

iSTEP Summer School  
Tsinghua University - Beijing  
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# Jet Reconstruction

## Jet algorithms and jet substructure

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Includes material from  
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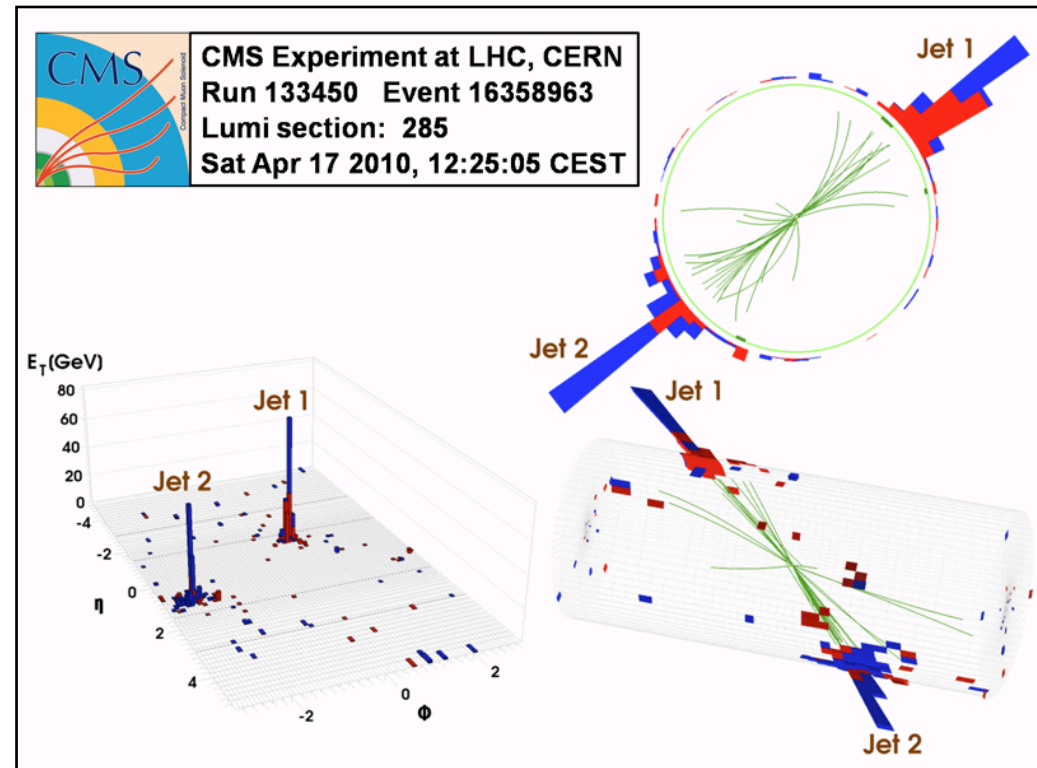
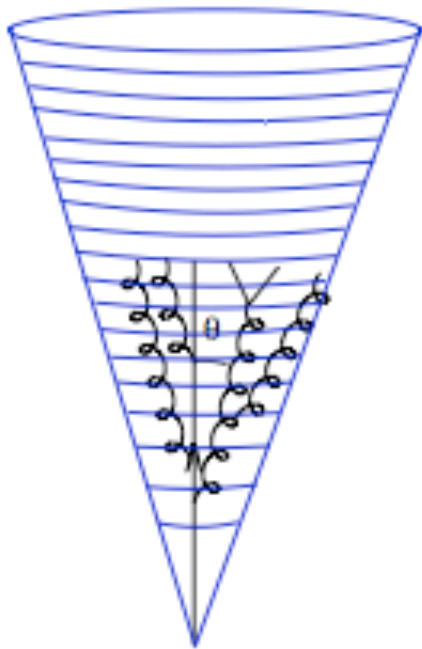
## ▶ Jet algorithms

- ▶ How are jets made

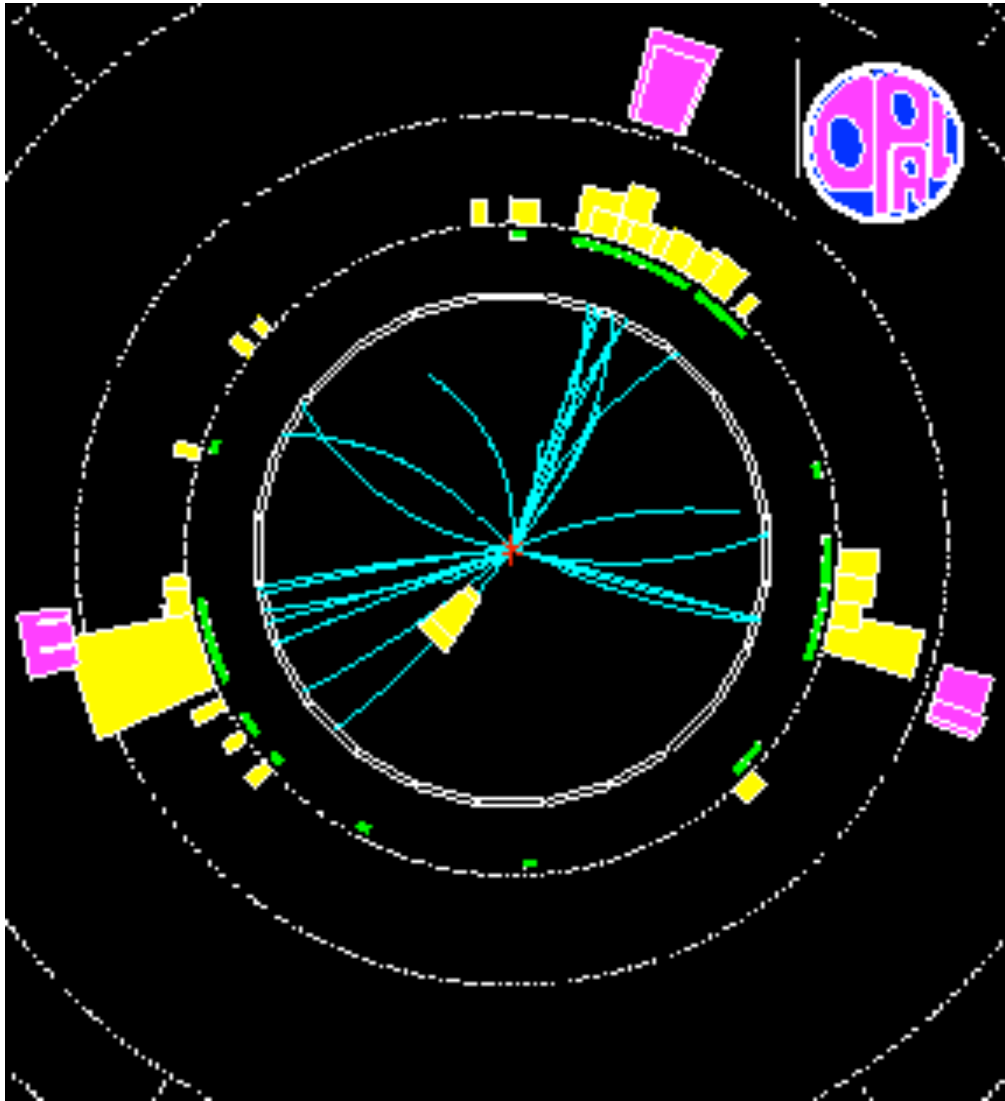
## ▶ Jet substructure

- ▶ What's inside them

# What is a jet?



# Why jets



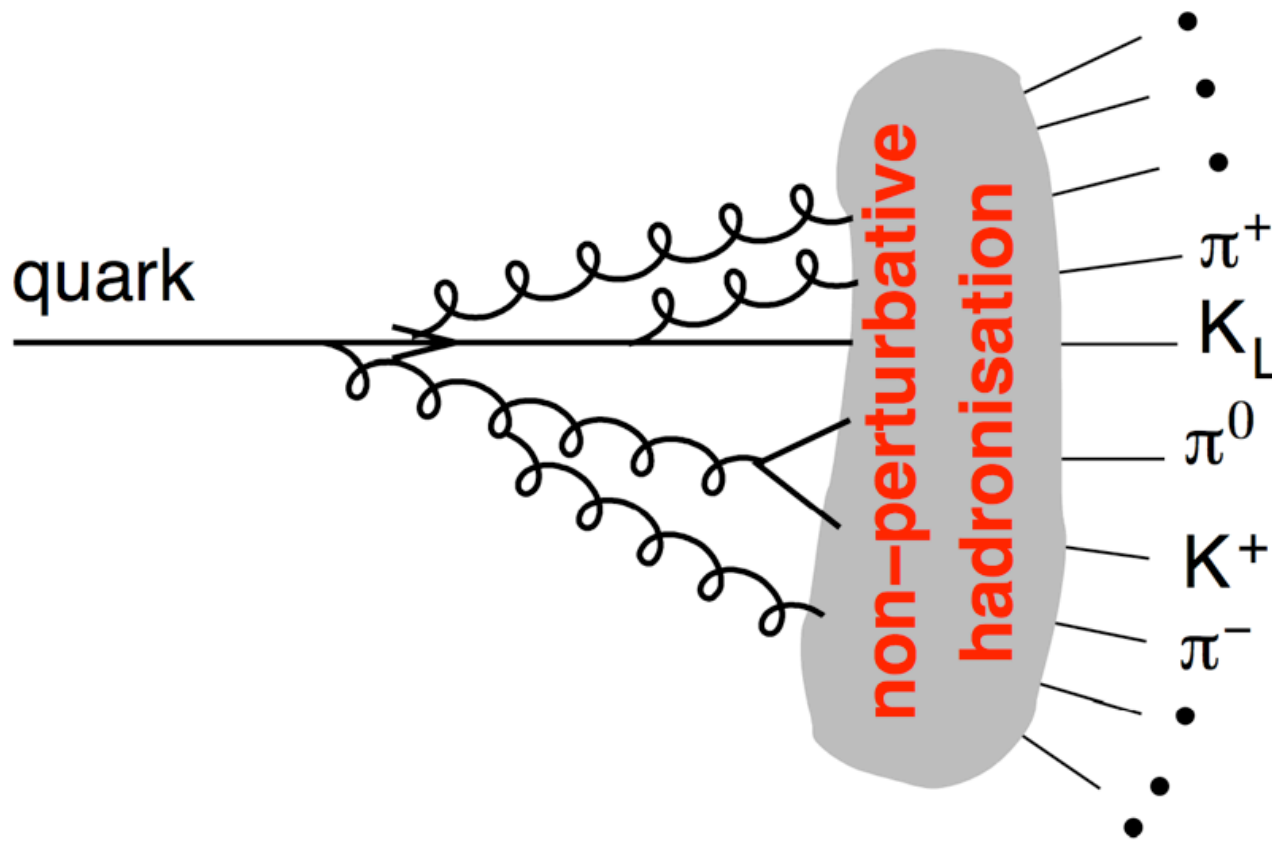
A **jet** is something that happens in high energy events: **a collimated bunch of hadrons flying roughly in the same direction**

We could eyeball the collimated bunches, but it becomes impractical with millions of events

The classification of particles into jets is best done using a **clustering algorithm**



# Why do jets happen?



Gluon emission

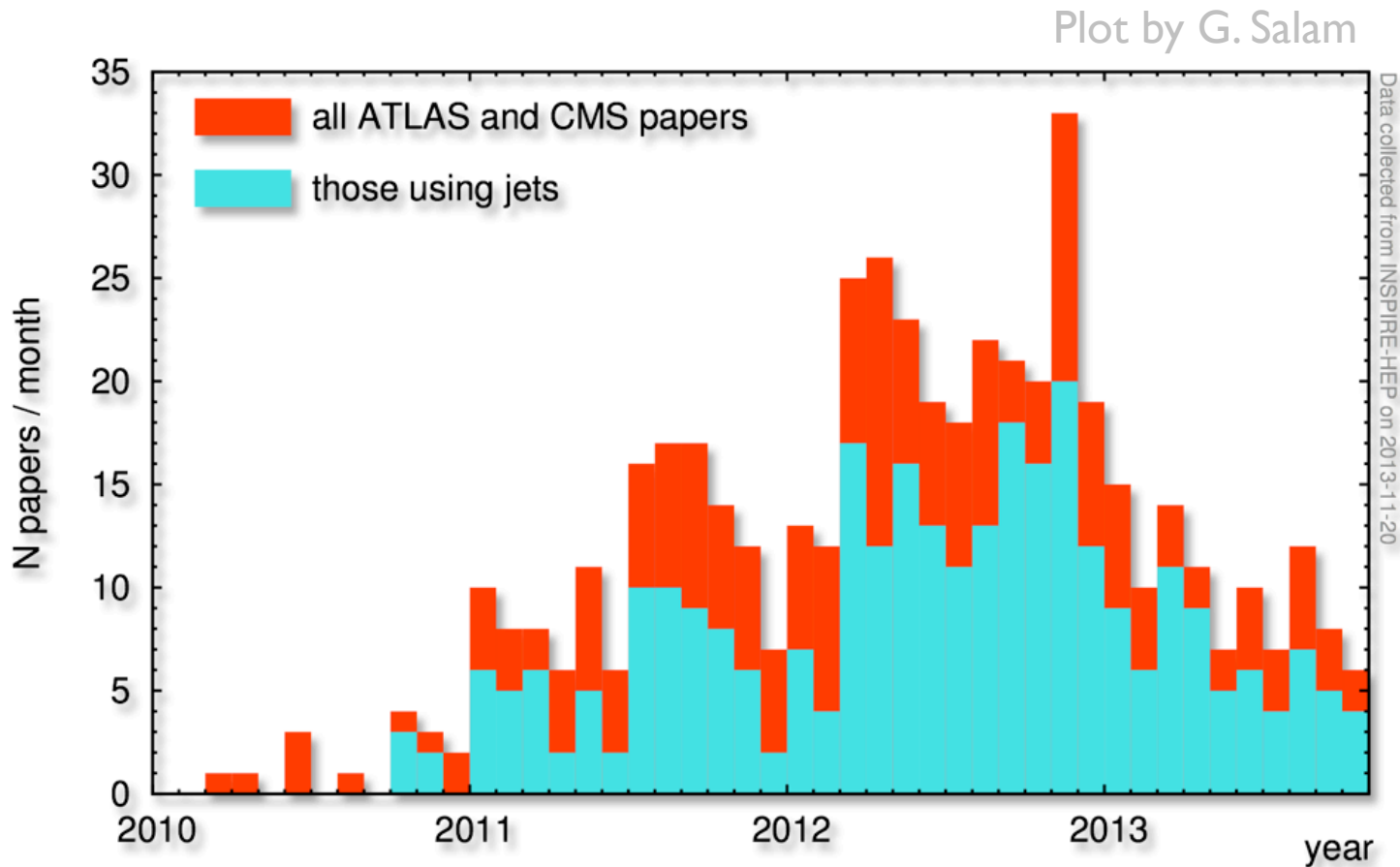
$$\int \alpha_s \frac{dE}{E} \frac{d\theta}{\theta} \gg 1$$

Non-perturbative physics

$$\alpha_s \sim 1$$

# The pervasiveness of jets

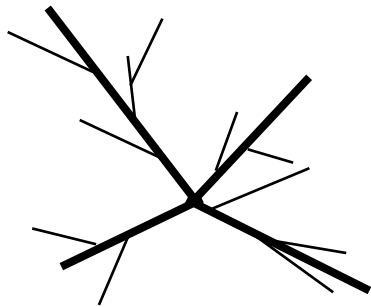
- ▶ ATLAS and CMS have each published **400+** papers since 2010
  - ▶ More than **half** of these papers make use of **jets**
  - ▶ **60%** of the **searches** papers makes use of **jets**



(Source: INSPIRE.  
Results may vary when  
employing different search  
keywords)

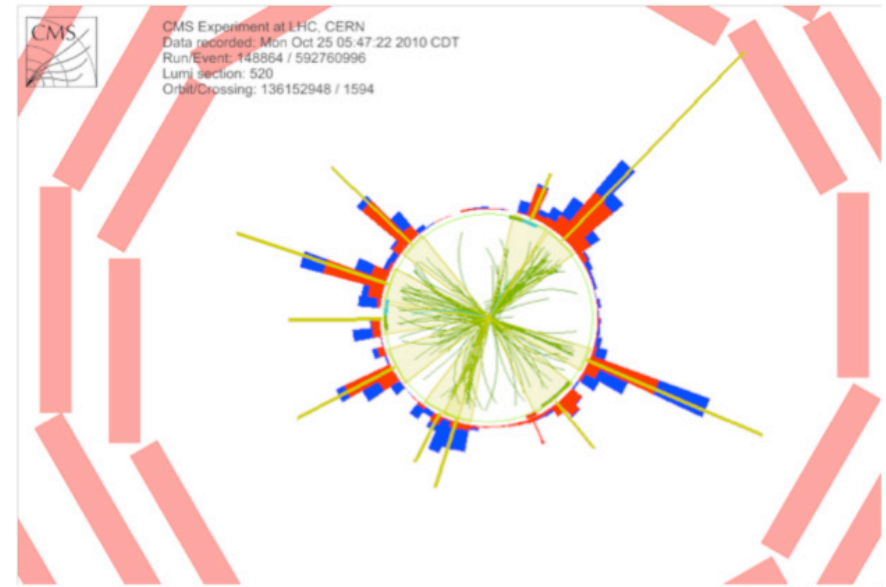
## Why are jets so important?

Multileg + PS



QCD predictions

??



Real data

Jets

One purpose of a 'jet clustering' algorithm is to **reduce the complexity** of the final state, simplifying many hadrons to **simpler objects** that one can hope to **calculate**

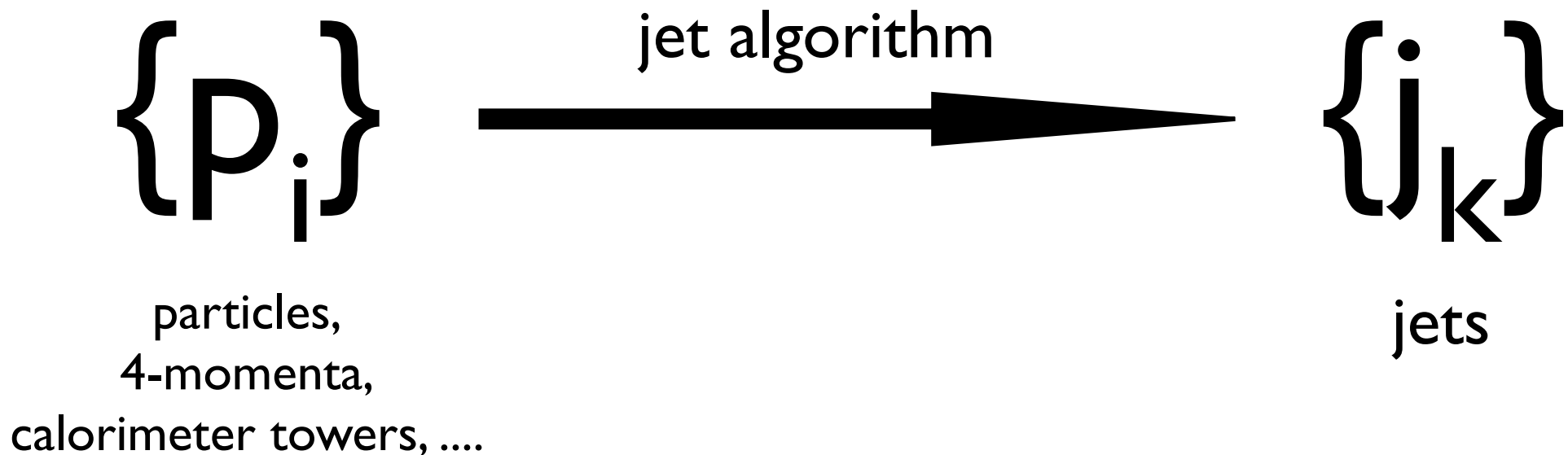
## Jets can serve **two** purposes

- ▶ They can be **observables**, that one can measure and calculate
- ▶ They can be **tools**, that one can employ to extract specific properties of the final state

Different clustering algorithms have different properties and characteristics that can make them more or less appropriate for each of these tasks

# Jet clustering algorithm

A **jet algorithm** maps the momenta of the final state particles into the momenta of a certain number of jets:

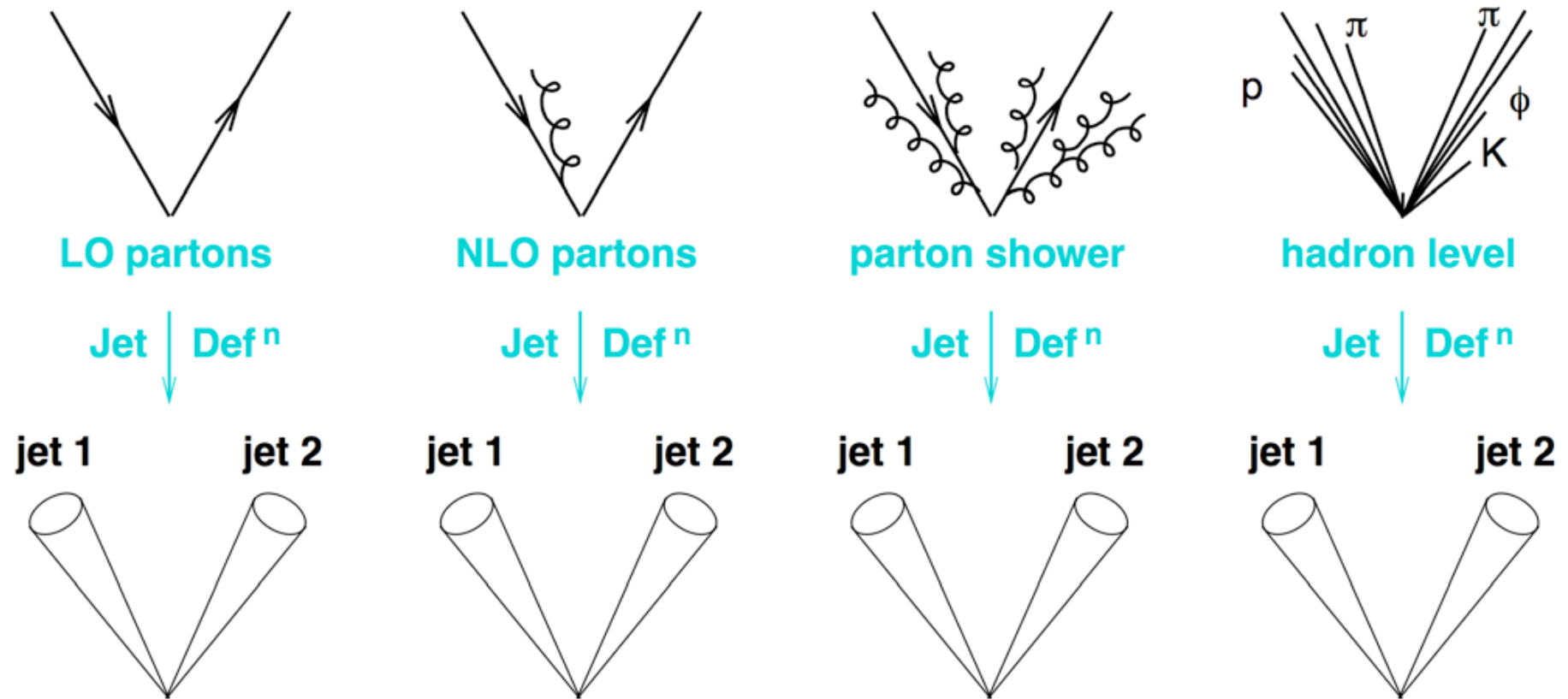


Most algorithms contain a resolution parameter, **R**, which controls the extension of the jet

A jet algorithm  
+  
its parameters (e.g. R)  
+  
a recombination scheme  
=  
a **Jet Definition**

```
/// define a jet definition  
JetDefinition jet_def(JetAlgorithm jet_algorithm,  
                      double R,  
                      RecombinationScheme rec_sch = E_scheme);
```

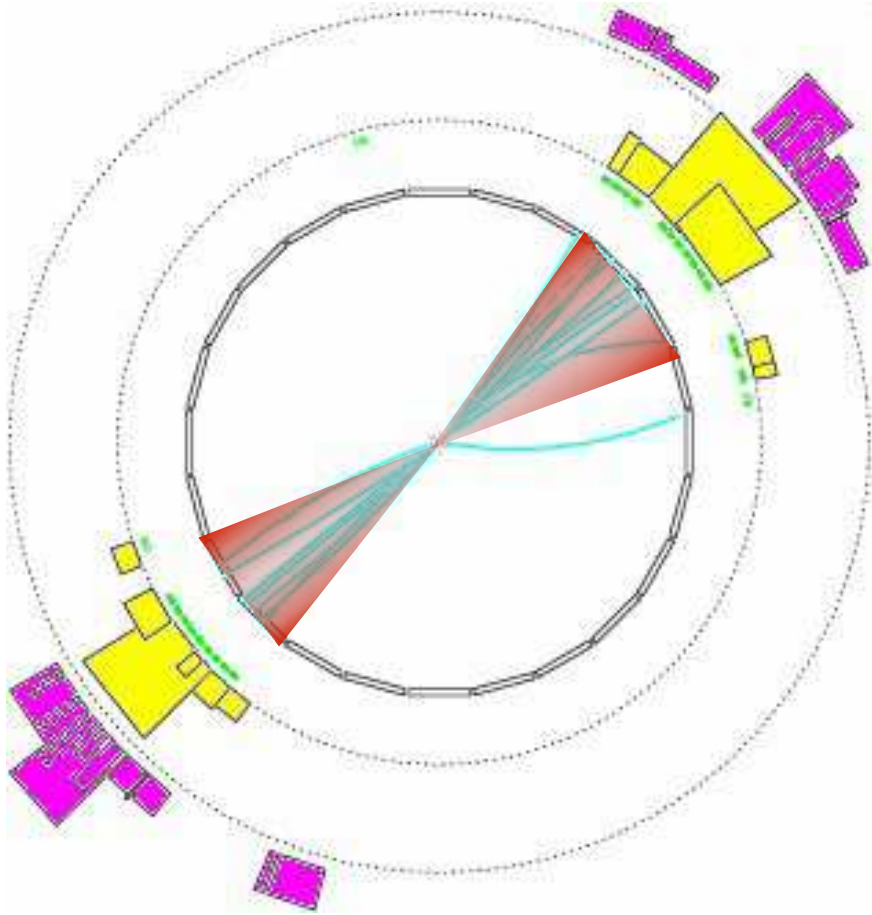
# Jet definitions as projections



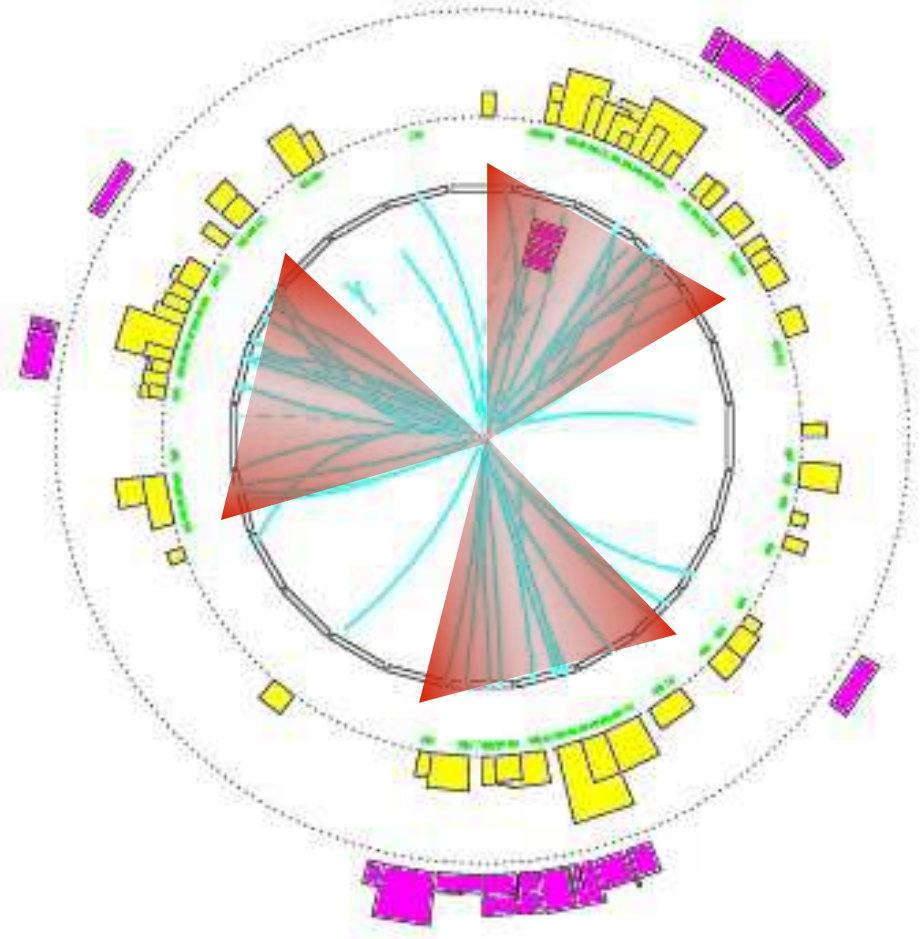
Projection to jets should be resilient to QCD effects

**NB: projections are NOT unique:  
a jet is NOT EQUIVALENT to a parton**

# Reconstructing jets is an ambiguous task



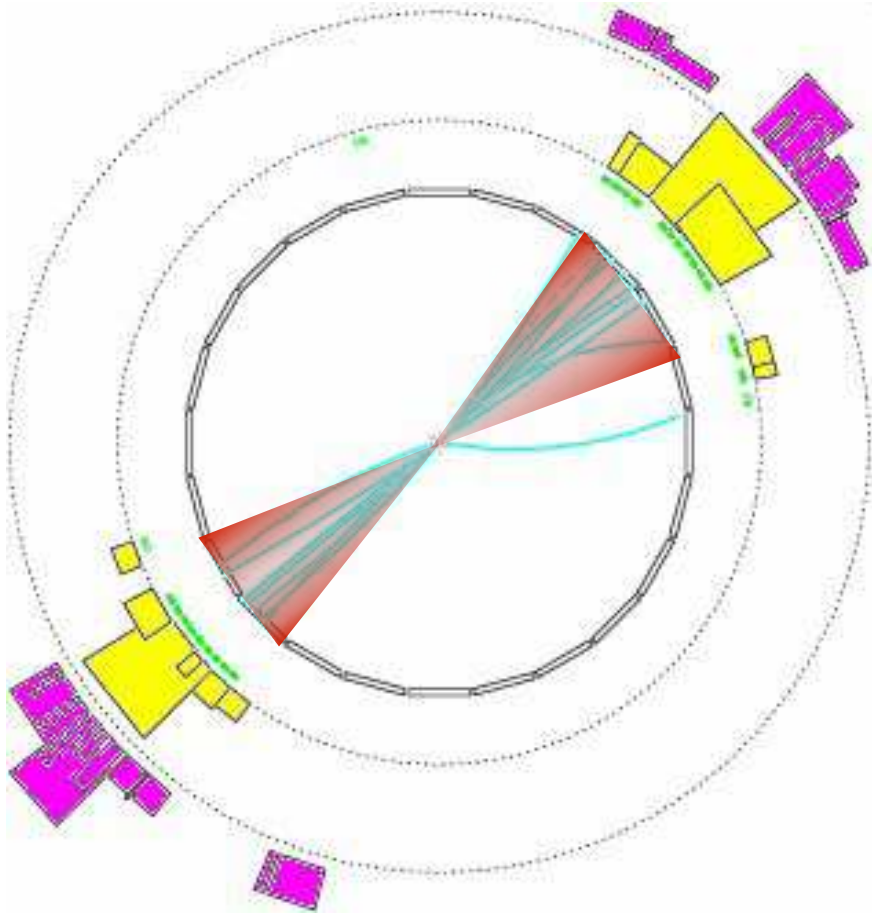
2 clear jets



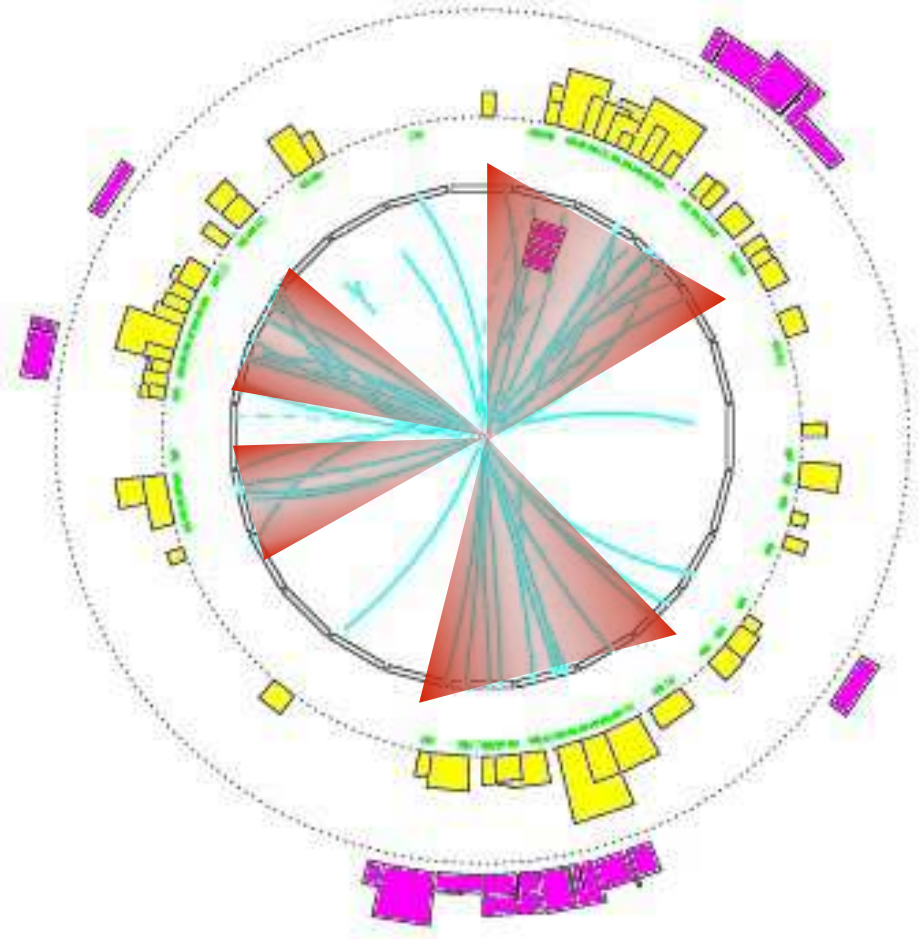
3 jets?



# Reconstructing jets is an ambiguous task



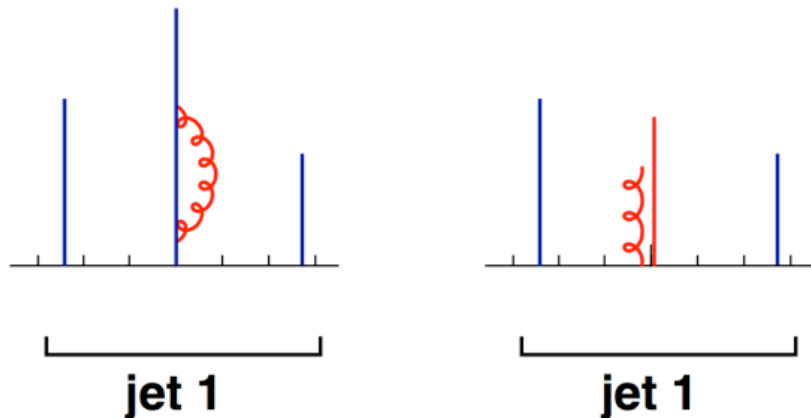
2 clear jets



3 jets?  
**or 4 jets?**

# Reconstructing jets must respect rules

## Collinear Safe

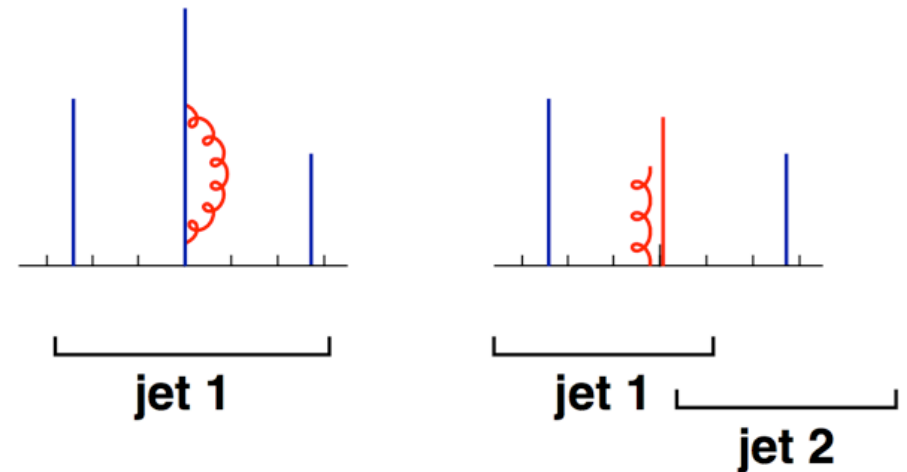


$$\alpha_S^n \times (-\infty)$$

$$\alpha_S^n \times (+\infty)$$

**Infinities cancel**

## Collinear Unsafe



$$\alpha_S^n \times (-\infty)$$

$$\alpha_S^n \times (+\infty)$$

**Infinities do not cancel**

Perturbative calculations of jet observable will only be possible with collinear (and infrared) safe jet definitions

An observable is **infrared and collinear safe** if, in the limit of a **collinear splitting**, or the **emission of an infinitely soft** particle, the observable remains **unchanged**:

$$O(X; p_1, \dots, p_n, p_{n+1} \rightarrow 0) \rightarrow O(X; p_1, \dots, p_n)$$

$$O(X; p_1, \dots, p_n \parallel p_{n+1}) \rightarrow O(X; p_1, \dots, p_n + p_{n+1})$$

This property ensures cancellation of **real** and **virtual** divergences in higher order calculations

If we wish to be able to calculate a jet rate in perturbative QCD the jet algorithm that we use must be IRC safe:  
**soft emissions and collinear splittings must not change the hard jets**

# Two main classes of jet algorithms

## ▶ **Sequential recombination algorithms**

Bottom-up approach: combine particles starting from **closest ones**

**How?** Choose a **distance measure**, iterate recombination until few objects left, call them jets

Works because of mapping closeness  $\Leftrightarrow$  QCD divergence

Examples: Jade,  $k_t$ , Cambridge/Aachen, anti- $k_t$ , .....

## ▶ **Cone algorithms**

Top-down approach: find coarse regions of energy flow.

**How?** Find **stable cones** (i.e. their axis coincides with sum of momenta of particles in it)

Works because QCD only modifies energy flow on small scales

Examples: JetClu, MidPoint, ATLAS cone, CMS cone, SISCone.....

# Two main classes of jet algorithms

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Examples: Jade,  $k_t$ , Cambridge/Aachen, anti- $k_t$ , .....

Usually trivially made IRC safe, but their algorithmic complexity scales like  $N^3$

## ▶ Cone algorithms

Top-down approach: find coarse regions of energy flow.

**How?** Find **stable cones** (i.e. their axis coincides with sum of momenta of particles in it)

Works because QCD only modifies energy flow on small scales

Examples: JetClu, MidPoint, ATLAS cone, CMS cone, SISCone.....

Can be programmed to be fairly fast, at the price of being complex and IRC unsafe

- ▶ Cone-type jets were introduced first in QCD in the 1970s (Sterman-Weinberg '77)
- ▶ In the 1980s cone-type jets were adapted for use in hadron colliders (SpS, Tevatron...) → iterative cone algorithms
- ▶ LEP was a golden era for jets: new algorithms and many relevant calculations during the 1990s
  - ▶ Introduction of the 'theory-friendly'  $k_t$  algorithm
    - ▶ sequential recombination type algorithm, IRC safe
    - ▶ it allows for all order resummation of jet rates
  - ▶ Several accurate calculations in perturbative QCD of jet properties: rates, jet mass, thrust, ....

# $e^+e^- k_t$ (Durham) algorithm

[Catani, Dokshitzer, Olsson, Turnock, Webber '91]

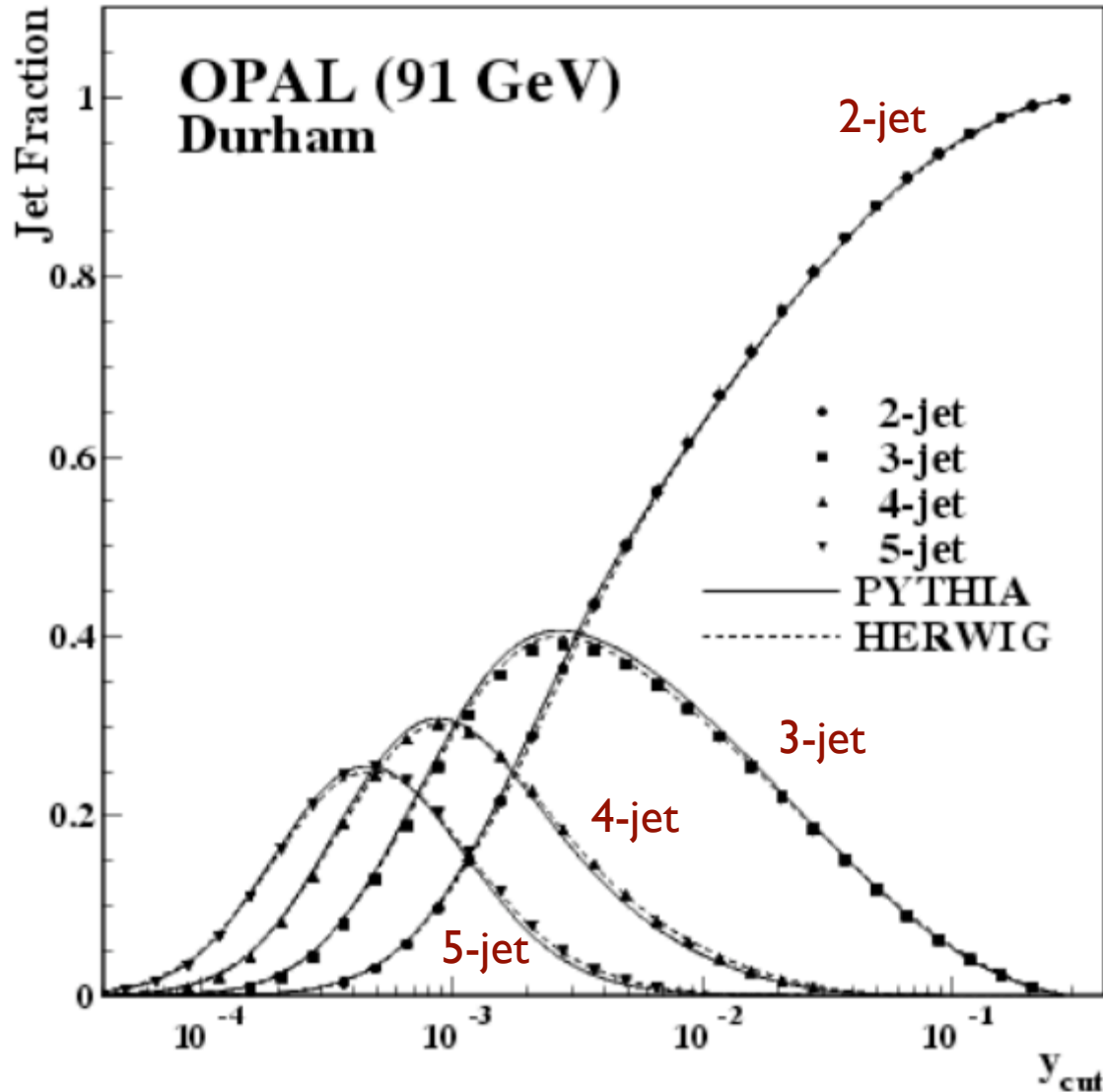
Distance:

$$y_{ij} = \frac{2 \min(E_i^2, E_j^2) (1 - \cos \theta_{ij})}{Q^2}$$

In the collinear limit, the numerator reduces to the **relative transverse momentum** (squared) of the two particles, hence the name of the algorithm

- ▶ Find the minimum  $y_{\min}$  of all  $y_{ij}$
- ▶ If  $y_{\min}$  is below some jet resolution threshold  $y_{\text{cut}}$ , recombine  $i$  and  $j$  into a single new particle ('pseudojet'), and repeat
- ▶ If no  $y_{\min} < y_{\text{cut}}$  are left, all remaining particles are jets

# $e^+e^- k_t$ (Durham) algorithm in action



Characterise events  
in terms of number of jets  
(as a function of  $y_{cut}$ )

Resummed calculations for distributions of  $y_{cut}$  doable with the  $k_t$  algorithm



# $e^+e^-$ $k_t$ (Durham) algorithm v. QCD

$k_t$  is a sequential recombination type algorithm

One key feature of the  $k_t$  algorithm is its relation to the structure of QCD divergences:

$$\frac{dP_{k \rightarrow ij}}{dE_i d\theta_{ij}} \sim \frac{\alpha_s}{\min(E_i, E_j)\theta_{ij}}$$

The  $y_{ij}$  distance is the inverse of the emission probability

- ▶ The  $k_t$  algorithm roughly inverts the QCD branching sequence (the pair which is recombined first is the one with the largest probability to have branched)
- ▶ The history of successive clusterings has **physical meaning**

# Jet challenges at the LHC

The LHC environment differs from the LEP one (and even the Tevatron) under many respects

- ▶ Number of final state particles much larger (order  $10^3$ )  
→ needs a fast algorithm
- ▶ Many higher order calculations (NLO, NNLO) available  
→ needs an IRC-safe algorithm
- ▶ Presence of background (underlying event and pileup)  
→ needs small/known susceptibility and/or ability to subtract background
- ▶ Jets often initiated by a large-momentum heavy particle  
→ needs capability to distinguish boosted object jet from QCD jet

## Two parameters, $R$ and $p_{t,min}$

(These are the two parameters in essentially every widely used hadron-collider jet algorithm)

$$d_{ij} = \min(p_{ti}^2, p_{tj}^2) \frac{\Delta R_{ij}^2}{R^2}, \quad \Delta R_{ij}^2 = (y_i - y_j)^2 + (\phi_i - \phi_j)^2$$

## Sequential recombination algorithm

1. Find smallest of  $d_{ij}$ ,  $d_{iB}$
2. If  $ij$ , recombine them
3. If  $iB$ , call  $i$  a jet and remove from list of particles
4. repeat from step 1 until no particles left

Only use jets with  $p_t > p_{t,min}$

## Inclusive $k_t$ algorithm

S.D. Ellis & Soper, 1993

Catani, Dokshitzer, Seymour & Webber, 1993

# The $k_t$ algorithm and its siblings

$$d_{ij} = \min(p_{ti}^{2p}, p_{tj}^{2p}) \frac{\Delta y^2 + \Delta \phi^2}{R^2} \quad d_{iB} = p_{ti}^{2p}$$

**p = 1**  $k_t$  algorithm

S. Catani, Y. Dokshitzer, M. Seymour and B. Webber, Nucl. Phys. B406 (1993) 187  
S.D. Ellis and D.E. Soper, Phys. Rev. D48 (1993) 3160

**p = 0** Cambridge/Aachen algorithm

Y. Dokshitzer, G. Leder, S. Moretti and B. Webber, JHEP 08 (1997) 001  
M. Wobisch and T. Wengler, hep-ph/9907280

**p = -1** anti- $k_t$  algorithm

MC, G. Salam and G. Soyez, arXiv:0802.1189

NB: in anti- $k_t$  pairs with a **hard** particle will cluster first: if no other hard particles are close by, the algorithm will give **perfect cones**

Quite ironically, a sequential recombination algorithm is the 'perfect' cone algorithm

# IRC safety of generalised- $k_t$ algorithms

$$d_{ij} = \min(p_{ti}^{2p}, p_{tj}^{2p}) \frac{\Delta y^2 + \Delta \phi^2}{R^2} \quad d_{iB} = p_{ti}^{2p}$$

## $p > 0$

New **soft** particle ( $p_t \rightarrow 0$ ) means that  $d \rightarrow 0 \Rightarrow$  clustered first, no effect on jets

New **collinear** particle ( $\Delta y^2 + \Delta \phi^2 \rightarrow 0$ ) means that  $d \rightarrow 0 \Rightarrow$  clustered first, no effect on jets

## $p = 0$

New **soft** particle ( $p_t \rightarrow 0$ ) can be new jet of zero momentum  $\Rightarrow$  no effect on hard jets

New **collinear** particle ( $\Delta y^2 + \Delta \phi^2 \rightarrow 0$ ) means that  $d \rightarrow 0 \Rightarrow$  clustered first, no effect on jets

## $p < 0$

New **soft** particle ( $p_t \rightarrow 0$ ) means  $d \rightarrow \infty \Rightarrow$  clustered last or new zero-jet, no effect on hard jets

New **collinear** particle ( $\Delta y^2 + \Delta \phi^2 \rightarrow 0$ ) means that  $d \rightarrow 0 \Rightarrow$  clustered first, no effect on jets

# IRC safe algorithms

$k_t$	<p>SR</p> $d_{ij} = \min(p_{ti}^2, p_{tj}^2) \Delta R_{ij}^2 / R^2$ <p>hierarchical in rel <math>p_t</math></p>	<p>Catani et al '91 Ellis, Soper '93</p>	$N \ln N$
Cambridge/ Aachen	<p>SR</p> $d_{ij} = \Delta R_{ij}^2 / R^2$ <p>hierarchical in angle</p>	<p>Dokshitzer et al '97 Wengler, Wobish '98</p>	$N \ln N$
anti- $k_t$	<p>SR</p> $d_{ij} = \min(p_{ti}^{-2}, p_{tj}^{-2}) \Delta R_{ij}^2 / R^2$ <p>gives perfectly conical hard jets</p>	<p>MC, Salam, Soyez '08 (Delsart, Loch)</p>	$N^{3/2}$
SISCone	<p>Seedless iterative cone with split-merge gives 'economical' jets</p>	<p>Salam, Soyez '07</p>	$N^2 \ln N$

'second-generation' algorithms

All are available in FastJet, <http://fastjet.fr>

(As well as many IRC unsafe ones)

# Jet clustering in FastJet

```
/// define a jet definition
JetDefinition jet_def(JetAlgorithm jet_algorithm,
                      double R,
                      RecombinationScheme rec_sch = E_scheme);
```

jet\_algorithm can be any one of the four IRC safe algorithms, or also most of the old IRC-unsafe ones, for legacy purposes

```
/// create a ClusterSequence, extract the jets
ClusterSequence cs(input_particles, jet_def);
vector<PseudoJet> jets = sorted_by_pt(cs.inclusive(jets));
...
// pt of hardest jet
double pt_hardest = jets[0].pt();
...
// constituents of hardest jet
vector<PseudoJet> constits = jets[0].constituents();
```

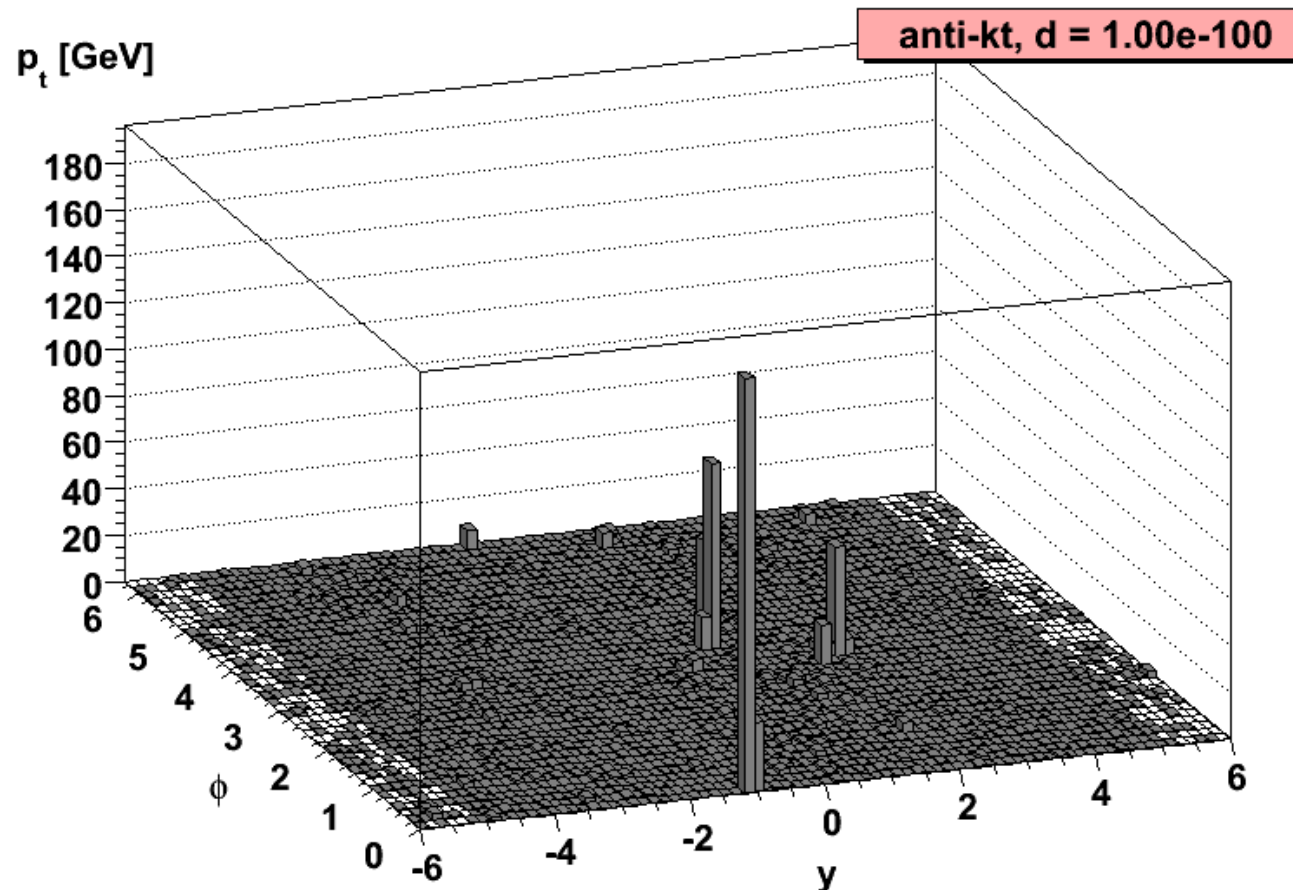
Clustering grows  
around hard cores

$$d_{ij} = \frac{1}{\max(p_{ti}^2, p_{tj}^2)} \frac{\Delta R_{ij}^2}{R^2}, \quad d_{iB} = \frac{1}{p_{ti}^2}$$



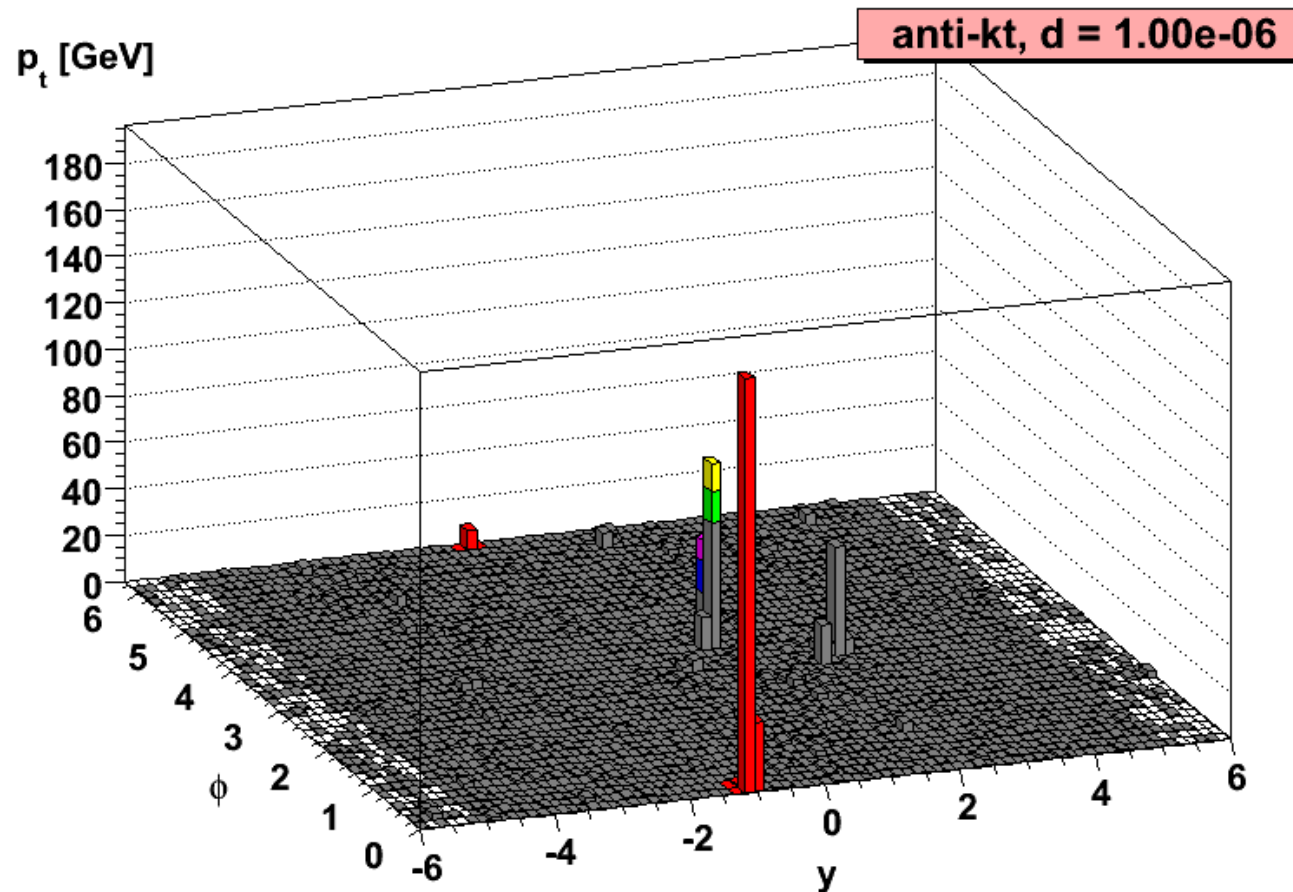
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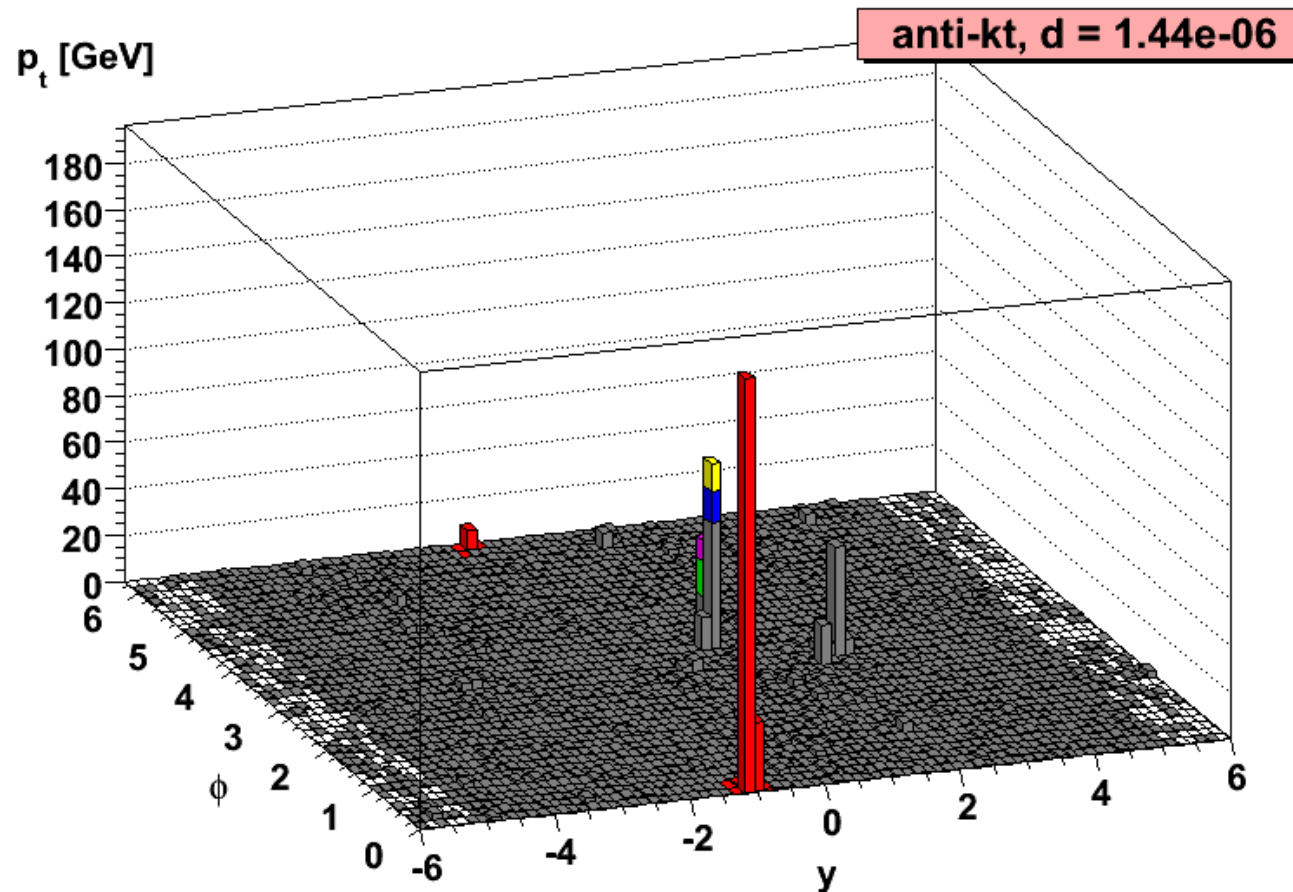
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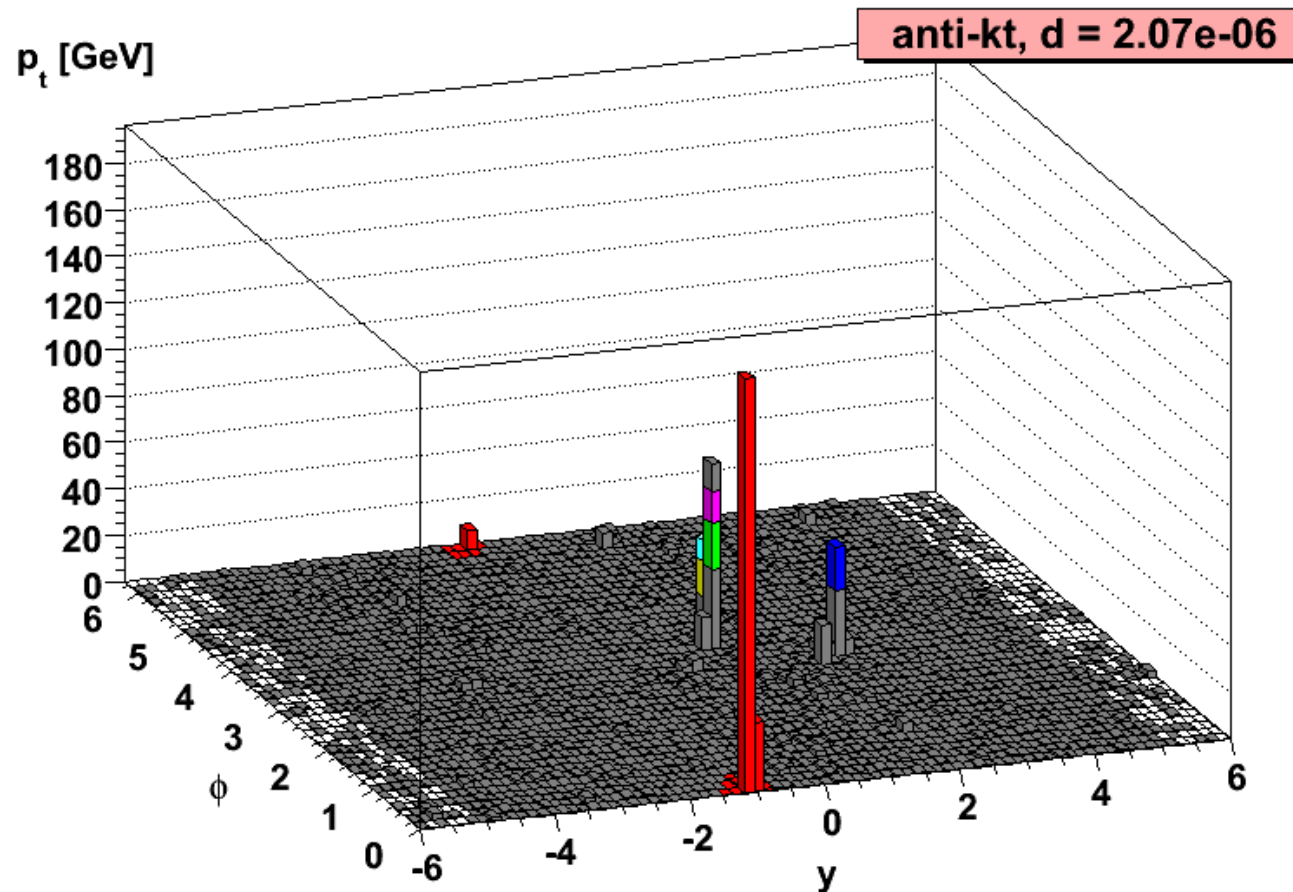
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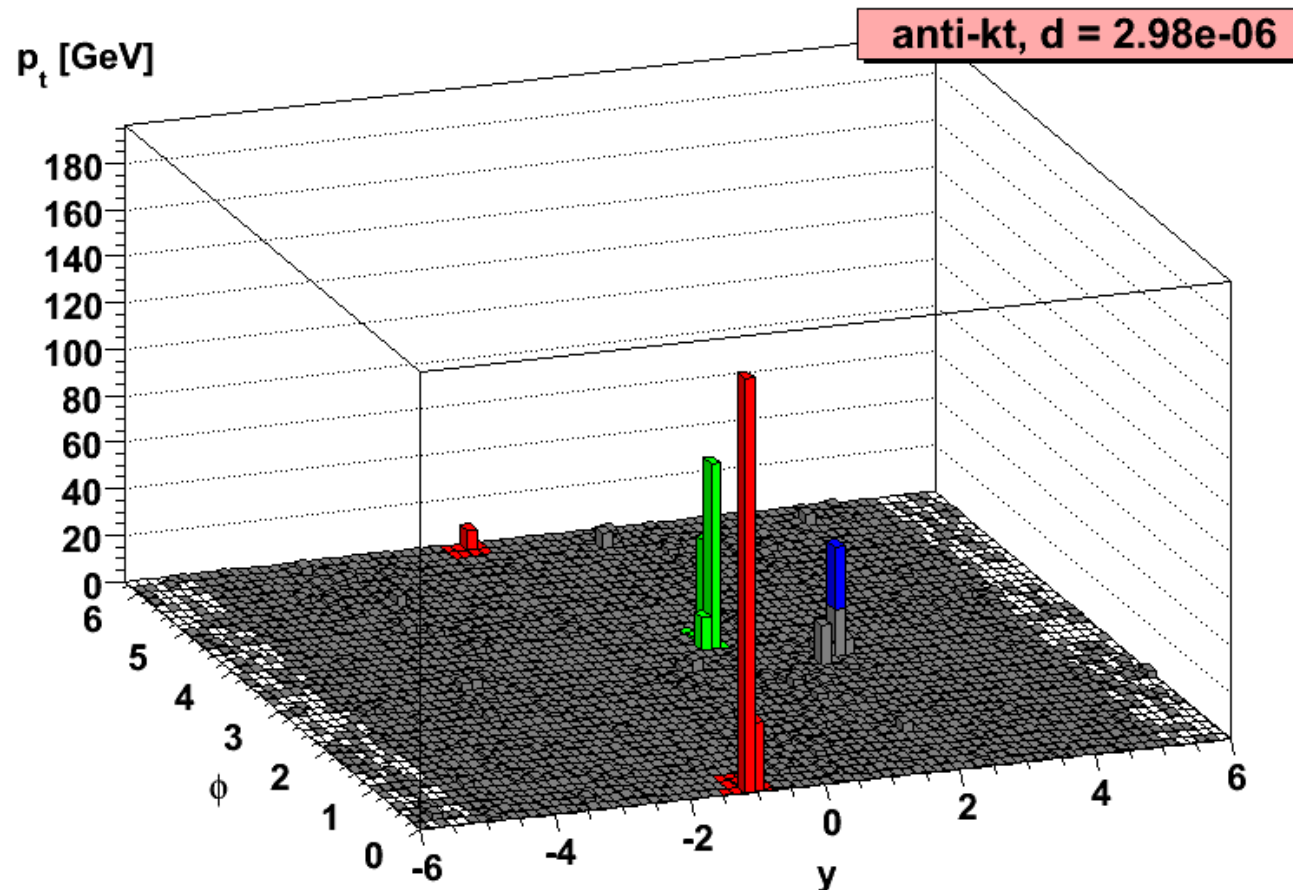
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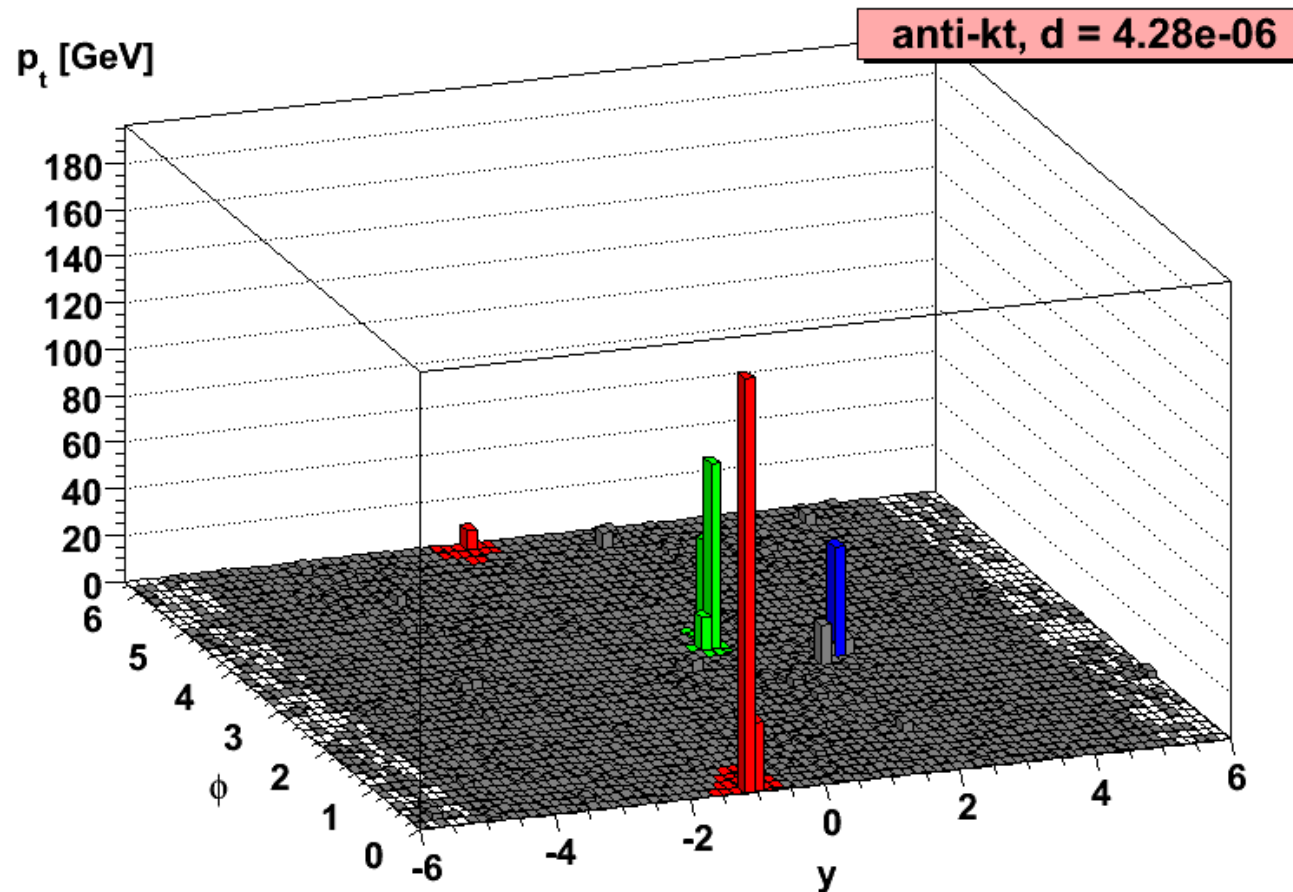
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Clustering grows  
around hard cores

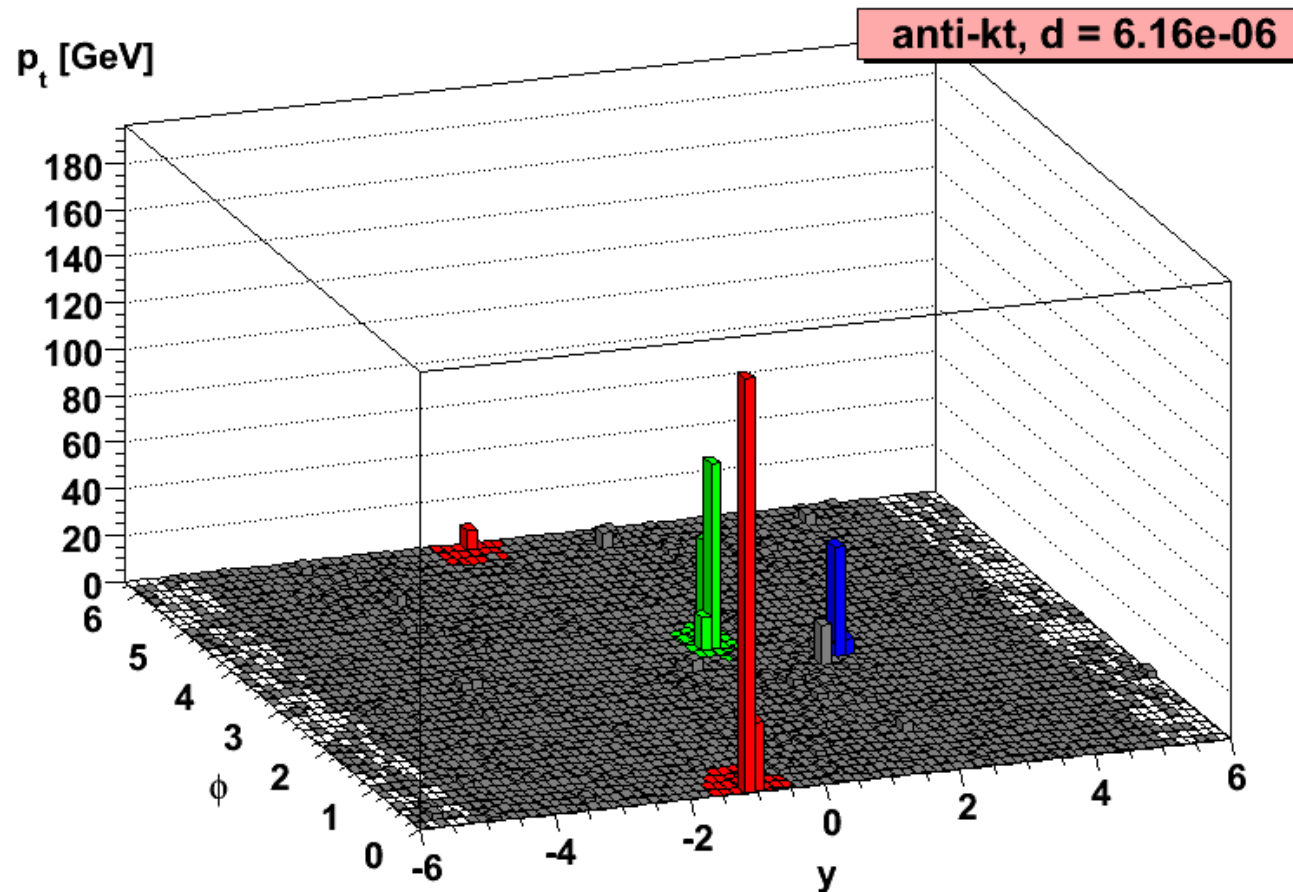
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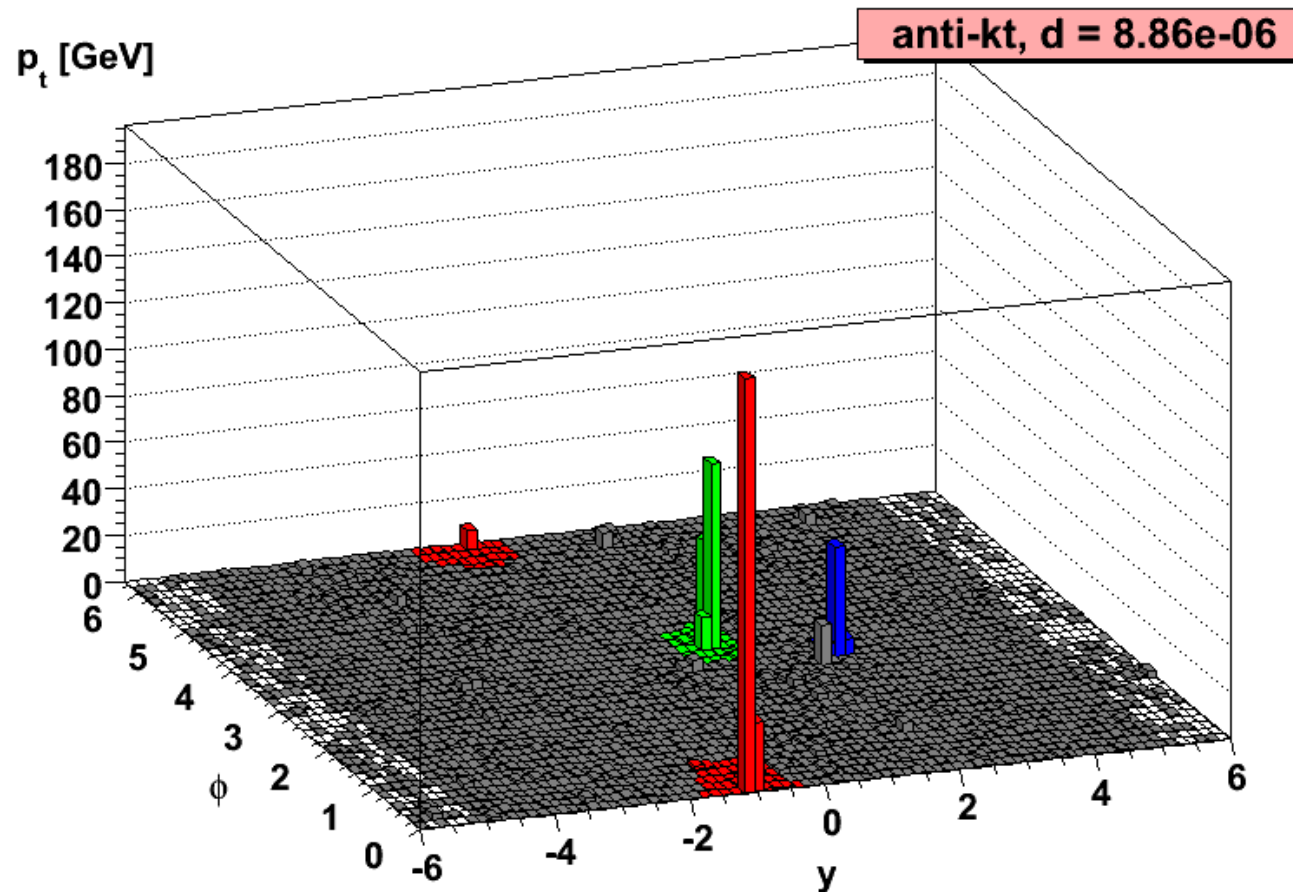
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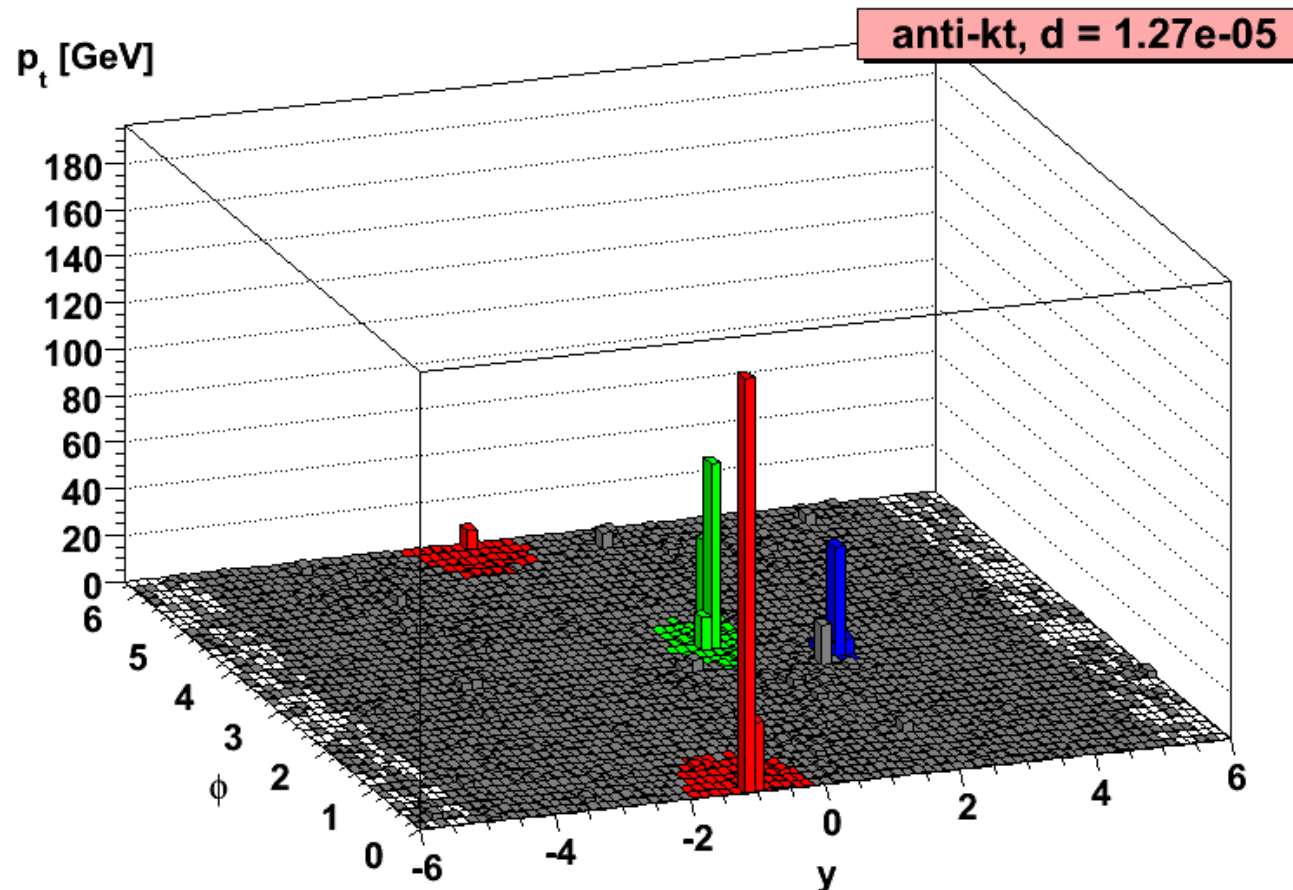
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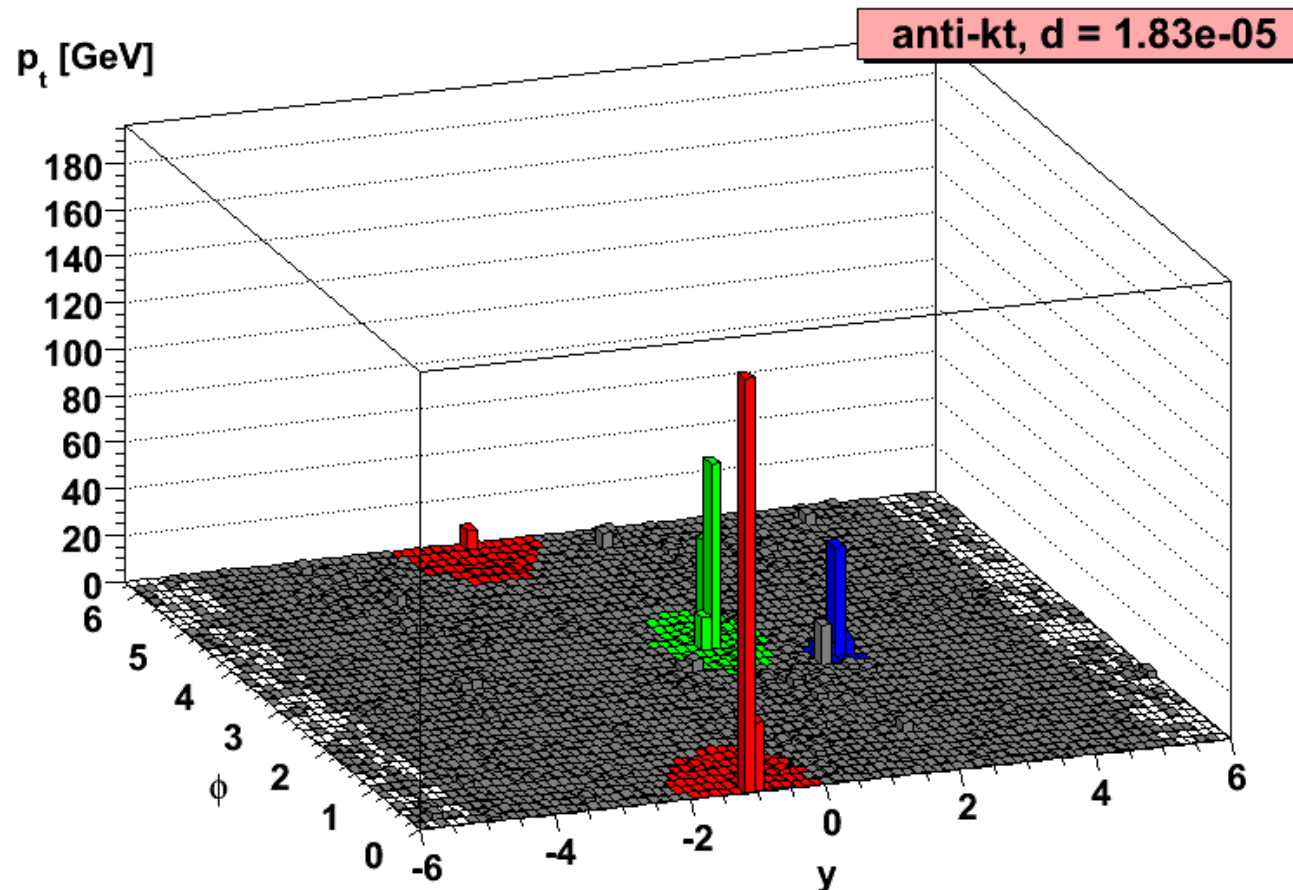
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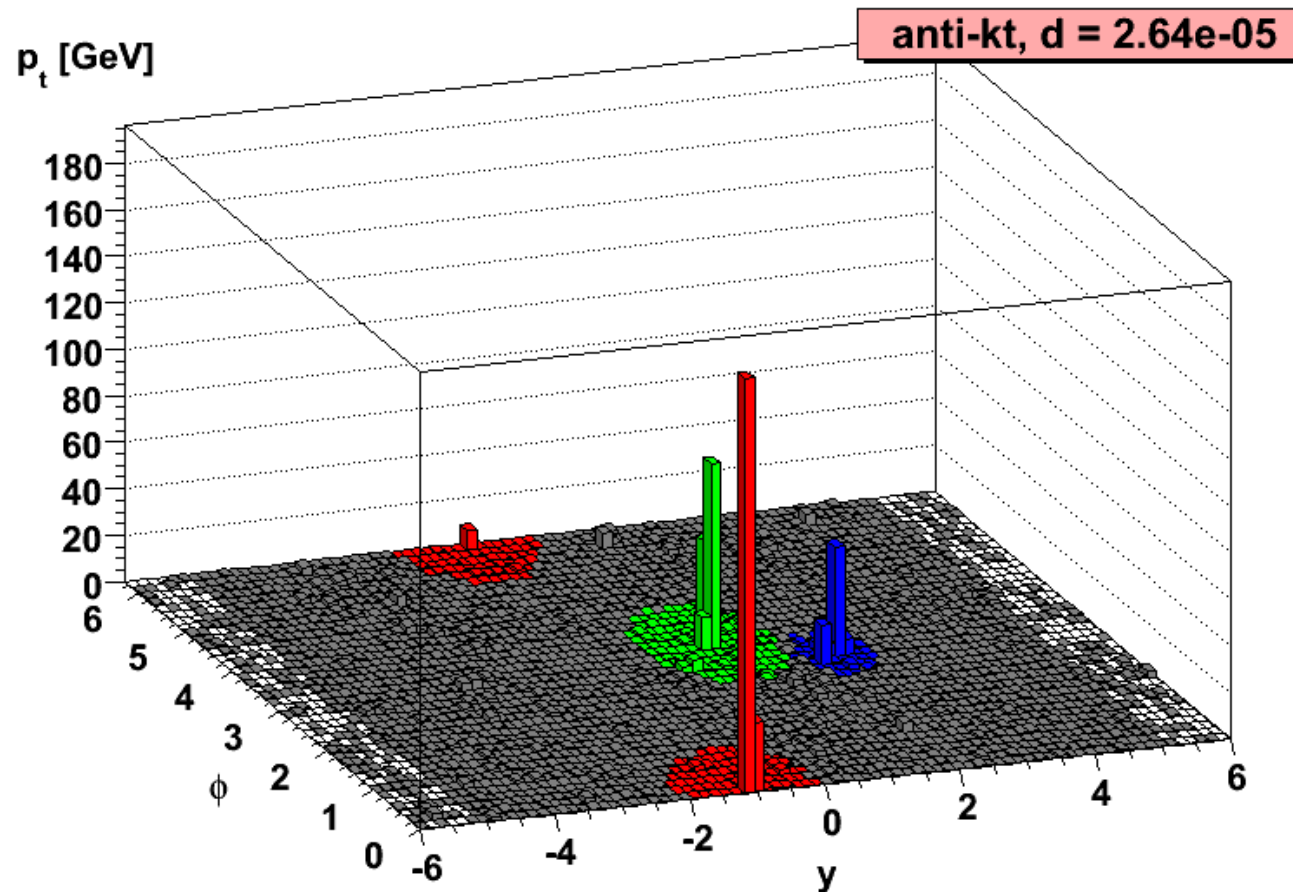
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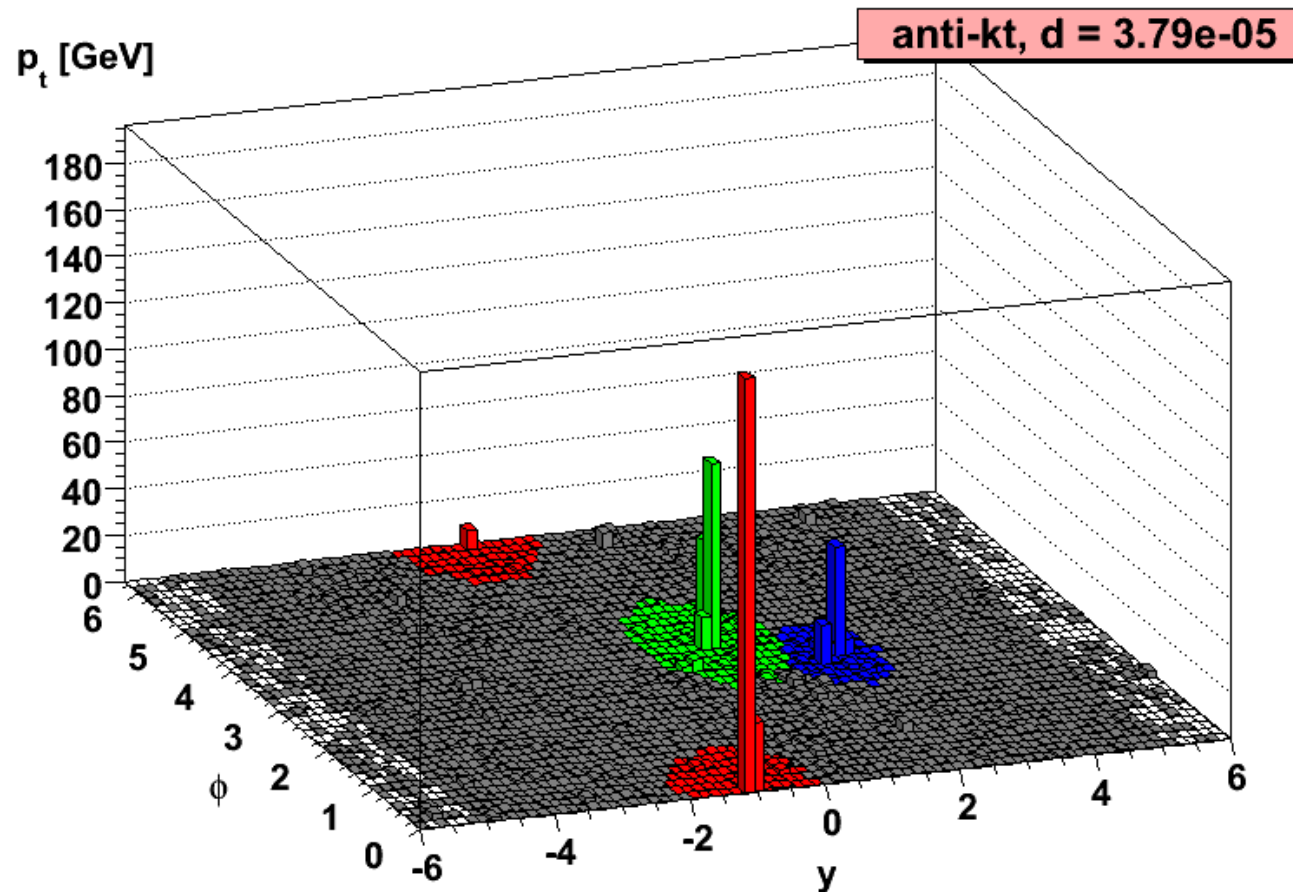
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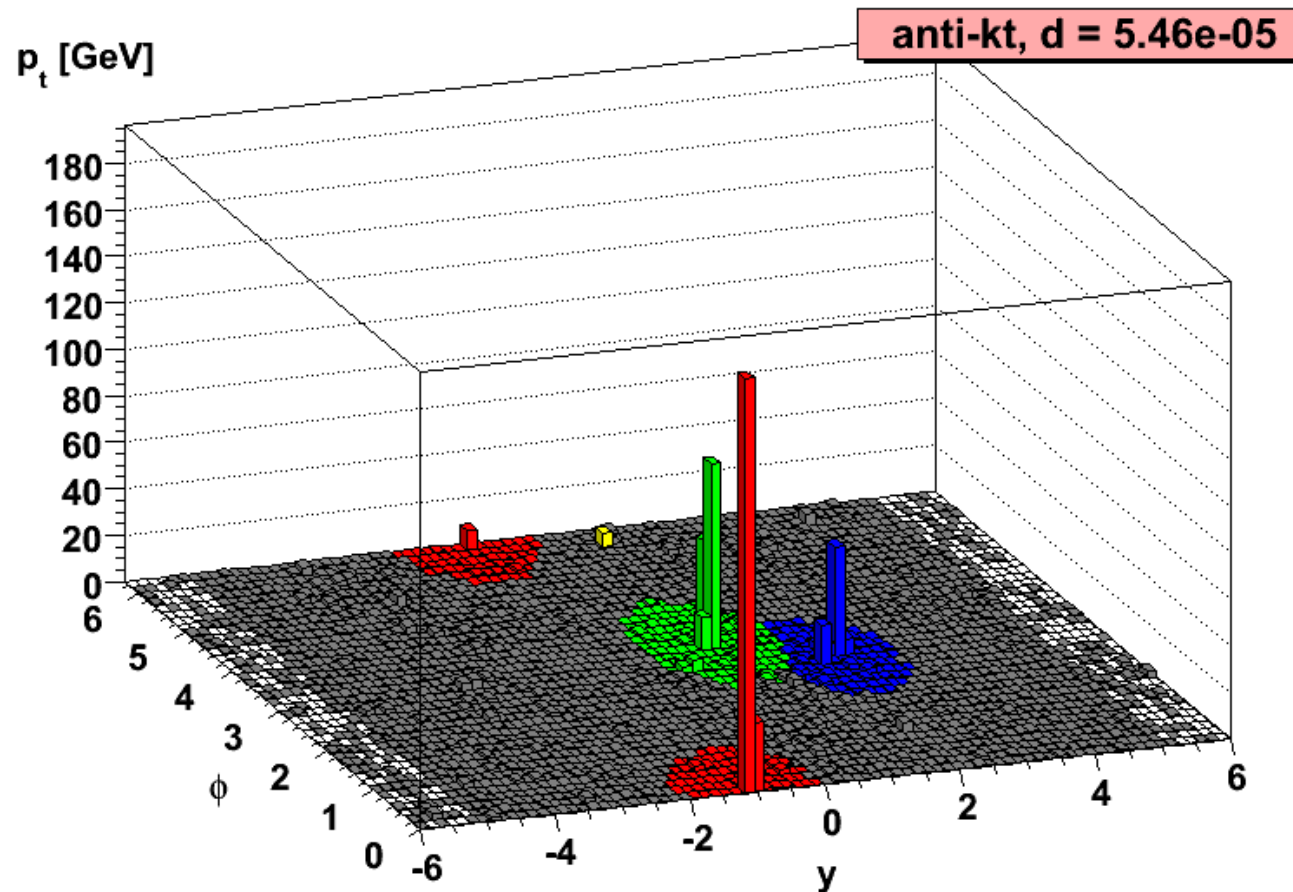
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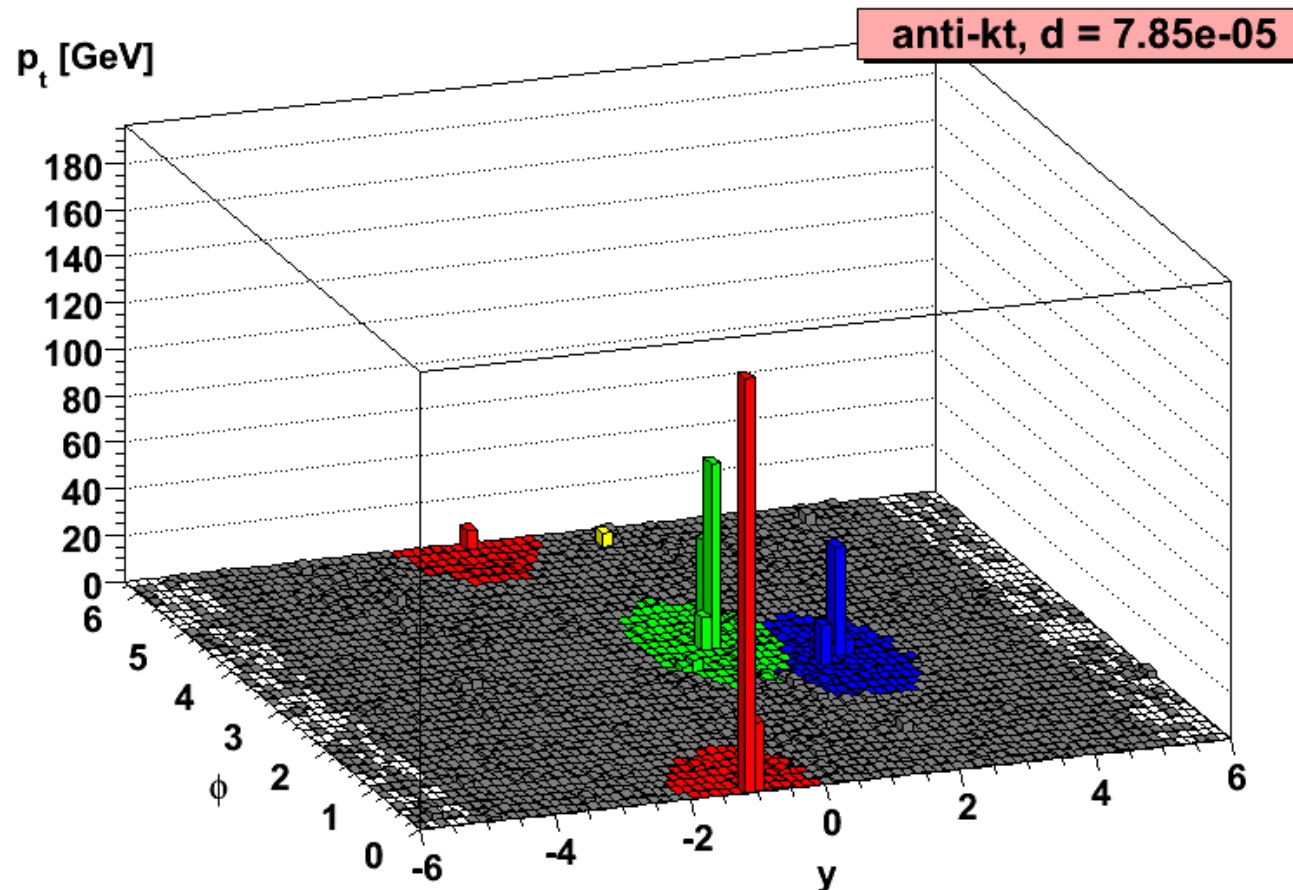
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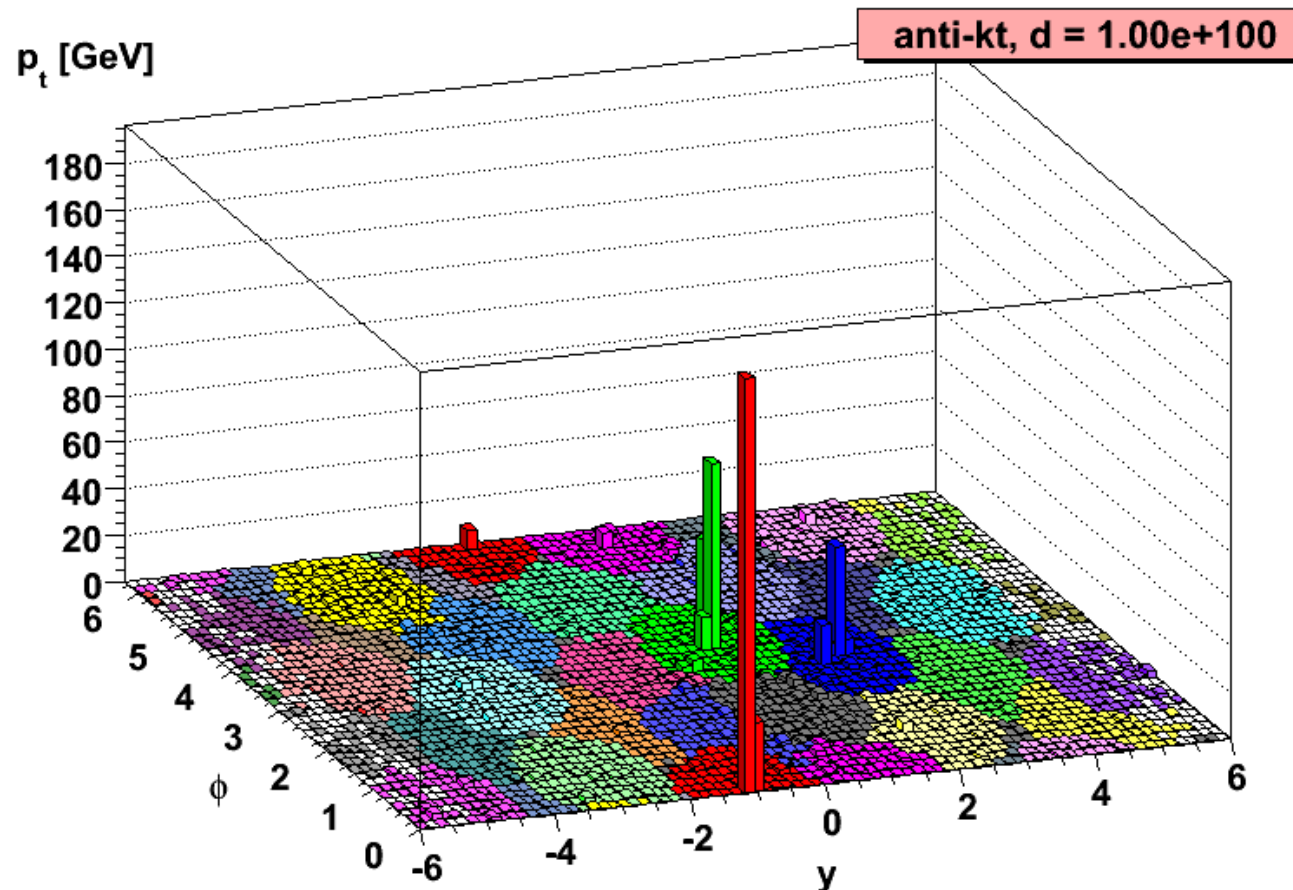
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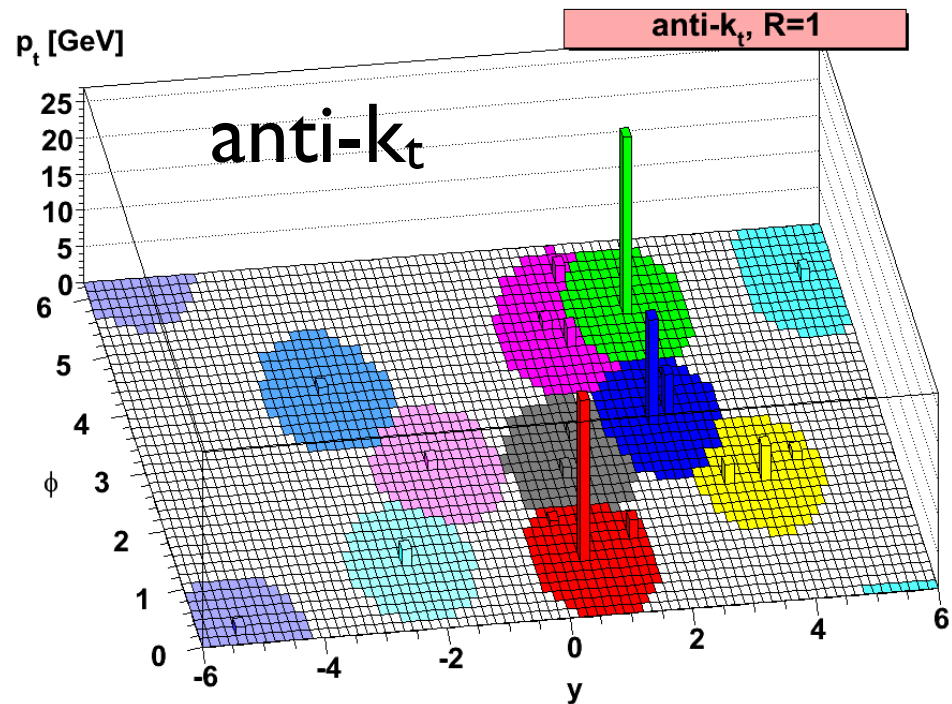
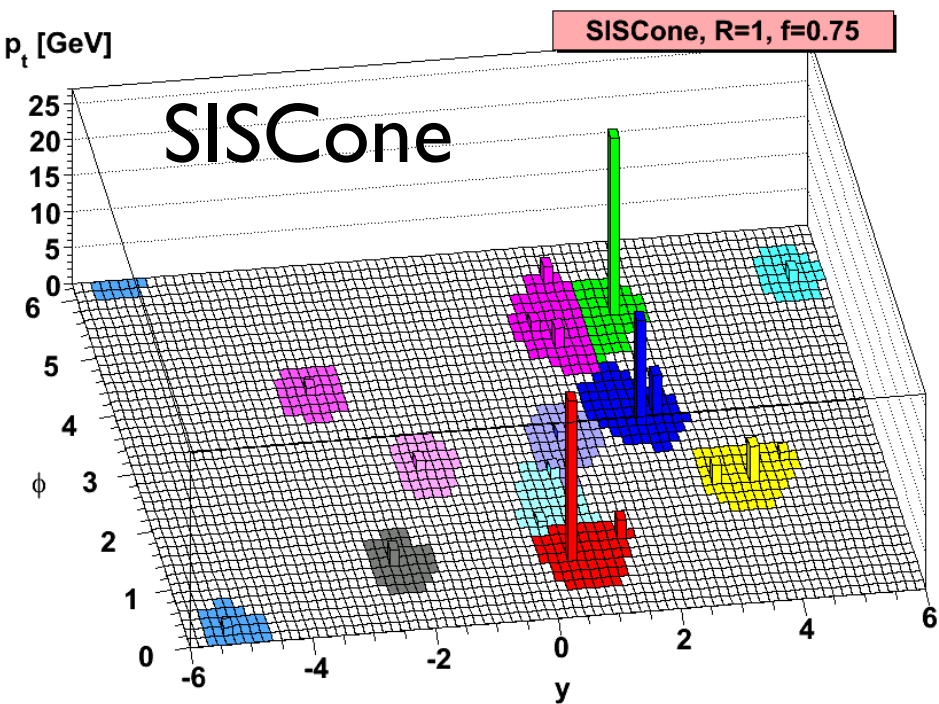
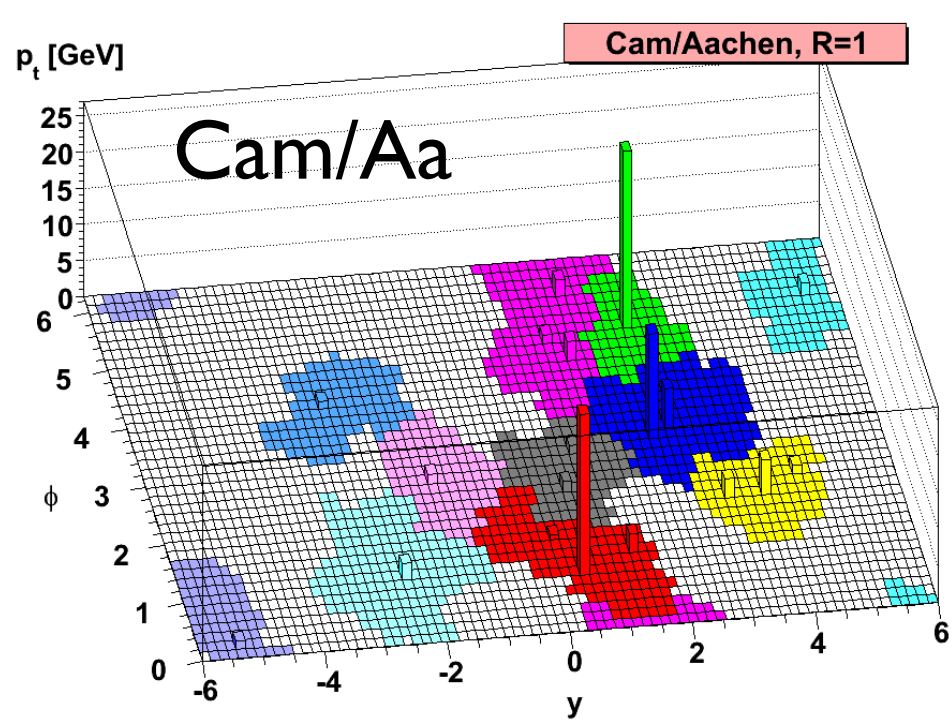
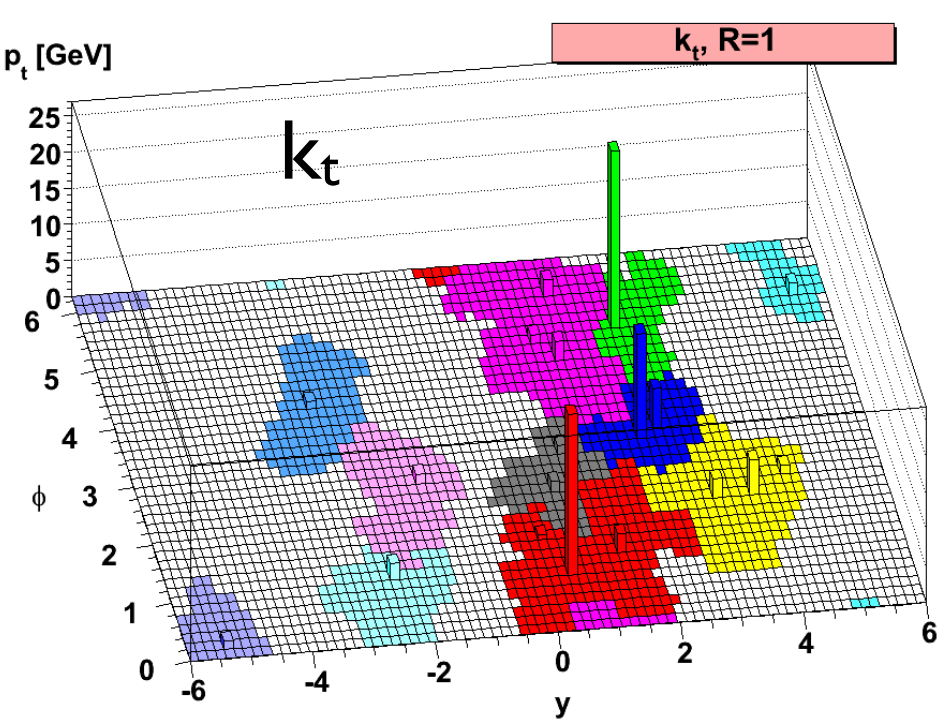


Clustering grows around hard cores

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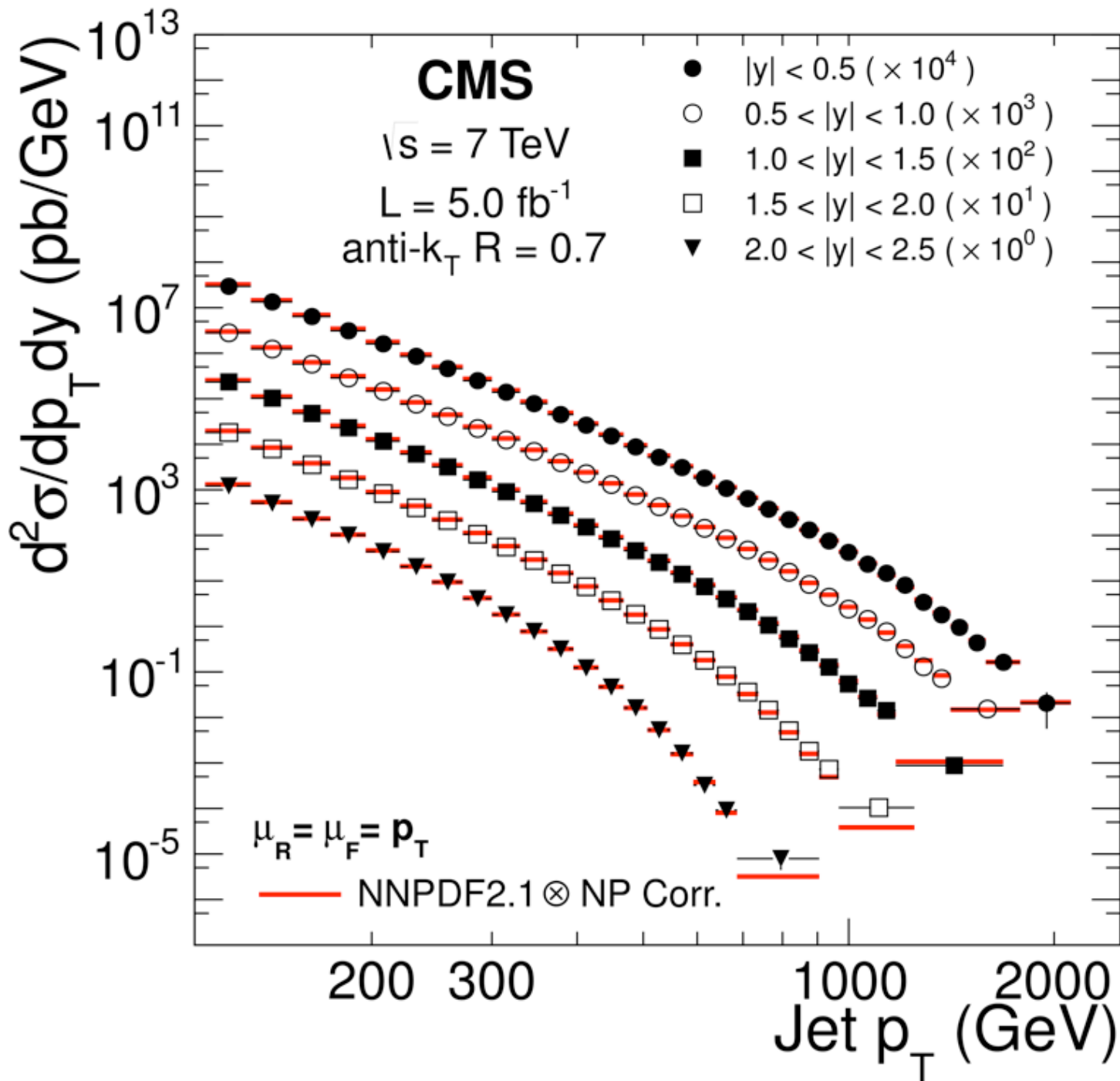


Anti- $k_t$  gives circular jets (“cone-like”) in a way that’s infrared safe





# Example of jet observable

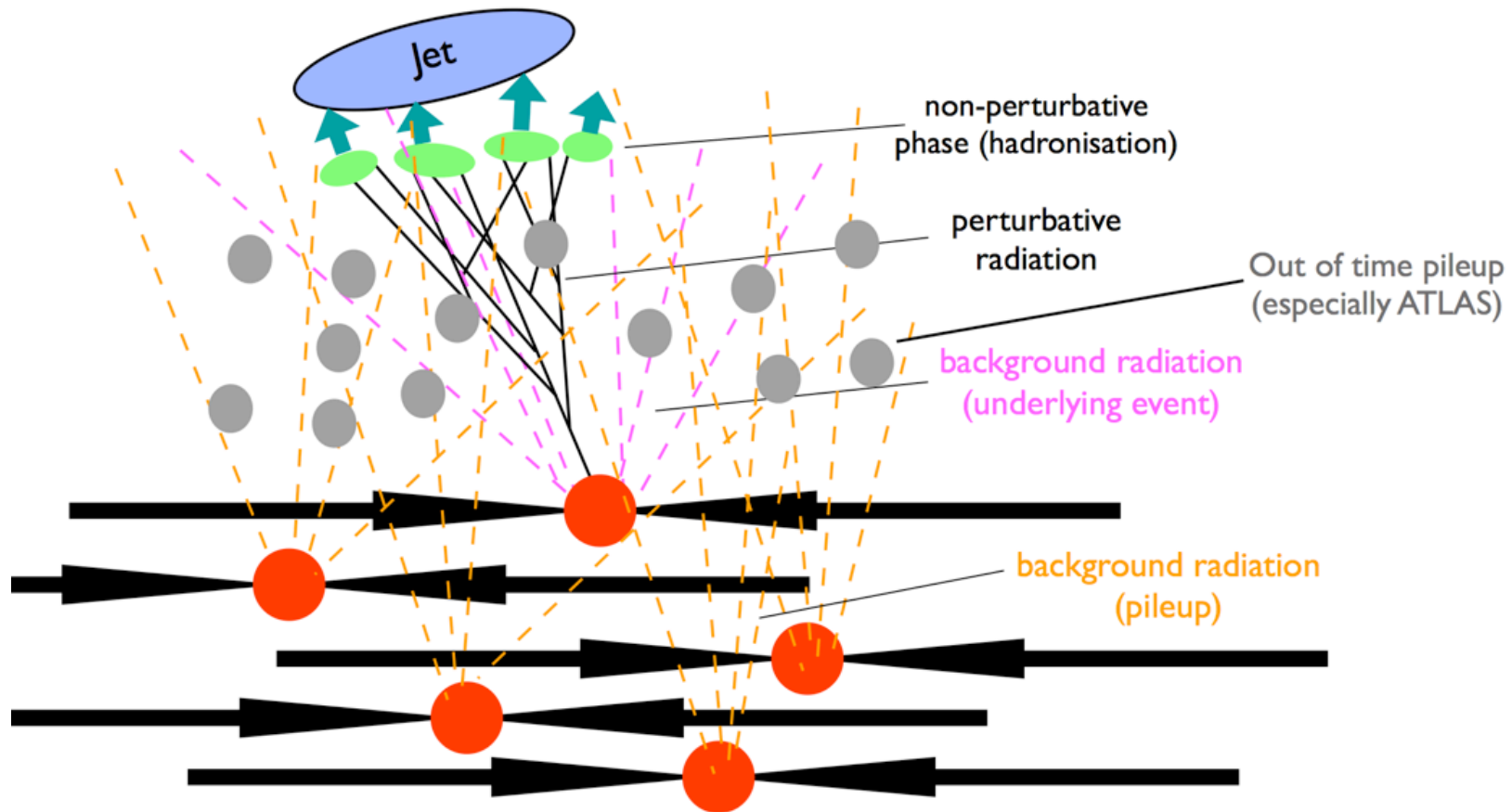


Inclusive  
jet cross  
section

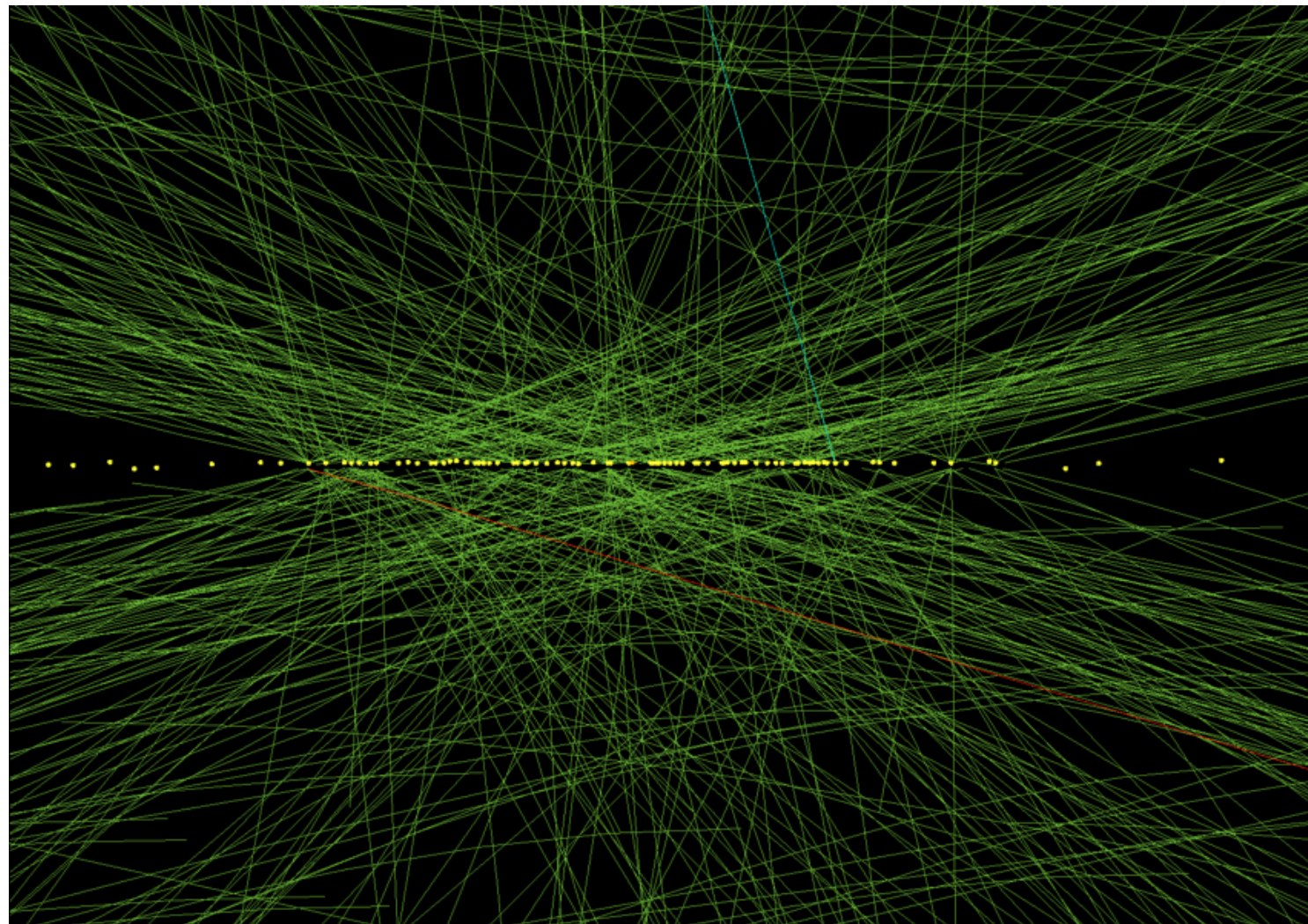
Excellent  
theory-data  
agreement over  
many orders of  
magnitude

# Background

Many 'things' can be clustered into (or lost from) a jet other than what we want (typically, perturbative radiation from a parton)



Ideally we'd like to be able to correct for these effects



78-vertices event  
from CMS

<https://cds.cern.ch/record/1479324>

Pileup can deposit several tens of GeV (or even hundreds, in a heavy ion collision) into a medium-sized jet

# Background subtraction

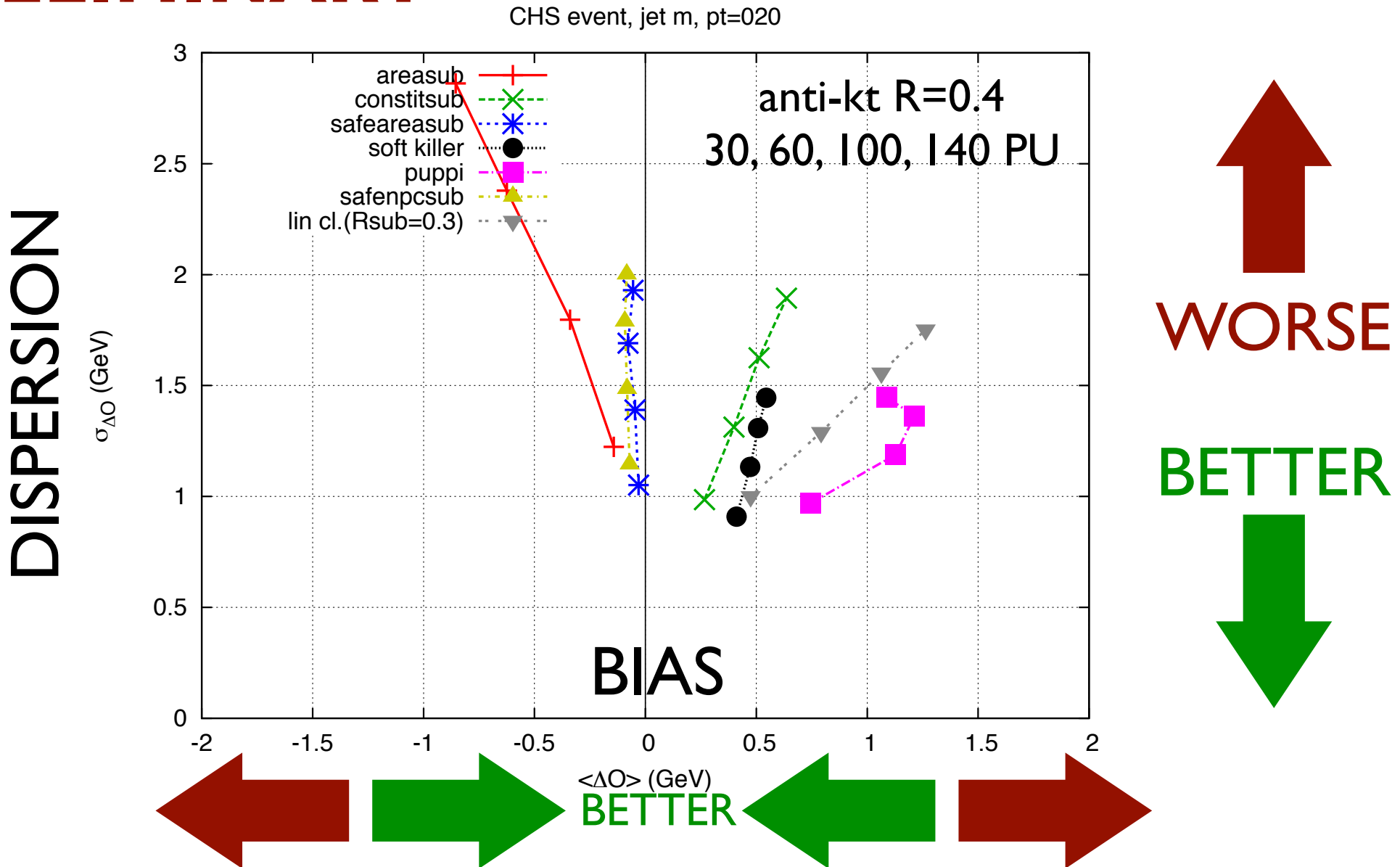
Many advanced tool have been developed in the past ten years to subtract background from jets

This can be done, with varying level of precision and bias, either at the **observable** level (measure a quantity, e.g. a jet  $p_t$ , and then correct its value), or at the **constituents** level (select only the ‘good’ constituents of the jet before measuring the quantity)

# Comparisons of pileup subtractions

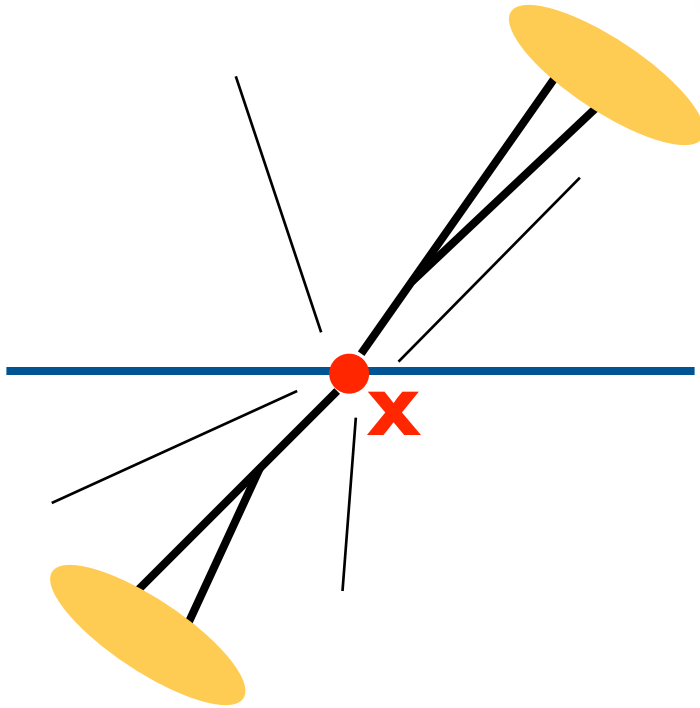
from the CERN pileup mitigation workshop, <https://indico.cern.ch/event/306155/>

**PRELIMINARY**



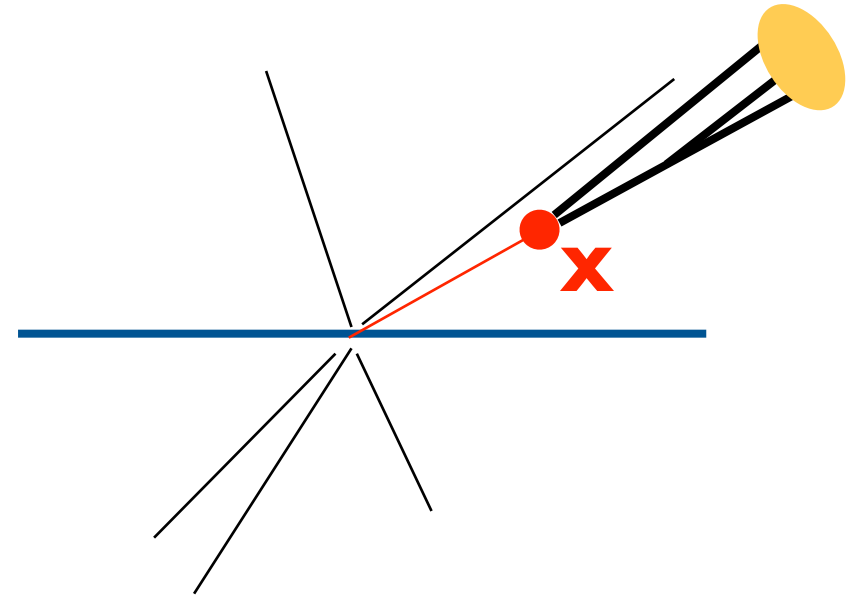


# Why boosted objects



Heavy particle X at **rest**

Easy to resolve jets and calculate invariant mass, but signal very likely swamped by background (eg  $H \rightarrow bb$  v.  $tt \rightarrow WbWb$ )

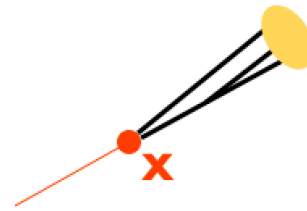
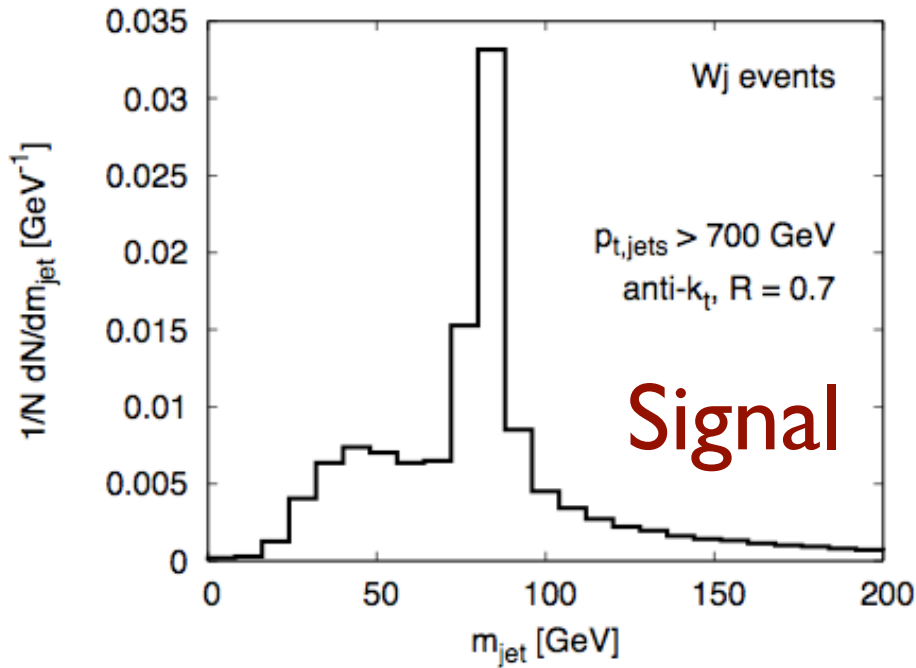


**Boosted** heavy particle X

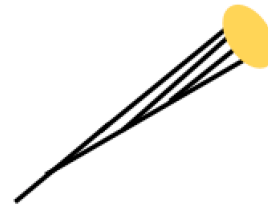
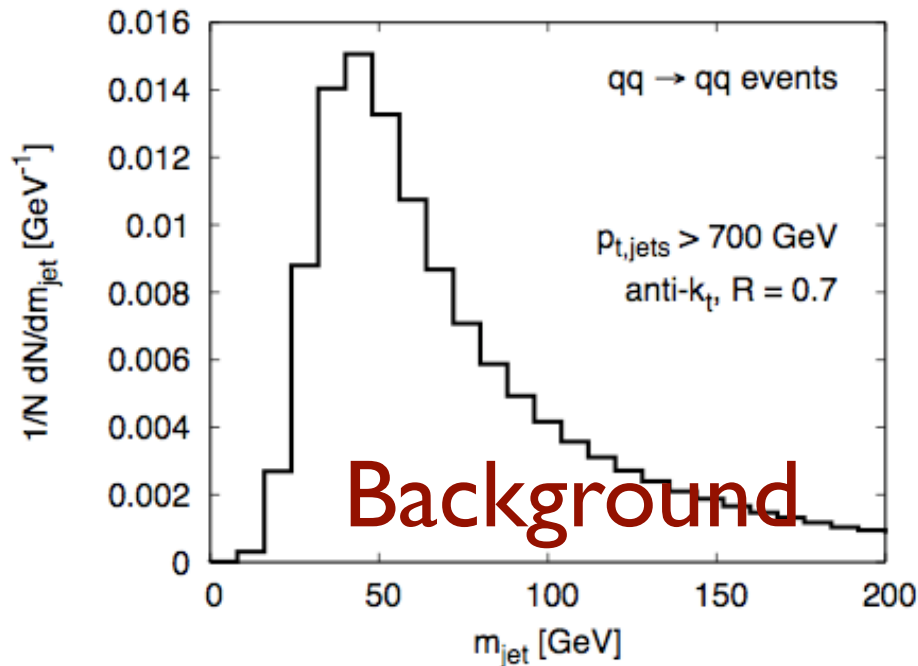
Cross section very much reduced, but acceptance better and some backgrounds smaller/reducible

# Mass of a single jet

G. Salam



A heavy object decaying into a single jet naturally gives it a mass...

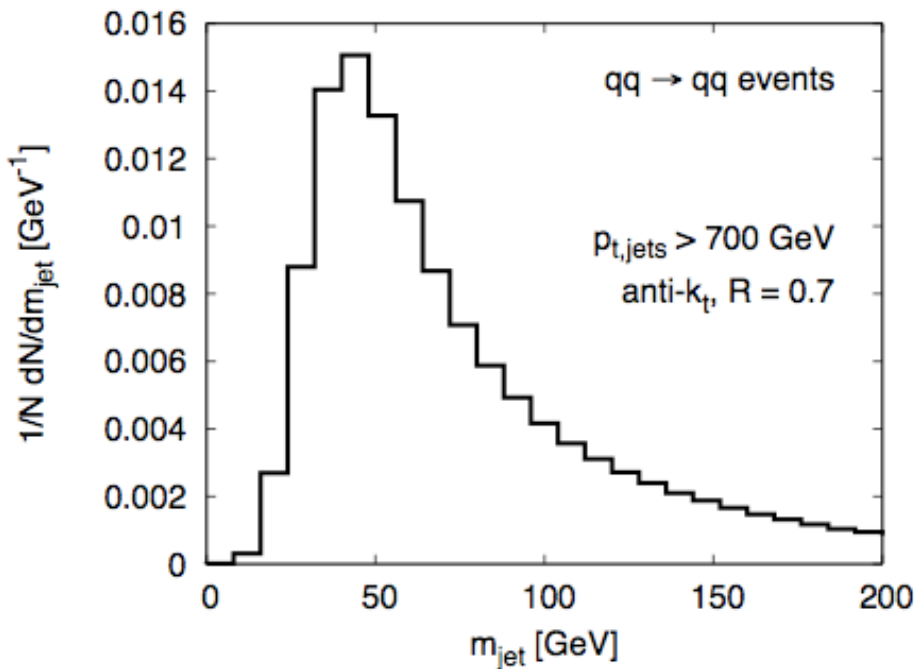


... but pure QCD jets can be massive too:

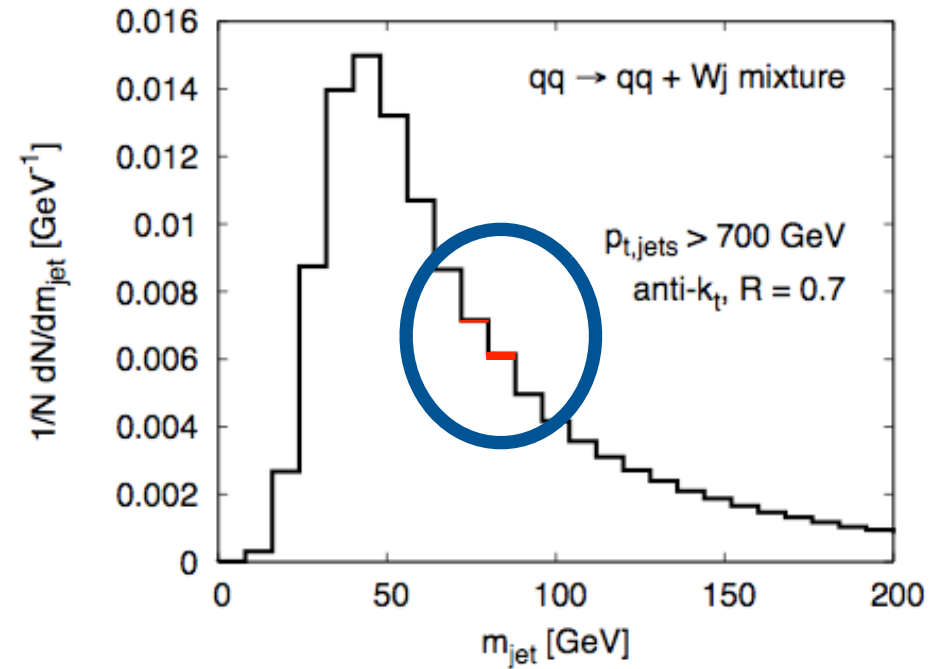
$$\frac{dN}{d \ln m} \sim \alpha_s \ln \frac{p_t R}{m} \times \text{Sudakov}$$

# Mass of a single jet

Summing 'signal' and 'background' (with appropriate cross sections) shows how much the background dominates



**Background only**



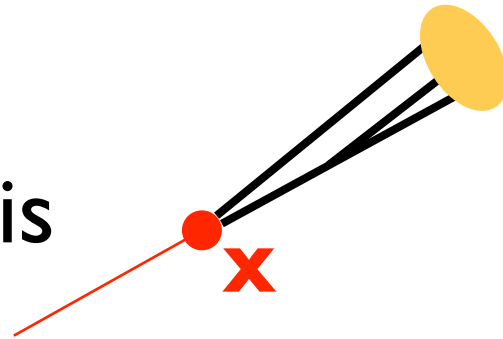
**Signal + background**

**Practically identical**

This means that one can't rely on the invariant mass only.  
An appropriate strategy must be found to reduce the background and enhance the signal

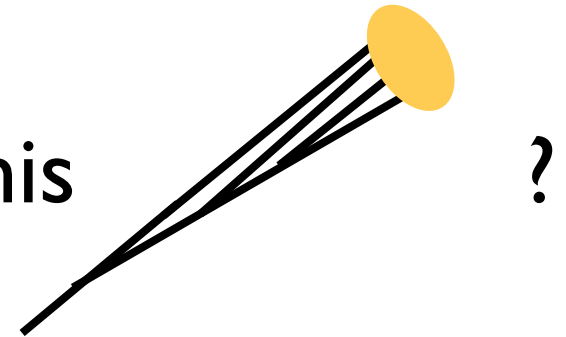


How to tell this



Decay of a heavy  
(boosted) object

from this



Light parton  
fragmentation

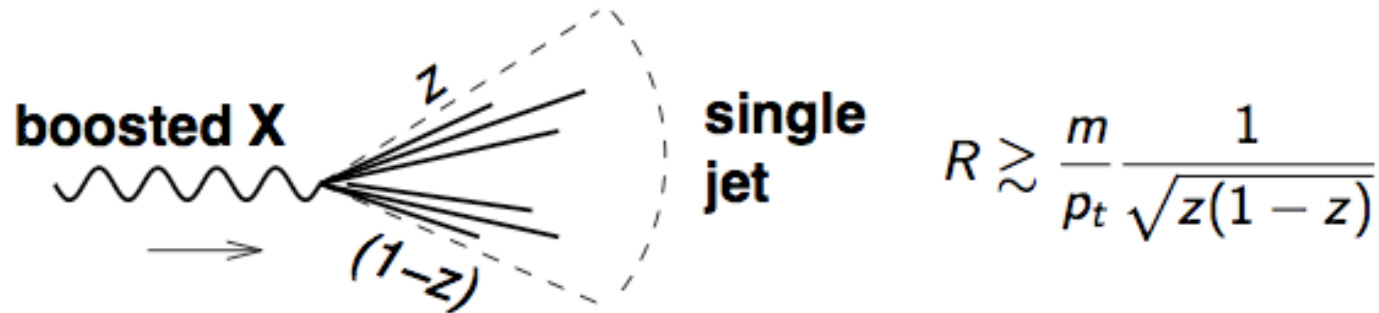
# Tagging and Grooming

- ▶ The substructure of a jet can be exploited to
  - ▶ **tag** a particular structure inside the jet, i.e. a massive particle
    - ▶ First examples: Higgs (2-prong decay), top (3-prong decay)
  - ▶ remove background contamination from the jet or its components, while keeping the bulk of the perturbative radiation (often generically denoted as **grooming**)
    - ▶ First examples: filtering, trimming, pruning

# Why substructure

Scales:  $m \sim 100 \text{ GeV}$ ,  $p_t \sim 500 \text{ GeV}$

(e.g. electroweak particle from decay of  $\sim 1 \text{ TeV}$  BSM particle)



- ▶ need **small R** ( $< 2m/p_t \sim 0.4$ ) to resolve **two** prongs
- ▶ need **large R** ( $> \sim 3m/p_t \sim 0.6$ ) to cluster into a **single** jet

## Possible strategies

- ▶ Use large  $R$ , get a single jet : **background large**
- ▶ Use small  $R$ , resolve the jets : **what is the right scale?**
  - ▶ Also: small jets lead to huge combinatorial issues

**Let an algorithm find the 'right' substructure**

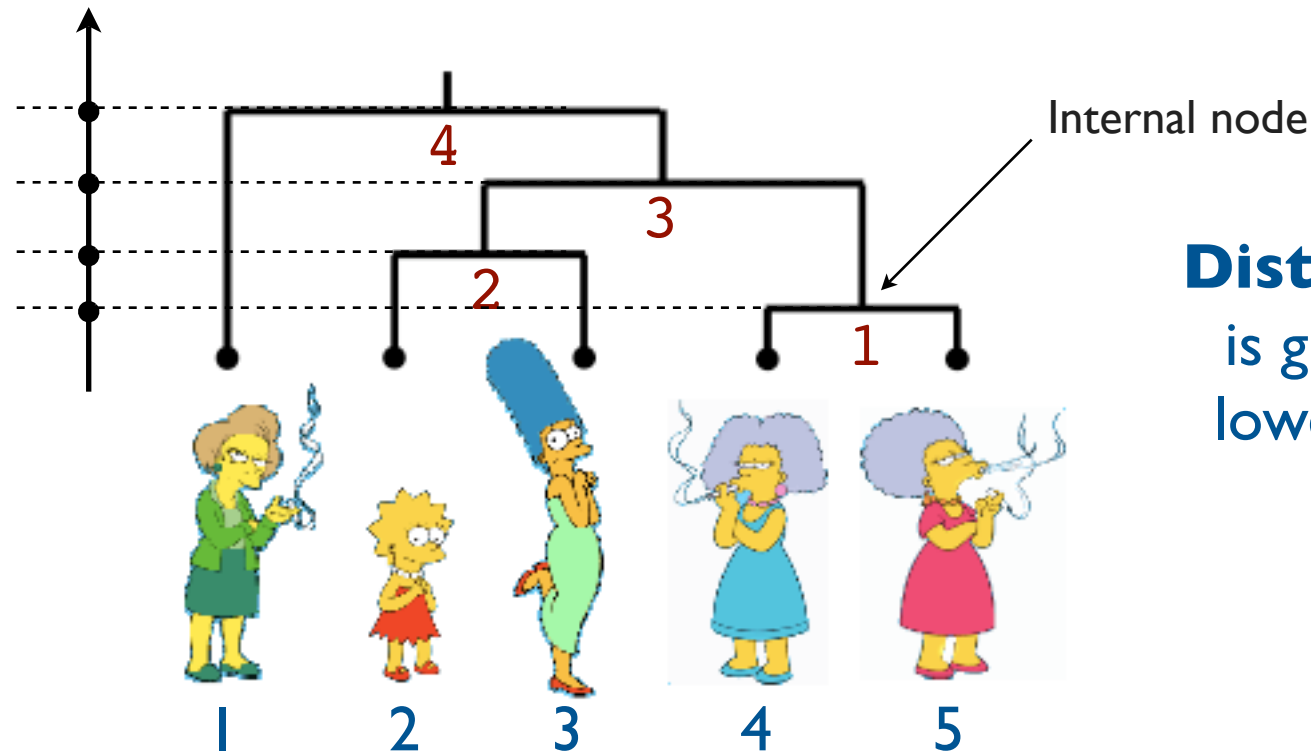
# What jets to use for substructure?

Different jet algorithms will give different ‘pictures’  
of what’s inside a jet

# Dendrogram

Used to represent graphically the sequence of clustering steps in a sequential recombination algorithm

Distance



**Distance** between two objects is given by the **height** of the lowest internal node that they share.

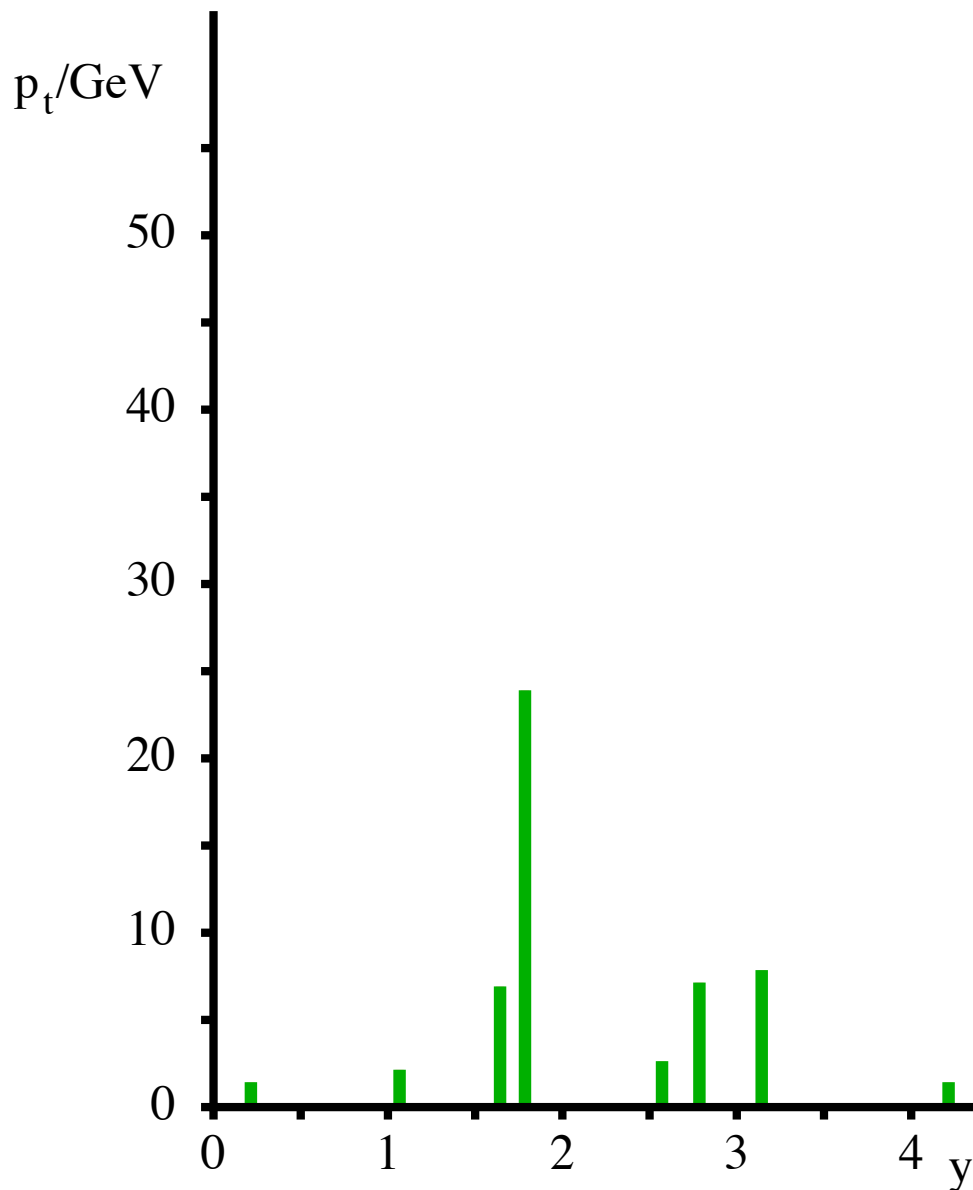
Order of clustering here is 1,2,3,4

The **clustering sequence** is 4-5 (1), 2-3 (2), 23-45 (3), 1-2345 (4)

anti-kt

# Identifying jet substructure: try out anti- $k_t$

## anti- $k_t$ algorithm



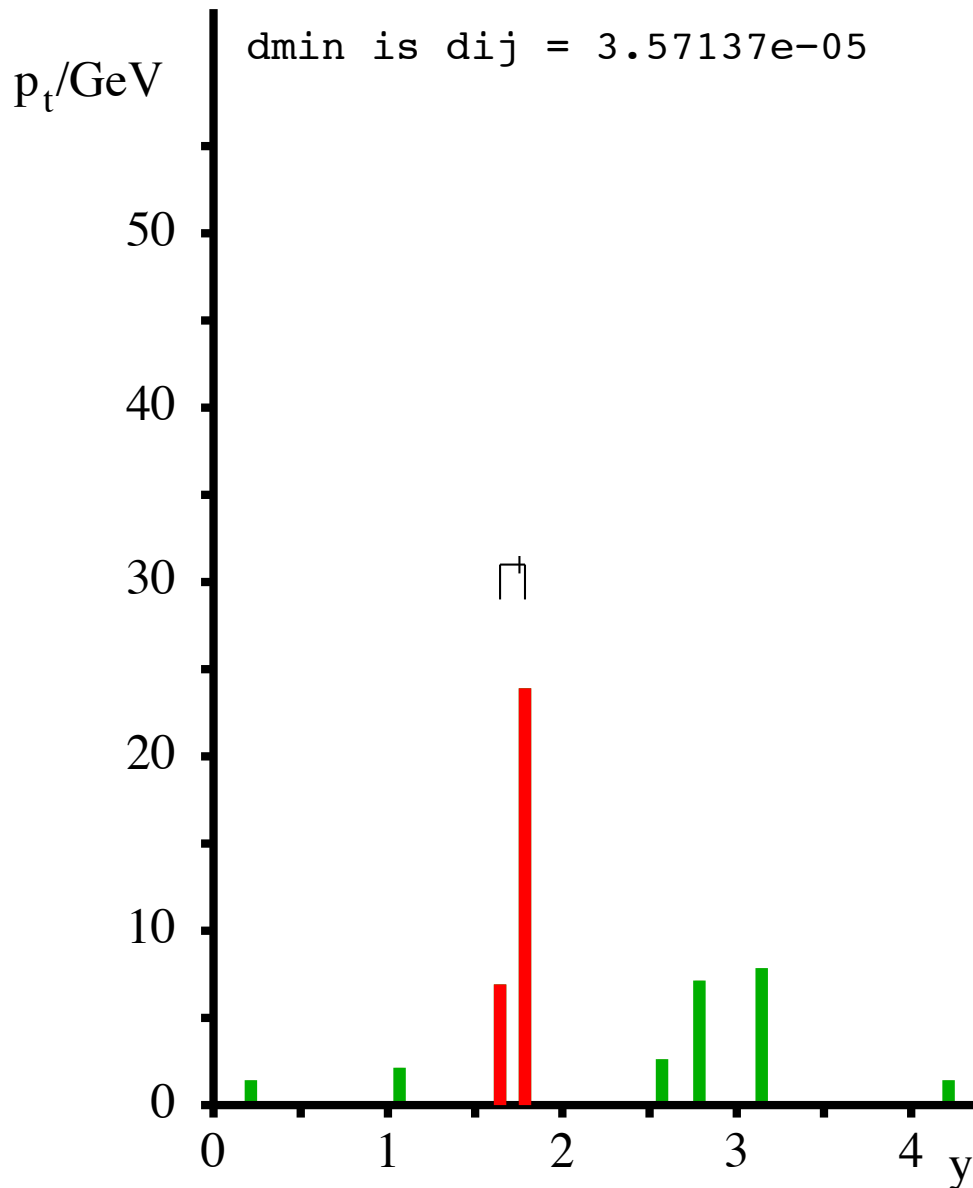
How well can an algorithm identify the “blobs” of energy inside a jet that come from different partons?

This is crucial for identifying the kinematic variables of the partons in the jet (e.g.  $z$ ).

# Identifying jet substructure: try out anti- $k_t$

## anti- $k_t$ algorithm

$d_{\min}$  is  $d_{ij} = 3.57137e-05$



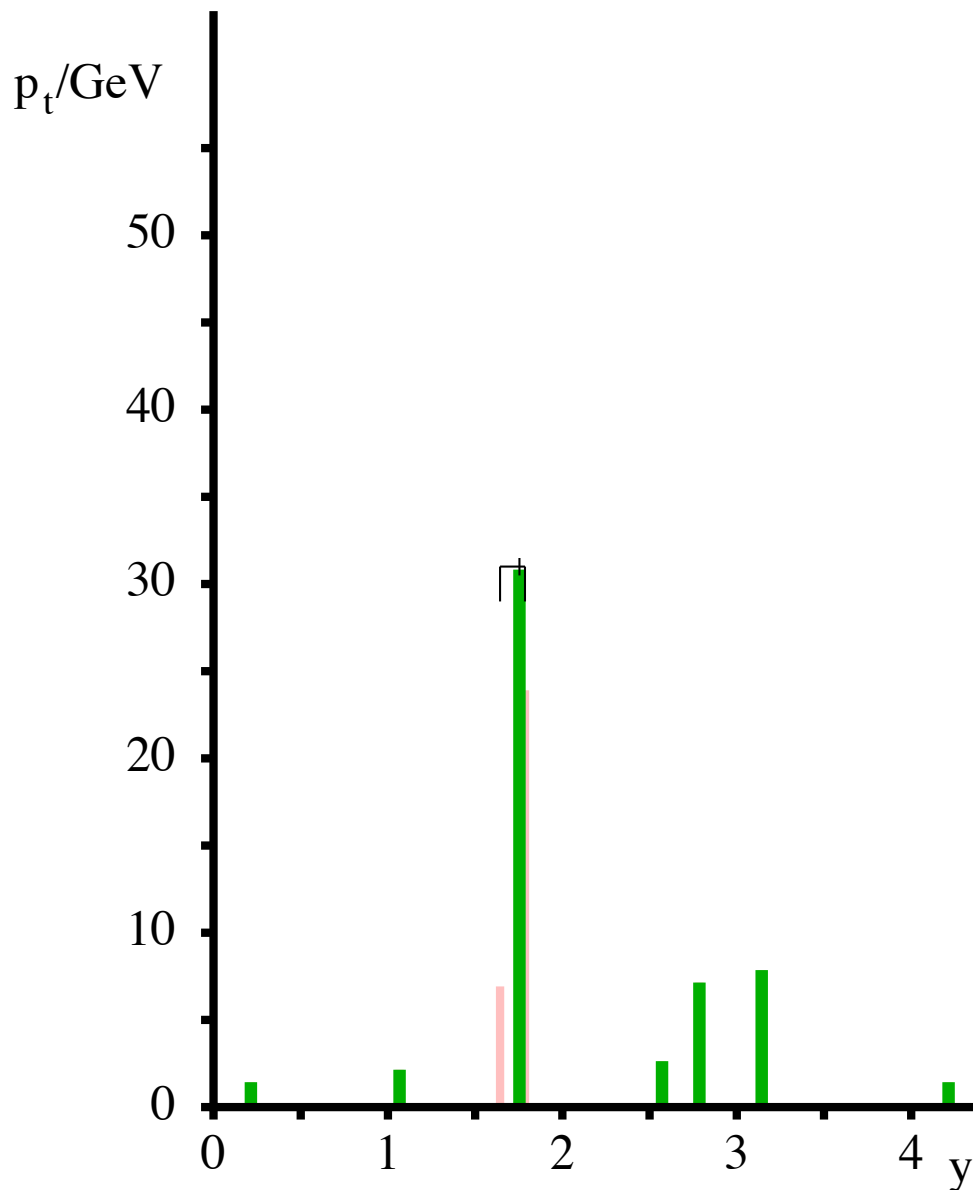
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## anti- $k_t$ algorithm



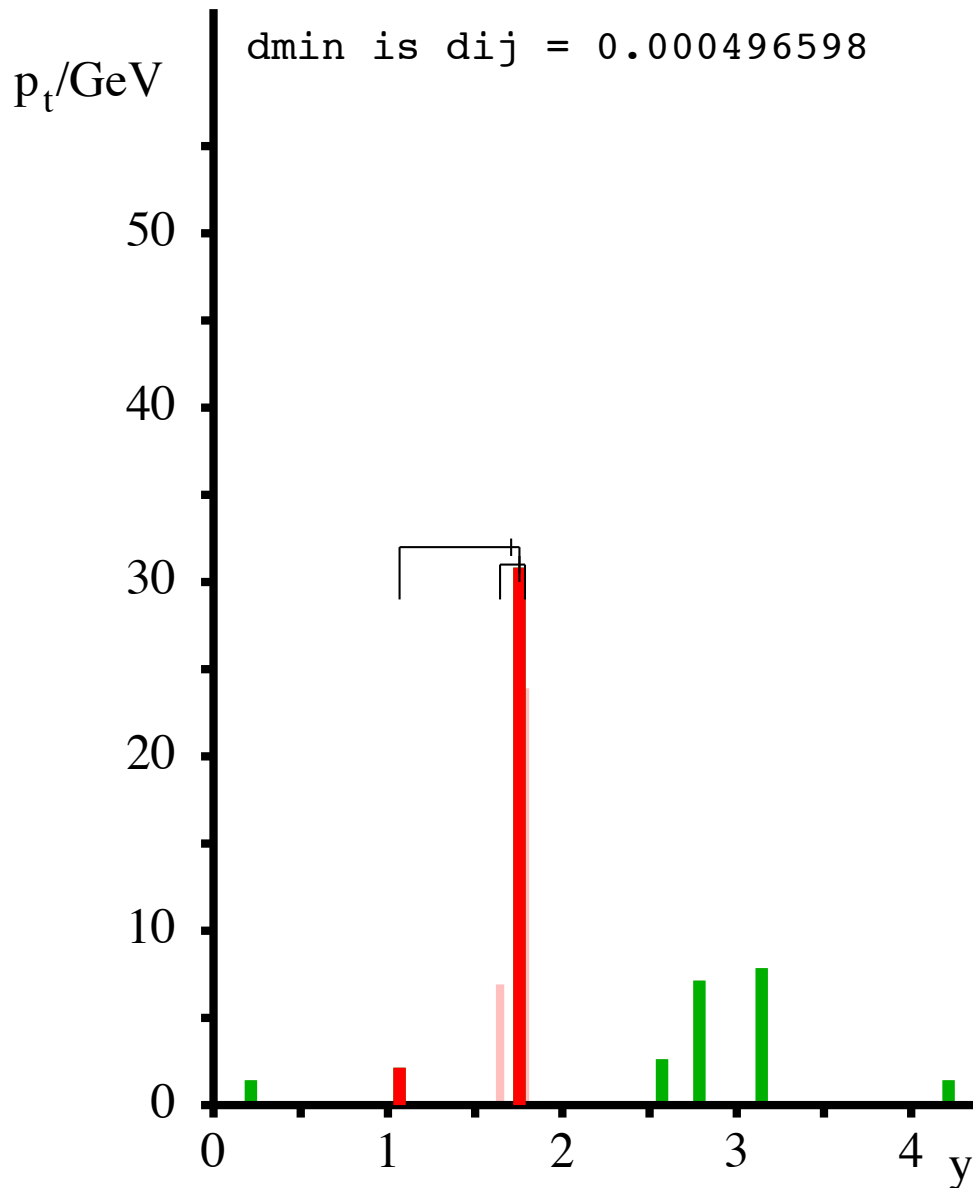
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# Identifying jet substructure: try out anti- $k_t$

## anti- $k_t$ algorithm

$d_{\min}$  is  $d_{ij} = 0.000496598$

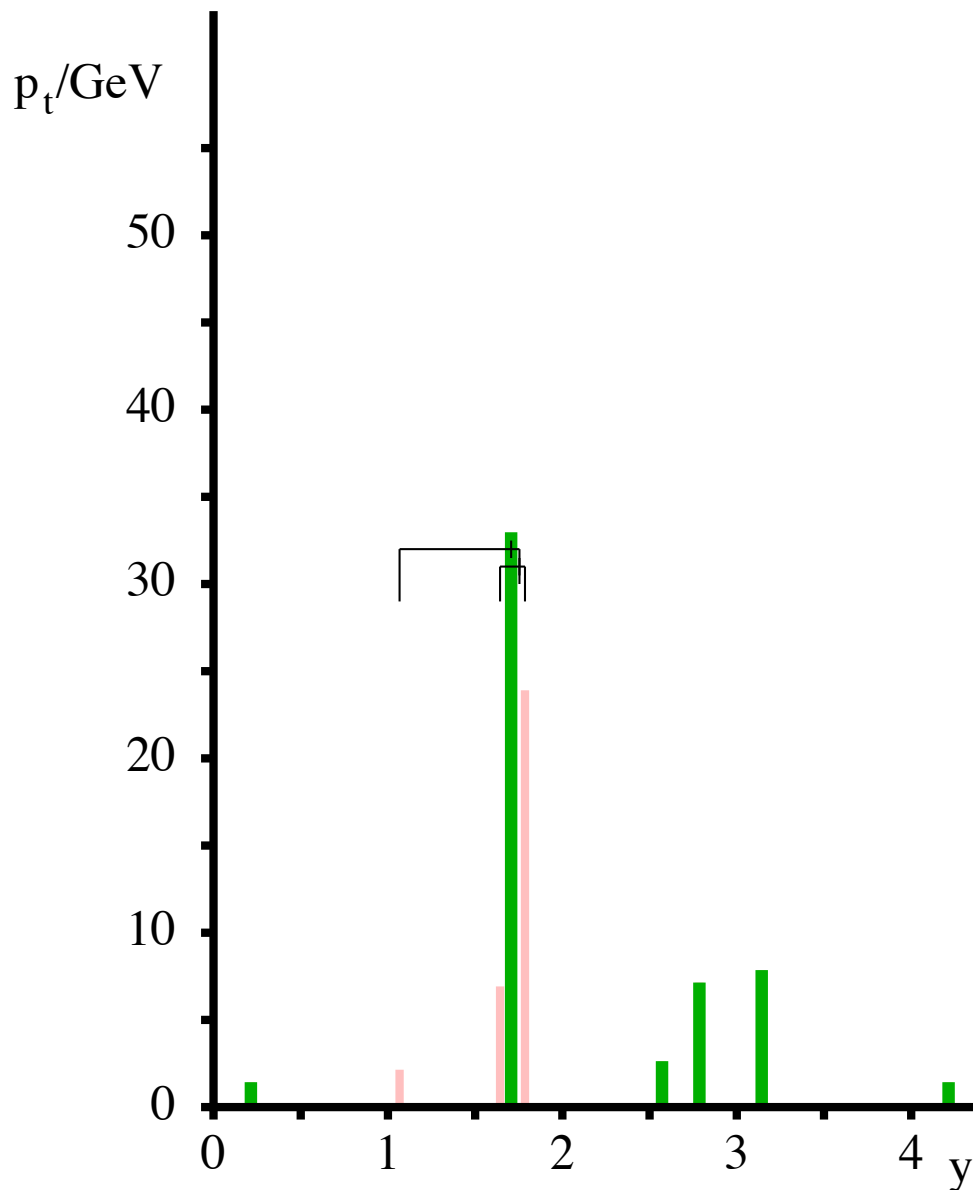


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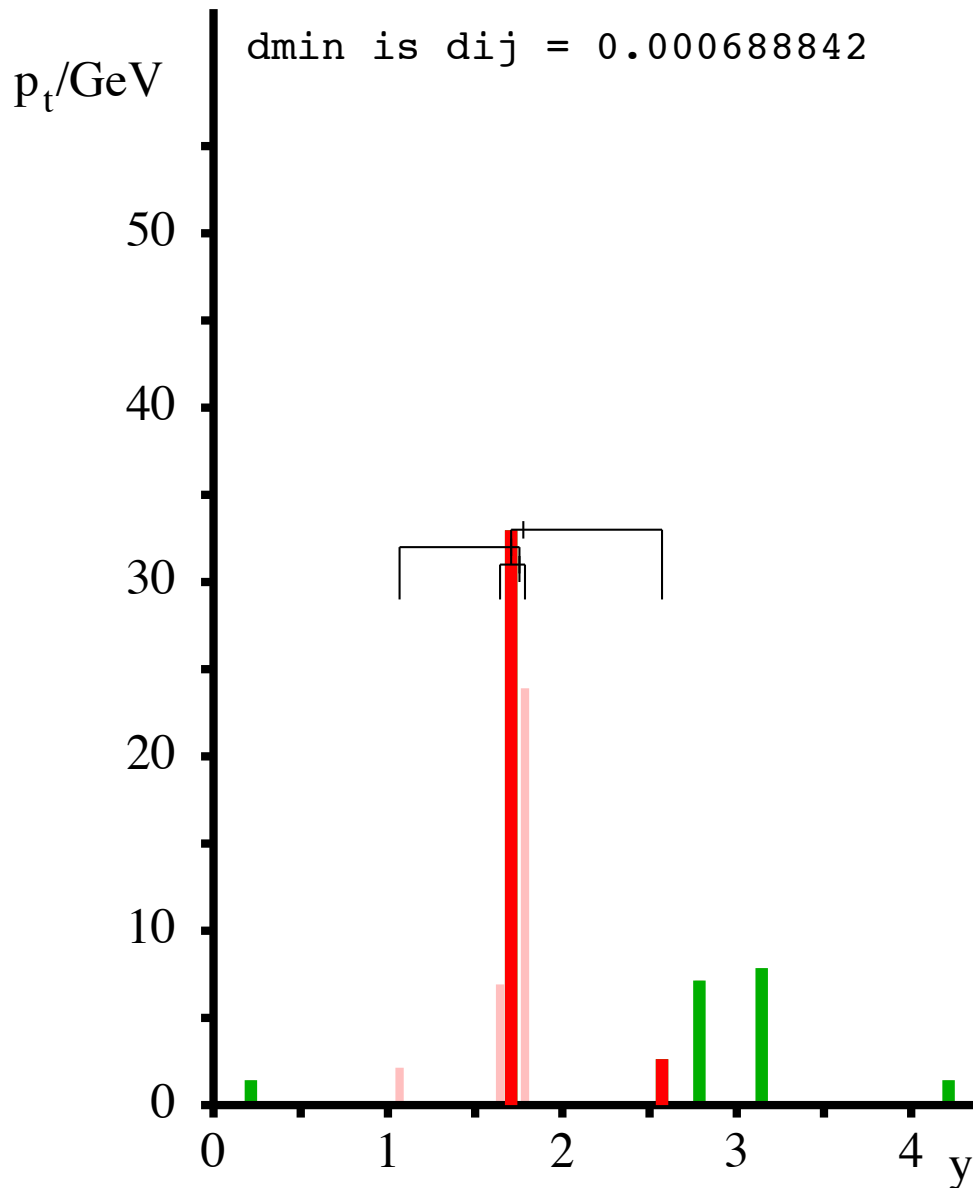
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# Identifying jet substructure: try out anti- $k_t$

## anti- $k_t$ algorithm

$d_{\min}$  is  $d_{ij} = 0.000688842$

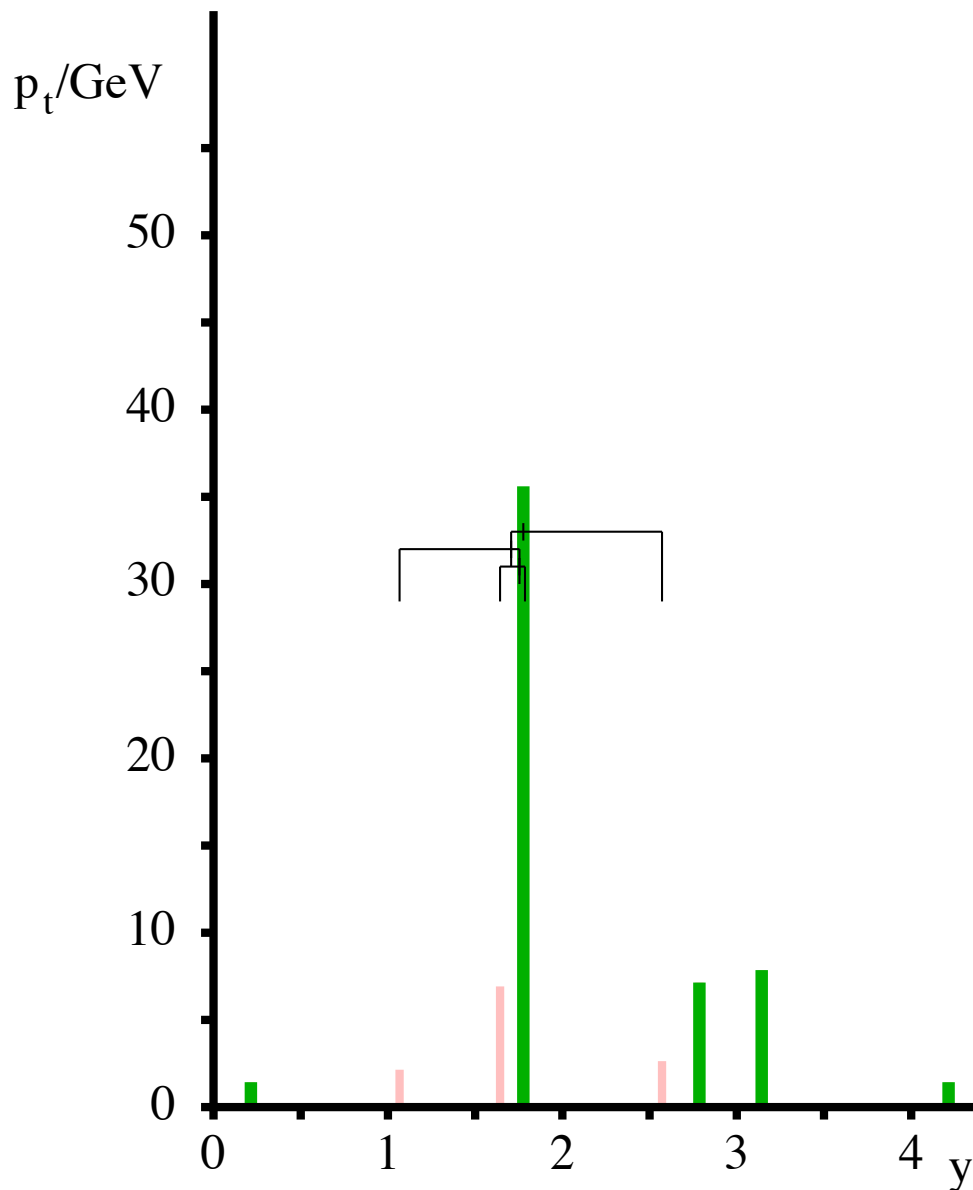


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# Identifying jet substructure: try out anti- $k_t$

## anti- $k_t$ algorithm



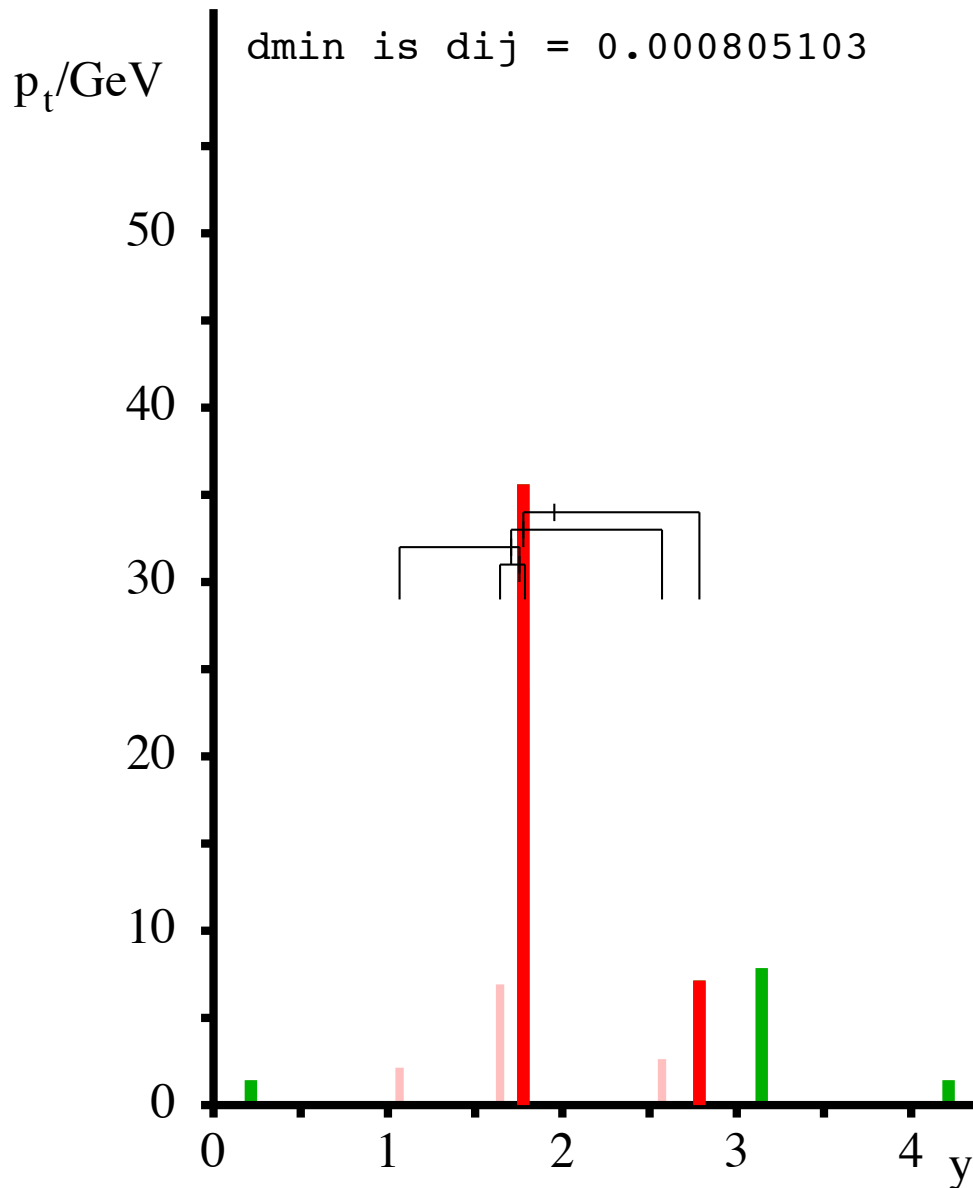
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# Identifying jet substructure: try out anti- $k_t$

## anti- $k_t$ algorithm

$d_{\min}$  is  $d_{ij} = 0.000805103$



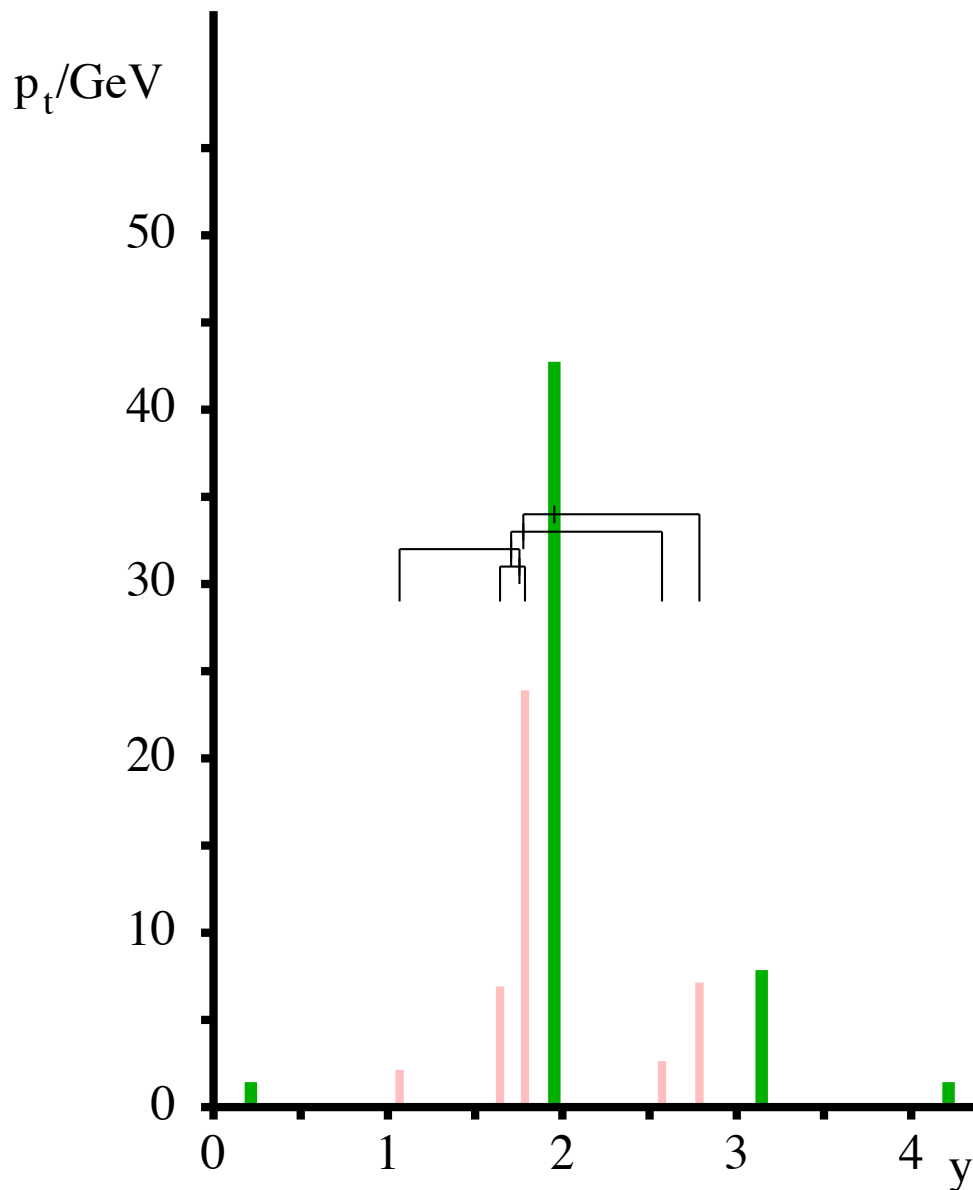
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*Anti- $k_t$  gradually makes its way through the secondary blob  $\rightarrow$  no clear identification of substructure associated with 2nd parton.*

# Identifying jet substructure: try out anti- $k_t$

## anti- $k_t$ algorithm



How well can an algorithm identify the “blobs” of energy inside a jet that come from different partons?

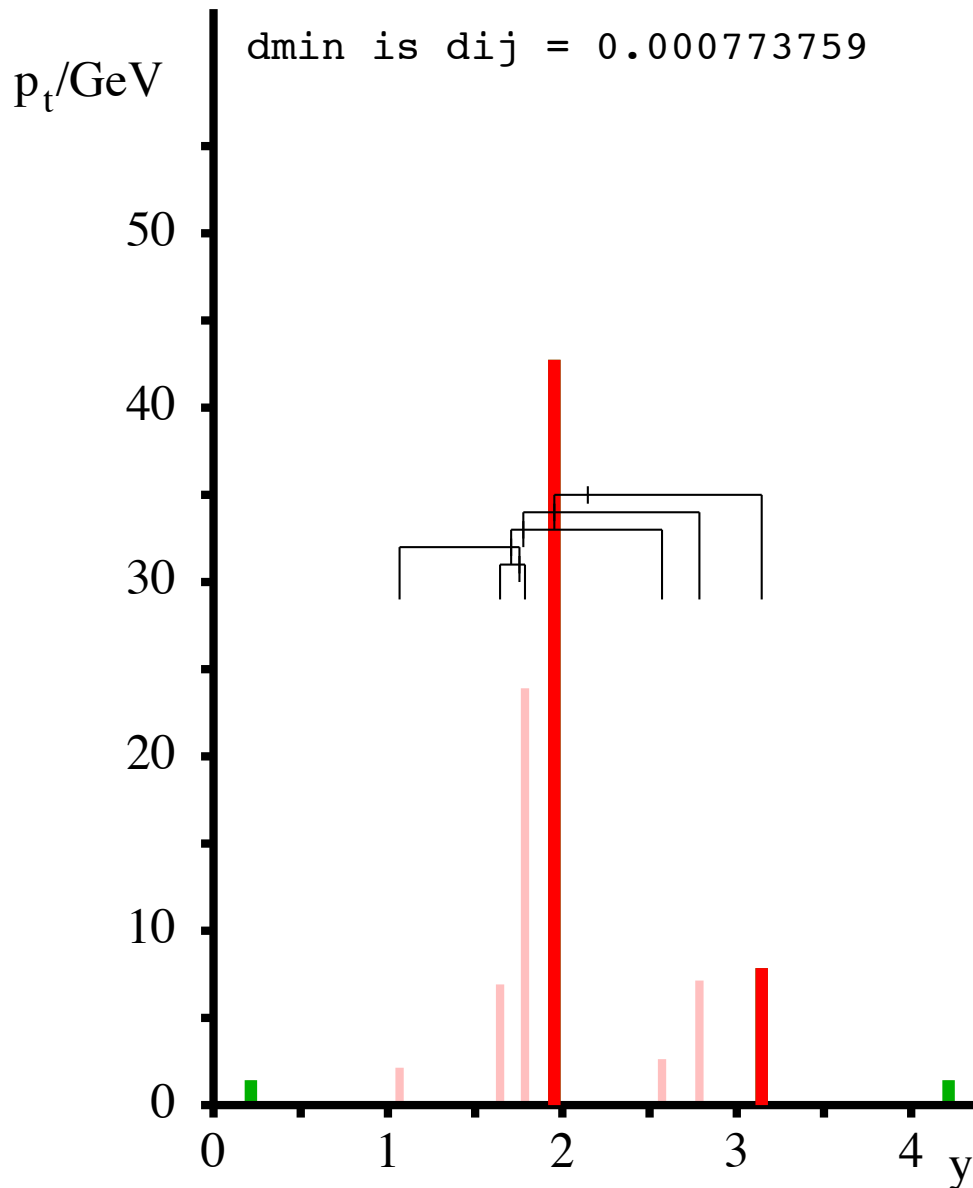
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# Identifying jet substructure: try out anti- $k_t$

## anti- $k_t$ algorithm

$d_{\min}$  is  $d_{ij} = 0.000773759$



How well can an algorithm identify the “blobs” of energy inside a jet that come from different partons?

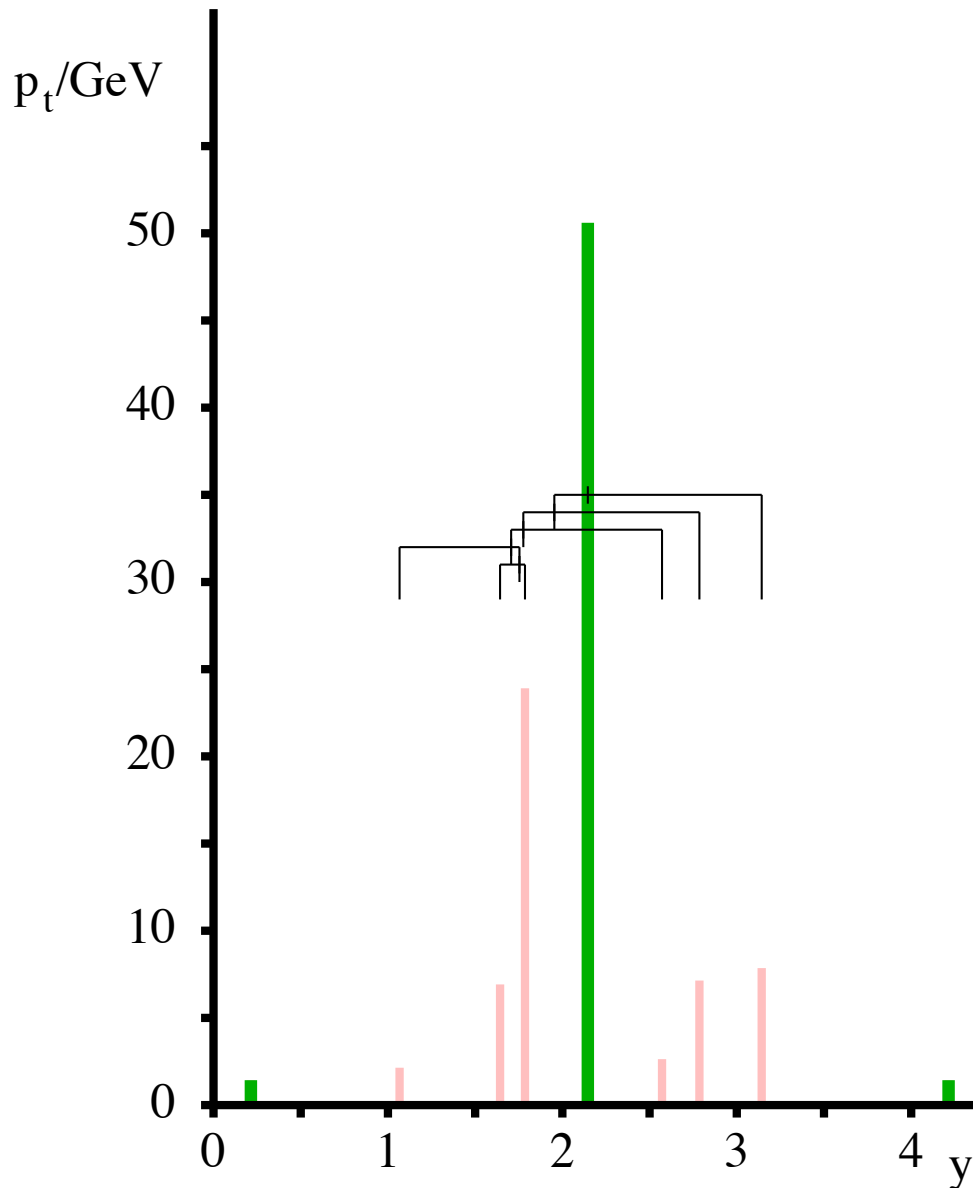
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## anti- $k_t$ algorithm



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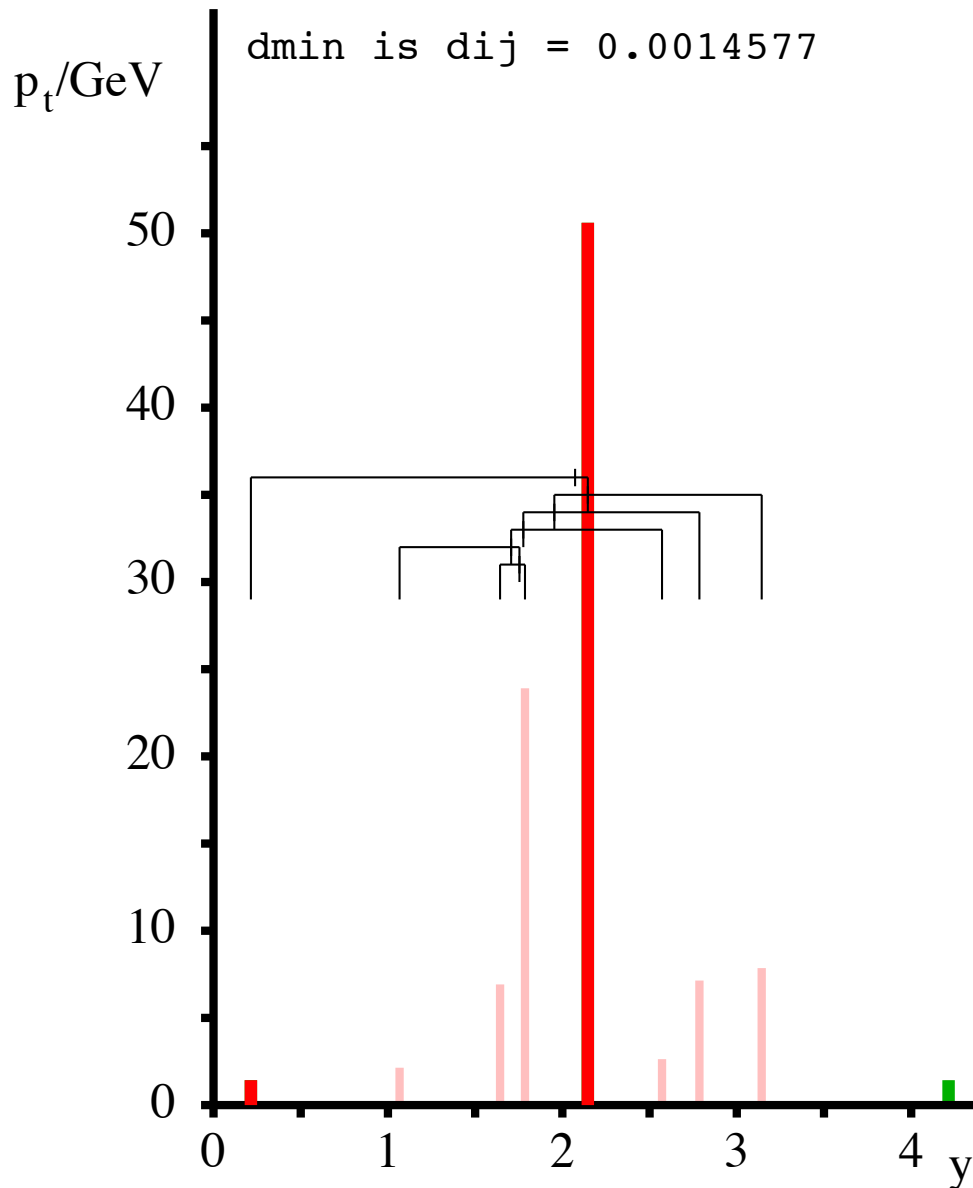
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# Identifying jet substructure: try out anti- $k_t$

## anti- $k_t$ algorithm

$d_{\min}$  is  $d_{ij} = 0.0014577$



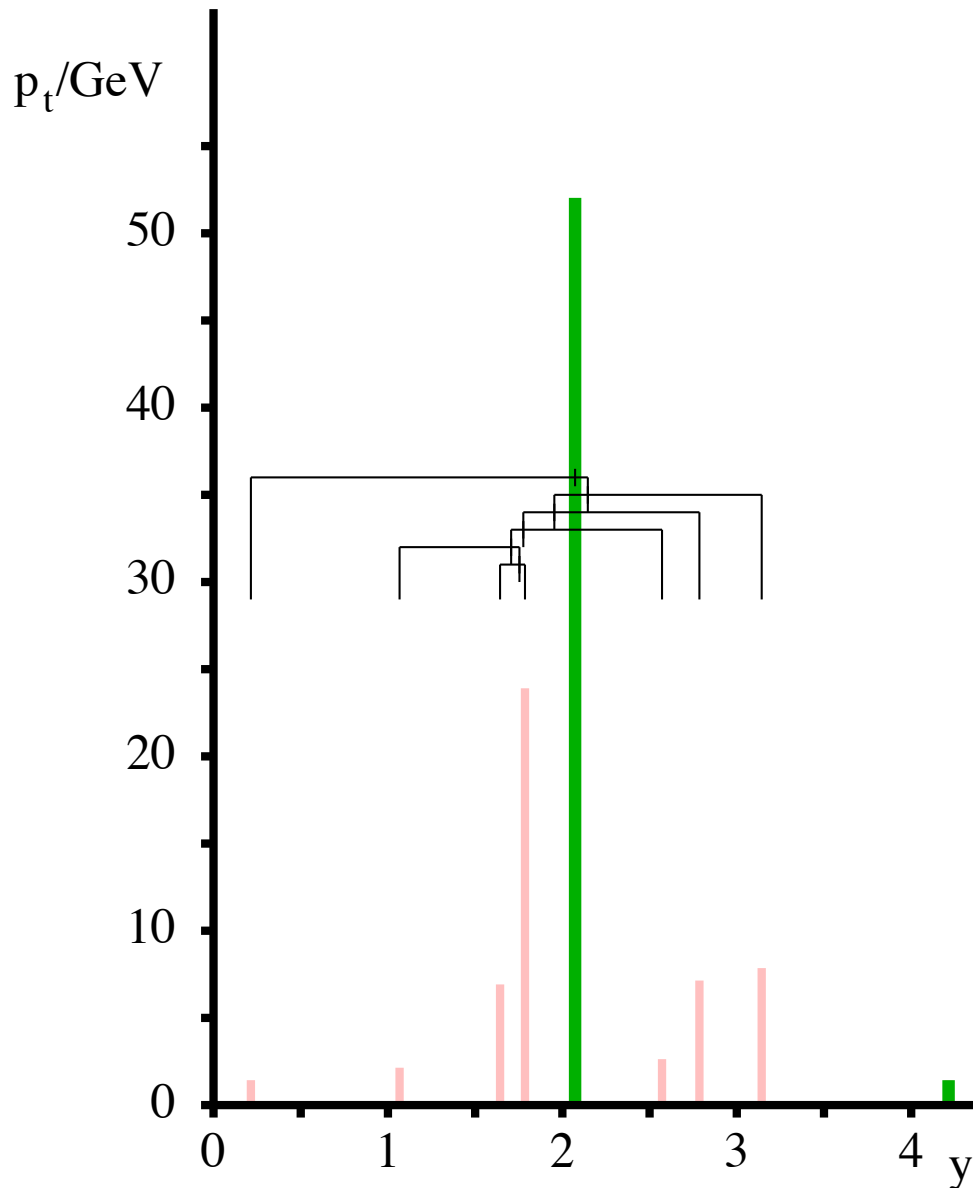
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# Identifying jet substructure: try out anti- $k_t$

## anti- $k_t$ algorithm



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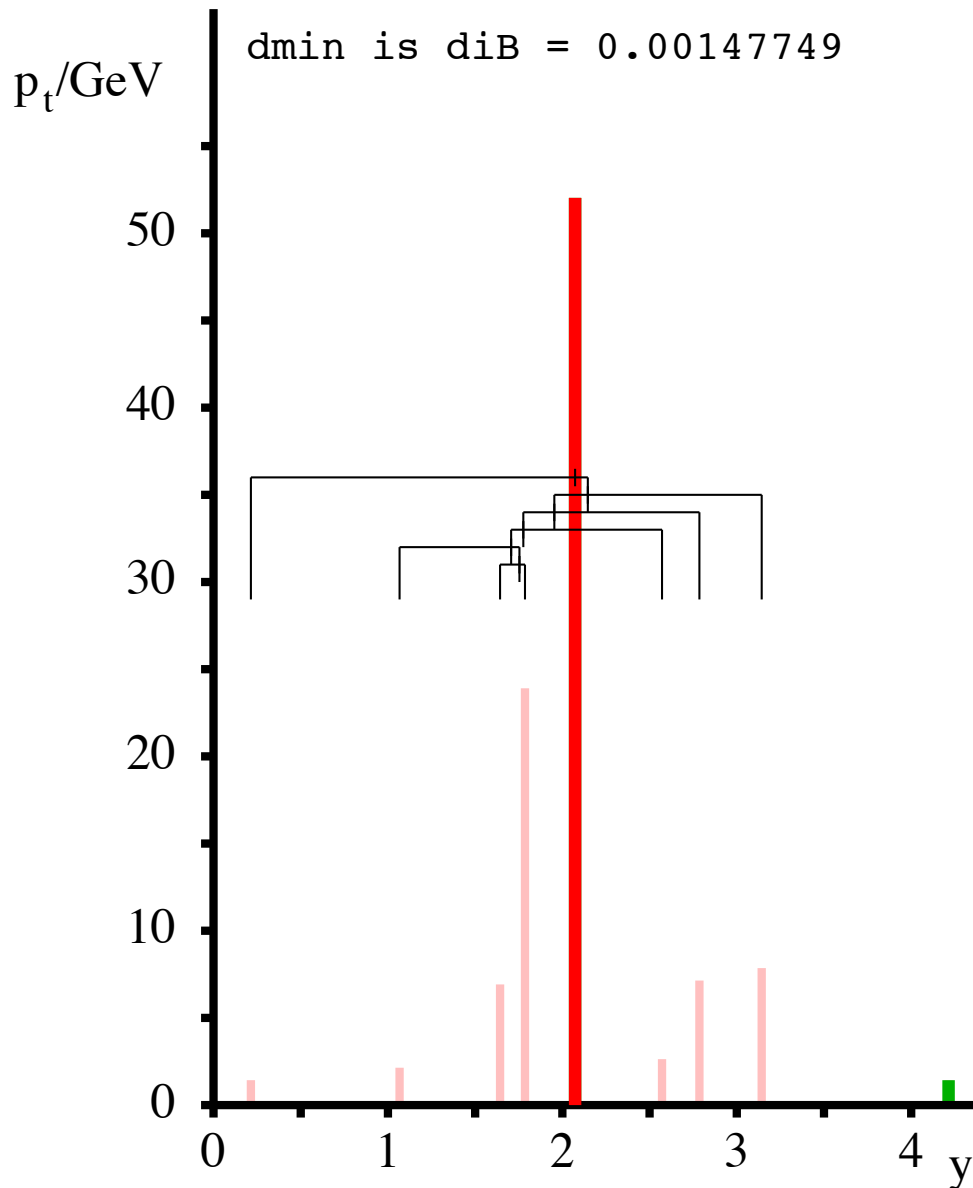
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# Identifying jet substructure: try out anti- $k_t$

## anti- $k_t$ algorithm

$d_{\min}$  is  $d_{iB} = 0.00147749$



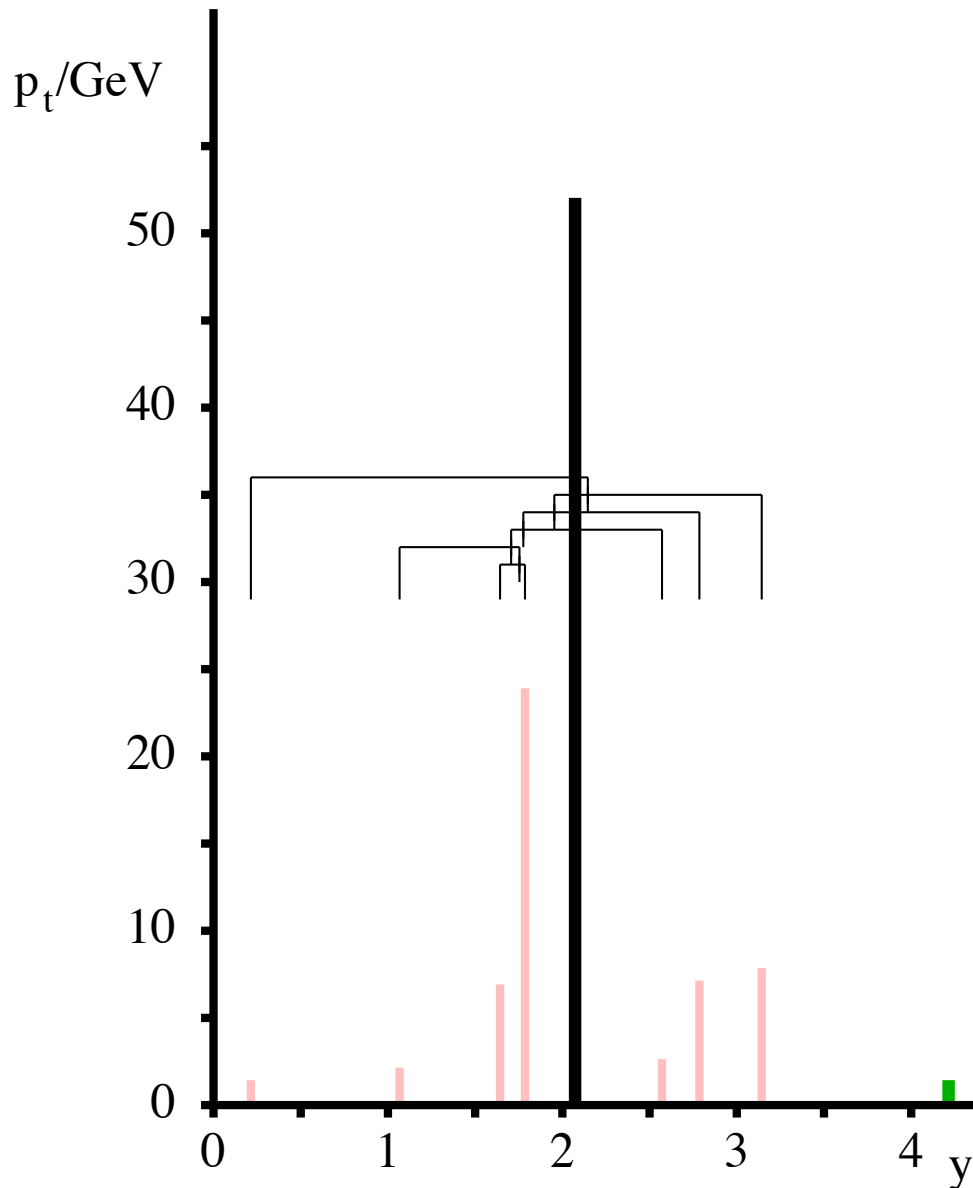
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# Identifying jet substructure: try out anti- $k_t$

## anti- $k_t$ algorithm



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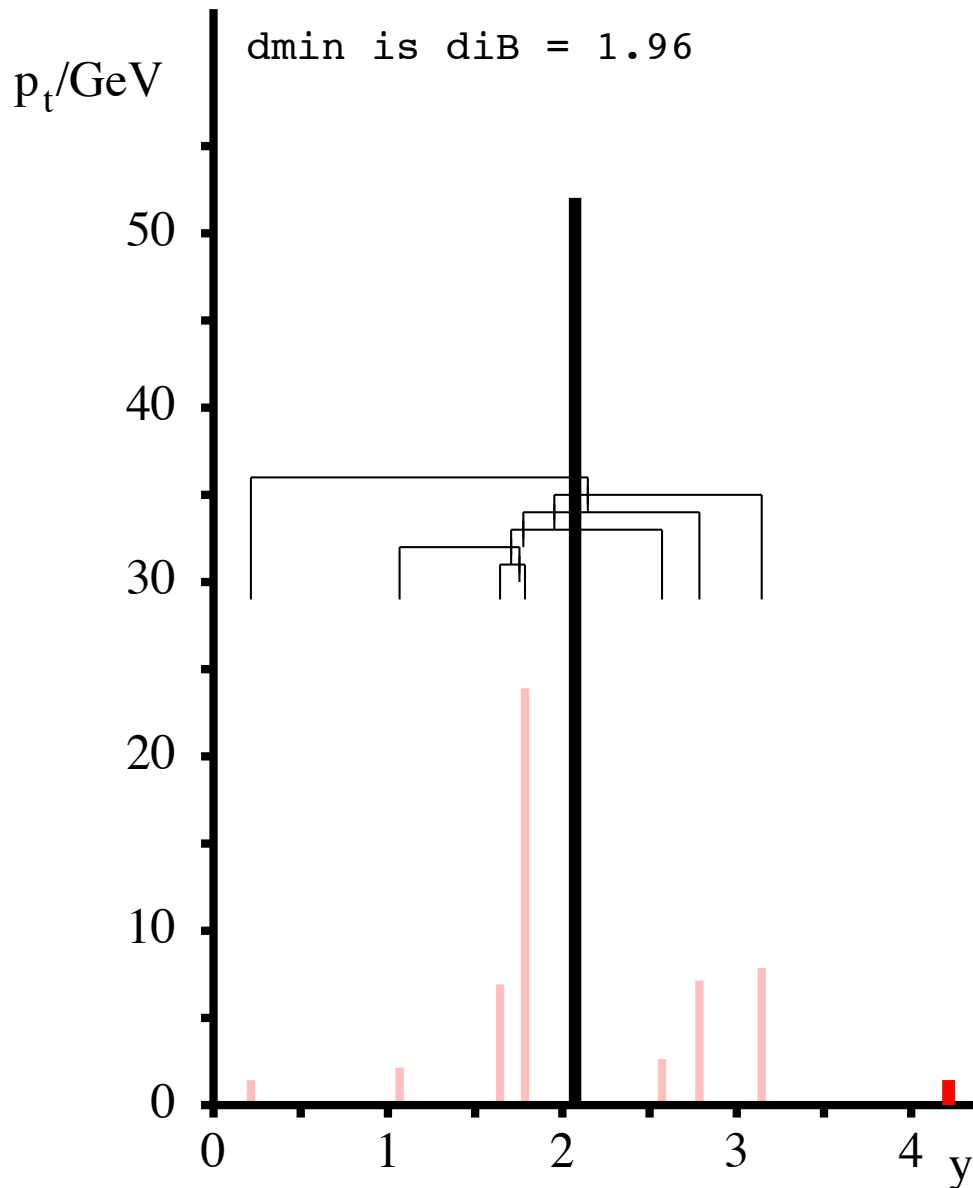
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# Identifying jet substructure: try out anti- $k_t$

## anti- $k_t$ algorithm

$d_{\min}$  is  $d_{iB} = 1.96$



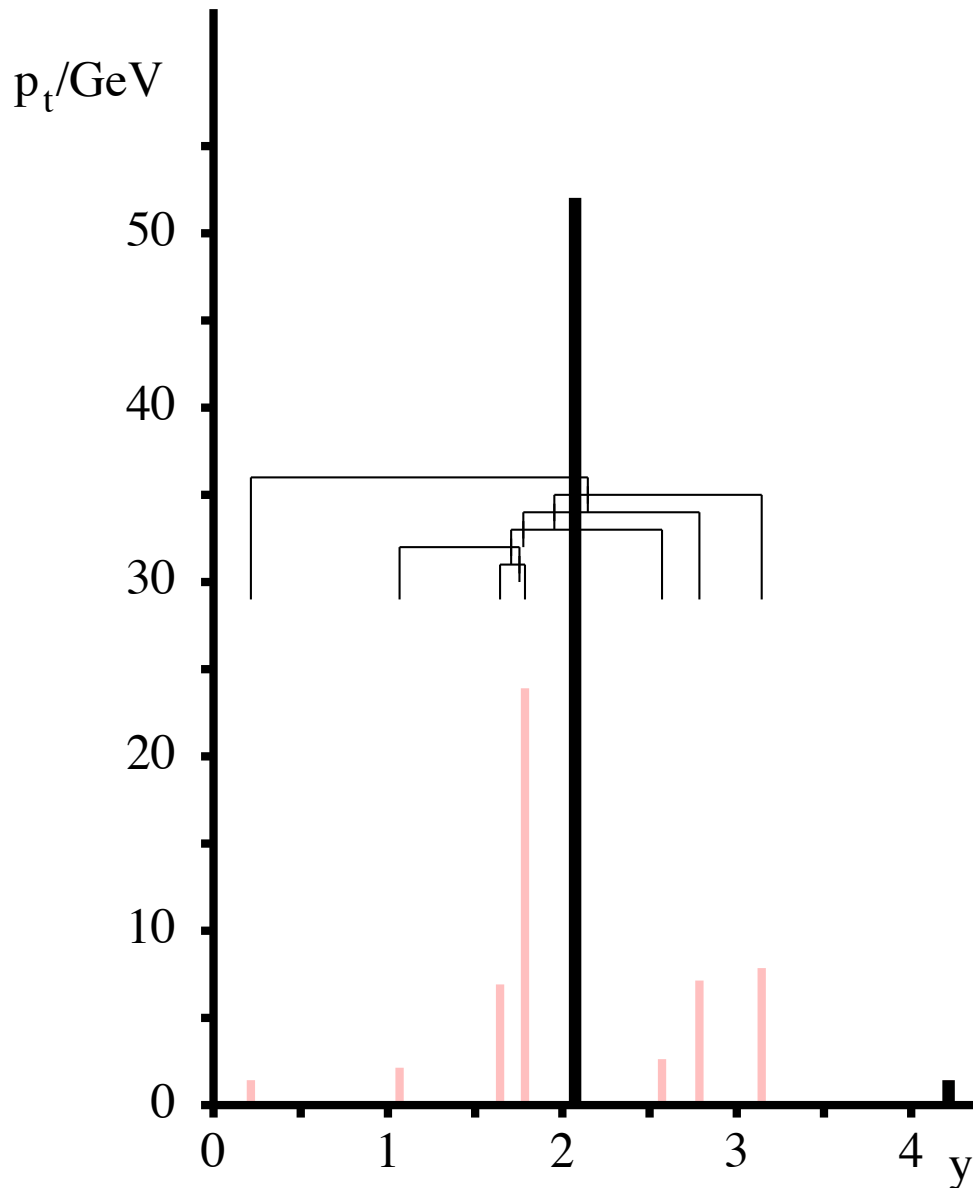
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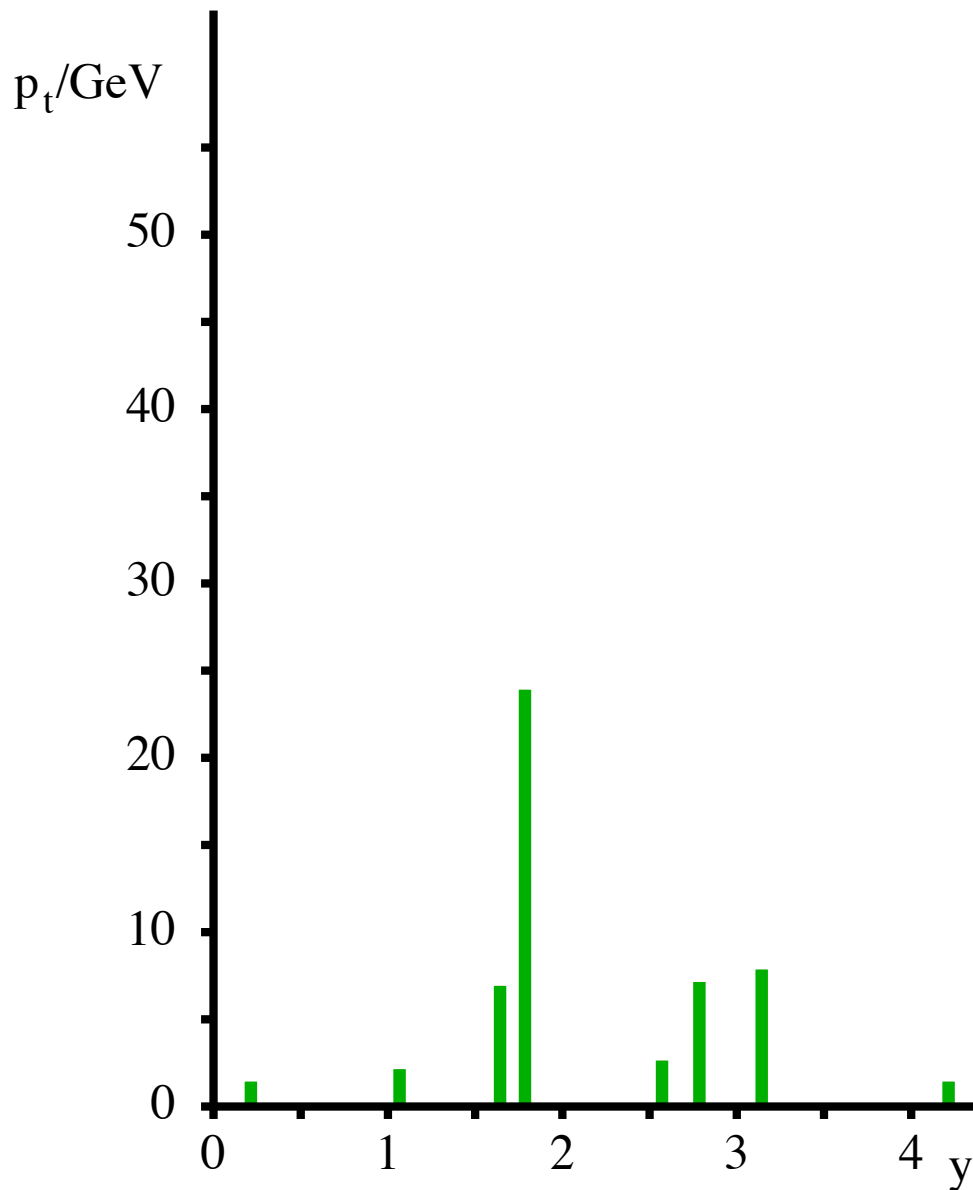
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kt



# Identifying jet substructure: try out $k_t$

## $k_t$ algorithm



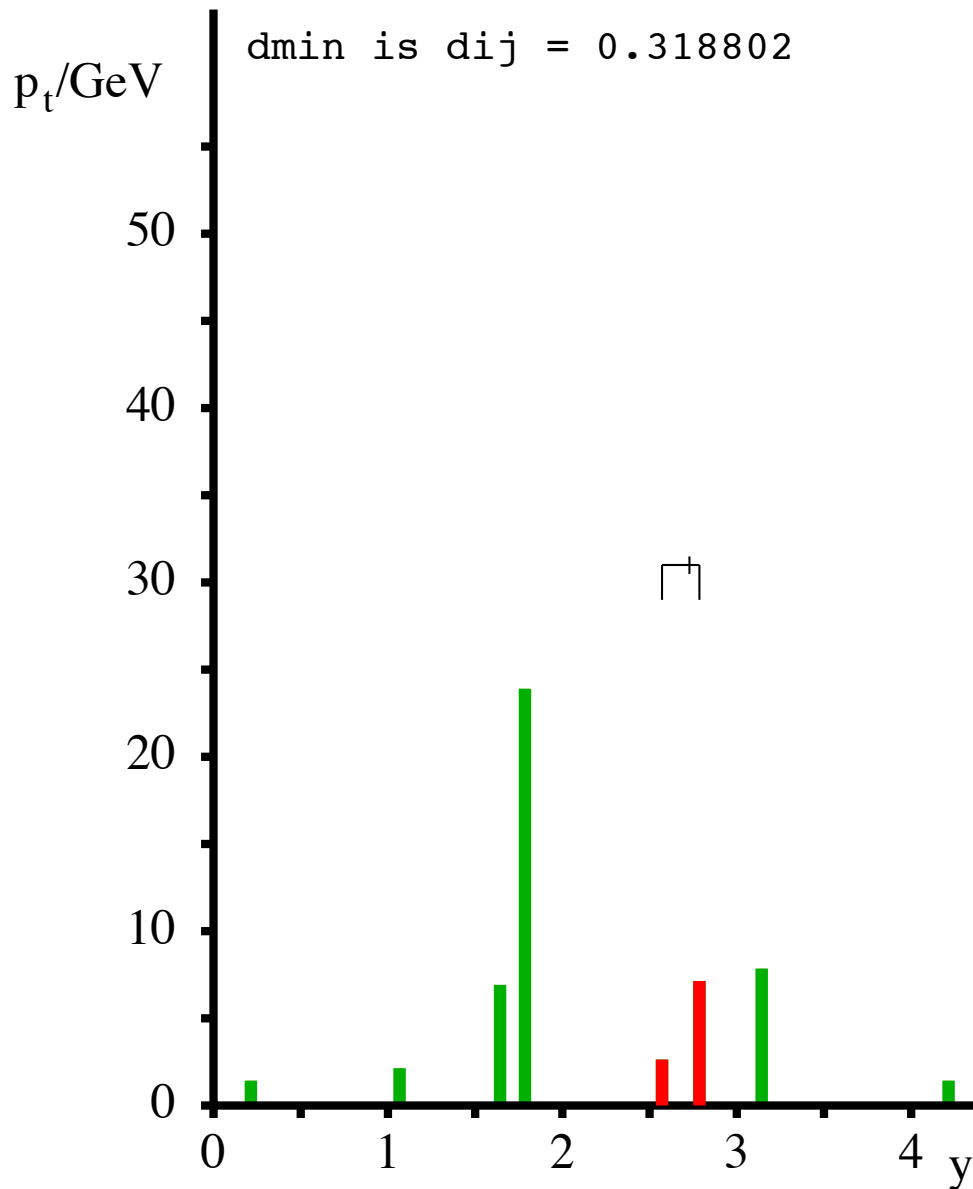
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# Identifying jet substructure: try out $k_t$

## $k_t$ algorithm

$d_{\min}$  is  $d_{ij} = 0.318802$

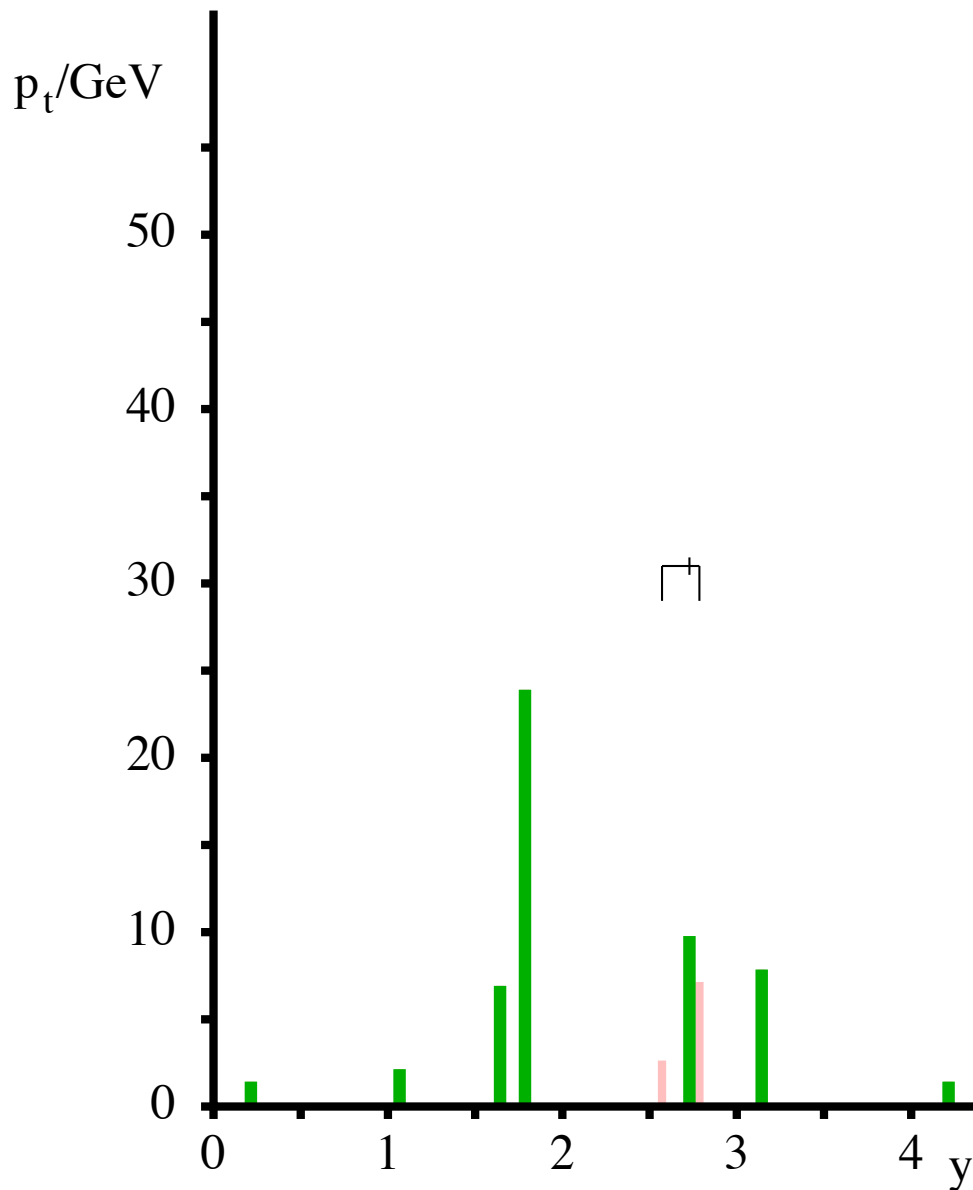


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# Identifying jet substructure: try out $k_t$

## $k_t$ algorithm



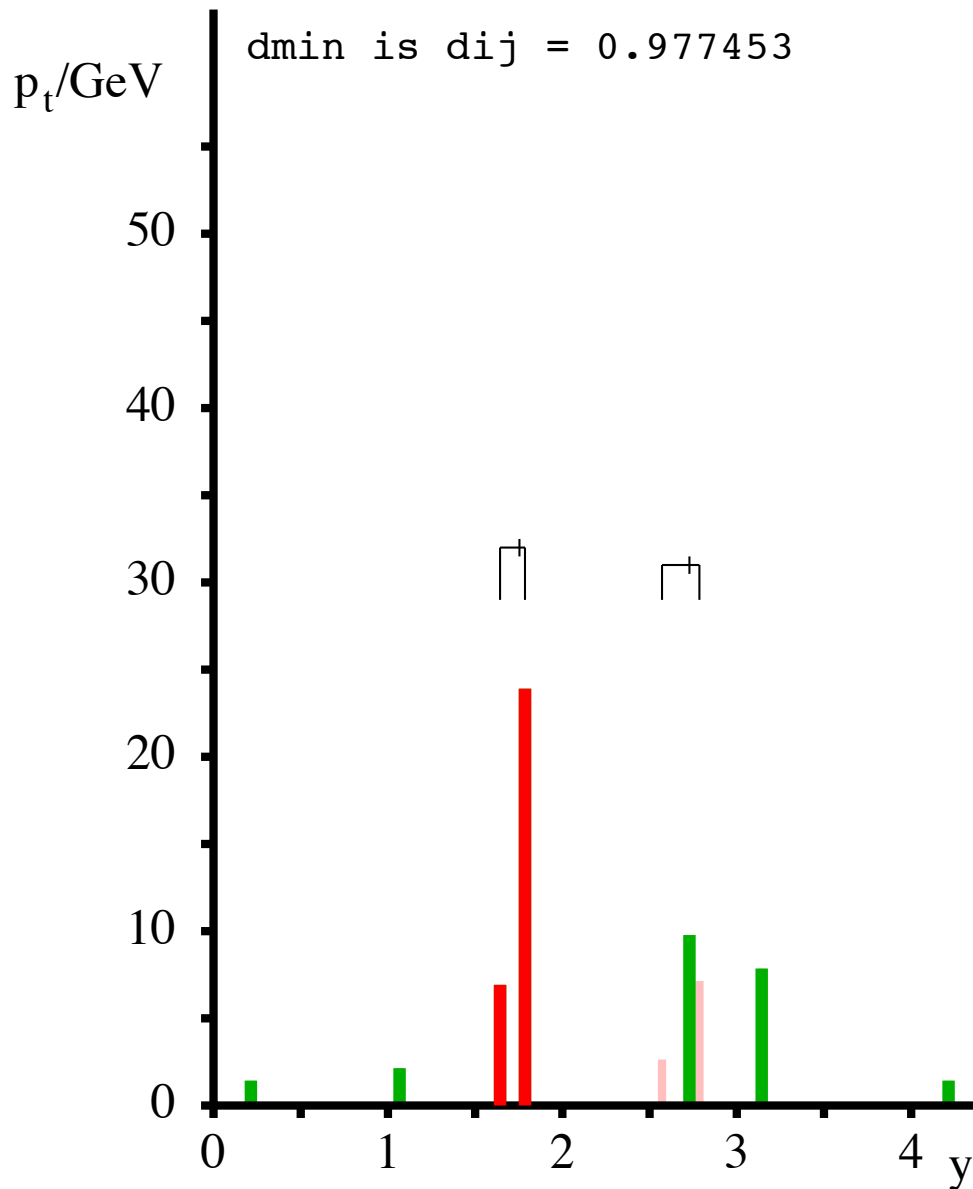
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# Identifying jet substructure: try out $k_t$

## $k_t$ algorithm

$d_{\min}$  is  $d_{ij} = 0.977453$

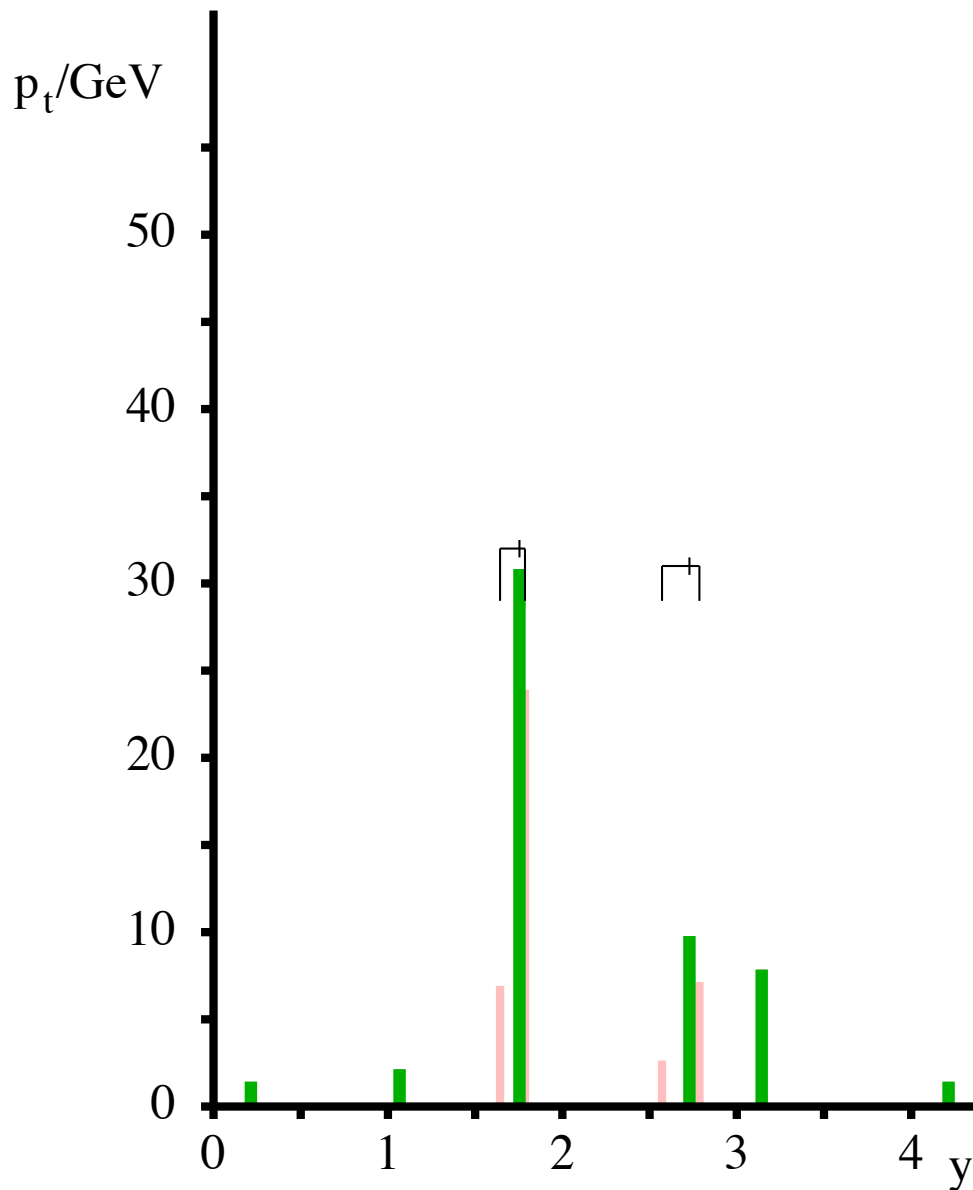


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## $k_t$ algorithm



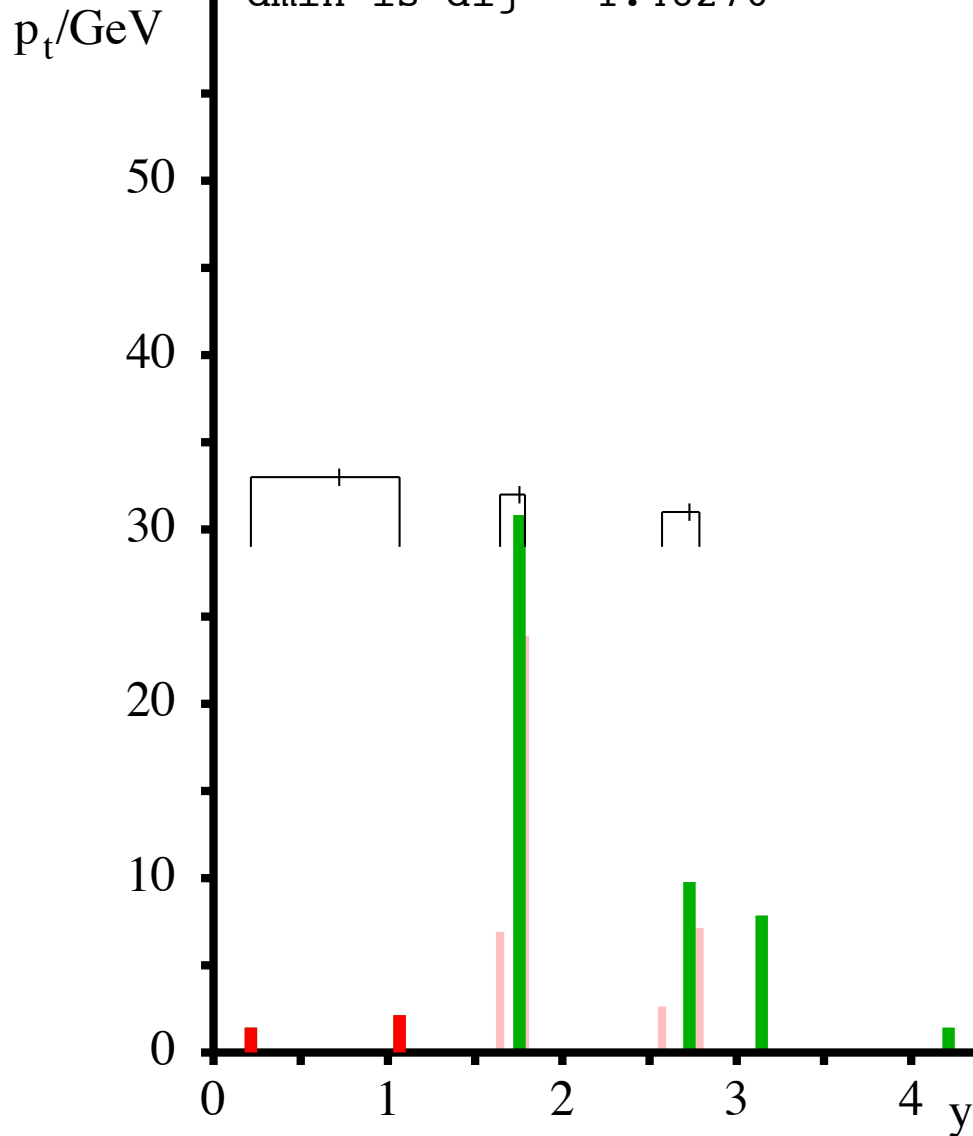
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# Identifying jet substructure: try out $k_t$

## $k_t$ algorithm

$d_{\min}$  is  $d_{ij} = 1.48276$



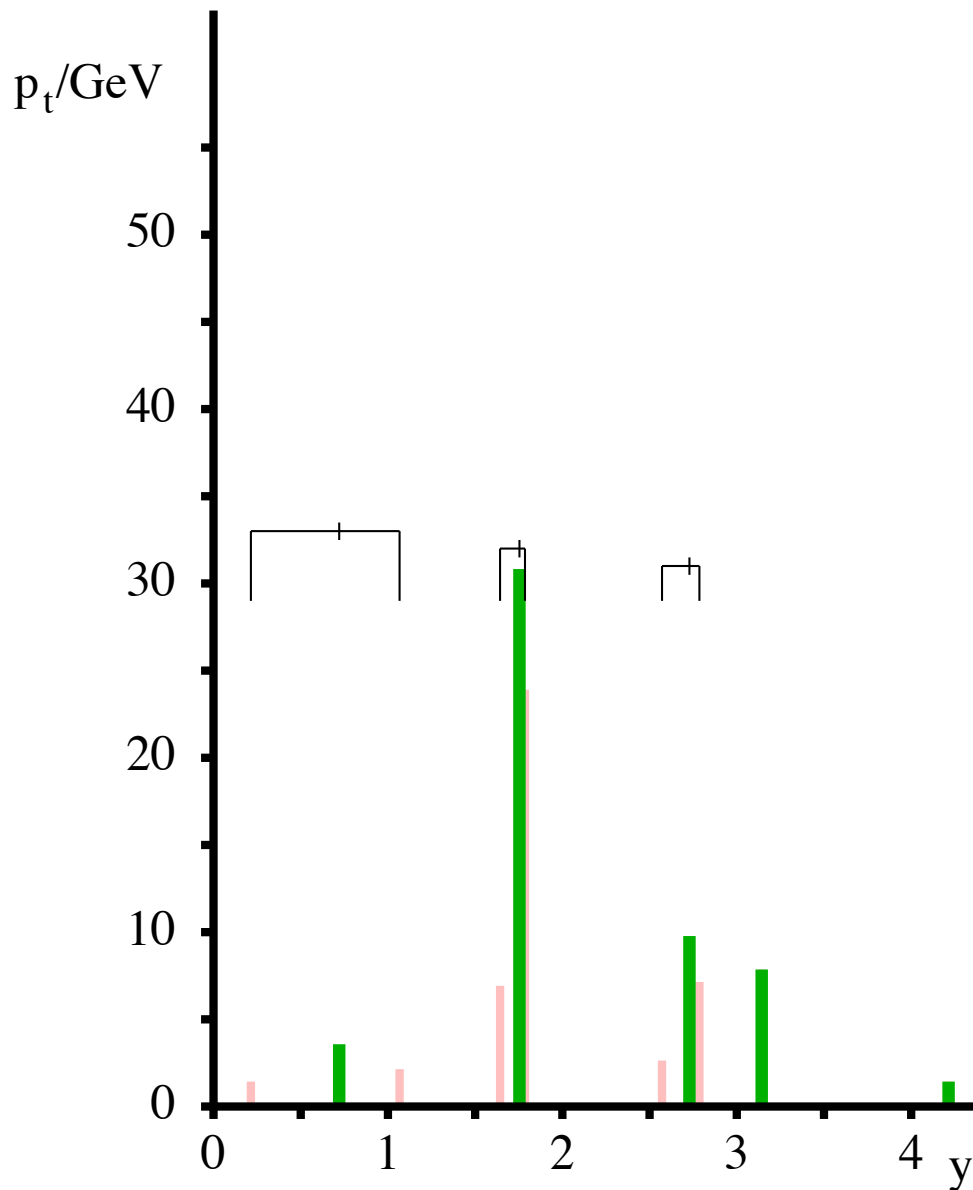
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$k_t$  clusters soft “junk” early on in the clustering

# Identifying jet substructure: try out $k_t$

## $k_t$ algorithm



How well can an algorithm identify the “blobs” of energy inside a jet that come from different partons?

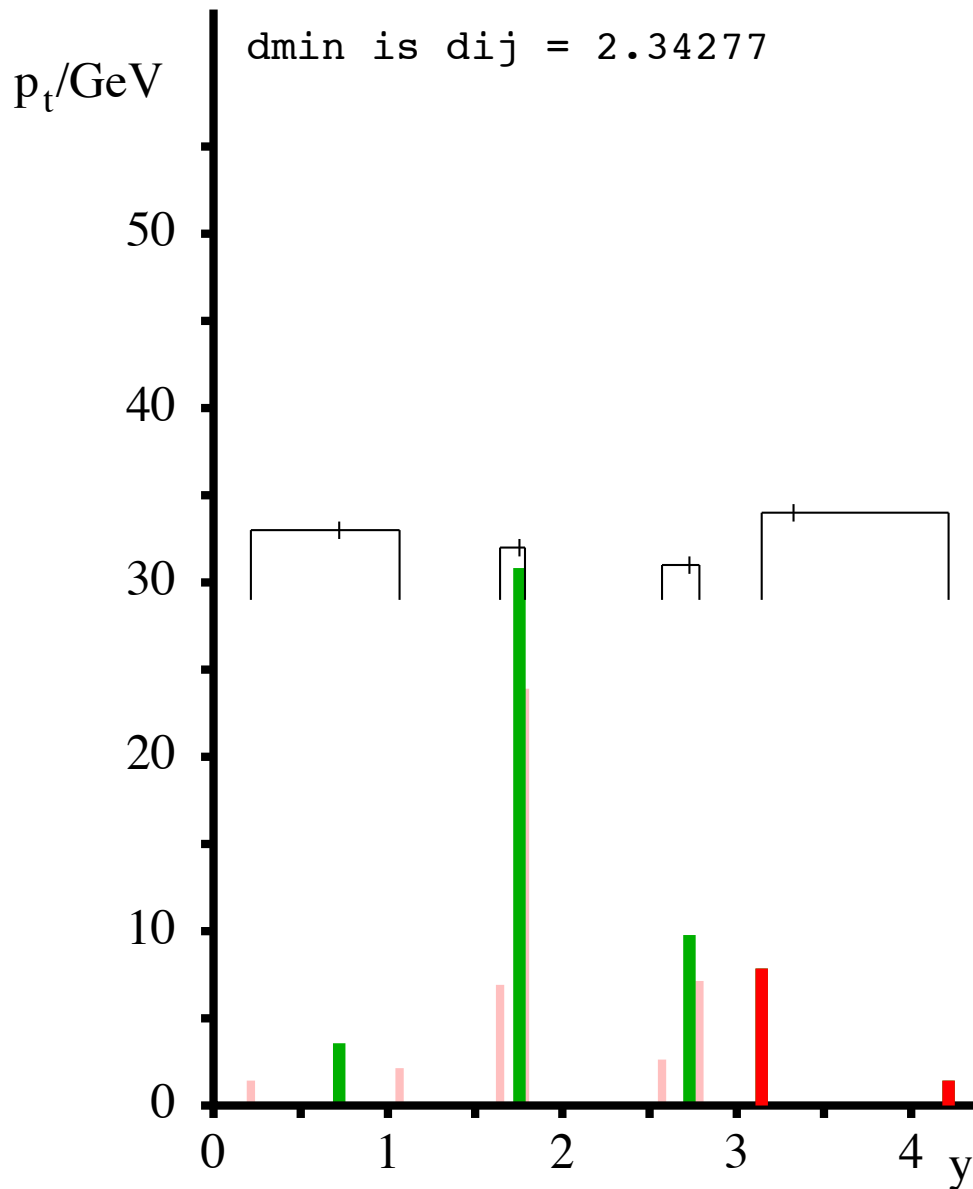
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# Identifying jet substructure: try out $k_t$

## $k_t$ algorithm

$d_{\min}$  is  $d_{ij} = 2.34277$



How well can an algorithm identify the “blobs” of energy inside a jet that come from different partons?

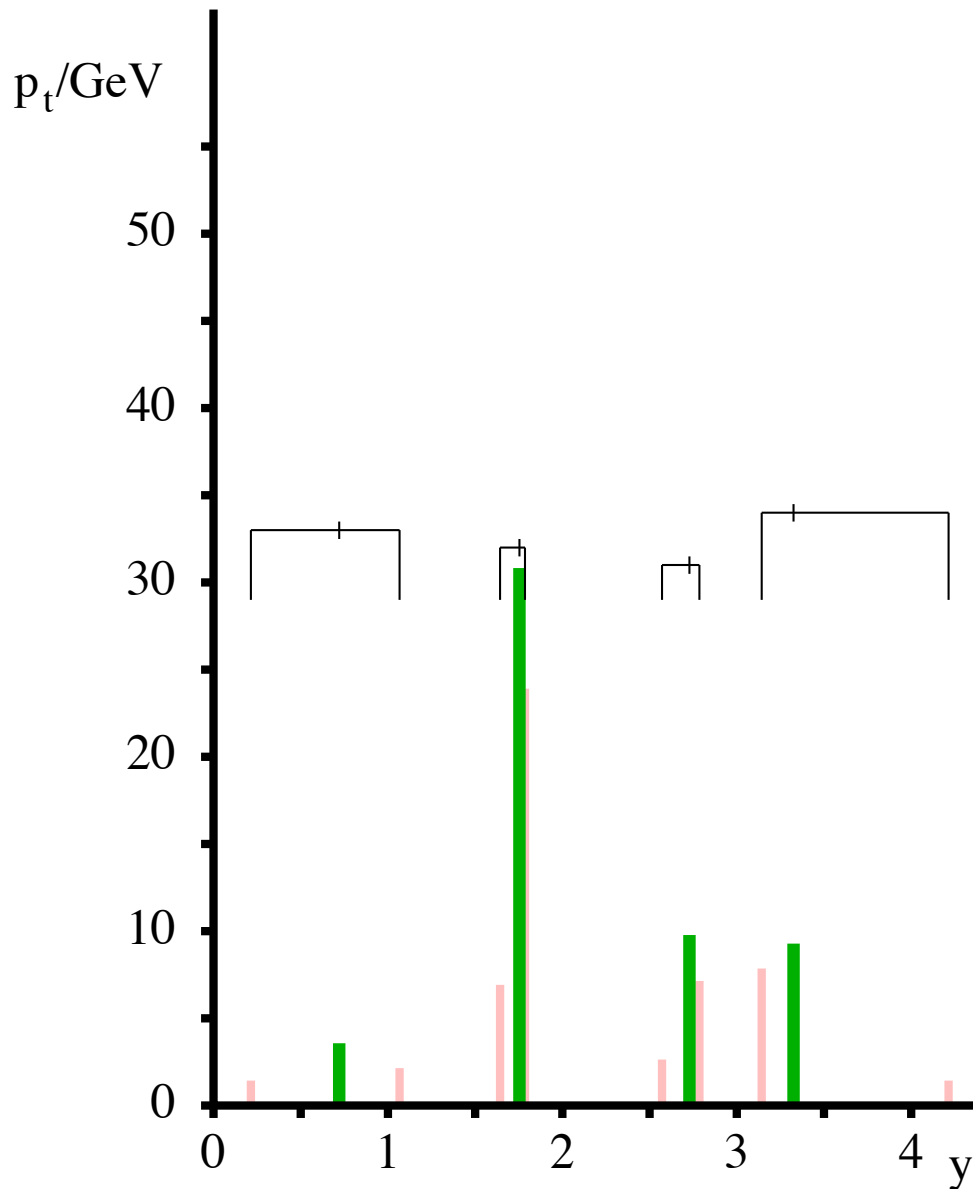
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# Identifying jet substructure: try out $k_t$

## $k_t$ algorithm



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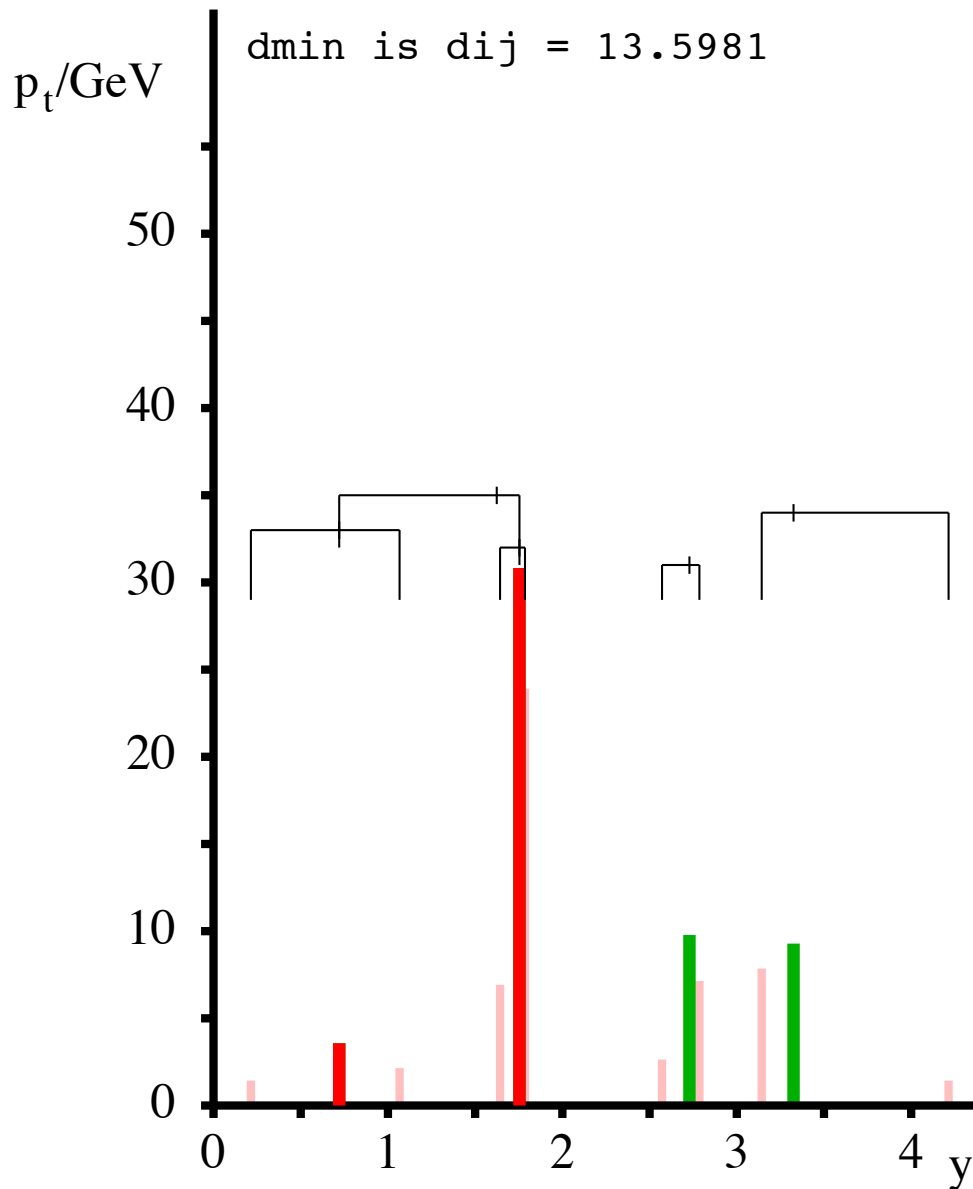
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# Identifying jet substructure: try out $k_t$

## $k_t$ algorithm

$d_{\min}$  is  $d_{ij} = 13.5981$



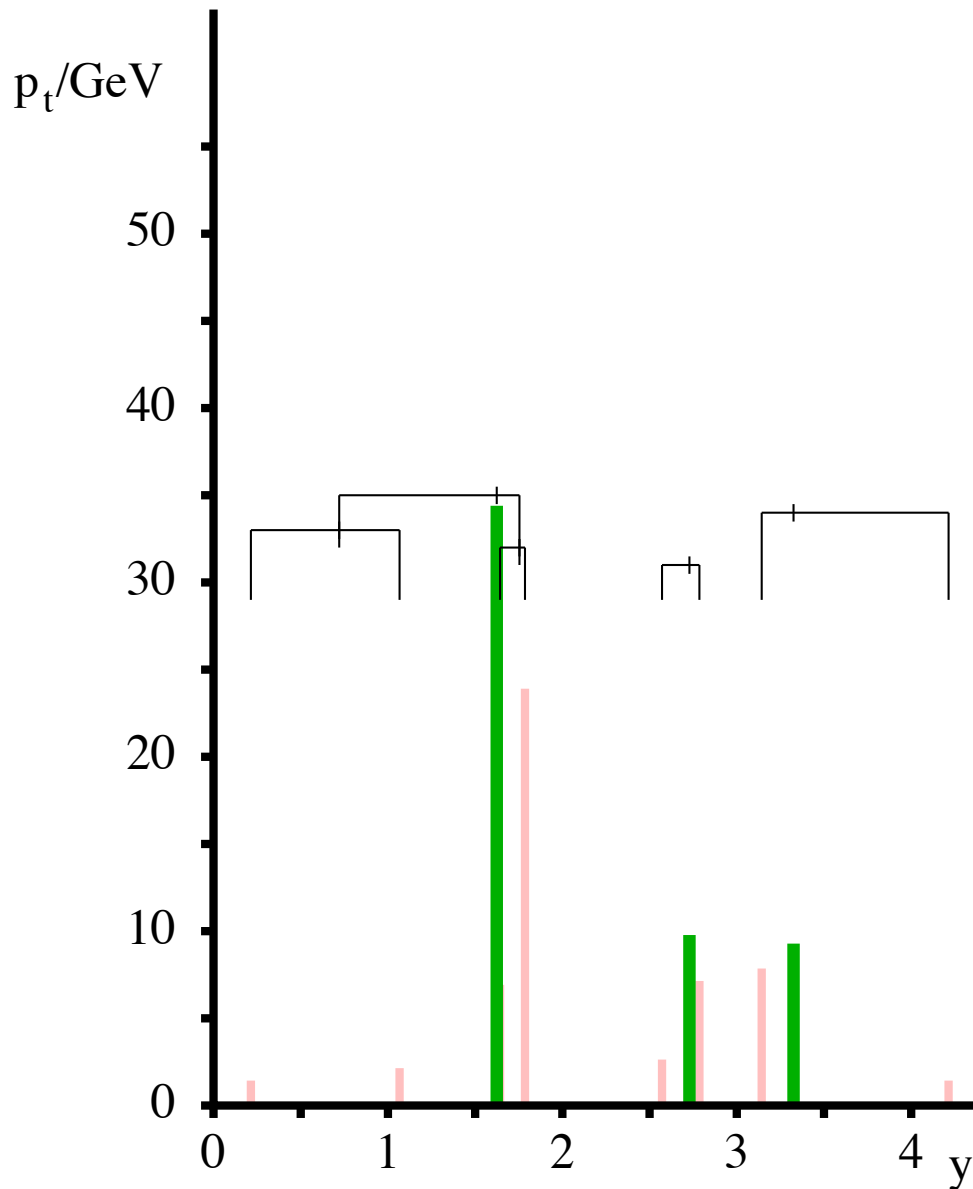
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## $k_t$ algorithm



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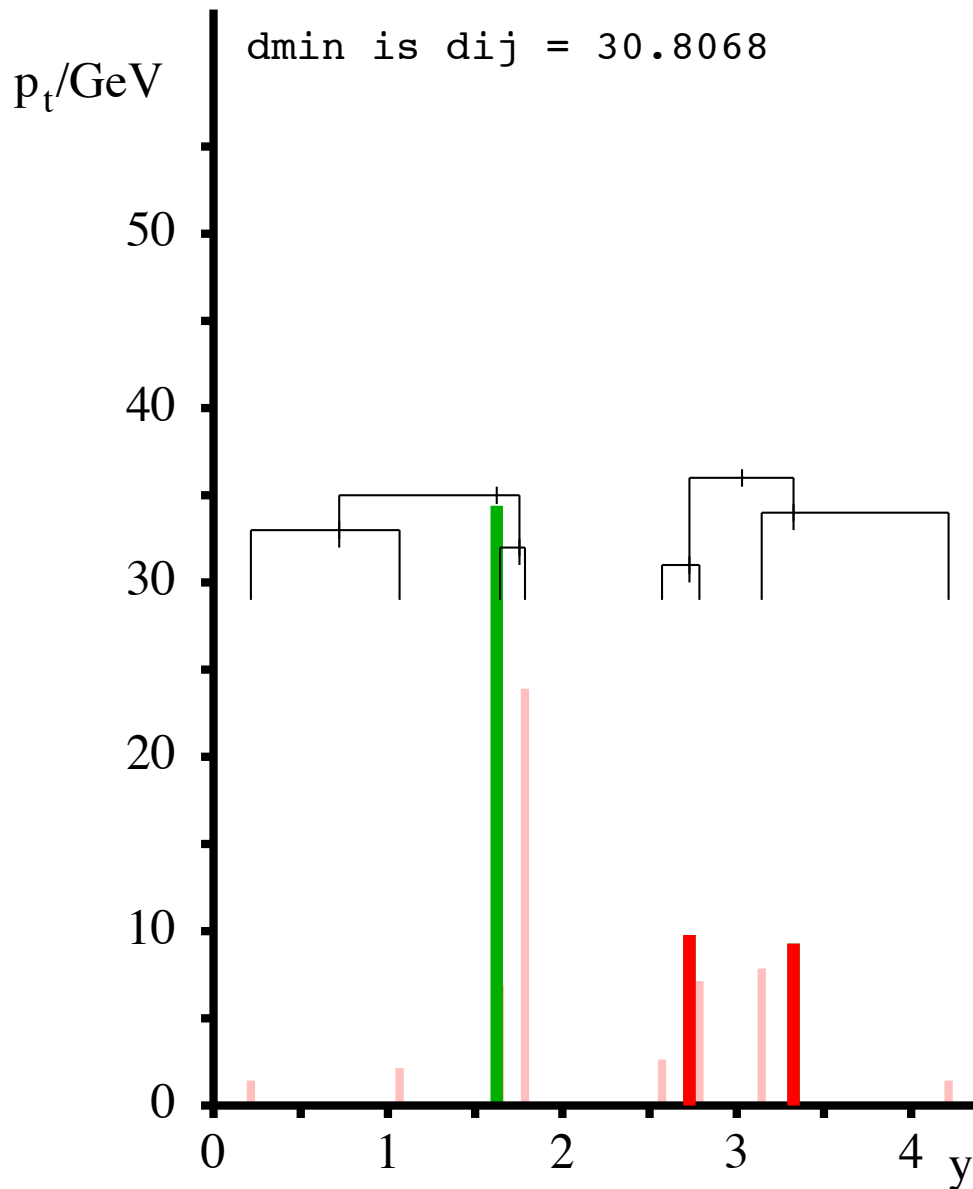
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# Identifying jet substructure: try out $k_t$

## $k_t$ algorithm

$d_{\min}$  is  $d_{ij} = 30.8068$



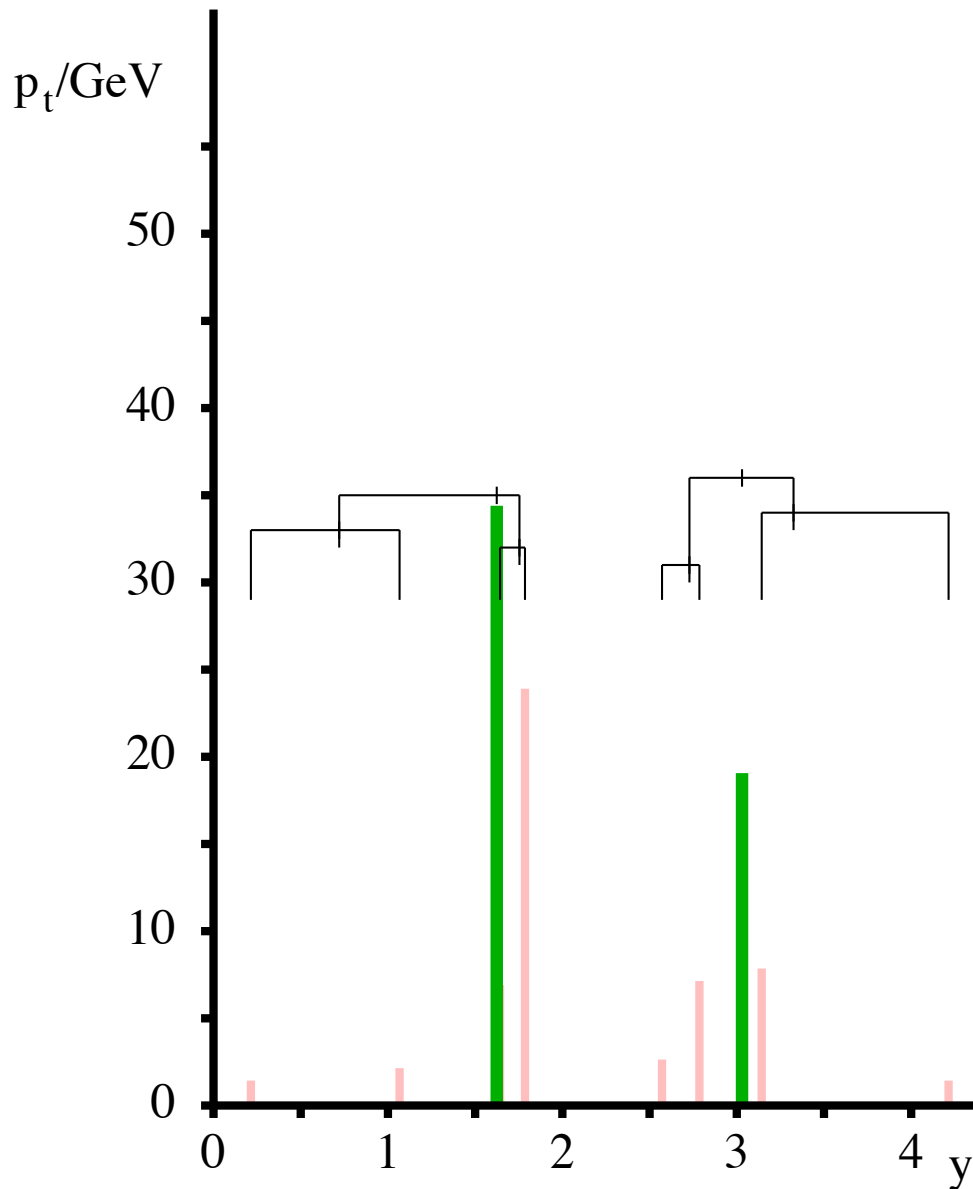
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## $k_t$ algorithm



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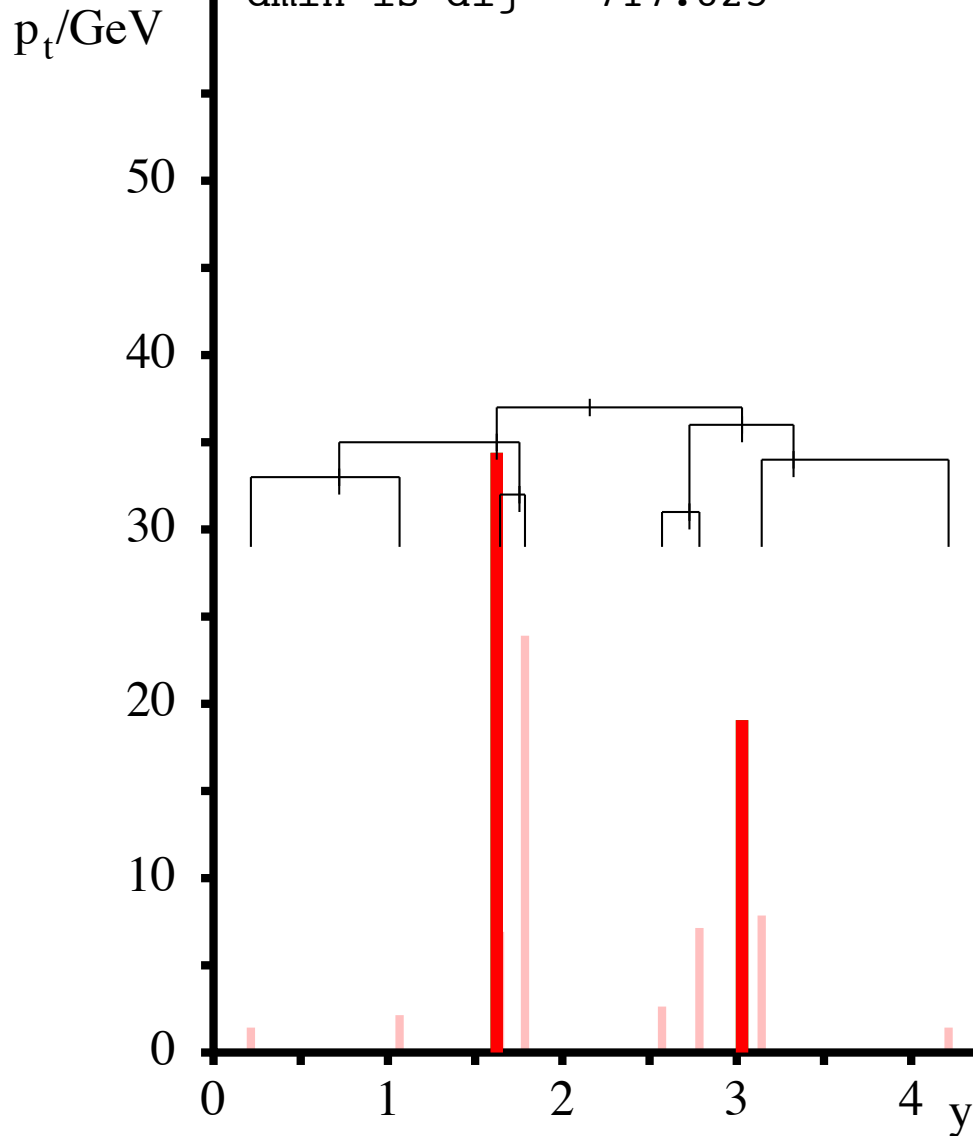
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$k_t$  clusters soft “junk” early on in the clustering

# Identifying jet substructure: try out $k_t$

## $k_t$ algorithm

$d_{\min}$  is  $d_{ij} = 717.825$



How well can an algorithm identify the “blobs” of energy inside a jet that come from different partons?

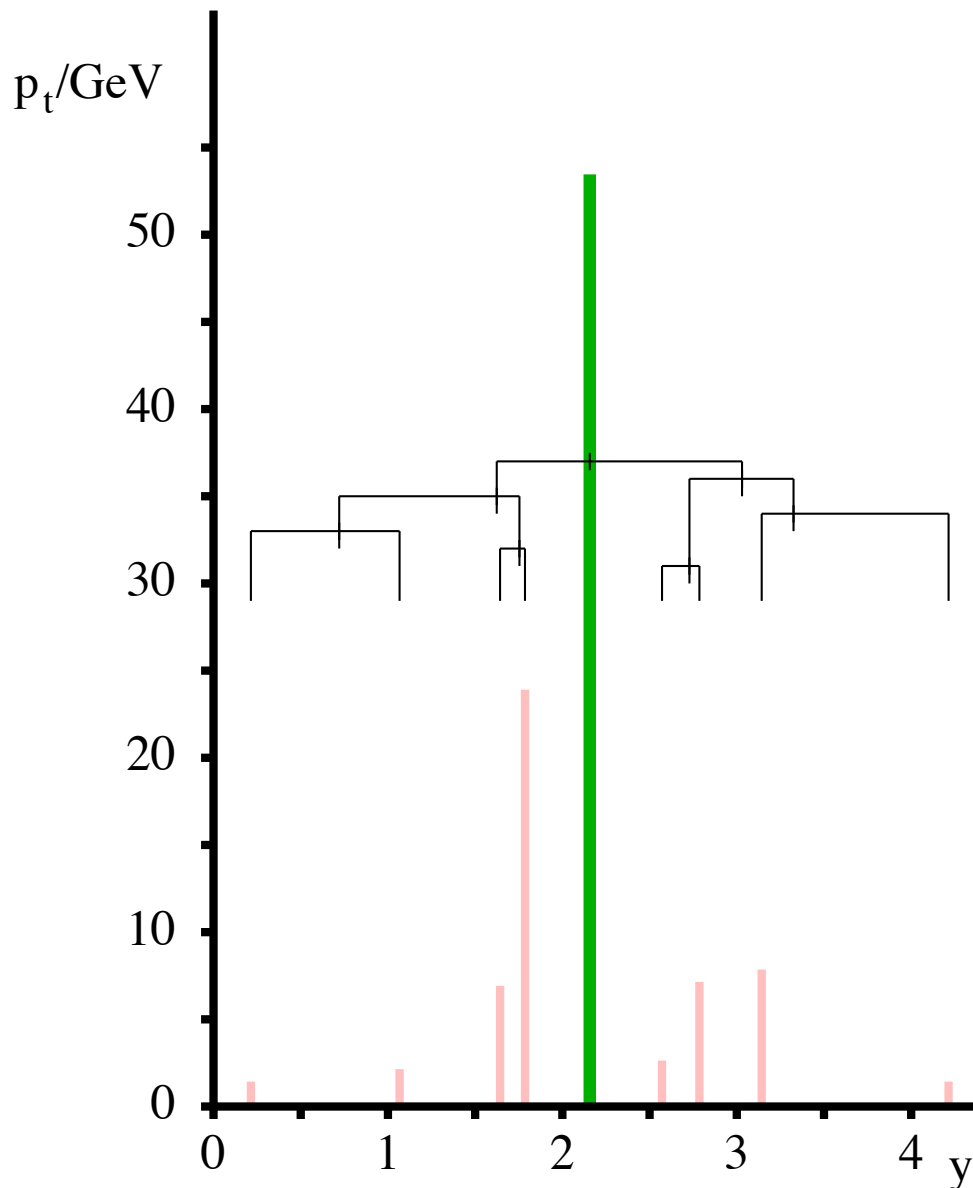
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$k_t$  clusters soft “junk” early on in the clustering

*Its last step is to merge two hard pieces. Easily undone to identify underlying kinematics*

# Identifying jet substructure: try out $k_t$

## $k_t$ algorithm



How well can an algorithm identify the “blobs” of energy inside a jet that come from different partons?

This is crucial for identifying the kinematic variables of the partons in the jet (e.g.  $z$ ).

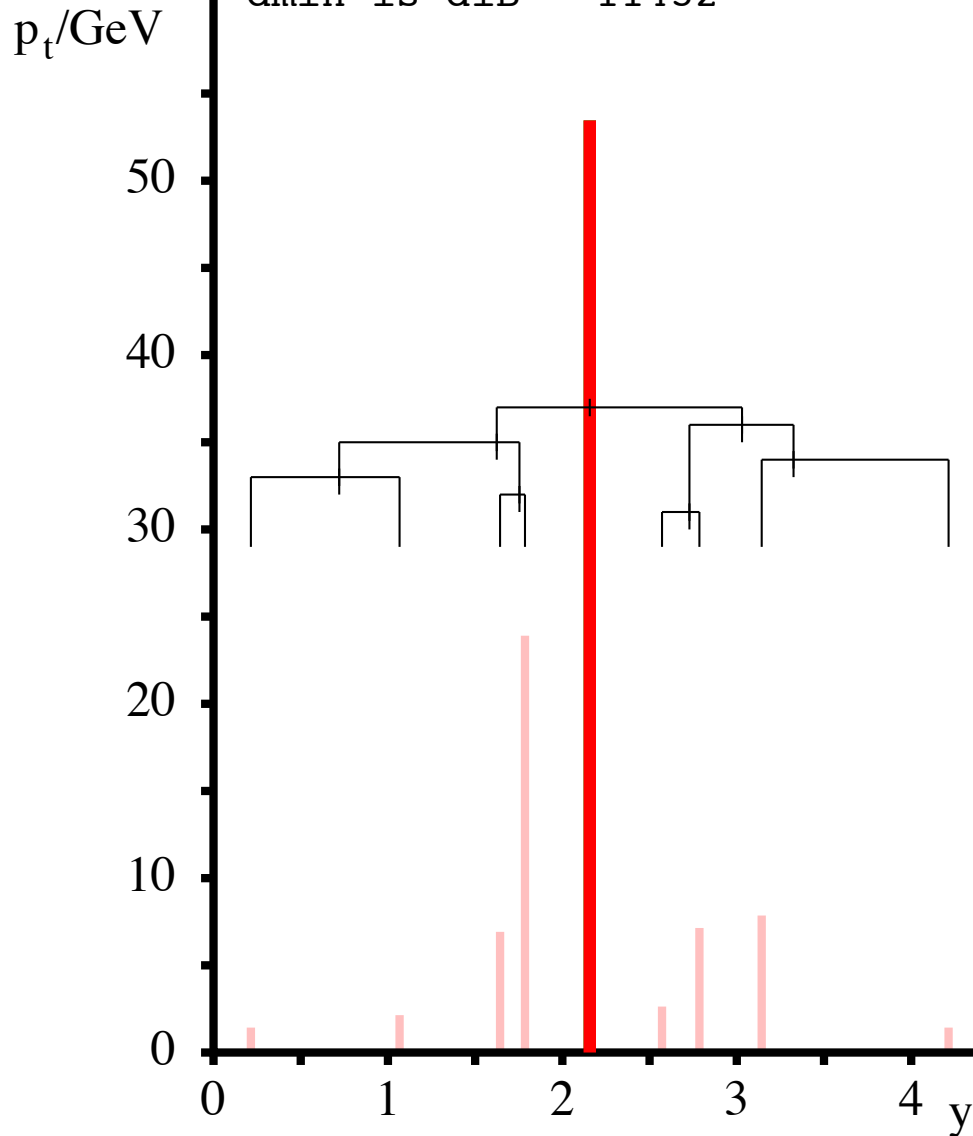
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# Identifying jet substructure: try out $k_t$

## $k_t$ algorithm

$d_{\min}$  is  $d_{iB} = 11432$



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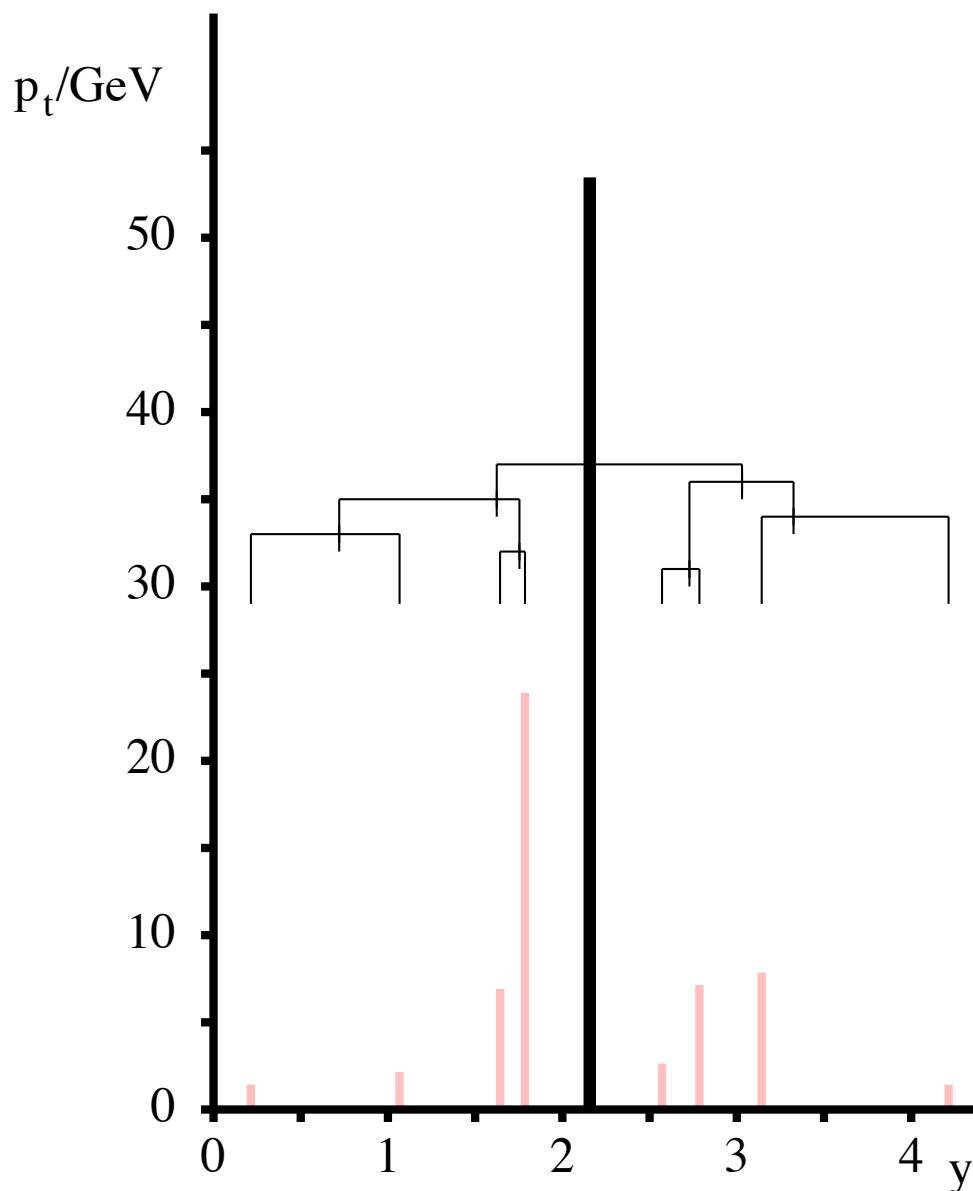
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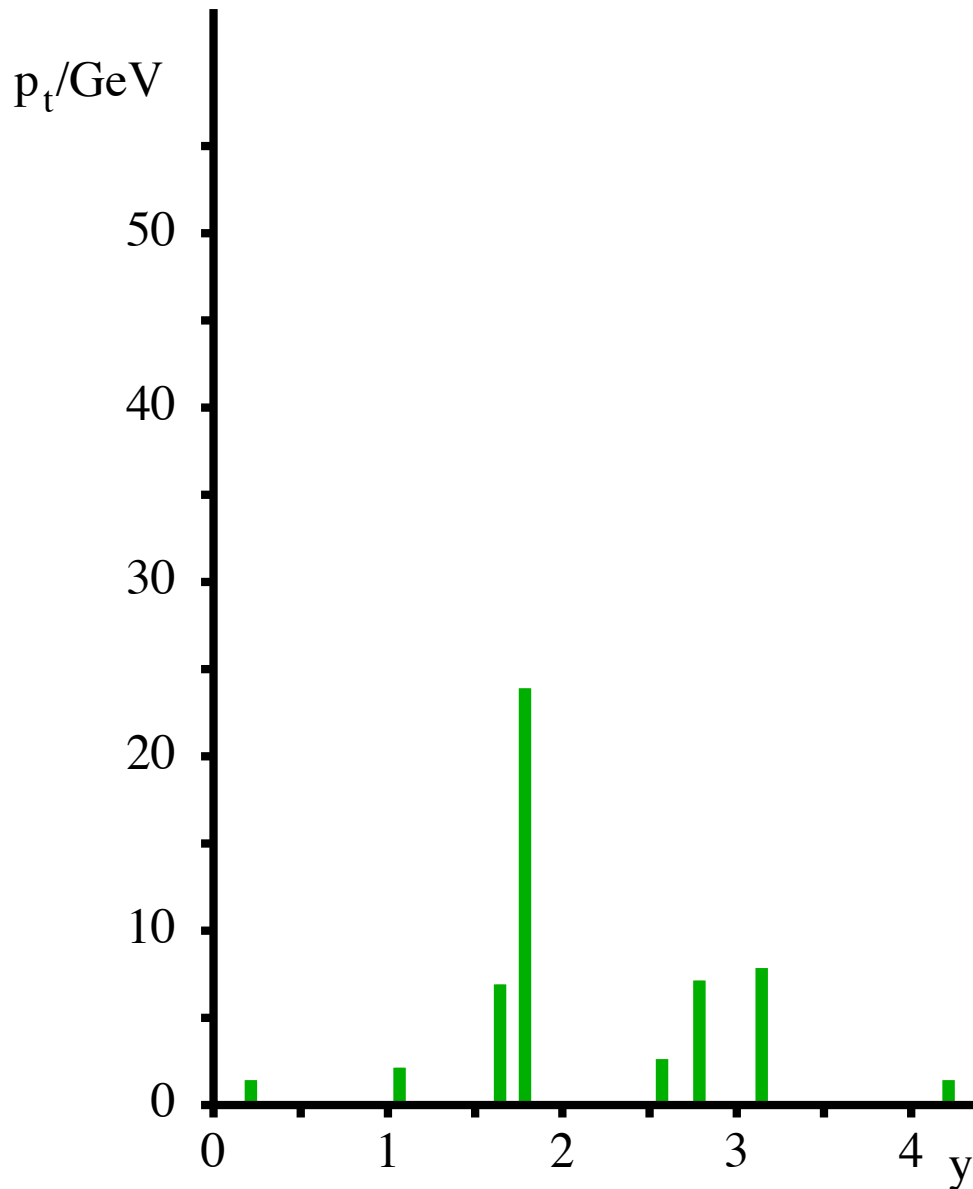
This meant it was the first algorithm to be used for jet substructure.

Seymour '93

Butterworth, Cox & Forshaw '02

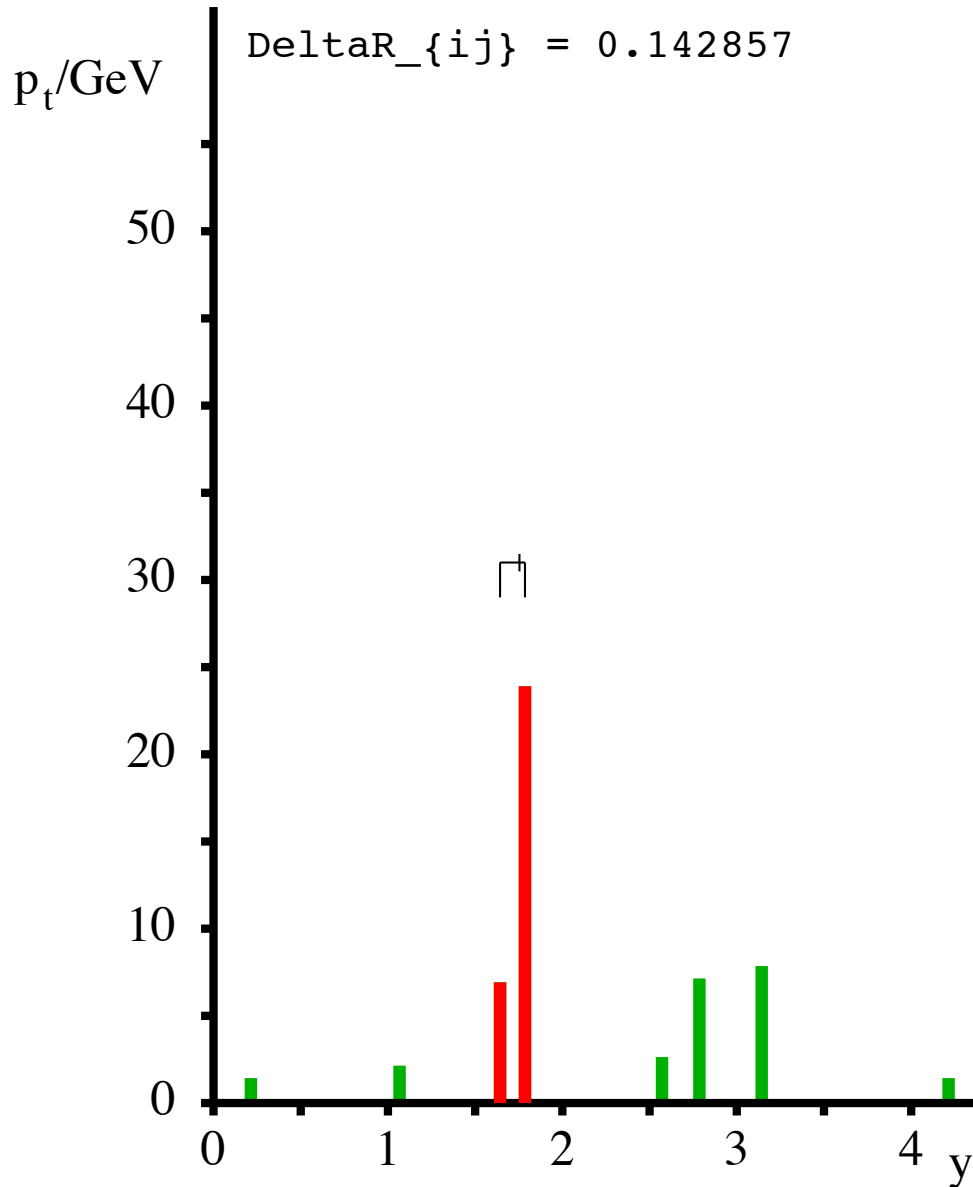
## Cambridge/Aachen

## Cambridge/Aachen algorithm



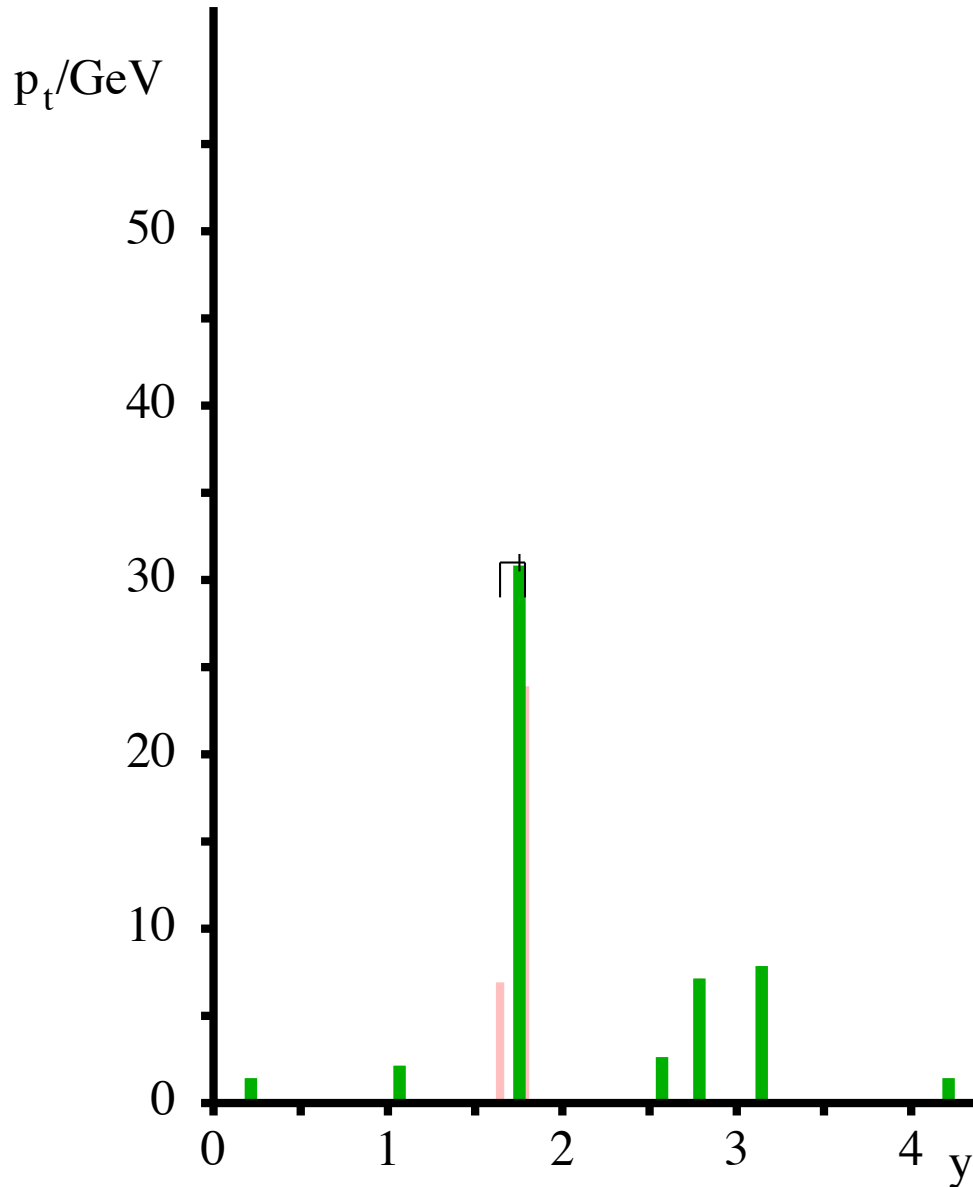
How well can an algorithm identify the “blobs” of energy inside a jet that come from different partons?

## Cambridge/Aachen algorithm



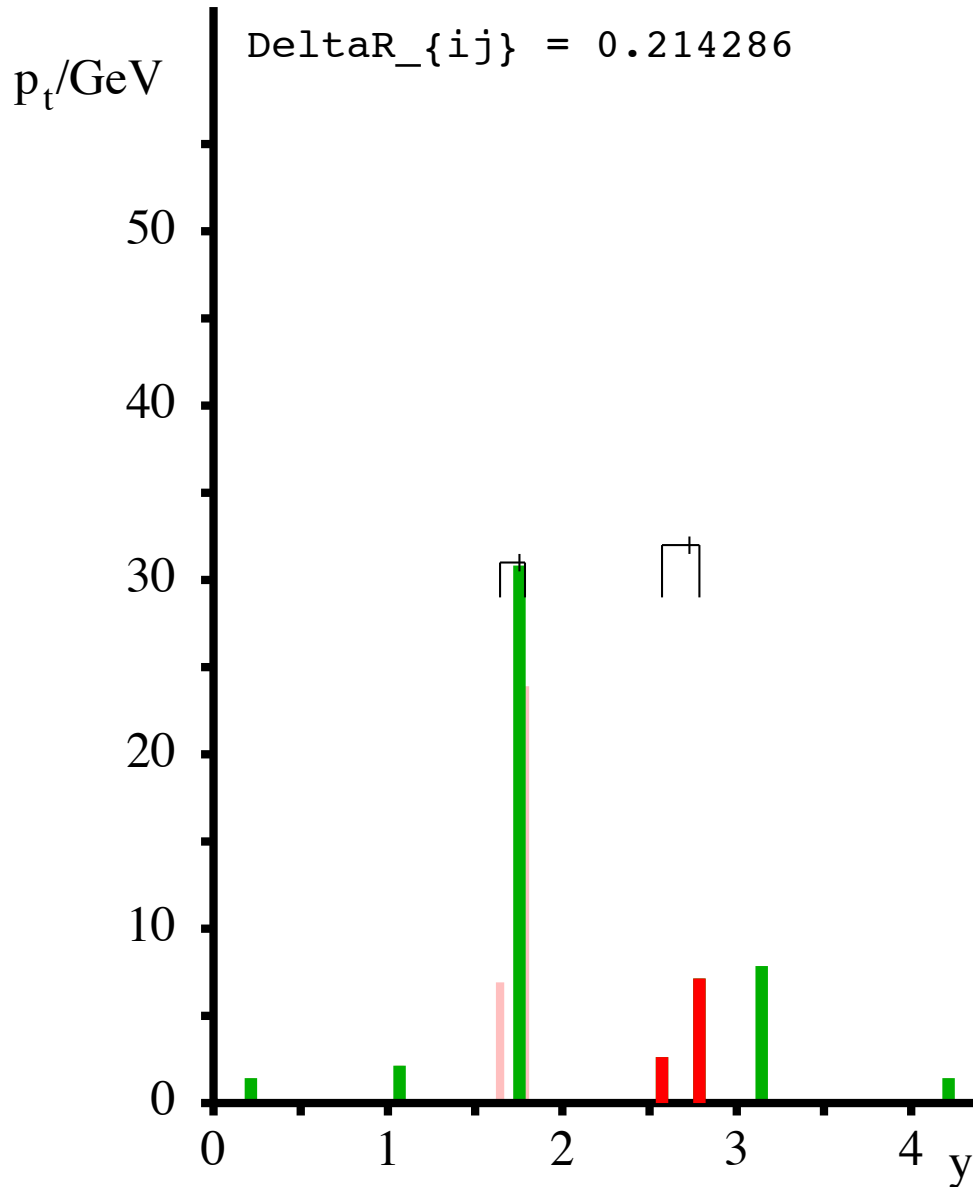
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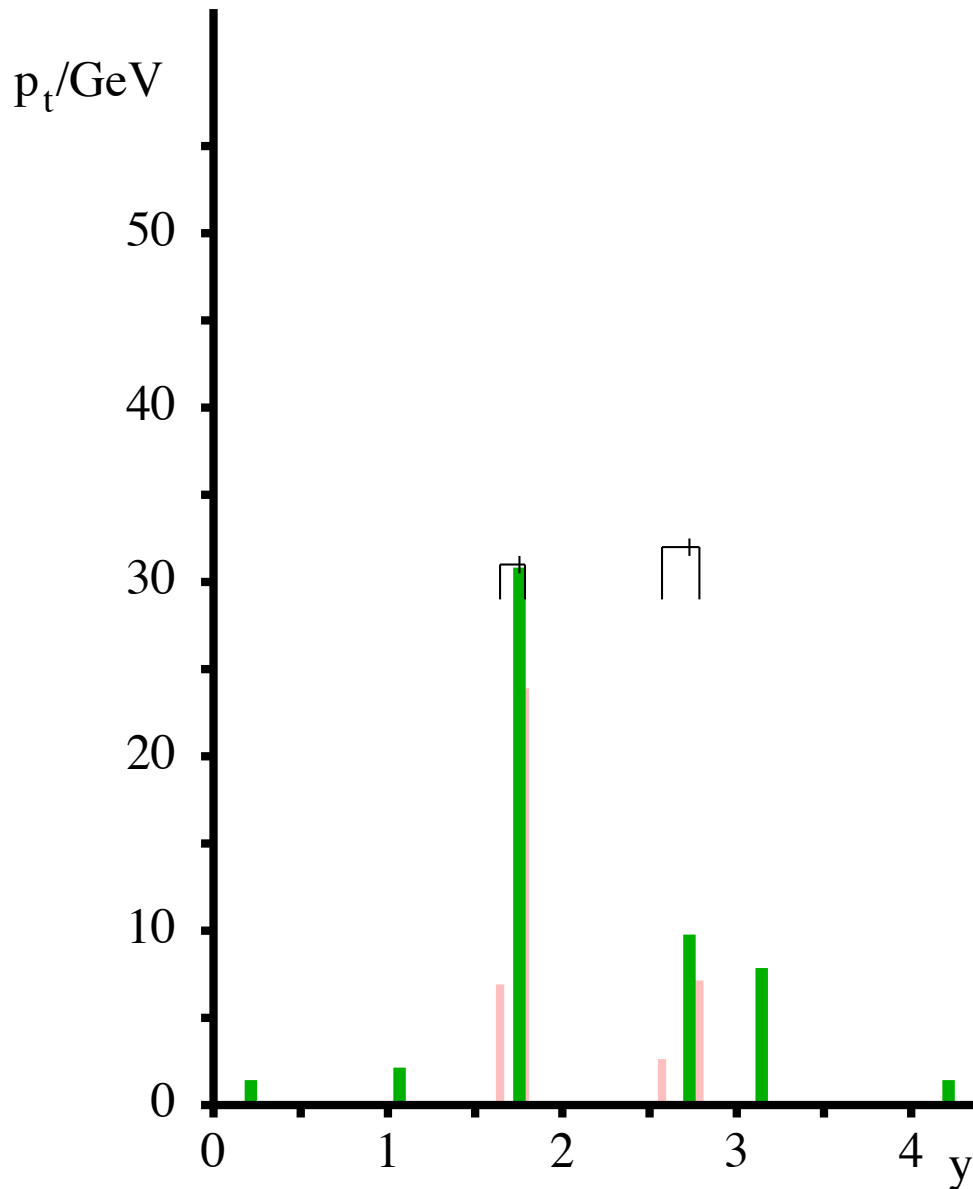
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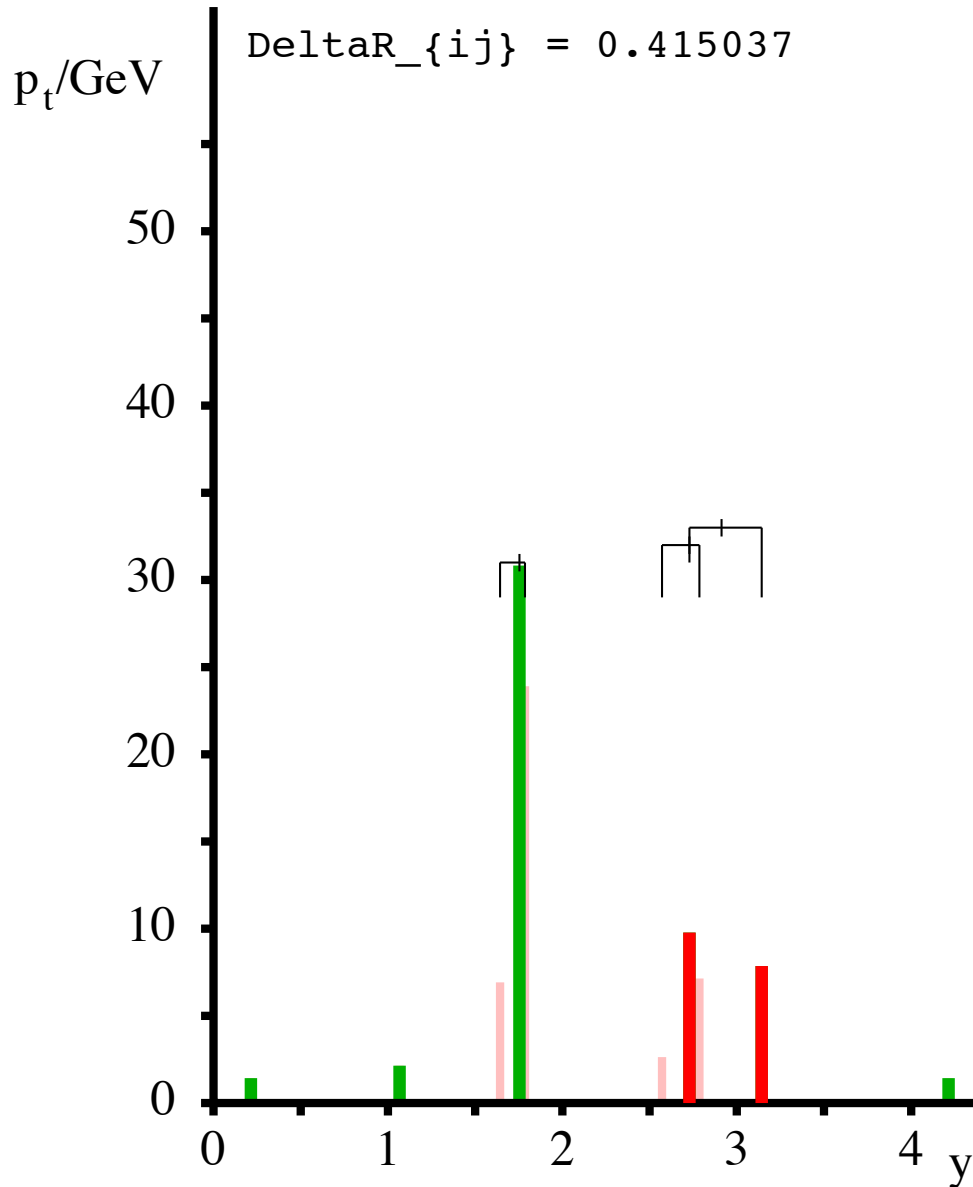
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## Cambridge/Aachen algorithm

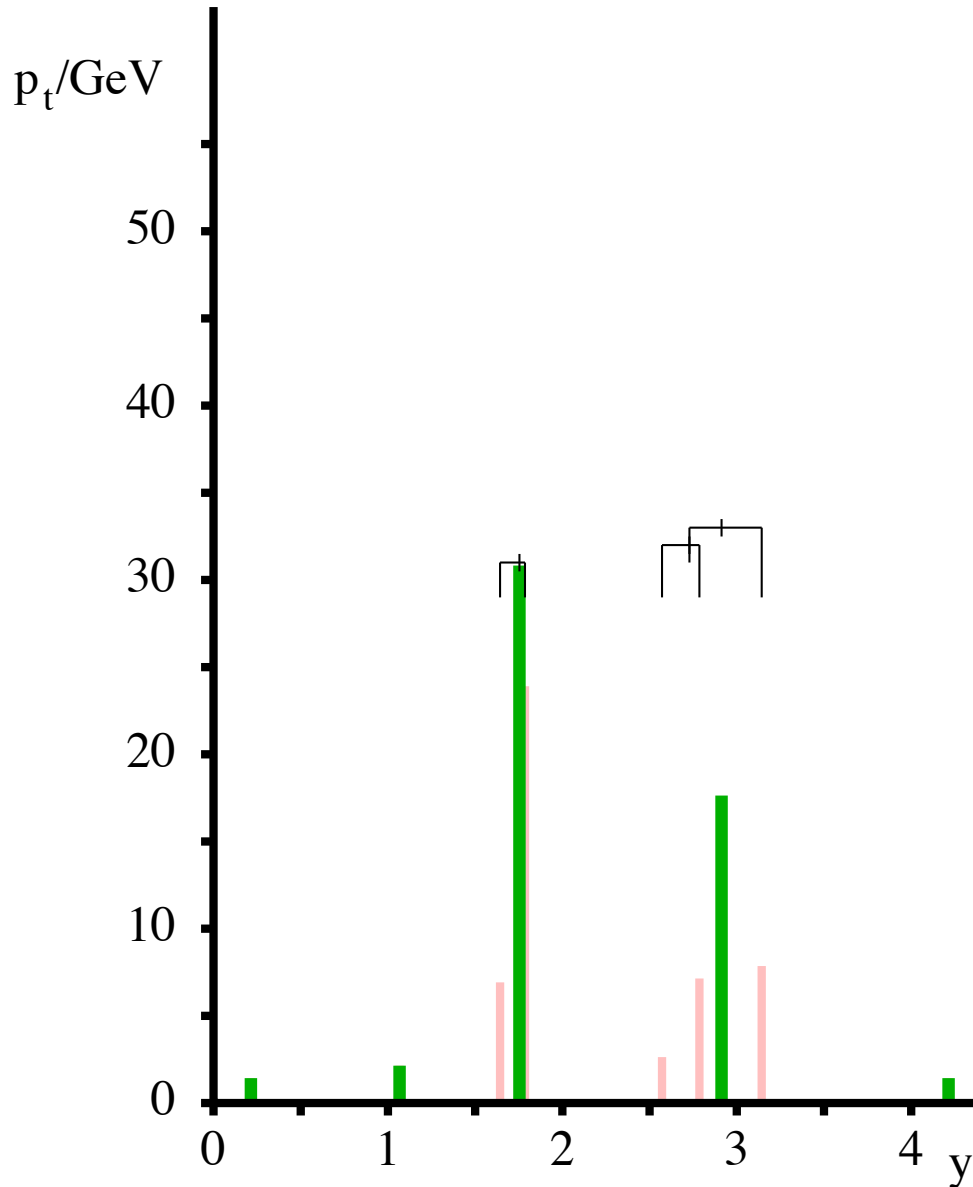


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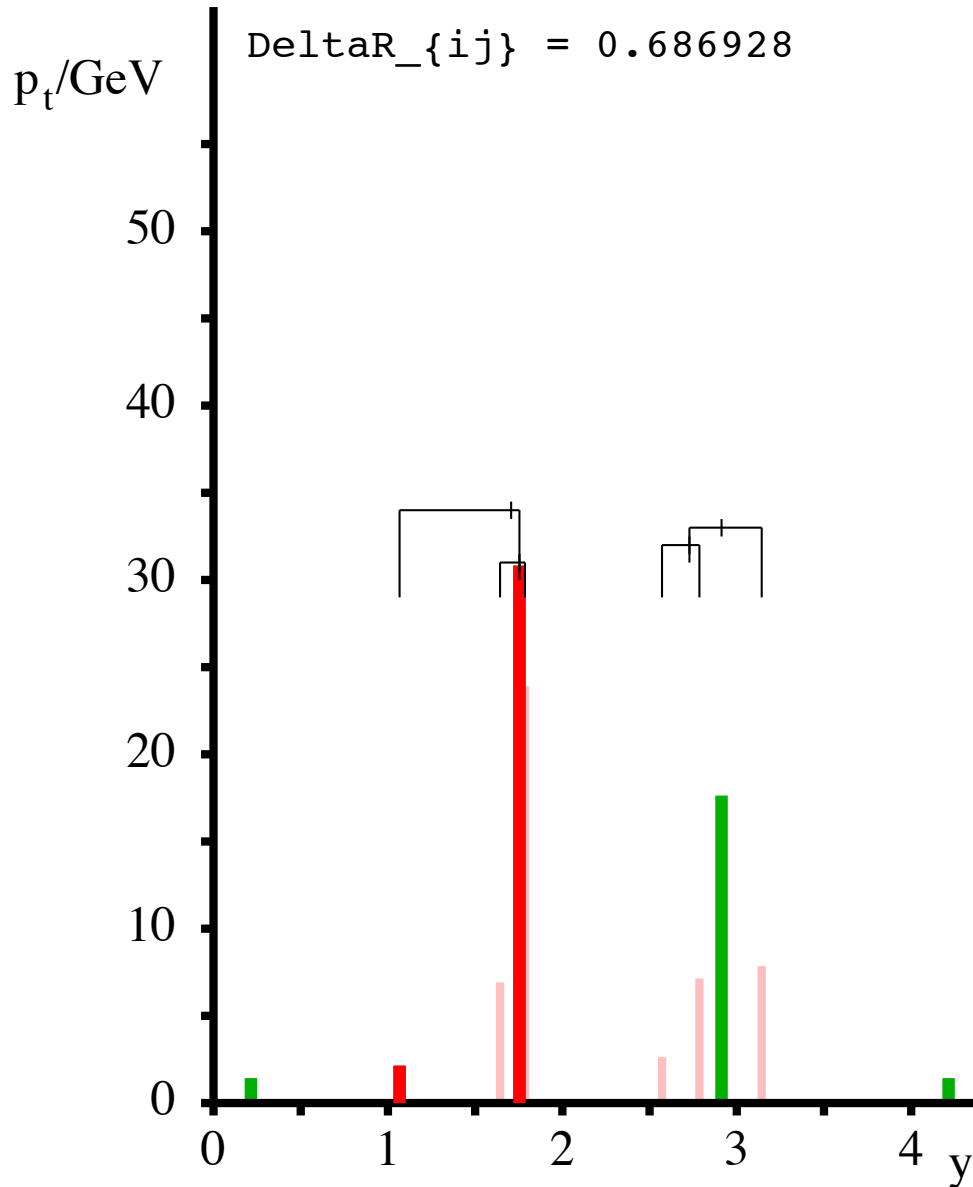
# Identifying jet substructure: Cam/Aachen

## Cambridge/Aachen algorithm



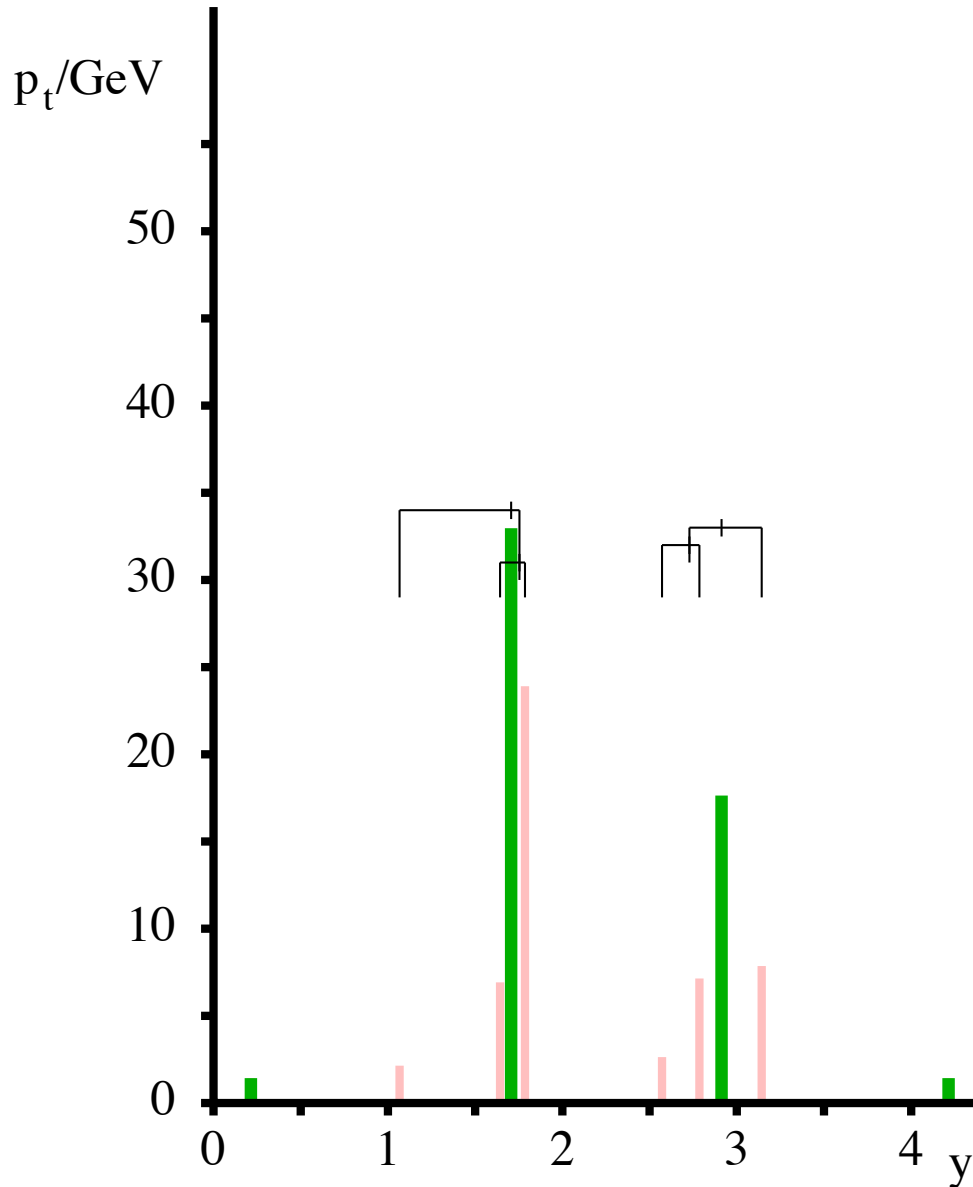
How well can an algorithm identify the “blobs” of energy inside a jet that come from different partons?

## Cambridge/Aachen algorithm



How well can an algorithm identify the “blobs” of energy inside a jet that come from different partons?

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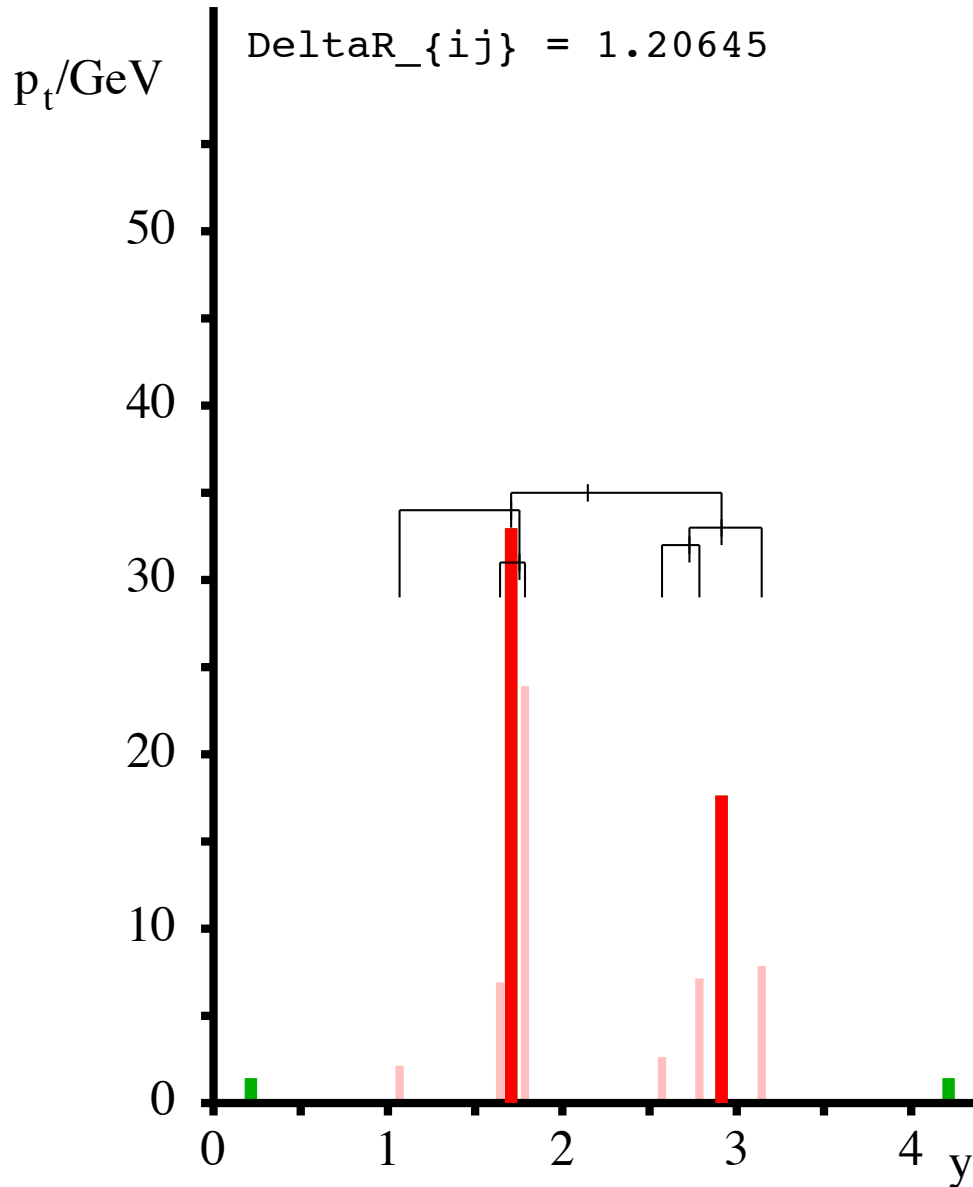


How well can an algorithm identify the “blobs” of energy inside a jet that come from different partons?

**C/A identifies two hard blobs with limited soft contamination**

# Identifying jet substructure: Cam/Aachen

## Cambridge/Aachen algorithm

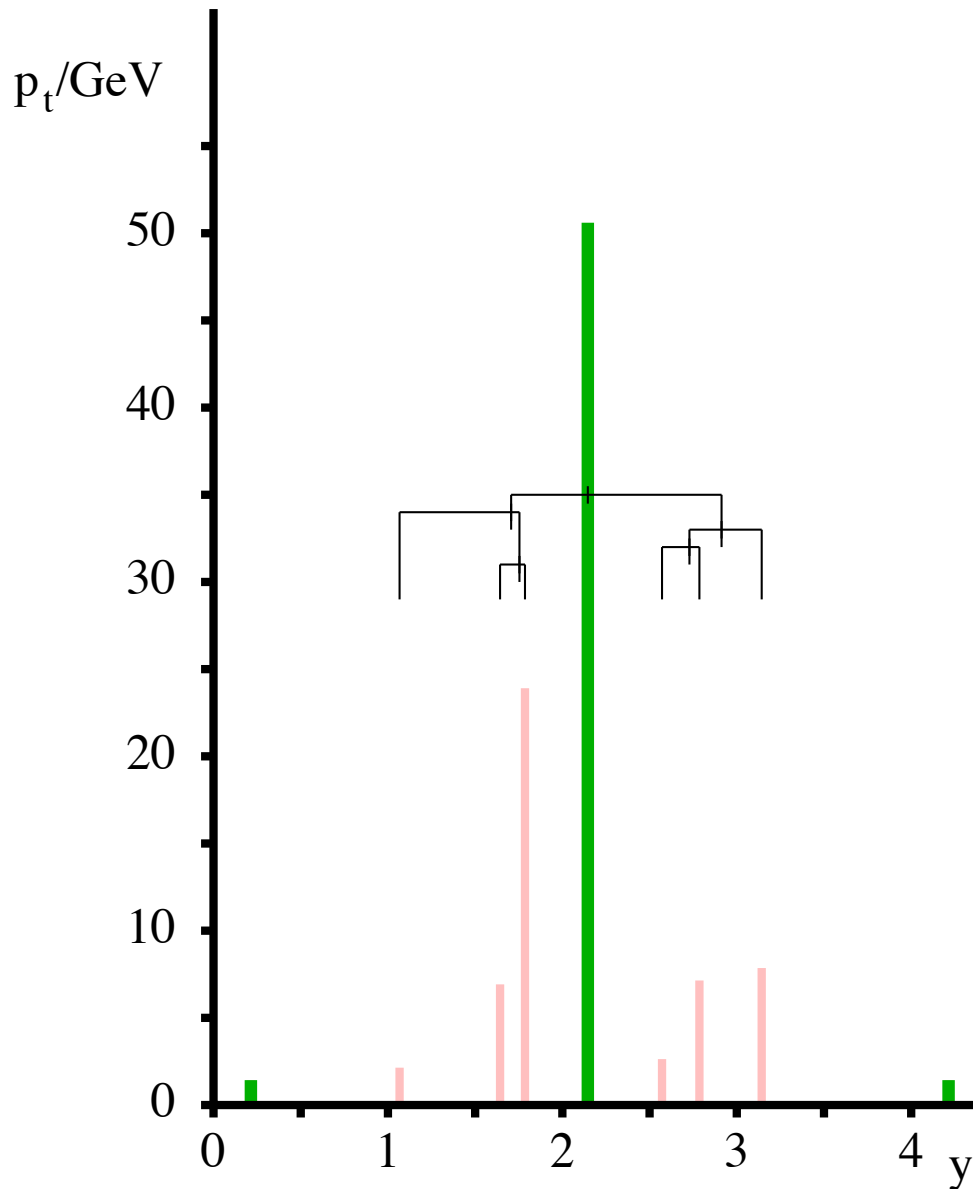


How well can an algorithm identify the “blobs” of energy inside a jet that come from different partons?

C/A identifies two hard blobs with limited soft contamination, **joins them**

# Identifying jet substructure: Cam/Aachen

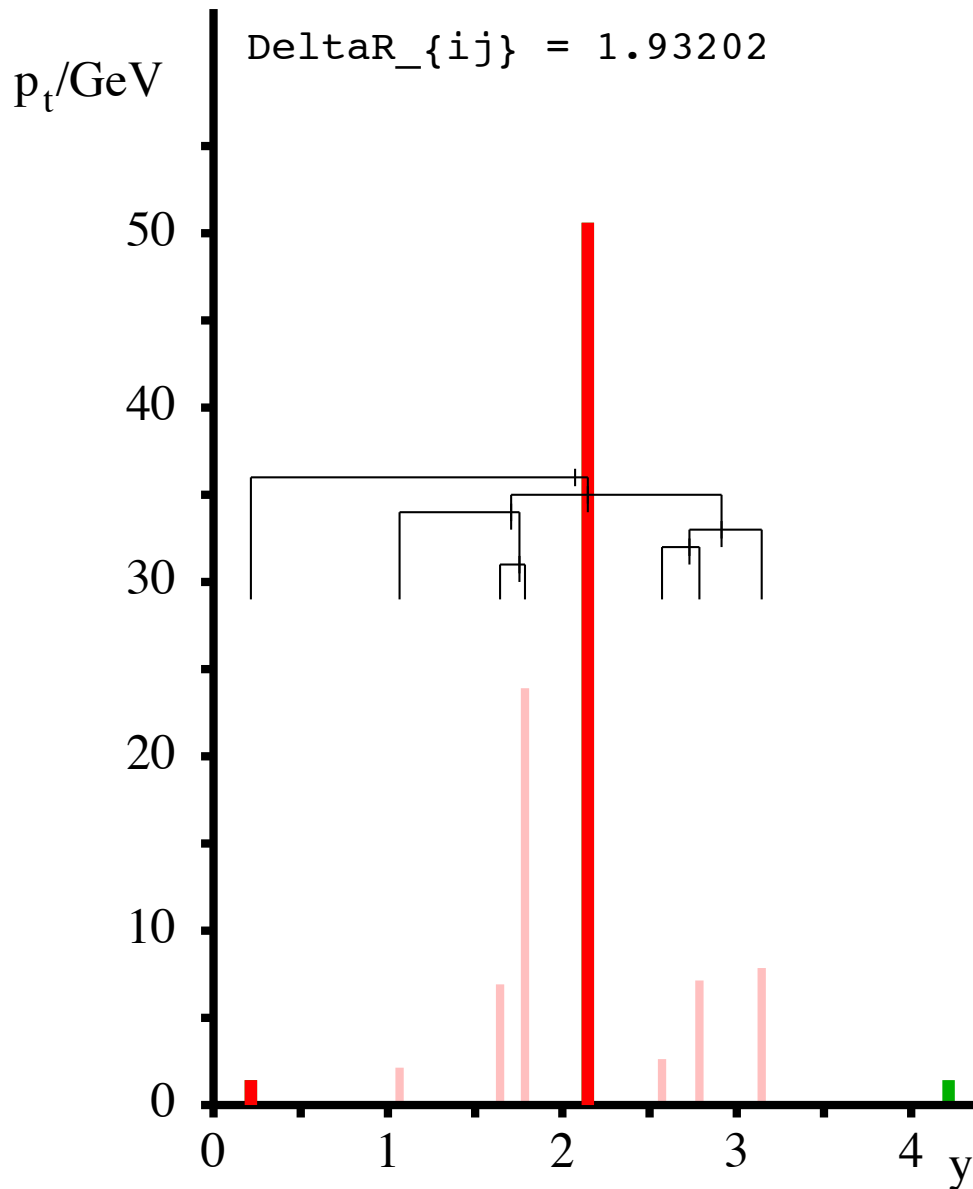
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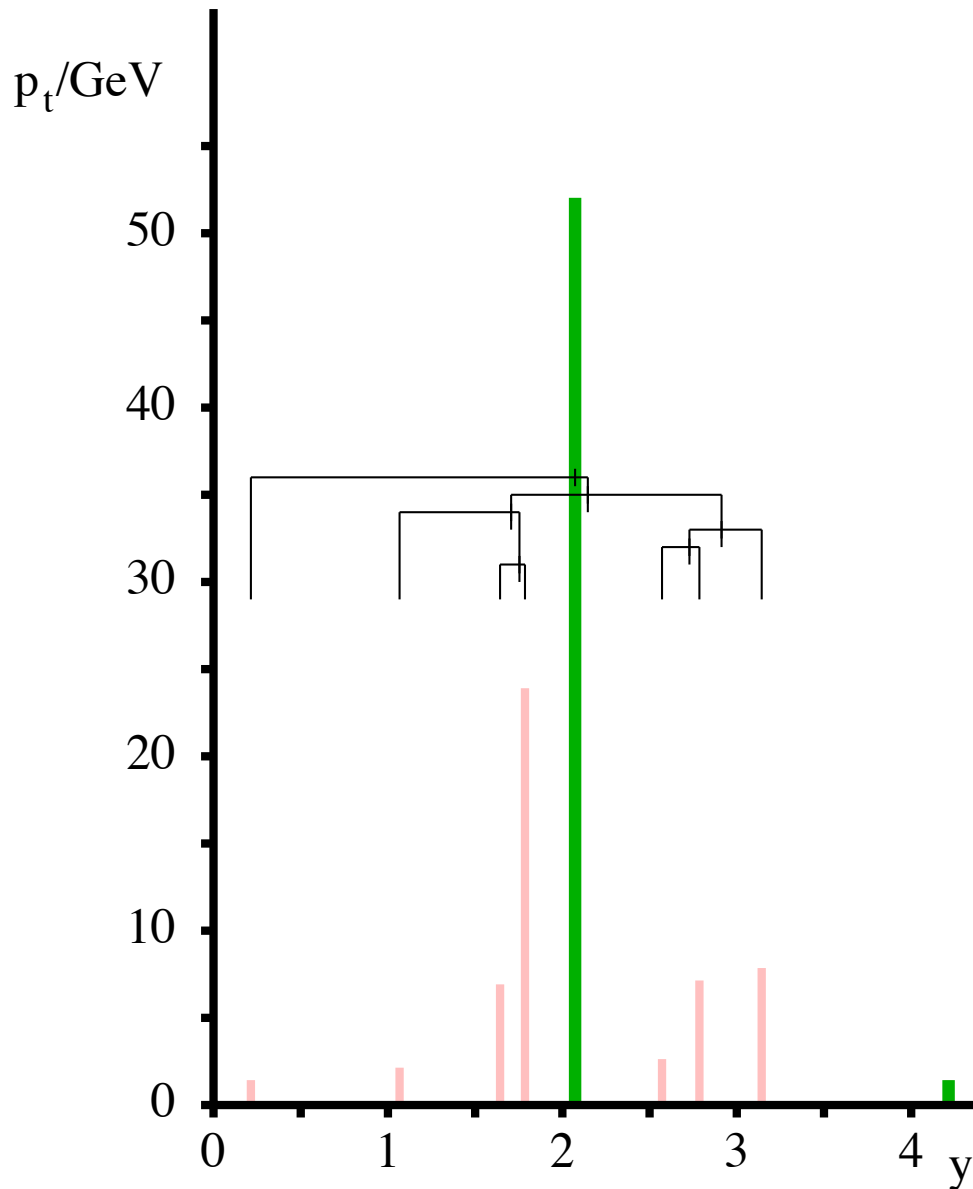
## Cambridge/Aachen algorithm



How well can an algorithm identify the “blobs” of energy inside a jet that come from different partons?

C/A identifies two hard blobs with limited soft contamination, joins them, and then adds in remaining soft junk

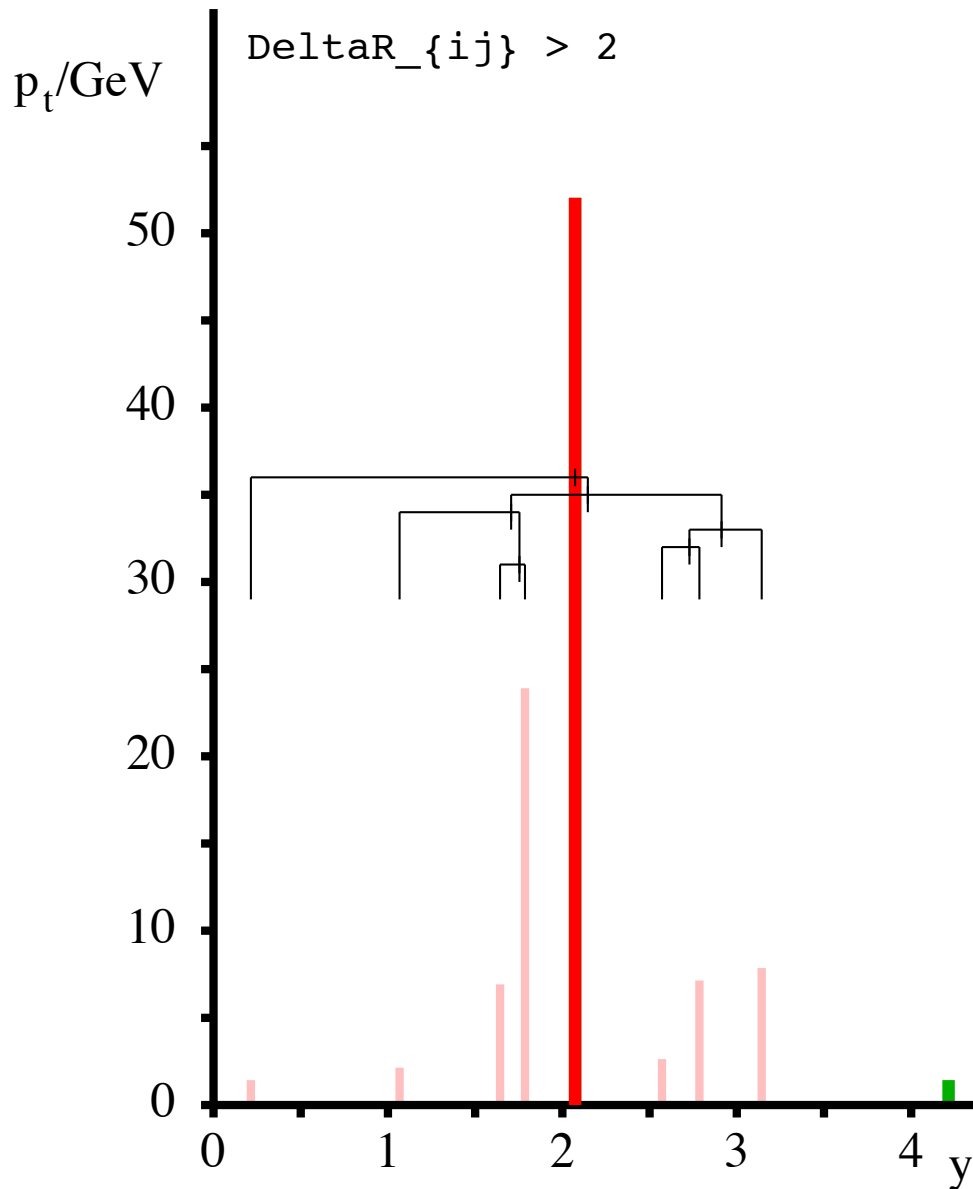
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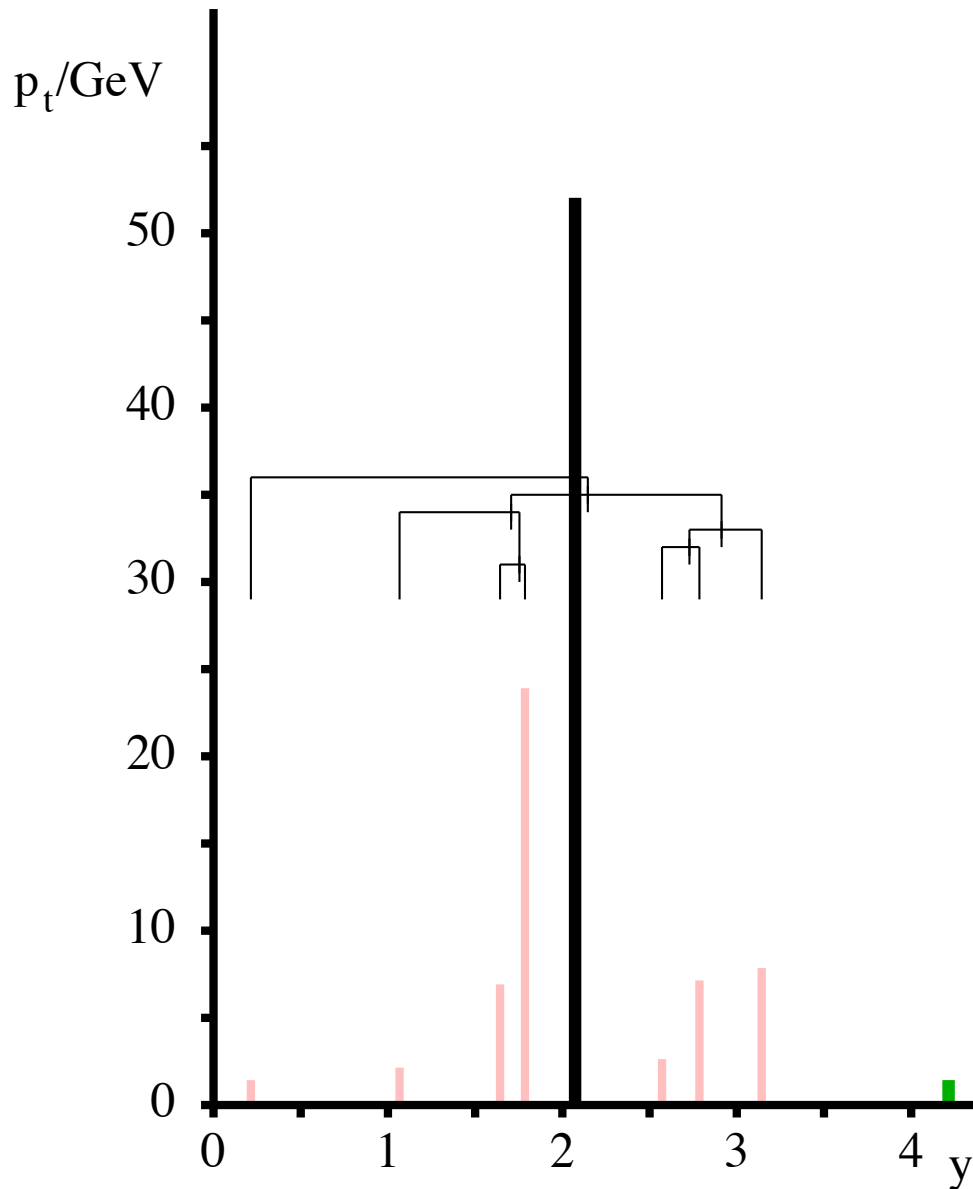


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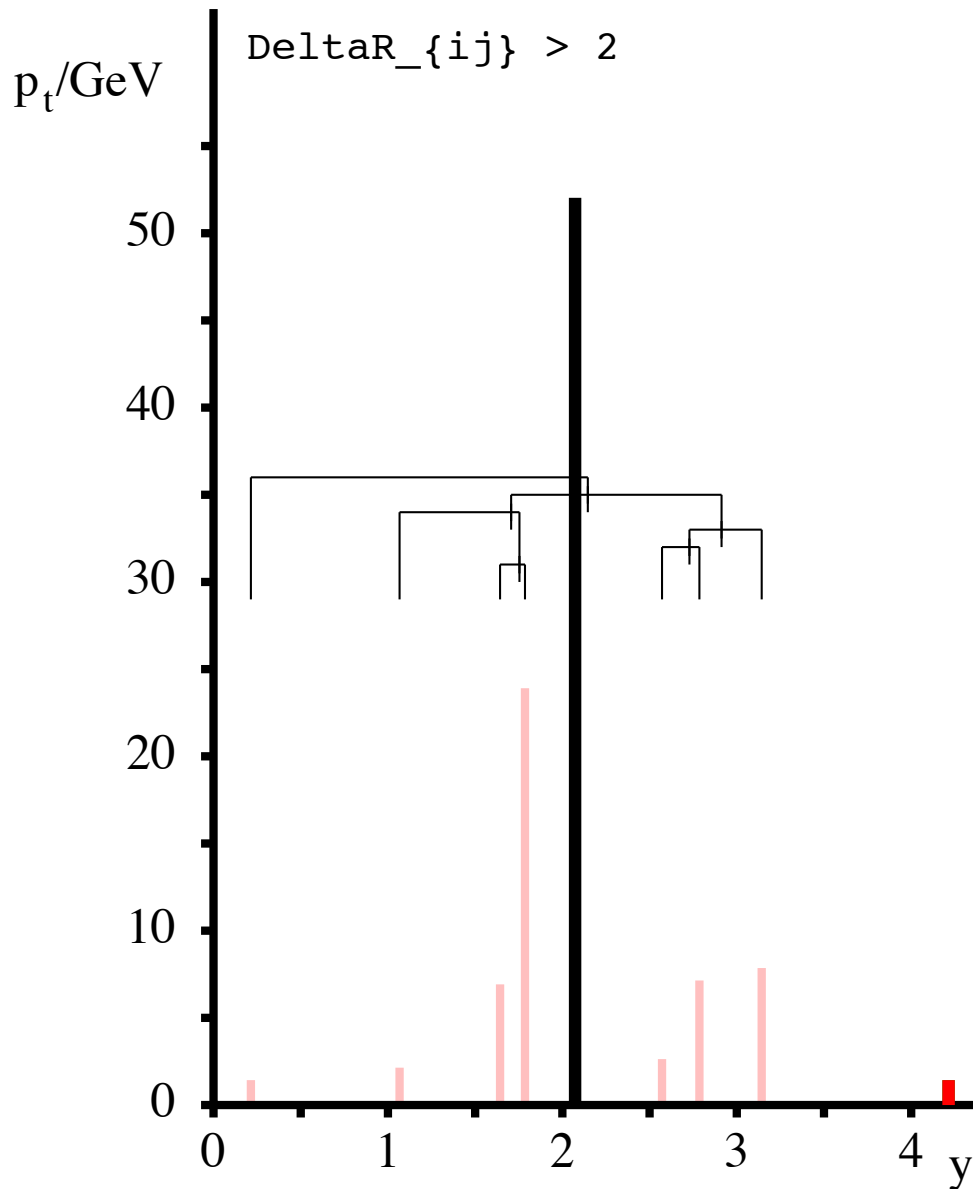
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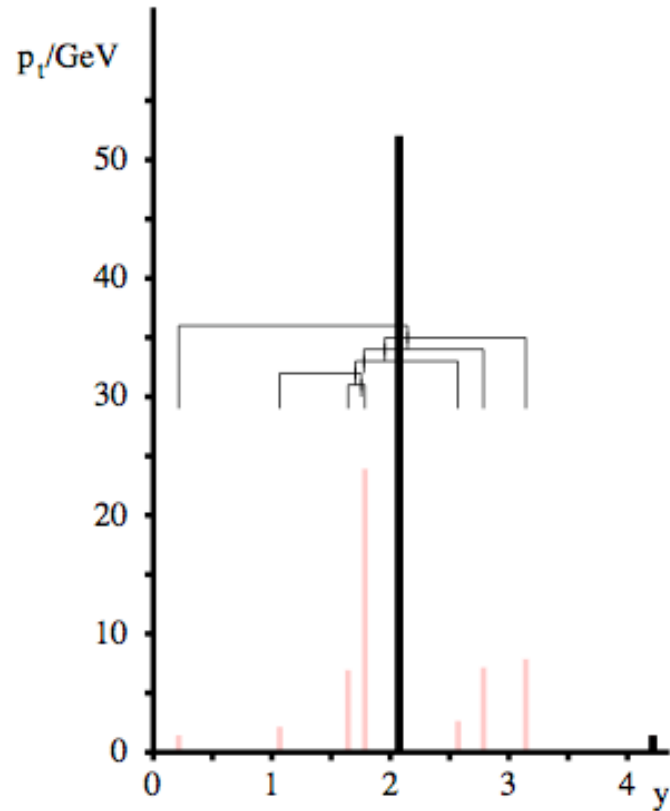
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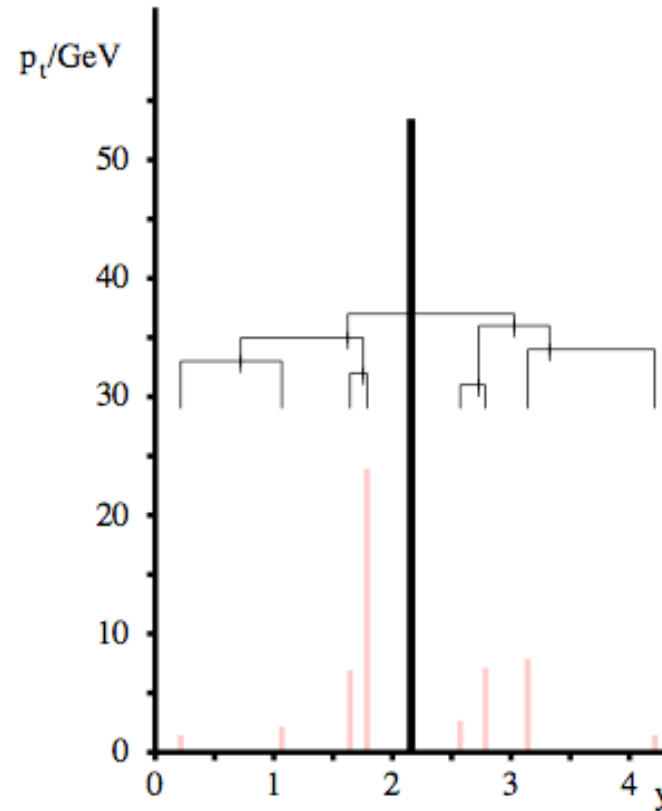


# Hierarchical substructure

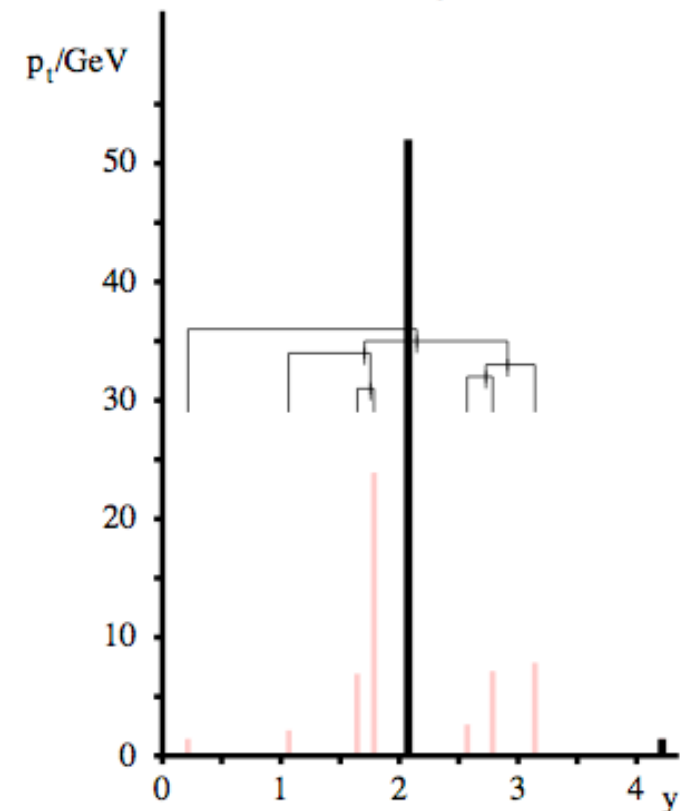
anti- $k_t$  algorithm



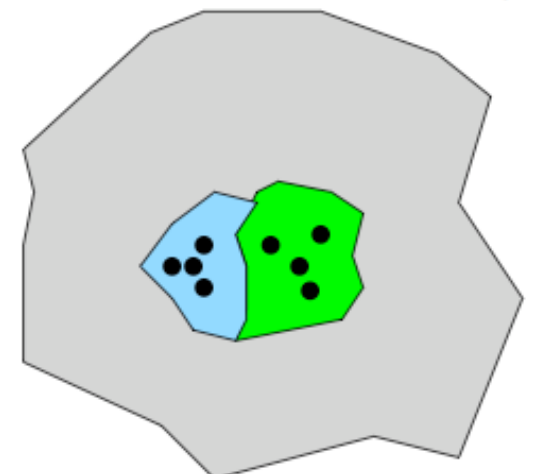
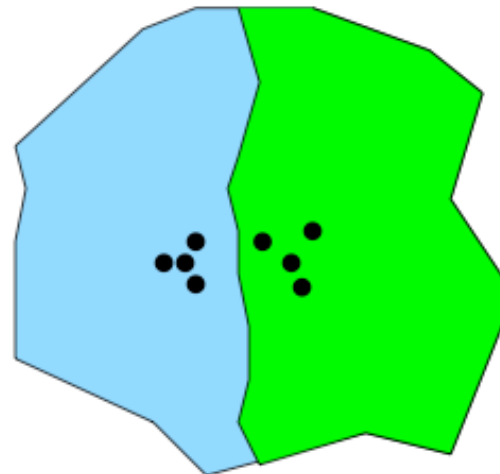
$k_t$  algorithm



Cambridge/Aachen



Undo the last  
clustering step(s)



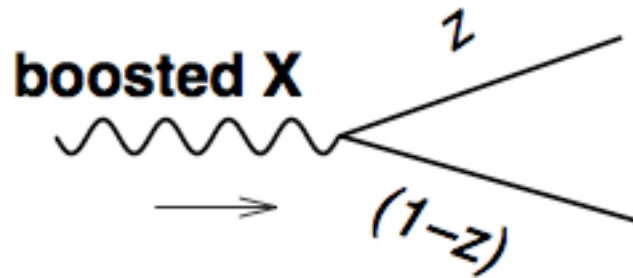
# The IRC safe algorithms

	Speed	Regularity	UE contamination	Backreaction	Hierarchical substructure
$k_t$	☺ ☺ ☺	☂	☂ ☂	☁ ☁	☺ ☺
Cambridge /Aachen	☺ ☺ ☺	☂	☂	☁ ☁	☺ ☺ ☺
anti- $k_t$	☺ ☺ ☺	☺ ☺	☁ / ☺	☺ ☺	✘
SISCone	☺	☁	☺ ☺	☁	✘

Array of tools with different characteristics.  
Pick the right one for the job

# QCD v. heavy decay

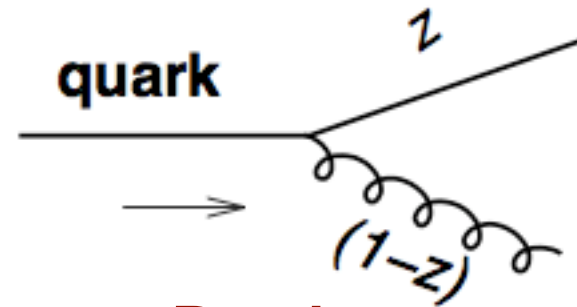
A possible approach for reducing the QCD background is to identify the two prongs of the heavy particle decay, and put a cut on their momentum fraction



**Signal:**

$$P(z) \sim 1$$

Will split mainly  
**symmetrically**



**Background:**

$$P(z) \sim \frac{1+z^2}{1-z} \qquad P(z) \sim \frac{1+(1-z)^2}{z}$$

Will split mainly  
**asymmetrically**

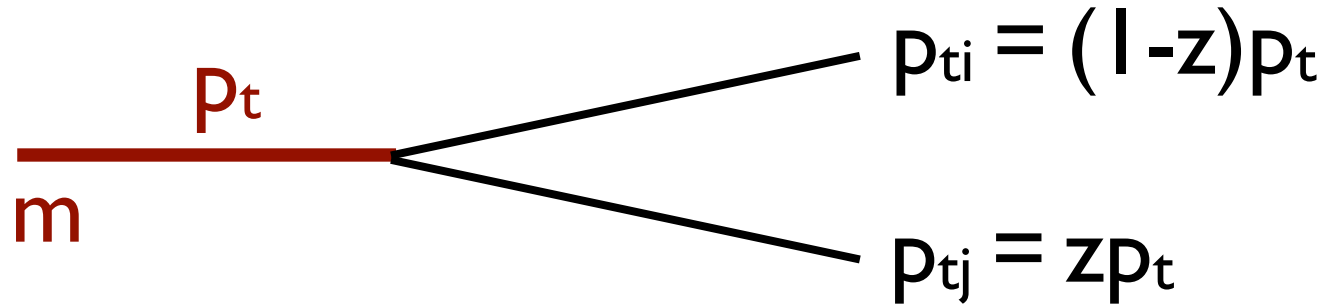
**Potential tagger: asymmetric splitting**

Possibly  
implemented  
via a cut on

$$y = \min(p_{ti}^2, p_{tj}^2) \frac{\Delta R_{ij}^2}{m^2} \simeq \frac{\min(p_{ti}, p_{tj})}{\max(p_{ti}, p_{tj})}$$

# Splittings and distances

Quasi-collinear  
splitting ( $p_{tj} < p_{ti}$ )



Invariant mass: 
$$m^2 \simeq p_{ti}p_{tj}\Delta R_{ij}^2 = (1-z)zp_t^2\Delta R_{ij}^2$$

$k_t$  distance: 
$$d_{ij}^{(p_{tj} < p_{ti})} \simeq z^2 p_t^2 \Delta R_{ij}^2 \simeq \frac{z}{1-z} m^2$$

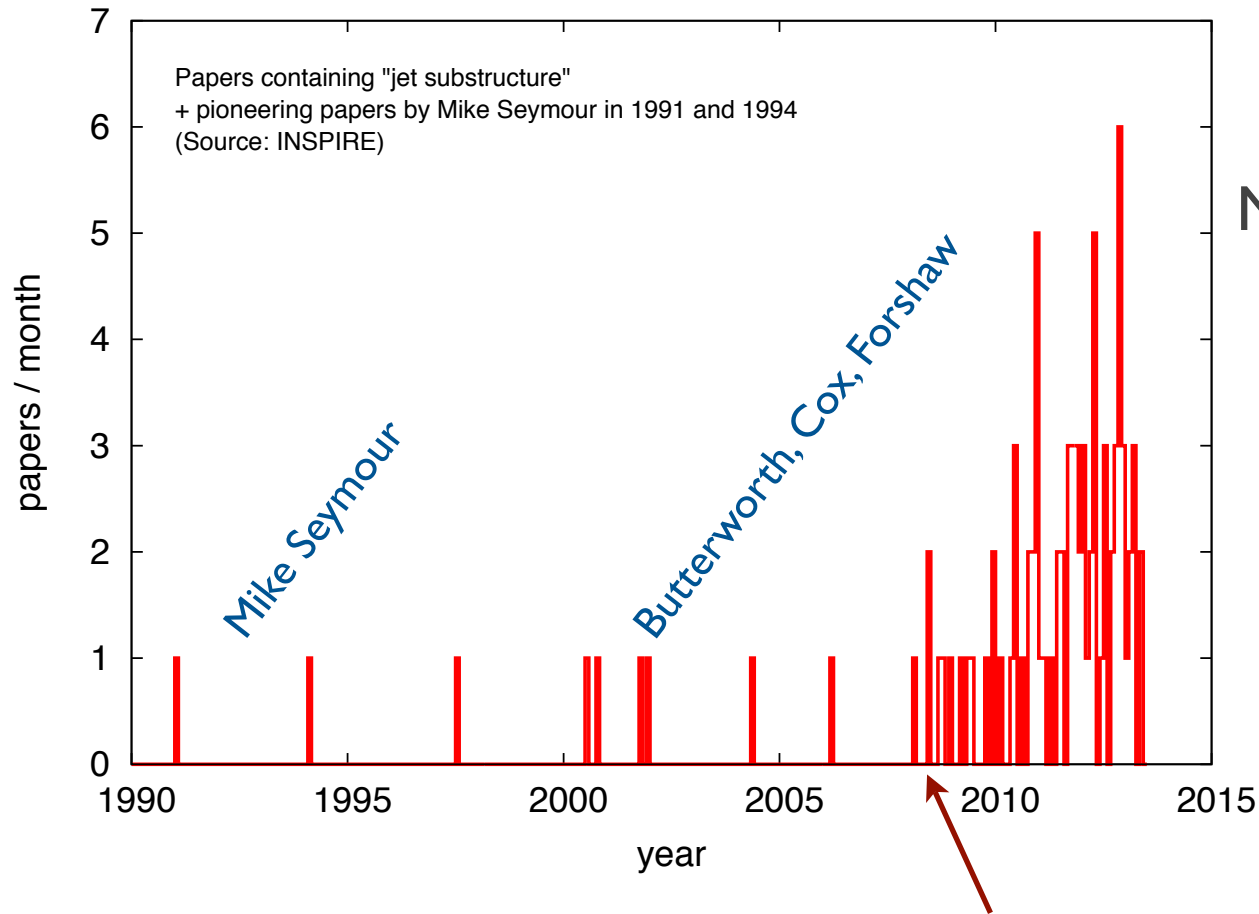
For a given mass, the **background** will have **smaller distance**  $d_{ij}$  than the signal,  
i.e. it will tend to **cluster earlier** in the  $k_t$  algorithm

## Potential tagger: last clustering in $k_t$ algorithm

This is where the hierarchy of the  $k_t$  algorithm becomes relevant.  
QCD radiation is clustered first, and only at the end the symmetric,  
large-angle splittings due to decays are reclustered

# 'Jet substructure' papers in INSPIRE

Number of papers containing the words 'jet substructure'



More than 100 papers since 2008  
(+ some background noise)

Pioneered by M. Seymour in the early  
'90s, rebooted by BDRS paper

## 15. Jet substructure as a new Higgs search channel at the LHC.

Jonathan M. Butterworth, Adam R. Davison (University Coll. London), Mathieu Rubin, Gavin P. Salam (Paris, LPTHE).

Published in *Phys.Rev.Lett.* **100** (2008) 242001

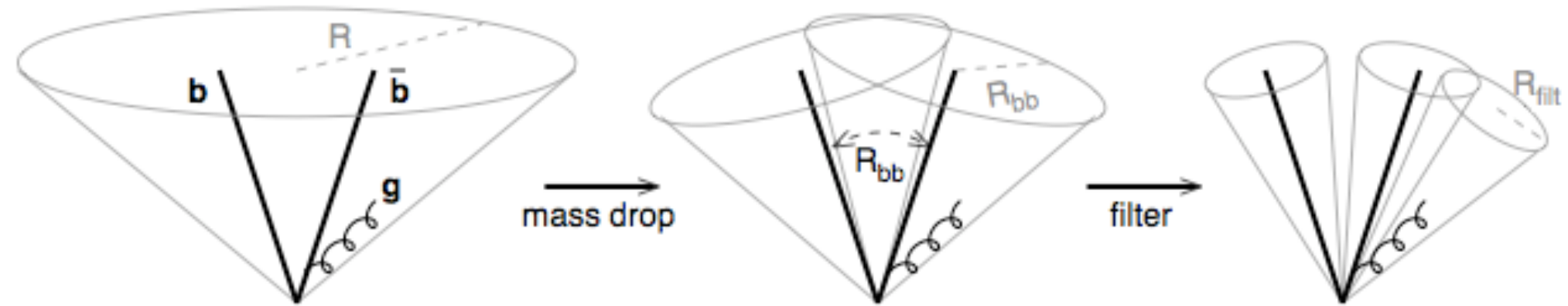
e-Print: [arXiv:0802.2470](https://arxiv.org/abs/0802.2470) [hep-ph]



$$pp \rightarrow ZH \rightarrow \nu\bar{\nu}b\bar{b}$$

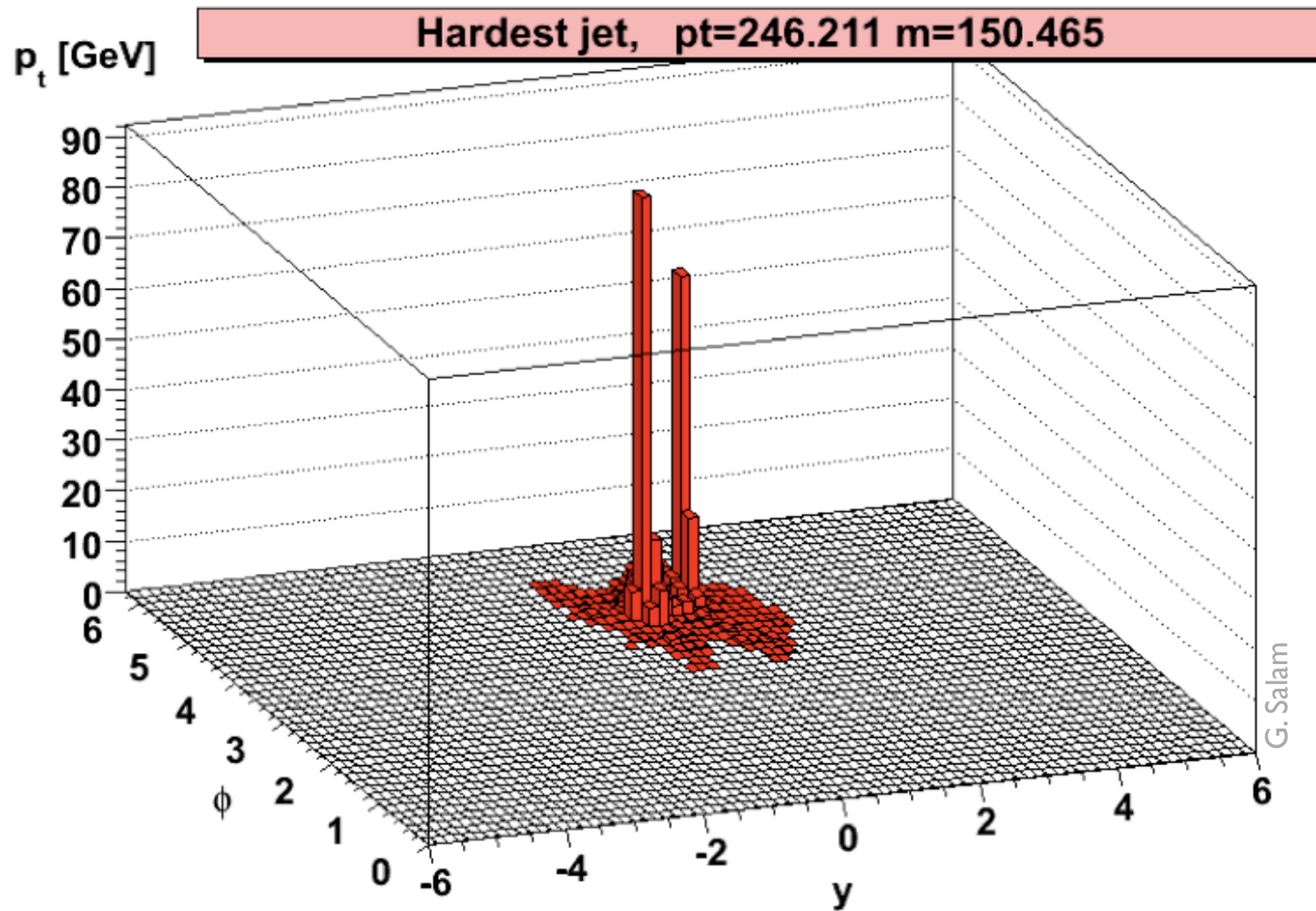
# The BDRS tagger/groomer

Butterworth, Davison, Rubin, Salam, 2008



- ▶ A two-prong tagger/groomer for boosted Higgs, which
  - ▶ Uses the **Cambridge/Aachen** algorithm (because it's 'physical')
  - ▶ Employs a **Mass-Drop** condition, as well as an **asymmetry cut** to find the **relevant splitting** (i.e. '**tag**' the heavy particle)
  - ▶ Includes a post-processing step, using '**filtering**' (introduced in the same paper) to clean as much as possible the resulting jets of UE contamination ('**grooming**')

$$pp \rightarrow ZH \rightarrow \nu\bar{\nu}b\bar{b}$$

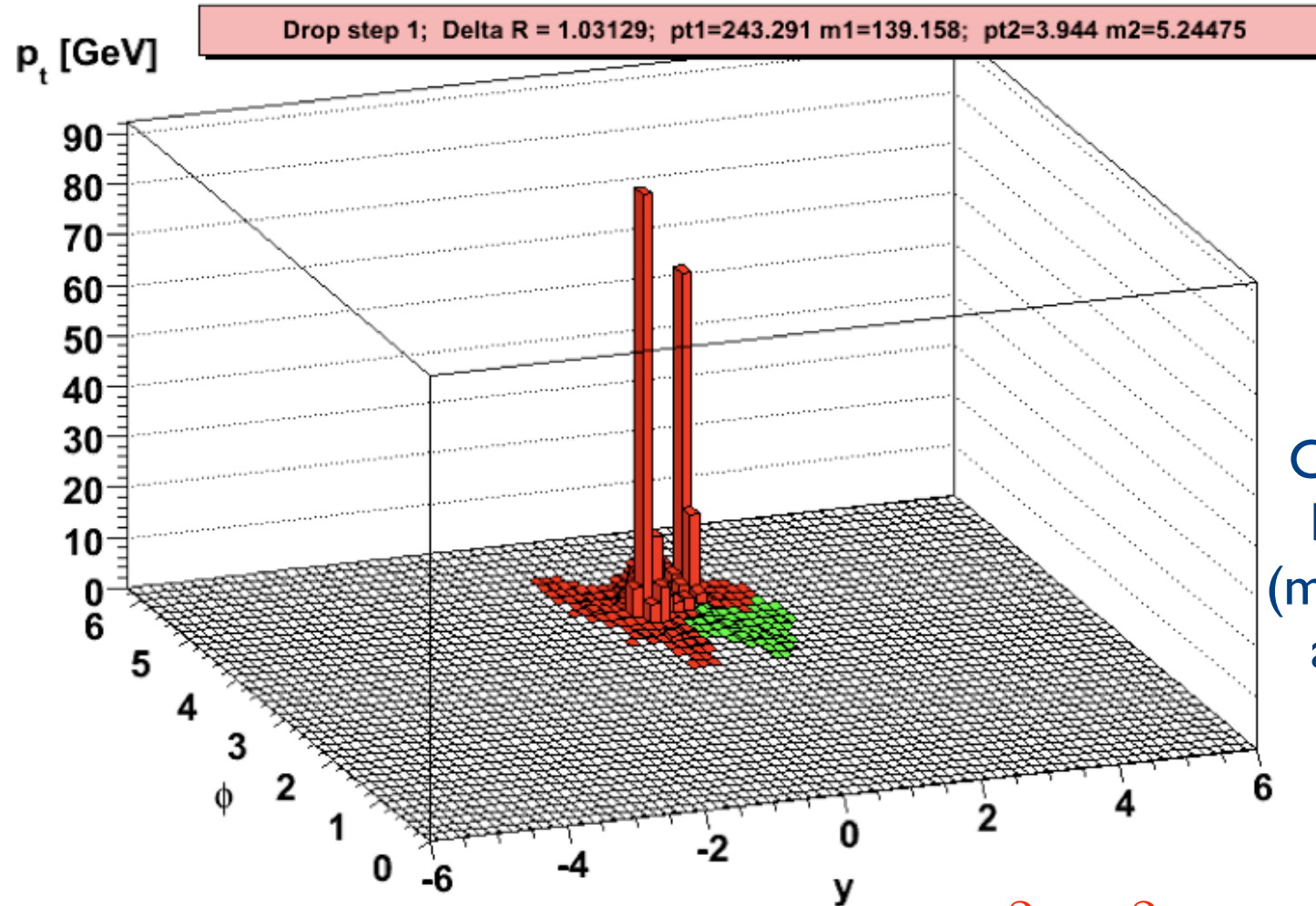


Start with the  
hardest jet

Use C/A with  
large  $R=1.2$

$m_j = 150$  GeV

$pp \rightarrow ZH \rightarrow \nu\nu b\bar{b}$

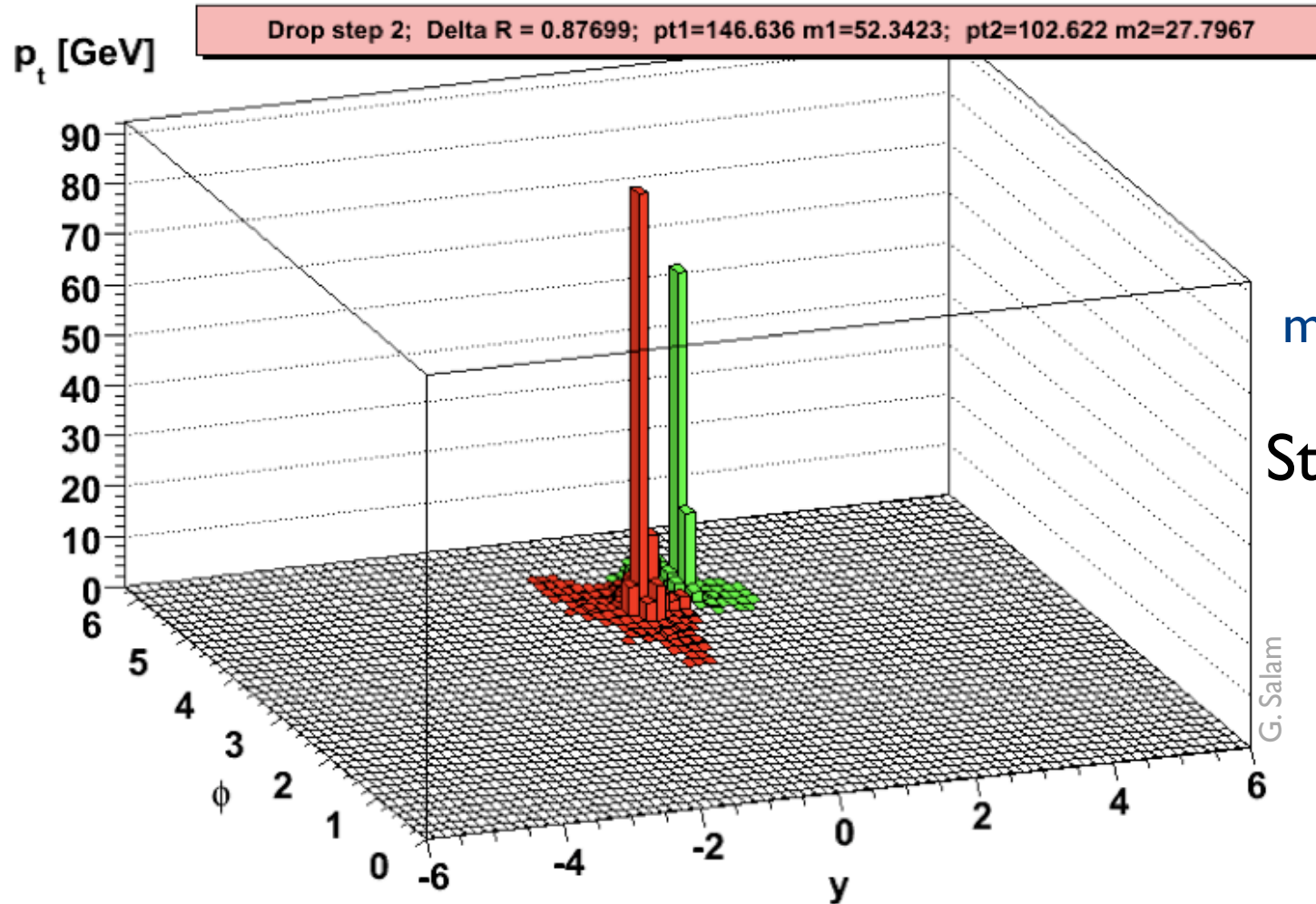


Undo last step of clustering

Check how the mass splits between the two subjects ( $m_1 = 139$  GeV,  $m_2 = 5$  GeV) and how asymmetric the splitting is

If  $\frac{\max(m_1, m_2)}{m_j} > \mu$  or  $\frac{\min(p_{t1}^2, p_{t2}^2)}{m_j^2} \Delta R_{12}^2 < y_{cut}$  repeat

$pp \rightarrow ZH \rightarrow \nu\nu b\bar{b}$



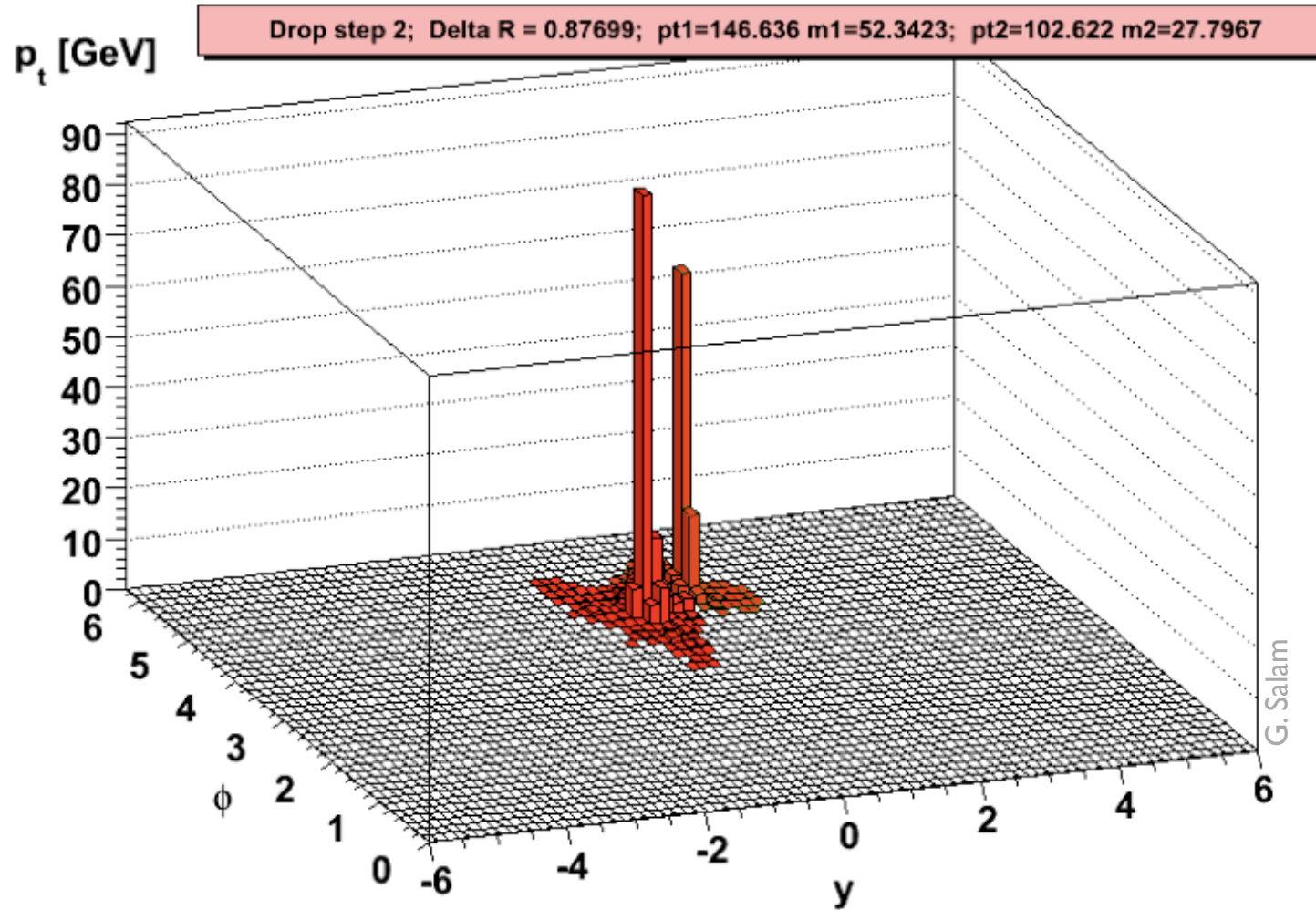
$m_1 = 52 \text{ GeV}, m_2 = 28 \text{ GeV}$

Stop when a **large mass drop** is observed  
(and **recombine** these two jets)

[NB. Parameters used  $\mu = 0.67$  and  $y_{\text{cut}} = 0.09$ ]

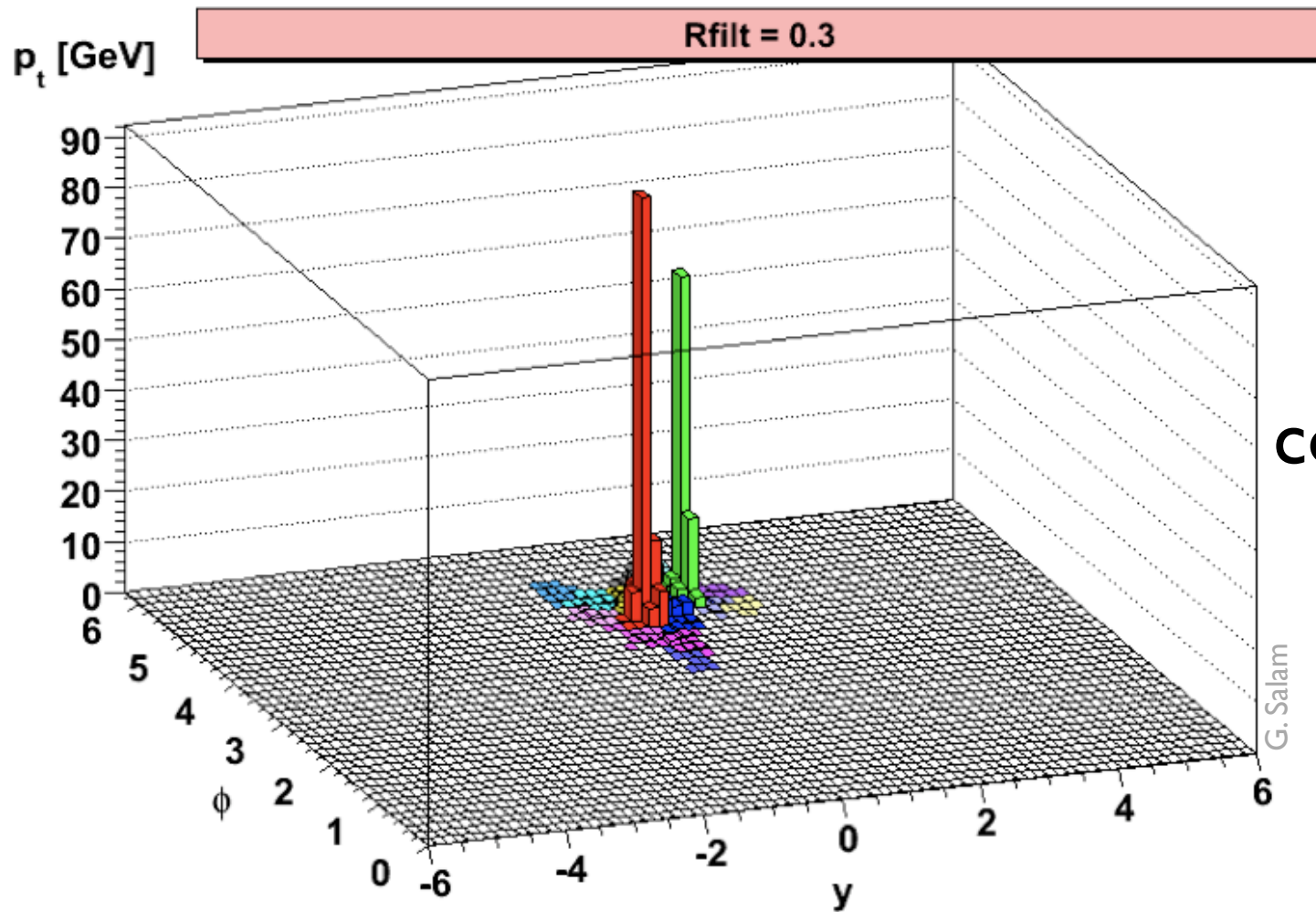


$pp \rightarrow ZH \rightarrow \nu\nu b\bar{b}$



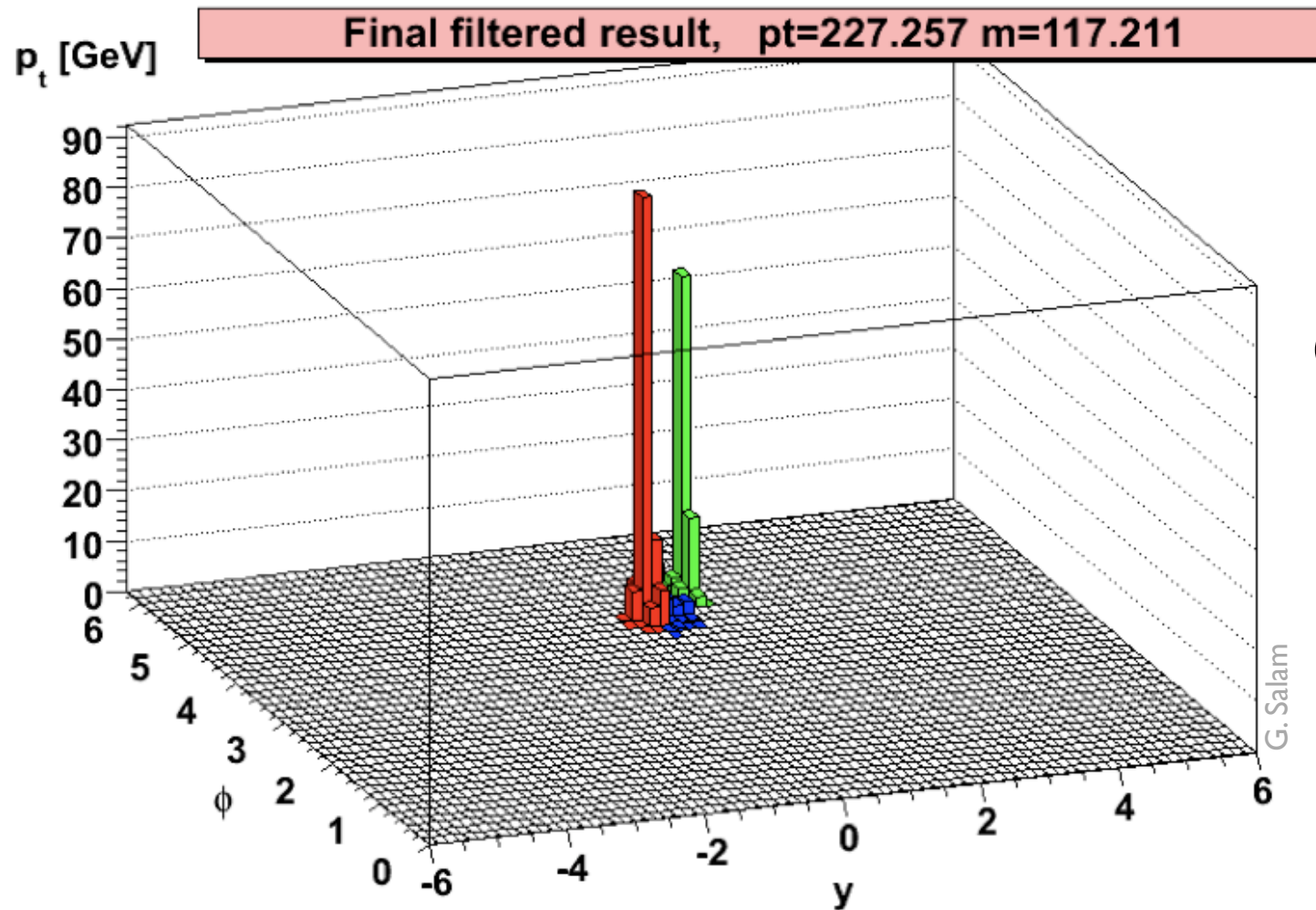
Start with the recombined jet

$pp \rightarrow ZH \rightarrow \nu\nu b\bar{b}$



Recluster the  
constituents with  $R_{\text{filt}}$

$pp \rightarrow ZH \rightarrow \nu\nu b\bar{b}$



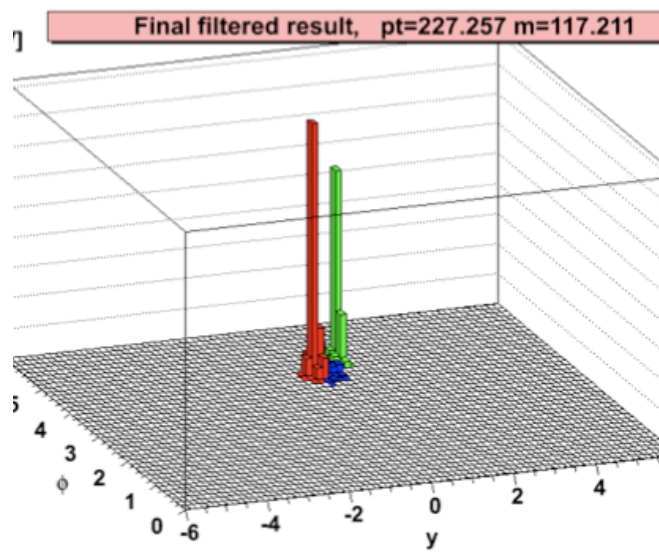
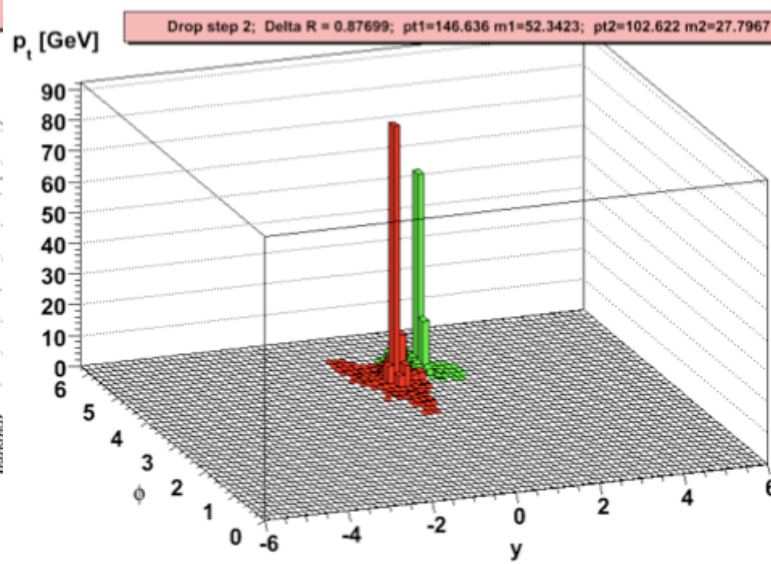
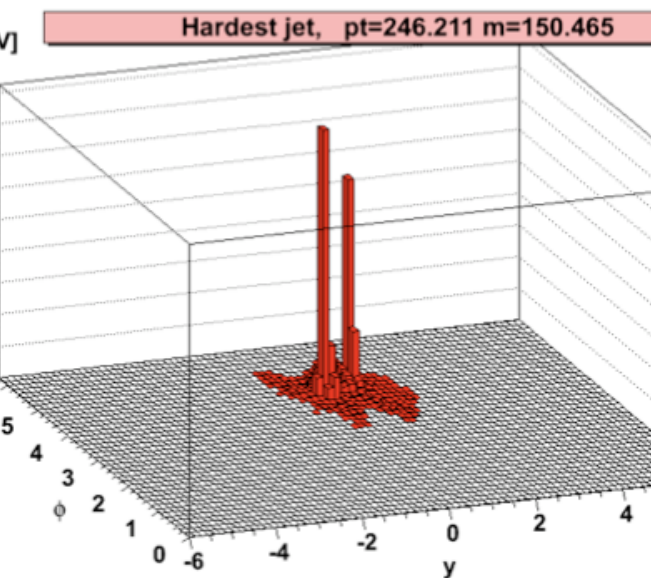
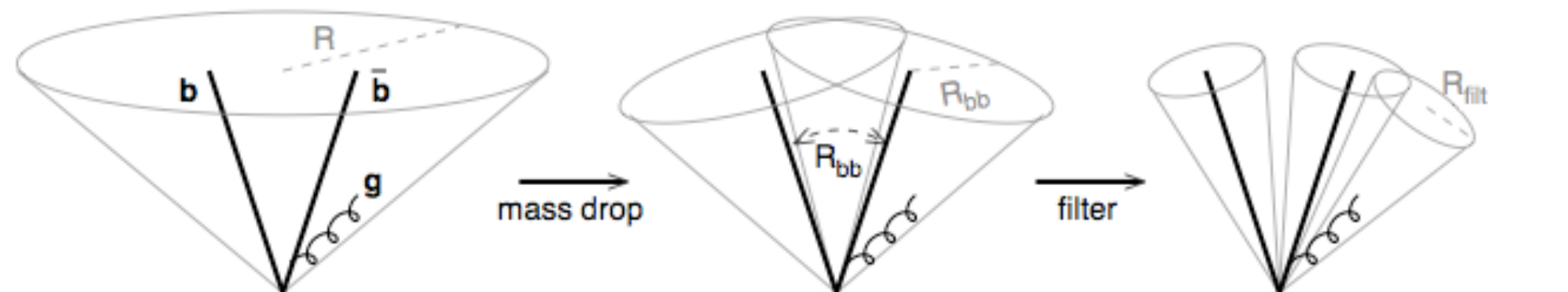
Only keep the  $n_{\text{filt}}$  hardest jets

The low-momentum stuff surrounding the hard particles has been removed

$$pp \rightarrow ZH \rightarrow \nu\bar{\nu}b\bar{b}$$

# Visualisation of BDRS

Butterworth, Davison, Rubin, Salam, 2008



Cluster with a large R

Undo the clustering into subjects, until a large asymmetry/mass drop is observed: tagging step

Re-cluster with smaller R, and keep only 3 hardest jets: grooming step



## In FastJet

```
#include "fastjet/tools/MassDropTagger.hh"
#include "fastjet/tools/Filter.hh"

JetDefinition jet_def(cambridge_algorithm, 1.2);
ClusterSequence cs(input_particles, jet_def);

// define the tagger and use it
MassDropTagger md_tagger(0.667, 0.09);
PseudoJet tagged = md_tagger(jets[0]);

// define the filter and use it
Filter filter(0.3, SelectorNHardest(3));
Pseudojet higgs = filter(tagged);      // this is the Higgs!!
```

The real analysis is slightly more refined (b-tagging, dynamical filter radius, etc)  
but the main features are already present here

# First taggers/groomers

## ► Mass Drop + Filtering

Butterworth, Davison, Rubin, Salam, 2008

Decluster with mass drop and asymmetry conditions

Recluster constituents into subjects at distance scale  $R_{\text{filt}}$ , retain  $n_{\text{filt}}$  hardest subjects

## ► Jet 'trimming'

Krohn, Thaler, Wang, 2009

Recluster constituents into subjects at distance scale  $R_{\text{trim}}$ ,

retain subjects with  $p_{t,\text{subject}} > \epsilon_{\text{trim}} p_{t,\text{jet}}$

## ► Jet 'pruning'

S. Ellis, Vermilion, Walsh, 2009

While building up the jet, discard softer subjects when  $\Delta R > R_{\text{prune}}$

and  $\min(p_{t1}, p_{t2}) < \epsilon_{\text{prune}} (p_{t1} + p_{t2})$

**Aim: limit contamination from QCD background while retaining bulk of perturbative radiation**

**Trimming and pruner are a priori groomers, but can become taggers when combined with an invariant mass window test (if you can groom everything then there's no heavy particle in the jet)**

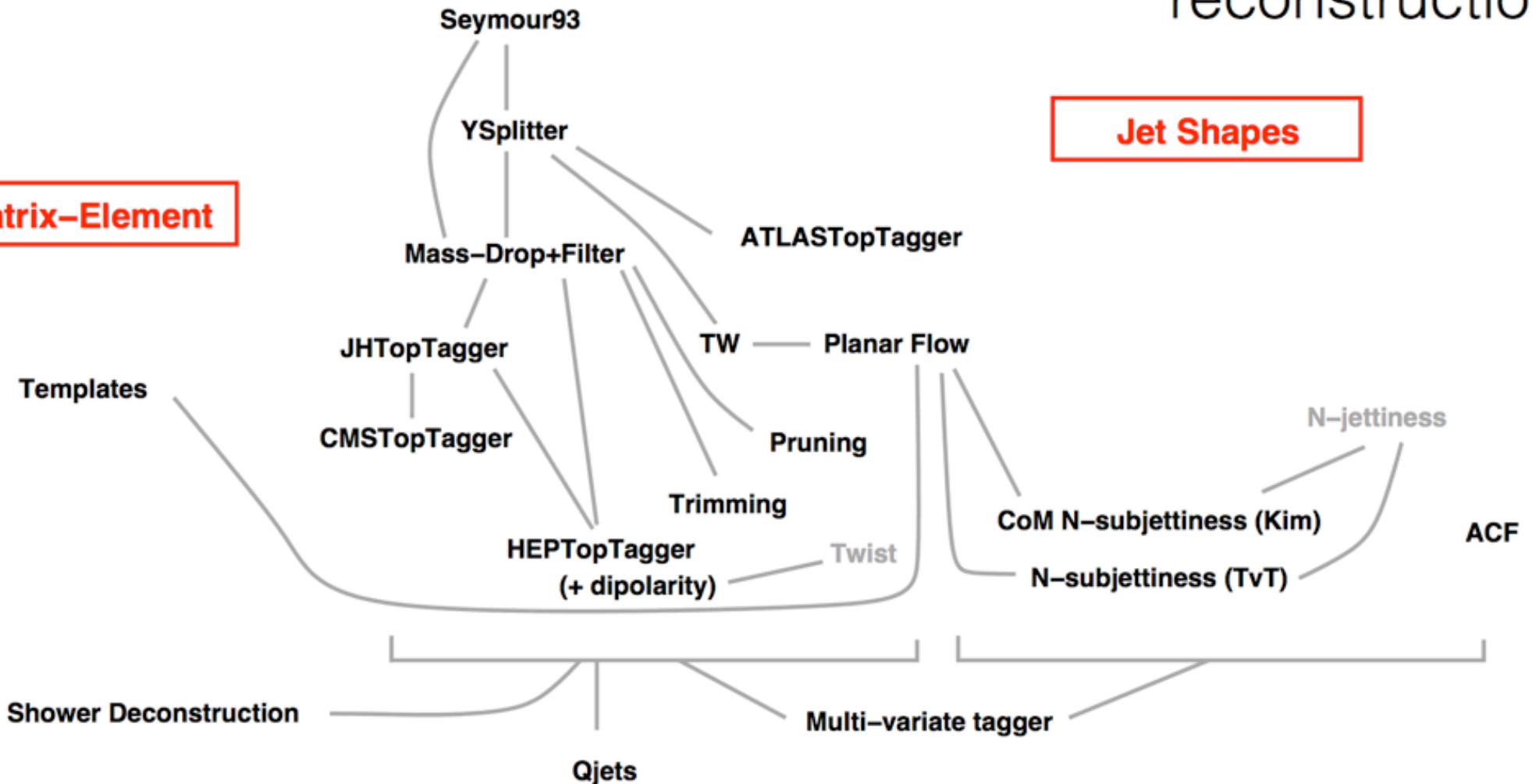
# The jet substructure maze

Some of the tools developed for boosted W/Z/H/top reconstruction

**Jet Declustering**

**Jet Shapes**

**Matrix-Element**



Slide by G. Salam, now a few years old

# Conclusion part I

- ▶ A number of different IRC-safe jet algorithms exist
  - ▶ They all try to be good proxies for hard partons, but they have different characteristics, especially with respect to soft particles
- ▶ Jets from all algorithms inevitably suffer from pileup contamination
  - ▶ Techniques exist to subtract it, either at jet-level, or at particle-level
- ▶ Both the jet algorithms and many pileup subtraction techniques are packaged either in FastJet or in fjcontrib contributions
  - ▶ **Use of standard algorithms and packages** (either directly or through interfaces) **should be privileged**, as it ensures reproducibility

<http://fastjet.fr>

<http://fastjet.hepforge.org/contrib/>

The big news of the past few years has been the emergence of jet-based taggers and groomers

- ▶ They have proven their worth in ‘Standard Model’ analyses
- ▶ They are being implemented in BSM searches

The tutorial will offer you the opportunity to play with clustering and substructure