The Standard Model of Electroweak & Strong Interactions

Theory and Practice

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Lecture Pre-requisites:

I: ABC of Quantum Field Theory:

Ref. "An Introduction to QFT"
by M. Peskin & D. Schroeder

A. Fermion fields: "f, ψ" (spin-1/2, e^-, quarks…)
B. Vector fields: "A^μ, V^μ" (spin-1, photon, W,Z…)
C. Scalar fields: "π, ϕ, H" (pions, Higgs …)
D. Lagrangian formulation; Perturbation Theo.
E. Feynman rules, Feynman diagrams
F. Field-theoretic calculations:
Amplitudes, cross sections, loops & renormalization, running couplings

II: ABC of Particle Physics:

Ref.1 “Introduction to Elementary Particles ”
by D. Griffiths (中译本 by Prof.王青)

Ref.2: “Quarks & Leptons”
by F. Halzen & A. Martin

A. Hadrons (p^+, n^0, $\pi^{0,\pm}$, $K^{0,\pm}$…)

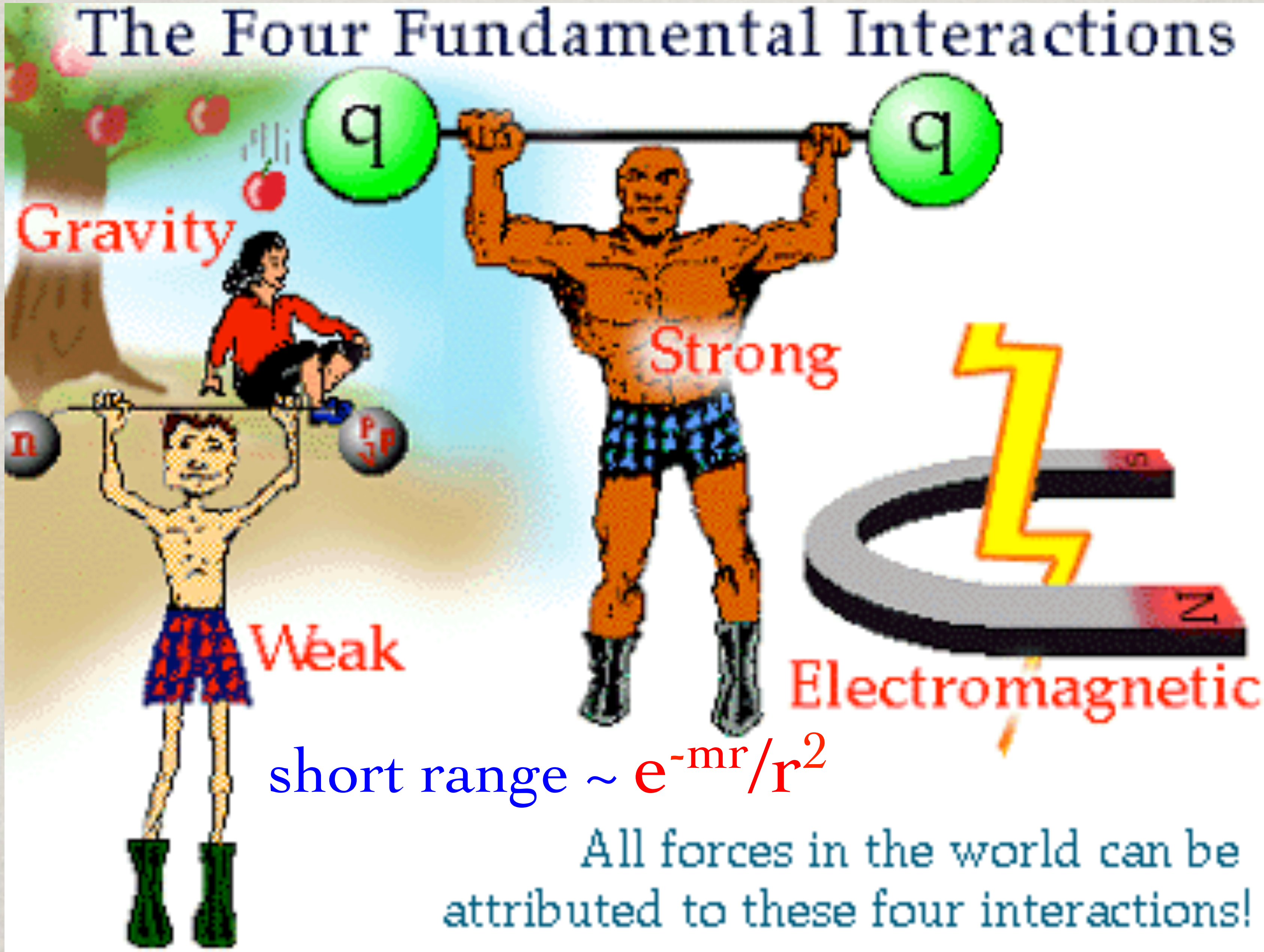
B. Quarks (constituents for hadrons)

C. Leptons (e, μ, τ, ν’s …still elementary)

Get prepared! Try the “Homework”!

The Nature of Forces:

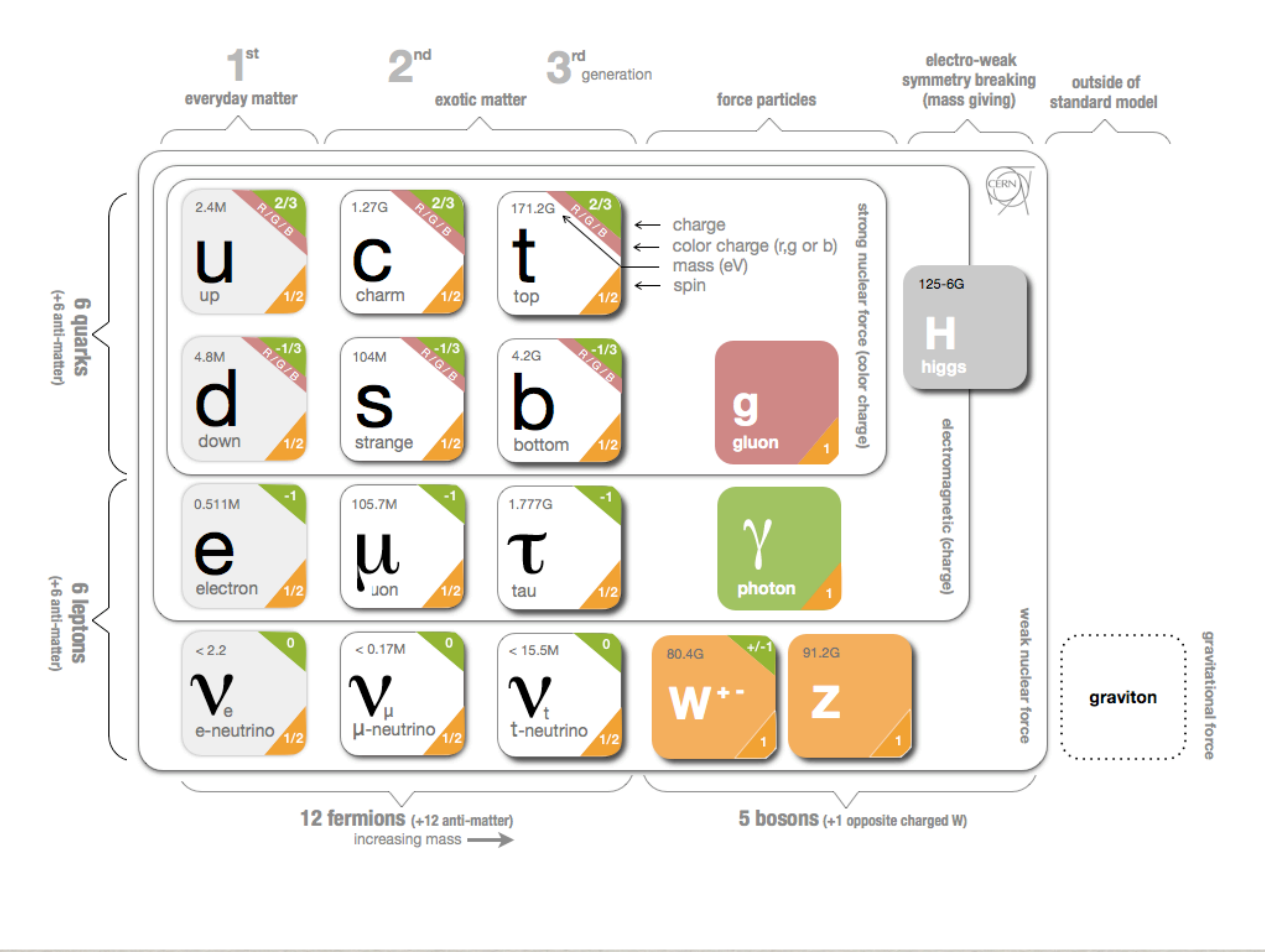
long range
~$(G_N m_1 m_2)/r^2$
→ GR



long range
~$(\alpha\, e_1 e_2)/r^2$
→E&M

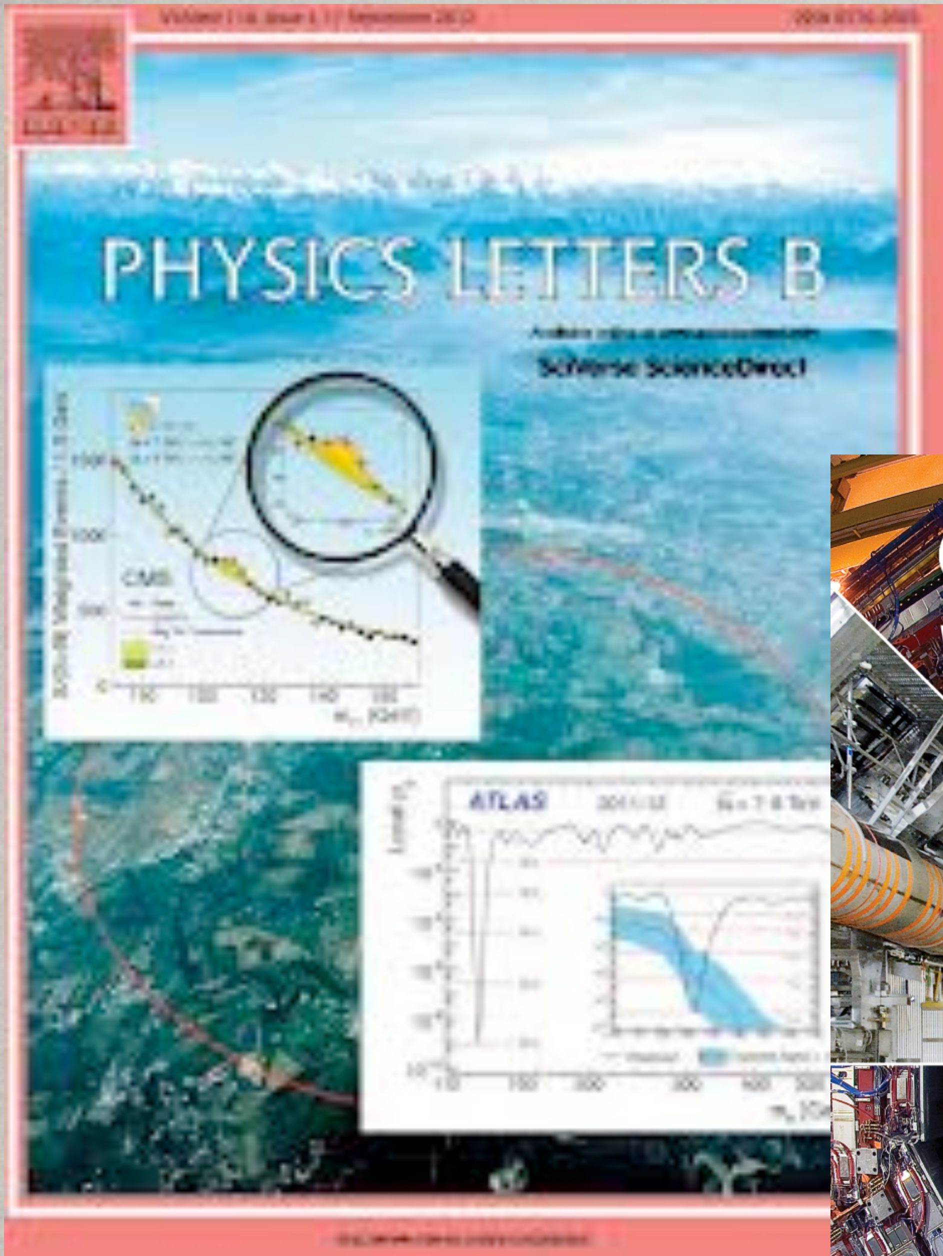
In these lectures, I will bring you to the foremost stage of our understanding of Nature, beyond the above.

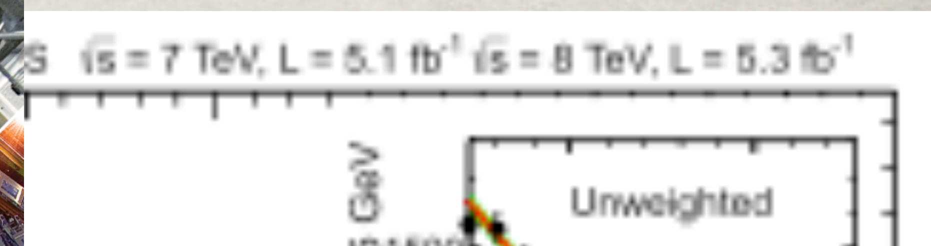
Standard Model of Particle Physics:



NOUNCEMENT 4TH OF JULY, 2012:

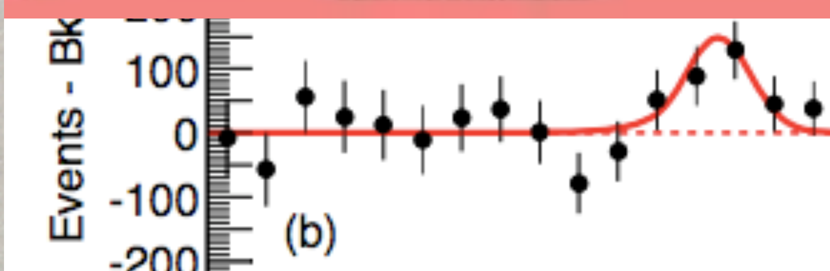
OSON DECAY TO TWO PHOTONS





Mosaic of the CMS and ATLAS detectors (as in 2007), part of the Large Hadron Collider at CERN. In 2012, resea teams used these detectors to fingerprint decay products from the long-sought Higgs boson and determine its ma successfully testing a key prediction of the standard model of particle physics.

Photos: Maximilien Brice and Claudia Marcelloni/CERN



The combined signa

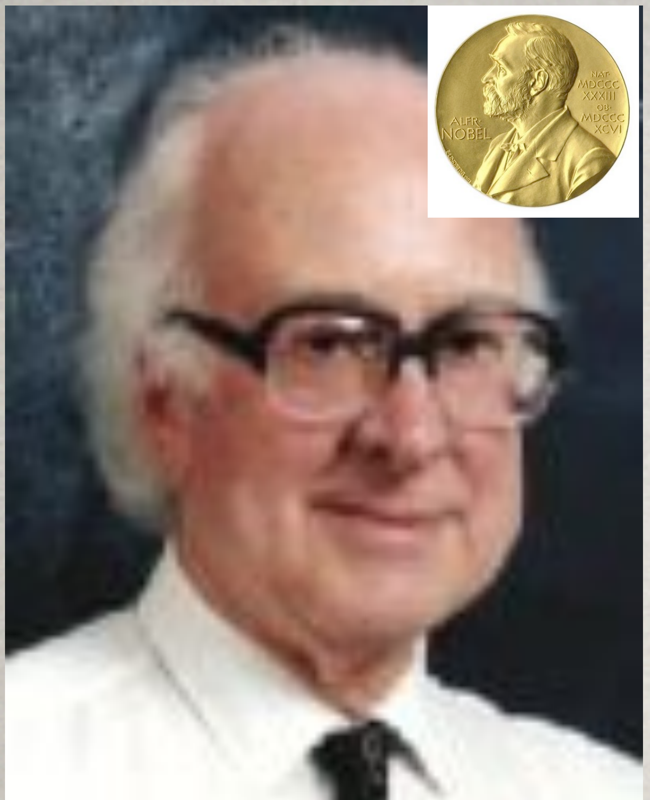
ATLAS: 5

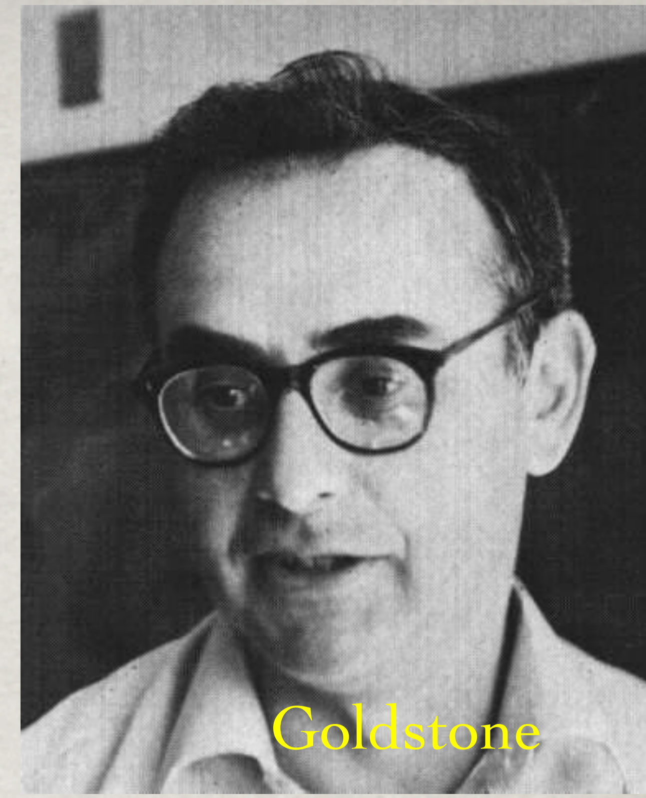
Phys. Lett. B716, 1 (2012)   Phys. L

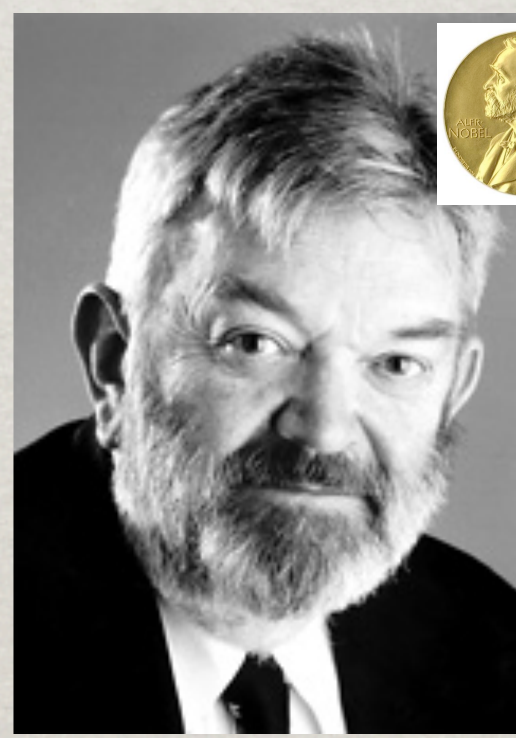
François Englert and Peter W. Higgs

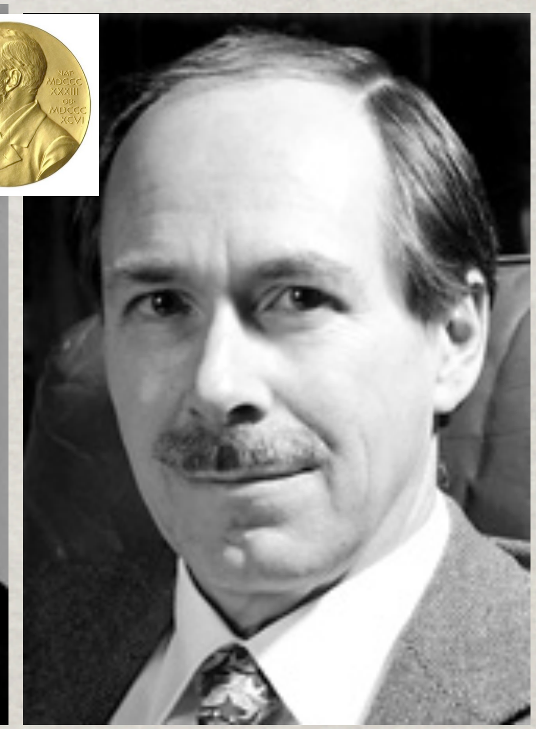
"for the theoretical discovery of a mechanism that contributes to our understanding of the origin of mass of subatomic particles, and which recently was confirmed through the discovery of the predicted fundamental particle, by the ATLAS and CMS experiments at CERN's Large Hadron Collider"

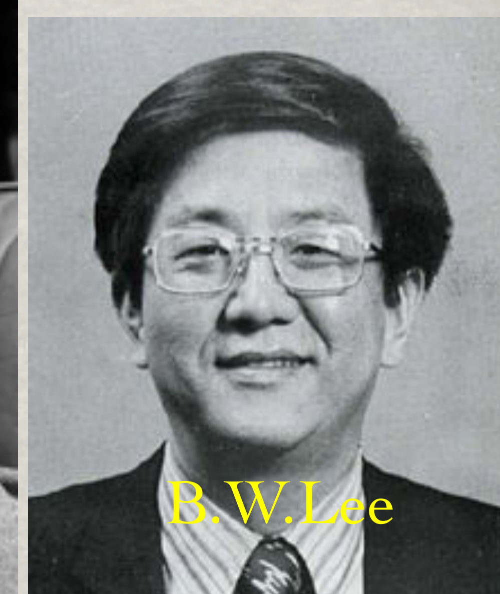




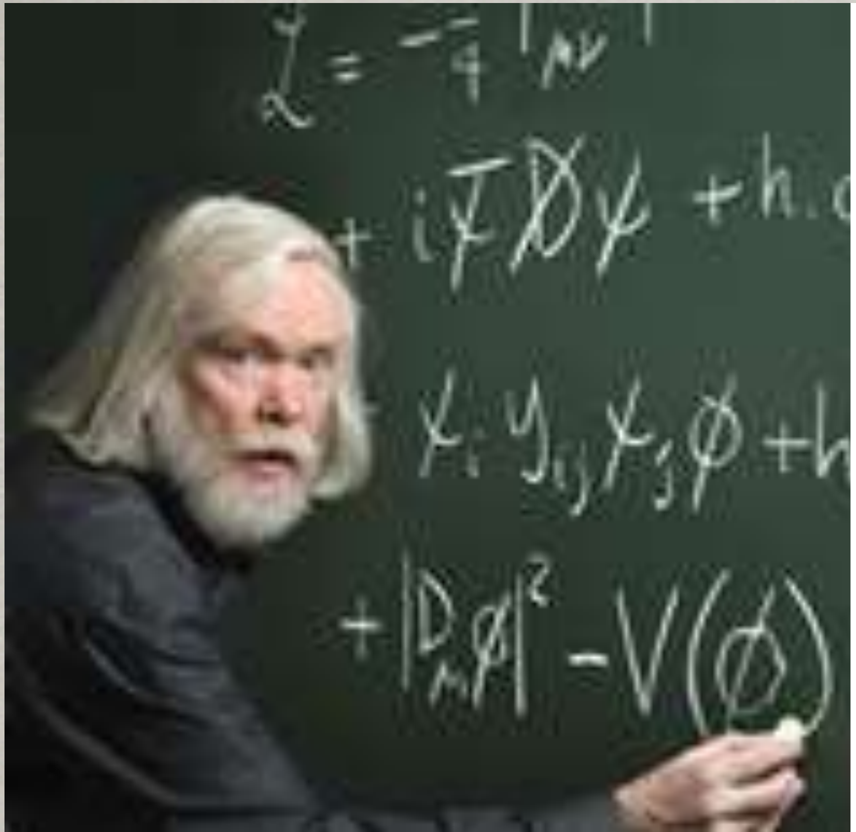
The Higgs mechanism (1964)







The Standard Model (1960-1967, 1972)



A PHENOMENOLOGICAL PROFILE OF THE HIGGS BOSON

John ELLIS, Mary K. GAILLARD * and D.V. NANOPOULOS **
CERN, Geneva

Received 7 November 1975

A discussion is given of the production, decay and observability of the scalar Higgs boson H expected in gauge theories of the weak and electromagnetic interactions such as the Weinberg-Salam model. After reviewing previous experimental limits on the mass of the Higgs boson, we give a speculative cosmological argument for a small mass. If its mass is similar to that of the pion, the Higgs boson may be visible in the reactions $\pi^- p \to Hn$ or $\gamma p \to Hp$ near threshold. If its mass is $\lesssim 300$ MeV, the Higgs boson may be present in the decays of kaons with a branching ratio $O(10^{-7})$, or in the decays of one of the new particles: $3.7 \to 3.1 + H$ with a branching ratio $O(10^{-4})$. If its mass is $\leqslant 4$ GeV, the Higgs boson may be visible in the reaction $pp \to H + X$, $H \to \mu^+\mu^-$. If the Higgs boson has a mass $\leqslant 2m_\mu$, the decays $H \to e^+e^-$ and $H \to \gamma\gamma$ dominate, and the lifetime is $O(6 \times 10^{-4}$ to $2 \times 10^{-12})$ seconds. As thresholds for heavier particles (pions, strange particles, new particles) are crossed, decays into them become dominant, and the lifetime decreases rapidly to $O(10^{-20})$ sec for a Higgs boson of mass 10 GeV. Decay branching ratios in principle enable the quark masses to be determined.

Higgs Phenomenology (70's)

50 years theory work!

Fermi National Accelerator Laboratory

FERMILAB-Pub-84/17-T
LBL-16875
DOE/ER/01545-345
February, 1984

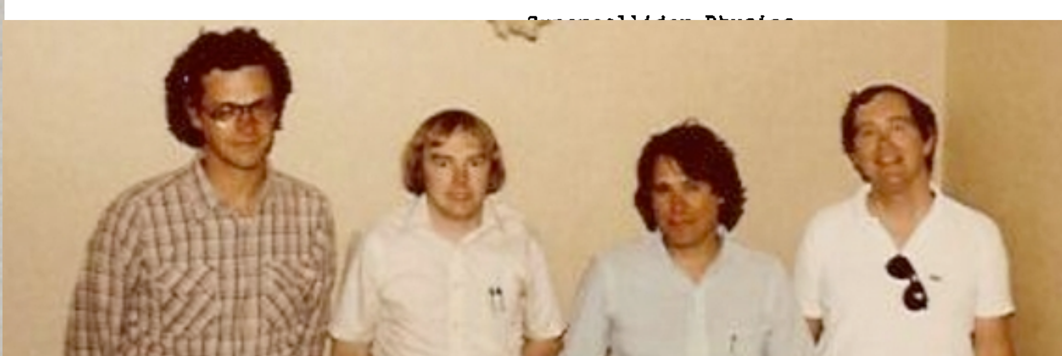
The "EHLQ" (80's)

FRONTIERS IN PHYSICS

COLLIDER PHYSICS

FRONTIERS IN PHYSICS

THE HIGGS HUNTER'S GUIDE



Lawrence Berkeley Laboratory†
Berkeley, CA 94720

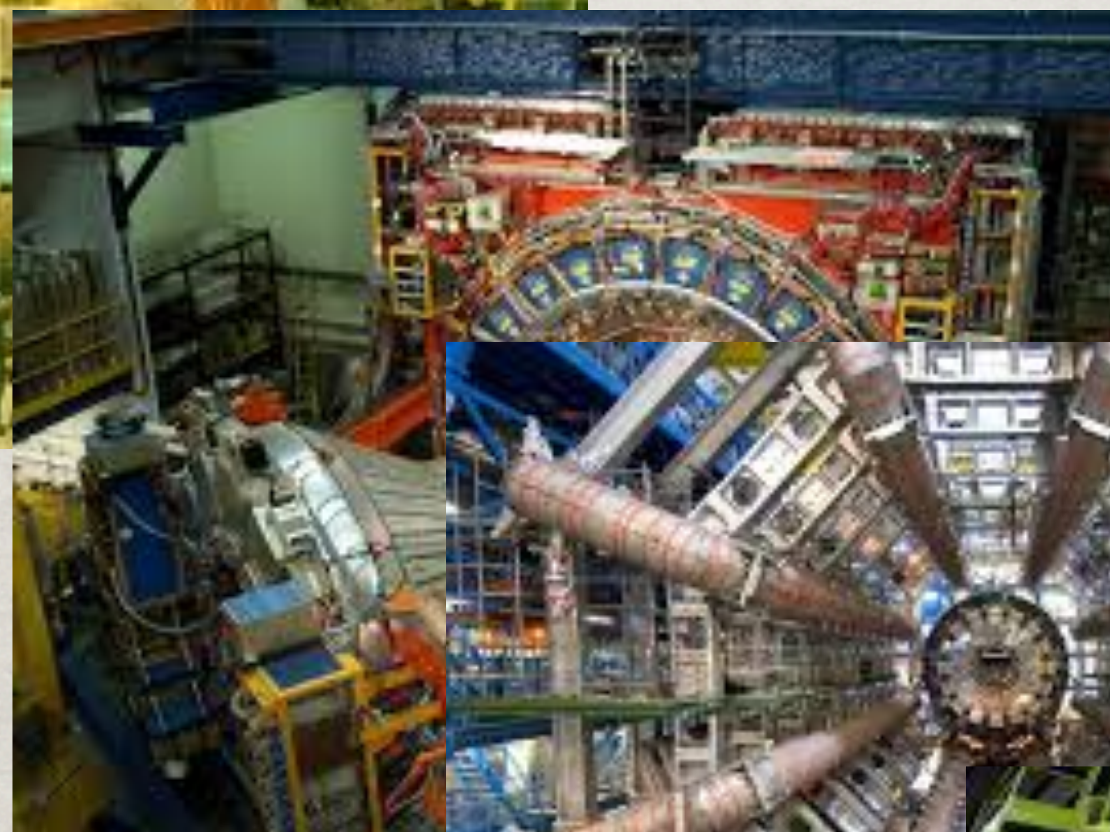
K. LANE
Ohio State University,† Columbus, OH 43210

C. QUIGG
Fermi National Accelerator Laboratory*
P.O. Box 500, Batavia, IL 60510

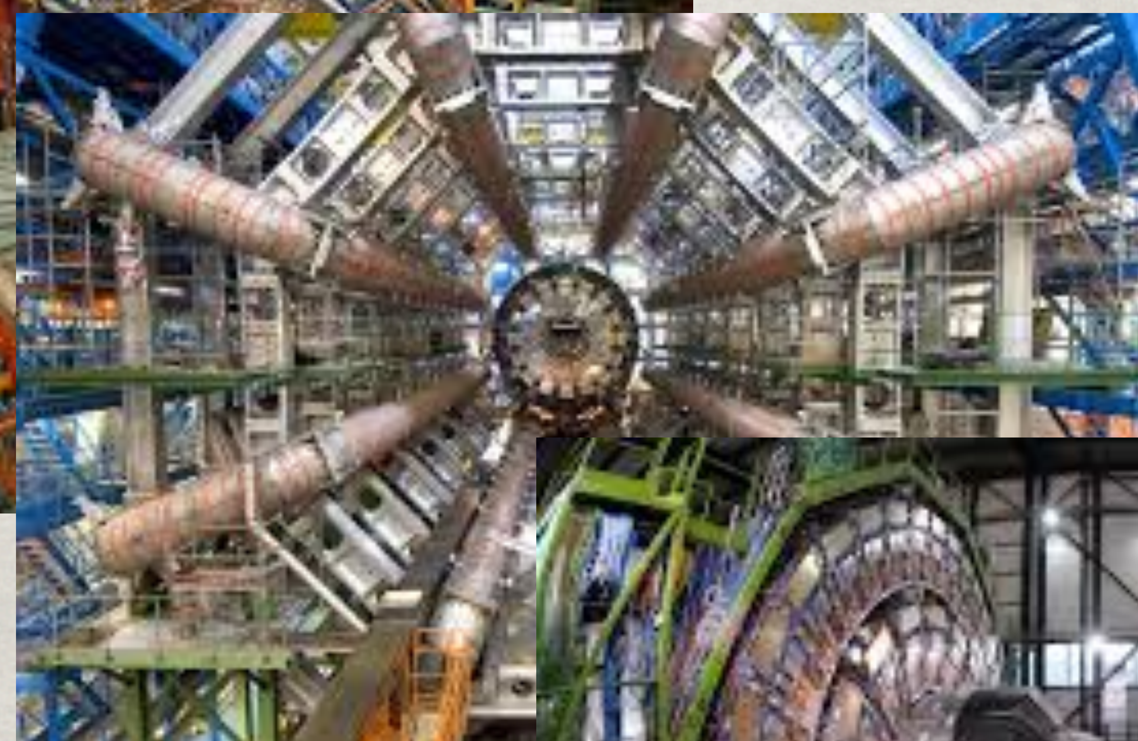
25 year's work by thousands experimenters

We made it !

CDF@Tevatron



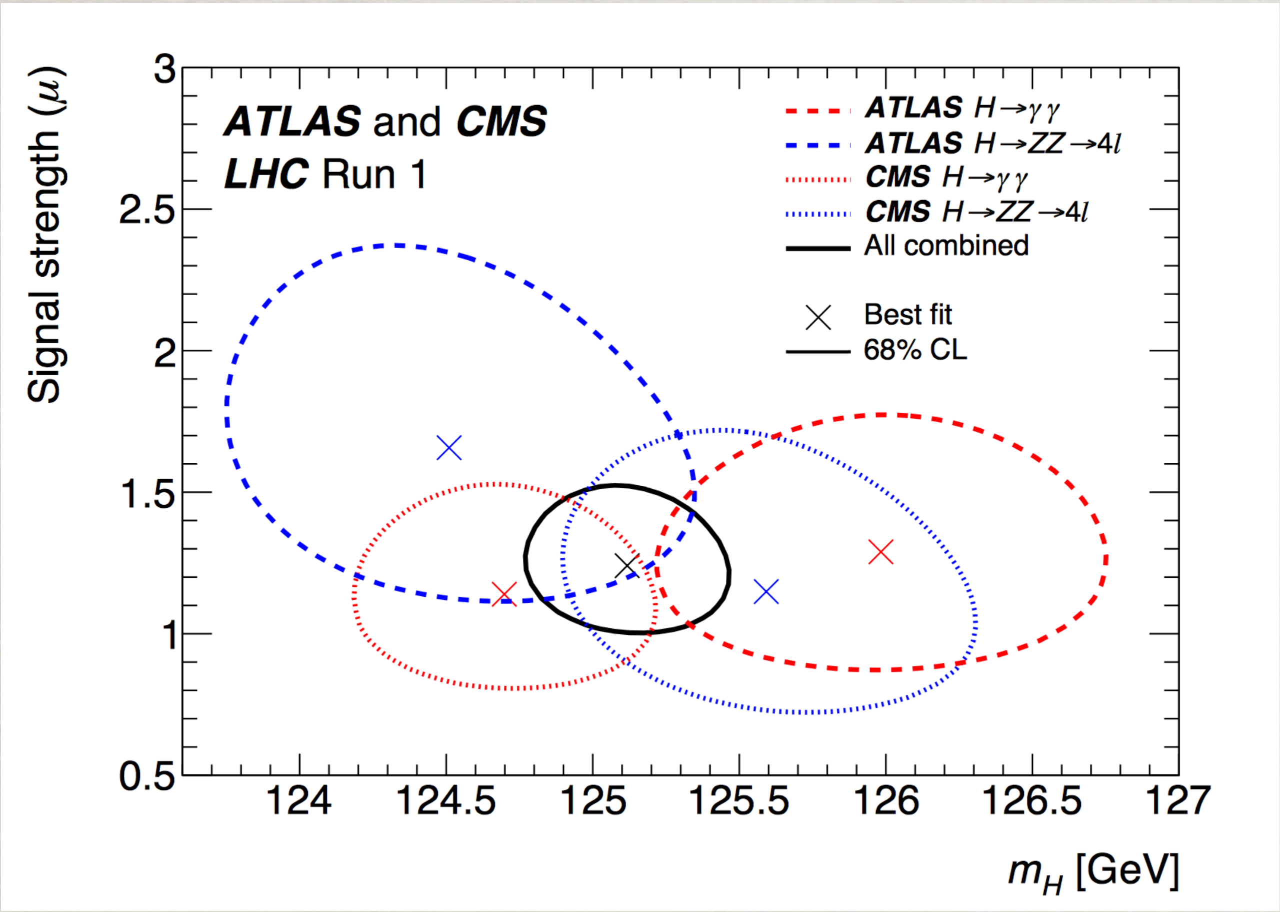
ALEPH@LEP

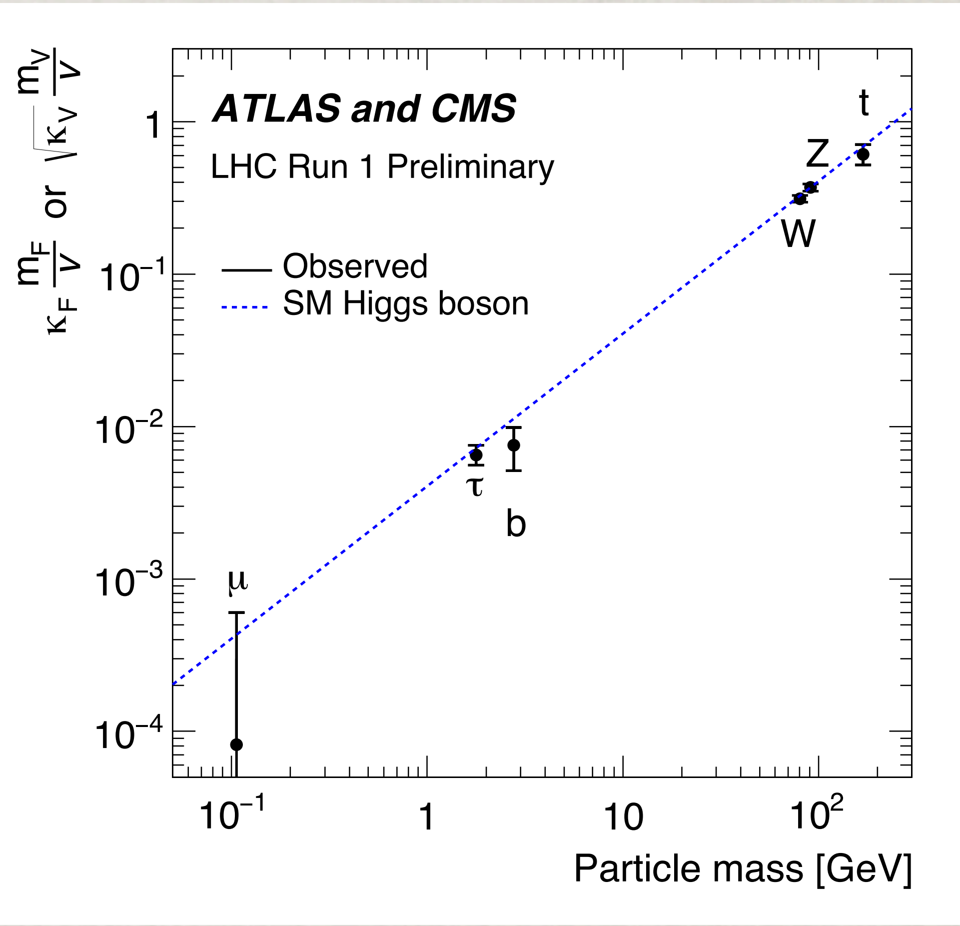


ATLAS

CMS

LHC Update 2016:



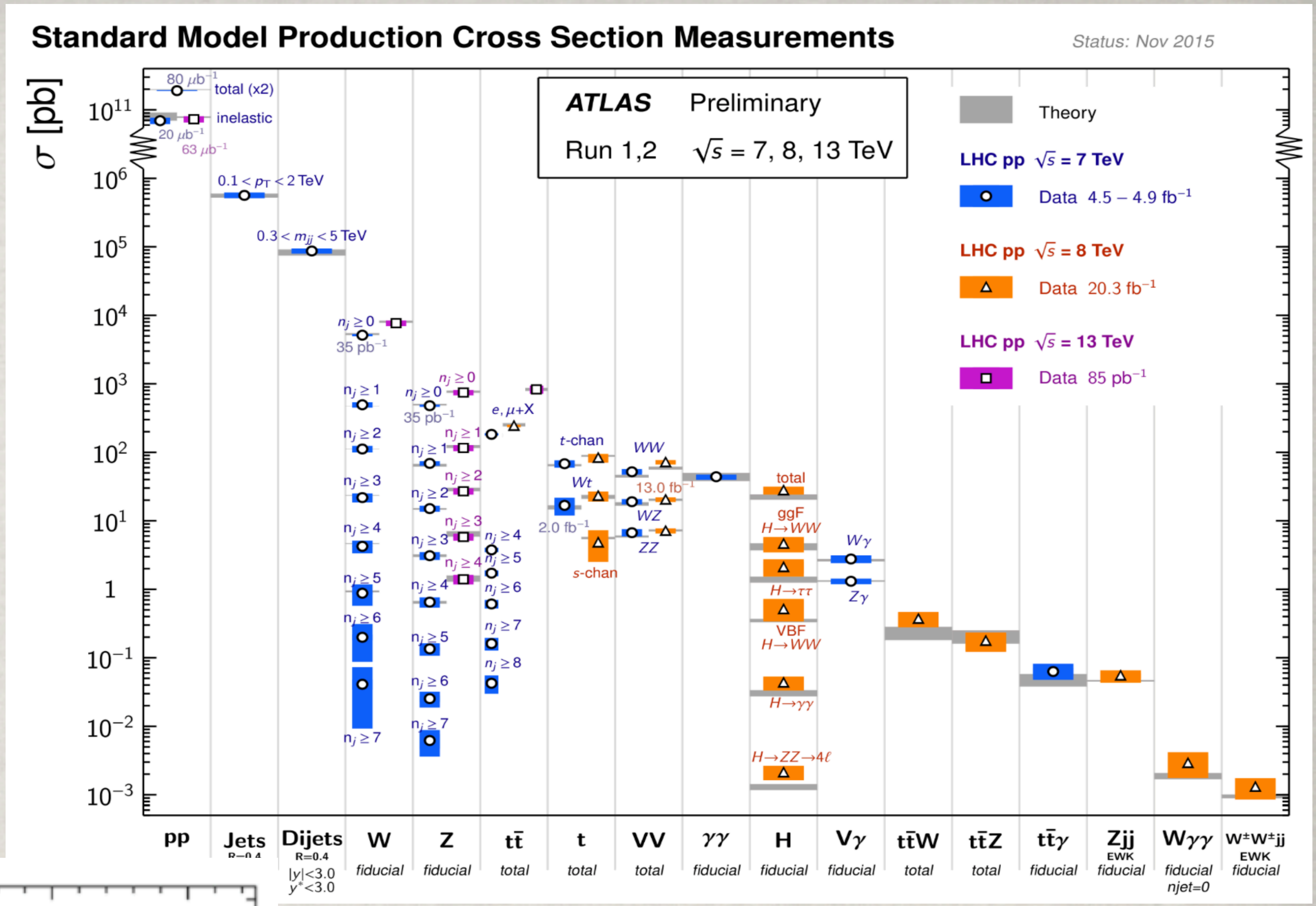


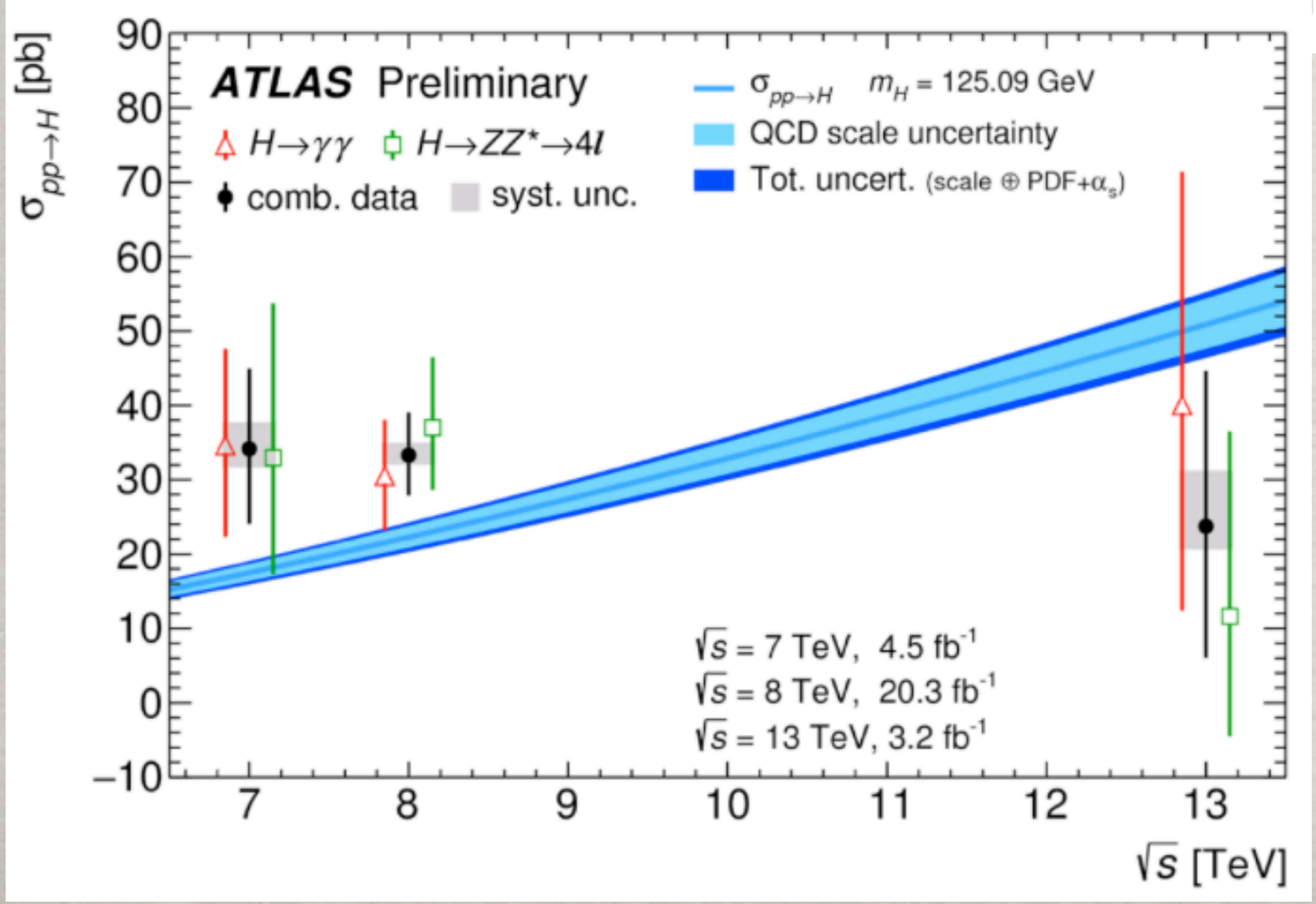
- **Mass accuracy** **0.2%: 125.09±0.21±0.11 GeV**
- 5σ for both fermion coupling h → ττ
- & bosonic coupling WW→h
- Couplings proportional to mass

All indications point to a SM-like Higgs boson "elementary" at a scale Λ < O(1 TeV)

SM re-discovered at LHC

Look forward to the LHC Run 2 Results! (see Profs. Mellado, Liang, Whiteson, Zhuang …)





Higgs to be re-discovered

What We Know

1. X → γγ :

- it's neutral, a boson
- can be spin-0
- cannot be spin-1 (Landau-Yang's theorem)
- can be spin-2 unlikely/disfavored

$$\frac{f_s}{\Lambda} H A^{\mu\nu} A_{\mu\nu}$$

$$\frac{f_T}{\Lambda} T_{\mu\mu'} \; g_{\nu\nu'} A^{\mu\nu} A^{\mu'\nu'}$$

2. X → ZZ, W^+W^- :

- Vacuum Q#: EWSB
- CP-odd part of gauge interaction must be small

$$(v+H)^2 \, g^2 V^\mu V_\mu$$

$$\frac{f_A}{\Lambda} A \, \tilde{V}^{\mu\nu} V_{\mu\nu}$$

3. X not to $\mu^+\mu^-$, e^+e^-, but $\tau^+\tau^-$ seen!

- Non-universal leptonic couplings unlike the gauge couplings

$$(1 + H/v)\, m_f \bar{\psi}_f \psi_f$$

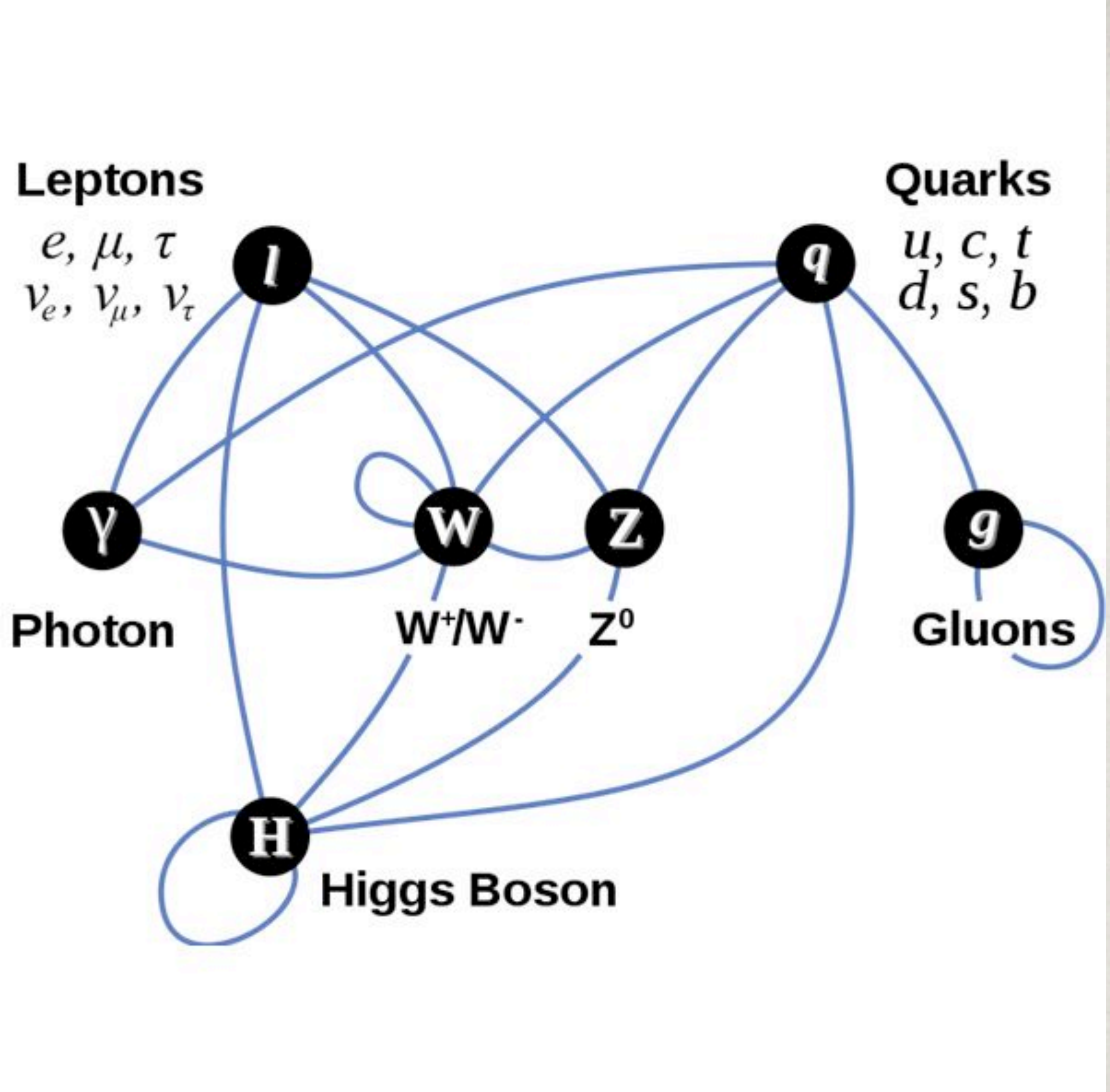
4. $Xt\bar{t}$ needed for gluon fusion

   $X \rightarrow b\bar{b}$ seen (vaguely)

- Non-universal quark couplings

It couples to mass, it is a new class.

It IS a Higgs!



Completion of the SM:

A perturbative, renormalizable theory, valid up to a scale

TeV ? ..., M_{Pl} ?

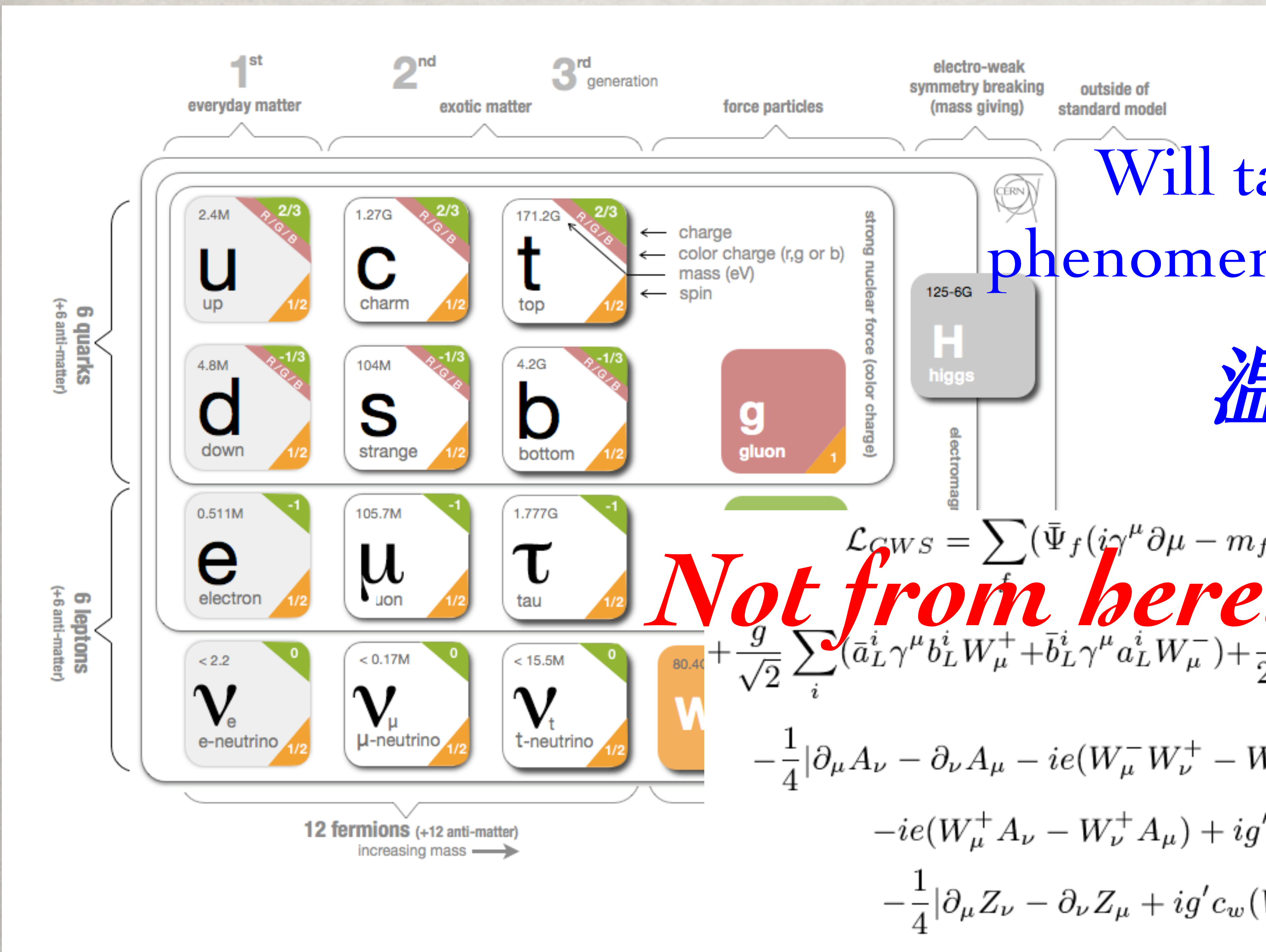
All known physics

$$W = \int_{k<\Lambda} [\mathcal{D}g \ldots] \; \exp\left\{ \frac{i}{\hbar} \int d^4x \sqrt{-g} \left[\frac{1}{16\pi G} R - \frac{1}{4} F^2 + \bar{\psi} i \not{D} \psi - \lambda \phi \bar{\psi} \psi + |D\phi|^2 - V(\phi) \right] \right\}$$

amplitude | current understanding | quantum mechanics | spacetime | gravity | strong & electroweak | matter | Higgs

These Lectures:

The Standard Model



Will take a historical, phenomenological approach:

温故知新

$$\mathcal{L}_{GWS} = \sum_f (\bar{\Psi}_f(i\gamma^\mu \partial\mu - m_f)\Psi_f - eQ_f\bar{\Psi}_f\gamma^\mu\Psi_f A_\mu) +$$
$$+\frac{g}{\sqrt{2}}\sum_i(\bar{a}^i_L\gamma^\mu b^i_L W^+_\mu + \bar{b}^i_L\gamma^\mu a^i_L W^-_\mu) + \frac{g}{2c_w}\sum_f \bar{\Psi}_f\gamma^\mu(I^3_f - 2s^2_w Q_f - I^3_f\gamma_5)\Psi_f Z_\mu +$$
$$-\frac{1}{4}|\partial_\mu A_\nu - \partial_\nu A_\mu - ie(W^-_\mu W^+_\nu - W^+_\mu W^-_\nu)|^2 - \frac{1}{2}|\partial_\mu W^+_\nu - \partial_\nu W^+_\mu +$$
$$-ie(W^+_\mu A_\nu - W^+_\nu A_\mu) + ig'c_w(W^+_\mu Z_\nu - W^+_\nu Z_\mu|^2 +$$
$$-\frac{1}{4}|\partial_\mu Z_\nu - \partial_\nu Z_\mu + ig'c_w(W^-_\mu W^+_\nu - W^+_\mu W^-_\nu)|^2 +$$
$$-\frac{1}{2}M^2_\eta\eta^2 - \frac{gM^2_\eta}{8M_W}\eta^3 - \frac{g'^2M^2_\eta}{32M_W}\eta^4 + |M_W W^+_\mu + \frac{g}{2}\eta W^+_\mu|^2 +$$
$$+\frac{1}{2}|\partial_\mu\eta + iM_Z Z_\mu + \frac{ig}{2c_w}\eta Z_\mu|^2 - \sum_f \frac{g}{2}\frac{m_f}{M_W}\bar{\Psi}_f\Psi_f\eta$$

Not from here!

Outline

Lecture I: The Making of the SM

A. Deep Root in E&M → QED
B. The Strong Nuclear Force → QCD
C. The Weak Nuclear Force
D. Electro-Weak Unification → The SM

Lecture II: Story of Mass-generation

A. Spontaneous Symmetry Breaking
B. The Nambu-Goldstone Theorem
C. The Higgs Mechanism
D. The Higgs Boson Interactions

Lecture III:
Supplemental materials: Higgs Physics

A. What Does THIS Higgs Tell us?
B. SM Higgs Sector at Higher Energies
   & the Need for New Physics

Advanced Topics:

C. Higgs Boson Decays
D. Higgs Physics at the LHC Colliders
E. Higgs Physics at an e+e- Collider

Lecture I: The Making of the SM

A. Deep Root in QED

B. The Strong Nuclear Force → QCD ← till here 90 min!

C. The Weak Nuclear Force

D. Electro-Weak Unification: the SM

Lecture II: Story of Mass-generation

A. Spontaneous Symmetry Breaking

B. The Nambu-Goldstone Theorem ← + 90 min

C. The Higgs Mechanism

D. The Higgs Boson Interactions

Lecture III:

A. What Does THIS Higgs Tell us?

B. SM Higgs Sector at Higher Energies & the Need for New Physics ← + 120 min

Skipe below …

C. Higgs Boson Decays

D. Higgs Physics at the LHC Colliders

E. Higgs Physics at an e+e- Collider

Lect I. The Making of the SM

A. Deep Root in QED

Maxwell Equations ➔
Lorentz invariance, U(1) Gauge Invariance

Electromagnetic fields can be treated by **E**(x,t), **B**(x,t)
the introduction of co-variant vector potential $A_\mu(x,t)$
makes the symmetries manifest (but redundant)

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}, \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

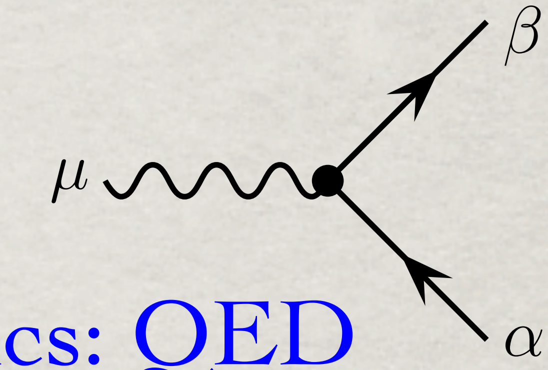
1). Lorentz/Local Gauge invariance manifest.
2). Classically, geometrical interpretation: fiber bundles…
3). Quantum-mechanically, wave function for the EM field.

Dirac's relativistic theory:

Lorentz/Local gauge invariant ➔ antiparticle e^+

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu D_\mu - m_e)\psi$$

$$D_\mu = \partial_\mu + ieA_\mu$$



$$= (+ie\gamma^\mu)_{\beta\alpha}$$

Quantum Electro-Dynamics: QED

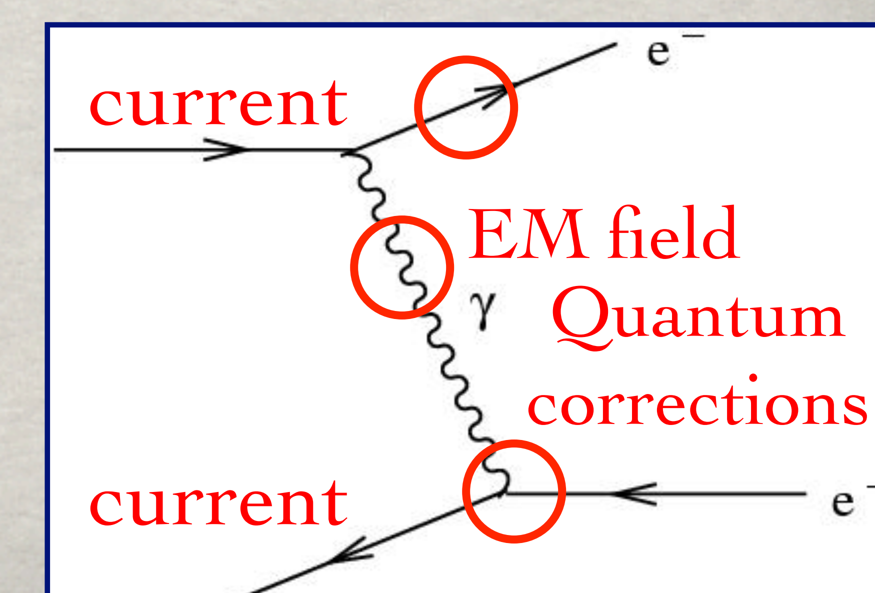
Feynman/Schwinger/Tomonaga ➔ Renormalization

$$a_e(\text{Schwinger}) \approx \frac{\alpha}{2\pi} \approx 0.0011614$$

$$a_e^{theo} = 0.001159652181643(763)$$

$$a_e^{exp} = 0.00115965218073(28)$$

QED becomes the most accurate theory in science!



Warmup Exercise 1:

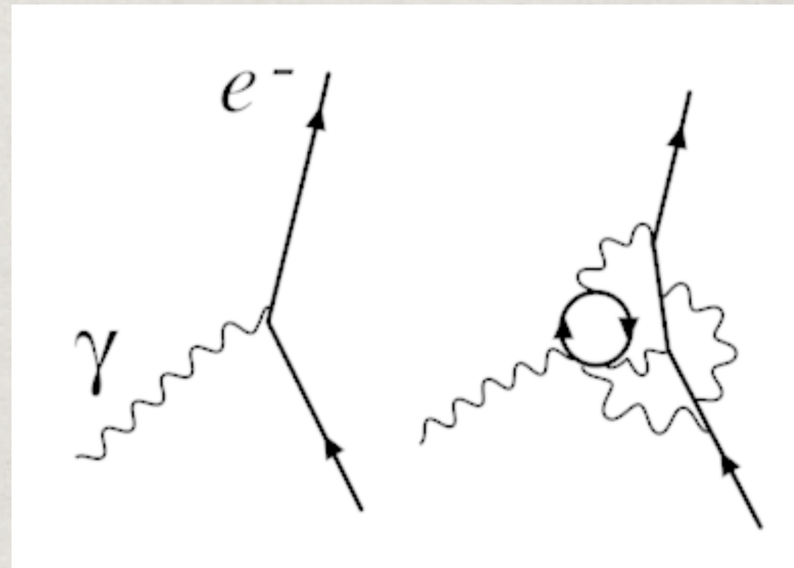
For charge scalar field $\phi^{\pm}$, construct the locally $U(1)_{em}$ gauge invariant Lagranian and derive the Feynman rules for its EM interactions.

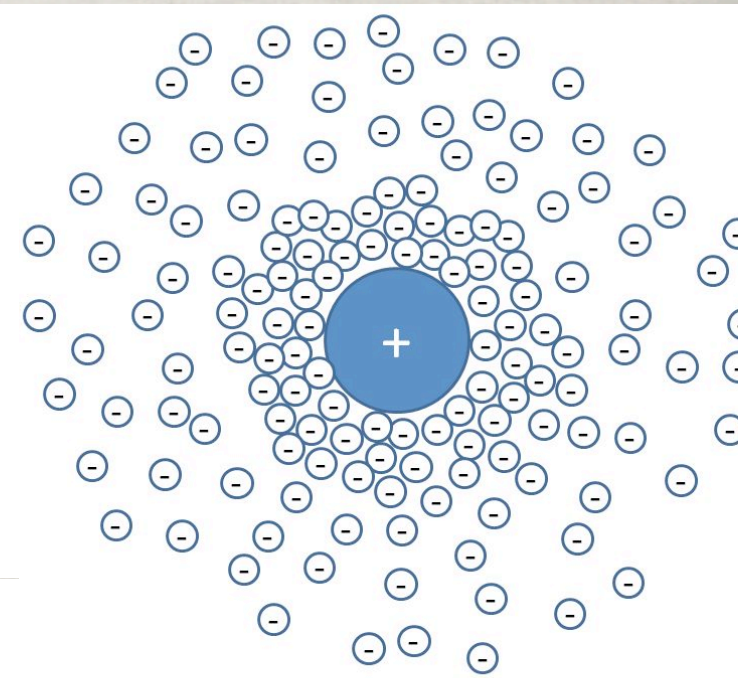
Sketch a calculation for the differential and total cross section for the process:

$$e^+ e^- \rightarrow \phi^+ \phi^-$$

The key effect of renormalization: Running of coupling with energies

$$\beta(g) = \mu\frac{\partial g}{\partial \mu} = \frac{\partial g}{\partial \ln \mu},$$





$$\beta(e) = \frac{e^3}{12\pi^2}$$

$$\alpha(Q^2) = \frac{\alpha(Q_0^2)}{1 - \frac{\alpha(Q_0^2)}{3\pi}\ln(Q^2/Q_0^2)}$$

$$\alpha_{QED}(keV) = 1/137$$
$$\alpha_{QED}(M_Z) = 1/128$$

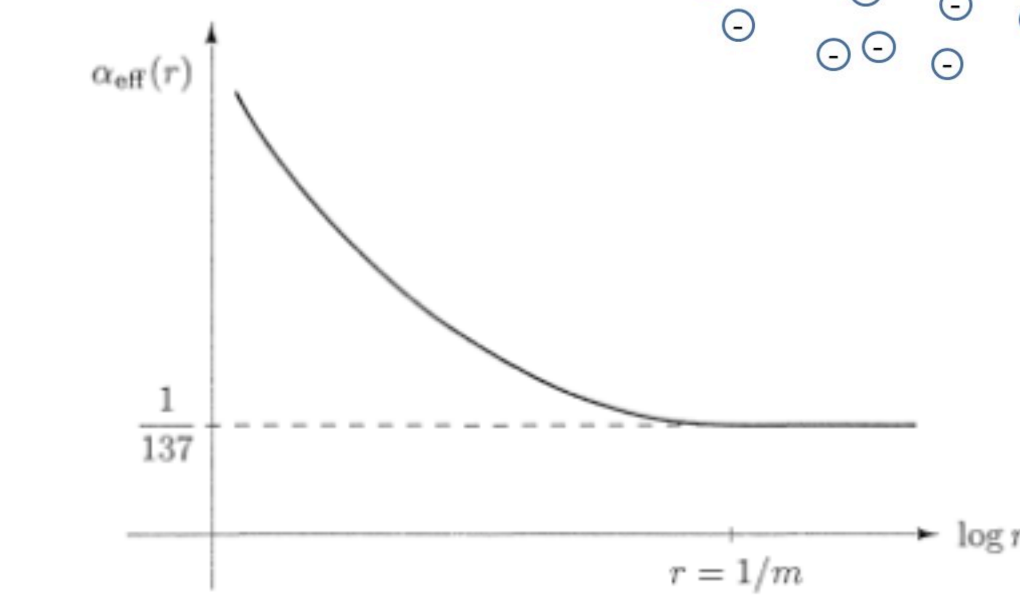


Figure 7.10. A qualitative sketch of the effective electromagnetic coupling constant generated by the one-loop vacuum polarization diagram, as a function of distance. The horizontal scale covers many orders of magnitude.

The Landau pole:
It blows up at high energies! Must be modified at UV.

QED: Most Successful in Theory & Practice!

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \bar{\psi}(i\gamma^\mu D_\mu - m_e)\psi$$

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu, \; D_\mu = \partial_\mu + ieA_\mu$$

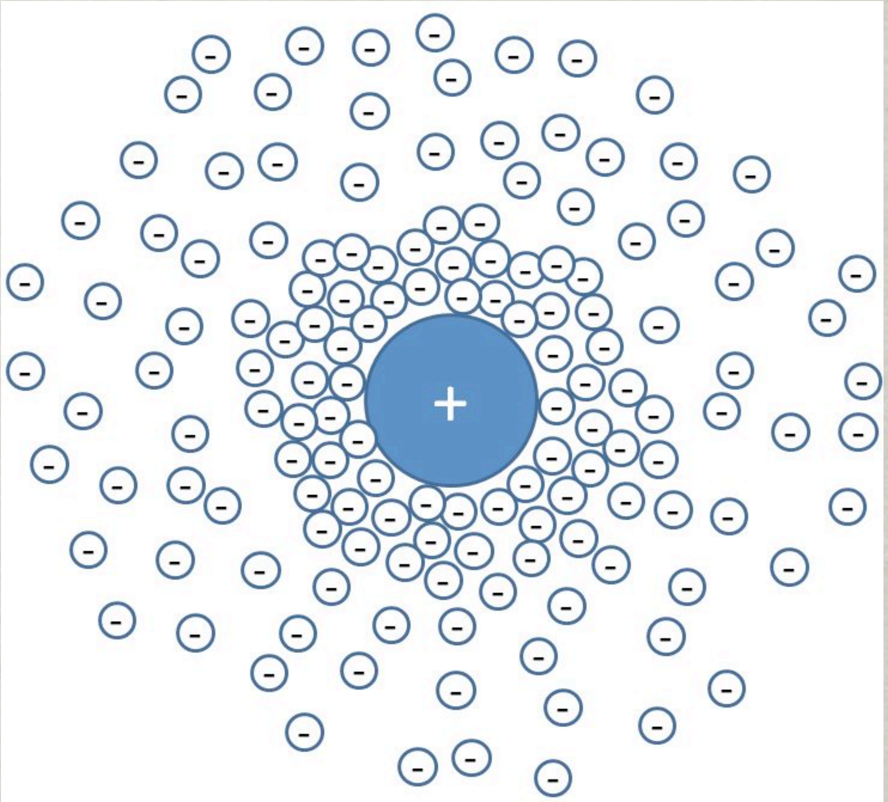
- At low energies → Maxwell's theory; vector-like coupling by a $U_{em}(1)$ gauge symmetry
- At high energies → Quantum-mechanical, renormalizable, most accurate (in science!): a part of trillion

$$a_e^{theo} = 0.001159652181643(763)$$
$$a_e^{exp} = 0.00115965218073(28)$$

- QED becomes strongly interacting asymptotically (screening effects):

Fine structure constant:

$$\alpha(Q^2) = \frac{\alpha(Q_0^2)}{1 - \frac{\alpha(Q_0^2)}{3\pi}\ln(Q^2/Q_0^2)}$$



At ultra-violet (UV) → theory is invalid: the "Landau pole".

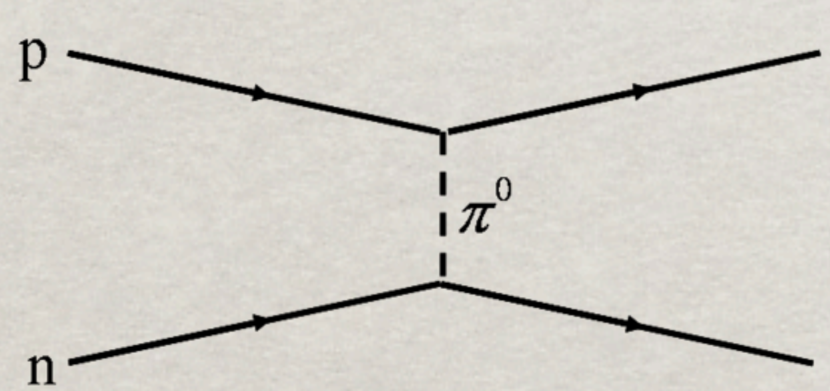
B. There Is a strong force!

Ever since Rutherford established the atomic nuclear model → a new force to bound p^+ to a nucleus.

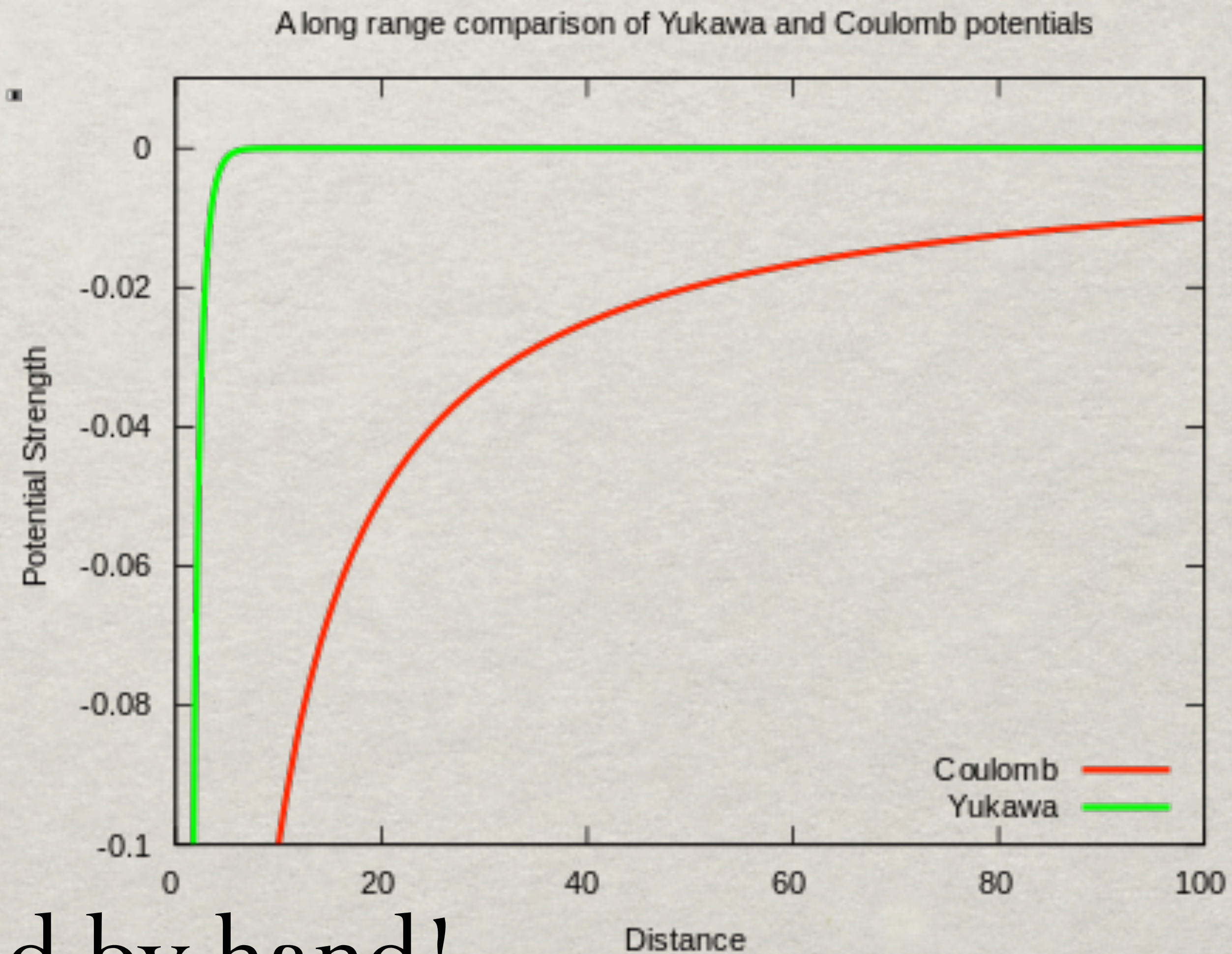
The discovery neutron (1932) → a charge-independent force:
Heisenberg → (p^+, n^0) "iso-spin" doublet

Yukawa (1935) → a universal attractive force via pions

$$\mathcal{L}_{\rm int}(x) = g\bar{\psi}(x)\phi(x)\psi(x).$$



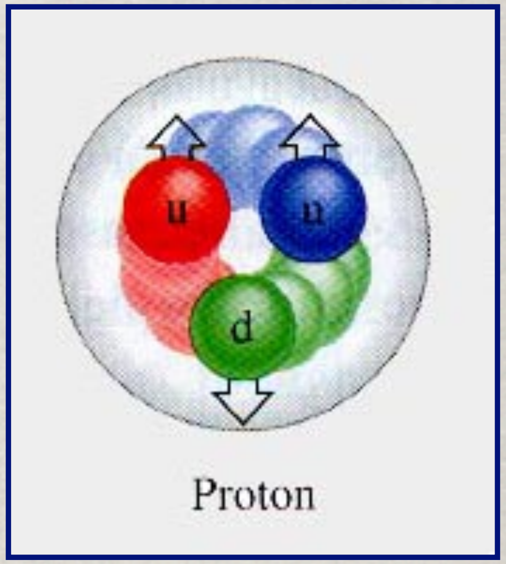
$$V_{\rm Yukawa}(r) = -g^2\,\frac{e^{-mr}}{r}$$



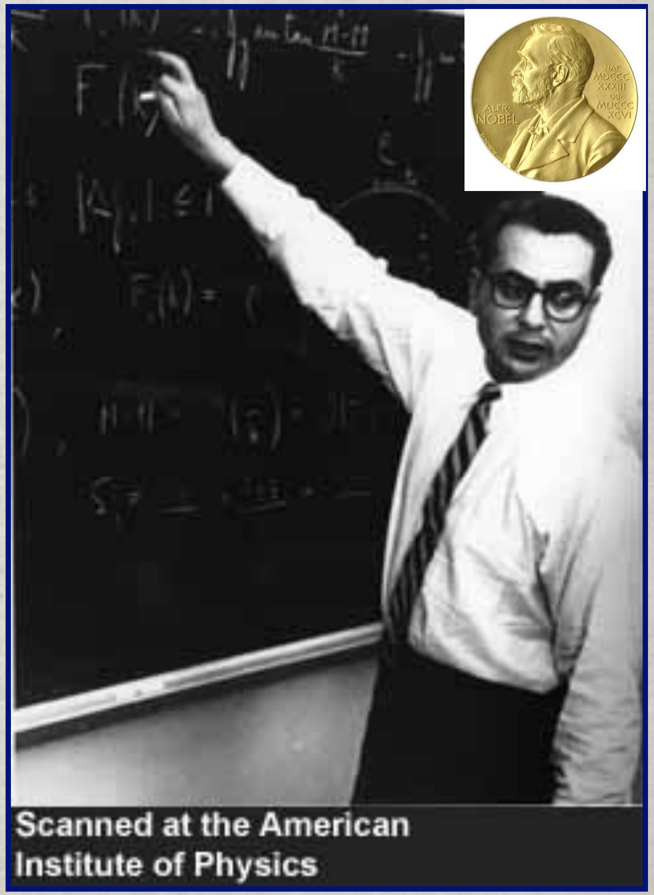
Discoveries & theory hand by hand!

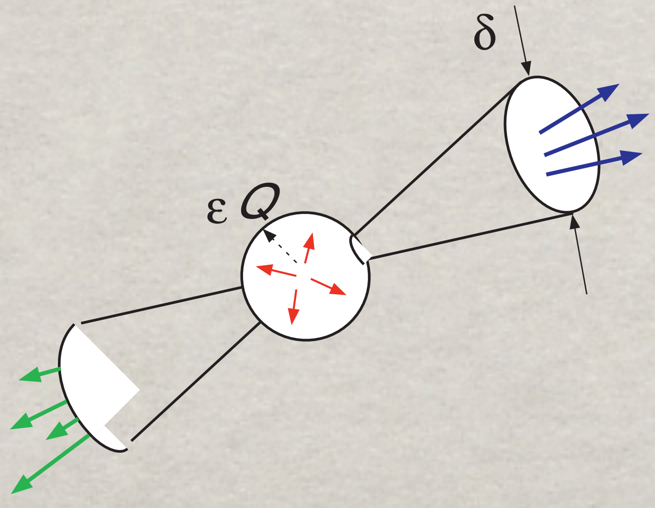
What IS the strong force?

Numerous "hadrons" discovered (50's):
Mesons: π, η, ρ, ω … Baryons: p, n, Δ, Λ, Σ …
How to understand/describe them?
→ Hadronic string theory developed.



- Gell-Mann – Zweig's "quarks" (1963)
- π → ϒϒ 3 colors (1964)
- Proton structure by DIS (1969)
- 2 or 3-jet structure (q: 1975, g: 1979)





SU(3)$_C$ gauge theory established (1973)

Quantum-Chromo-Dynamics (QCD)

H. Fritzsch, M. Gell-Mann, H. Leutwyler (1973)

$$\mathcal{L} = \sum_f^{n_f} \bar{q}_f (i\gamma^\mu \partial_\mu - g_s \gamma^\mu A_\mu + m_f) q_f - \frac{1}{2} \mathrm{Tr} F_{\mu\nu}^2$$

Direct analogue of QED

$$F_{\mu\nu} = \partial\mu A_\nu - \partial\mu A_\nu + ig_s[A_\mu, A_\nu]$$

← Non-Abelian

$$A^\mu(x) = \sum_1^8 A(x)_a^\mu \, T^a, \quad [T^a, T^b] = if_{abc}T^c.$$

QED analogue:

- Similar gauge principles;
- Tempting for perturbative renormalization calculations

Non-Abelian gauge theory: Yang-Mills

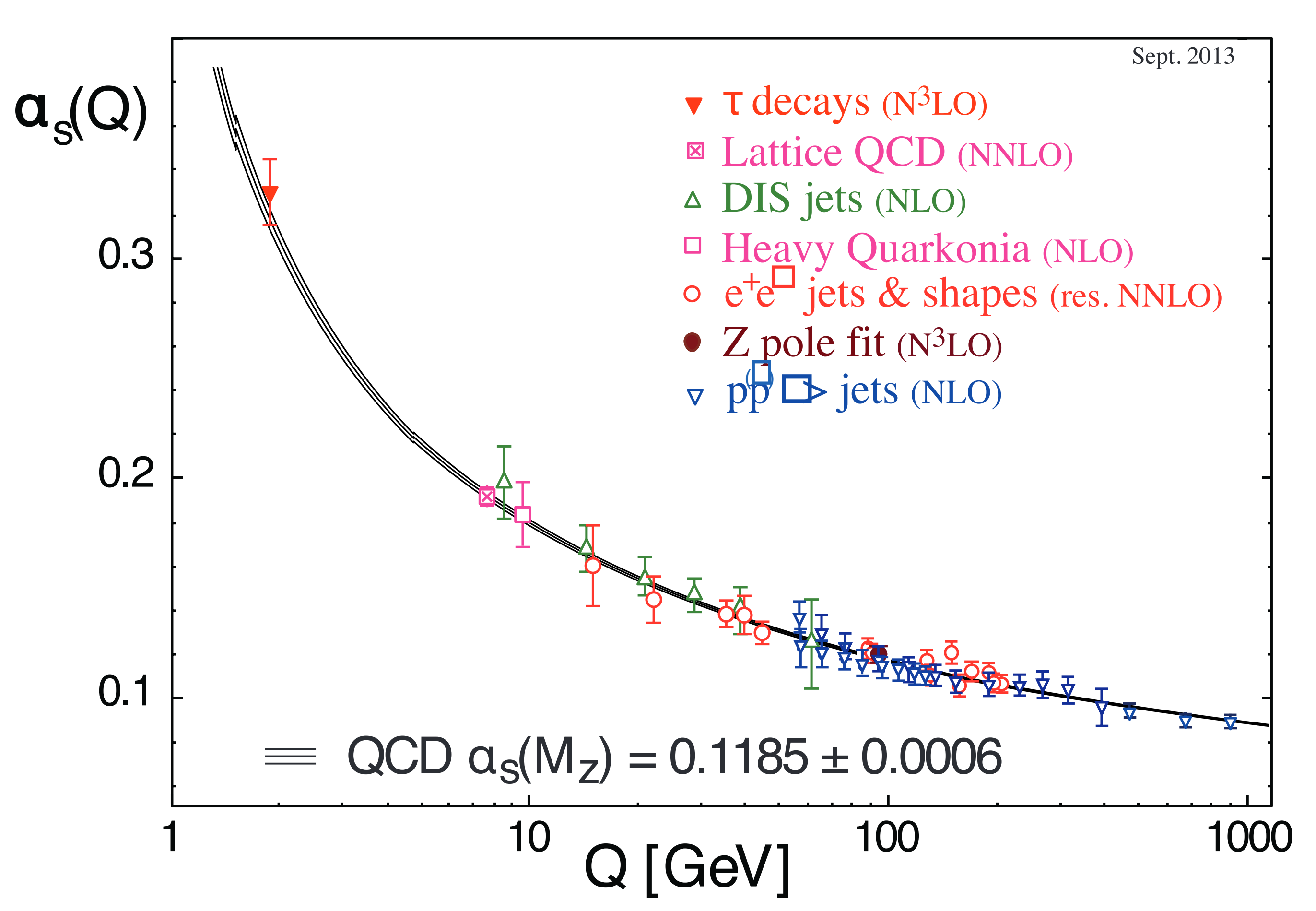
- Self gauge interactions among 8 gluons;
- Coupling rather strong, may invalidate perturbation theory

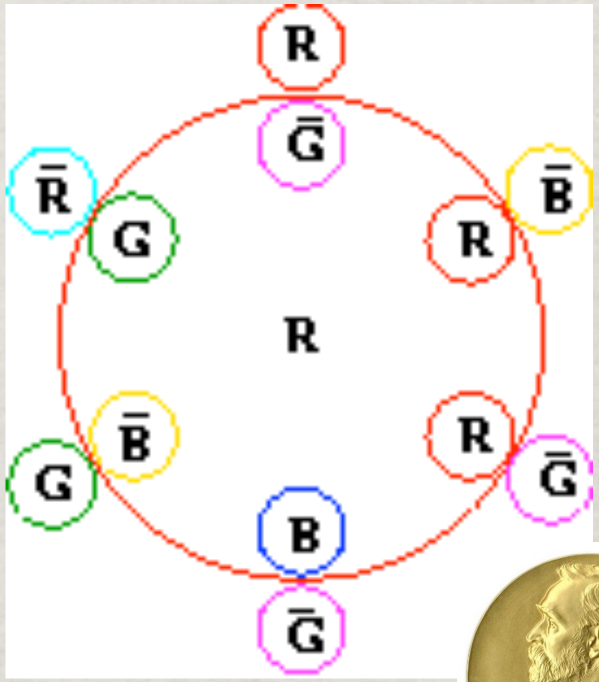
Remarkable Features:

IR confinement & UV asymptotic freedom

Interaction strength changes fast with energy/distance scale:

$$\alpha_s(Q^2) = \frac{\alpha_s(Q_0^2)}{1 + \frac{(33-2n_f)\alpha_s(Q_0^2)}{12\pi}\ln(Q^2/Q_0^2)}$$



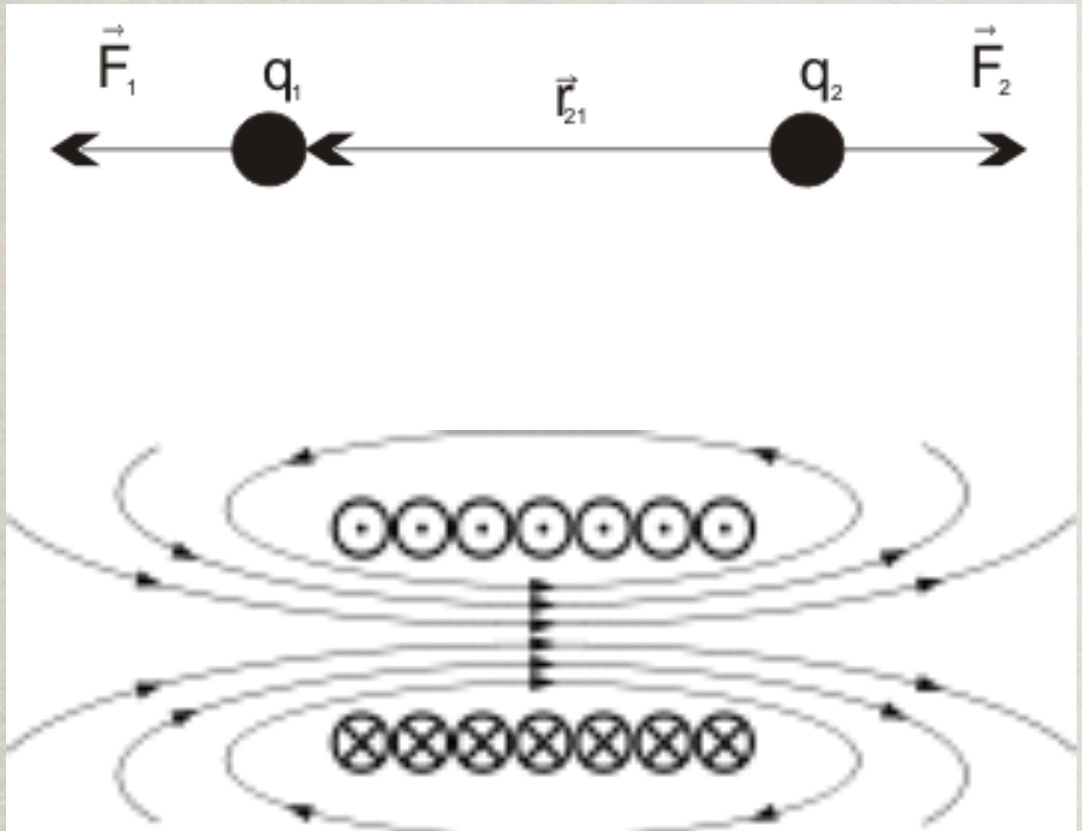


D. Gross, F. Wilczek, D. Politzer (2004)

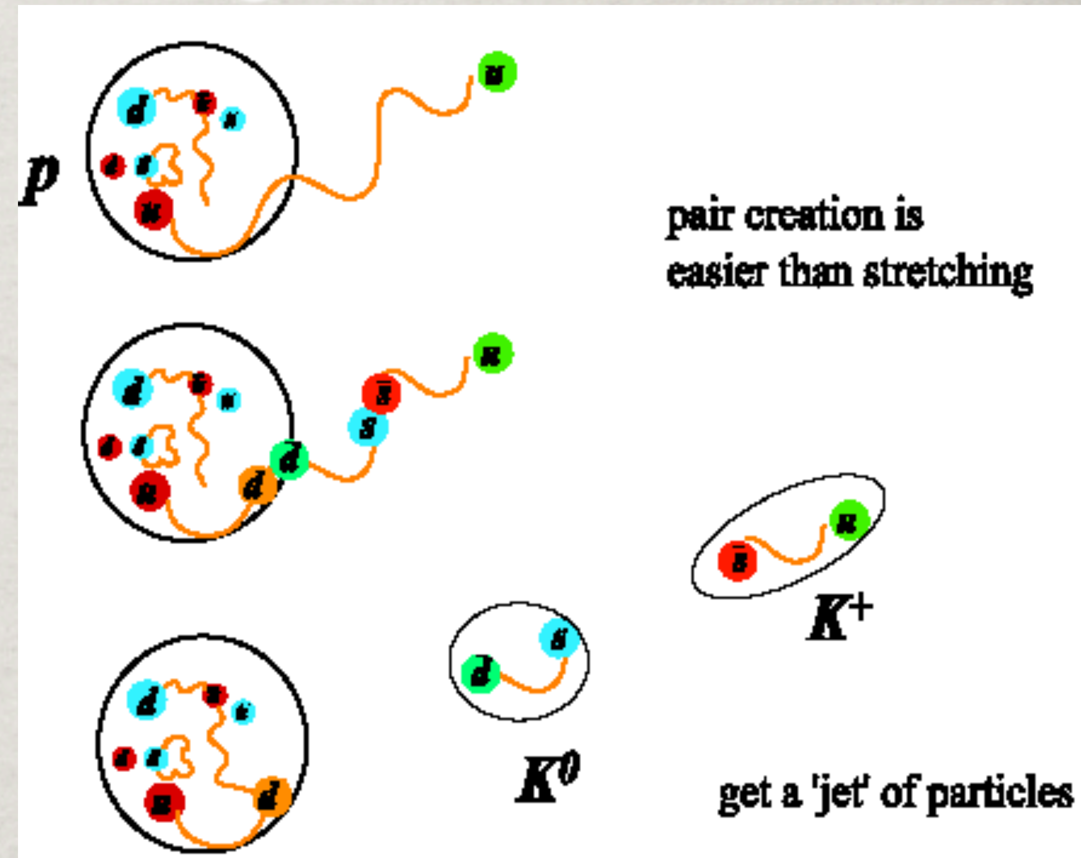
T. Han

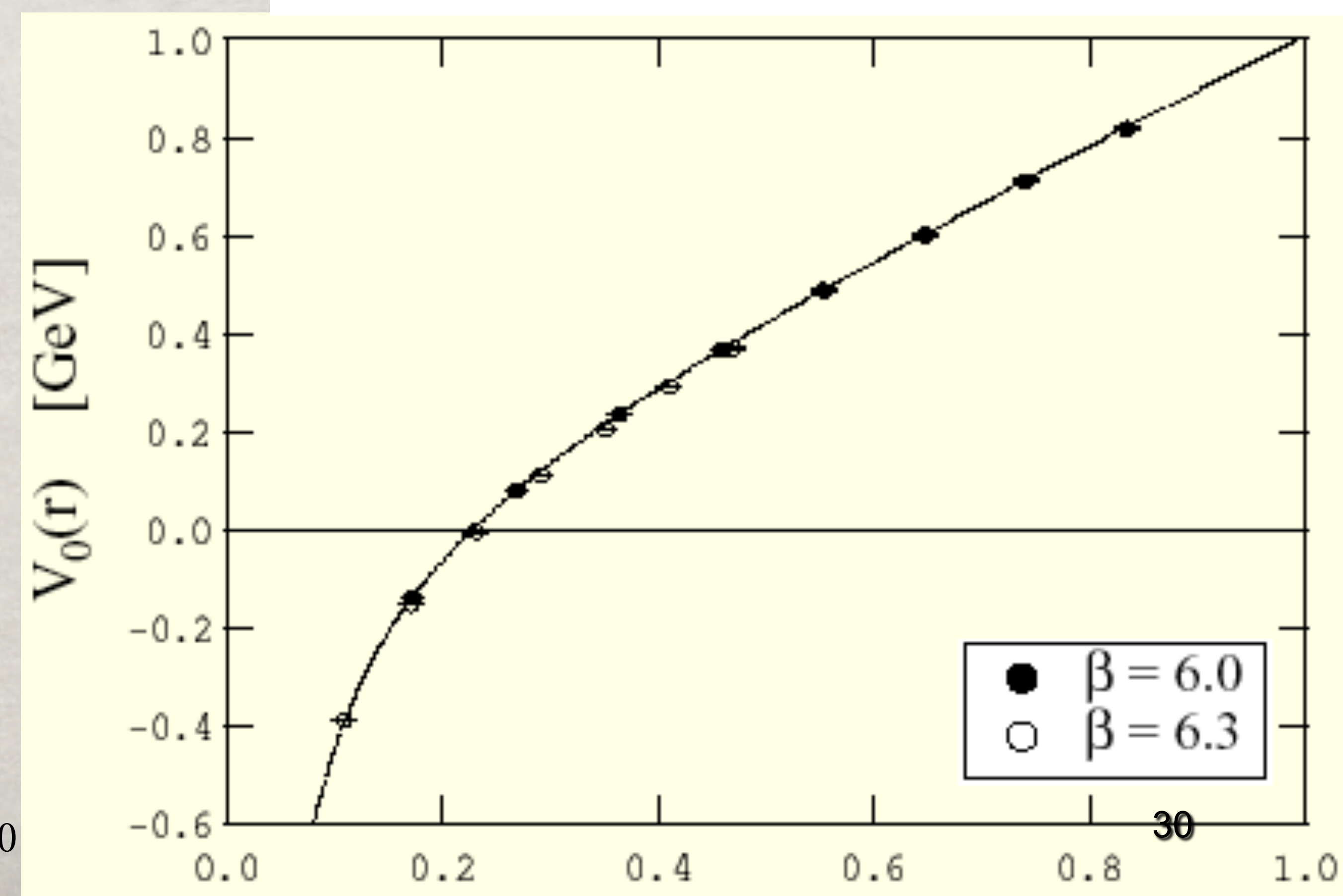
QED versus QCD

Electromagnetism vs. Strong force



Photons
vs.
gluons





QED: $V(r) = -\alpha_{em}/r$

QCD: $V(r) = -\alpha_s/r + k\,r$

In long distances, we see charged particles, but not colored particles!

QCD at Low Energies: Quark condensation

Consider the two-flavor massless QCD

$$-\frac{1}{4}G^a_{\mu\nu}G_a^{\mu\nu} - \sum_{u,d}(\bar{q}_L\gamma^\mu D_\mu q_L + \bar{q}_R\gamma^\mu D_\mu q_R)$$

$$\begin{pmatrix} u \\ d \end{pmatrix} \to \left(U_L\,\gamma_L + U_R\,\gamma_R\right)\begin{pmatrix} u \\ d \end{pmatrix} \Rightarrow SU(2)_L \otimes SU(2)_R$$

QCD exhibits a L-R chiral symmetry.

Below Λ_{QCD}, QCD becomes strongly interacting and forms condensate: $\langle \bar{q}_L q_R + \bar{q}_R q_L \rangle \sim v^3$

$$SU(2)_L \otimes SU(2)_R \Rightarrow SU(2)_V, \text{thus } U_L = U_R.$$

Chiral symmetry is broken to iso-spin.

The Spontaneous Symmetry Breaking

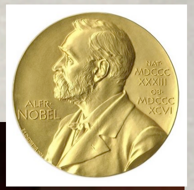
" The Lagrangian of the system may display an symmetry, but the ground state does not respect the same symmetry."

The concept of SSB: profound, common.

Known Example: Ferromagnetism

Above a critical temperature, the system is symmetric, magnetic dipoles randomly oriented. Below a critical temperature, the ground state is a completely ordered configuration in which all dipoles are ordered in some arbitrary direction,

SO(3) → SO(2)



Y. Nambu was the first one to have formulated the spontaneous symmetry breaking in a relativistic quantum field theory (1960).

He is the one to propose the understanding of the nucleon mass by dynamical chiral symmetry breaking: The Nambu-Jona-Lasinio Model.

2008 Nobel Prize in physics: "for the discovery of the mechanism of spontaneous broken symmetry in subatomic physics"

Be aware of the difference between the dynamical mass for baryons (you and me) and that of elementary particles by the Higgs mechanism.

"Pseudo-Nambu-Goldstone Bosons"

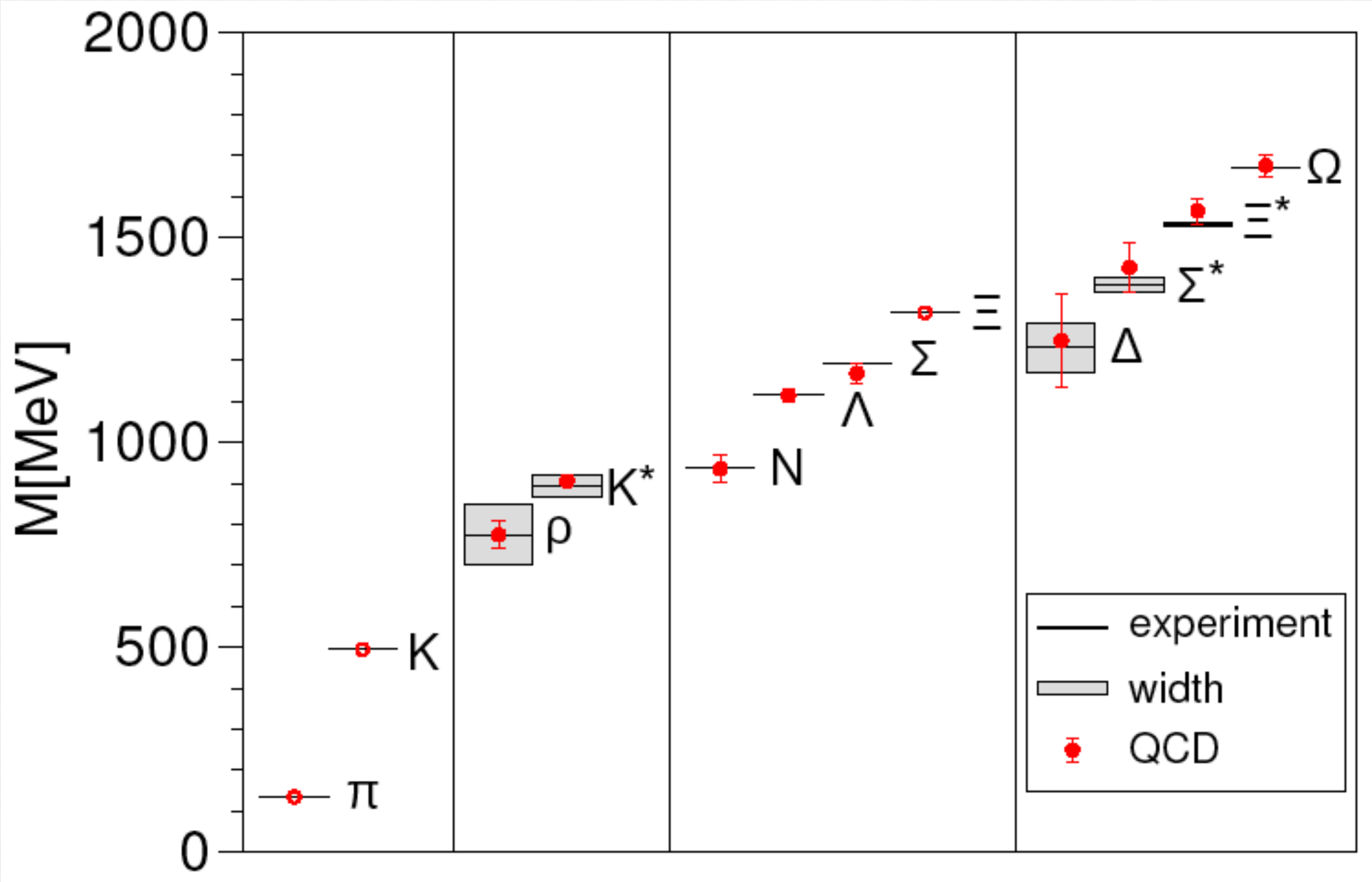
When a continuous symmetry is broken both explicitly AND spontaneously, and if the effect of the explicit breaking is much smaller than the SSB, then the Goldstone are massive, governed by the explicit breaking, thus called:
"Pseudo-Nambu-Goldstone bosons".

The pions are NOT massless, due to explicit symmetry breaking. They are "Pseudo-Nambu-Goldstone bosons".

Except the photon, no massless boson (a long-range force carrier) has been seen in particle physics!

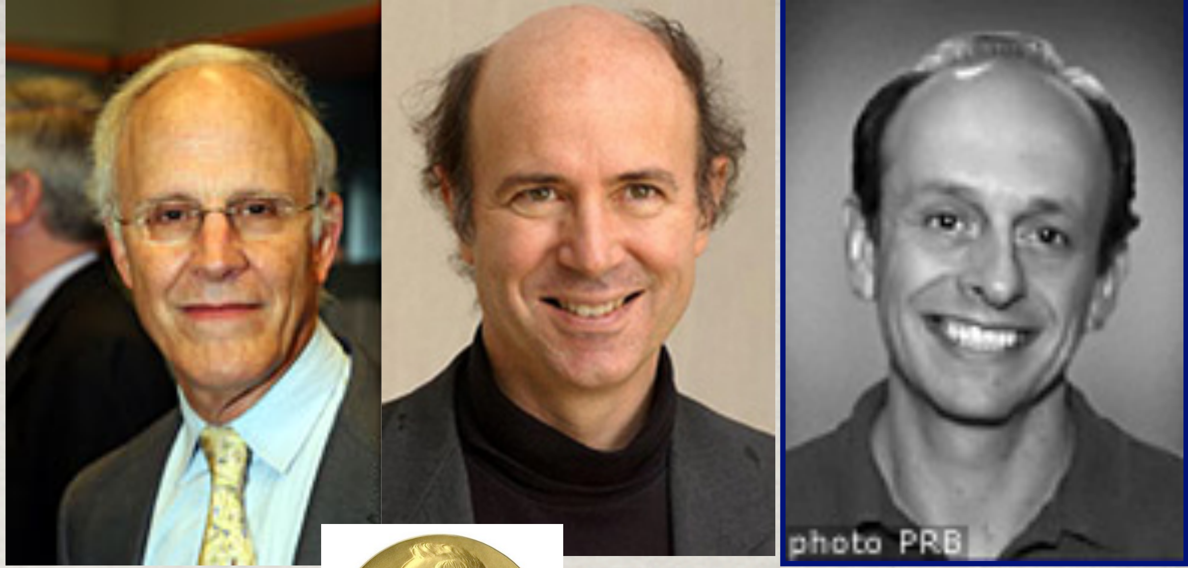
Most Mass due to QCD:

From quark constituents to hadrons:
(From PDG, based on lattice QCD)



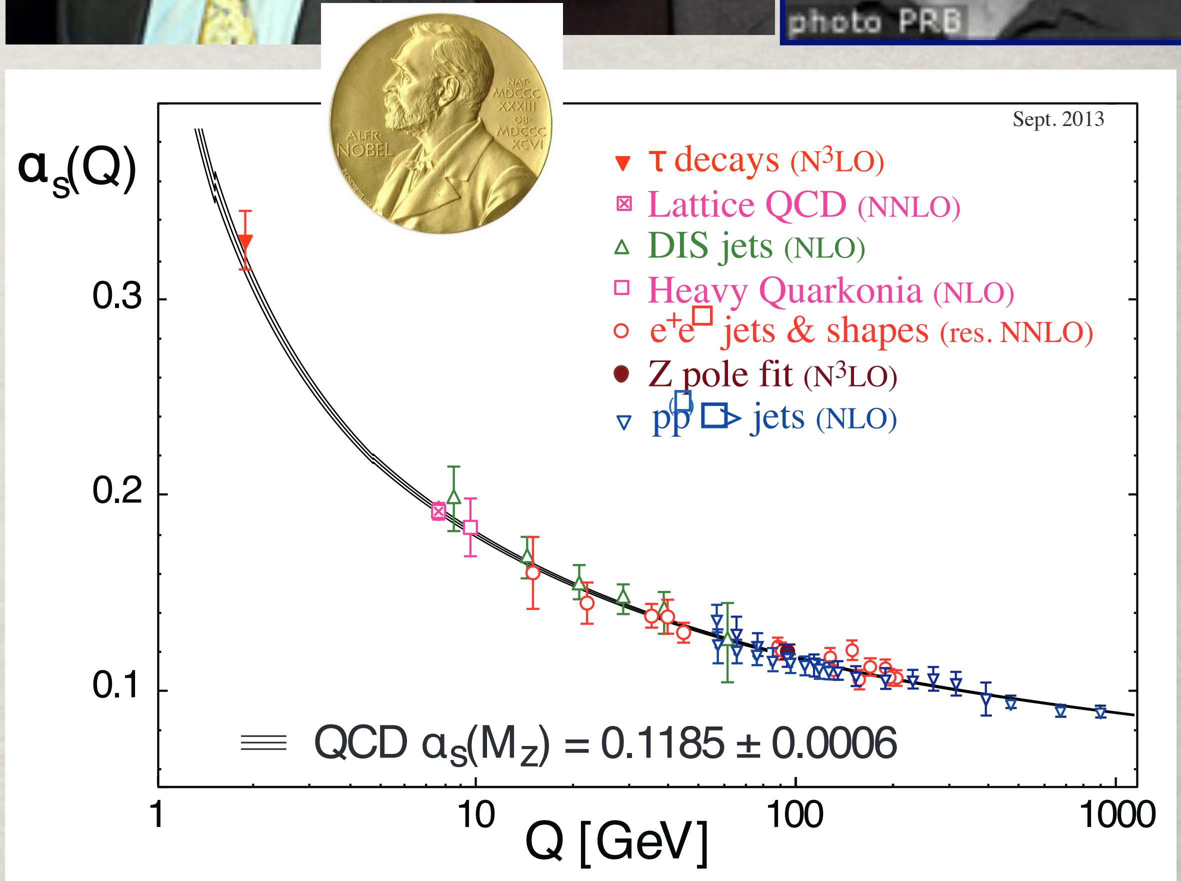
Majority of the (luminous) mass around us is of dynamical origin, from strong interactions (u, d quarks + gluons).
It is not from the Higgs mechanism!.

QCD at High Energies



Interaction strength changes fast with energy/distance scale:

$$\alpha_s(Q^2) = \frac{\alpha_s(Q_0^2)}{1 + \frac{(33-2n_f)\alpha_s(Q_0^2)}{12\pi}\ln(Q^2/Q_0^2)}$$



At high energies, E >> Λ_{QCD} ,
QCD is weakly interacting!
"Asymptotic freedom"

- Perturbative → prediction for high energy experiments (ee, ep, pp etc. LHC …)
- Think about higher energy physics at M_{GUT}, M_{PL}
- Early universe cosmology at high T.

QCD Factorization Theorem:

J. Collins, D. Soper, G. Sterman (1985)

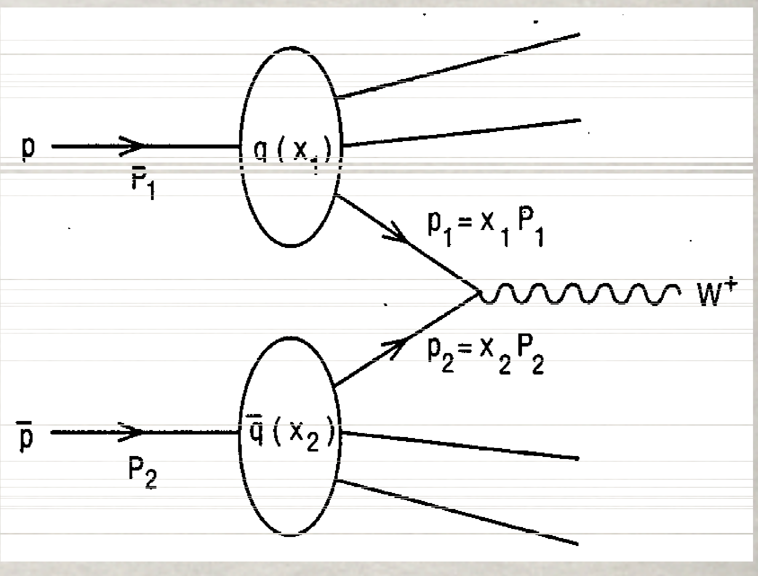
In high energy collisions involving a hadron, the total cross sections can be factorized into two factors:

(1). hard subprocess of parton scattering with a large scale $\mu^2 \gg \Lambda^2_{QCD}$;

(2). "parton distribution functions" (hadronic structure with $Q^2 < \mu^2$.)

Observable cross sections at hadron level:

$$\sigma_{pp}(S) = \int dx_1 dx_2 P_1(x_1, Q^2) P_2(x_2, Q^2)\ \hat{\sigma}_{parton}(s).$$

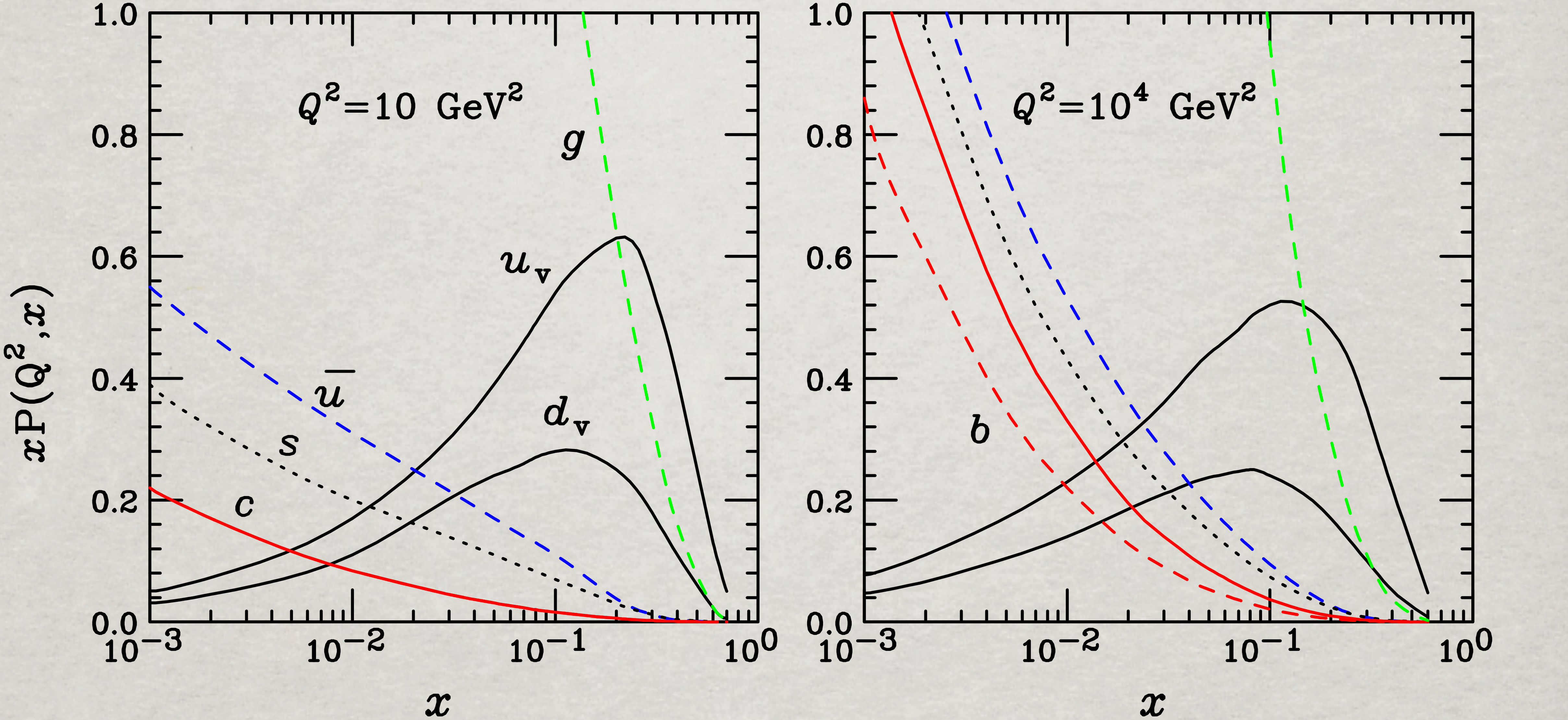


† $\hat{\sigma}_{parton}(s)$ is theoretically calculated by perturbation theory (in the SM or models beyond the SM).

Ultra violet (UV) divergence (beyond leading order) is renormalized;
Infra-red (IR) divergence is cancelled by soft gluon emissions;
Co-linear divergence (massless) is factorized into PDF

PDF's: $q(x, Q^2)$, $g(x, Q^2)$, ...

Typical quark/gluon parton distribution functions:



(CTEQ-5)

Quarks carry ½ momentum; gluons carry the other ½!

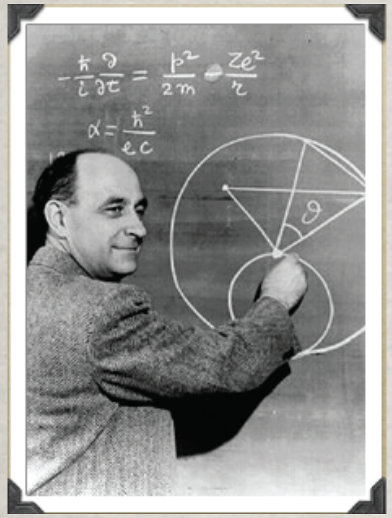
 CTEQ, MRS (Durham), NLOPDF etc.

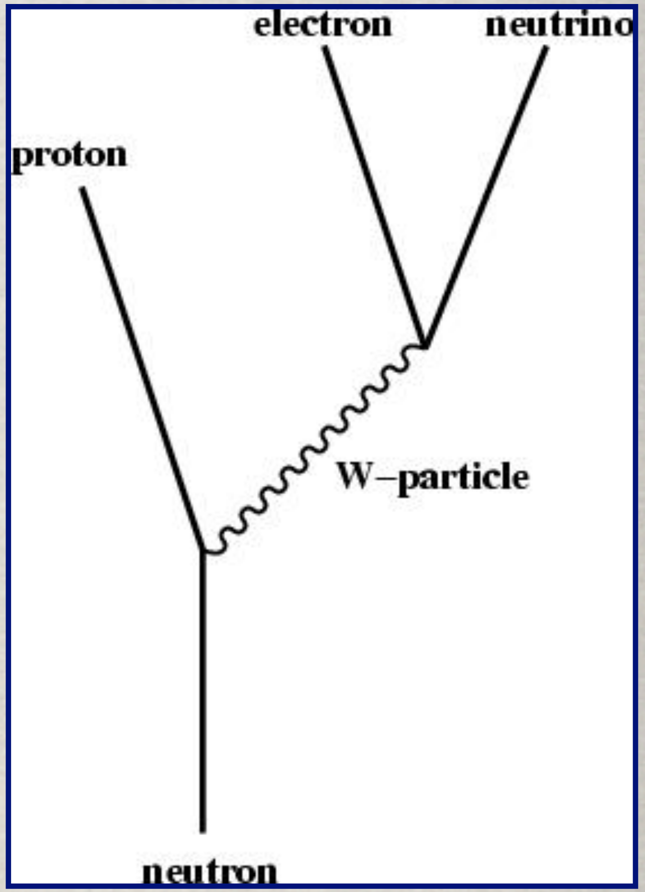
C. The Weak Nuclear Force

beta decay n → $p^+ e^- \nu$ ➔ Charged current interaction: $W^\pm$

ν N → ν N ➔ Neutral current interaction

via: Z^0 (1973)





$$-\mathcal{L}_{eff}^{cc} = \frac{G_F}{\sqrt{2}} J_W^\mu J_{W\mu}^\dagger, \qquad -\mathcal{L}_{eff}^{NC} = \frac{G_F}{\sqrt{2}} J_Z^\mu J_{Z\mu}.$$

$$J_\lambda^{(\pm)} = \sum_i \bar{\psi}_i \tau_\pm \gamma_\lambda (1-\gamma_5)\psi_i,$$

- Beyond E&M, Fermi was inspired by the current-current interactions to construct the weak interaction (1934).
- parity violation ➔ V-A interactions (1957).

The fact $G_F = (300\ \text{GeV})^{-2}$ implies that:

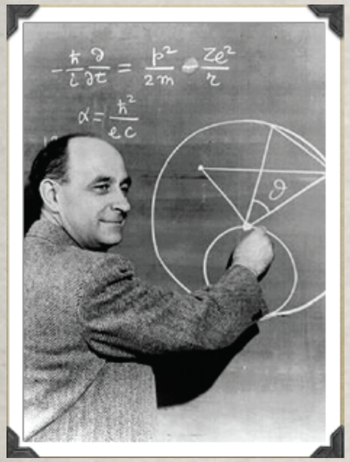
1. A new mass scale to show up at O(100 GeV).
2. Partial-wave Unitarity requires new physics below

$E < 300$ GeV

The Weak force: Quark & Lepton Flavor Transitions

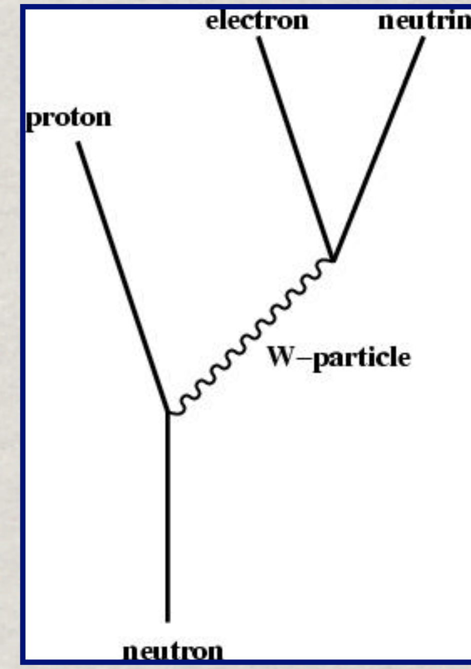
Beta decay $n \rightarrow p^+ e^- \nu$ ➔ Charged current interaction: $W^\pm$

Inspired by EM current-current interactions, Fermi proposed (1934)



$$\mathcal{L}_{weak} = -\frac{G_F}{\sqrt{2}} J^\mu(p^+ n) J_\mu(e^- \nu)$$

$$\text{force range} \sim \sqrt{G_F} \sim M_W^{-1} \sim 10^{-18}\text{m}$$



Weak interaction based on $SU(2)_L \times U(1)$: (Glashow, '63)

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

$$W^i_{\mu\nu} = \partial_\mu W^i_\nu - \partial_\nu W^i_\mu - g\epsilon_{ijk} W^j_\mu W^k_\nu$$

$$-\frac{g}{2\sqrt{2}} \sum_i \overline{\Psi}_i \gamma^\mu (1-\gamma^5)(T^+ W^+_\mu + T^- W^-_\mu) \Psi_i$$

$$-e \sum_i q_i \overline{\psi}_i \gamma^\mu \psi_i A_\mu$$

$$-\frac{g}{2\cos\theta_W} \sum_i \overline{\psi}_i \gamma^\mu (g^i_V - g^i_A \gamma^5) \psi_i Z_\mu \, .$$

However,

The local gauge symmetry prevents gauge bosons masses!

$$\frac{1}{2} M_A^2 A_\mu A^\mu \rightarrow \frac{1}{2} M_A^2 (A_\mu - \frac{1}{e}\partial_\mu \alpha)(A^\mu - \frac{1}{e}\partial^\mu \alpha) \neq \frac{1}{2} M_A^2 A_\mu A^\mu$$

Pauli's rejection to the Yang-Mills theory.

Exercise:

Assume that the $\nu\, e \rightarrow \nu\, e$ scattering amplitude to be

$$\mathcal{M} = G_F\, E_{cm}^2$$

estimate the unitarity bound on the c.m. energy.

Partial wave expansion:

$$a_{I\ell}(s) = \frac{1}{64\pi}\int_{-1}^{1} d\cos\theta\; P_\ell(\cos\theta)\; \mathcal{M}^I(s,t)$$

Partial wave unitarity:

$$Im\,(a_{I\ell}) = |a_{I\ell}|^2 < 1, \quad Re\,(a_{I\ell}) < \frac{1}{2}$$

D. The Idea of Unification:

Within a frame work of relativistic, quantum, gauge field theory

PARTIAL-SYMMETRIES OF WEAK INTERACTIONS

SHELDON L. GLASHOW †

Institute for Theoretical Physics, University of Copenhagen, Copenhagen, Denmark

Received 9 September 1960

Abstract: Weak and electromagnetic interactions of the leptons are examined under the hypothesis that the weak interactions are mediated by vector bosons. With only an isotopic triplet of leptons coupled to a triplet of vector bosons (two charged decay-intermediaries and the photon) the theory possesses no partial-symmetries. Such symmetries may be established if additional vector bosons or additional leptons are introduced. Since the latter possibility yields a theory disagreeing with experiment, the simplest partially-symmetric model reproducing the observed electromagnetic and weak interactions of leptons requires the existence of at least four vector-boson fields (including the photon). Corresponding partially-conserved quantities suggest leptonic analogues to the conserved quantities associated with strong interactions: strangeness and isobaric spin.

The birth of the Standard Model:

VOLUME 19, NUMBER 21 PHYSICAL REVIEW LETTERS 20 NOVEMBER 1967

[11] In obtaining the expression (11) the mass difference between the charged and neutral has been ignored.

[12]M. Ademollo and R. Gatto, Nuovo Cimento 44A, 282

bra is slightly larger than that (0.23%) obtained from the ρ-dominance model of Ref. 2. This seems to be true also in the other case of the ratio $\Gamma(\eta \to \pi^+\pi^-\gamma)/$ ...) calculated in Refs. 12 and 14.

. M. Brown and P. Singer, Phys. Rev. Letters 8, (1962).

A MODEL OF LEPTONS*

Steven Weinberg†

Physics Department,
...ambridge, Massachusetts
... 1967)

Leptons interact only with photons, and with the intermediate bosons that presumably mediate weak interactions. What could be more natural than to unite[1] these spin-one bosons into a multiplet of gauge fields? Standing in the way of this synthesis are the obvious differences in the masses of the photon and intermediate meson, and in their couplings. We might hope to understand these differences

... on a right-handed singlet

$$R = [\tfrac{1}{2}(1-\gamma_5)]e.$$

The EW Unification I:
Particle representation

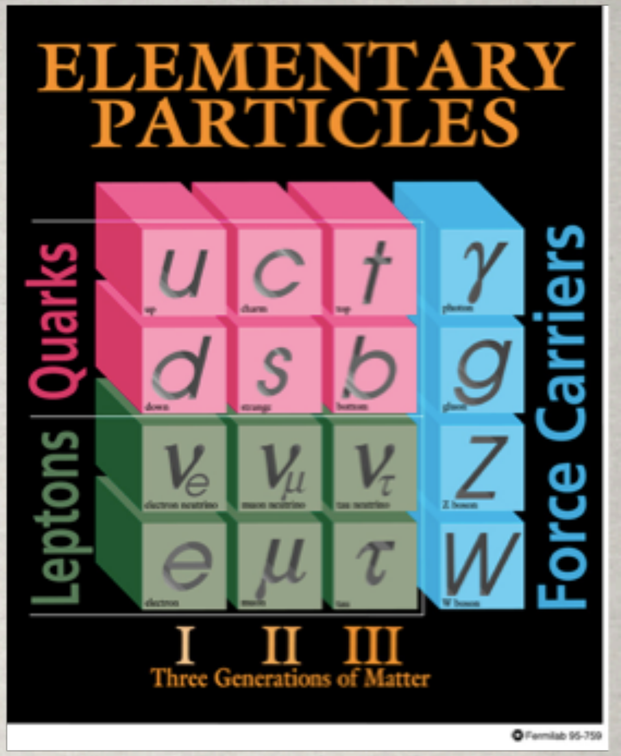
- Simple structure and particle contents:

Leptons:

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L, \quad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L, \quad e_R, \ \mu_R, \ \tau_R, \quad (\nu_R' s\ ?)$$

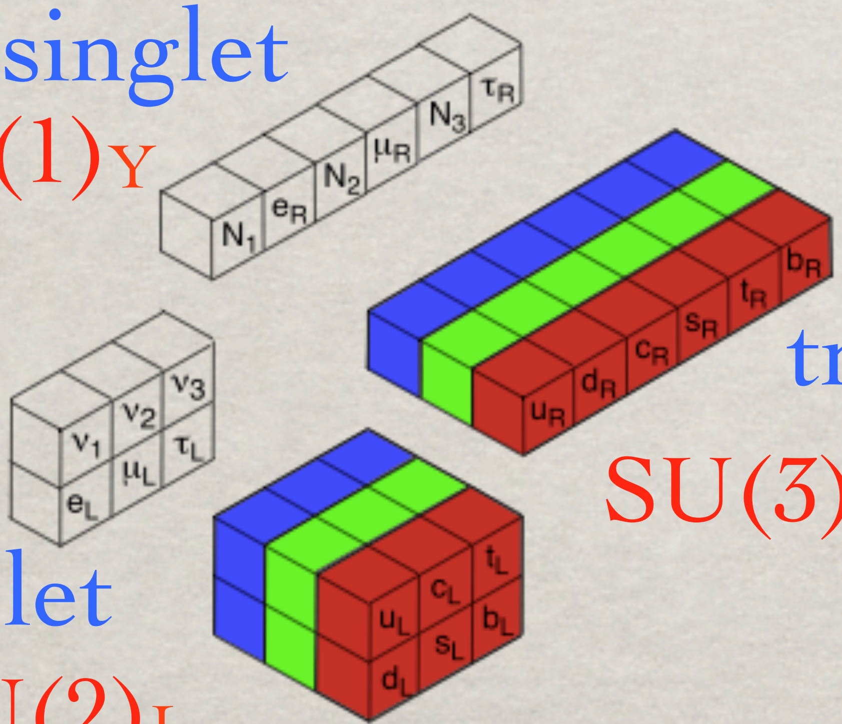
Quarks:

$$\begin{pmatrix} u \\ d \end{pmatrix}_L, \quad \begin{pmatrix} c \\ s \end{pmatrix}_L, \quad \begin{pmatrix} t \\ b \end{pmatrix}_L, \quad u_R, \ d_R, \ c_R, \ s_R, \ t_R, \ b_R$$



singlet

$U(1)_Y$



triplet

$SU(3)_c$

doublet

$SU(2)_L$

(1979 Nobel)

The EW Unification II:
The Interactions

$$-\frac{g}{2\sqrt{2}}\sum_i \overline{\Psi}_i\,\gamma^\mu\,(1-\gamma^5)(T^+\,W_\mu^+ + T^-\,W_\mu^-)\,\Psi_i$$

$$-\,e\sum_i q_i\,\overline{\psi}_i\,\gamma^\mu\,\psi_i\,A_\mu \quad -\frac{1}{4}W^i_{\mu\nu}W^{\mu\nu i} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}$$

$$-\frac{g}{2\cos\theta_W}\sum_i \overline{\psi}_i\,\gamma^\mu(g_V^i - g_A^i\gamma^5)\,\psi_i\,Z_\mu\ .$$

$$M_W = \frac{1}{2}\,g\,v = \frac{e\,v}{2\sin\theta_W},$$

$$M_Z = \frac{1}{2}\sqrt{g^2+g'^2}\,v = \frac{e\,v}{2\sin\theta_W\cos\theta_W} = \frac{M_W}{\cos\theta_W},$$

$$M_\gamma = 0.$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

$$W^i_{\mu\nu} = \partial_\mu W^i_\nu - \partial_\nu W^i_\mu - g\epsilon_{ijk}W^j_\mu W^k_\nu$$

$SU(2)_L$: Non-Abelian gauge theory, asymptotically free

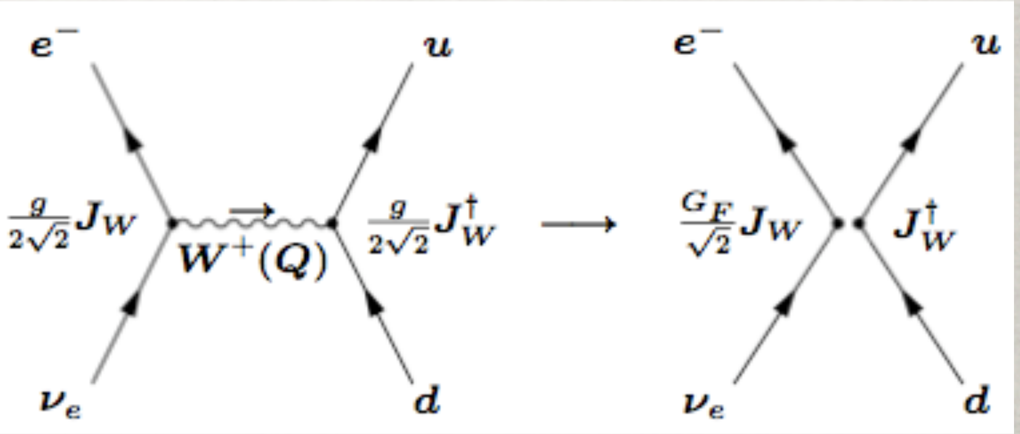
$U(1)_Y$: Non-asymptotically free → Landau pole!

"Weak force" NOT Weak!

$\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$ interactions.

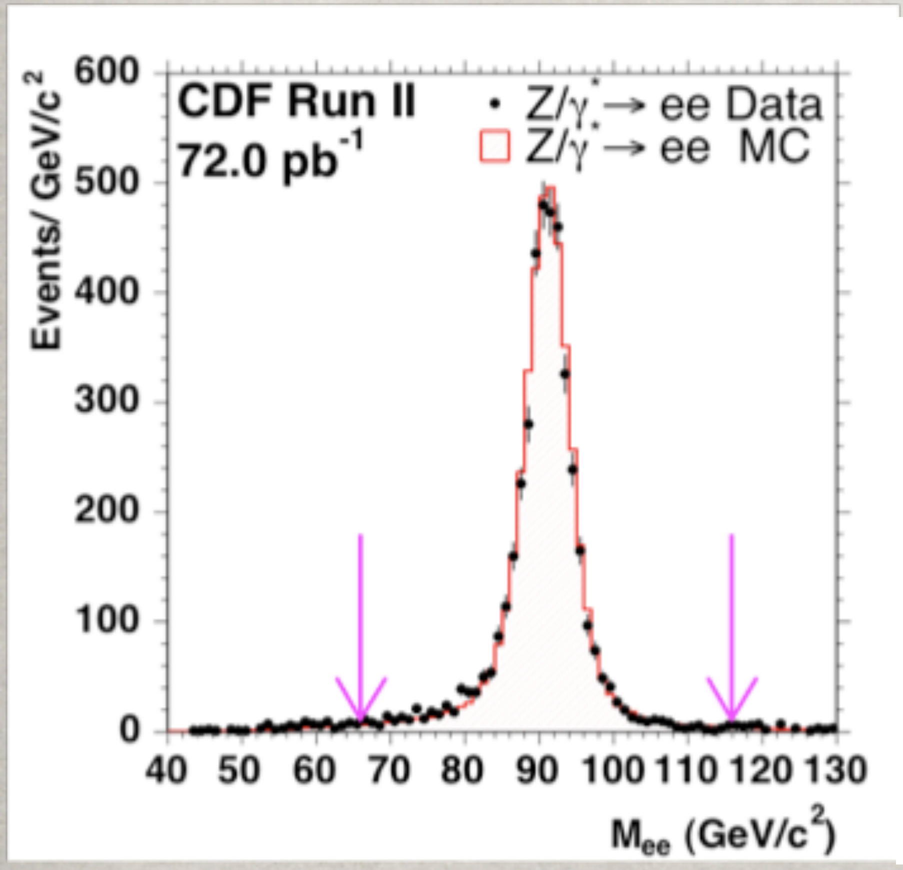
$e = g \sin\theta_W$ coupling unification

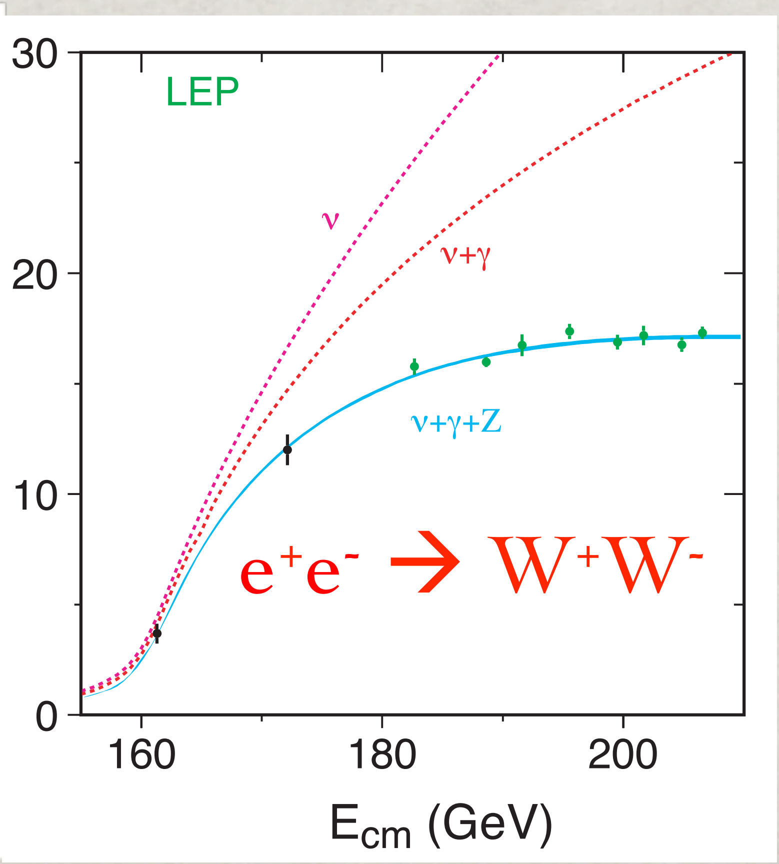
$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2}$$ short – range scale.

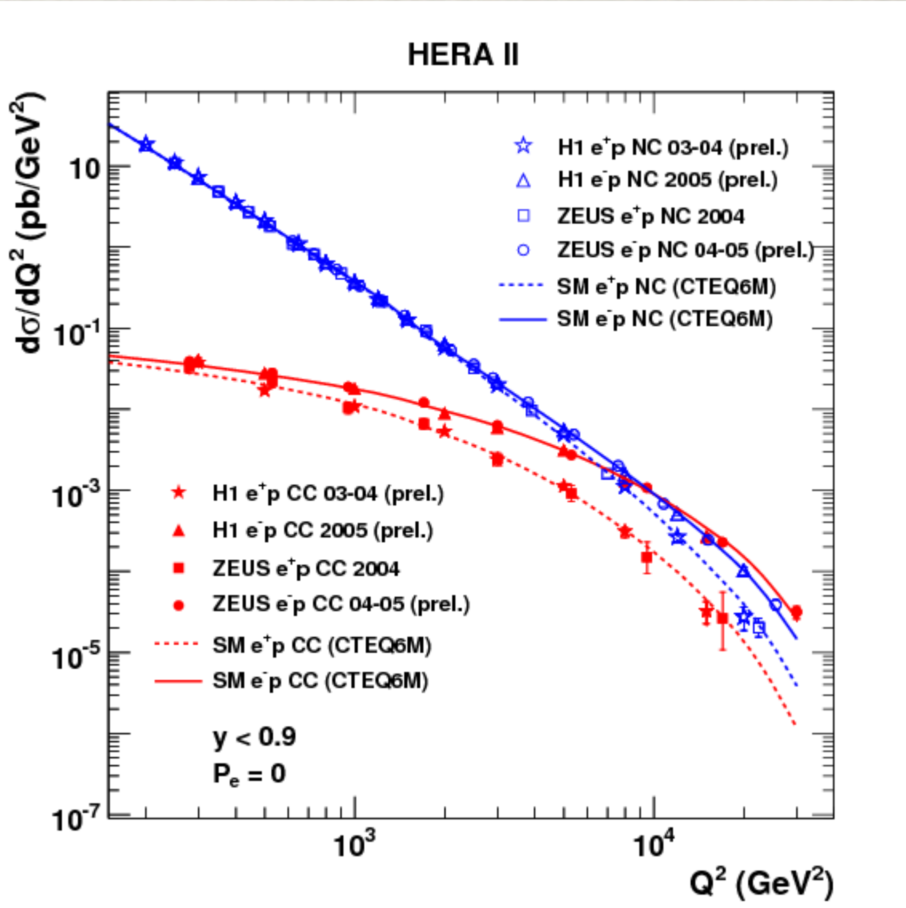


The EW couplings merging:

The EW scale is fully open up:







Quantity	Value	Standard Model	Pull	Dev.
M_Z [GeV]	91.1876 ± 0.0021	91.1874 ± 0.0021	0.1	0.0
Γ_Z [GeV]	2.4952 ± 0.0023	2.4961 ± 0.0010	−0.4	−0.2
$\Gamma(\text{had})$ [GeV]	1.7444 ± 0.0020	1.7426 ± 0.0010	—	—
$\Gamma(\text{inv})$ [MeV]	499.0 ± 1.5	501.69 ± 0.06	—	—
$\Gamma(\ell^+\ell^-)$ [MeV]	83.984 ± 0.086	84.005 ± 0.015	—	—
σ_{had}[nb]	41.541 ± 0.037	41.477 ± 0.009	1.7	1.7
R_e	20.804 ± 0.050	20.744 ± 0.011	1.2	1.3
R_μ	20.785 ± 0.033	20.744 ± 0.011	1.2	1.3
R_τ	20.764 ± 0.045	20.789 ± 0.011	−0.6	−0.5
R_b	0.21629 ± 0.00066	0.21576 ± 0.00004	0.8	0.8
R_c	0.1721 ± 0.0030	0.17227 ± 0.00004	−0.1	−0.1
$A_{FB}^{(0,e)}$	0.0145 ± 0.0025	0.01633 ± 0.00021	−0.7	−0.7
$A_{FB}^{(0,\mu)}$	0.0169 ± 0.0013		0.4	0.6
$A_{FB}^{(0,\tau)}$	0.0188 ± 0.0017		1.5	1.6
$A_{FB}^{(0,b)}$	0.0992 ± 0.0016	0.1034 ± 0.0007	−2.6	−2.3
$A_{FB}^{(0,c)}$	0.0707 ± 0.0035	0.0739 ± 0.0005	−0.9	−0.8
$A_{FB}^{(0,s)}$	0.0976 ± 0.0114	0.1035 ± 0.0007	−0.5	−0.5
$\bar{s}_\ell^2(A_{FB}^{(0,q)})$	0.2324 ± 0.0012	0.23146 ± 0.00012	0.8	0.7
	0.23200 ± 0.00076		0.7	0.6
	0.2287 ± 0.0032		−0.9	−0.9
A_e	0.15138 ± 0.00216	0.1475 ± 0.0010	1.8	2.1
	0.1544 ± 0.0060		1.1	1.3
	0.1498 ± 0.0049		0.5	0.6
A_μ	0.142 ± 0.015		−0.4	−0.3
A_τ	0.136 ± 0.015		−0.8	−0.7
	0.1439 ± 0.0043		−0.8	−0.7
A_b	0.923 ± 0.020	0.9348 ± 0.0001	−0.6	−0.6
A_c	0.670 ± 0.027	0.6680 ± 0.0004	0.1	0.1
A_s	0.895 ± 0.091	0.9357 ± 0.0001	−0.4	− 0.4

(nearly) perfect agreement between SM theory & expts!

Gauge coupling universality

Some tension!

Only: 3σ discrepancy!

$$a_\mu^{exp} = (1165920.80 \pm 0.63) \times 10^{-9}$$

$$a_\mu^{SM} = (1165918.41 \pm 0.48) \times 10^{-9}$$

Lecture II: Story of Mass-generation

A. Spontaneous Symmetry Breaking
B. The Nambu-Goldstone Theorem
C. The Higgs Mechanism
D. The Higgs Boson Interactions

A Problem! Pauli's Criticism:

An Anecdote by Yang: SU(2) gauge symmetry

Wolfgang Pauli (1900-1958) was spending the year in Princeton, and was deeply interested in symmetries and interactions.... Soon after my seminar began, when I had written on the blackboard,

Wolfgang Pauli and C. N. Yang

$(\partial_\mu - i\epsilon \mathbf{B}_\mu)\psi$

Pauli asked, "What is the mass of this field $\mathbf{B}_\mu$?" I said we did not know. Then I resumed my presentation but soon Pauli asked the same question again. I said something to the effect that it was a very complicated problem, we had worked on it and had come to no definite conclusions. I still remember his repartee: "That is not sufficient excuse". I was so taken aback that I decided, after a few moments' hesitation, to sit down. There was general embarrassment. Finally Oppenheimer, who was chairman of the seminar, said "We should let Frank proceed". I then resumed and Pauli did not ask any more questions during the seminar.

The local gauge symmetry prevents gauge bosons masses!

$$\frac{1}{2}M_A^2 A_\mu A^\mu \to \frac{1}{2}M_A^2(A_\mu - \frac{1}{e}\partial_\mu \alpha)(A^\mu - \frac{1}{e}\partial^\mu \alpha) \neq \frac{1}{2}M_A^2 A_\mu A^\mu$$

Even worse:

"The Left- and right-chiral electrons carry different Weak charges" (Lee & Yang)

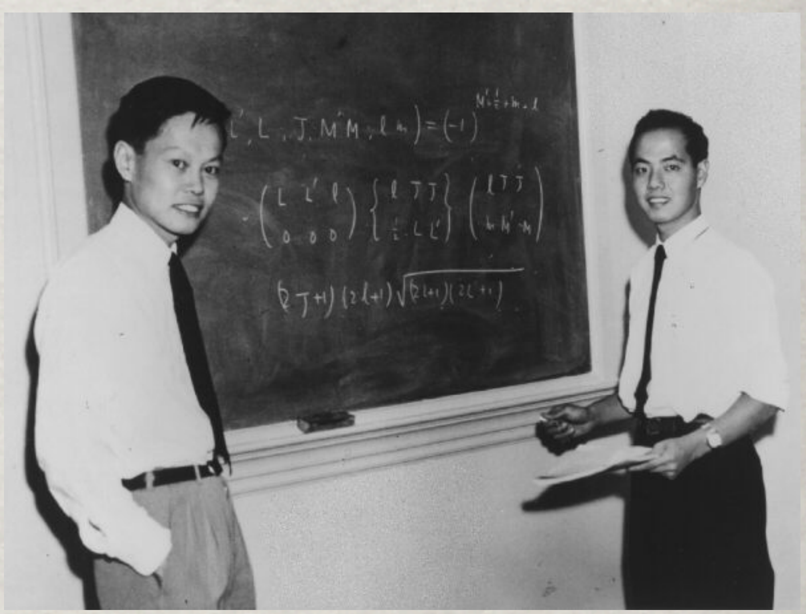
- Simple structure and particle contents:

Leptons:

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L, \quad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L, \quad e_R,\ \mu_R,\ \tau_R, \quad (\nu'_R s\ ?)$$

Quarks:

$$\begin{pmatrix} u \\ d \end{pmatrix}_L, \quad \begin{pmatrix} c \\ s \end{pmatrix}_L, \quad \begin{pmatrix} t \\ b \end{pmatrix}_L, \quad u_R,\ d_R,\ c_R,\ s_R,\ t_R,\ b_R$$



Fermion masses also forbidden by gauge symmetry!

$$-m_e\bar{e}e = -m_e\bar{e}\left(\frac{1}{2}(1-\gamma_5)+\frac{1}{2}(1+\gamma_5)\right)e = -m_e(\bar{e}_R e_L + \bar{e}_L e_R)$$

Electroweak gauge theory → massless!

A. The Spontaneous Symmetry Breaking
-- Nature May Not be THAT Symmetric:

"The Lagrangian of the system may display an symmetry, but the ground state does not respect the same symmetry."

Exercise 3:

Find (or make up) other examples for spontaneous symmetry breaking.

Also, think about the relations between the fundamental theoretical formalisms (Newton's Law; Maxwell Equations; Einstein Equation; Lagrangians...) and specific states for a given system (initial and boundary conditions of a system).

B. The Nambu-Goldstone Theorem
-- A show stopper or helper?

"If a continuous symmetry of the system is spontaneously broken, then there will appear a massless degree of freedom, called the Nambu-Goldstone boson."

Symmetry: $[Q, H] = QH - HQ = 0$

Vacuum state: $H\,|0> = Emin\,|0>$ But: $Q\,|0> \neq 0 = |0'>$

$(QH - HQ)|0> = 0 = (Emin - H)|0'>$,

thus: $H\,|0'> = Emin\,|0'>$

There is a new, non-symmetric state |0'>,
that has a degenerate energy with vacuum |0>,
thus massless: the Nambu-Goldstone boson.

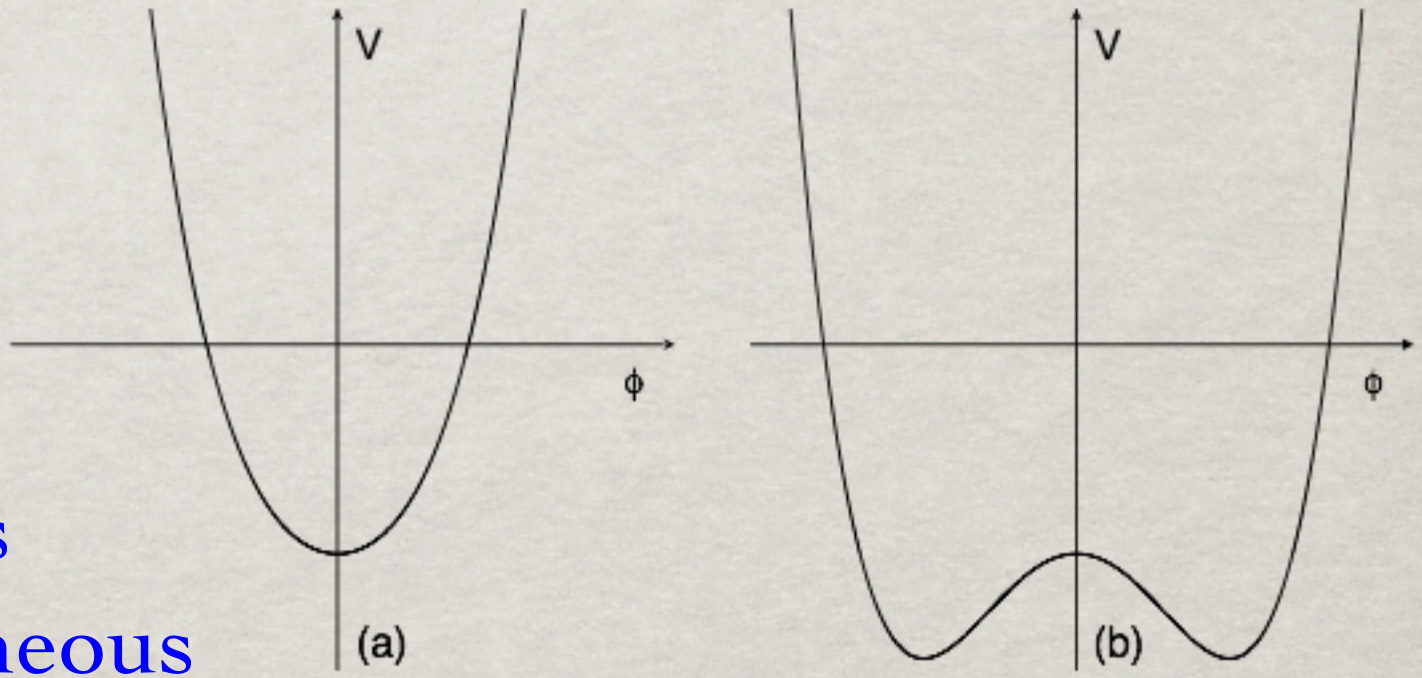
An illustrative (Goldstone's original) Model:

(a). Background complex scalar field Φ:

$$\mathcal{L} = \partial^\mu \phi^* \partial_\mu \phi - V(\phi^* \phi) \qquad V = \frac{\lambda}{4}\left(\phi^*\phi - \frac{\mu^2}{\lambda}\right)^2$$

Invariant under a U(1) global transformation:

$$\phi \rightarrow e^{i\alpha} \phi$$



For $\mu^2 > 0$, the vacuum is shifted, and thus spontaneous symmetry breaking.

$$v = \langle 0|\phi|0\rangle = \mu/\sqrt{\lambda}.$$

(b). Field Φ Re-definition:

[+] C. Burgges, hep-ph/9812468

Weinberg's 1st Law of Theoretical Physics[+]:

"You can use whatever variables you like. But if you used the wrong one, you'd be sorry."

Define:

$$\phi(x) = \chi(x)\, e^{i\theta(x)},$$

$$\mathcal{L} = -\partial_\mu \chi \partial^\mu \chi - \chi^2 \partial_\mu \theta \partial^\mu \theta - V(\chi^2).$$

(this is like from the rectangular *form* to the *polar form*.)

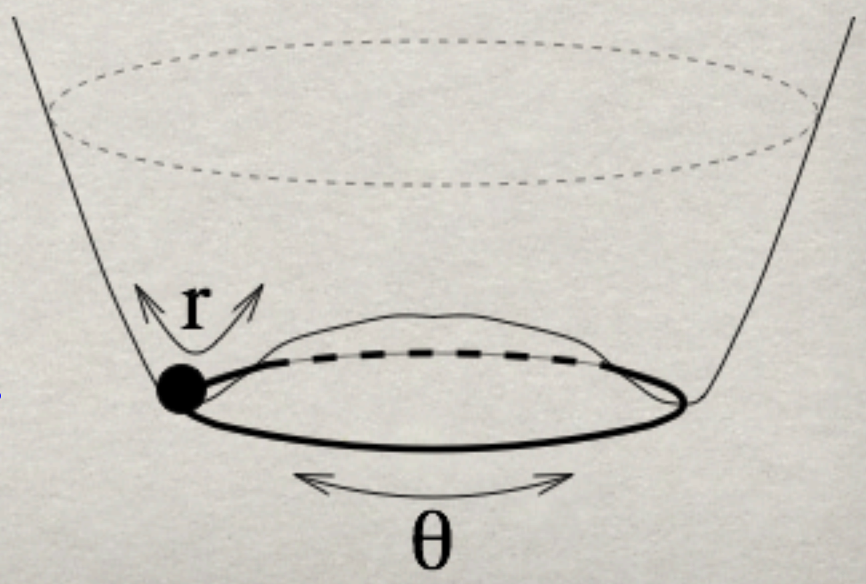
We then see that:

* the θ field is only derivatively coupled, and thus decoupled at low energies
* the θ field respects an inhomogeneous transformation

$$\theta \to \theta + \alpha, \quad \phi = v e^{i\theta(x)}$$

a phase rotation from the vacuum:

* the $\chi(x)$ is massive radial excitation.



"Nambu-Goldstone Bosons"

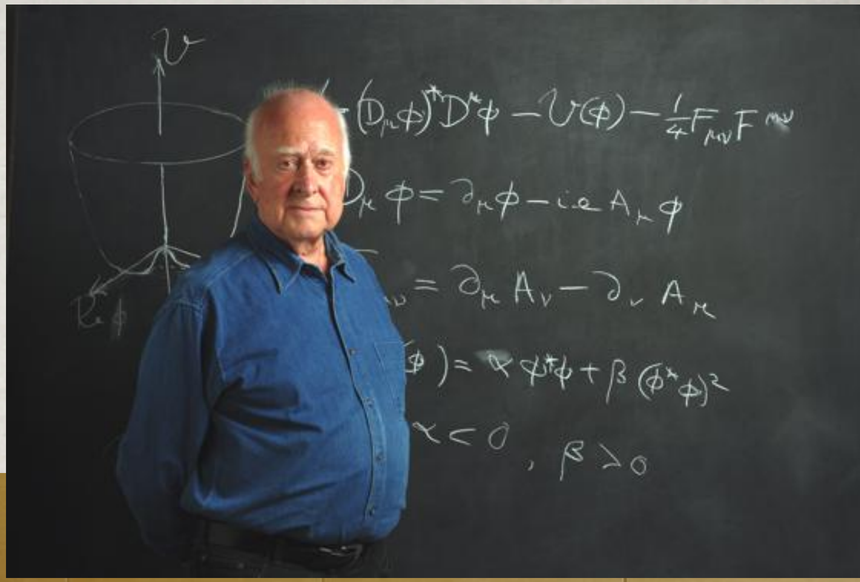
Except the photon, no massless boson (a long-range force carrier) has been seen in particle physics!

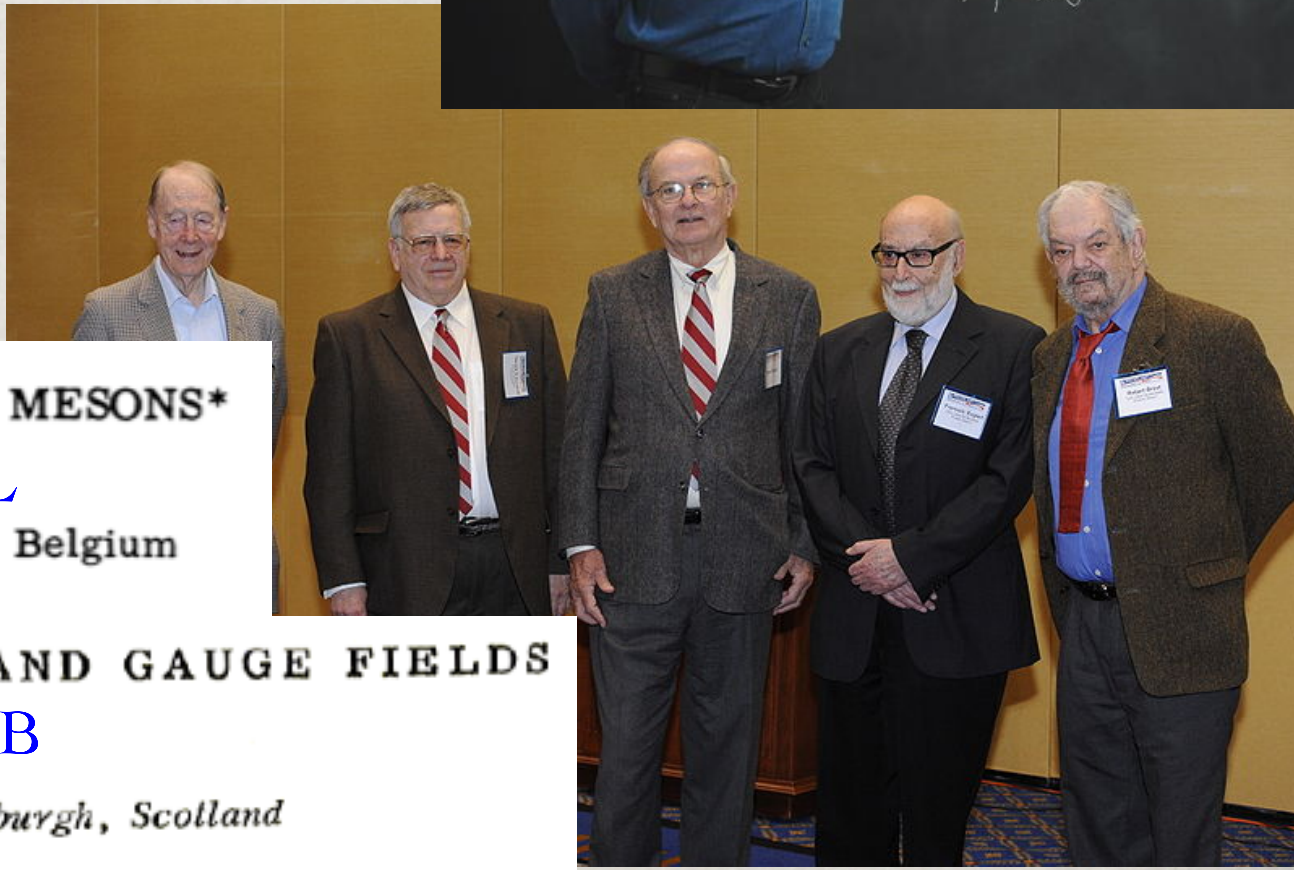
(Recall Pauli's criticism)

The Spontaneous Symmetry Breaking: Brilliant idea & common phenomena, confronts the Nambu-Goldstone theorem!

C. The Magic in 1964: The "Higgs Mechanism"

"If a LOCAL gauge symmetry is spontaneously broken, then the gauge boson acquires a mass by absorbing the Goldstone mode."





BROKEN SYMMETRY AND THE MASS OF GAUGE VECTOR MESONS*

F. Englert and R. Brout
Faculté des Sciences, Université Libre de Bruxelles, Bruxelles, Belgium
(Received 26 June 1964)

PRL

BROKEN SYMMETRIES, MASSLESS PARTICLES AND GAUGE FIELDS

P. W. HIGGS
Tait Institute of Mathematical Physics, University of Edinburgh, Scotland

Received 27 July 1964

PLB

BROKEN SYMMETRIES AND THE MASSES OF GAUGE BOSONS

Peter W. Higgs
Tait Institute of Mathematical Physics, University of Edinburgh, Edinburgh, Scotland
(Received 31 August 1964)

PRL

GLOBAL CONSERVATION LAWS AND MASSLESS PARTICLES*

G. S. Guralnik,† C. R. Hagen,‡ and T. W. B. Kibble
Department of Physics, Imperial College, London, England
(Received 12 October 1964)

PRL

An illustrative (original) Model:¶

$$\mathscr{L} = |\mathscr{D}^\mu \phi|^2 - \mu^2 |\phi|^2 - |\lambda| (\phi^* \phi)^2 - \tfrac{1}{4} F_{\mu\nu} F^{\mu\nu},$$

where

$$\phi = \frac{\phi_1 + i\phi_2}{\sqrt{2}}$$

is a complex scalar field[4] and as usual

$$\mathscr{D}_\mu \equiv \partial_\mu + iqA_\mu$$

and

$$F_{\mu\nu} \equiv \partial_\nu A_\mu - \partial_\mu A_\nu.$$

The Lagrangian (5.3.1) is invariant under U(1) rotations

$$\phi \to \phi' = e^{i\theta} \phi$$

and under the local gauge transformations

$$\phi(x) \to \phi'(x) = e^{iq\alpha(x)} \phi(x),$$

$$A_\mu(x) \to A'_\mu(x) = A_\mu(x) - \partial_\mu \alpha(x).$$

¶ C. Quigg, Gauge Theories of the Strong ...

An illustrative (original) Model:¶

After the EWSB, parameterized in terms of

$$\langle \phi \rangle_0 = v/\sqrt{2}, \qquad \phi = e^{i\zeta/v}(v+\eta)/\sqrt{2}$$
$$\approx (v+\eta+i\zeta)/\sqrt{2}.$$

Then the Lagrangian appropriate for the study of small oscillations is

$$\mathcal{L}_{\text{so}} = \tfrac{1}{2}[(\partial_\mu \eta)(\partial^\mu \eta) + 2\mu^2\eta^2] + \tfrac{1}{2}[(\partial_\mu \zeta)(\partial^\mu \zeta)]$$
$$-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + qvA_\mu(\partial^\mu \zeta) + \frac{q^2v^2}{2}A_\mu A^\mu + \cdots$$

The gauge field acquires a mass, mixes with the Goldstone boson.

Upon diagonalization: $$\frac{q^2v^2}{2}\left(A_\mu + \frac{1}{qv}\partial_\mu \zeta\right)\left(A^\mu + \frac{1}{qv}\partial^\mu \zeta\right),$$

a form that pleads for the gauge transformation

$$A_\mu \to A'_\mu = A_\mu + \frac{1}{qv}\partial^\mu \zeta,$$

which corresponds to the phase rotation on the scalar field

$$\phi \to \phi' = e^{-i\zeta(x)/v}\phi(x) = (v+\eta)/\sqrt{2}.$$

the resultant Lagrangian is then:

$$\mathscr{L}_{so} = \tfrac{1}{2}[(\partial_\mu \eta)(\partial^\mu \eta) + 2\mu^2\eta^2] - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{q^2v^2}{2}A'_\mu A'^\mu$$

- an η-field, with $(\text{mass})^2 = -2\mu^2 > 0$; the Higgs boson!
- a massive vector field A'_μ, with mass $= qv$
- no ζ-field.

- By virtue of a gauge choice - the unitary gauge, the ζ-field disappears in the spectrum: a massless photon "swallowed" the massless NG boson!

Degrees of freedom count:

Before EWSB: After:

2 (scalar)+2 (gauge pol.); 1 (scalar)+3 (gauge pol.)

- Two problems provide cure for each other!

massless gauge boson + massless NG boson

→ massive gauge boson + no NG boson

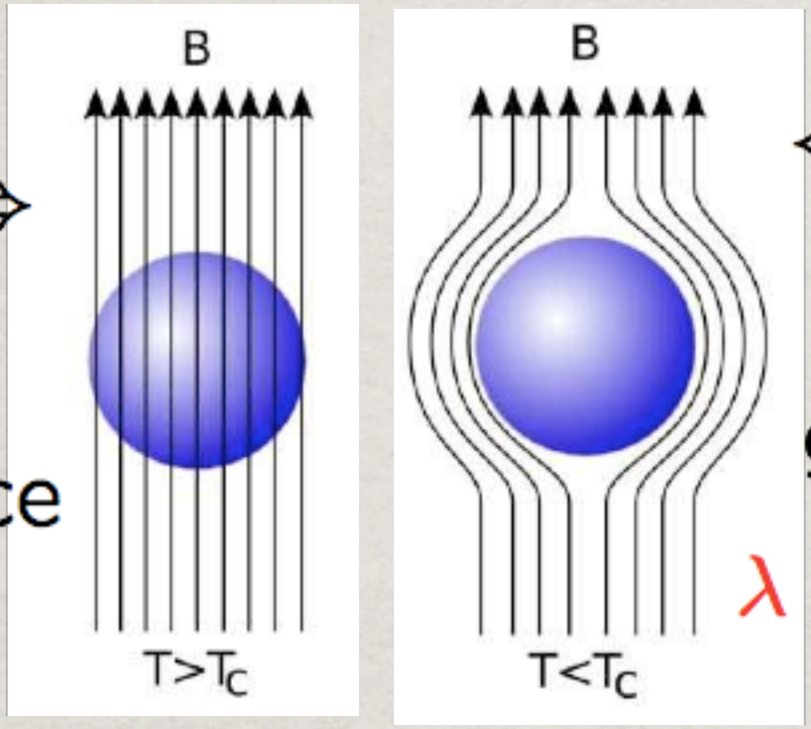
This is truly remarkable!

Known example: Superconductivity

Normal phase⇒

$E^2 = p^2c^2$

Long-range force



⇐Superconducting phase

$E^2 = p^2c^2 + m^2c^4$

gap leads to $\sim exp(-r/\lambda)$

$\lambda \sim m^{-1}$ penetration depth

In "conventional" electro-magnetic superconductivity:

$m_\gamma \sim m_e/1000$, $T_c^{em} \sim \mathcal{O}(\text{few } K)$. BCS theory.

In "electro-weak superconductivity":

$m_w \sim G_F^{-\frac{1}{2}} \sim 100$ GeV, $T_c^w \sim 10^{15} K!$

As for the name ...

1972: Ben Lee (Rochester Conf. at FNAL) named "Higgs boson" and the "Higgs mechanism".§

§ Peter Higgs: My Life as a Boson.

Font Size:

The New York Review of Books

The Crisis of Big Science

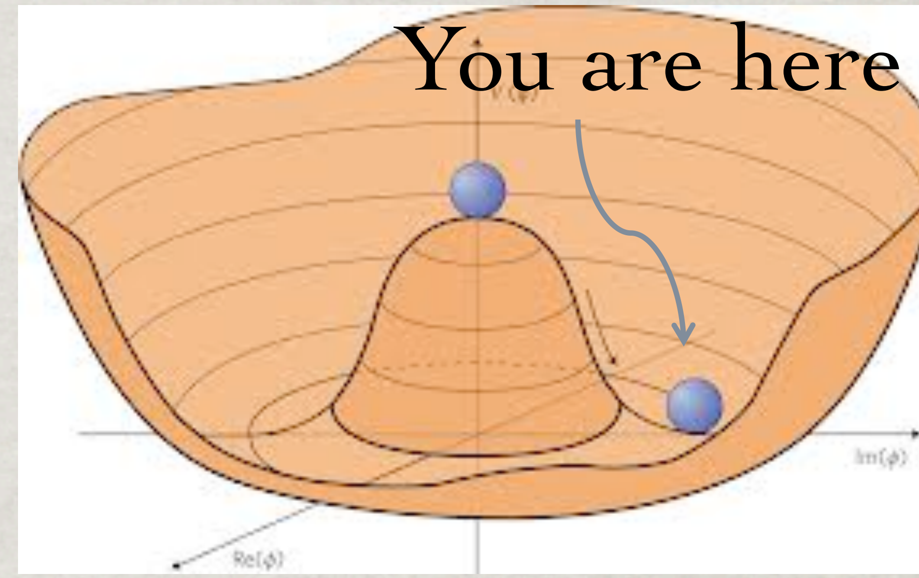
MAY 10, 2012

Steven Weinberg

As to my responsibility for the name "Higgs boson," because of a mistake in reading the dates on these three earlier papers, I thought that the earliest was the one by Higgs, so in my 1967 paper I cited Higgs first, and have done so since then. Other physicists apparently have followed my lead. But as Close points out, the earliest paper of the three I cited was actually the one by Robert Brout and François Englert. In extenuation of my mistake, I should note that Higgs and Brout and Englert did their work independently and at about the same time, as also did the third group (Gerald Guralnik, C.R. Hagen, and Tom Kibble). But the name "Higgs boson" seems to have stuck. ↩

It's like Landau-Ginzburg
It's NOT Landau-Ginzburg

In the SM:



$$V(|\Phi|) \quad = \quad -\mu^2\Phi^\dagger\Phi + \lambda(\Phi^\dagger\Phi)^2$$

$$<|\Phi|> \; = \; v = (\sqrt{2}G_F)^{-1/2} \approx 246 \text{ GeV}$$

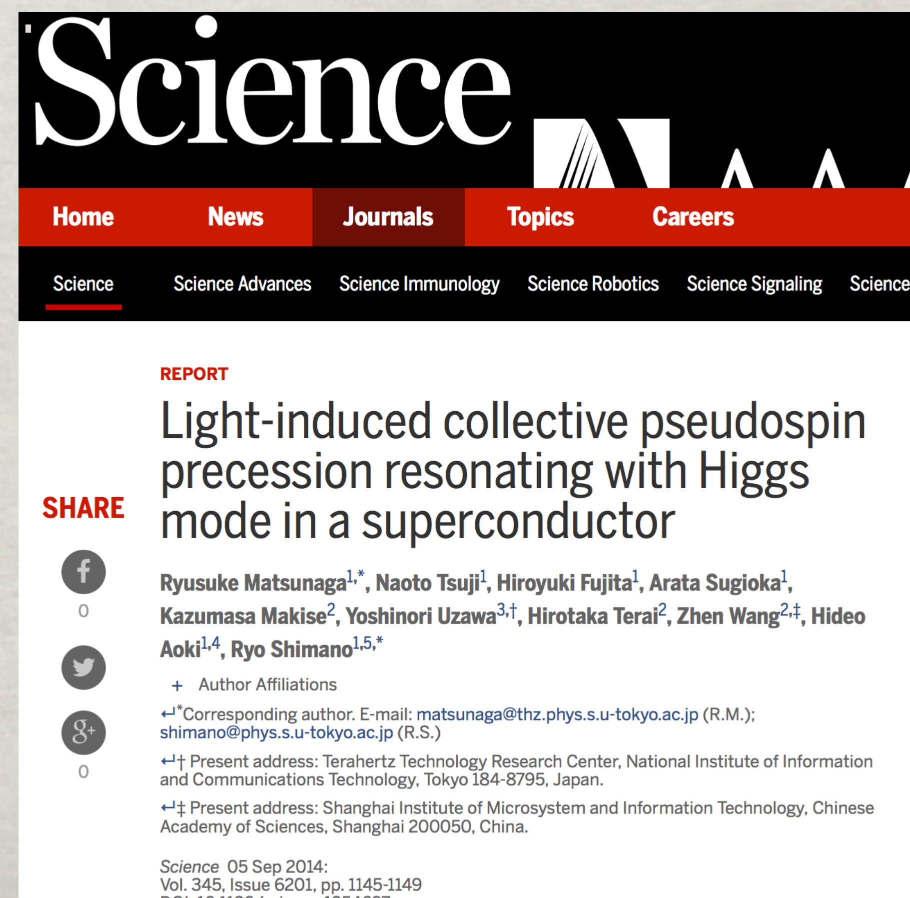
$$m_H \approx 126 \text{ GeV}$$

It is a weakly coupled, very narrow particle $(\Gamma/m \approx 10^{-5})$
elementary at a scale >1000 GeV!

Landau-Ginzburg:

Similar parameterization, but BCS as the underlying theory!

A collective mode of TeraHertz $(10^{-3}$ eV) vibration observed!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REPORT

SHARE

Light-induced collective pseudospin precession resonating with Higgs mode in a superconductor

Ryusuke Matsunaga[1,*], Naoto Tsuji[1], Hiroyuki Fujita[1], Arata Sugioka[1], Kazumasa Makise[2], Yoshinori Uzawa[3,†], Hirotaka Terai[2], Zhen Wang[2,‡], Hideo Aoki[1,4], Ryo Shimano[1,5,*]

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D. Higgs Boson Interactions

1. The SM Lagrangian: $\mathcal{L}_{SU(2)\times U(1)} = \mathcal{L}_{gauge} + \mathcal{L}_\phi + \mathcal{L}_f + \mathcal{L}_{Yuk}.$

The gauge part is **Pure gauge sector:**

$$\mathcal{L}_{gauge} = -\frac{1}{4}W^i_{\mu\nu}W^{\mu\nu i} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu},$$

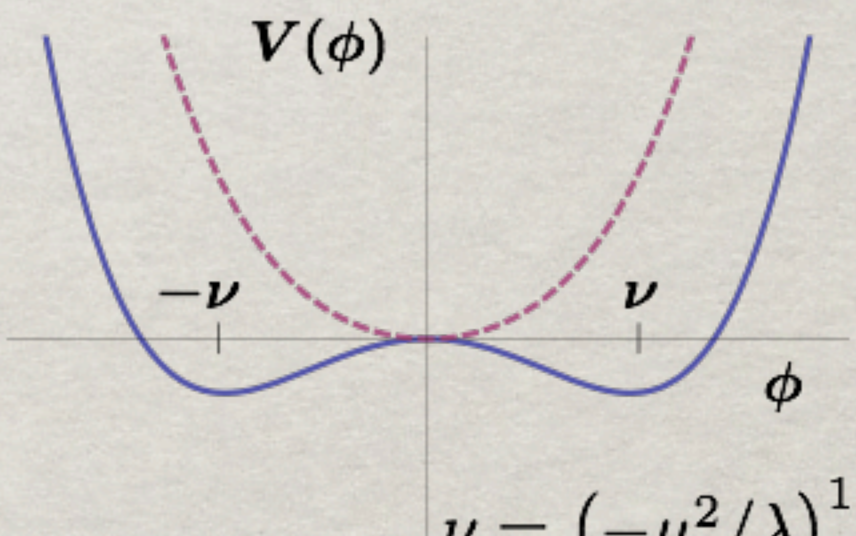
The scalar part of the Lagrangian is

The Higgs: $\mathcal{L}_\phi = (D^\mu\phi)^\dagger D_\mu\phi - V(\phi)$ $\quad D_\mu\phi = \left(\partial_\mu + ig\frac{\tau^i}{2}W^i_\mu + \frac{ig'}{2}B_\mu\right)\phi,$

$$V(\phi) = +\mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2.$$

$$\phi = \frac{1}{\sqrt{2}}e^{i\sum\xi^i L^i}\begin{pmatrix}0\\ \nu+H\end{pmatrix}$$

$$\phi \to \phi' = e^{-i\sum\xi^i L^i}\phi = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ \nu+H\end{pmatrix}$$



$$\nu = (-\mu^2/\lambda)^{1/2}$$

$$\mathcal{L}_\phi = (D^\mu\phi)^\dagger D_\mu\phi - V(\phi)$$

$$= M_W^2 W^{\mu+}W^-_\mu\left(1+\frac{H}{\nu}\right)^2 + \frac{1}{2}M_Z^2 Z^\mu Z_\mu\left(1+\frac{H}{\nu}\right)^2$$

$$M_H^2 = -2\mu^2 = 2\lambda v^2$$

$$+\frac{1}{2}(\partial_\mu H)^2 - V(\phi). \qquad V(\phi) = -\frac{\mu^4}{4\lambda} - \mu^2 H^2 + \lambda\nu H^3 + \frac{\lambda}{4}H^4.$$

The Fermions:[§]

$$\mathcal{L}_f = \sum_{m=1}^{F} \left(\bar{q}^0_{mL} i \not{D} q^0_{mL} + \bar{l}^0_{mL} i \not{D} l^0_{mL} + \bar{u}^0_{mR} i \not{D} u^0_{mR} + \bar{d}^0_{mR} i \not{D} d^0_{mR} + \bar{e}^0_{mR} i \not{D} e^0_{mR} + \bar{\nu}^0_{mR} i \not{D} \nu^0_{mR} \right)$$

$$D_\mu q^0_{mL} = \left(\partial_\mu + \frac{ig}{2} \vec{\tau} \cdot \vec{W}_\mu + \frac{ig'}{6} B_\mu \right) q^0_{mL} \qquad D_\mu u^0_{mR} = \left(\partial_\mu + \frac{2ig'}{3} B_\mu \right) u^0_{mR}$$

$$D_\mu l^0_{mL} = \left(\partial_\mu + \frac{ig}{2} \vec{\tau} \cdot \vec{W}_\mu - \frac{ig'}{2} B_\mu \right) l^0_{mL} \qquad D_\mu d^0_{mR} = \left(\partial_\mu - \frac{ig'}{3} B_\mu \right) d^0_{mR}$$

$$D_\mu e^0_{mR} = \left(\partial_\mu - ig' B_\mu \right) e^0_{mR}$$

$$D_\mu \nu^0_{mR} = \partial_\mu \nu^0_{mR},$$

Gauge invariant, massless.

However, a fermion mass must flip chirality:

$$m_f (\bar{f}_L f_R + \bar{f}_R f_L)$$

and thus not SM gauge invariant L ≠ R !

Need something like a doublet:

$$y_f (\bar{f}_1, f_2)_L \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}_L f_R$$

that's the Higgs doublet!

[§] P. Langacker: TASI Lectures 2007.

The gauge invariant Yukawa interactions:

Need a doublet with a flip Y: $\tilde{\phi} = i\sigma_2\phi^*$

$$\mathcal{L}_{Yuk} = -\sum_{m,n=1}^{F}\left[\Gamma^u_{mn}\bar{q}^{\,0}_{mL}\tilde{\phi}u^0_{nR} + \Gamma^d_{mn}\bar{q}^{\,0}_{mL}\phi d^0_{nR}\right.$$
$$\left. + \Gamma^e_{mn}\bar{l}^{\,0}_{mn}\phi e^0_{nR} + \Gamma^\nu_{mn}\bar{l}^{\,0}_{mL}\tilde{\phi}\nu^0_{nR}\right] + h.c.,$$

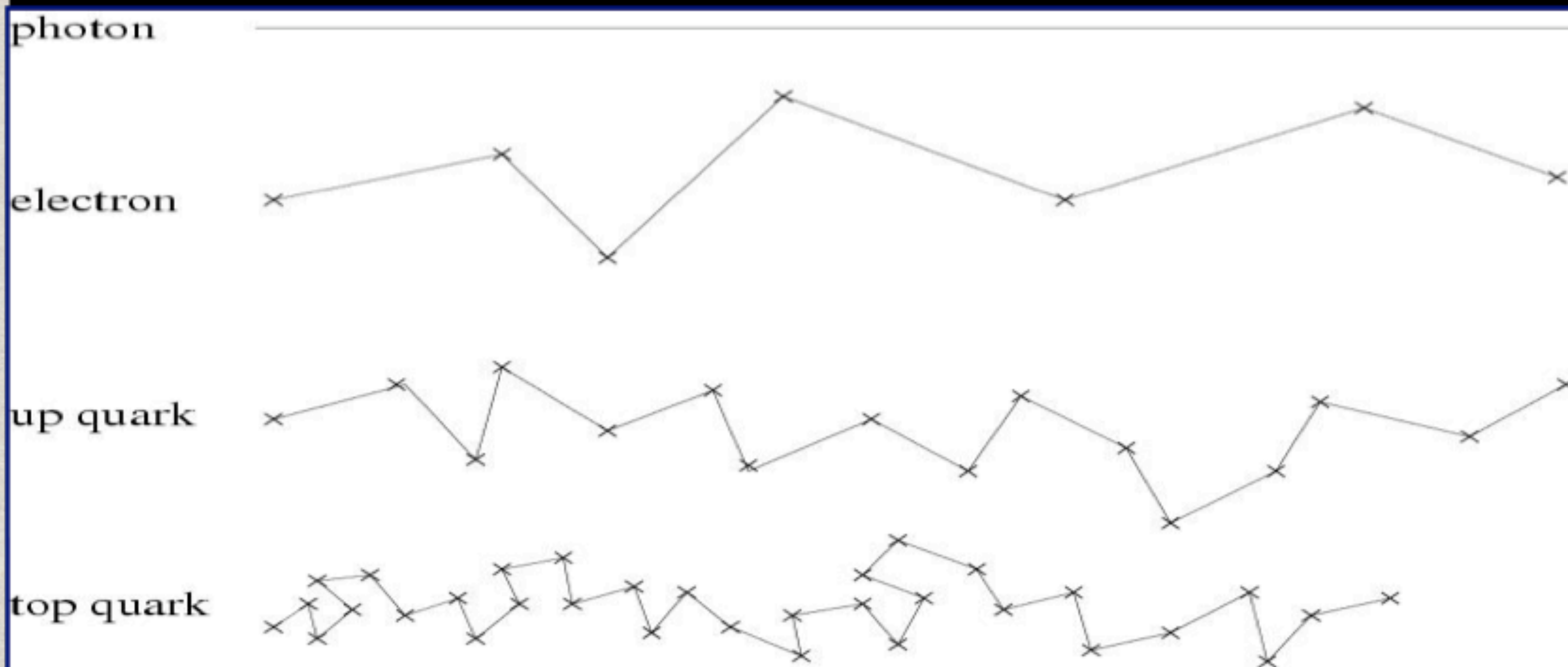
After the EWSB,

$$-\mathcal{L}_{Yuk} \rightarrow \sum_{m,n=1}^{F}\bar{u}^0_{mL}\Gamma^u_{mn}\left(\frac{\nu + H}{\sqrt{2}}\right)u^0_{mR} + (d, e, \nu) \text{ terms}$$
$$= \bar{u}^0_L\left(M^u + h^u H\right)u^0_R + (d, e, \nu) \text{ terms } + h.c.,$$

$$-\mathcal{L}_{Yuk} = \sum_i m_i\bar{\psi}_i\psi_i\left(1 + \frac{g}{2M_W}H\right) = \sum_i m_i\bar{\psi}_i\psi_i\left(1 + \frac{H}{\nu}\right)$$

Higgs Boson Couplings:

Masses determined by interactions with vacuum:



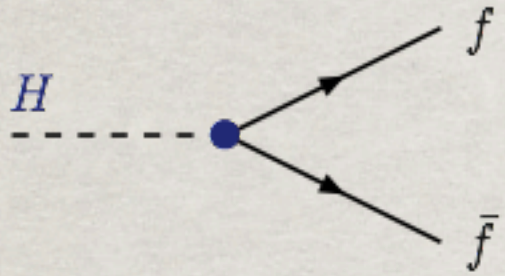
$$M_{W,Z} = \frac{1}{2} g_V v, \quad m_f = \frac{g_f}{\sqrt{2}} v, \quad v^{-2} = \sqrt{2}\, G_F.$$

Thus, where ever is mass, there will be H!

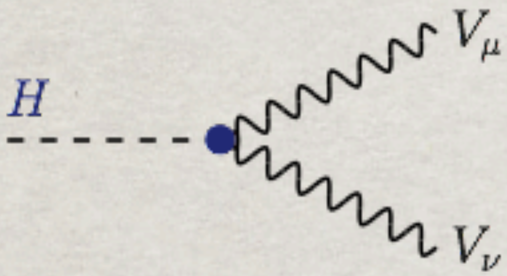
The Low-Energy-theorem:

$$m_i \to m_i \left(1 + \frac{H}{v}\right) \text{ for } p_H < v.$$

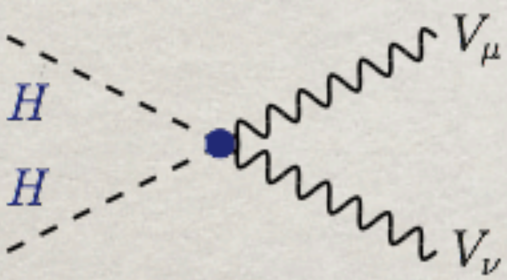
Feynman rules:



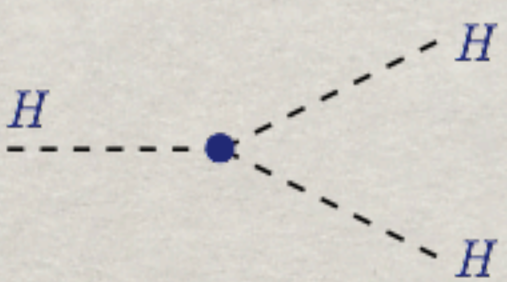
$$g_{Hff} \;=\; m_f/v \;=\; (\sqrt{2}G_\mu)^{1/2}\, m_f \qquad \times\ (i)$$



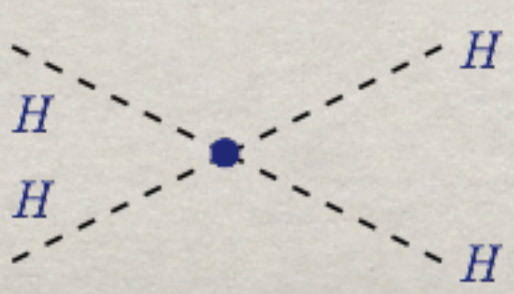
$$g_{HVV} \;=\; 2M_V^2/v \;=\; 2(\sqrt{2}G_\mu)^{1/2}\, M_V^2 \qquad \times\ (-ig_{\mu\nu})$$



$$g_{HHVV} \;=\; 2M_V^2/v^2 \;=\; 2\sqrt{2}G_\mu\, M_V^2 \qquad \times\ (-ig_{\mu\nu})$$



$$g_{HHH} \;=\; 3M_H^2/v \;=\; 3(\sqrt{2}G_\mu)^{1/2}\, M_H^2 \qquad \times\ (i)$$



$$g_{HHHH} = 3M_H^2/v^2 \;=\; 3\sqrt{2}G_\mu\, M_H^2 \qquad \times\ (i)$$

Lecture III: Higgs Physics and Beyond

A. What Does THIS Higgs Tell us?
B. SM Higgs Sector at Higher Energies
   & the Need for New Physics

C. Higgs Boson Decays
D. Higgs Physics at the LHC Colliders
E. Higgs Physics at an e+e- Collider

This discovery opens up a new era in HEP!

In these Lectures, I wish to convey to you:

- This is truly an "LHC Revolution", ever since the "November Revolution" in 1974 for the J/ψ discovery!

- It strongly argues for new physics beyond the Standard Model:
  Under the Higgs lamp post.

A. A Weakly Coupled Light Higgs?

1. The Higgs Mechanism DOES NOT require a Higgs boson!

"If a LOCAL gauge symmetry is spontaneously broken, then the gauge boson acquires a mass by absorbing the Goldstone mode."

The Non-Linear realization:

$$\Phi = \frac{1}{\sqrt{2}}(v+H)U\,, \qquad U = \exp[i\pi^a\tau^a/v]$$

$\mathcal{G}_{\rm SM} = SU(2)_L \otimes U(1)_Y$, as

$$U \to U' = g_L U g_Y^\dagger\,, \qquad H \to H' = H\,,$$

$$g_L = \exp[-i\theta_L^a\tau^a/2]\,, \qquad g_Y = \exp[-i\theta_Y\tau^3/2]\,.$$

Then leave out the singlet H, the SM gauge symmetry spontaneously broken:

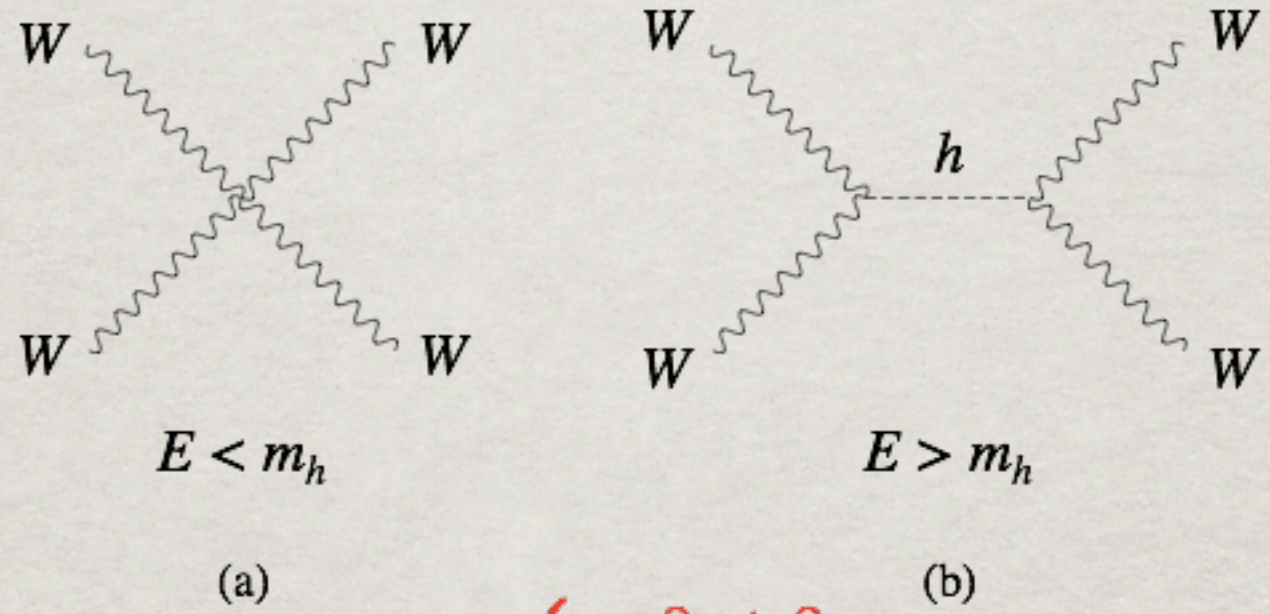
$$D_\mu U = \partial_\mu U + igW_\mu^a\frac{\tau^a}{2}U - ig'UB_\mu\frac{\tau^3}{2}$$

$$\mathcal{L} = \frac{1}{2}Tr(D_\mu\Phi^\dagger D^\mu\Phi) \Rightarrow \frac{v^2}{4}Tr(D_\mu U^\dagger D^\mu U) \longrightarrow \frac{v^2}{4}(\sum_i g^2W_i^2 + g'^2B^2)$$

(fermion masses can be accommodated similarly)

Higgs boson could be absent, but:

Consider the massive gauge boson scattering:



$$\mathcal{M}(W_L W_L \to W_L W_L) \sim \begin{cases} E_{cm}^2/v^2 & \text{no light Higgs,} \\ m_h^2/v^2 & \text{with a SM Higgs.} \end{cases}$$

Partial-wave unitarity demands

$$a_0 = \frac{1}{16\pi} \frac{m_h^2 \text{ or } E_{cm}^2}{v^2} \lesssim 1$$
$$\Rightarrow m_h \text{ or } E_{cm} \lesssim \mathcal{O}(1 \text{ TeV}).$$

Exercise 11: Verify this unitarity bound by an explicit partial wave analysis.

2. Natural dynamics prefers a heavier, broad Higgs boson!

In low-energy QCD, a generic dynamical mass is

$m \sim 4\pi f_\pi \sim 1$ GeV:

$m(f_0) \sim 0.4$ -1.2 GeV, $\Gamma \sim 0.6$ - 1.0 GeV !

$m(\rho^{\pm,0}) \sim 0.77$ GeV, $\Gamma \sim 0.15$ GeV.

Lessons from QCD and other strong dynamical models (Technicolor-like, composite, dilaton...) argue the dynamical mass to be of the order

$4\pi v \approx 2$ TeV!

And typically strong interacting: Γ(total) $\geq$ 20%M !

--- except the pseudo Goldstone bosons.

The fact that we do have observed a rather light, weakly coupled boson:

m_h = 125-126 GeV, $\Gamma < 1$ GeV,

is truly revolutionary!

We have just discovered a "fifth (weak) force":

$\lambda \approx 1/8$! ← $m_H^2/2v^2$ in the SM

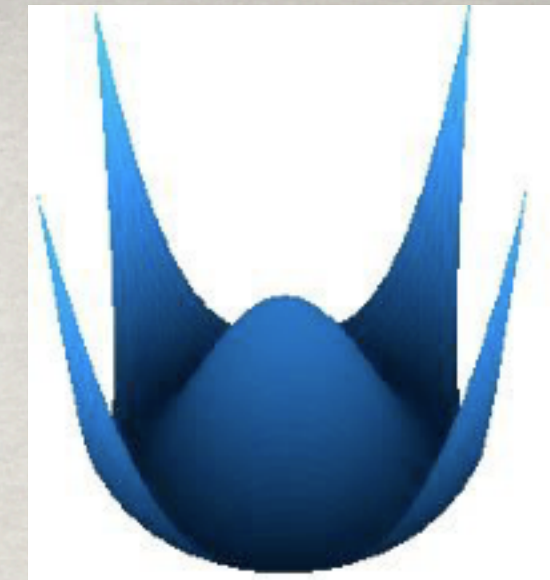
Hopes for uncovering a deeper theory:

- λ determined by other couplings like in SUSY?

where $\lambda = (g_1^2 + g_2^2)/8$

- or dynamically generated by a new strong force?

B. SM Higgs Sector at Higher Energies



Recall the SM Higgs sector: $V(\Phi) = \mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2$

$$\begin{aligned} \mathcal{L}_H &= \frac{1}{2}(\partial_\mu H)(\partial^\mu H) - V \\ &= \frac{1}{2}(\partial^\mu H)^2 - \lambda v^2 H^2 - \lambda v H^3 - \frac{\lambda}{4} H^4 \end{aligned}$$

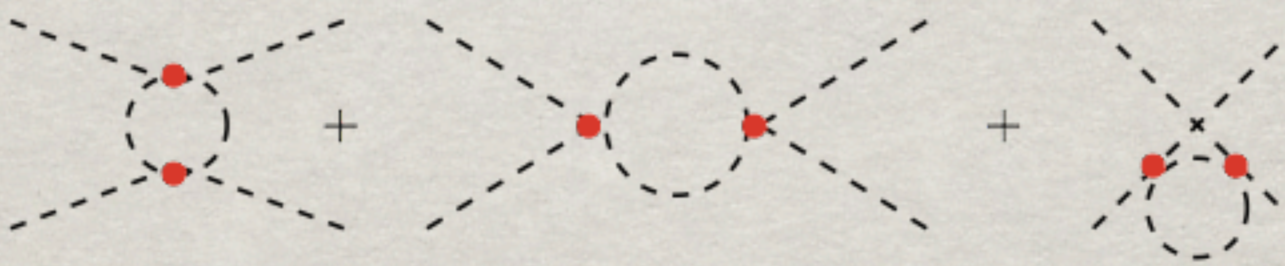
$$M_H^2 = 2\lambda v^2 = -2\mu^2$$

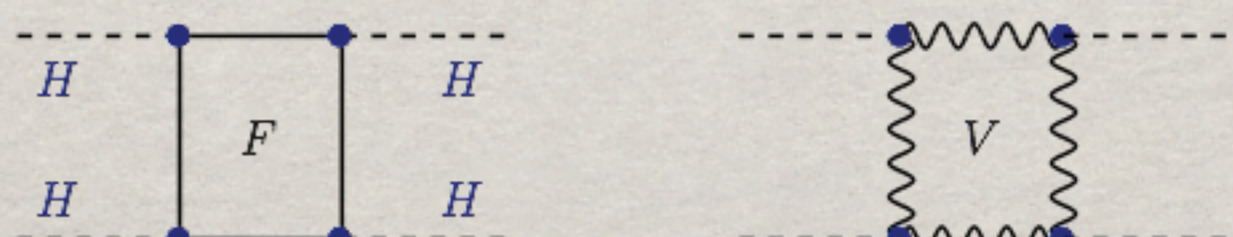
Crucial conditions: $\mu^2(Q^2) < 0, \quad \lambda(Q^2) > 0$

Renormalization Group Equation Evolution at NLO:

$$32\pi^2 \frac{d\lambda}{dt} = 24\lambda^2 - (3g'^2 + 9g^2 - 24y_t^2)\lambda + \frac{3}{8}g'^4 + \frac{3}{4}g'^2 g^2 + \frac{9}{8}g^4 - 24y_t^4 + \cdots$$

$$t = \ln(Q^2/Q_0^2)$$





1. Triviality bound

How large M_H (λ) can be dragged up?

$$V(\Phi) = \mu^2 \Phi^\dagger \Phi + \lambda(\Phi^\dagger \Phi)^2$$

$$M_H^2 = 2\lambda v^2 = -2\mu^2$$

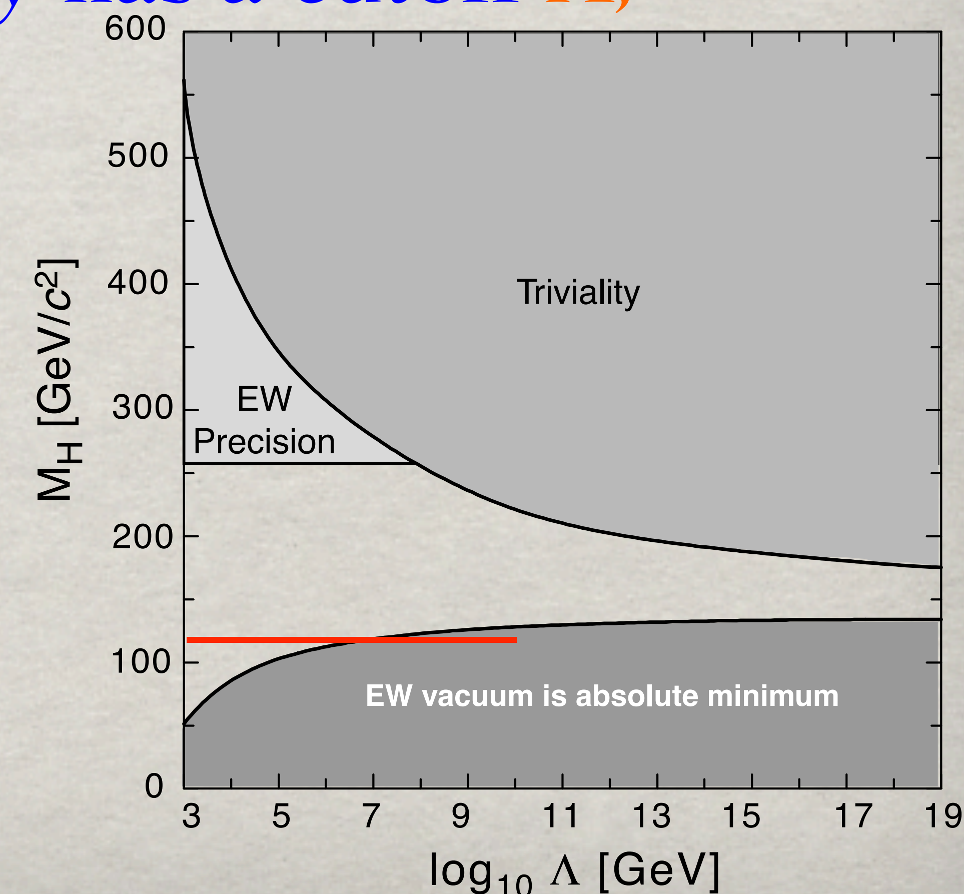
$$32\pi^2 \frac{d\lambda}{dt} = 24\lambda^2 \qquad \lambda(Q) = \frac{\lambda(Q_0)}{1 - \frac{3}{4\pi^2}\lambda(Q_0)\ln\left(\frac{Q^2}{Q_0^2}\right)}$$

There is a (famous) Landau Pole!

(present in all but non-Abelian gauge theories)

1. If SM valid to infinite energy, then $\lambda(Q_0) = 0$,
   a non-interacting trivial theory!
2. Since M_H is non-zero, then the theory has a cutoff Λ, translate to a M_H upper bound:

$$M_H^2 < \frac{8\pi^2 v^2}{3\log\left(\frac{\Lambda^2}{v^2}\right)}$$



For M_H = 125 GeV, the cutoff is over M_{PL}.

2. Vacuum stability bound

For small λ, the Top-Yukawa dominates:

$$32\pi^2 \frac{d\lambda}{dt} = -24 y_t^4 \qquad \lambda(\Lambda) = \lambda(v) - \frac{3}{4\pi^2} y_t^4 \log\left(\frac{\Lambda^2}{v^2}\right)$$

To have a stable vacuum,

$$\lambda(\Lambda) > 0 \quad \longrightarrow \quad M_H^2 > \frac{3v^2}{2\pi^2} y_t^4 \log\left(\frac{\Lambda^2}{v^2}\right)$$

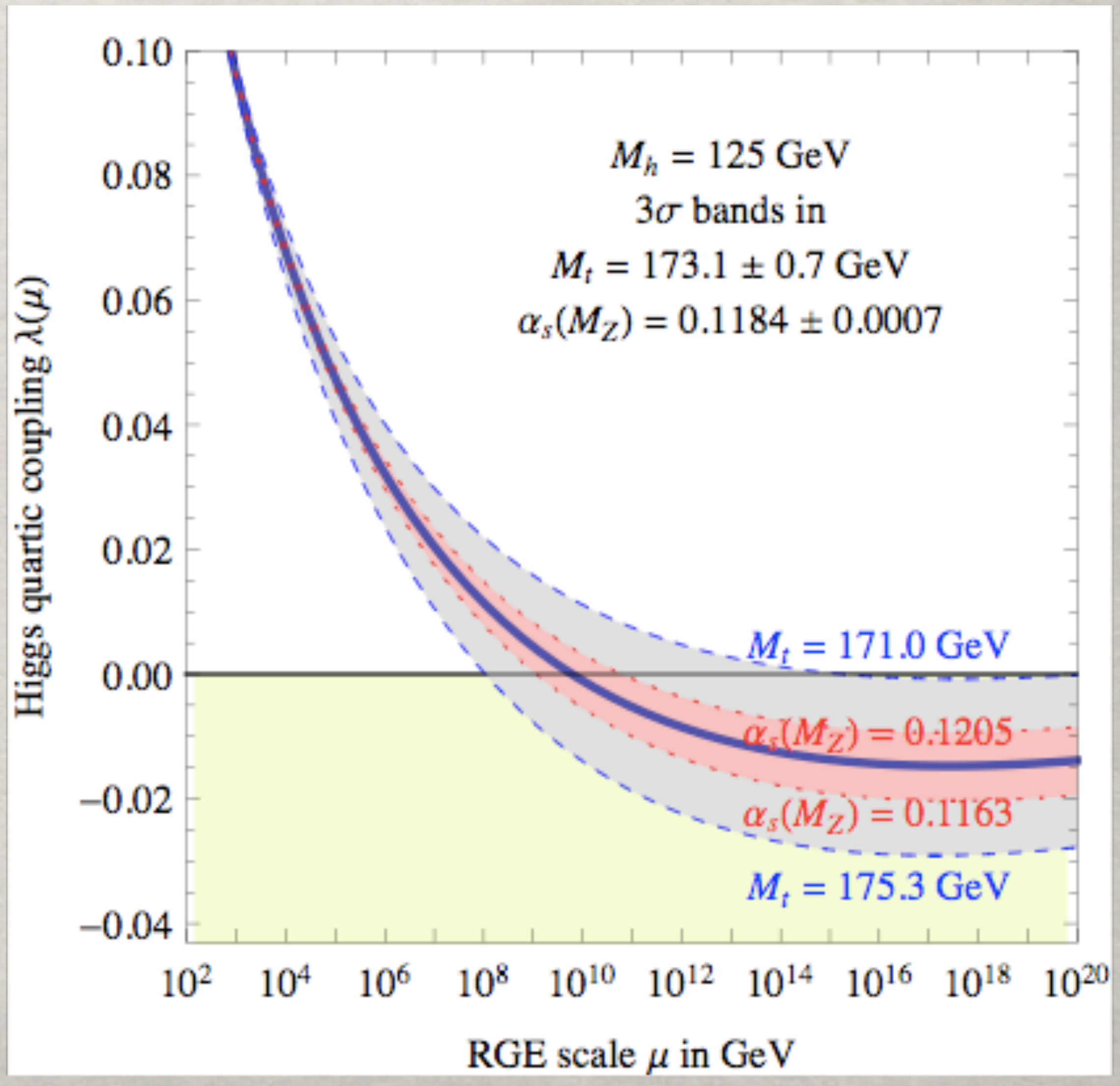
$$\Lambda_C \sim 10^3 \text{ GeV} \quad \Rightarrow \quad M_H \gtrsim 70 \text{ GeV}$$

$$\Lambda_C \sim 10^{16} \text{ GeV} \quad \Rightarrow \quad M_H \gtrsim 130 \text{ GeV}$$

Much renewed interest, updates:$

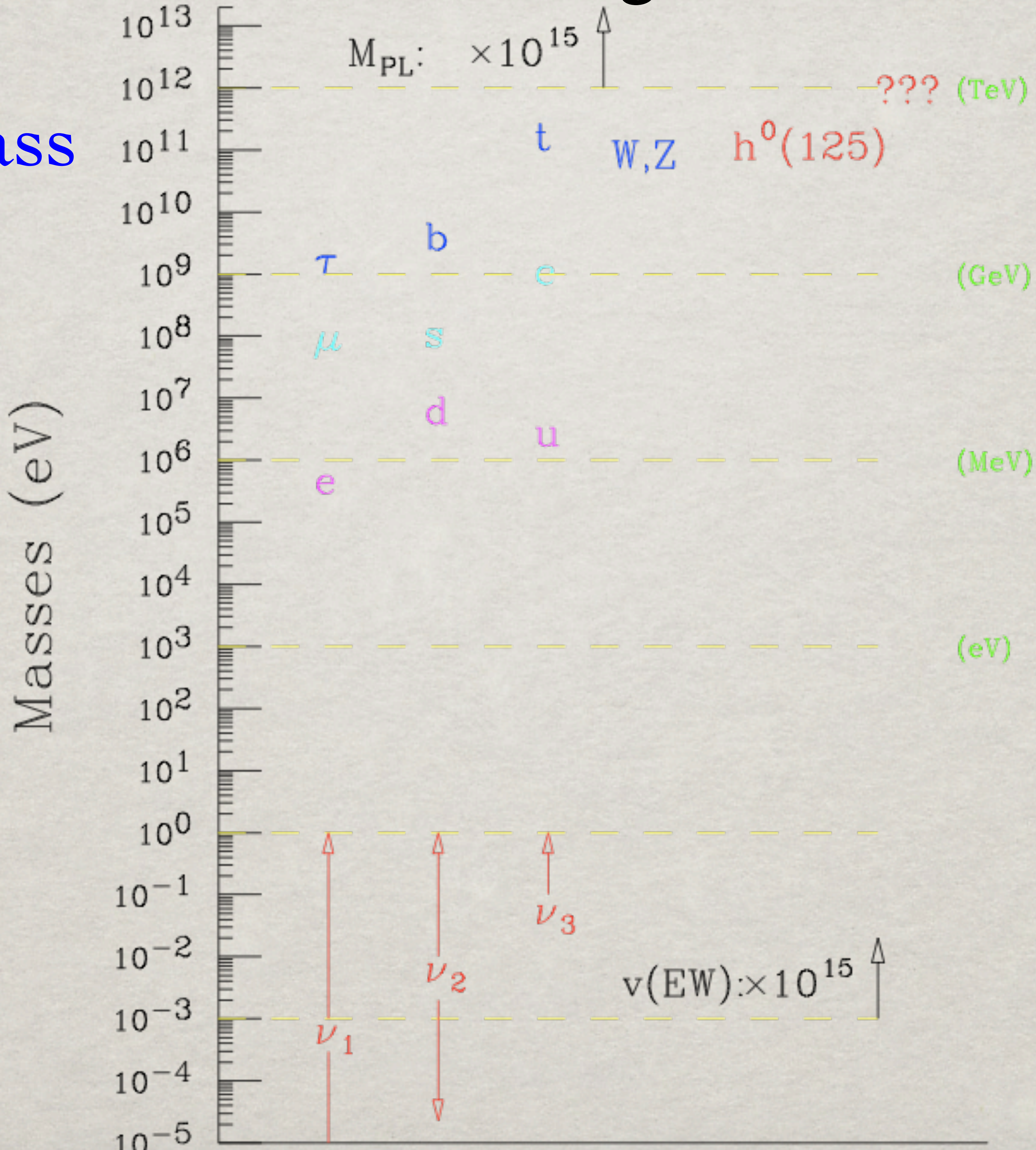
$ G. Degrassi et al., arXiv:1205.6497.

For M_H = 125 GeV,
then $\Lambda(m_t=175) < 10^7$ GeV.
(but m_t=171 GeV would be fine)



3. “Naturalness” argument:

Particle mass hierarchy:



Since all the masses are generated like $\sim g\, v$,
the natural scale should be just v.

Thus, except M_W, M_Z, M_H, $m_t \sim g\, v$,
all others are unnatural: (to some extent)
$m_b \sim 5$ GeV, $m_e \sim 0.5$ MeV, $m_\nu < 0.2$ eV ...

But, they are "technically natural":

For a given mass, if the quantum corrections are merely logarithmically dependent upon the high energy scale, then the mass parameter is said technically natural.

t'Hooft statement for "technical naturalness":

If a parameter is turned off (set to 0), the system results in an enlarged symmetry, then this parameter must be technically natural.

$$m_e \sim m^0_e\, [1 + 3\alpha/4\pi\, \ln(\Lambda/m_e)]$$

If m^0_e is turned off, the system possesses a chiral symmetry.

Dynamical scale generation is natural!

Recall in QCD: coupling runs logarithmically between vastly separated scales:

$$\alpha_s(\Lambda^2) \approx \frac{1}{\ln \frac{\Lambda^2}{\Lambda_{QCD}^2}} \quad e.g. \quad (\frac{\Lambda}{\Lambda_{QCD}})^2 \approx (\frac{E_{LHC}}{\Lambda_{QCD}})^2 \approx 10^8.$$

Dynamical scale can be generated by "dimensional transmutation":

$$\Lambda_{TC} \approx \Lambda \ \exp(-\frac{1}{2\alpha_{TC}}) \approx 4\pi v.$$

However, this picture (Technicolor and variations) doesn't work (well) in EW:

* It is strong interaction, not seen in EW physics.
* Fermion masses/mixing a real killer.
* No fundamental scalar (at least not a light one).

"… scalar particles are the only kind of free particles whose mass term does not break either an internal or a gauge symmetry." -- Ken Wilson, 1970

Quantum corrections to the potential or to $\mathbf{m_H}^2$

Tree-level SM Higgs potential:

$$V(H) = -\mu^2|\Phi|^2 + \lambda|\Phi|^4$$

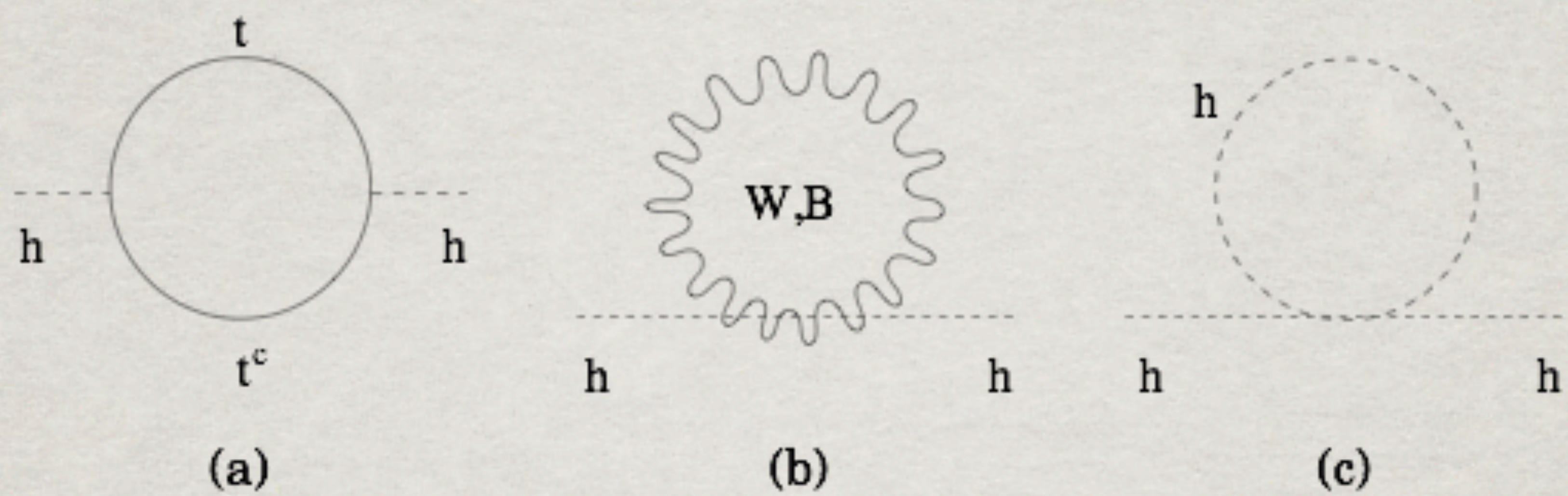
$$m_H^2 = 2\mu^2 = 2\lambda v^2 \quad \Rightarrow \quad \mu \approx 89 \text{ GeV}, \quad \lambda \approx \frac{1}{8}.$$

Quantum corrections to y_t:

$$\delta\mu^2 = -\frac{3y_t^2}{8\pi^2}\Lambda^2$$

It is "un-natural": quadratic (not log) correction!

The "naturalness" problem?



$$m_H^2 = m_{H0}^2 \quad - \frac{3}{8\pi^2} y_t^2 \Lambda^2 \quad + \frac{1}{16\pi^2} g^2 \Lambda^2 \quad + \frac{1}{16\pi^2} \lambda^2 \Lambda^2$$

If $\Lambda^2 \gg m_H^2$, then unnaturally large cancellations must occur.

Cancelation in perspective:

m_H^2 = 36,127,890,984,789,307,394,520,932,878,928,933,023

–36,127,890,984,789,307,394,520,932,878,928,917,398

= $(125\ \mathrm{GeV})^2$! ?

Amazing !

Unnatural: Fine-tuned to

0.05 mm/0.5 cm ~ 10^{-2}

A light Higgs is unnatural

"Naturalness" argument strongly indicates the existence of TeV scale new physics:

Requiring less 90% cancellation → $\Lambda_t < 3$ TeV

If you give up this belief, you are subscribing the "anthropic principle".

Cancellation Mechanisms ?

- Super-symmetry (SUSY) (symmetry between *opposite* spin & statistics)

  Natural cancellations: $\tilde{t}$ versus t

  $\tilde{W}$ versus W

  $\tilde{H}$ versus H

  H_d versus H_u,

$$\Delta m_H^2 \sim (M_{SUSY}^2 - M_{SM}^2)\,\frac{\lambda_f^2}{16\pi^2}\ln\left(\frac{\Lambda}{M_{SUSY}}\right).$$

Weak scale SUSY is natural if $M_{SUSY} \sim \mathcal{O}(1\ \text{TeV})$.

- The Little Higgs idea – Strongly interacting dynamics:

  An alternative way to keep H light (naturally).

  Again, predicting new states:

$$W^\pm, Z, B \leftrightarrow W_H^\pm, Z_H, B_H; \quad t \leftrightarrow T; \quad H \leftrightarrow \Phi.$$

(cancellation among same spin states!)

A light Higgs implies new physics near 1 TeV!

Or

Summary:

- The Standard Model based on the gauge structure

  $SU(3)_c \times SU(2)_L \times U(1)_Y$

  describe our microscopic world very well: 0.1% or better up to a scale O(1 TeV)!

  And could be valid all the way to M_{PL} .

- The revolutionary discovery of the Higgs boson verified the idea of spontaneous EW symmetry breaking & the Higgs mechanism.

- The "Naturalness" argument indicates the need for new physics at the O(1 TeV): Go LHC & beyond!

We are a lucky generation to participate in the exciting journey!