General scan in flavor parameter space in the models with vector quark doublets and an enhancement in $B \to X_s\gamma$ process

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4. Summary

The problem:

In the SM $M_U,\ M_D$ for the up and down type fermions.

$$Z_U^\dagger M_U U_U = M_U^D = \text{diag.}[m_u, m_c, m_t],$$
$$Z_D^\dagger M_D U_D = M_D^D = \text{diag.}[m_d, m_s, m_b],$$

and the so called CKM matrix

$$V_{\text{CKM}} = U_U^\dagger U_D.$$

Since $M_U,\ M_D$ come from separate Yukawa couplings, we can always set one of the matrices diagonal, for example M_U, and use the CKM matrix to get the Yukawa couplings

$$Z_D \begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix} V_{\text{CKM}}^\dagger = \begin{pmatrix} Y_{11}^D v & Y_{12}^D v & Y_{13}^D v \\ Y_{21}^D v & Y_{22}^D v & Y_{23}^D v \\ Y_{31}^D v & Y_{32}^D v & Y_{33}^D v \end{pmatrix}$$

In the model with a vector doublet,

$$Q : \mathbf{3},\ \mathbf{2},\ \frac{\mathbf{1}}{\mathbf{6}} \quad \bar{Q} : \bar{\mathbf{3}},\ \mathbf{2},\ -\frac{\mathbf{1}}{\mathbf{6}}$$

resulting bilinear term in the lagrangian

$$M^V Q \cdot \bar{Q}.$$

$$M_U = \begin{pmatrix} Y^U_{11}v & Y^U_{12}v & Y^U_{13}v & \cdots \\ Y^U_{21}v & Y^U_{22}v & Y^U_{23}v & \cdots \\ Y^U_{31}v & Y^U_{32}v & Y^U_{33}v & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ M^V_{41} & M^V_{42} & M^V_{43} & \cdots \end{pmatrix},$$

$$M_D = \begin{pmatrix} Y^D_{11}v & Y^D_{12}v & Y^D_{13}v & \cdots \\ Y^D_{21}v & Y^D_{22}v & Y^D_{23}v & \cdots \\ Y^D_{31}v & Y^D_{32}v & Y^D_{33}v & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ -M^V_{41} & -M^V_{42} & -M^V_{43} & \cdots \end{pmatrix}$$

Though this is just a numerical problem, when one treats the VLP contributions to the flavor physics seriously, diagonalization of quark matrices will be the first and important step.

The Trick of diagonalization of vector quark doublet

diagonalization of $N \times N$ matrix M_U and M_D:

$$Z_U^\dagger M_U U_U = M_U^D,\ Z_D^\dagger M_D U_D = M_D^D$$

in which M_U^D, M_D^D are the diagonal mass matrices for up and down type quark, respectively.
Adding two matrices

$$M_U + M_D = \left(Z_U M_U^D U_{\text{CKMN}} + Z_D M_D^D\right) U_D^\dagger$$

The left side of the equation is

$$\begin{pmatrix} Y_{11}^U v + Y_{11}^D v & Y_{12}^U v + Y_{12}^D v & Y_{13}^U v + Y_{13}^D v & \cdots & M_{U1N} + M_{D1N} \\ Y_{21}^U v + Y_{21}^D v & Y_{22}^U v + Y_{22}^D v & Y_{23}^U v + Y_{23}^D v & \cdots & M_{U2N} + M_{D2N} \\ Y_{31}^U v + Y_{31}^D v & Y_{32}^U v + Y_{32}^D v & Y_{33}^U v + Y_{33}^D v & \cdots & M_{U3N} + M_{D3N} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & M_{UNN} + M_{DNN} \end{pmatrix}$$

We can denote the matrix in the form as

$$M_U + M_D = M_{UD} = \begin{pmatrix} \mathbf{M_A} & \mathbf{M_B} \\ \mathbf{M_0} & M_C \end{pmatrix}$$

in which $\mathbf{M_A}$, $\mathbf{M_B}$, $\mathbf{M_0}$ are $(N-1)\times(N-1)$, $(N-1)\times 1$ and $1\times(N-1)$ matrices correspondingly.
Input is $(m_u, m_c, m_t, \cdots m_X)$, $(m_d, m_s, m_b, \cdots, m_Y)$ and a matrix $U_{\rm CKMN}$

$$\begin{aligned} U_{\rm CKMN} &= U_U^\dagger U_D = \begin{pmatrix} (U_{\rm CKM})_{3\times3} & \cdots \\ \cdots & U_{NN} \end{pmatrix} \\ &= \begin{pmatrix} \begin{pmatrix} U_{ud} & U_{us} & U_{ub} \\ U_{cd} & U_{cs} & U_{cb} \\ U_{td} & U_{ts} & U_{tb} \end{pmatrix} & \cdots \\ \cdots & U_{NN} \end{pmatrix} \end{aligned}$$

$(U_{\rm CKM})_{3\times3}$ is not an ordinary CKM matrix $V_{\rm CKM}$ which is non-unitary in this case.

In the similar way we denote U_D as

$$U_D = \begin{pmatrix} \mathbf{U_{DA}} & \mathbf{U_{DB}} \\ \mathbf{U_{D0}} & U_{DNN} \end{pmatrix}$$

$$\begin{aligned} M_{UD}U_D &= \begin{pmatrix} \mathbf{M_A}\mathbf{U_{DA}} + \mathbf{M_B}\mathbf{U_{D0}} & \mathbf{M_A}\mathbf{U_{DB}} + \mathbf{M_B}U_{DNN} \\ M_C\mathbf{U_{D0}} & M_C U_{DNN} \end{pmatrix} \\ &= (Z_U M_U^D U_{\mathrm{CKMN}} + Z_D M_D^D) \end{aligned}$$

We can get the last line of U_D simply by inputting M_U^D, M_D^D, U_{CKMN} and random Z_U, Z_D:

$$\begin{aligned} (Z_U M_U^D U_{\mathrm{CKMN}} + Z_D M_D^D)_{\text{last line}} &= \begin{pmatrix} M_C\mathbf{U_{D0}} & M_C U_{DNN} \end{pmatrix} \\ &= M_C \mathbf{U}_{\mathbf{D}N} \end{aligned}$$

where

$$\mathbf{U}_{\mathbf{D}N} = \begin{pmatrix} U_{DN1} & U_{DN2} & \cdots & U_{DNN} \end{pmatrix}$$

is a unit vector in N dimension.

Next we use the unit vector to generate total U_D.
$\mathbf{U_D}_{N-1}$can be determined as

$$\mathbf{U_D}_{N-1} = \begin{pmatrix} -\frac{U^*_{DN2}}{\sqrt{|U_{DN1}|^2+|U_{DN2}|^2}} & \frac{U^*_{DN1}}{\sqrt{|U_{DN1}|^2+|U_{DN2}|^2}} & 0 & \cdots & 0 \end{pmatrix}$$

Then we use the first three elements of $\mathbf{U_D}_N$ and $\mathbf{U_D}_{N-1}$ to generate $\mathbf{U_D}_{N-2}$: Normalize the algebraic complements of first line of the 3×3 matrix.
Step by step, we can finally get $(\mathbf{U_D}_1, \mathbf{U_D}_2, \cdots, \mathbf{U_D}_{N-1})$ and form a special U_D^S

$$\begin{pmatrix} \mathbf{U_D}_1 \\ \cdots \\ \mathbf{U_D}_{N-2} \\ \mathbf{U_D}_{N-1} \\ \mathbf{U_D}_N \end{pmatrix} = \begin{pmatrix} U_{D11} & U_{D12} & U_{D13} & \cdots & U_{D1N} \\ \cdots & \cdots & \cdots & \cdots & 0 \\ U_{D(N-2)1} & U_{D(N-2)2} & U_{D(N-2)3} & \cdots & 0 \\ U_{D(N-1)1} & U_{D(N-1)2} & 0 & \cdots & 0 \\ U_{DN1} & U_{DN2} & U_{DN3} & \cdots & U_{DNN} \end{pmatrix}$$

From above steps, we can see that $(\mathbf{U_{D}}_1, \mathbf{U_{D}}_2, \cdots, \mathbf{U_{D}}_{N-1})$ can be rotated into any other orthogonal $N-1$ vectors to construct random matrix $\mathbf{M_A}$ and $\mathbf{M_B}$, only $\mathbf{U_{D}}_N$ must be kept unchanged. Therefore, a general unitary matrix can be realized by timesing a unitary $N \times N$ matrix U_R,

$$U_D = U_R U_D^S = \begin{pmatrix} \mathbf{U_{R}}_{N-1} & \mathbf{0} \\ \mathbf{0} & 1 \end{pmatrix} U_D^S$$

in which $\mathbf{U_{R}}_{N-1}$ is a $(N-1) \times (N-1)$ unitary matrix. We finish the work by

$$\begin{aligned} U_U^\dagger &= U_{\mathrm{CKMN}} U_D^\dagger \\ M_U &= Z_U M_U^D U_U^\dagger \\ M_D &= Z_D M_D^D U_D^\dagger \end{aligned}$$

Generation of a $N \times N$ unitary matrix:

$$
\begin{aligned}
U_{N\times N} &= \begin{pmatrix}
\cos\theta_{12} & \sin\theta_{12} & 0 & \cdots & \cdots & \cdots \\
-\sin\theta_{12} & \cos\theta_{12} & 0 & \cdots & \cdots & \cdots \\
0 & 0 & 1 & \cdots & \cdots & \cdots \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots
\end{pmatrix} \\
&\times \quad U(\theta_{13}) \cdots U(\theta_{N-1,N})
\end{aligned}
$$

Summary of the method:

- **STEP 1:** Chose $(m_u, m_c, m_t, \cdots, m_X, m_d, m_s, m_b, \cdots, m_Y)$ and $U_{\rm CKMN}$ and input random unitary matrices Z_U and Z_D;
- **STEP 2:** Determine the last line of matrix $Z_U M_U^D U_{\rm CKMN} + Z_D M_D^D$ as

$$M_C \left(\begin{array}{cccc} U_{DN1} & U_{DN2} & \cdots & U_{DNN} \end{array} \right)$$

  and normalize it into a unit vector $\mathbf{U_{D}}_N$.
- **STEP 3:** Use the unit vector $\mathbf{U_{D}}_N$ to generate other $N-1$ to form a special U_D^S

$$U_D^S \;=\; \left(\begin{array}{ccccc} \mathbf{U_{D1}} & \cdots & \mathbf{U_{D}}_{N-2} & \mathbf{U_{D}}_{N-1} & \mathbf{U_{D}}_N \end{array} \right)^T .$$

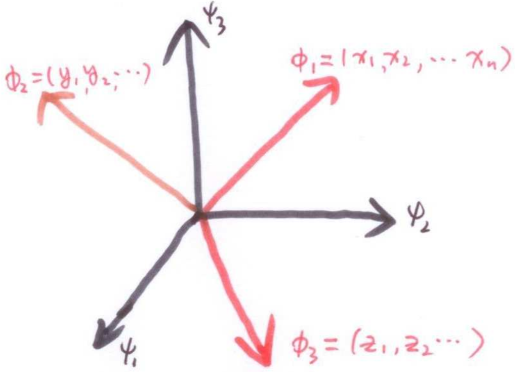
- **STEP 4:** Generate a $N-1$ unitary matrix $\mathbf{U_{R}}_{N-1}$ and a general U_D is obtained by

$$U_D = U_R U_D^S.$$

- **STEP 5:** Use these equations

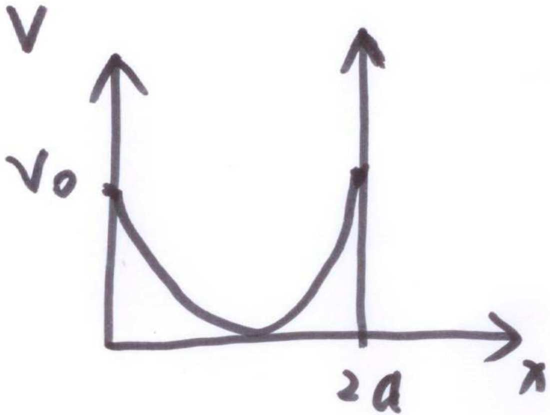
$$U_U^\dagger \;=\; U_{\rm CKMN} U_D^\dagger, \;\; M_U = Z_U M_U^D U_U^\dagger, \;\; M_D = Z_D M_D^D U_D^\dagger,$$

  to get the inputs for the flavor physics.



变分原理求解薛定谔方程

One dimension well



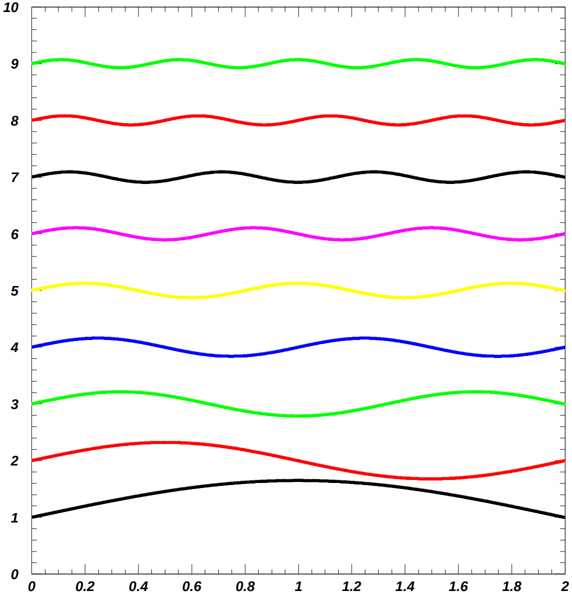


Figure: Wave of one dimension well

$B \to X_s\gamma$ process in extension of the SM with one vector like quark doublet

Motivation of vector like particles:

- Anomalies generated by the VLPs cancel automatically, and vector quarks can be heavy naturally.
- RGE

Table: A simple extension of the standard model with one vector like quarks doublet

	$\mathbf{SU(3),\ SU(2),\ U(1)}$
$Q = \begin{pmatrix} U \\ D \end{pmatrix}_L$	$\mathbf{3,\ 2,\ \frac{1}{6}}$
u_R	$\mathbf{3,\ 1,\ \frac{2}{3}}$
d_R	$\mathbf{3,\ 1,\ -\frac{1}{3}}$

	$\mathbf{SU(3),\ SU(2),\ U(1)}$
$V_Q = \begin{pmatrix} \bar{V}_d \\ \bar{V}_u \end{pmatrix}_R$	$\mathbf{\bar{3},\ 2,\ -\frac{1}{6}}$
$\bar{V}_{uL}$	$\mathbf{\bar{3},\ 1,\ -\frac{2}{3}}$
$\bar{V}_{dL}$	$\mathbf{\bar{3},\ 1,\ \frac{1}{3}}$

The lagrangian for two quarks of the model is written as:

$$\mathcal{L} = \quad Y_d \bar{Q} H d_R + Y_u \bar{Q} \cdot \bar{H} u_R + Y_{Vu} \bar{V_Q} H \bar{V}_{uL} + Y_{Vd} \bar{V_Q} \cdot \bar{H} \bar{V}_{dL} + M_Q V_q \cdot Q + M_u \bar{V}_{uL} u_R + M_d \bar{V}_{dL} d_R + h.c.,$$

in which $A \cdot B = \epsilon^{ij} A_i B_j$.

The mass matrices of up and down quarks in the basis of $(u,\ c,\ t,\ V_u)$ and $(d,\ s,\ b,\ V_d)$:

$$M_U = \begin{pmatrix} Y_u^{11}v & Y_u^{12}v & Y_u^{13}v & M_u^1 \\ Y_u^{21}v & Y_u^{22}v & Y_u^{23}v & M_u^2 \\ Y_u^{31}v & Y_u^{32}v & Y_u^{33}v & M_u^3 \\ -M_Q^1 & -M_Q^2 & -M_Q^3 & Y_{V_u}v \end{pmatrix},\ M_D = \begin{pmatrix} Y_d^{11}v & Y_d^{12}v & Y_d^{13}v & M_d^1 \\ Y_d^{21}v & Y_d^{22}v & Y_d^{23}v & M_d^2 \\ Y_d^{31}v & Y_d^{32}v & Y_d^{33}v & M_d^3 \\ M_Q^1 & M_Q^2 & M_Q^3 & Y_{V_d}v \end{pmatrix}$$

These two matrices can be diagonalized by unitary matrices U and Z,

$$Z_u^\dagger M_U U_u = \mathrm{diag.}[m_u, m_c, m_t, m_X],$$
$$Z_d^\dagger M_D U_d = \mathrm{diag.}[m_d, m_s, m_b, m_Y].$$

Product of the two matrices is denoted as

$$U_{\mathrm{CKM4}} = U_u^\dagger U_d,$$

which is unitary 4×4 matrix.

Gauge interactions:

$$\frac{1}{2}\sum_{\alpha=1}^{3}\bar{Q}_L^{\alpha}\begin{vmatrix} g_2 \not{A}^3 + \frac{g_1}{3}\not{B} & g_2(\not{A}^1 - i\not{A}^2) \\ g_2(\not{A}^1 + i\not{A}^2) & -g_2\not{A}^3 + \frac{g_1}{3}\not{B} \end{vmatrix} Q_L^{\alpha}$$

$$+\frac{2}{3}g_1\sum_{\beta=1}^{3}\bar{u}_R^{\beta}\not{B}\,u_R^{\beta} - \frac{1}{3}g_1\sum_{\beta=1}^{3}\bar{d}_R^{\beta}\not{B}\,d_R^{\beta}$$

$$-\frac{1}{2}\bar{V_Q}_R\begin{vmatrix} g_2 \not{A}^3 - \frac{g_1}{3}\not{B} & g_2(\not{A}^1 - i\not{A}^2) \\ g_2(\not{A}^1 + i\not{A}^2) & -g_2\not{A}^3 - \frac{g_1}{3}\not{B} \end{vmatrix} {V_Q}_R$$

$$+\frac{2}{3}g_1\bar{V_u}_L\not{B}\,{V_u}_L - \frac{1}{3}g_1\bar{V_d}_L\not{B}\,{V_d}_L$$

Feynman rules for the interaction of $\bar{u}_l d_j W^+$

$$\mathrm{i}\frac{g}{\sqrt{2}}\gamma^{\mu}\left[g_L^W(i,j)P_L + g_R^W(i,j)P_R\right],$$

where

$$g_L^W(i,j) = \sum_{m=1}^{3} U_u^{*mi}U_d^{m,j}, \quad g_R^W(i,j) = Z_u^{*4i}Z_d^{4j},$$

Feynman rules for the interaction of $\bar{u}_l d_j G^+$ and $\bar{d}_l d_j Z$ in the Feynman gauge:

$$\mathrm{i}\frac{g}{\sqrt{2}m_W}\left[g_L^G(i,j)P_L + g_R^G(i,j)P_R\right], \quad \mathrm{i}\frac{g}{\sqrt{2}}\gamma^\mu\left[g_L^Z(i,j)P_L + g_R^Z(i,j)P_R\right],$$

where

$$\begin{aligned}
g_L^G(i,j) &= \sum_{k,m=1}^{3} Y_u^{km} v Z_u^{*ki} U_d^{mj} + Y_{Vd} v Z_u^{*4i} U_d^{4j}, \\
g_R^G(i,j) &= -\sum_{k,m=1}^{3} Y_d^{*mk} v Z_d^{*kj} U_u^{mi} - Y_{Vu}^{*} v Z_d^{*4j} U_d^{4i}. \\
g_L^Z(i,j) &= -\frac{1}{\sqrt{2}\cos\theta_W}\left[\left(1-\frac{2}{3}\sin^2\theta_W\right)\delta^{ij} - U_d^{*4i}U_d^{4j}\right], \\
g_R^Z(i,j) &= -\frac{1}{\sqrt{2}\cos\theta_W}\left[-\frac{2}{3}\sin^2\theta_W\delta^{ij} + Z_d^{*4i}Z_d^{4j}\right].
\end{aligned}$$

The Yukawa terms can not be written into the simple form in the SM such as

$$g_L^{G,\mathrm{SM}}(i,2) = m_{u_i}V_{is}, \; g_R^{G,\mathrm{SM}}(i,3) = -m_b V_{ib}.$$

Two points:

- The CKM matrix is got from the $W^+\bar{u}_i d_j$ vertex in Eq. (1)

$$V^{ij}_{\rm CKM4} = \sum_{m=1}^{3} U_u^{*mi} U_d^{mj} = U^{ij}_{\rm CKM4} - U_u^{*4i} U_d^{4j}.$$

  which is non-unitary for that the indexes i, j range form 1 to 4, but the summation of index m is from 1 to 3. $V^{ij}_{\rm CKM4}$ is also a 4×4 matrix of which the upper left elements $(i, j\nu_e 4)$ are physical measurable value of CKM matrix V as in the SM.
- The tail terms violate the gauge universality of fermions and cause tree-level FCNC processes induced by the processes such as $b \to s\ell^+\ell^-$, then the constraints on the parameter space need to be explored.

Table: The CKM matrix elements constrained by the tree-level B decays.

	absolute value	direct measurement from
V_{ud}	0.97425 ± 0.00022	nuclear beta decay
V_{us}	0.2252 ± 0.0009	semi-leptonic K-decay
V_{ub}	0.00415 ± 0.00049	semi-leptonic B-decay
V_{cd}	0.230 ± 0.011	semi-leptonic D-decay
V_{cs}	1.006 ± 0.023	(semi-)leptonic D-decay
V_{cb}	0.0409 ± 0.0011	semi-leptonic B-decay
V_{tb}	0.89 ± 0.07	(single) top-production

In order to keep gauge universality of quarks, the tail terms in the Feynman rules must be much smaller than the SM like terms, namely

$$|Z^{4i}_{u,d}|^2_{i=1,2,3},\ |U^{4i}_{u,d}|^2_{i=1,2,3} \ll \sin^2\theta_W$$

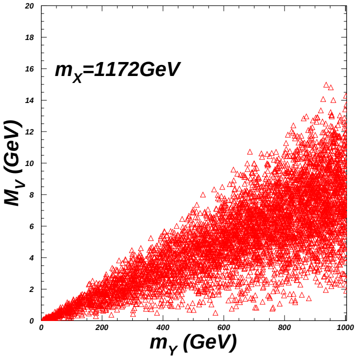


Figure: M_V versus M_Y under constraints $|Z_{u,d}^{4i}|^2_{i=1,2,3},\ |U_{u,d}^{4i}|^2_{i=1,2,3} < 10^{-4}$.

M_V Increases as m_Y growing up, it is much smaller than m_X and m_Y. The deviation from unitarity is suppressed by the ratio $m/m_{X,Y}$ where m denotes generically the standard quark masses, which is a typical result of VLP models.

Implication on B physcs: the Hamiltionian

$$\mathcal{H}_{\text{eff}} = -\frac{G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=1}^{10} [C_i(\mu) O_i(\mu) + C_i'(\mu) O_i'(\mu)] \ ,$$

in which the operators in SM are:

$$\begin{aligned}
O_1 &= (\bar{s}_i c_j)_{V-A} (\bar{c}_j b_i)_{V-A} \\
O_2 &= (\bar{s} c)_{V-A} (\bar{c} b)_{V-A} \\
O_3 &= (\bar{s} b)_{V-A} \sum_q (\bar{q} q)_{V-A} \\
O_4 &= (\bar{s}_i b_j)_{V-A} \sum_q (\bar{q}_j q_i)_{V-A} \\
O_5 &= (\bar{s} b)_{V-A} \sum_q (\bar{q} q)_{V+A} \\
O_6 &= (\bar{s}_i b_j)_{V-A} \sum_q (\bar{q}_j q_i)_{V+A} \\
O_7 &= \frac{e}{8\pi^2} m_b \bar{s}_i \sigma^{\mu\nu} (1+\gamma_5) b_i F_{\mu\nu} \\
O_8 &= \frac{g}{8\pi^2} m_b \bar{s}_i \sigma^{\mu\nu} (1+\gamma_5) T_{ij}^a b_j G_{\mu\nu}^a \\
O_9 &= (\bar{s} b)_{V-A} (\bar{l} l)_V \\
O_{10} &= (\bar{s} b)_{V-A} (\bar{l} l)_A
\end{aligned}$$

New operators and the implication

The chirality-flipped operators O_i' are obtained from O_i by the replacement $\gamma_5 \to -\gamma_5$ in quark current.

- CKM matrix is replaced by a 4×4 matrix. In our analysis we take a reasonable assumption that the deviation from unitary is not large.
- The effective coefficient $C_9^{eff}(\mu_b)$ have the same as the SM.
- The coefficient of operator $O_2' = (\overline{s}c)_{V+A}(\overline{c}b)_{V-A}$, is proportional to the elements of quark mixing matrix V_u^{4j} or U_d^{4i}. $C_9^{',eff}(\mu_b)$ receives contributions mainly from the tree-level diagrams,

- The Wilson coefficient C_9, C_{10}, C_9', and C_{10}' at the matching scale is

$$\begin{aligned} C_9 &= \frac{P(x_t)-Q(x_t)}{\sin^2\theta_W} + 4Q(x_t) \\ &- \frac{2\pi}{\alpha_{em}}\frac{U_d^{*42}U_d^{43}}{V_{tb}V_{ts}^*}\left(\frac{1}{4}-\sin^2\theta_W\right) \\ &+ \frac{1}{V_{tb}V_{ts}^*}\left\{\sum_{i=3}^{5}\left[R(x_i)g_L^{W*}(i,2)g_L^W(i,3) + S(x_i)g_R^{G*}(i,2)g_L^G(i,3)\right]\right. \\ &+ \sum_{i=1}^{5}\frac{m_W}{m_{u_i}}T(x_i)\left[g_L^{W*}(i,2)g_L^G(i,3) + g_R^{G*}(i,2)g_L^W(i,3)\right] \\ &+ \left.\frac{x_i}{y_i}S(y_i)g_R^{h*}(i,2)g_L^h(i,3)]\right\} + \frac{4}{9}. \end{aligned}$$

$$C_{10} = -\frac{P(x_t)-Q(x_t)}{\sin^2\theta_W} + \frac{2\pi}{\alpha_{em}}\frac{1}{4}\frac{U_d^{*42}U_d^{43}}{V_{tb}V_{ts}^*},$$

$$C_9' = \left(\frac{1}{4}-\sin^2\theta_W\right)\frac{2\pi}{\alpha_{em}}\frac{V_d^{*42}V_d^{43}}{V_{tb}V_{ts}^*},$$

$$C_{10}' = -\frac{2\pi}{\alpha_{em}}\frac{1}{4}\frac{V_d^{*42}V_d^{43}}{V_{tb}V_{ts}^*}.$$

- The contributions from loop diagrams to $C_{9,10}'$ can be neglected safely.

Calculation of Branching ratios:

1. $B \to X_s \gamma$

$$\mathrm{Br}(B \to X_s\gamma) = \mathrm{Br}^{\mathrm{ex}}(B \to X_c e \overline{\nu_e}) \frac{|V_{ts}^* V_{tb}|^2}{|V_{cb}|^2} \frac{6\alpha}{\pi f(z)} |C_7^{eff}(\mu_b)|^2.$$

2. $B \to X_s \ell^+ \ell^-$

$$\begin{aligned} \frac{d\Gamma(B \to X_s \ell^+\ell^-)}{ds} &= \frac{G_F^2 m_b^5}{768\pi^5} \alpha_{em}^2 |V_{tb}V_{ts}^*|^2 (1-s)^2 (1-\frac{4r}{s})^{1/2} \\ &\times \Big\{ 4|C_7^{eff}|^2 (1+\frac{2}{s}) + (|C_9^{eff}|^2 + |C_9'|^2)(1+2s) \\ &+ (|C_{10}|^2 + |C_{10}'|^2)(1+2s) + 12Re(C_7^{eff} C_9^{eff*}) \Big\} \;, \end{aligned}$$

where $s = (p_{\ell^+} + p_{\ell^-})^2 / m_b^2$.

3. $B_s \to \mu^+ \mu^-$

$$\Gamma(B_s \to \mu^+\mu^-) = \kappa \frac{\alpha_{em}^2 G_F^2}{16\pi^3} |V_{tb}V_{ts}^*|^2 f_{B_s}^2 m_{B_s} m_\mu^2 |C_{10} - C_{10}'|^2 \;, \qquad (1)$$

where f_{B_s} is the decay constant for B_s determined by $\langle 0|\overline{q}\gamma_\mu\gamma_5 b|B_q\rangle = -if_{B_q} p_\mu$.

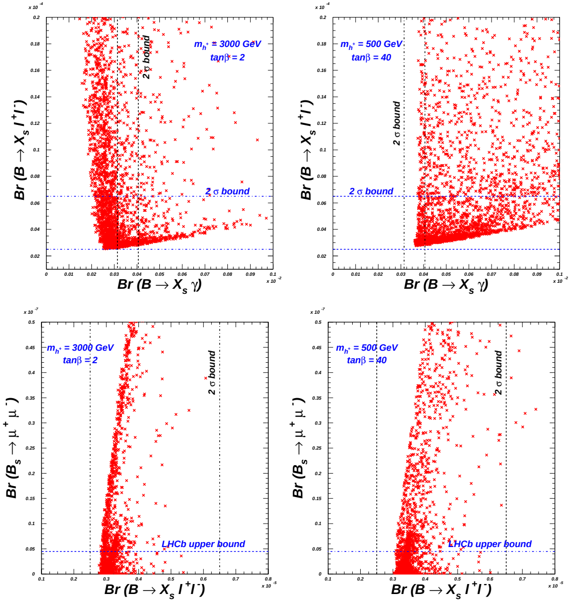


Figure: Branching ratios of $B \to X_s\gamma$, $B_s \to \mu^+\mu^-$ versus $B \to X_s\ell^+\ell^-$.

We concentrate on $B \to X_s\gamma$

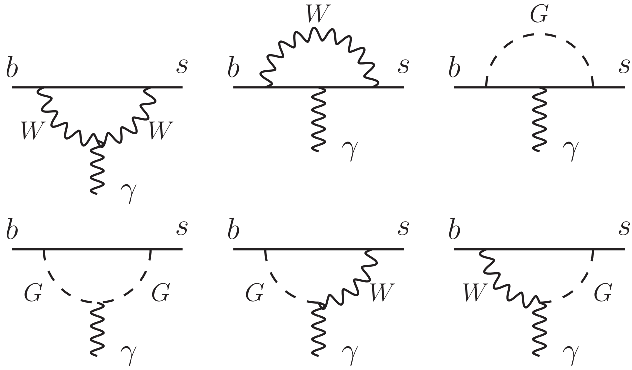


Figure: Leading order Feynman diagram of $B \to X_s\gamma$ process.

$$
\begin{aligned}
C_7(m_W) &= \frac{1}{V_{tb}V_{ts}^*}\sum_{i=1}^{4}\Big[g_L^{W*}(i,2)g_L^W(i,3)A(x_i)+\frac{g_L^{G*}(i,2)g_L^G(i,3)}{m_{u_i}^2}x_iB(x_i)\\
&\quad +\frac{g_L^{G*}(i,2)g_R^G(i,3)}{m_{u_i}m_b}x_iC(x_i)+\frac{g_L^{W*}(i,2)g_R^G(i,3)}{m_b}D(x_i)\\
&\quad +\frac{m_{u_i}}{m_b}g_L^{W*}(i,2)g_R^W(i,3)E(x_i)+\frac{g_L^{G*}(i,2)g_R^W(i,3)}{m_b}D(x_i)\Big]
\end{aligned}
$$

where $x_i = m_{u_i}^2/m_W^2$.

$$
\begin{aligned}
\frac{g_L^G(4,2)}{m_X} &= U_{CKM4}^{42}+\frac{1}{m_X}\left[\sum_{m=1}^{3}\left(M_Q^m U_d^{m2}Z_u^{*44}-M_u^m U_d^{42}Z_u^{*m4}\right)+\cdots\right],\\
\frac{g_R^G(4,3)}{m_b} &= -U_{CKM4}^{*43}-\frac{1}{m_b}\left[\sum_{m=1}^{3}\left(M_Q^{*m}U_u^{*m4}Z_d^{43}+M_d^{*m}U_u^{*44}Z_d^{m3}\right)+\cdots\right]
\end{aligned}
$$

Note that

$$
\frac{g_L^{G,\mathrm{SM}}(i,2)}{m_{u_i}} = V_{is},\quad \frac{g_R^{G,\mathrm{SM}}(i,3)}{m_b} = -V_{ib}
$$

In the SM4 the term $V_{4b}V_{4s}^*$ satisfying the unitary constraint

$$V_{ub}V_{us}^* + V_{cb}V_{cs}^* + V_{tb}V_{ts}^* + V_{4b}V_{4s}^* = 0$$

In the VLP models such relation does not exists. The suppression of Z_d^{43} (order of $m/m_{X,Y}$) are enhanced by terms with factor such as $\frac{Y_{Vu}v}{m_b}$, etc., resulting

$$\frac{g_R^G(4,3)}{m_b} \gg V_{\mathrm{CKM4}}^{43}.$$

In order to do the comparison We define two factors

$$\begin{aligned} K_1 &= \frac{g_R^G(4,3)g_L^G(4,2)^*}{m_X m_b V_{tb}V_{ts}^*} = \frac{g_R^{GVb}g_L^{GVs*}}{m_X m_b V_{tb}V_{ts}^*} \\ K_2 &= \frac{U^{43}U^{42*}}{V_{tb}V_{ts}^*} = \frac{U^{Vb}U^{Vs*}}{V_{tb}V_{ts}^*} \end{aligned}$$

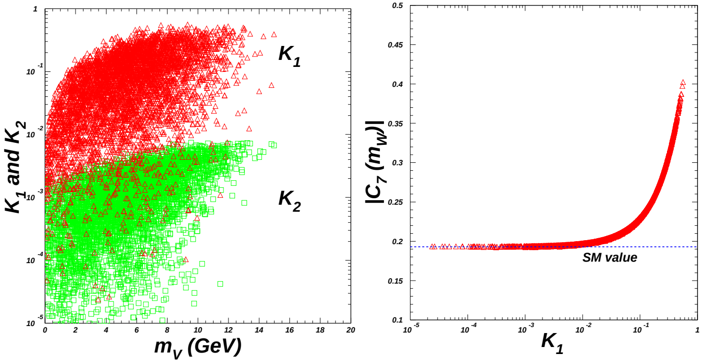


Figure: K_1, (red $\triangle$) K_2 (green $\square$) versus M_V and enhancement of $|C_7(m_W)|$ case of $|Z^{4i}_{u,d}|^2_{i=1,2,3},\ |U^{4i}_{u,d}|^2_{i=1,2,3} < 10^{-4}$ (color online).

We can see that though K_2 increase as M_V increases, it is still much smaller than $V_{tb}V^*_{ts}$, implying that deviation of unitarity are negligible. However the factor K_1 can be enhanced up to order $\mathcal{O}(1)$ by the increase of M_V.

In the numerical scan, we vary $Z_{u,d}$ and $U_{u,d}$ randomly, keeping the constraints of $|V_{u,d}^{4i}|^2_{i=1,2,3}$, $|U_{u,d}^{4i}|^2_{i=1,2,3}$, scan m_X and m_Y in the range of $(1, 2000)\mathrm{GeV}$.
The branching ratio of $B \to X_s\gamma$ is normalized by the process $B \to X_c e\bar{\nu}_e$:

$$\begin{aligned} \mathrm{Br}(B \to X_s\gamma) &= \mathrm{Br}^{\mathrm{ex}}(B \to X_c e\bar{\nu}_e)\frac{|V_{ts}^* V_{tb}|^2}{|V_{cb}|^2}\frac{6\alpha}{\pi f(z)} \\ &\times [|C_7^{\mathrm{eff}}(\mu_b)|^2 + |C_7^{\prime,\mathrm{eff}}(\mu_b)|^2]. \end{aligned}$$

We use the following bounds on the calculation

$$\mathrm{Br}^{\mathrm{ex}}(b \to ce\overline{\nu}_e) = (10.72 \pm 0.13) \times 10^{-2},$$
$$\mathrm{Br}^{\mathrm{ex}}(B \to X_s\gamma) = (3.55 \pm 0.24 \pm 0.09) \times 10^{-4}.$$

The numerical results show that the $C_7^{\prime,\mathrm{eff}}(\mu_b)$ is much smaller than $C_7^{\mathrm{eff}}(\mu_b)$, therefore we do not present the formula of $C_7^{\prime,\mathrm{eff}}(m_W)$ here.

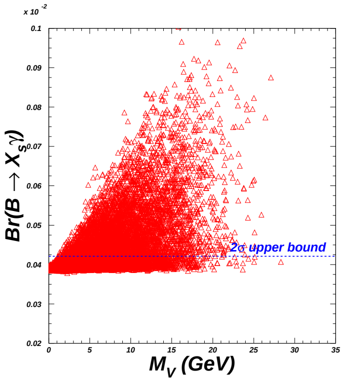


Figure: $B \to X_s\gamma$ prediction in random scan.

$\mathrm{Br}(B \to X_s\gamma)$ can be enhanced much greater than the experiment bound. Then the measurements of FCNC process can give a stringent constraint on the vector like quark model, especially when the masses of vector quark are much greater than the electro-weak scale.

Two remarks

- There is one point of view on the unitarity of the CKM matrix which is that the 3×3 ordinary quark mixing matrix is regarded as nearly unitary, deviation from unitarity is suppressed by heavy particle in the new physics beyond the SM. All the new physical effects should decouple from the flavor sector and what should be checked is that if 3×3 unitariry is consistent in all kinds of flavor processes.
- Another one is that the 3×3 ordinary quark mixing matrix elements are only extracted by experiments in the measurements of tree and loop level precesses. The unitarity should be checked, experiment measurements on the elements of matrix can be used as the constraints to the new physics beyond the SM. In the numerical analysis, the elements of CKM matrix are regarded as inputs. Thus what should be done is to scan the parameter space generally under these constraints, no prejudice should be imposed.

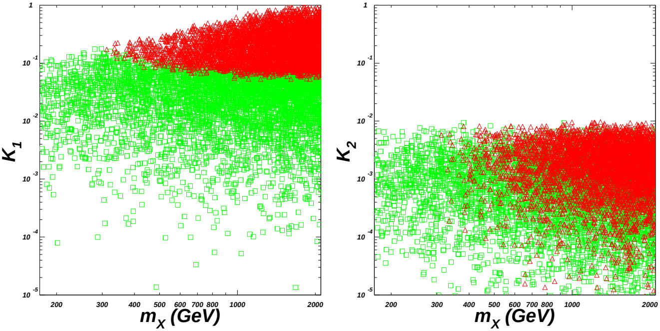


Figure: Enhancement factor and deviation from unitarity versus m_X, red $\triangle$ are excluded by the bound of $B \to X_s\gamma$ measurement which the green $\square$ are the survived points.

We can see that deviation from unitarity are very small and almost irrelevant with m_X since we are doing a general scan of $Z_{u,d}$ and $U_{u,d}$. However as However, as m_X increases up, $\mathrm{Br}(B \to X_s\gamma)$ measurement will constrain the enhancement factor and then constrain the input parameter of m_X.

Summary:

- We find a trick to deal with the scan in the model with vector doublets in which there exist bilinear terms in the lagrangian. Our scan method are exactly and the more efficient.
- Even the deviations from the unitarity of quark mixing matrix are small, the enhancement to rare B decay from VLPs are still significant. The enhanced effect is an important feature in the vector like particle model.

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Thank you!