# Insight into $f_0(980)$ through the $B_{(s)}$ charmed decays

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# Introduction

- The quark-level substructure of scalar mesons is still not well understood.
- The structure of the slight scalar mesons, such as $f_0(500)$, $f_0(980)$,$K_0^*(800)$,and $a_0(980)$, is still uncertainty (two-quark states or tetrquark states).
- If one considers these light scalar mesons as $q\bar{q}$ states, many experiments support that

$$|f_0(980)\rangle = |s\bar{s}\rangle \cos\theta + |n\bar{n}\rangle \sin\theta, \quad (1)$$

$$|f_0(500)\rangle = -|s\bar{s}\rangle \sin\theta + |n\bar{n}\rangle \cos\theta. \quad (2)$$

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# Introduction

- There are several different values for the mixing angle.

Experimental implications for the mixing angle:

$$J/\Psi \to f_0\phi, f_0\omega \Rightarrow \theta = (34 \pm 6)^\circ \text{or} \theta = (146 \pm 6)^\circ, \tag{3}$$

$$R = 4.03 \pm 0.14 \Rightarrow \theta = (25.1 \pm 0.5)^\circ \text{or} \theta = (164 \pm 0.2)^\circ, \tag{4}$$

$$R = 1.63 \pm 0.46 \Rightarrow \theta = (42.3^{+8.3}_{-5.5})^\circ \text{or} \theta = (158 \pm 2)^\circ, \tag{5}$$

$$\phi \to f_0\gamma, f_0 \to \gamma\gamma \Rightarrow \theta = (5 \pm 5)^\circ \text{or} \theta = (138 \pm 6)^\circ, \tag{6}$$

$$\text{QCD sum rules and} f_0 \text{data} \Rightarrow \theta = (27 \pm 13)^\circ \text{or} \theta = (153 \pm 13)^\circ, \tag{7}$$

$$\text{QCD sum rules and} a_0 \text{data} \Rightarrow \theta = (41 \pm 11)^\circ \text{or} \theta = (139 \pm 11)^\circ, \tag{8}$$

where $R = g^2_{f_0 K^+ K^-}/g^2_{f_0 \pi^+ \pi^-}$ is the ratio of the $f_0(980)$ coupling to $K^+K^-$ and $\pi^+\pi^-$. In short, $\theta$ lies in the ranges of $25^\circ < \theta < 40^\circ$ and $140^\circ < \theta < 165^\circ$.

# Introduction

There are several different values for the mixing angle.

- By using PQCD and QCDF approaches, Li, Du, and Lu analysised the decays

$$B_s \to J/\Psi f_0(980), J/\Psi\sigma \Rightarrow \theta \sim 34^\circ \text{or} \theta \sim 146^\circ.(2012) \tag{9}$$

- Through determining the ratio of form factors in the decay $B_s^0 \to J/\Psi f_0(980)$ with respect to $B^0 \to J/\Psi f_0(500) \Rightarrow$ S.Stone, L.Zhang obtained the mixing angle $\theta < 29^\circ$ at 90% confidence level.(2013)

- By averaging over several decay processes$\Rightarrow$ Ochs considered that $\theta = 30^\circ \pm 3^\circ$.(2013)

# Introduction

Recently, the decays were measured by the LHCb Collaboration(2015).

$$\mathcal{B}(B^0 \to \bar{D}^0 f_0(500)) = (11.2 \pm 0.8 \pm 0.5 \pm 2.1 \pm 0.5) \times 10^{-5}, \quad (10)$$
$$\mathcal{B}(B^0 \to \bar{D}^0 f_0(980)) = (1.34 \pm 0.25 \pm 0.10 \pm 0.46 \pm 0.06) \times 10^{-5}, \quad (11)$$
$$\mathcal{B}(B_s^0 \to \bar{D}^0 f_0(980)) = (1.7 \pm 1.0 \pm 0.5 \pm 0.1) \times 10^{-6}. \quad (12)$$

# Decay constants and distribution amplitudes



- For the wave functions of the $B_{(s)}$ meson,

$$\Phi_{B_{(s)}}(x,b) = \frac{1}{\sqrt{2N_c}}(\not{P}_{B_{(s)}} + m_{B_{(s)}})\gamma_5 \phi_{B_{(s)}}(x,b). \tag{13}$$

Here only the contribution of the Lorentz structure $\phi_{B_{(s)}}(x,b)$ is taken into account.

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$$\phi_{B_{(s)}}(x,b) = N_{B_{(s)}} x^2(1-x)^2 \exp\left[-\frac{M_{B_{(s)}}^2 x^2}{2\omega_b^2} - \frac{1}{2}(\omega_b b)^2\right], \tag{14}$$

where $\omega_b$ is a free parameter and taken to be $\omega_b = 0.4 \pm 0.04 (0.5 \pm 0.05)$ GeV for $B(B_s)$ in numerical calculations, and $N_B = 101.445$ ($N_{B_s} = 63.671$) is the normalization factor for $\omega_b = 0.4$ (0.5).

# Decay constants and distribution amplitudes

For the wave functions of the $D$ meson,

$$\int \frac{d^4\omega}{(2\pi)^4} e^{ik\cdot\omega} \langle 0|\bar{c}_\beta(0) u_\gamma(\omega)|\bar{D}^0\rangle = -\frac{i}{\sqrt{2N_c}} [(\not{P}_D + m_D)\gamma_5]_{\gamma\beta} \phi_D(x,b), \quad (15)$$

$$\int \frac{d^4\omega}{(2\pi)^4} e^{ik\cdot\omega} \langle 0|\bar{c}_\beta(0) u_\gamma(\omega)|\bar{D}^{*0}\rangle = -\frac{i}{\sqrt{2N_c}} [(\not{P}_{D^*} + m_{D^*})\not{\epsilon}_L]_{\gamma\beta} \phi^L_{D^*}(x,b), \quad (16)$$

where $\not{\epsilon}_L$ is the longitudinal polarization vector.

$$\phi_D(x,b) = \frac{f_D}{2\sqrt{2N_c}} 6x(1-x)[1 + C_D(1-2x)] \exp[\frac{-\omega^2 b^2}{2}]. \quad (17)$$

$f_D = 204.6$ MeV, $f_{D_s} = 257.5$ MeV, and $C_{D_{(s)}} = 0.5\ (0.4)$, $\omega_{D_{(s)}} = 0.1\ (0.2)$, $f_{D^*} = 270$ MeV and $f_{D_s^*} = 310$ MeV.

# Decay constants and distribution amplitudes

For the neutral scalar meson $f_0(980)$,vector current

$$\langle f_0(p)|\bar{q}_2\gamma_\mu q_1|0\rangle = 0, \tag{18}$$

Scalar current

$$\langle f_0(p)|\bar{q}_2 q_1|0\rangle = m_S\bar{f}_S. \tag{19}$$

Taking the $f_0(980) - \sigma$ mixing into account,

$$\langle f_0^n|d\bar{d}|0\rangle = \langle f_0^n|u\bar{u}|0\rangle = \frac{1}{\sqrt{2}}m_{f_0}\tilde{f}_{f_0}^n, \quad \langle f_0^s|s\bar{s}|0\rangle = m_{f_0}\tilde{f}_{f_0}^s, \tag{20}$$

where $\tilde{f}_{f_0}^n = \tilde{f}_{f_0}^s = \bar{f}_{f_0}$.

# Decay constants and distribution amplitudes

The twist-2 and twist-3 LCDAs for the different components of $f_0(980)$ are defined by:

$$\langle f_0(p)|\bar{q}(z)_l q(0)_j|0\rangle = \frac{1}{\sqrt{2N_c}}\int_0^1 dx e^{ixp\cdot z}\{\not{p}\Phi_{f_0}(x) + m_{f_0}\Phi^S_{f_0}(x) \tag{21}$$

$$+m_{f_0}(\not{n}_+\not{n}_- - 1)\Phi^T_{f_0}(x)\}_{jl}, \tag{22}$$

where we assume $f_0^n(p)$ and $f_0^s(p)$ are the same and denote them as $f_0(p)$, $n_+$ and $n_-$ are light-like vectors: $n_+ = (1,0,0_T), n_- = (0,1,0_T)$. And

$$\int_0^1 dx\Phi_{f_0}(x) = \int_0^1 dx\Phi^T_{f_0}(x) = 0, \quad \int_0^1 dx\Phi^S_{f_0}(x) = \frac{\bar{f}_{f_0}}{2\sqrt{2N_c}}. \tag{23}$$

# Decay constants and distribution amplitudes

- The twist-2 LCDA $\Phi_{f_0}(x)$:

$$\Phi_{f_0}(x) = \frac{1}{2\sqrt{2N_c}}\bar{f}_{f_0} 6x(1-x)\left[B_0 + \sum_{m=1} B_m C_n^{3/2}(2x-1)\right], \tag{24}$$

where $\bar{f}_{f_0} = 0.18 \pm 0.015$ GeV, $B_1 = -0.78 \pm 0.08$.

- For the twist-3 LCDA, $\Phi^S_{f_0}(x)$ and $\Phi^T_{f_0}(x)$,

$$\Phi^S_{f_0}(x) = \frac{1}{2\sqrt{2N_c}}\bar{f}_{f_0}, \quad \Phi^T_{f_0}(x) = \frac{1}{2\sqrt{2N_c}}\bar{f}_{f_0}(1-2x). \tag{25}$$

# The perturbative QCD calculation



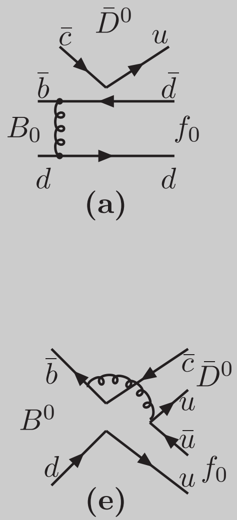


Figure 1: Diagrams contributing to the $B^0 \to \bar{D}^0 f_0(980)$ decay.

# The perturbative QCD calculation

- The weak effective Hamiltonian $H_{eff}$ for the charmed $B_{(s)}$ decays $B_{(s)} \to \bar{D} f_0(980)$, $B_{(s)} \to \bar{D}^* f_0(980)$, is composed only by the tree operators and given by:

$$H_{eff} = \frac{G_F}{\sqrt{2}} V_{cb}^* V_{uq} [C_1(\mu) O_1(\mu) + C_2(\mu) O_2(\mu)], \tag{26}$$

- the tree operators are writen as:

$$O_1 = (\bar{c}_\alpha b_\beta)_{V-A} (\bar{d}_\beta u_\alpha)_{V-A}, \quad O_2 = (\bar{c}_\alpha b_\alpha)_{V-A} (\bar{d}_\beta u_\alpha)_{V-A}, \tag{27}$$

where with $d$ represents $d(s)$.

## The perturbative QCD calculation



$$
\begin{aligned}
\mathcal{F}^{\bar{D}}_{B\to f_0} &= 8\pi C_F M_B^4 f_D \int_0^1 dx_1 dx_2 \int_0^\infty b_1 db_1 b_2 db_2\, \phi_B(x_1,b_1)[(1+x_2)\phi_{f_0}(x_2) \\
&+ r_{f_0}(1-2x_2)(\phi^s_{f_0}(x_2)+\phi^t_{f0}(x_2))]E_e(t_a)h_e(x_1,x_2(1-r_D^2),b_1,b_2) \\
&\times S_t(x_2) + 2r_{f_0}\phi_{f_s}(x_2)E_e(t_b)\ h_e(x_2,x_1(1-r_D^2),b_2,b_1)S_t(x_1)], \qquad (28)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{M}^{\bar{D}}_{B\to f_0} &= 32\pi C_f m_B^4/\sqrt{2N_C} \int_0^1 dx_{i=1,2,3} \int_0^\infty b_{i=1,2,3}\, \phi_B(x_1,b_1)\phi_D(x_3,b_3) \\
&\times \left\{ \left[(x_3-1)\phi_{f_0}(x_2) + r_{f_0}x_2(\phi^s_{f_0}(x_2)-\phi^t_{f0}(x_2)) - 4r_{f_0}r_c r_D \phi^s_{f0}(x_2)\right] \right. \\
&\times E_{en}(t_c)h^c_{en}(x_1,x_2(1-r_D^2),x_3,b_1,b_3) \\
&+ E_{en}(t_d)h^d_{en}(x_1,x_2(1-r_D^2),x_3,b_1,b_3) \\
&\times \left. \left[(x_2+x_3)\phi_{f_0}(x_2) - r_{f_0}x_2(\phi^s_{f_0}(x_2)+\phi^t_{f0}(x_2))\right] \right\}, \qquad (29)
\end{aligned}
$$

with the mass ratios $r_{f_0} = m_{f_0}/M_B, r_D = m_D/M_B$, and $r_c = m_c/M_B$.

# The perturbative QCD calculation

- The evolution factors evolving the scale $t$:

$$E_e(t) = \alpha_s(t)\exp[-S_B(t) - S_{f_0}(t)], \tag{30}$$

$$E_{en}(t) = \alpha_s(t)\exp[-S_B(t) - S_{f_0}(t) - S_D(t)|_{b_1=b_2}] \tag{31}$$

- Hard scales:

$$t_a = \max(\sqrt{x_2(1-r_D^2)}m_B, 1/b_1, 1/b_2), \tag{32}$$

$$t_b = \max(\sqrt{x_1(1-r_D^2)}m_B, 1/b_1, 1/b_2), \tag{33}$$

$$t_{c,d} = \max(\sqrt{x_1 x_2(1-r_D^2)}m_B, \sqrt{|A^2_{c,d}|}m_B, 1/b_1, 1/b_3). \tag{34}$$

# The perturbative QCD calculation



- The hard functions:

$$h_e = K_0(\sqrt{x_1 x_2} m_B b_1)\left[\theta(b_1 - b_2) K_0(\sqrt{x_2} m_B b_1) \times I_0(\sqrt{x_2 m_B b_2}) + \theta(b_2 - b_1) K_0(\sqrt{x_2 m_B b_2}) I_0(\sqrt{x_2 m_B b_1})\right], \quad (35)$$

$$h_{en}^j = \left[\theta(b_1 - b_3) K_0(\sqrt{x_1 x_2 (1 - r_D^2)} m_B b_1) I_0(\sqrt{x_1 x_2 (1 - r_D^2)} m_B b_3) + (b_1 \leftrightarrow b_3)\right] \begin{pmatrix} K_0(A_j m_B b_3) & \text{for} A_j^2 \geq 0 \\ \frac{i\pi}{2} H_0^{(1)}(\sqrt{|A_j^2|} m_B b_3) & \text{for} A_j^2 \leq 0 \end{pmatrix}, \quad (36)$$

with the variables $A_j^2 (j = c, d)$ listed as:

$$A_c^2 = r_c^2 - (1 - x_1 - x_3)(x_2(1 - r_D^2) + r_D^2), \quad (37)$$

$$A_d^2 = (x_1 - x_3) x_2 (1 - r_D^2). \quad (38)$$

- Threshold resummation leads to the quark jet function:

$$S_t(x) = \frac{2^{1+2c}\Gamma(3/2 + c)}{\sqrt{\pi}\Gamma(1 + c)} [x(1 - x)]^c, \quad (39)$$

with $c = 0.32$.

# The perturbative QCD calculation

$$\mathcal{A}(B^0 \to \bar{D}^0 f_0) = \frac{G_F}{\sqrt{2}} V_{cb}^* V_{ud} (F_{B\to f_0}^{\bar{D}} a_2 + M_{B\to f_0}^{\bar{D}} C_2 + M_{ann}^{\bar{D}} C_2 + F_{ann}^{\bar{D}} a_2),$$

$$\mathcal{A}(B^0 \to D^0 f_0) = \frac{G_F}{\sqrt{2}} V_{ub}^* V_{cd} (F_{B\to f_0}^{D} a_2 + M_{B\to f_0}^{D} C_2 + M_{ann}^{f_0} C_2 + F_{ann}^{f_0} a_2),$$

$$\mathcal{A}(B_s^0 \to \bar{D}^0 f_0) = \frac{G_F}{\sqrt{2}} V_{cb}^* V_{us} (F_{B\to f_0}^{\bar{D}} a_2 + M_{B\to f_0}^{\bar{D}} C_2 + M_{ann}^{\bar{D}} C_2 + F_{ann}^{\bar{D}} a_2),$$

$$A(B_s^0 \to D^0 f_0) = \frac{G_F}{\sqrt{2}} V_{ub}^* V_{cs} (F_{B\to f_0}^{D} a_2 + M_{B\to f_0}^{D} C_2 + M_{ann}^{f_0} C_2 + F_{ann}^{f_0} a_2),$$

$$\mathcal{A}(B^+ \to D^+ f_0) = \frac{G_F}{\sqrt{2}} V_{ub}^* V_{cd} (F_{B\to f_0}^{D} a_1 + M_{B\to f_0}^{D} C_2/3 + M_{ann}^{f_0} C_2/3 + F_{ann}^{f_0} a_1),$$

$$\mathcal{A}(B^+ \to D_s^+ f_0) = \frac{G_F}{\sqrt{2}} V_{ub}^* V_{cs} (F_{B\to f_0}^{D} a_1 + M_{B\to f_0}^{D} C_2/3 + M_{ann}^{f_0} C_2/3 + F_{ann}^{f_0} a_1).$$

## Numerical results and discussions

- Input parameters

$$f_B = 190MeV, f_{B_s} = 230MeV, M_B = 5.28GeV, M_{B_s} = 5.37GeV,$$
$$\tau_B^{\pm} = 1.638 \times 10^{-12}s, \tau_{B^0} = 1.519 \times 10^{-12}s, \tau_{B_s} = 1.512 \times 10^{-12}s,$$
$$M_{D^0} = 1.869GeV, M_{D_s^+} = 1.968GeV, M_{D^{*0}} = 2.007GeV, M_{D_s^{*+}} = 2.112GeV.$$

- CKM matrix elements

$$A = 0.814, \lambda = 0.22537, \bar{\rho} = 0.117 \pm 0.021, \bar{\eta} = 0.353 \pm 0.013.$$

# Numerical results and discussions



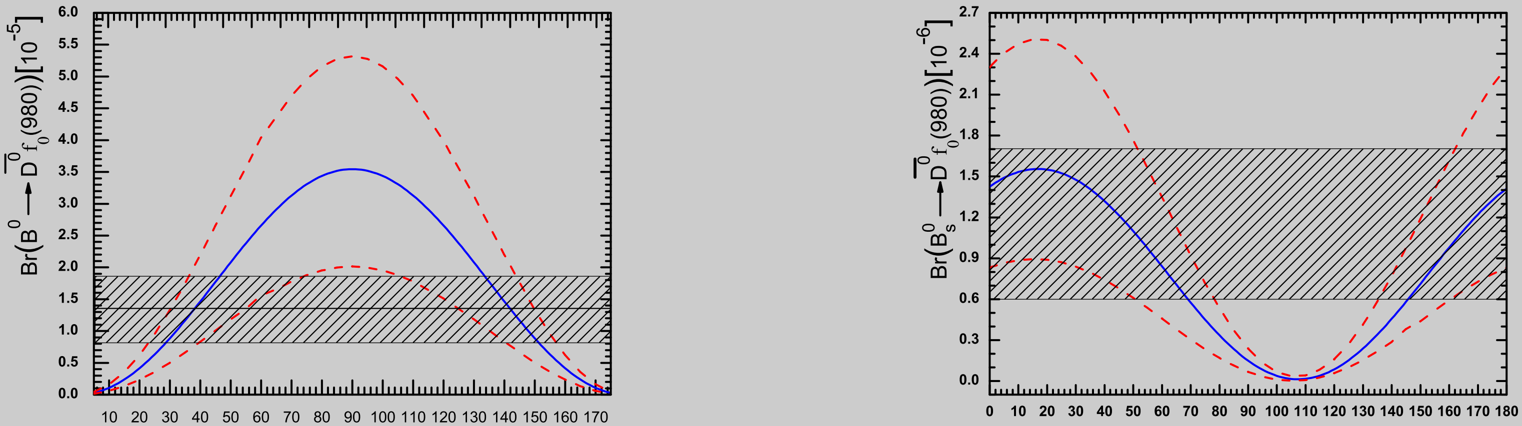


Figure 2: Dependencies of the branching ratios $\mathcal{BR}(B^0 \to \bar{D}^0 f_0(980))$(left) and $\mathcal{BR}(B_s \to \bar{D}^0 f_0(980))$ (right) on the mixing angle $\theta$. The experimental data $\mathcal{BR}(B^0 \to \bar{D}^0 f_0(980)) = (1.34 \pm 0.54) \times 10^{-5}, \mathcal{BR}(B_s \to \bar{D}^0 f_0(980)) = (1.7 \pm 1.1) \times 10^{-6}$.

## Numerical results and discussions

- $135^\circ < \theta < 158^\circ$ in the large angle region.($\theta \sim 146^\circ$)
- To explain the data of the decay $B_s \to \bar{D}^0 f_0(980)$, which is about $(1.7 \pm 1.1) \times 10^{-6}$, the small mixing angle is needed.
- $\mathcal{BR}(B_s \to \bar{D}^0 f_0(980)) = 1.56 \times 10^{-6}$ corresponds to the mixing angle $\theta = 19^\circ$.
- But too small mixing angle will make the branching ratio of the decay $B^0 \to \bar{D}^0 f_0(980)$ undershoot the shaded band from the experiment.
- Too large mixing angle, say larger than $70^\circ$, both the decays $B_s \to \bar{D}^0 f_0(980)$ and $B^0 \to \bar{D}^0 f_0(980)$ will deviate from the data even with the large errors taken into account.

# Numerical results and discussions



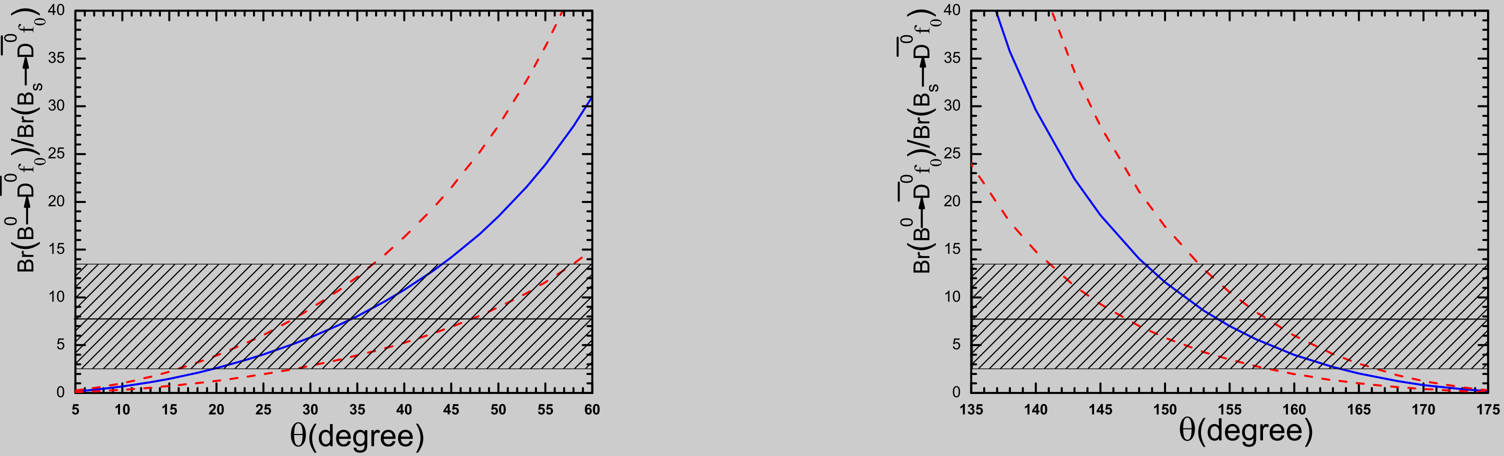


Figure 3: Dependencies of the ratio between $\mathcal{BR}(B^0 \to \bar{D}^0 f_0(980))$ and $\mathcal{BR}(B_s \to \bar{D}^0 f_0(980))$ on the mixing angle $\theta$ at different regions. The shaded band shows the allowed region and the horizontal bisector the central value of $\mathcal{BR}(B^0 \to \bar{D}^0 f_0(980))/\mathcal{BR}(B_s \to \bar{D}^0 f_0(980)) = 7.88 \pm 5.60$ for the data.

# Numerical results and discussions

- There are some advantages in considering the ratio, because one can eliminate the systematic errors on the experimental side.
- one can avoid the hadronic uncertainties, such as the decay constants and the Gegenbauer moments of the final states on the theoretical side.
- $\mathcal{BR}(B^0 \to \bar{D}^0 f_0(980))/\mathcal{BR}(B_s \to \bar{D}^0 f_0(980)) = 7.88 \pm 5.60$. The uncertainty is mainly from the statistical error in the decay $B_s \to \bar{D}^0 f_0(980)$.
- If combining these four panels, one will get two further shrunken mixing angle ranges $22^\circ < \theta < 58^\circ$ and $141^\circ < \theta < 158^\circ$.

# Numerical results and discussions

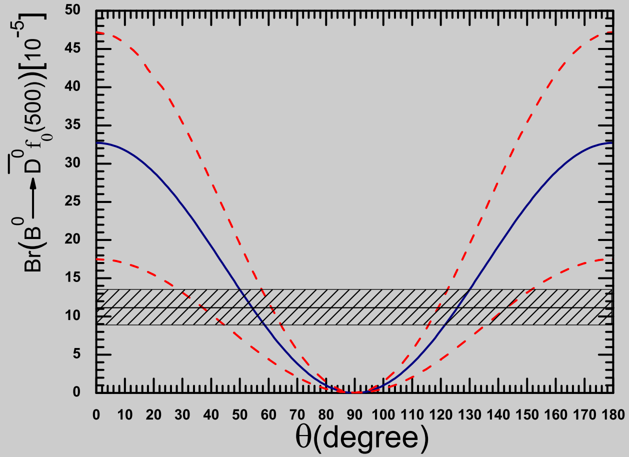


Figure 4: Dependence of the branching ratio $\mathcal{BR}(B^0 \to \bar{D}^0 f_0(500))$ on the mixing angle $\theta$. The solid (blue) curve represents the central value of the theoretical prediction, and the two dashed (red) curves correspond to the upper and lower limits. The shaded band shows the allowed region and the horizontal bisector the central value of $\mathcal{BR}(B^0 \to \bar{D}^0 f_0(500)) = (11.2 \pm 2.4) \times 10^{-5}$ for data.

## Numerical results and discussions

- there also exist two allowed mixing angle regions $28^\circ \sim 64^\circ$ and $116^\circ \sim 152^\circ$, where the former region can overlap (mostly) with the allowed region $22^\circ \sim 58^\circ$ and $141^\circ < \theta < 158^\circ$obtained from the analysis of $B^0 \to \bar{D}^0 f_0(980)$ and $B_s \to \bar{D}^0 f_0(980)$ decays.
- In order to predict other $B_{(s)}$ charmed decays, the mixing angle is taken as two values $34^\circ$ and $38^\circ$ (certainly, one can get similar branching ratios by taking $\theta = 142^\circ$ and $154^\circ$, if they can not be excluded by the future data).

# Numerical results and discussions

Table 1: The CP-averaged branching ratios ($\times 10^{-6}$) of $B \to D f_0(980)$ obtained by taking the mixing angle $\theta = 34^\circ$ and $38^\circ$, respectively. The uncertainties come from $\omega_b = 0.4 \pm 0.1 (0.5 \pm 0.1)$,$\bar{f}_{f_0} = 0.18 \pm 0.015$ GeV ,$B_1 = -0.78 \pm 0.08$,$C_{D_{(s)}} = 0.5(0.4) \pm 0.1$ respectively.

| | $34^\circ$ | $38^\circ$ |
|---|---|---|
| $\mathcal{BR}(B \to D^0 f_0)[\times 10^{-9}]$ | $4.45^{+2.25+0.96+0.71+0.35}_{-1.42-0.85-0.63-0.33}$ | $5.39^{+2.72+1.16+0.86+0.43}_{-1.71-1.04-0.77-0.41}$ |
| $\mathcal{BR}(B_s \to D^0 f_0)[\times 10^{-7}]$ | $1.32^{+1.02+0.30+0.21+0.19}_{-0.60-0.27-0.20-0.17}$ | $1.29^{+0.99+0.29+0.21+0.18}_{-0.55-0.27-0.19-0.16}$ |
| $\mathcal{BR}(B^+ \to D^+ f_0)[\times 10^{-7}]$ | $1.00^{+0.37+0.16+0.06+0.01}_{-0.26-0.15-0.06-0.01}$ | $1.22^{+0.45+0.19+0.08+0.01}_{-0.32-0.18-0.08-0.01}$ |
| $\mathcal{BR}(B^+ \to D_s^+ f_0)[\times 10^{-6}]$ | $2.30^{+0.96+0.32+0.11+0.07}_{-0.67-0.30-0.11-0.06}$ | $2.97^{+1.20+0.43+0.16+0.07}_{-0.83-0.40-0.15-0.07}$ |

# Numerical results and discussions



Table 2: The CP-averaged branching ratios ($\times 10^{-6}$) of $B \to D f_0(980)$ obtained by taking the mixing angle $\theta = 34^\circ$ and $38^\circ$, respectively. The uncertainties come from $\omega_b = 0.4 \pm 0.1(0.5 \pm 0.1)$,$\bar{f}_{f_0} = 0.18 \pm 0.015$ GeV ,$B_1 = -0.78 \pm 0.08$,$C_{D_{(s)}} = 0.5(0.4) \pm 0.1$ respectively.

| | $34^\circ$ | $38^\circ$ |
|---|---|---|
| $\mathcal{BR}(B \to \bar{D}^{*0} f_0)[\times 10^{-6}]$ | $7.40^{+2.33+1.32+2.32+0.75}_{-1.84-1.26-1.78-0.73}$ | $8.97^{+2.83+1.60+2.82+0.91}_{-2.23-1.52-2.16-0.89}$ |
| $\mathcal{BR}(B_s \to \bar{D}^{*0} f_0)[\times 10^{-6}]$ | $1.63^{+0.72+0.31+0.48+0.20}_{-0.50-0.29-0.38-0.17}$ | $1.43^{+0.62+0.27+0.42+0.17}_{-0.44-0.25-0.33-0.15}$ |
| $\mathcal{BR}(B \to D^{*0} f_0)[\times 10^{-9}]$ | $6.48^{+3.57+1.37+0.64+0.33}_{-2.24-1.23-0.56-0.31}$ | $7.86^{+4.33+1.66+0.78+0.40}_{-2.72-1.49-0.68-0.37}$ |
| $\mathcal{BR}(B_s \to D^{*0} f_0)[\times 10^{-7}]$ | $2.06^{+1.79+0.46+0.20+0.20}_{-0.98-0.41-0.18-0.17}$ | $1.94^{+1.63+0.44+0.19+0.19}_{-0.90-0.39-0.17-0.16}$ |
| $\mathcal{BR}(B^+ \to D^{*+} f_0)[\times 10^{-7}]$ | $2.07^{+0.69+0.38+0.16+0.02}_{-0.49-0.34-0.15-0.02}$ | $2.51^{+0.84+0.46+0.19+0.02}_{-0.60-0.42-0.19-0.03}$ |
| $\mathcal{BR}(B^+ \to D_s^{*+} f_0)[\times 10^{-6}]$ | $5.00^{+1.68+0.94+0.37+0.07}_{-1.21-0.88-0.39-0.06}$ | $6.10^{+2.04+1.14+0.45+0.08}_{-1.47-1.06-0.47-0.10}$ |

# Numerical results and discussions

- From our calculations, we find that the branching ratios of the $B_s$ decays are not very sensitive to the mixing angle $\theta$, especially for $\mathcal{BR}(B_s \to D^0 f_0)$, its value changes in the range of $(1.2 \sim 1.8) \times 10^{-7}$ when the mixing angle varies from $0°$ to $180°$.
- By contrast, the branching ratios of the B decays are more sensitive to the mixing angle $\theta$.
- The branching ratios of all the $B$ decay modes are dependent on the mixing angle via $\sin\theta$ with a initial phase, while those of $B_s$ decay modes are dependent on the mixing angle via $\cos\theta$ with a initial phase

# Summary

- We analyze the decays $B \to \bar{D}^0 f_0(980)$ and $B_s \to \bar{D}^0 f_0(980)$ carefully in the PQCD factorization approach and find two possible regions for the mixing angle $\theta$, one is centered at $34^\circ \sim 38^\circ$ and the other is near $142^\circ \sim 154^\circ$.
- Our analyses support that the two quark component in $f_0(980)$ is dominant in B decay dynamic mechanism, and the $s\bar{s}$ component is more important than the $q\bar{q}$ component.
- The four-quark structure is dominant in explaining the mass degeneracy of $f_0(980)$ and $a_0(980)$, and the narrower decay width of $f_0(980)$ than that of $f_0(500)$.
- Other components, such as gluon, and $K\bar{K}$ threshold effect may also give some more or less influences.

Thank you for your attention!