h.c.}$$

Type (2): SM + 1 Higgs triplet (Konetschny, Kummer 77; Magg, Wetterich 80; Schechter, Valle 80; Cheng, Li 80; Lazarides et al 80; Mohapatra, Senjanovic 80)

$$-\mathcal{L}_{\rm lepton} = \overline{l_{\rm L}} Y_l H E_{\rm R} + \frac{1}{2} \overline{l_{\rm L}} Y_\Delta \Delta i\sigma_2 l^c_{\rm L} - \lambda_\Delta M_\Delta H^T i\sigma_2 \Delta H + {\rm h.c.}$$

variations

Type (3): SM + 3 triplet fermions (Foot, Lew, He, Joshi 89)

$$-\mathcal{L}_{\rm lepton} = \overline{l_{\rm L}} Y_l H E_{\rm R} + \overline{l_{\rm L}} \sqrt{2} Y_\Sigma \Sigma^c \tilde{H} + \frac{1}{2} {\rm Tr} \left(\overline{\Sigma} M_\Sigma \Sigma^c \right) + {\rm h.c.}$$

combinations

Seesaw mechanisms

Weinberg operator: the unique dimension-five operator of ν-masses after integrating out heavy degrees of freedom.

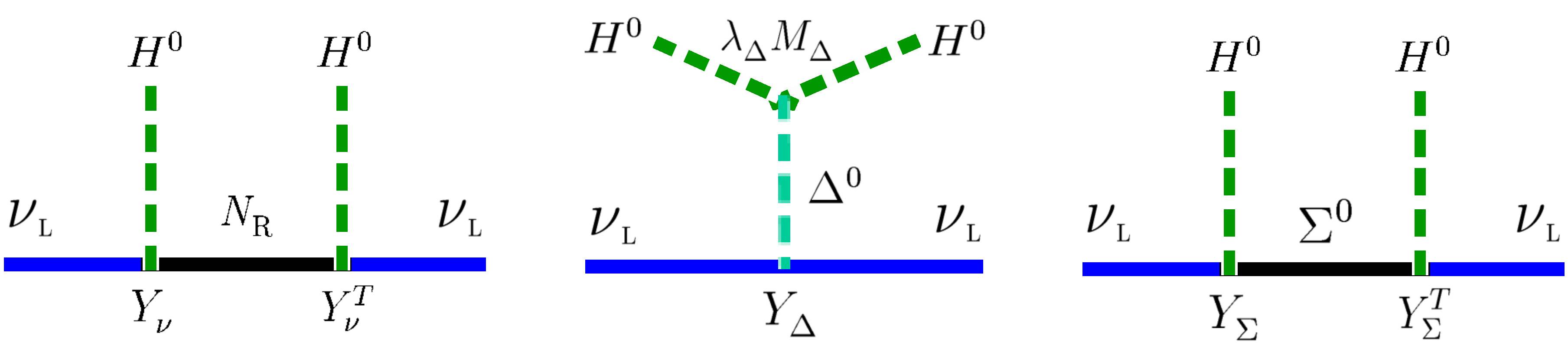
$$\frac{\mathcal{L}_{\rm d=5}}{\Lambda} = \begin{cases} \frac{1}{2}\left(Y_\nu M_{\rm R}^{-1} Y_\nu^T\right)_{\alpha\beta} \overline{l_{\alpha \rm L}} \tilde{H} \tilde{H}^T l^c_{\beta \rm L} + {\rm h.c.} \\ -\frac{\lambda_\Delta}{M_\Delta} \left(Y_\Delta\right)_{\alpha\beta} \overline{l_{\alpha \rm L}} \tilde{H} \tilde{H}^T l^c_{\beta \rm L} + {\rm h.c.} \\ \frac{1}{2}\left(Y_\Sigma M_\Sigma^{-1} Y_\Sigma^T\right)_{\alpha\beta} \overline{l_{\alpha \rm L}} \tilde{H} \tilde{H}^T l^c_{\beta \rm L} + {\rm h.c.} \end{cases}$$

$$M_\nu = \begin{cases} -\frac{1}{2} Y_\nu \frac{v^2}{M_{\rm R}} Y_\nu^T & ({\rm Type\ 1}) \\ \lambda_\Delta Y_\Delta \frac{v^2}{M_\Delta} & ({\rm Type\ 2}) \\ -\frac{1}{2} Y_\Sigma \frac{v^2}{M_\Sigma} Y_\Sigma^T & ({\rm Type\ 3}) \end{cases}$$

After SSB, a Majorana neutrino mass term is

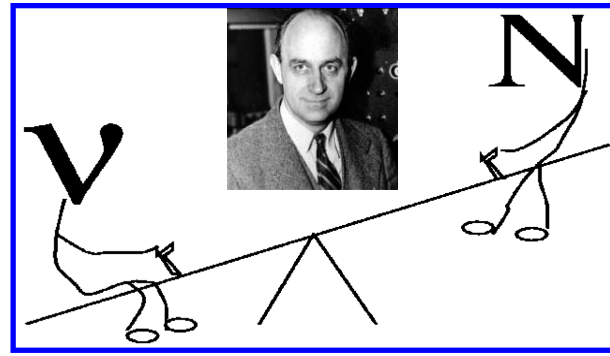
$$-\mathcal{L}_{\rm mass} = \frac{1}{2} \overline{\nu_{\rm L}} M_\nu \nu^c_{\rm L} + {\rm h.c.}$$

$$\langle \tilde{H} \rangle = v/\sqrt{2}$$



The seesaw scale (1)

What is the energy scale at which the seesaw mechanism works and new physics come in?



Planck

GUT **to unify strong, weak & electromagnetic forces**

Conventional Seesaws: heavy degrees of freedom near GUT

This appears to be rather reasonable, since one often expects new physics to appear around a fundamental scale

Naturalness √

Testability ×

Fermi

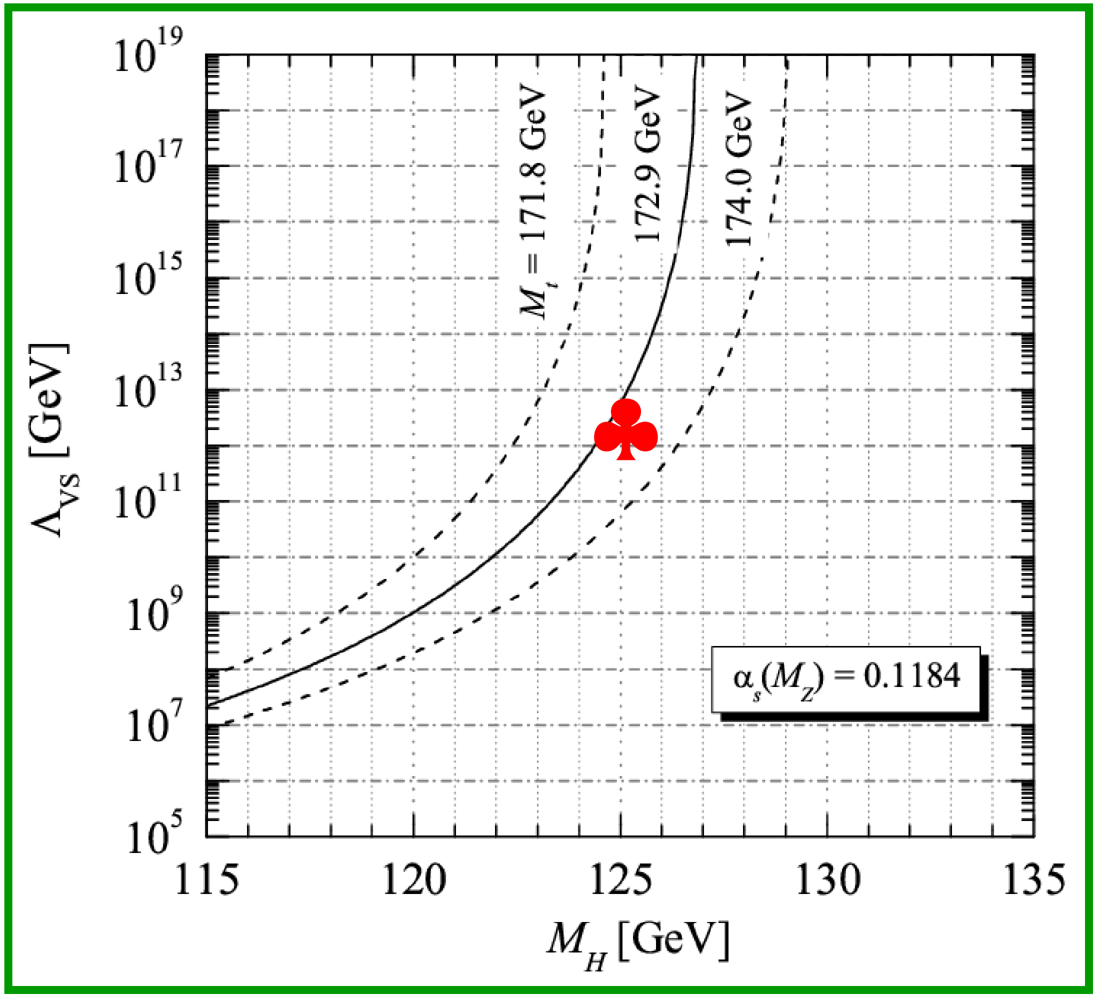
Uniqueness ×

Hierarchy ×

The seesaw scale (2)

Planck scale	$\Lambda \sim 10^{19}\,\mathrm{GeV}$
GUT scale?	$\Lambda \sim 10^{16}\,\mathrm{GeV}$
Seesaw scale?	$\Lambda \sim 10^{12}\,\mathrm{GeV}$
TeV / SUSY?	$\Lambda \sim 10^{3}\,\mathrm{GeV}$
Fermi scale	$\Lambda \sim 10^{2}\,\mathrm{GeV}$
QCD scale	$\Lambda \sim 10^{2}\,\mathrm{MeV}$

The SM vacuum stability for a light Higgs



Seesaws could make life easier?

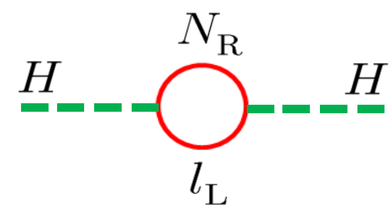
Elias-Miro et al., arXiv:1112.3022; Xing, Zhang, Zhou, arXiv:1112.3112; …….

New hierarchy problem

Seesaw-induced fine-tuning problem: the Higgs mass is very sensitive to quantum corrections from the heavy degrees of freedom induced in the seesaw mechanisms (Vissani 98; Casas et al 04; Abada et al 07)

Type 1:

$$\delta m_H^2 = -\frac{y_i^2}{8\pi^2}\left(\Lambda^2 + M_i^2 \ln\frac{M_i^2}{\Lambda^2}\right)$$

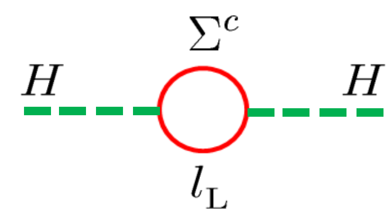


Type 2:

$$\delta m_H^2 = \frac{3}{16\pi^2}\left[\lambda_3\left(\Lambda^2 + M_\Delta^2 \ln\frac{M_\Delta^2}{\Lambda^2}\right) + 4\lambda_\Delta^2 M_\Delta^2 \ln\frac{M_\Delta^2}{\Lambda^2}\right]$$

Type 3:

$$\delta m_H^2 = -\frac{3y_i^2}{8\pi^2}\left(\Lambda^2 + M_i^2 \ln\frac{M_i^2}{\Lambda^2}\right)$$



here y_i & M_i are eigenvalues of Y_ν (or Y_Σ) & M_R (or M_Σ), respectively.

An illustration of fine-tuning

$$M_i \sim \left[\frac{(2\pi v)^2|\delta m_H^2|}{m_i}\right]^{1/3} \sim \underline{10^7\mathrm{GeV}}\left[\frac{0.2\ \mathrm{eV}}{m_i}\right]^{1/3}\left[\frac{|\delta m_H^2|}{0.1\ \mathrm{TeV}^2}\right]^{1/3}$$

Possible way out: (1) Supersymmetric seesaw? (2) TeV-scale seesaw?

TeV neutrino physics?

to discover the SM Higgs boson **OK**

to verify Yukawa interactions **OK**

to pin down heavy seesaw particles

to single out a seesaw mechanism

to measure all low-energy effects

Let's **H**ave **C**offee!

Collider signature (1)

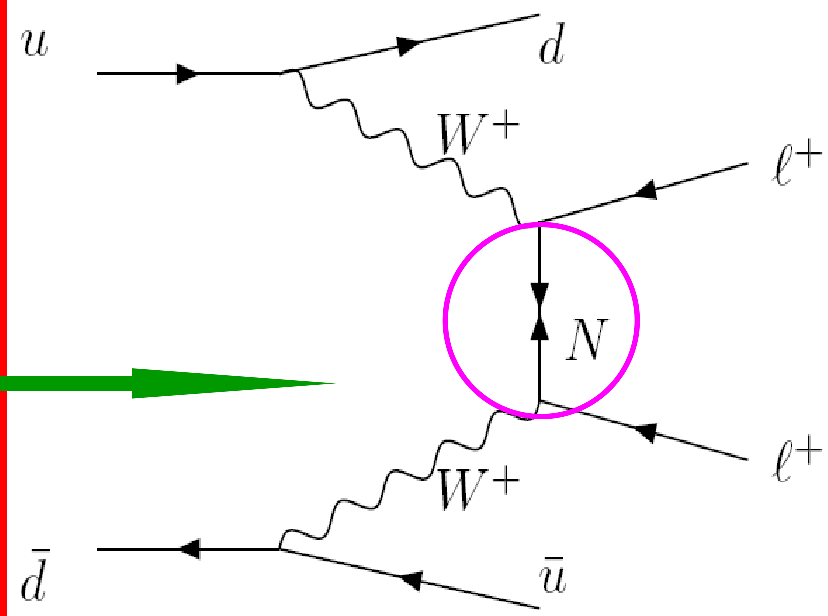
Lepton number violation: **like-sign dilepton events at hadron colliders, such as Tevatron (~2 TeV) and LHC (~14 TeV).**

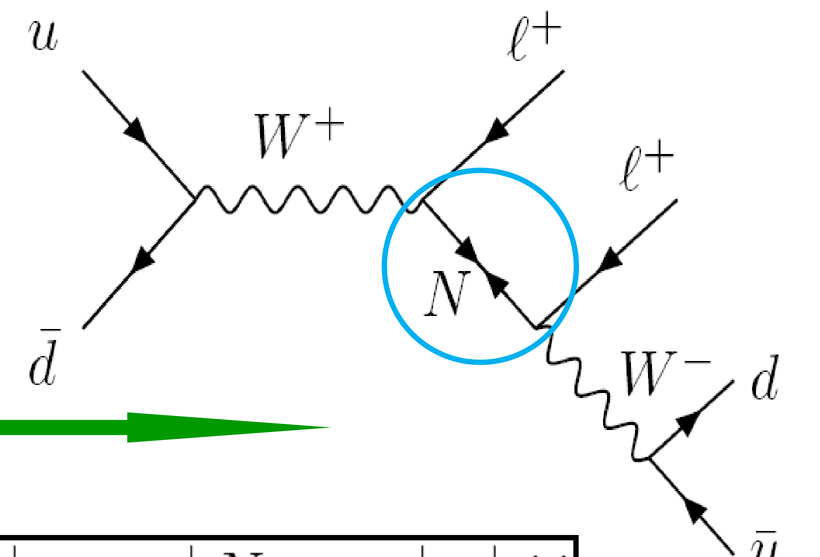
T. Han, Z.G. Si, K. Wang, B. Zhang, ...

collider analogue to 0νββ decay

dominant channel

N **can be produced on resonance**

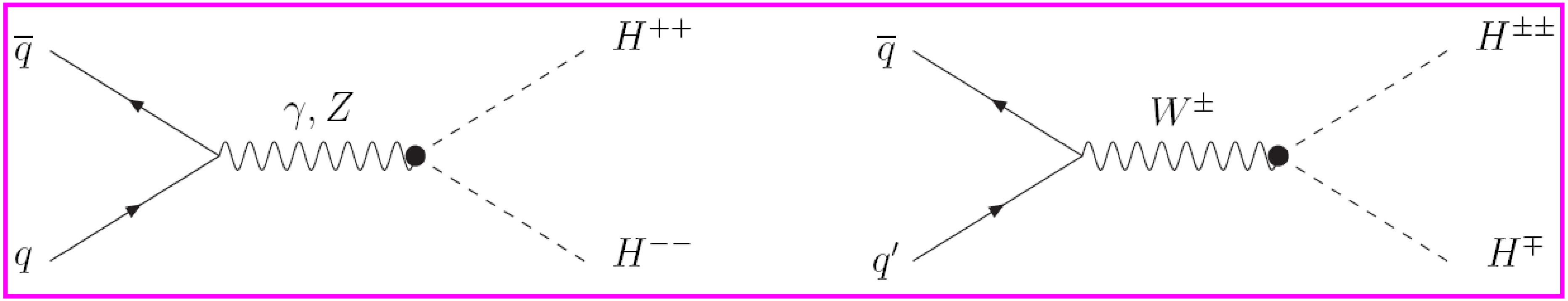




$pp \to W^\pm W^\pm \to \mu^\pm \mu^\pm jj$ and $pp \to W^\pm \to \mu^\pm N \to \mu^\pm \mu^\pm jj$

Collider signature (2)

From a viewpoint of **direct tests**, the triplet seesaw has an advantage: The **SU(2)_L** Higgs triplet contains a **doubly-charged scalar** which can be produced at colliders: it is dependent on its mass but independent of the (small) Yukawa coupling.



Typical LNV signatures: $\boxed{H^{\pm\pm} \longrightarrow l_\alpha^{\pm} l_\beta^{\pm}}$ $\boxed{H^{+} \longrightarrow l_\alpha^{+} \bar{\nu}_\beta}$ $\boxed{H^{-} \longrightarrow l_\alpha^{-} \nu}$

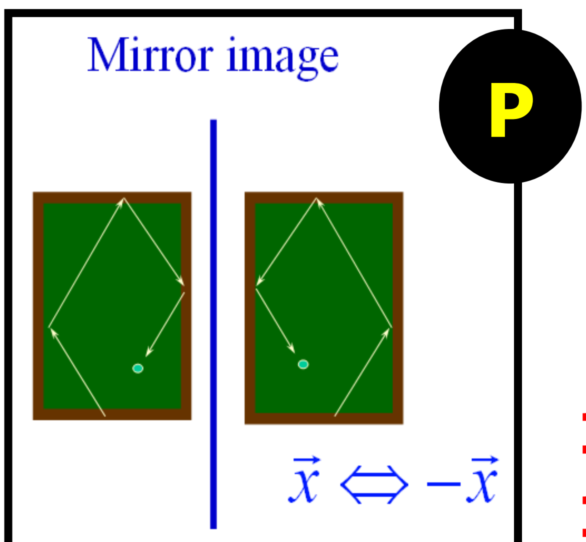
$$\boxed{\mathcal{B}(H^{\pm\pm} \to l_\alpha^{\pm} l_\beta^{\pm}) = \frac{(2-\delta_{\alpha\beta})|(M_{\rm L})_{\alpha\beta}|^2}{\sum\limits_{\rho,\sigma} |(M_{\rm L})_{\rho\sigma}|^2} \,, \qquad \mathcal{B}(H^{+} \to l_\alpha^{+} \overline{\nu}) = \frac{\sum\limits_{\beta} |(M_{\rm L})_{\alpha\beta}|^2}{\sum\limits_{\rho,\sigma} |(M_{\rm L})_{\rho\sigma}|^2}}$$

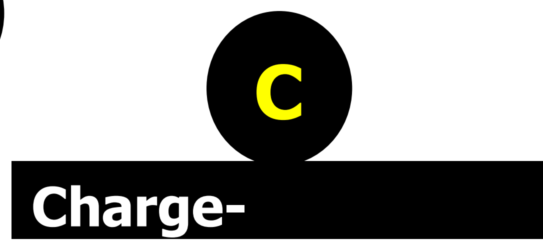
Flavor mixing

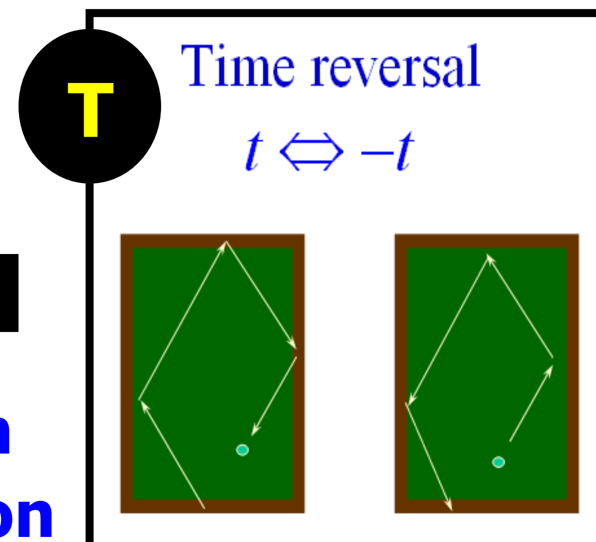
Flavor mixing: mismatch between **weak/flavor** eigenstates and **mass** eigenstates of fermions due to coexistence of **2** types of interactions.

Weak eigenstates: members of weak isospin doublets transforming into each other through the interaction with the ***W*** boson;
Mass eigenstates: states of definite masses that are created by the interaction with the Higgs boson (**Yukawa** interactions).

CP violation: **matter** and **antimatter**, or a reaction & its CP-conjugate process, are distinguishable --- coexistence of **2** types of interactions.







1957: **P violation**
1964: **CP violation**

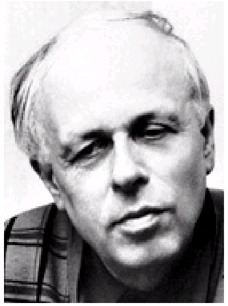
Towards the KM paper

1964: **Discovery of CP violation in K decays (J.W. Cronin, Val L. Fitch)**

NP 1980

1967: **Sakharov conditions for cosmological matter-antimatter asymmetry (A. Sakharov)**

NP 1975



1967: **The standard model of electromagnetic and weak interactions without quarks (S. Weinberg)**

0 citation for the first 4 yrs

NP 1979

1971: **The first proof of the renormalizability of the standard model (G. 't Hooft)**

NP 1999

KM in 1972

Progress of Theoretical Physics, Vol. 49, No. 2, February 1973

CP-Violation in the Renormalizable Theory of Weak Interaction

Makoto KOBAYASHI and Toshihide MASKAWA

Department of Physics, Kyoto University, Kyoto

(Received September 1, 1972)

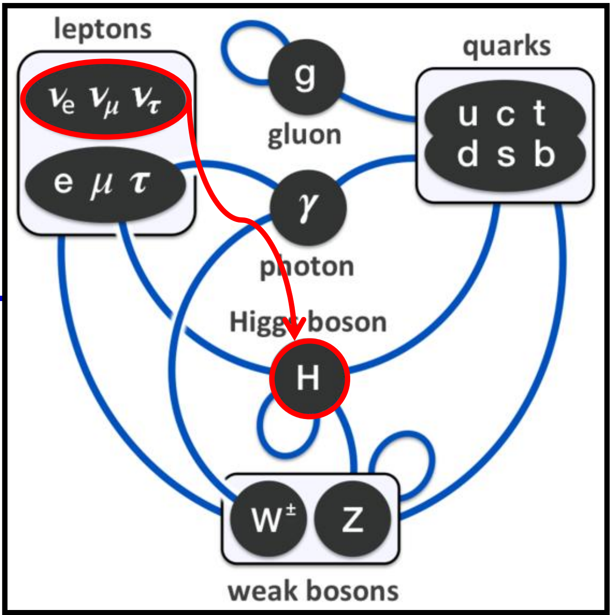
In a framework of the renormalizable theory of weak interaction, problems of *CP*-violation are studied. It is concluded that no realistic models of *CP*-violation exist in the quartet scheme without introducing any other new fields. Some possible models of *CP*-violation are also discussed.

3 families allow for CP violation: Maskawa's bathtub idea!

"as I was getting out of the bathtub, an idea came to me"

Diagnosis of CP violation

In the SM (+ 3 right-handed ν's) , the KM mechanism is responsible for CPV.



$$\mathcal{L}_{\nu\mathrm{SM}} = \mathcal{L}_{\mathrm{G}} + \mathcal{L}_{\mathrm{H}} + \mathcal{L}_{\mathrm{F}} + \mathcal{L}_{\mathrm{Y}}$$

$$\mathcal{L}_{\mathrm{G}} = -\frac{1}{4}\left(W^{i\mu\nu}W^{i}_{\mu\nu} + B^{\mu\nu}B_{\mu\nu}\right)$$

$$\mathcal{L}_{\mathrm{H}} = \left(D^{\mu}H\right)^{\dagger}\left(D_{\mu}H\right) - \mu^2 H^{\dagger}H - \lambda\left(H^{\dagger}H\right)^2$$

$$\mathcal{L}_{\mathrm{F}} = \overline{Q_{\mathrm{L}}} i \not{D} Q_{\mathrm{L}} + \overline{\ell_{\mathrm{L}}} i \not{D} \ell_{\mathrm{L}} + \overline{U_{\mathrm{R}}} i \not{\partial}' U_{\mathrm{R}} + \overline{D_{\mathrm{R}}} i \not{\partial}' D_{\mathrm{R}} + \overline{E_{\mathrm{R}}} i \not{\partial}' E_{\mathrm{R}} + \underline{\overline{N_{\mathrm{R}}} i \not{\partial}' N_{\mathrm{R}}}$$

$$\mathcal{L}_{\mathrm{Y}} = -\overline{Q_{\mathrm{L}}} Y_{\mathrm{u}} \tilde{H} U_{\mathrm{R}} - \overline{Q_{\mathrm{L}}} Y_{\mathrm{d}} H D_{\mathrm{R}} - \overline{\ell_{\mathrm{L}}} Y_{l} H E_{\mathrm{R}} - \underline{\overline{\ell_{\mathrm{L}}} Y_{\nu} \tilde{H} N_{\mathrm{R}}} + \mathrm{h.c.}$$

ν's Dirac mass

The strategy of diagnosis: given proper CP transformations of gauge, Higgs and fermion fields, we may prove that the 1st, 2nd and 3rd terms are formally invariant, and hence the 4th term can be invariant only if provided the corresponding Yukawa coupling matrices are real. (Note that the SM spontaneous symmetry breaking itself doesn't affect CP.)

The source

The **Yukawa** interactions of fermions are **formally invariant** under **CP** if and only if

$$Y_{\rm u} = Y_{\rm u}^* \; , \qquad Y_{\rm d} = Y_{\rm d}^*$$
$$Y_l = Y_l^* \; , \qquad Y_\nu = Y_\nu^*$$

If the effective **Majorana** mass term is added into the SM, then the **Yukawa** interactions of leptons can be **formally invariant** under **CP** if

$$M_{\rm L} = M_{\rm L}^* \; , \qquad Y_l = Y_l^*$$

If the **flavor eigenstates** are transformed into the **mass eigenstates**, flavor mixing and **CP** violation will show up in the **CC** interactions:

quarks

$$\mathcal{L}_{\rm cc} = \frac{g}{\sqrt{2}} \overline{(u \; c \; t)_{\rm L}} \; \gamma^\mu V \begin{pmatrix} d \\ s \\ b \end{pmatrix}_{\rm L} W_\mu^+ + {\rm h.c.}$$

leptons

$$\mathcal{L}_{\rm cc} = \frac{g}{\sqrt{2}} \overline{(e \; \mu \; \tau)_{\rm L}} \; \gamma^\mu U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_{\rm L} W_\mu^- + {\rm h.c.}$$

Comment A: flavor mixing and **CP** violation take place since fermions interact with both the **gauge bosons** and the **Higgs boson**.

Comment B: both the **CC** and **Yukawa** interactions have been verified.

Comment C: the **CKM** matrix ***V*** is unitary, the **PMNS** matrix ***U*** is too?

Physical phases

If massive neutrinos are the Dirac particles, then the 3×3 lepton flavor mixing matrix can be parametrized as:

Dirac neutrinos: 3 angles + 1 phase

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

If neutrinos are the Majorana particles, their left- & right-handed fields should be correlated. In this case the lepton flavor mixing matrix contains 3 nontrivial phases:

Majorana neutrinos: 3 angles + 3 phases

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} e^{i\rho} & 0 & 0 \\ 0 & e^{i\sigma} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Global fit of current data

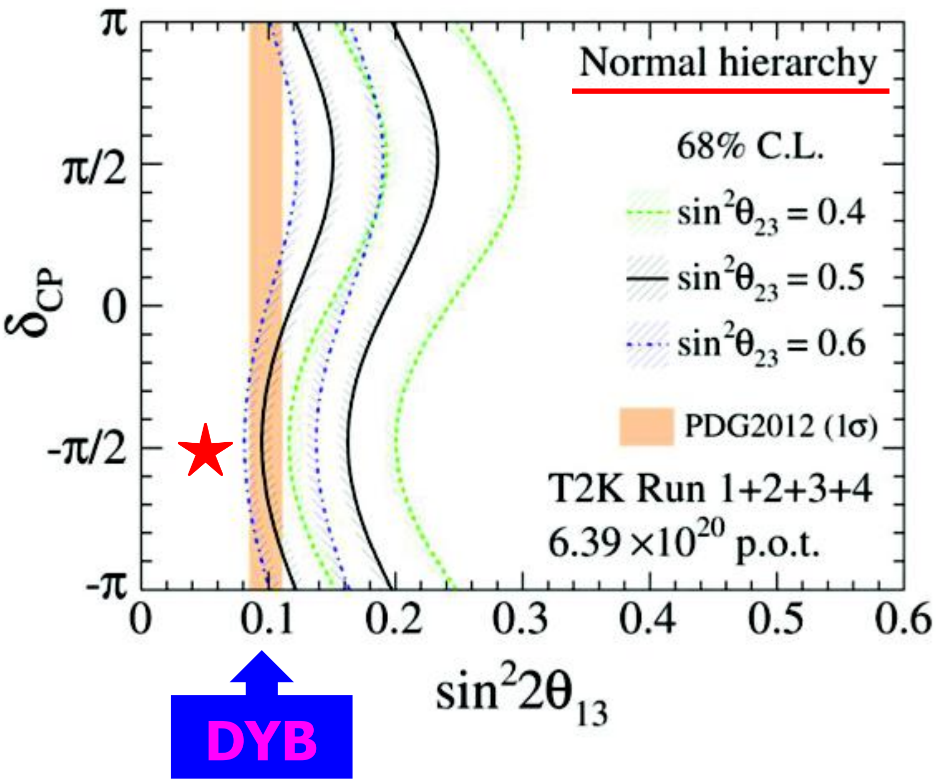
F. Capozzi et al (2014) —— the standard parametrization:

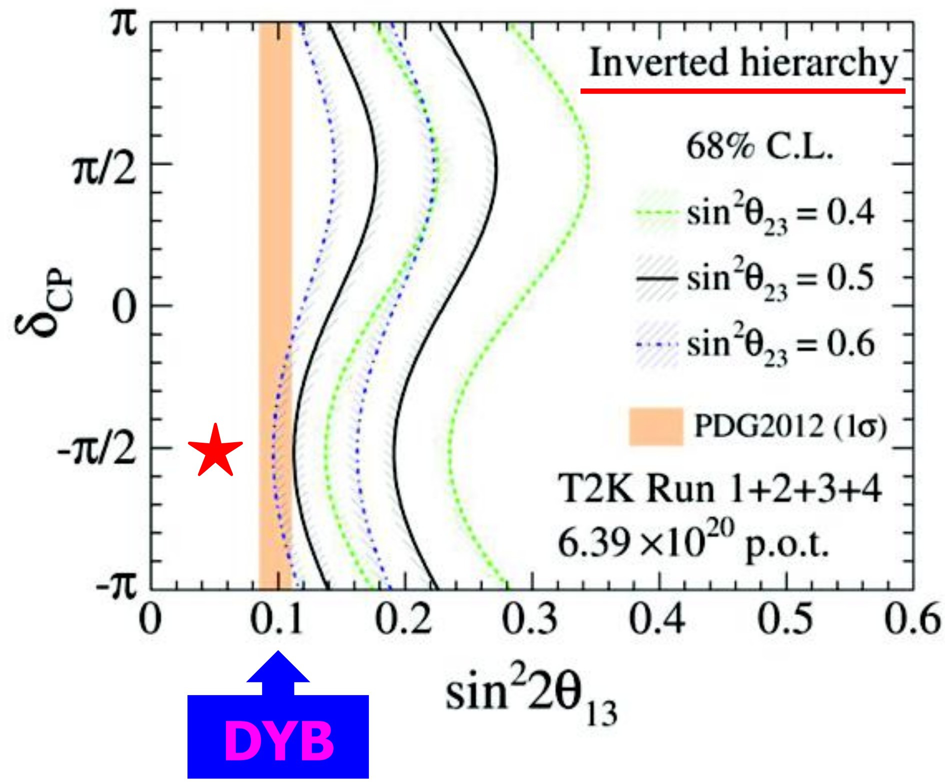
Parameter	Best fit	1σ range	2σ range	3σ range
Normal neutrino mass ordering $(m_1 < m_2 < m_3)$				
$\Delta m^2_{21}/10^{-5}$ eV^2	7.54	7.32 — 7.80	7.15 — 8.00	6.99 — 8.18
$\Delta m^2_{31}/10^{-3}$ eV^2	2.47	2.41 — 2.53	2.34 — 2.59	2.26 — 2.65
$\sin^2\theta_{12}/10^{-1}$	3.08	2.91 — 3.25	2.75 — 3.42	2.59 — 3.59
$\sin^2\theta_{13}/10^{-2}$	2.34	2.15 — 2.54	1.95 — 2.74	1.76 — 2.95
$\sin^2\theta_{23}/10^{-1}$	4.37	4.14 — 4.70	3.93 — 5.52	3.74 — 6.26
$\delta/180°$	1.39	1.12 — 1.77	0.00 — 0.16 $\oplus$ 0.86 — 2.00	0.00 — 2.00
Inverted neutrino mass ordering $(m_3 < m_1 < m_2)$				
$\Delta m^2_{21}/10^{-5}$ eV^2	7.54	7.32 — 7.80	7.15 — 8.00	6.99 — 8.18
$\Delta m^2_{13}/10^{-3}$ eV^2	2.42	2.36 — 2.48	2.29 — 2.54	2.22 — 2.60
$\sin^2\theta_{12}/10^{-1}$	3.08	2.91 — 3.25	2.75 — 3.42	2.59 — 3.59
$\sin^2\theta_{13}/10^{-2}$	2.40	2.18 — 2.59	1.98 — 2.79	1.78 — 2.98
$\sin^2\theta_{23}/10^{-1}$	4.55	4.24 — 5.94	4.00 — 6.20	3.80 — 6.41
$\delta/180°$	1.31	0.98 — 1.60	0.00 — 0.02 $\oplus$ 0.70 — 2.00	0.00 — 2.00

The neutrino mass ordering unknown: normal or inverted?

Hint for the CP phase

The T2K observation of a relatively strong $\nu_\mu \to \nu_e$ appearance plays a crucial role in the global fit to make θ_{13} consistent with the Daya Bay result and drive a slight but intriguing preference for $\delta \sim -\pi/2$.





DYB's good news: θ_{13} unsuppressed

T2K's good news: δ unsuppressed

precision measurements

Life is easier for probing CP violation, ν mass hierarchy ....

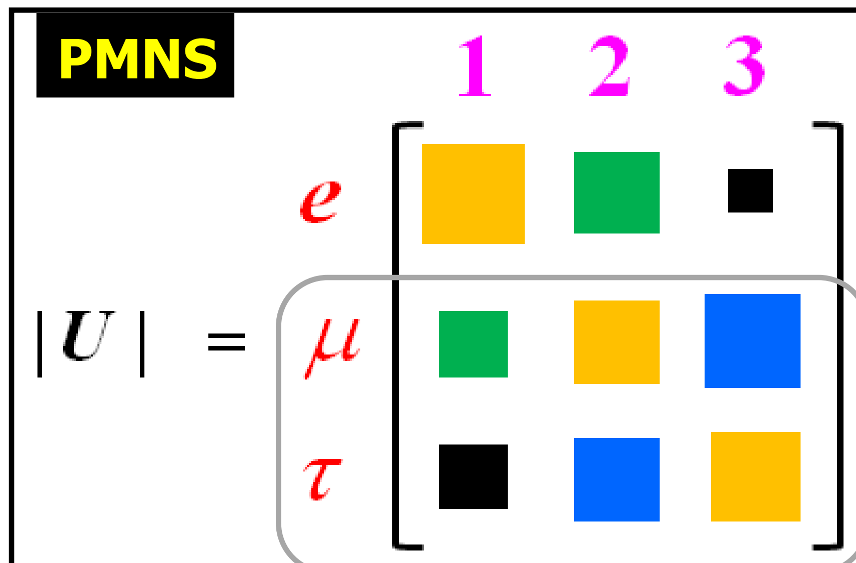
What the data tell?

Given the global-fit results at the 3σ level, the elements of the PMNS matrix are:

The normal ordering: $$|U| \simeq \begin{pmatrix} 0.79-0.85 & 0.50-0.59 & 0.13-0.17 \\ 0.19-0.56 & 0.41-0.74 & 0.60-0.78 \\ 0.19-0.56 & 0.41-0.74 & 0.60-0.78 \end{pmatrix}$$

The inverted ordering: $$|U| \simeq \begin{pmatrix} 0.89-0.85 & 0.50-0.59 & 0.13-0.17 \\ 0.19-0.56 & 0.40-0.73 & 0.61-0.79 \\ 0.20-0.56 & 0.41-0.74 & 0.59-0.78 \end{pmatrix}$$

$$\boxed{\begin{aligned} |U_{\mu 1}| &\simeq |U_{\tau 1}| \\ |U_{\mu 2}| &\simeq |U_{\tau 2}| \\ |U_{\mu 3}| &\simeq |U_{\tau 3}| \end{aligned}}$$



Behind the PMNS matrix

Behind the observed pattern of lepton flavor mixing is an approximate (or a partial) μ-τ flavor symmetry!

$$|U_{\mu 1}| \simeq |U_{\tau 1}| \; , \;\; |U_{\mu 2}| \simeq |U_{\tau 2}| \; , \;\; |U_{\mu 3}| \simeq |U_{\tau 3}|$$

It is very likely that the PMNS matrix possesses an exact μ-τ symmetry at a given energy scale, and this symmetry must be softly broken —— shed light on flavor structures

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta} & c_{13}c_{23} \end{pmatrix} P_\nu$$

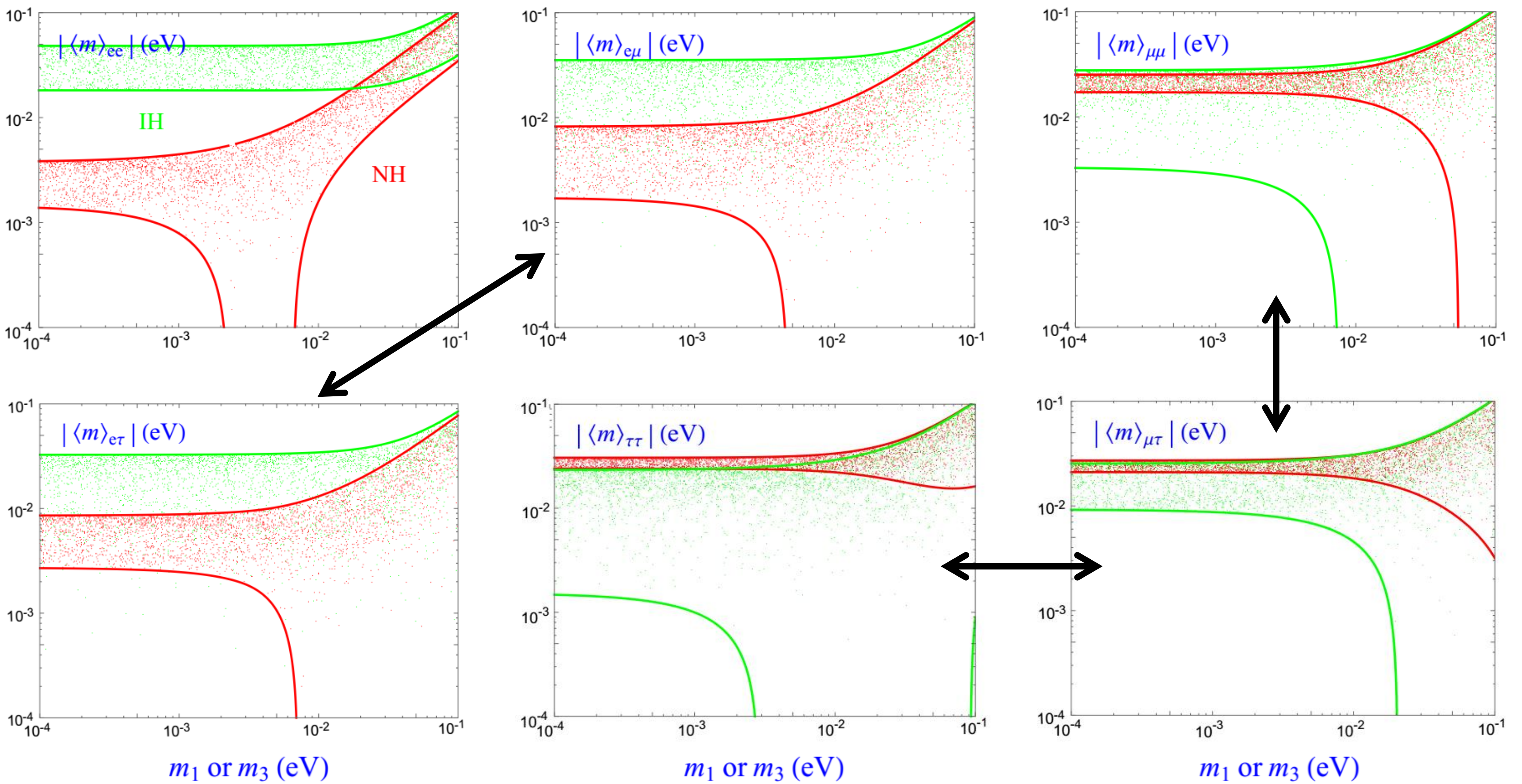
Conditions for the exact μ-τ symmetry in the PMNS matrix:

$$|U_{\mu i}| = |U_{\tau i}| \quad \Longrightarrow \left\{ \begin{array}{l} \theta_{13} = 0 \\ \theta_{23} = \pi/4 \end{array} \right. \text{or} \left\{ \begin{array}{l} \delta = +\pi/2 \\ \theta_{23} = \pi/4 \end{array} \right. \text{or} \left\{ \begin{array}{l} \delta = -\pi/2 \\ \theta_{23} = \pi/4 \end{array} \right.$$

Current data: **ruled out** **not sure** **favored**

Neutrino mass matrix

$$M_\nu = \begin{pmatrix} M_{ee} & M_{e\mu} & M_{e\tau} \\ M_{e\mu} & M_{\mu\mu} & M_{\mu\tau} \\ M_{e\tau} & M_{\mu\tau} & M_{\tau\tau} \end{pmatrix} = U \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} U^T = \begin{pmatrix} \langle m\rangle_{ee} & \langle m\rangle_{e\mu} & \langle m\rangle_{e\tau} \\ \langle m\rangle_{e\mu} & \langle m\rangle_{\mu\mu} & \langle m\rangle_{\mu\tau} \\ \langle m\rangle_{e\tau} & \langle m\rangle_{\mu\tau} & \langle m\rangle_{\tau\tau} \end{pmatrix}$$



μ-τ flavor symmetry

In the flavor basis, the **Majorana** ν mass matrix can be reconstructed:

$$M_\nu = \begin{pmatrix} M_{ee} & M_{e\mu} & M_{e\tau} \\ M_{e\mu} & M_{\mu\mu} & M_{\mu\tau} \\ M_{e\tau} & M_{\mu\tau} & M_{\tau\tau} \end{pmatrix} = U \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} U^T$$

μ-τ symmetry

μ-τ permutation symmetry

$$M_\nu = \begin{pmatrix} C & D & D \\ D & A & B \\ D & B & A \end{pmatrix}$$

$\nu_e \quad \nu_\mu \leftrightarrow \nu_\tau$

$$\begin{cases} \theta_{13} = 0 \\ \theta_{23} = \pi/4 \end{cases}$$

T. Fukuyama, H. Nishiura
hep-ph/9702253

Bimaximal, Tribimaximal …

μ-τ reflection symmetry

$$M_\nu = \begin{pmatrix} C & D & D^* \\ D & A & B \\ D^* & B & A^* \end{pmatrix}$$

$\nu_e \quad \nu_\mu \leftrightarrow \nu_\tau^c$

$$\begin{cases} \delta = \pm\pi/2 \\ \theta_{23} = \pi/4 \end{cases}$$

K. Babu, E. Ma, J. Valle
hep-ph/0206292

TM1, Tetramaximal …

$$\boxed{\frac{1}{2}\,\overline{\left(\nu_e \;\; \nu_\mu \;\; \nu_\tau\right)_{\rm L}}\, M_\nu \begin{pmatrix} \nu_e^c \\ \nu_\mu^c \\ \nu_\tau^c \end{pmatrix}_{\rm R}}$$

Current data

Larger ⟵ μ-τ **symmetry breaking** ⟶ **Softer**

A proof: permutation

A generic (symmetric) Majorana neutrino mass term reads as follows:

$$
\begin{aligned}
-\mathcal{L}_{\rm mass} = \; & M_{ee}\overline{\nu_{e\rm L}}(\nu_{e\rm L})^c + \underline{M_{e\mu}\overline{\nu_{e\rm L}}(\nu_{\mu\rm L})^c} + M_{e\tau}\overline{\nu_{e\rm L}}(\nu_{\tau\rm L})^c \\
& + M_{e\mu}\overline{\nu_{\mu\rm L}}(\nu_{e\rm L})^c + \underline{M_{\mu\mu}\overline{\nu_{\mu\rm L}}(\nu_{\mu\rm L})^c} + M_{\mu\tau}\overline{\nu_{\mu\rm L}}(\nu_{\tau\rm L})^c \\
& + M_{e\tau}\overline{\nu_{\tau\rm L}}(\nu_{e\rm L})^c + M_{\mu\tau}\overline{\nu_{\tau\rm L}}(\nu_{\mu\rm L})^c + M_{\tau\tau}\overline{\nu_{\tau\rm L}}(\nu_{\tau\rm L})^c + {\rm h.c.}
\end{aligned}
$$

Under μ-τ permutation, the above term changes to

$$\boxed{\nu_{\mu\rm L} \leftrightarrow \nu_{\tau\rm L}}$$

$$
\begin{aligned}
-\mathcal{L}_{\rm mass} = \; & M_{ee}\overline{\nu_{e\rm L}}(\nu_{e\rm L})^c + M_{e\mu}\overline{\nu_{e\rm L}}(\nu_{\tau\rm L})^c + \underline{M_{e\tau}\overline{\nu_{e\rm L}}(\nu_{\mu\rm L})^c} \\
& + M_{e\mu}\overline{\nu_{\tau\rm L}}(\nu_{e\rm L})^c + M_{\mu\mu}\overline{\nu_{\tau\rm L}}(\nu_{\tau\rm L})^c + M_{\mu\tau}\overline{\nu_{\tau\rm L}}(\nu_{\mu\rm L})^c \\
& + M_{e\tau}\overline{\nu_{\mu\rm L}}(\nu_{e\rm L})^c + M_{\mu\tau}\overline{\nu_{\mu\rm L}}(\nu_{\tau\rm L})^c + \underline{M_{\tau\tau}\overline{\nu_{\mu\rm L}}(\nu_{\mu\rm L})^c} + {\rm h.c.}
\end{aligned}
$$

Invariance of this transformation requires: $\underline{M_{e\mu} = M_{e\tau}}$ and $\underline{M_{\mu\mu} = M_{\tau\tau}}$

$$
M_\nu = \begin{array}{c} \begin{pmatrix} C & D & D \\ D & A & B \\ D & B & A \end{pmatrix} \\ \nu_e \quad \nu_\mu \leftrightarrow \nu_\tau \end{array}
\quad \Longrightarrow \quad
\begin{cases} \theta_{13} = 0 \\ \theta_{23} = \pi/4 \end{cases}
$$

reflection

A generic Majorana neutrino mass term reads as follows:

$$
\begin{aligned}
-\mathcal{L}_{\rm mass} = \; & M_{ee}\overline{\nu_{e\mathrm{L}}}(\nu_{e\mathrm{L}})^c + M_{e\mu}\overline{\nu_{e\mathrm{L}}}(\nu_{\mu\mathrm{L}})^c + M_{e\tau}\overline{\nu_{e\mathrm{L}}}(\nu_{\tau\mathrm{L}})^c \\
& + M_{e\mu}\overline{\nu_{\mu\mathrm{L}}}(\nu_{e\mathrm{L}})^c + M_{\mu\mu}\overline{\nu_{\mu\mathrm{L}}}(\nu_{\mu\mathrm{L}})^c + M_{\mu\tau}\overline{\nu_{\mu\mathrm{L}}}(\nu_{\tau\mathrm{L}})^c \\
& + M_{e\tau}\overline{\nu_{\tau\mathrm{L}}}(\nu_{e\mathrm{L}})^c + M_{\mu\tau}\overline{\nu_{\tau\mathrm{L}}}(\nu_{\mu\mathrm{L}})^c + M_{\tau\tau}\overline{\nu_{\tau\mathrm{L}}}(\nu_{\tau\mathrm{L}})^c \\
& + M^*_{ee}\overline{(\nu_{e\mathrm{L}})^c}\nu_{e\mathrm{L}} + M^*_{e\mu}\overline{(\nu_{\mu\mathrm{L}})^c}\nu_{e\mathrm{L}} + M^*_{e\tau}\overline{(\nu_{\tau\mathrm{L}})^c}\nu_{e\mathrm{L}} \\
& + M^*_{e\mu}\overline{(\nu_{e\mathrm{L}})^c}\nu_{\mu\mathrm{L}} + M^*_{\mu\mu}\overline{(\nu_{\mu\mathrm{L}})^c}\nu_{\mu\mathrm{L}} + M^*_{\mu\tau}\overline{(\nu_{\tau\mathrm{L}})^c}\nu_{\mu\mathrm{L}} \\
& + M^*_{e\tau}\overline{(\nu_{e\mathrm{L}})^c}\nu_{\tau\mathrm{L}} + M^*_{\mu\tau}\overline{(\nu_{\mu\mathrm{L}})^c}\nu_{\tau\mathrm{L}} + M^*_{\tau\tau}\overline{(\nu_{\tau\mathrm{L}})^c}\nu_{\tau\mathrm{L}}
\end{aligned}
$$

Under μ-τ reflection, the mass term is

$$
\boxed{\begin{aligned}
\nu_{e\mathrm{L}} &\leftrightarrow (\nu_{e\mathrm{L}})^c \\
\nu_{\mu\mathrm{L}} &\leftrightarrow (\nu_{\tau\mathrm{L}})^c \\
\nu_{\tau\mathrm{L}} &\leftrightarrow (\nu_{\mu\mathrm{L}})^c
\end{aligned}}
$$

$$
\begin{aligned}
-\mathcal{L}_{\rm mass} = \; & M_{ee}\overline{(\nu_{e\mathrm{L}})^c}\nu_{e\mathrm{L}} + M_{e\mu}\overline{(\nu_{e\mathrm{L}})^c}\nu_{\tau\mathrm{L}} + M_{e\tau}\overline{(\nu_{e\mathrm{L}})^c}\nu_{\mu\mathrm{L}} \\
& + M_{e\mu}\overline{(\nu_{\tau\mathrm{L}})^c}\nu_{e\mathrm{L}} + M_{\mu\mu}\overline{(\nu_{\tau\mathrm{L}})^c}\nu_{\tau\mathrm{L}} + M_{\mu\tau}\overline{(\nu_{\tau\mathrm{L}})^c}\nu_{\mu\mathrm{L}} \\
& + M_{e\tau}\overline{(\nu_{\mu\mathrm{L}})^c}\nu_{e\mathrm{L}} + M_{\mu\tau}\overline{(\nu_{\mu\mathrm{L}})^c}\nu_{\tau\mathrm{L}} + M_{\tau\tau}\overline{(\nu_{\mu\mathrm{L}})^c}\nu_{\mu\mathrm{L}} \\
& + M^*_{ee}\overline{\nu_{e\mathrm{L}}}(\nu_{e\mathrm{L}})^c + M^*_{e\mu}\overline{\nu_{\tau\mathrm{L}}}(\nu_{e\mathrm{L}})^c + M^*_{e\tau}\overline{\nu_{\mu\mathrm{L}}}(\nu_{e\mathrm{L}})^c \\
& + M^*_{e\mu}\overline{\nu_{e\mathrm{L}}}(\nu_{\tau\mathrm{L}})^c + M^*_{\mu\mu}\overline{\nu_{\tau\mathrm{L}}}(\nu_{\tau\mathrm{L}})^c + M^*_{\mu\tau}\overline{\nu_{\mu\mathrm{L}}}(\nu_{\tau\mathrm{L}})^c \\
& + M^*_{e\tau}\overline{\nu_{e\mathrm{L}}}(\nu_{\mu\mathrm{L}})^c + M^*_{\mu\tau}\overline{\nu_{\tau\mathrm{L}}}(\nu_{\mu\mathrm{L}})^c + M^*_{\tau\tau}\overline{\nu_{\mu\mathrm{L}}}(\nu_{\mu\mathrm{L}})^c
\end{aligned}
$$

Invariance of this transformation:

$$M_{ee} = M^*_{ee}$$

$$M_{\mu\tau} = M^*_{\mu\tau}$$

$$M_{e\mu} = M^*_{e\tau}$$

$$M_{\mu\mu} = M^*_{\tau\tau}$$

$$
M_\nu = \begin{pmatrix} C & D & D^* \\ D & A & B \\ D^* & B & A^* \end{pmatrix} \Longrightarrow \begin{cases} \delta = \pm\pi/2 \\ \theta_{23} = \pi/4 \end{cases}
$$

$\nu_e \quad \nu_\mu \leftrightarrow \nu_\tau^c$

Model building strategies

The flavor symmetry is a powerful guiding principle of model building.

The flavor symmetry could be

- ♣ Abelian or non-Abelian
- ♣ Continuous or discrete
- ♣ Local or global
- ♣ Spontaneously or explicitly broken

S_3 , S_4 , A_4 , Z_2 ,
$U(1)_F$, $SU(2)_F$, ...

Advantages of choosing a global + discrete flavor symmetry group G_F.

- ♣ No Goldstone bosons
- ♣ No additional gauge bosons mediating harmful FCNC processes
- ♣ No family-dependent D-terms contributing to sfermion masses
- ♣ Discrete G_F could come from some string compactifications
- ♣ Discrete G_F could be embedded in a continuous symmetry group

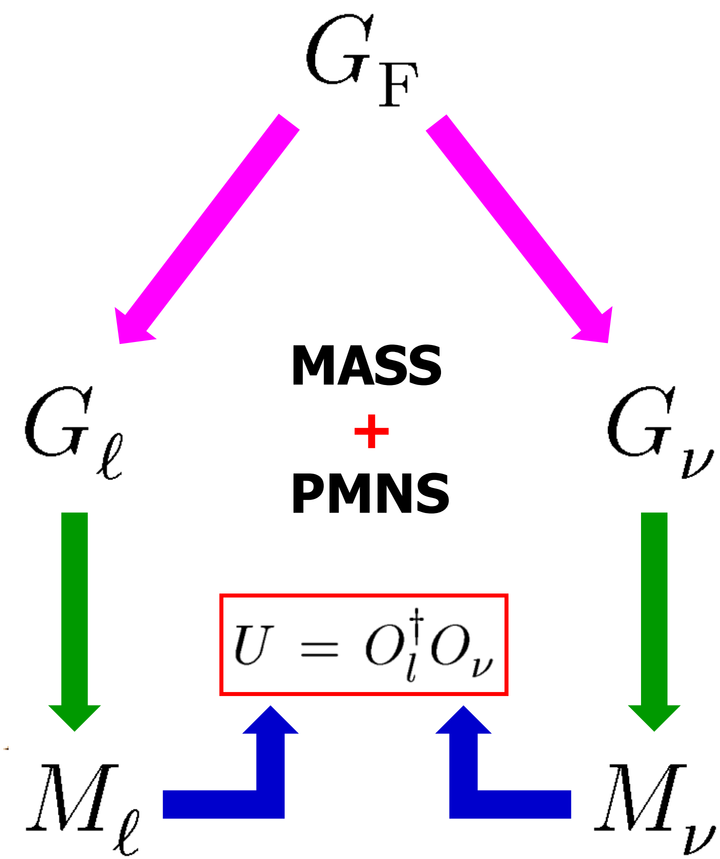
SUSY

Flavor symmetry groups

Some small discrete groups for model building (Altarelli, Feruglio 2010).

Group	d	Irreducible representation
$D_3 \sim S_3$	6	1, 1′, 2
D_4	8	$1_1, \ldots, 1_4$, 2
D_7	14	1, 1′, 2, 2′, 2″
A_4	12	1, 1′, 1″, 3
$A_5 \sim PSL_2(5)$	60	1, 3, 3′, 4, 5
T'	24	1, 1′, 1″, 2, 2′, 2″, 3
S_4	24	1, 1′, 2, 3, 3′
$\Delta(27) \sim Z_3 \rtimes Z_3$	27	$1_1, 1_9, 3, \bar{3}$
$PSL_2(7)$	168	$1, 3, \bar{3}, 6, 7, 8$
$T_7 \sim Z_7 \rtimes Z_3$	21	$1, 1', \bar{1}', 3, \bar{3}$

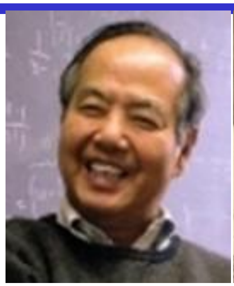
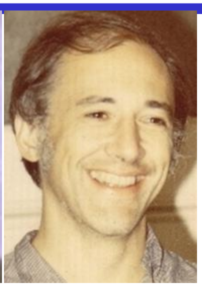
Too many possibilities, but the μ-τ symmetry inclusive



Generalized CP combined with flavor symmetry to predict the phase δ.

The Friedberg-Lee ansatz (1)

A simple example is the Friedberg-Lee ansatz. In the Majorana case the neutrino mass term (2006):

$$
\left.\begin{aligned}
-\mathcal{L}_{\rm mass} = \; & a(\overline{\nu_{\tau\rm L}} - \overline{\nu_{\mu\rm L}})(\nu^c_{\tau\rm L} - \nu^c_{\mu\rm L}) \\
& + b(\overline{\nu_{\mu\rm L}} - \overline{\nu_{e\rm L}})(\nu^c_{\mu\rm L} - \nu^c_{e\rm L}) \\
& + c(\overline{\nu_{e\rm L}} - \overline{\nu_{\tau\rm L}})(\nu^c_{e\rm L} - \nu^c_{\tau\rm L})
\end{aligned}\right\}
$$

Invariant under

$$\boxed{\nu_\alpha \to \nu_\alpha + z \;\; (\text{for } \alpha = e, \mu, \tau)}$$

$$+ m_0(\overline{\nu_{e\rm L}}\nu^c_{e\rm L} + \overline{\nu_{\mu\rm L}}\nu^c_{\mu\rm L} + \overline{\nu_{\tau\rm L}}\nu^c_{\tau\rm L}) + \text{h.c.}$$

Symmetry breaking

where z is a space-time-independent constant element of the Grassmann algebra

The corresponding neutrino mass matrix reads

$$
M_\nu = m_0 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \begin{pmatrix} b+c & -b & -c \\ -b & a+b & -a \\ -c & -a & a+c \end{pmatrix}
$$

Its structure will be further constrained by the μ-τ permutation or reflection symmetry.

μ-τ permutation

$$\boxed{\nu_{\mu\rm L} \leftrightarrow \nu_{\tau\rm L}} \Longrightarrow b = c$$

μ-τ reflection

$$
\boxed{\begin{aligned}
\nu_{e\rm L} &\leftrightarrow (\nu_{e\rm L})^c \\
\nu_{\mu\rm L} &\leftrightarrow (\nu_{\tau\rm L})^c \\
\nu_{\tau\rm L} &\leftrightarrow (\nu_{\mu\rm L})^c
\end{aligned}}
$$

$$b = c^*, \;\; {\rm Im}(a) = {\rm Im}(m_0) = 0$$

The Friedberg-Lee ansatz (2)

Consequently, the neutrino mixing matrix takes the following form:

Case A: all the parameters are real:

$$U_{\rm FL} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \cos\frac{\theta}{2} & 0 & \sin\frac{\theta}{2} \\ 0 & 1 & 0 \\ -\sin\frac{\theta}{2} & 0 & \cos\frac{\theta}{2} \end{pmatrix} \qquad \tan\theta = \frac{\sqrt{3}\,(b-c)}{(b+c)-2a}$$

If $b = c$, one recovers the μ-τ permutation symmetry limit: $\left\{ \begin{array}{l} \theta_{13} = 0 \\ \theta_{23} = \pi/4 \end{array} \right.$

Case B: $b = c^*$ (complex), and the other parameters are real:

$$U_{\rm FL} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \cos\frac{\theta}{2} & 0 & \pm{\rm i}\sin\frac{\theta}{2} \\ 0 & 1 & 0 \\ \pm{\rm i}\sin\frac{\theta}{2} & 0 & \cos\frac{\theta}{2} \end{pmatrix}$$

$$\tan\theta = \frac{\sqrt{3}{\rm Im}\,(b)}{m_0 + a + 2{\rm Re}\,(b)}$$

In this case we'll reach the μ-τ reflection symmetry limit: $\left\{ \begin{array}{l} \delta = \pm\pi/2 \\ \theta_{23} = \pi/4 \end{array} \right.$

Phenomenology (1)

Matter effects: the behavior of neutrino oscillations is modified due to the coherent forward scattering induced by the weak charged-current interactions. The effective Hamiltonian for neutrino propagation:

$$\widetilde{\mathcal{H}}_{\rm eff} = \frac{1}{2E}\left[\widetilde{U}\begin{pmatrix}\widetilde{m}_1^2 & 0 & 0\\ 0 & \widetilde{m}_2^2 & 0\\ 0 & 0 & \widetilde{m}_3^2\end{pmatrix}\widetilde{U}^\dagger\right] = \frac{1}{2E}\left[U\begin{pmatrix}m_1^2 & 0 & 0\\ 0 & m_2^2 & 0\\ 0 & 0 & m_3^2\end{pmatrix}U^\dagger + \begin{pmatrix}A & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}\right]$$

in matter **in vacuum** **correction**

Sum rules between matter and vacuum:

$$A = 2\sqrt{2}\, G_{\rm F} N_e E$$

$$\sum_{i=1}^{3}\widetilde{m}_i^2\widetilde{U}_{\alpha i}\widetilde{U}_{\beta i}^* = \sum_{i=1}^{3} m_i^2 U_{\alpha i}U_{\beta i}^* + \underline{A\delta_{\alpha e}\delta_{e\beta}}$$

$$\sum_{i=1}^{3}\widetilde{m}_i^4\widetilde{U}_{\alpha i}\widetilde{U}_{\beta i}^* = \sum_{i=1}^{3} m_i^2\left[m_i^2 + \underline{A\left(\delta_{\alpha e}+\delta_{e\beta}\right)}\right]U_{\alpha i}U_{\beta i}^* + \underline{A^2\delta_{\alpha e}\delta_{e\beta}}$$

$$\sum_{i=1}^{3}\widetilde{U}_{\alpha i}\widetilde{U}_{\beta i}^* = \sum_{i=1}^{3} U_{\alpha i}U_{\beta i}^* = \delta_{\alpha\beta}$$

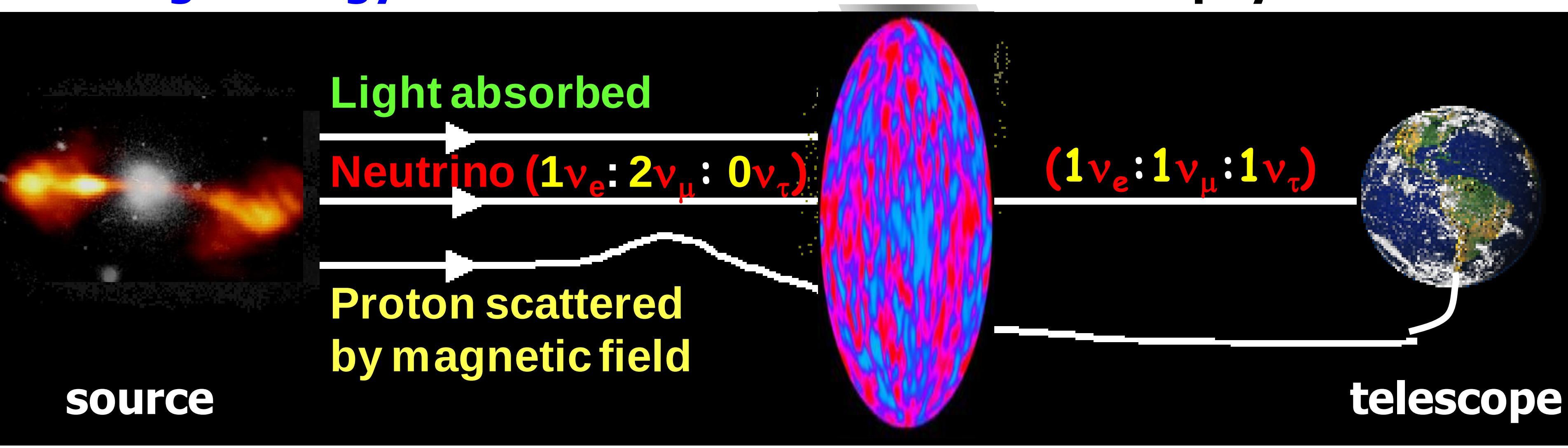
disappear when α, β = μ, τ

A proper phase convention leads us to $|\widetilde{U}_{\mu i}| = |\widetilde{U}_{\tau i}|$ from $|U_{\mu i}| = |U_{\tau i}|$**.**

Namely, matter effects (a constant profile) respect the μ-τ symmetry.

Phenomenology (2)

Ultrahigh-energy cosmic neutrinos from distant astrophysical sources



A conventional UHE cosmic neutrino source ($p + p$ or $p + \gamma$ collisions)

$$\Phi^{\rm T}_{\mu} - \Phi^{\rm T}_{\tau} = \frac{\Phi_0}{3} \sum_i \left(|U_{\mu i}|^2 - |U_{\tau i}|^2\right)^2$$

sensitive to μ-τ flavor symmetry

$$\Phi^{\rm T}_{e} : \Phi^{\rm T}_{\mu} : \Phi^{\rm T}_{\tau} = (1 + D_e) : (1 + D_\mu) : (1 + D_\tau)$$

with $D_e = -2\Delta$, $D_\mu = \Delta + \overline{\Delta}$ and $D_\tau = \Delta - \overline{\Delta}$

$$\Delta \simeq \frac{1}{2} \sin^2 2\theta_{12} \sin\varepsilon - \frac{1}{4} \sin 4\theta_{12} \sin\theta_{13} \cos\delta$$

$\varepsilon \equiv \theta_{23} - \pi/4$

$$\overline{\Delta} \simeq \left(4 - \sin^2 2\theta_{12}\right) \sin^2\varepsilon + \sin^2 2\theta_{12} \sin^2\theta_{13} \cos^2\delta + \sin 4\theta_{12} \sin\varepsilon \sin\theta_{13} \cos\delta$$

Summary

Z.Z.X., Z.H. Zhao (1512.04207)

—— A review of mu-tau flavor symmetry in neutrino physics

Report on Progress in Physics in printing, with ~ 350 references.

C.S. Wu: It is easy to do the right thing once you have the right ideas.

I.I. Rabi: Physics needs new ideas. But to have a new idea is a very difficult task…. (Berezhiani's talk)

L.C. Pauling: The best way to have a good idea is to have a lot of ideas.

Open questions

We've learnt a lot from ν oscillations:

$$\boxed{\Delta m^2_{21}\ ,\ \left|\Delta m^2_{31}\right|\ ,\ \theta_{12}\ ,\ \theta_{13}\ ,\ \theta_{23}}$$

It's more exciting that the SM is incomplete, although the Higgs has been discovered.

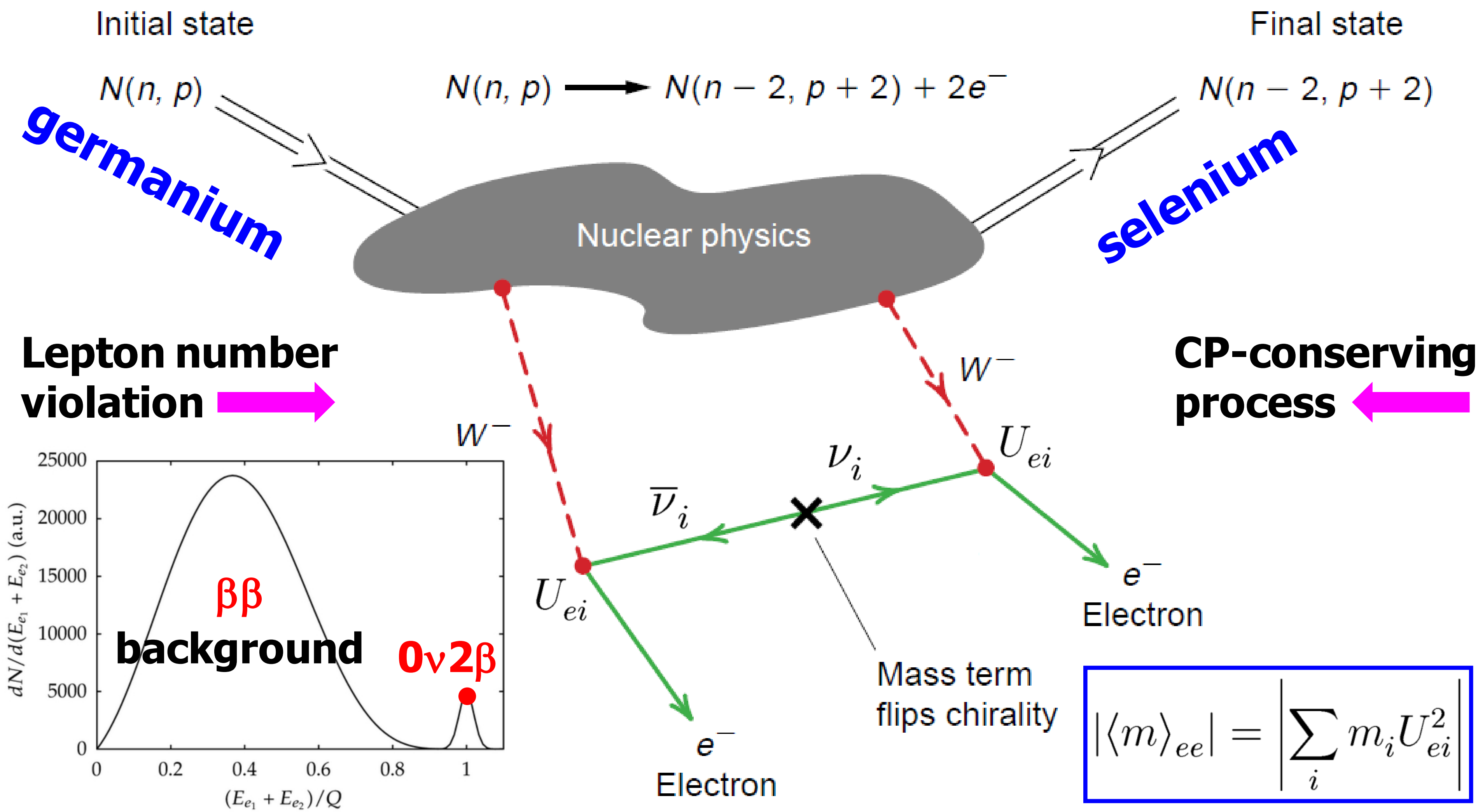
But a number of open questions:

- **the Majorana nature?**
- **the absolute ν mass scale?**
- **the ν mass hierarchy?**
- **the octant of θ_{23}?**
- **the Dirac phase δ?**
- **the Majorana phases? ......**

There are many other open questions about ν's in particle physics, cosmology, astrophysics .....

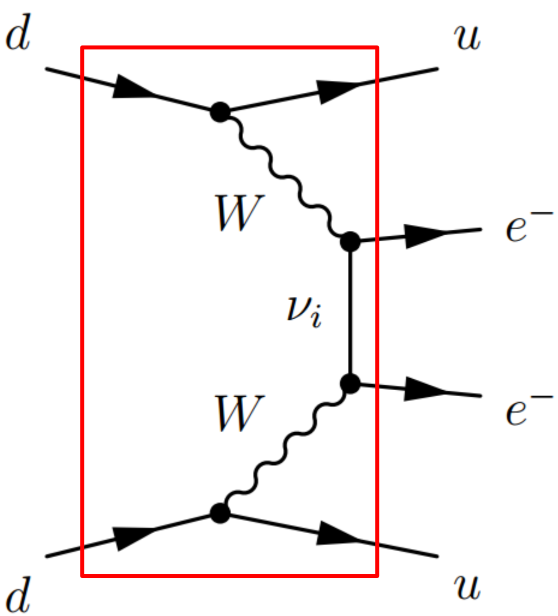
Majorana: 0ν2β decays

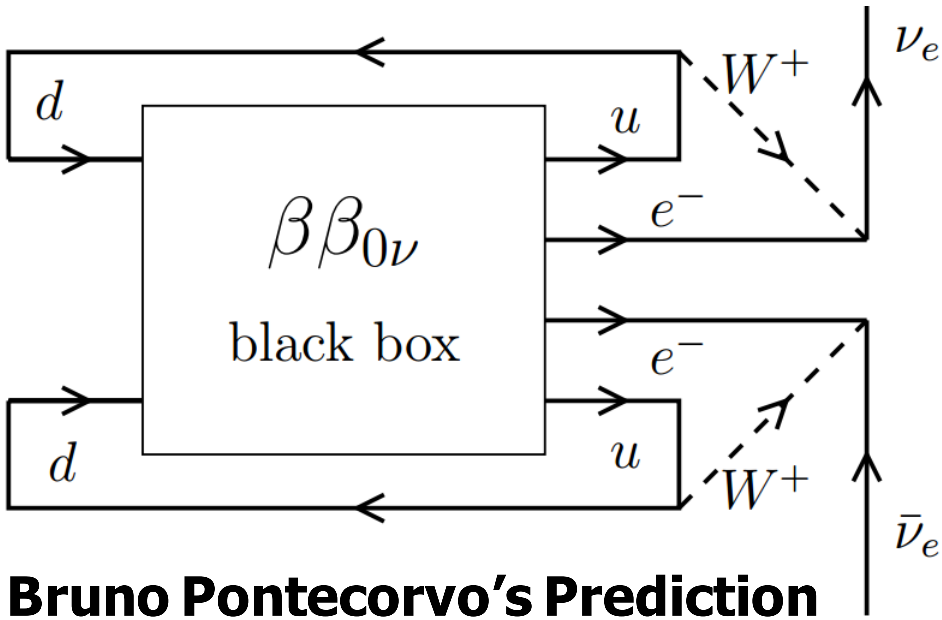
The neutrinoless double beta decay can happen if massive neutrinos are the Majorana particles (W.H. Furry 1939):



Schechter-Valle theorem

THEOREM (1982): if a 0νββ decay happens, there must be an effective Majorana mass term.





Bruno Pontecorvo's Prediction

指导我们试验的理论基础是SV定理

Four-loop ν mass:

$$\boxed{\delta m_\nu = \mathcal{O}(10^{-24}\,\mathrm{eV})}$$

(Duerr, Lindner, Merle, 2011)

Note: The black box can in principle have many different processes (new physics). Only in the simplest case, which is most interesting, it's likely to constrain neutrino masses

YES or NO?

QUESTION: are massive neutrinos the **Majorana** particles?

One might be able to answer **YES** through a measurement of the $0\nu\beta\beta$ decay or other **LNV** processes someday, but how to answer with **NO**?

The same question: how to distinguish between **Dirac** and **Majorana** neutrinos in a realistic experiment?

Answer 1: The $0\nu\beta\beta$ decay is currently the only possibility.

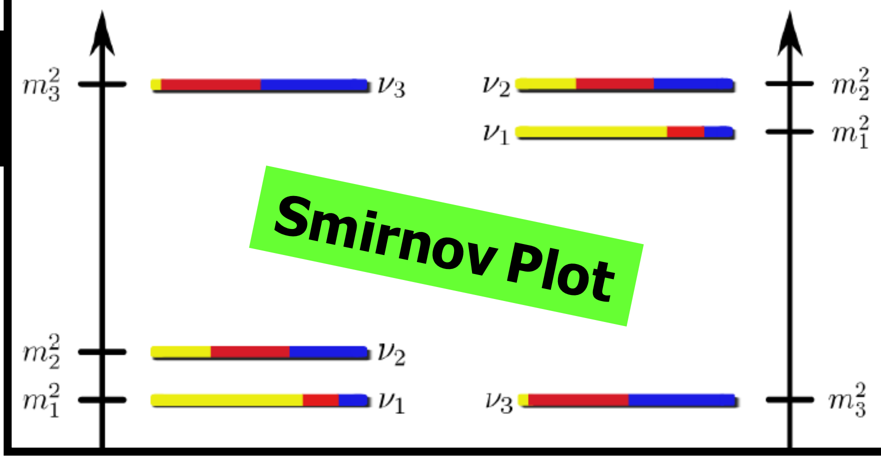
Answer 2: In principle their dipole moments are different.

Answer 3: They show different behavior if nonrelativistic.

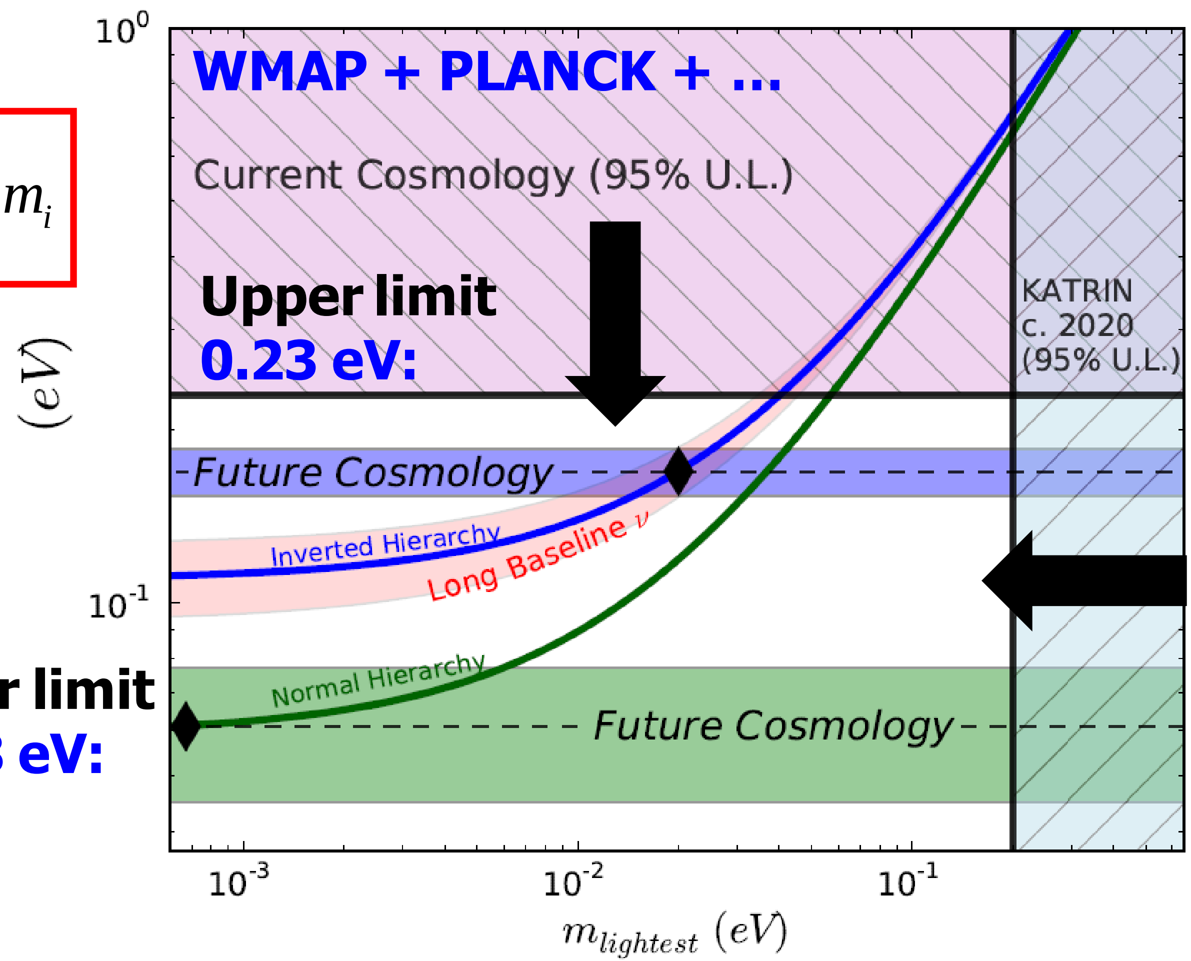
Light neutrino masses

Three ways to probe absolute ν mass:

- ★ **the β decay,**
- ★ **the 0νββ decay,**
- ★ **cosmology (CMB + LSS).**



$$\boxed{\sum_{i=1}^{3} m_i}$$



Lower limit
0.058 eV:

mass scale ≤ O(0.1) eV

Why so tiny?

arXiv:1309.5383

Stage-4 CMB

$$\sigma\left(\sum m_\nu\right) = 16\ \text{meV}$$
$$\sigma\left(N_{\text{eff}}\right) = 0.020\,.$$

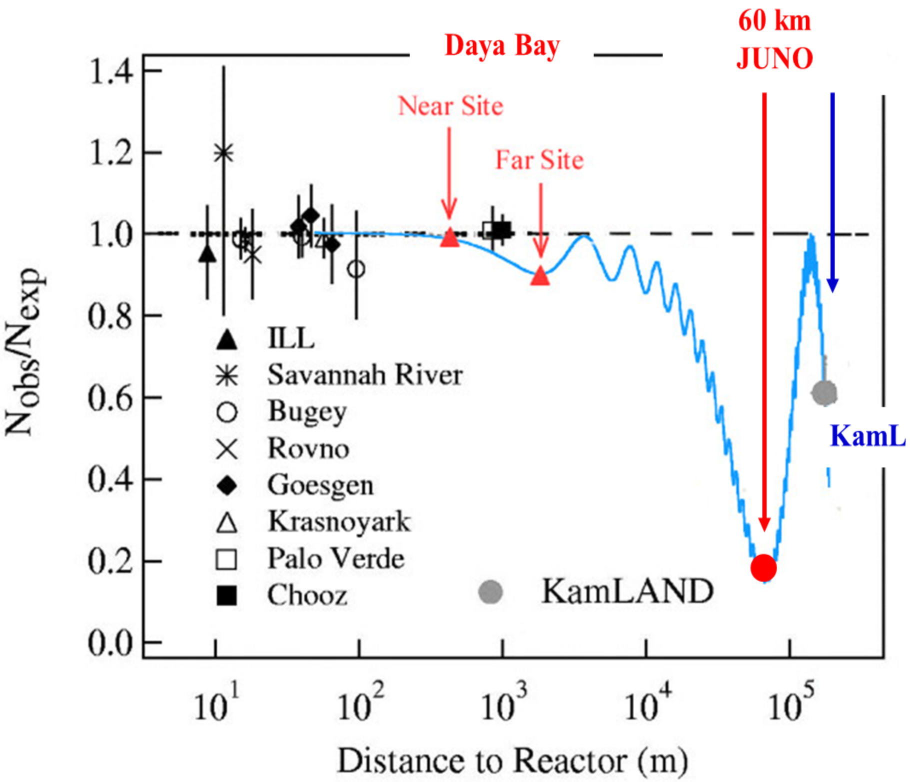
Neutrino mass hierarchy

Accelerator/atmospheric: terrestrial matter effects play crucial roles.

$$\boxed{\Delta m^2_{31} \mp 2\sqrt{2} G_F N_e E}$$

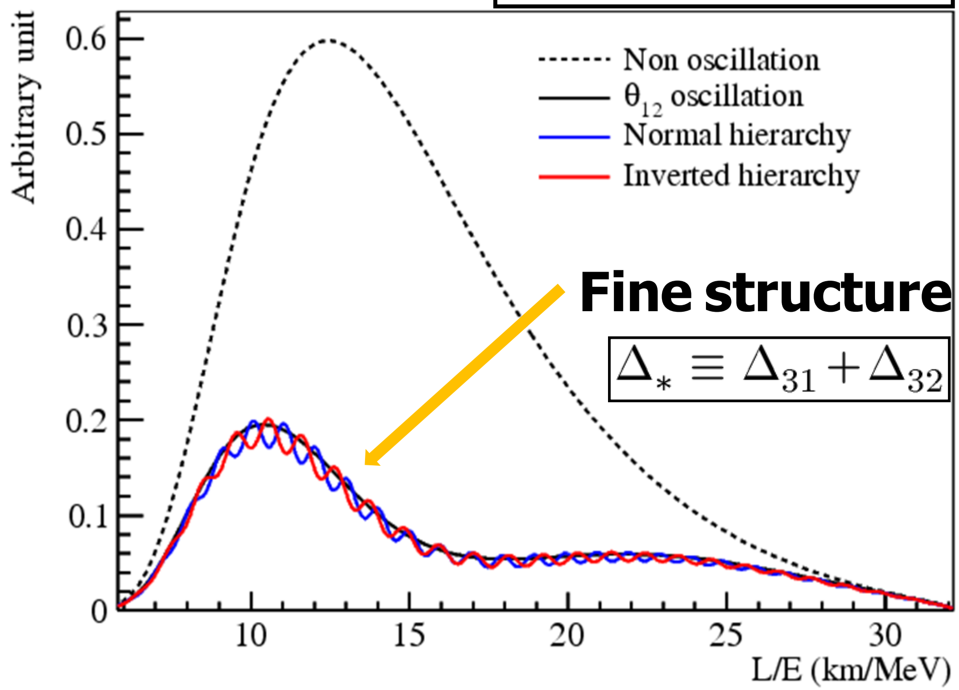
T2K, NOνA, SK, PINGU, INO, …

Reactor (JUNO): Optimum baseline at the valley of Δm^2_{21} oscillations, corrected by the fine structure of Δm^2_{31} oscillations.



JUNO / RENO-50

$$\boxed{F_{ji} \equiv \Delta_{ji} L / (4E)}$$



$$\boxed{\Delta_* \equiv \Delta_{31} + \Delta_{32}}$$

$$\boxed{P(\overline{\nu}_e \to \overline{\nu}_e) = 1 - \sin^2 2\theta_{12} \cos^4 \theta_{13} \sin^2 F_{21} - \frac{1}{2} \sin^2 2\theta_{13} \left(1 - \cos F_* \cos F_{21} + \cos 2\theta_{12} \sin F_* \sin F_{21}\right)}$$

JUNO in progress

	Daya Bay	Yangjiang	Taishan
Status	running	construction	construction
Power/GW	17.4	17.4	18.4

Roman Mythology

- Idea in 2008
- 20 kton LS detector
  3% E-resolution
- Approved in 2/2013
  ~ 2 billion CNY

JUNO collaboration

Europe (24)

France(5)
APC Paris
CPPM Marseille
IPHC Strasbourg
LLR Paris
Subatech Nantes

Czech(1)
Charles U

Finland(1)
U.Oulu

Russia(2)
INR Moscow
JINR

Italy(7)
INFN-Frascati
INFN-Ferrara
INFN-Milano
INFN-Mi-Bicocca
INFN-Padova
INFN-Perugia
INFN-Roma 3

Armenia(1)
Yerevan Phys. Inst.

Belgium(1)
ULB

Germany(6)
FZ Julich
RWTH Aachen
TUM
U.Hamburg
U.Mainz
U.Tuebingen

America(3)

US(2)
UMD
UMD-Geo

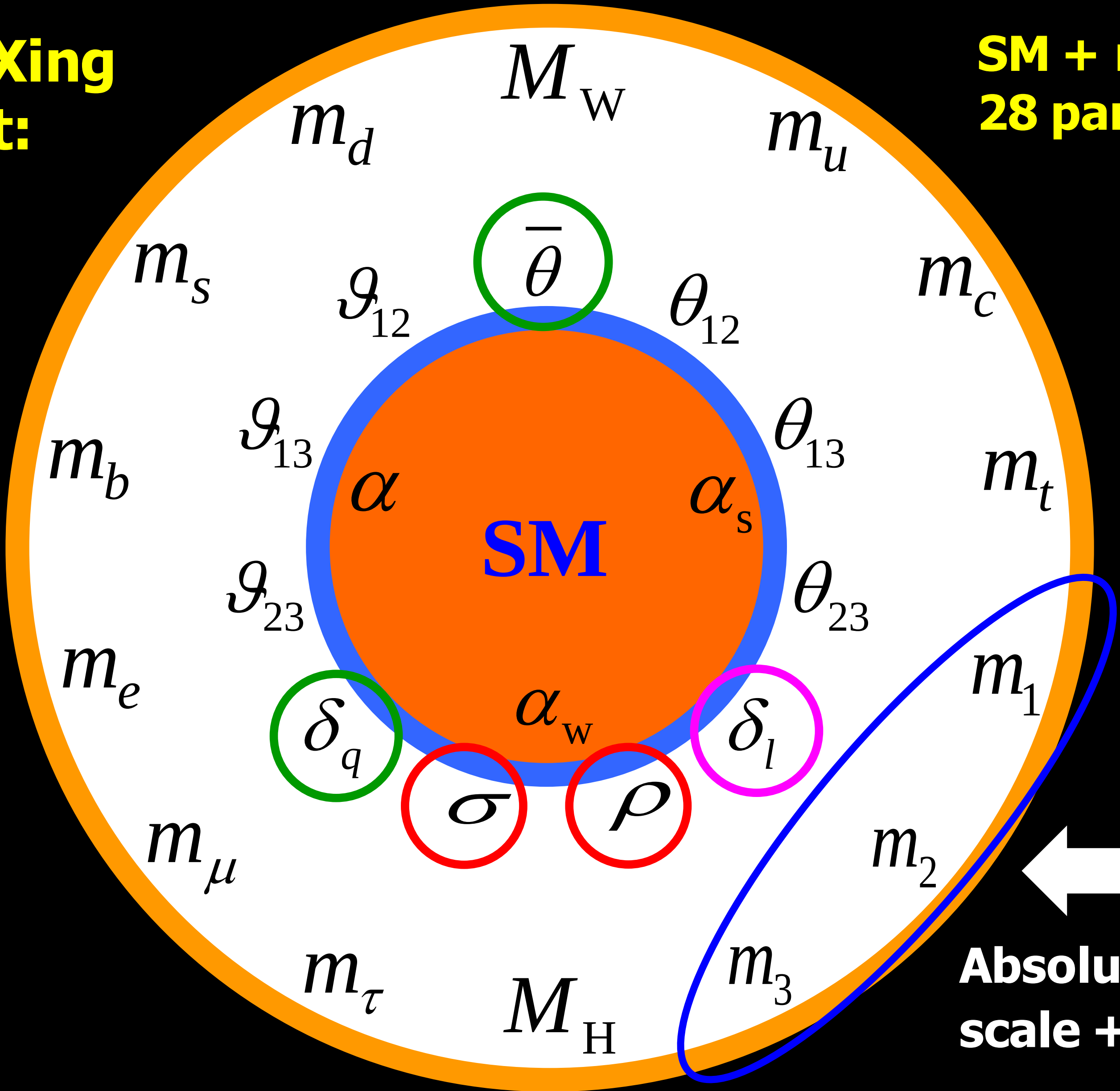
Chile(1)
Catholic Univ. of Chile

Asia (28)

BJ Nor. U.
CAGS
Chongqing U.
CIAE
DGUT
ECUST
Guangxi U.
HIT
IHEP
Jilin U.
Nanjing U.
Nankai U.
Chiao-Tung U.
Taiwan U.
United U.
NCEPU
Pekin U.
Shandong U.
Shanghai JT U.
Sichuan U.
SYSU
Tsinghua U.
UCAS
USTC
Wuhan U.
Wuyi U.
Xi'an JT U.
Xiamen U.

Outlook: Success in flavors is still a long way off

Fritzsch-Xing pizza plot:



SM + massive ν's
28 parameters

Absolute ν mass scale + hierarchy

Martinus Veltman: Anyway, we go on until we go wrong