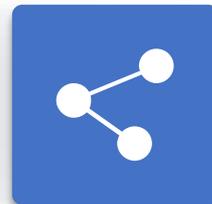


最大共形原理的 单能标实现方案

申建明

重庆大学



指导老师： 吴兴刚

全国第十四届重味物理和CP破坏研讨会



研究背景

Research Backgrounds

1

最大共形原理

Principle of Maximum Conformality, PMC

2

PMC单能标方案

Systematic single-scale approach to achieve PMC

3

4

实例: $R_{e^+e^-}$, $H \rightarrow b\bar{b}$

Examples

5

总结与展望

Summary and Outlook



1

研究背景



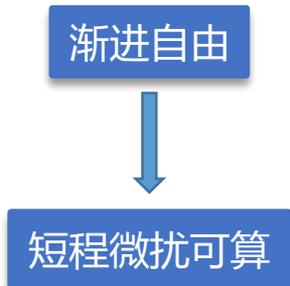
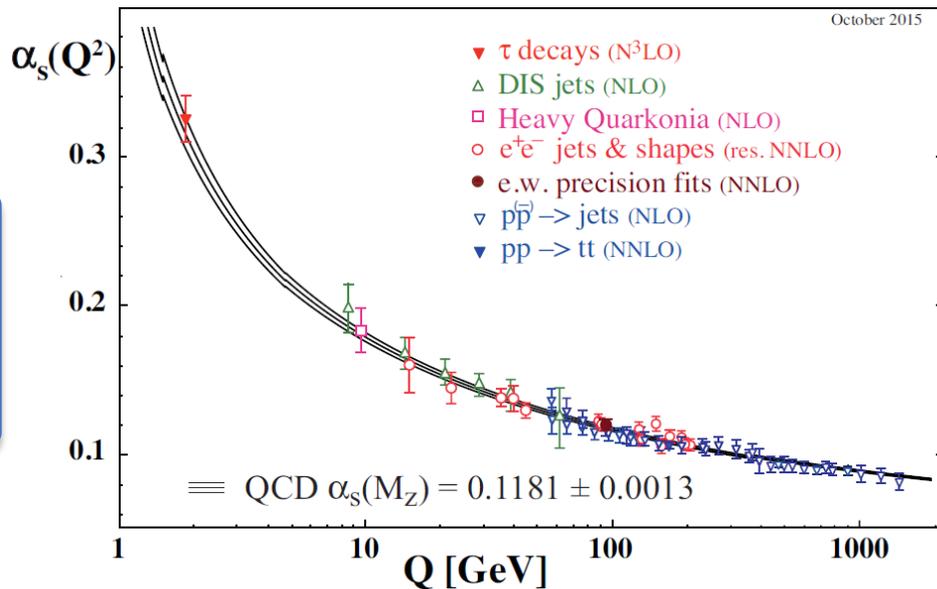
四种基本
相互作用

作用	强度	力程	媒介子	作用粒子	束缚态
强	1	$10^{-15} m$	胶子	夸克, 胶子	强子
电磁	1/137	$F \propto 1/r^2$	光子	带电粒子	原子
弱	10^{-5}	$< 10^{-17} m$	W^\pm, Z_0	费米子	无
引力	10^{-39}	$F \propto 1/r^2$	引力子?	所有粒子	星系

肖振军、吕才典著《粒子物理学导论》

$$\alpha_s = g_s^2 / (4\pi)$$

量子色动力学
(QCD) 描述
强相互作用



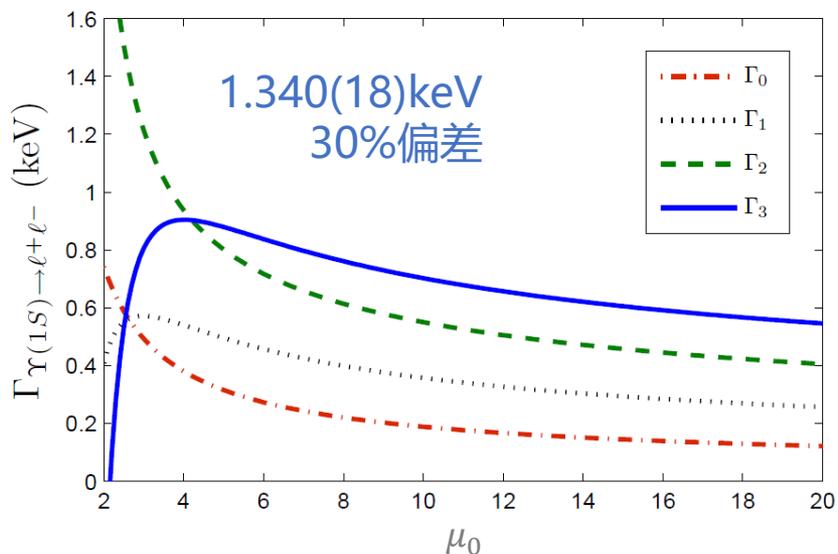


高阶微扰论计算中的重整化能标设定问题

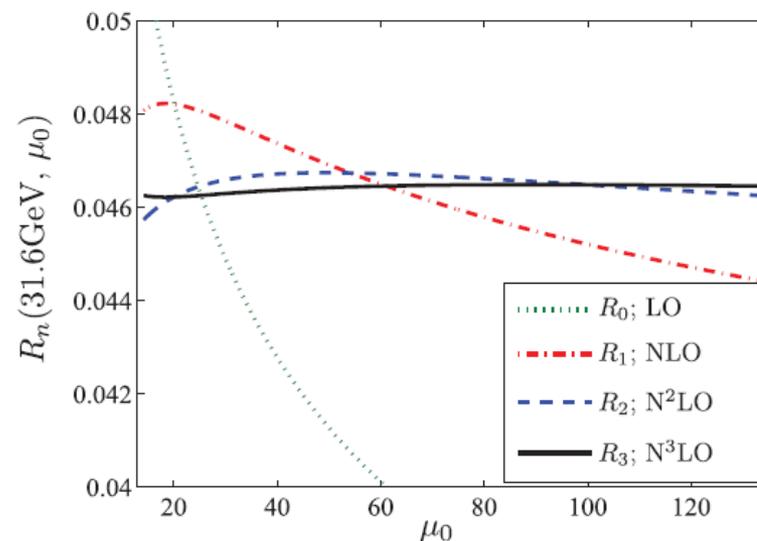
重整化群不变性
的基本要求

$$\frac{\partial \rho(Q)}{\partial \ln \mu^2} \equiv 0, \quad \frac{\partial \rho(Q)}{\partial RS} \equiv 0.$$

任意有限阶微扰计算中，理论预言都依赖于重整化方案和重整化能标的选择



M. Beneke et al., Phys. Rev. Lett. 112 (2014) 151801



Baikov et al., Phys. Rev. Lett. 101 (2008) 012002

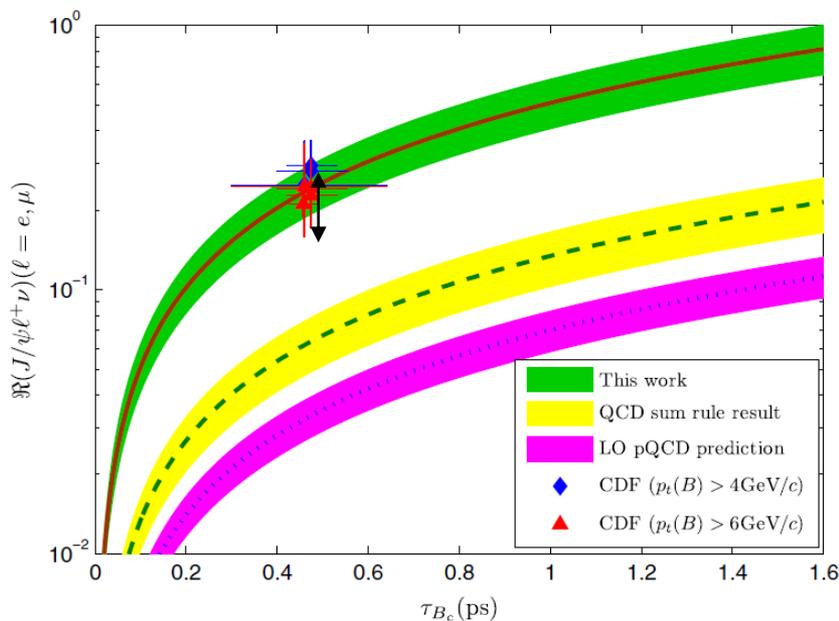


解决的重整化能标不确定性问题：最大共形原理

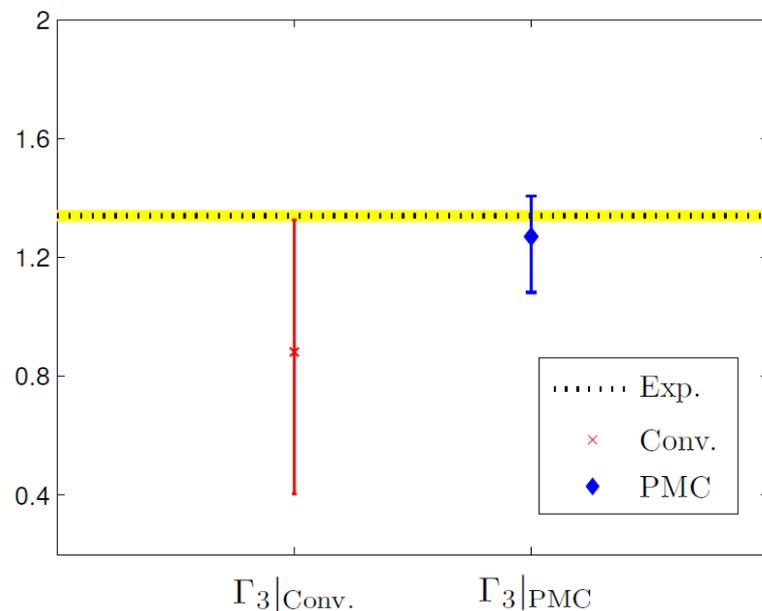
$$\mathfrak{R}(J/\psi\ell^+\nu) = \frac{\sigma(B_c^+) \text{BR}(B_c^+ \rightarrow J/\psi\ell^+\nu)}{\sigma(B^+) \text{BR}(B^+ \rightarrow J/\psi K^+)}$$

$$\Gamma_{\Upsilon(1S) \rightarrow \ell^+\ell^-} |_{\text{PMC}} = 1.270^{+0.130+0.043}_{-0.182-0.042} \pm 0.015 \text{ keV}$$

$$\Gamma_{\Upsilon(1S) \rightarrow e^+e^-} |_{\text{Exp.}} = 1.340(18) \text{ keV}$$



Jian-Ming Shen, et al, PRD90(2014)034025



Jian-Ming Shen, et al, JHEP06(2015)169



Front. Phys. 11(1), 111201 (2016)
DOI 10.1007/s11467-015-0518-5

Progress in Particle and Nuclear Physics 72 (2013) 44–98

REVIEW ARTICLE

Importance of pro



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Rep. Prog. Phys. 78 (2015) 126201 (15pp)

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Review

Renormalization group invariance and optimal QCD renormalization scale-setting: a key issues review

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Xing-Gang Wu¹, Yang Ma¹, Sheng-Quan Wang¹, Hai-Bing Fu¹,
Hong-Hao Ma¹, Stanley J Brodsky² and Matin Mojaza³

¹ Department of Physics, Chongqing University, Chongqing 401331, People's Republic of China

² SLAC National Accelerator Laboratory, Stanford University, Stanford, CA 94309, USA

³ CP3-Origins, Danish Institute for Advanced Studies, University of Southern Denmark, DK-5230,



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最大共形原理



基本思想

物理量的pQCD展开级数中包含共形项和非共形项(β 项), 基于重整化群不变性和重整化群方程(RGE), 可根据 β 项确定微扰每一阶的强耦合常数的准确跑动行为, 从而确定重整化能标

$$\text{RGE} \quad \mu_\delta^2 \frac{d\alpha}{d\mu_\delta^2} = \beta(\alpha) = -\alpha(\mu_\delta)^2 \sum_{i=0}^{\infty} \beta_i \alpha(\mu_\delta)^i \quad \alpha = \frac{\alpha_s}{4\pi}$$

BLM机制(1983年)

PMC思想(2011年)
(Brodsky, Wu)PMC I (2012年)
(Brodsky, Wu)PMC II(2013年)
(Brodsky, Mojaza, Wu)

基本性质

- 满足重整化群不变性基本要求;
- 消除重整化能标不确定性;
- 给出每一微扰阶的准确预言;
- 求和了高阶 β 项, 给出更精确的理论预言;
- 改善微扰级数的收敛性



PMC多能标方案

根据RGE，分别确定每一阶的耦合常数的行为

$$\rho(Q) = \underline{r_{1,0}}\alpha(\mu)^p + [\underline{r_{2,0}} + p\underbrace{\beta_0 r_{2,1}}] \alpha(\mu)^{p+1} + \left[\underline{r_{3,0}} + p\underbrace{\beta_1 r_{2,1}} + (p+1)\beta_0 r_{3,1} + \frac{p(p+1)}{2}\beta_0^2 r_{3,2} \right] \alpha(\mu)^{p+2}$$

$$+ \left[\underline{r_{4,0}} + p\underbrace{\beta_2 r_{2,1}} + (p+1)\beta_1 r_{3,1} + \frac{p(3+2p)}{2}\beta_1 \beta_0 r_{3,2} + (p+2)\beta_0 r_{4,1} + \frac{(p+1)(p+2)}{2}\beta_0^2 r_{4,2} \right. \\ \left. + \frac{p(p+1)(p+2)}{3!}\beta_0^3 r_{4,3} \right] \alpha(\mu)^{p+3} + \dots,$$

$$r_{1,0}\alpha^p(Q_1) \xleftarrow{\text{PMC}} r_{1,0}\alpha^p(\mu) + p\beta_0 r_{2,1}\alpha^{p+1}(\mu) + \left[p\beta_1 r_{2,1} + \frac{p(p+1)}{2}\beta_0^2 r_{3,2} \right] \alpha^{p+2}(\mu) + \left[p\beta_2 r_{2,1} + \frac{p(3+2p)}{2}\beta_1 \beta_0 r_{3,2} + \frac{p(p+1)(p+2)}{3!}\beta_0^3 r_{4,3} \right] \alpha^{p+3}(\mu) + \dots$$

.....

$$N^{n-1}LO \text{ 的PMC能标 } Q_n \quad \ln \frac{Q_n^2}{\mu^2} + \Delta_n^{(1)} \ln^2 \frac{Q_n^2}{\mu^2} + \Delta_n^{(2)} \ln^3 \frac{Q_n^2}{\mu^2} + \dots = R_{n,1} + \Delta_n^{(1)} R_{n,2} + \Delta_n^{(2)} R_{n,3} + \dots$$

$$R_{n,j} = (-1)^j \frac{r_{n+j,j}}{r_{n,0}}, \quad \Delta_n^{(1)} = \frac{1}{2} \left[\frac{\partial \beta}{\partial \alpha} + (p+n-2) \frac{\beta}{\alpha} \right], \quad \Delta_n^{(2)} = \frac{1}{3!} \left[\left(\frac{\partial \beta}{\partial \alpha} \right)^2 + \beta \frac{\partial^2 \beta}{(\partial \alpha)^2} + 3(p+n-2) \frac{\beta}{\alpha} \frac{\partial \beta}{\partial \alpha} + (p+n-2)(p+n-3) \frac{\beta^2}{\alpha^2} \right]$$

PMC理论预言 $\rho(Q) = r_{1,0}\alpha^p(Q_1) + r_{2,0}\alpha^{p+1}(Q_2) + r_{3,0}\alpha^{p+2}(Q_3) + r_{4,0}\alpha^{p+3}(Q_4) + \dots$

对应于共形理论，消除（大大抑制）了理论预言对重整化能标和方案的依赖

- 残留的能标依赖(很小)：A、各阶PMC能标的残留依赖；B、最后一阶的能标的不确定性



3

PMC单能标方案



基本想法

根据RGE，确定等效的所有阶耦合常数的行为

$$\rho(Q) = r_{1,0}\alpha(\mu)^p + [r_{2,0} + p\beta_0 r_{2,1}]\alpha(\mu)^{p+1} + \left[r_{3,0} + p\beta_1 r_{2,1} + (p+1)\beta_0 r_{3,1} + \frac{p(p+1)}{2}\beta_0^2 r_{3,2} \right] \alpha(\mu)^{p+2} \\ + \left[r_{4,0} + p\beta_2 r_{2,1} + (p+1)\beta_1 r_{3,1} + \frac{p(3+2p)}{2}\beta_1\beta_0 r_{3,2} + (p+2)\beta_0 r_{4,1} + \frac{(p+1)(p+2)}{2}\beta_0^2 r_{4,2} \right. \\ \left. + \frac{p(p+1)(p+2)}{3!}\beta_0^3 r_{4,3} \right] \alpha(\mu)^{p+3} + \dots,$$

$$\rho(Q) = \sum_{n \geq 1} r_{n,0} \alpha(\mu)^{n+p-1} + \sum_{n \geq 1} [(n+p-1)\alpha(\mu)^{n+p-2}\beta] \sum_{j \geq 1} (-1)^j \Delta_n^{(j-1)} r_{n+j,j}$$

根据重整化群不变性, $r_{i,j} = \sum_{k=0}^j C_j^k \tilde{r}_{i-k,j-k} L^k$ 其中, $\tilde{r}_{i,j} = r_{i,j}|_{\mu=Q}$ $L = \ln \mu^2 / Q^2$

$$\rho(Q) = \sum_{n \geq 1} \tilde{r}_{n,0} \alpha(\mu)^{n+p-1} + \sum_{n \geq 1} [(n+p-1)\alpha(\mu)^{n+p-2}\beta] \sum_{j \geq 1} (-1)^j \Delta_n^{(j-1)} \tilde{r}_{n+j,j} \\ + \sum_{k \geq 1} L^k \sum_{n \geq 1} [(n+p-1)\alpha(\mu)^{n+p-2}\beta] \sum_{j \geq k} (-1)^j C_j^k \Delta_n^{(j-1)} \tilde{r}_{n+j-k,j-k},$$

PMC单能标方法
给出的理论预言

$$\rho(Q) = \sum_{n \geq 1} \tilde{r}_{n,0} \alpha(Q_*)^{n+p-1}$$



PMC能标满足如下方程，

$$\sum_{n \geq 1} [(n+p-1)\alpha(Q_\star)^{n+p-2}\beta(\alpha(Q_\star))] \sum_{j \geq 1} (-1)^{j-1} \tilde{r}_{n+j,j} \Delta_n^{(j-1)}(\alpha(Q_\star))$$

$$= \sum_{k \geq 1} \ln^k \frac{Q_\star^2}{Q^2} \sum_{n \geq 1} [(n+p-1)\alpha(Q_\star)^{n+p-2}\beta(\alpha(Q_\star))] \sum_{j \geq k} (-1)^j C_j^k \tilde{r}_{n+j-k,j-k} \Delta_n^{(j-1)}(\alpha(Q_\star))$$

PMC能标 (微扰解) $\ln \frac{Q_\star^2}{Q^2} = S_0 + S_1 \alpha(Q_\star) + S_2 \alpha^2(Q_\star) + \dots$

NLO, $S_0 = -\frac{\tilde{r}_{2,1}}{\tilde{r}_{1,0}}$ 与PMC多能标方案一致!

NNLO, $S_1 = \frac{(p+1)(\tilde{r}_{2,0}\tilde{r}_{2,1} - \tilde{r}_{1,0}\tilde{r}_{3,1})}{p\tilde{r}_{1,0}^2} + \frac{(p+1)(\tilde{r}_{2,1}^2 - \tilde{r}_{1,0}\tilde{r}_{3,2})}{2\tilde{r}_{1,0}^2} \beta_0,$

NNNLO, $S_2 = -\frac{(p+1)^2\tilde{r}_{2,0}^2\tilde{r}_{2,1} - (p+1)^2\tilde{r}_{1,0}\tilde{r}_{2,0}\tilde{r}_{3,1} + p(p+2)\tilde{r}_{1,0}(\tilde{r}_{1,0}\tilde{r}_{4,1} - \tilde{r}_{2,1}\tilde{r}_{3,0})}{p^2\tilde{r}_{1,0}^3}$

$$-\frac{p(p+1)(2p+3)\tilde{r}_{2,0}\tilde{r}_{2,1}^2 - p(p+1)^2\tilde{r}_{1,0}\tilde{r}_{2,0}\tilde{r}_{3,2} + p(p+1)(p+2)\tilde{r}_{1,0}(\tilde{r}_{1,0}\tilde{r}_{4,2} - 2\tilde{r}_{2,1}\tilde{r}_{3,1})}{2p^2\tilde{r}_{1,0}^3} \beta_0$$

$$-\frac{(p+1)(p+2)(2\tilde{r}_{2,1}^3 - 3\tilde{r}_{1,0}\tilde{r}_{2,1}\tilde{r}_{3,2} + \tilde{r}_{1,0}^2\tilde{r}_{4,3})}{6\tilde{r}_{1,0}^3} \beta_0^2 - \frac{(p+2)(\tilde{r}_{1,0}\tilde{r}_{3,2} - \tilde{r}_{2,1}^2)}{2\tilde{r}_{1,0}^2} \beta_1,$$



4

实例: $R_{e^+e^-}$, $H \rightarrow b\bar{b}$



$$R_{e^+e^-}(Q) = \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} = 3 \sum_q e_q^2 [1 + R(Q)]$$

$$R_n(Q, \mu) = \sum_{i=0}^n C_i(Q, \mu) \alpha^{i+1}(\mu)$$

$Q = 31.6$ GeV时的数值结果。对于Conv. (传统方法) 和PMC (多能标), 输入 $\mu = Q$

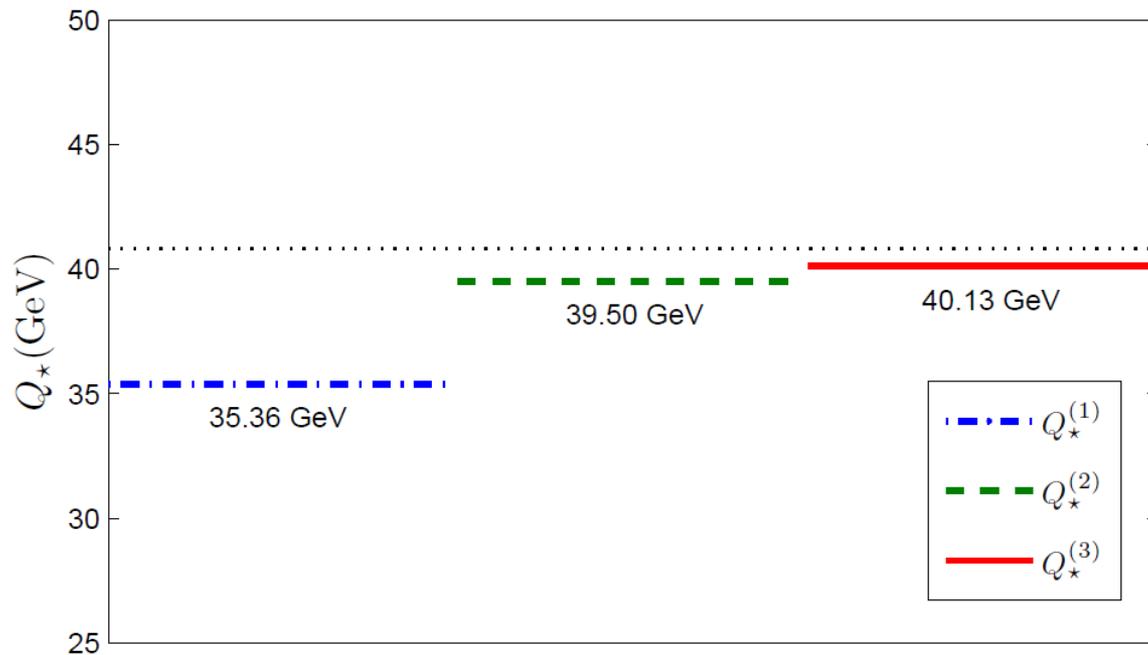
	LO	NLO	N ² LO	N ³ LO	total			
各阶 贡献	Conv.	0.04482	0.00283	-0.00115	-0.00033	0.04617		
	PMC	0.04275	0.00350	-0.00004	-0.00002	0.04619	等效性	
	PMC-s	0.04295	0.00340	-0.00008	-0.00004	0.04623		
	R_1	R_2	R_3	κ_1	κ_2	κ_3		
	Conv.	0.04763	0.04648	0.04617	7.36%	-2.43%	-0.66%	$\kappa_n = \frac{R_n - R_{n-1}}{R_{n-1}}$
	PMC	0.04745	0.04649	0.04619	6.96%	-2.03%	-0.64%	
	PMC-s	0.04745	0.04638	0.04623	6.96%	-2.25%	-0.35%	

随着计算阶数的增加, 理论预言快速收敛

微扰级数收敛性的体现!



$$\ln \frac{Q_*^2}{Q^2} = 0.22489 + 20.570\alpha(Q_*) + 281.16\alpha^2(Q_*)$$



Q=31.6GeV时，计算到不同精度下的PMC能标

随着计算精度的提高(LO→NLO→NNLO)，PMC能标快速收敛

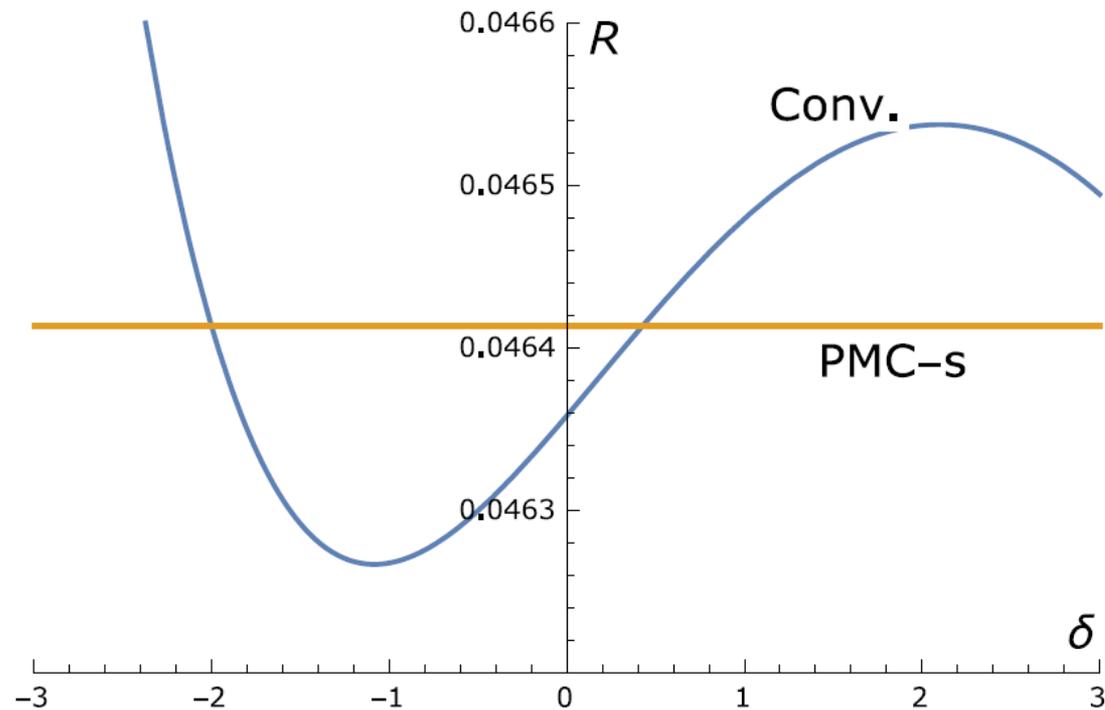


MS-like方案 \mathcal{R}_δ $\ln(4\pi) - \gamma_E - \delta$

$$\mathcal{R}_0 = \overline{\text{MS}},$$

$$\mathcal{R}_{\ln 4\pi - \gamma_E} = \text{MS},$$

$$\mathcal{R}_{-2} = \text{G},$$



$R_{e^+e^-}$ 的QCD修正部分 (4-loop)



$$\Gamma_{H \rightarrow b\bar{b}} = \frac{3G_F M_H m_b^2(M_H)}{4\sqrt{2}\pi} (1 + \tilde{R}) \quad \tilde{R}_n = \sum_{i=0}^n \tilde{C}_i a_s^{i+1}(M_H)$$

数值结果。 $M_H = 126$ GeV。对于Conv. (传统方法) 和PMC (多能标), 输入 $\mu = Q = M_H$

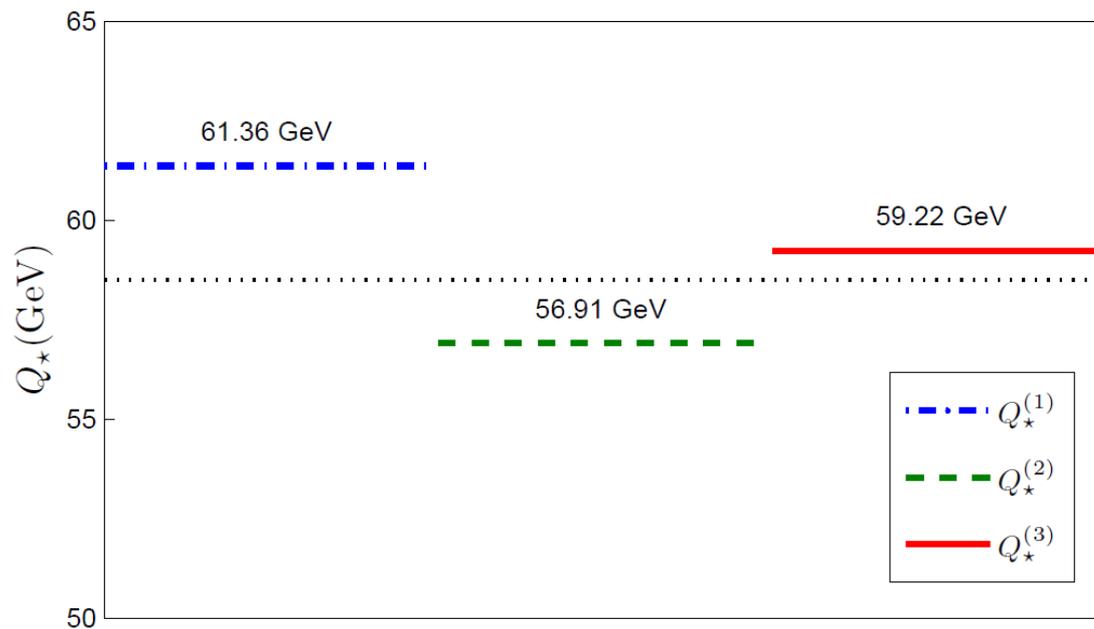
		\tilde{R}_1	\tilde{R}_2	\tilde{R}_3	$\tilde{\kappa}_1$	$\tilde{\kappa}_2$	$\tilde{\kappa}_3$
$\tilde{\kappa}_n = \frac{\tilde{R}_n - \tilde{R}_{n-1}}{\tilde{R}_{n-1}}$	Conv.	0.24056	0.24247	0.24109	18.19%	0.80%	-0.57%
	PMC	0.24823	0.24039	0.24017	21.96%	-3.16%	-0.09%
	PMC-s	0.24823	0.24253	0.23984	21.96%	-2.30%	-1.11%

	LO	NLO	N ² LO	N ³ LO	<i>total</i>
Conv.	0.20309	0.03744	0.00192	-0.00136	0.24109
PMC	0.22599	0.02474	-0.00934	-0.00121	0.24017
PMC-s	0.22795	0.02187	-0.00885	-0.00113	0.23984

PMC单能标方案和多能标方案的等效性



$$\ln \frac{\tilde{Q}_*^2}{M_H^2} = -1.4389 - 14.887\alpha(\tilde{Q}_*) + 776.20\alpha^2(\tilde{Q}_*)$$



不同计算精度下的PMC能标 (对应于2loop、3loop、4loop的能标)

随着计算阶数的增加，PMC能标快速收敛



5

总结与展望



PMC多能标实现方案

确定每一阶 α_s 的行为

- PMC能标的每一高阶均存在残留能标依赖
- 最后一阶的能标无法确定
- 残留依赖小

(基于RGI、RGE)

最大共形原理

通过系统的 β 项重求和确定能标，给出与方案-能标无关的理论预言

确定等效的所有阶 α_s 的行为

- PMC能标的每一微扰阶都与重整化方案和能标无关
- 等效性、更简单易用
- 无残留依赖

PMC单能标实现方案



最大共形原理(PMC)的相关问题

方案相关：

- CSR(Commensurate Scale Relations)关系及方案依赖问题
- 不同方案之间QCD标度 Λ_{QCD} 的关系(CG relation)
- 方案变化的表征(比如用一个参数表示方案变化)

规范相关：MOM-like方案的规范依赖问题

耦合常数在低能区的行为

如何更自洽的估算未知高阶项的贡献(如, 取已知最高阶)

PMC自动化：总截面，微分截面，

Thank You



全国第十四届重味物理和CP破坏研讨会