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TWO-BODY D TO VP DECAYS

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HY Cheng and CWC, PRD **81**, 074021(2010)
HY Cheng, CWC and AL Kuo, PRD **93**, 114010 (2016)

OUTLINE OF OUR WORK

- An update of previous work D → VP decays
- Use of SU(3) symmetry as working assumption
- A global χ^2 fit to Cabibbo-favored modes
- Extraction of weak annihilation amplitudes for the first time and seeing their importance
- Predictions for all D → VP branching fractions
- A test of flavour SU(3) symmetry

PECULIARITIES OF CHARM SYSTEMS

- Resides at an awkward place in mass spectrum
  ➠ no suitable effective theory to work with, particularly for hadronic decays
- Too light to grant reliable heavy-quark expansions; yet too heavy to use chiral perturbation theory
- Strong QCD coupling regime
  ➠ perturbative QCD calculations expected to fail
- Many resonances around
  ➠ nonperturbative rescattering effects kicking in
- Flavor SU(3) symmetry for decays to light mesons
- Good realm to test all these approaches

DOMINANT CHARM DECAYS

- D mesons decay dominantly (~84%) into hadronic final states, 3/4 of which are two-body modes.
  ➠ cf. B meson decays

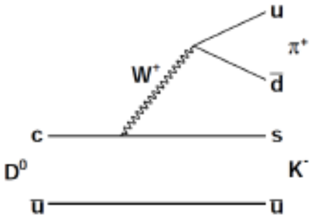
Mode	BR
PP	$\sim 10\%$
VP	$\sim 28\%$
VV	$\sim 10\%$
SP	$\sim 4.2\%$
AP	$\sim 10\%$
TP	$\sim 0.3\%$
2-body	$\sim 63\%$
hadronic	$\sim 84\%$
semileptonic	$\sim 16\%$

– most dominant ones

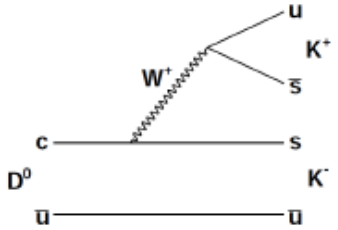
P: pseudoscalar meson
V: vector meson
A: axial vector meson
T: tensor meson

TWO-BODY HADRONIC CHARM DECAYS

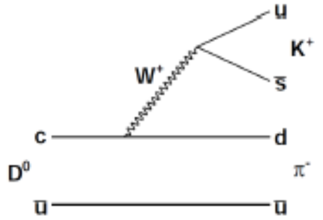
- Cabibbo-favored (CF):
  involving $V_{ud}{}^*V_{cs} \sim 1-\lambda^2 \sim 0.95$



- Singly Cabibbo-suppressed (SCS):
  involving $V_{us}{}^*V_{cs}$ / $V_{ud}{}^*V_{cd} \sim \lambda \sim 0.22$



- Doubly Cabibbo-suppressed (DCS):
  involving $V_{us}{}^*V_{cd} \sim \lambda^2 \sim 0.05$



- Only SCS decays can possibly involve diagrams with different CKM phases and thus possibly have CPA's:

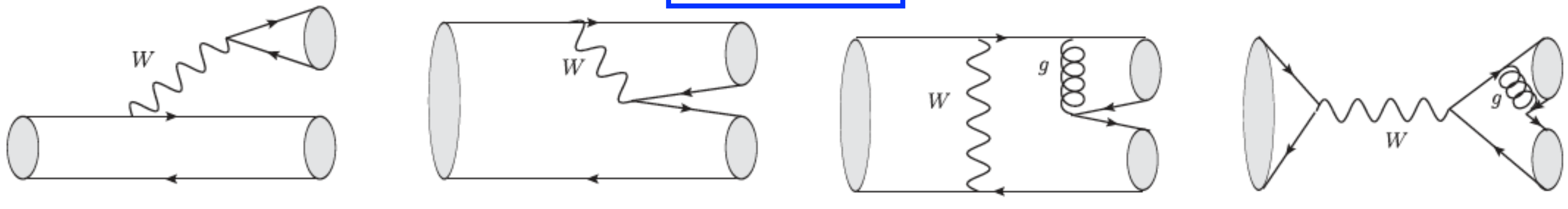
$$\mathrm{Amp} = V_{cd}^* V_{ud}(\text{trees} + \text{penguins}) \\ + V_{cs}^* V_{us}(\text{trees} + \text{penguins})$$

FLAVOR DIAGRAMS

- Diagrams for 2-body hadronic D meson decays can be classified according to flavor topology into the tree- and loop-types:

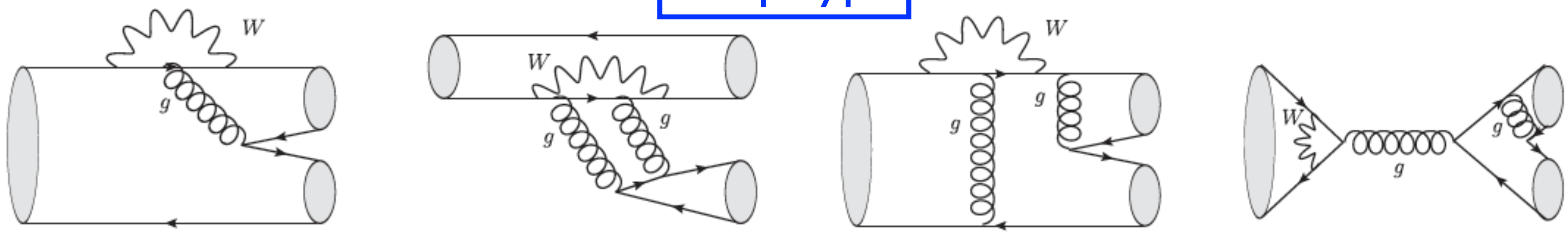
Zeppenfeld 1981
Chau and Cheng 1986, 1987, 1991
Savage and Wise 1989
Grinstein and Lebed 1996
Gronau et. al. 1994, 1995, 1995
Cheng and Oh 2011

Tree-type



(a) T (b) C (e) E (f) A

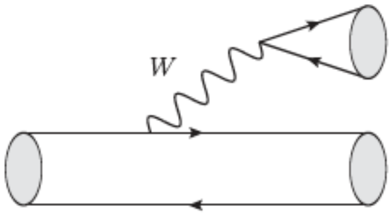
Loop-type



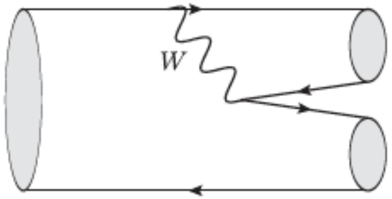
(c) $P,\ P_{\rm EW}^{C}$ (d) $S,\ P_{\rm EW}$ (g) $PE,\ PE_{\rm EW}$ (h) $PA,\ PA_{\rm EW}$

FLAVOR DIAGRAMS

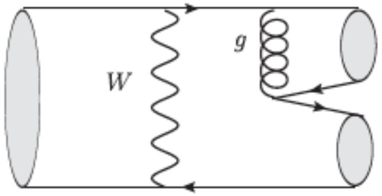
- Penguin diagrams negligible for BR’s because of GIM $V_{cd}V_{ud}^* = -V_{cs}V_{us}^*$ and $V_{cb}V_{ub}^* \sim A^2\lambda^5$.
- For current analysis, we only need to consider the tree-type diagrams:



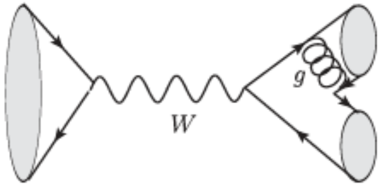
(a) T



(b) C



(e) E



(f) A

- Because the spectator quark may end up in P or V meson in the final state, these two kinds of diagrams of the same flavor topology have no relation a priori and should be distinguished.
- For example, T → T_P or T_V.

OUR APPROACH

- We perform a χ^2 fit to the branching fractions of all Cabibbo-favored (CF) modes, extracting magnitudes and phases of all flavor diagrams.
- Since what are fitted are branching fractions, there are degeneracies in χ^2-minimum solutions when all the strong phases simultaneously flip signs.
- Using the extracted information, we make predictions of branching fractions for singly Cabibbo-suppressed (SCS) and doubly Cabibbo-suppressed (DCS) modes.
  ➠ check against available data to test $SU(3)_F$

PARTIAL WIDTH

- The partial decay width of D → VP can be expressed in two different ways:

$$\frac{p_c^3}{8\pi m_D^2}|\tilde{\mathcal{M}}|^2 \quad \text{or} \quad \frac{p_c}{8\pi m_D^2}\sum_{\text{pol.}}|\mathcal{M}|^2$$

scheme A

scheme S

with the relation

$$\tilde{\mathcal{M}}(\epsilon \cdot p_D) = \frac{m_D}{m_V}\mathcal{M}$$

- Although the amplitudes obtained in the two schemes apparently have different magnitudes, they are expected to have similar strong phases.

QUARK CONTENTS IN MESONS

- Phase convention of quark contents in light pseudoscalar and vector mesons are taken as follows:

π^+	π^0	π^-	K^+	K^0	$\bar{K}^0$	K^-	η_q	η_s
$u\bar{d}$	$\frac{d\bar{d}-u\bar{u}}{\sqrt{2}}$	$-d\bar{u}$	$u\bar{s}$	$d\bar{s}$	$s\bar{d}$	$-s\bar{u}$	$\frac{u\bar{u}+d\bar{d}}{\sqrt{2}}$	$s\bar{s}$
ρ^+	ρ^0	ρ^-	K^{*+}	K^{*0}	$\bar{K}^{*0}$	K^{*-}	ω	ϕ
$u\bar{d}$	$\frac{d\bar{d}-u\bar{u}}{\sqrt{2}}$	$-d\bar{u}$	$u\bar{s}$	$d\bar{s}$	$s\bar{d}$	$-s\bar{u}$	$\frac{u\bar{u}+d\bar{d}}{\sqrt{2}}$	$s\bar{s}$

- The physical η and η' mesons are related to η_q and η_s via a mixing angle:

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} \eta_q \\ \eta_s \end{pmatrix}$$

with $\phi = 43.5°$ in our numerical calculations.

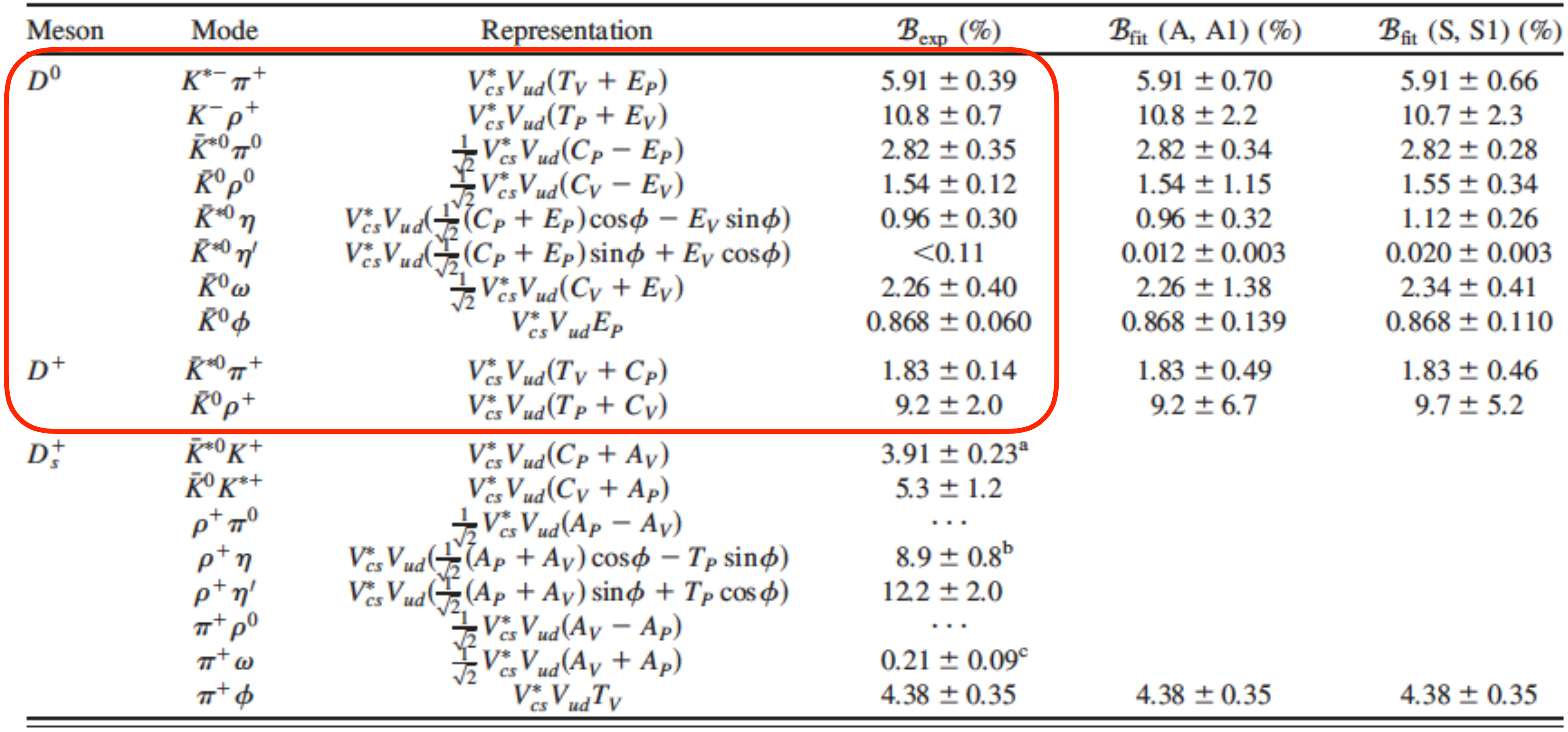
LHCb 2015

PREVIOUS ANALYSIS

Cheng and CWC 2010

- With only CF $D^{0,+}$ decays, there are two disjoint amplitude sets: $\{T_V, C_P, E_P\}$ and $\{T_P, C_V, E_V\}$.

  ➠ a connection in the D_s^+ decays (and CS modes)

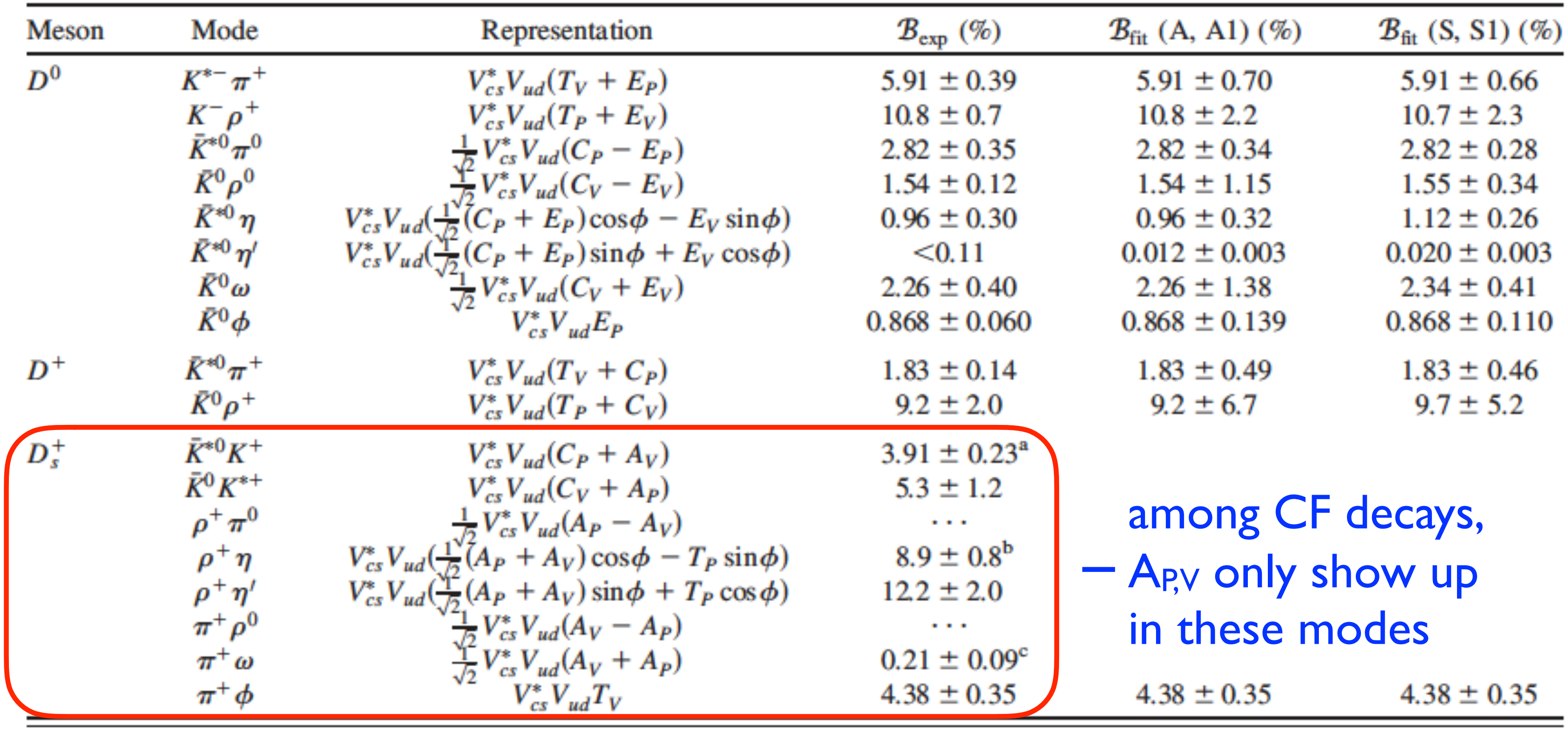


Meson	Mode	Representation	$\mathcal{B}_{\rm exp}$ (%)	$\mathcal{B}_{\rm fit}$ (A, A1) (%)	$\mathcal{B}_{\rm fit}$ (S, S1) (%)
D^0	$K^{*-}\pi^+$	$V^*_{cs}V_{ud}(T_V+E_P)$	5.91 ± 0.39	5.91 ± 0.70	5.91 ± 0.66
	$K^-\rho^+$	$V^*_{cs}V_{ud}(T_P+E_V)$	10.8 ± 0.7	10.8 ± 2.2	10.7 ± 2.3
	$\bar{K}^{*0}\pi^0$	$\frac{1}{\sqrt{2}}V^*_{cs}V_{ud}(C_P-E_P)$	2.82 ± 0.35	2.82 ± 0.34	2.82 ± 0.28
	$\bar{K}^0\rho^0$	$\frac{1}{\sqrt{2}}V^*_{cs}V_{ud}(C_V-E_V)$	1.54 ± 0.12	1.54 ± 1.15	1.55 ± 0.34
	$\bar{K}^{*0}\eta$	$V^*_{cs}V_{ud}(\frac{1}{\sqrt{2}}(C_P+E_P)\cos\phi-E_V\sin\phi)$	0.96 ± 0.30	0.96 ± 0.32	1.12 ± 0.26
	$\bar{K}^{*0}\eta'$	$V^*_{cs}V_{ud}(\frac{1}{\sqrt{2}}(C_P+E_P)\sin\phi+E_V\cos\phi)$	<0.11	0.012 ± 0.003	0.020 ± 0.003
	$\bar{K}^0\omega$	$\frac{1}{\sqrt{2}}V^*_{cs}V_{ud}(C_V+E_V)$	2.26 ± 0.40	2.26 ± 1.38	2.34 ± 0.41
	$\bar{K}^0\phi$	$V^*_{cs}V_{ud}E_P$	0.868 ± 0.060	0.868 ± 0.139	0.868 ± 0.110
D^+	$\bar{K}^{*0}\pi^+$	$V^*_{cs}V_{ud}(T_V+C_P)$	1.83 ± 0.14	1.83 ± 0.49	1.83 ± 0.46
	$\bar{K}^0\rho^+$	$V^*_{cs}V_{ud}(T_P+C_V)$	9.2 ± 2.0	9.2 ± 6.7	9.7 ± 5.2
D_s^+	$\bar{K}^{*0}K^+$	$V^*_{cs}V_{ud}(C_P+A_V)$	3.91 ± 0.23[a]		
	$\bar{K}^0K^{*+}$	$V^*_{cs}V_{ud}(C_V+A_P)$	5.3 ± 1.2		
	$\rho^+\pi^0$	$\frac{1}{\sqrt{2}}V^*_{cs}V_{ud}(A_P-A_V)$	⋯		
	$\rho^+\eta$	$V^*_{cs}V_{ud}(\frac{1}{\sqrt{2}}(A_P+A_V)\cos\phi-T_P\sin\phi)$	8.9 ± 0.8[b]		
	$\rho^+\eta'$	$V^*_{cs}V_{ud}(\frac{1}{\sqrt{2}}(A_P+A_V)\sin\phi+T_P\cos\phi)$	12.2 ± 2.0		
	$\pi^+\rho^0$	$\frac{1}{\sqrt{2}}V^*_{cs}V_{ud}(A_V-A_P)$	⋯		
	$\pi^+\omega$	$\frac{1}{\sqrt{2}}V^*_{cs}V_{ud}(A_V+A_P)$	0.21 ± 0.09[c]		
	$\pi^+\phi$	$V^*_{cs}V_{ud}T_V$	4.38 ± 0.35	4.38 ± 0.35	4.38 ± 0.35

PREVIOUS ANALYSIS

Cheng and CWC 2010

- 6 years ago, $A_{P,V}$ could not be fixed by available data. ➠ many of the D^+ and D_s^+ decays involving these amplitudes could not be predicted within the framework



Meson	Mode	Representation	$\mathcal{B}_{\rm exp}$ (%)	$\mathcal{B}_{\rm fit}$ (A, A1) (%)	$\mathcal{B}_{\rm fit}$ (S, S1) (%)
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	$K^-\rho^+$	$V_{cs}^*V_{ud}(T_P+E_V)$	10.8 ± 0.7	10.8 ± 2.2	10.7 ± 2.3
	$\bar{K}^{*0}\pi^0$	$\frac{1}{\sqrt{2}}V_{cs}^*V_{ud}(C_P-E_P)$	2.82 ± 0.35	2.82 ± 0.34	2.82 ± 0.28
	$\bar{K}^0\rho^0$	$\frac{1}{\sqrt{2}}V_{cs}^*V_{ud}(C_V-E_V)$	1.54 ± 0.12	1.54 ± 1.15	1.55 ± 0.34
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D^+	$\bar{K}^{*0}\pi^+$	$V_{cs}^*V_{ud}(T_V+C_P)$	1.83 ± 0.14	1.83 ± 0.49	1.83 ± 0.46
	$\bar{K}^0\rho^+$	$V_{cs}^*V_{ud}(T_P+C_V)$	9.2 ± 2.0	9.2 ± 6.7	9.7 ± 5.2
D_s^+	$\bar{K}^{*0}K^+$	$V_{cs}^*V_{ud}(C_P+A_V)$	3.91 ± 0.23[a]		
	$\bar{K}^0K^{*+}$	$V_{cs}^*V_{ud}(C_V+A_P)$	5.3 ± 1.2		
	$\rho^+\pi^0$	$\frac{1}{\sqrt{2}}V_{cs}^*V_{ud}(A_P-A_V)$	$\cdots$		
	$\rho^+\eta$	$V_{cs}^*V_{ud}(\frac{1}{\sqrt{2}}(A_P+A_V)\cos\phi-T_P\sin\phi)$	8.9 ± 0.8[b]		
	$\rho^+\eta'$	$V_{cs}^*V_{ud}(\frac{1}{\sqrt{2}}(A_P+A_V)\sin\phi+T_P\cos\phi)$	12.2 ± 2.0		
	$\pi^+\rho^0$	$\frac{1}{\sqrt{2}}V_{cs}^*V_{ud}(A_V-A_P)$	$\cdots$		
	$\pi^+\omega$	$\frac{1}{\sqrt{2}}V_{cs}^*V_{ud}(A_V+A_P)$	0.21 ± 0.09[c]		
	$\pi^+\phi$	$V_{cs}^*V_{ud}T_V$	4.38 ± 0.35	4.38 ± 0.35	4.38 ± 0.35

among CF decays,
— $A_{P,V}$ only show up
in these modes

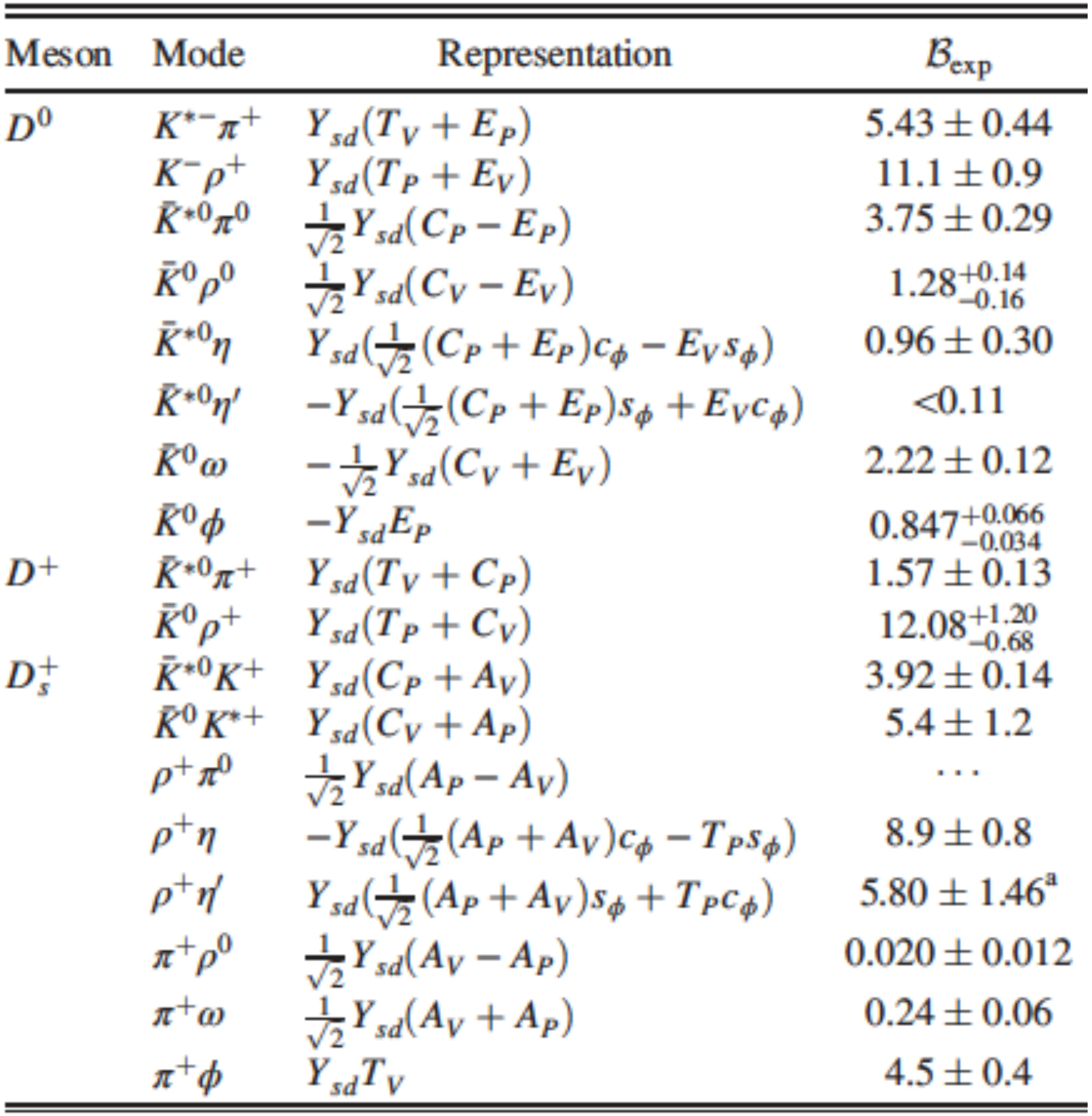
RECENT MEASUREMENTS

- It is now possible to fix $A_{P,V}$, thanks particularly to the recent measurement of $BR(D_s^+ \to \pi^+\rho^0)$ which involves the combination $A_P - A_V$.
- In addition to new measurements, several modes have better determinations than before.
  ➠ time for an updated $SU(3)_F$ analysis
- For example, $BR(D_s^+ \to \rho^+\eta') = (12.2\pm2.0)\%$ by CLEO had long been conjectured to be overestimated and problematic
  ➠ updated measurement is $(5.80\pm1.46)\%$ by BES-III is significantly smaller

BES-III 2015

CF MODES

compared to
2010 data



Meson	Mode	Representation	$\mathcal{B}_{\rm exp}$
D^0	$K^{*-}\pi^+$	$Y_{sd}(T_V+E_P)$	5.43 ± 0.44
	$K^-\rho^+$	$Y_{sd}(T_P+E_V)$	11.1 ± 0.9
	$\bar{K}^{*0}\pi^0$	$\frac{1}{\sqrt{2}}Y_{sd}(C_P-E_P)$	3.75 ± 0.29
	$\bar{K}^0\rho^0$	$\frac{1}{\sqrt{2}}Y_{sd}(C_V-E_V)$	$1.28^{+0.14}_{-0.16}$
	$\bar{K}^{*0}\eta$	$Y_{sd}(\frac{1}{\sqrt{2}}(C_P+E_P)c_\phi-E_Vs_\phi)$	0.96 ± 0.30
	$\bar{K}^{*0}\eta'$	$-Y_{sd}(\frac{1}{\sqrt{2}}(C_P+E_P)s_\phi+E_Vc_\phi)$	<0.11
	$\bar{K}^0\omega$	$-\frac{1}{\sqrt{2}}Y_{sd}(C_V+E_V)$	2.22 ± 0.12
	$\bar{K}^0\phi$	$-Y_{sd}E_P$	$0.847^{+0.066}_{-0.034}$
D^+	$\bar{K}^{*0}\pi^+$	$Y_{sd}(T_V+C_P)$	1.57 ± 0.13
	$\bar{K}^0\rho^+$	$Y_{sd}(T_P+C_V)$	$12.08^{+1.20}_{-0.68}$
D_s^+	$\bar{K}^{*0}K^+$	$Y_{sd}(C_P+A_V)$	3.92 ± 0.14
	$\bar{K}^0K^{*+}$	$Y_{sd}(C_V+A_P)$	5.4 ± 1.2
	$\rho^+\pi^0$	$\frac{1}{\sqrt{2}}Y_{sd}(A_P-A_V)$	$\cdots$
	$\rho^+\eta$	$-Y_{sd}(\frac{1}{\sqrt{2}}(A_P+A_V)c_\phi-T_Ps_\phi)$	8.9 ± 0.8
	$\rho^+\eta'$	$Y_{sd}(\frac{1}{\sqrt{2}}(A_P+A_V)s_\phi+T_Pc_\phi)$	$5.80\pm1.46^{\rm a}$
	$\pi^+\rho^0$	$\frac{1}{\sqrt{2}}Y_{sd}(A_V-A_P)$	0.020 ± 0.012
	$\pi^+\omega$	$\frac{1}{\sqrt{2}}Y_{sd}(A_V+A_P)$	0.24 ± 0.06
	$\pi^+\phi$	$Y_{sd}T_V$	4.5 ± 0.4

– not updated
– not updated

– somewhat increased

– not updated
– significantly reduced
– new

SOME REMARKS

- We have found many possible solutions with local χ^2 minima; some of them are not well separated by sufficiently high "χ^2 barriers" to render good 1σ ranges.
  ➠ in such cases, we stop the 1σ range scan at the obvious boundary
- We only present those whose predicted BFs for SCS modes have better agreement with data.
- In particular, in the effort of discarding irrelevant solutions, the SCS $D^0 \to \pi^0\omega$ mode plays a significant role.

SOLUTIONS IN SCHEME A

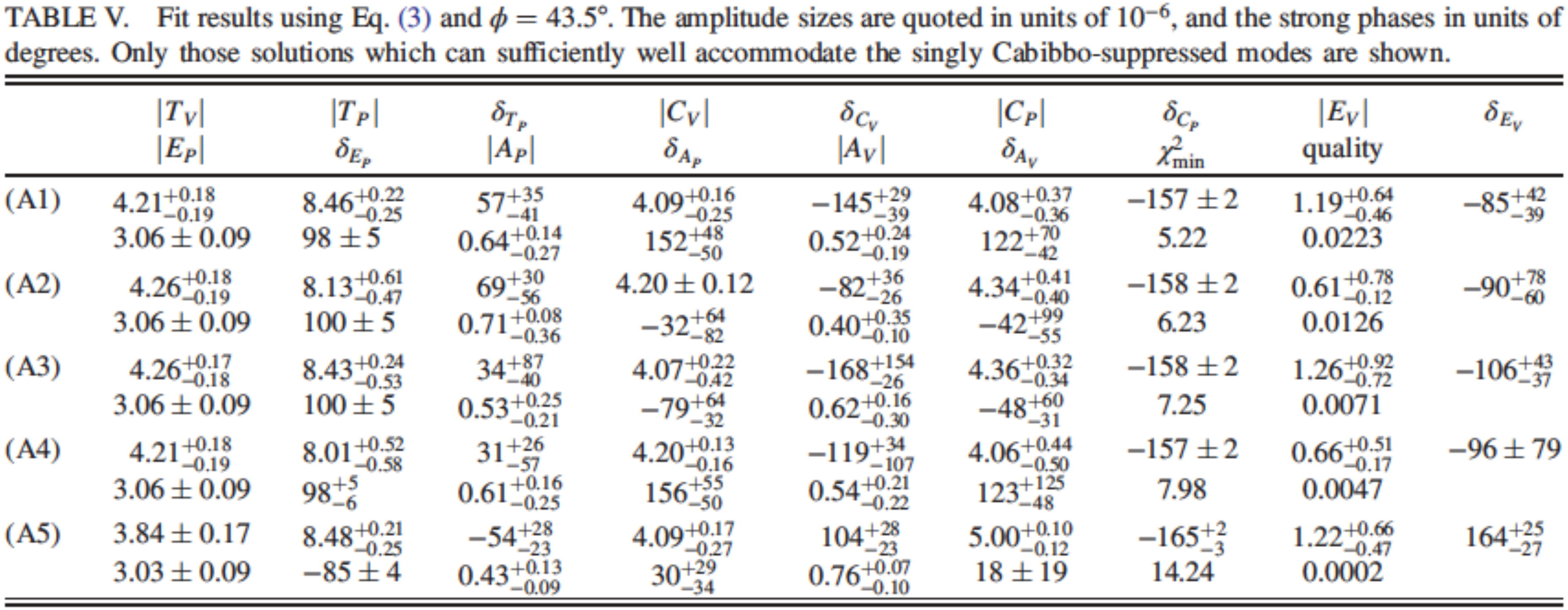


TABLE V. Fit results using Eq. (3) and $\phi = 43.5°$. The amplitude sizes are quoted in units of 10^{-6}, and the strong phases in units of degrees. Only those solutions which can sufficiently well accommodate the singly Cabibbo-suppressed modes are shown.

	$\lvert T_V\rvert$	$\lvert T_P\rvert$	δ_{T_P}	$\lvert C_V\rvert$	δ_{C_V}	$\lvert C_P\rvert$	δ_{C_P}	$\lvert E_V\rvert$	δ_{E_V}
	$\lvert E_P\rvert$	δ_{E_P}	$\lvert A_P\rvert$	δ_{A_P}	$\lvert A_V\rvert$	δ_{A_V}	$\chi^2_{\min}$	quality	
(A1)	$4.21^{+0.18}_{-0.19}$	$8.46^{+0.22}_{-0.25}$	57^{+35}_{-41}	$4.09^{+0.16}_{-0.25}$	-145^{+29}_{-39}	$4.08^{+0.37}_{-0.36}$	-157 ± 2	$1.19^{+0.64}_{-0.46}$	-85^{+42}_{-39}
	3.06 ± 0.09	98 ± 5	$0.64^{+0.14}_{-0.27}$	152^{+48}_{-50}	$0.52^{+0.24}_{-0.19}$	122^{+70}_{-42}	5.22	0.0223	
(A2)	$4.26^{+0.18}_{-0.19}$	$8.13^{+0.61}_{-0.47}$	69^{+30}_{-56}	4.20 ± 0.12	-82^{+36}_{-26}	$4.34^{+0.41}_{-0.40}$	-158 ± 2	$0.61^{+0.78}_{-0.12}$	-90^{+78}_{-60}
	3.06 ± 0.09	100 ± 5	$0.71^{+0.08}_{-0.36}$	-32^{+64}_{-82}	$0.40^{+0.35}_{-0.10}$	-42^{+99}_{-55}	6.23	0.0126	
(A3)	$4.26^{+0.17}_{-0.18}$	$8.43^{+0.24}_{-0.53}$	34^{+87}_{-40}	$4.07^{+0.22}_{-0.42}$	-168^{+154}_{-26}	$4.36^{+0.32}_{-0.34}$	-158 ± 2	$1.26^{+0.92}_{-0.72}$	-106^{+43}_{-37}
	3.06 ± 0.09	100 ± 5	$0.53^{+0.25}_{-0.21}$	-79^{+64}_{-32}	$0.62^{+0.16}_{-0.30}$	-48^{+60}_{-31}	7.25	0.0071	
(A4)	$4.21^{+0.18}_{-0.19}$	$8.01^{+0.52}_{-0.58}$	31^{+26}_{-57}	$4.20^{+0.13}_{-0.16}$	-119^{+34}_{-107}	$4.06^{+0.44}_{-0.50}$	-157 ± 2	$0.66^{+0.51}_{-0.17}$	-96 ± 79
	3.06 ± 0.09	98^{+5}_{-6}	$0.61^{+0.16}_{-0.25}$	156^{+55}_{-50}	$0.54^{+0.21}_{-0.22}$	123^{+125}_{-48}	7.98	0.0047	
(A5)	3.84 ± 0.17	$8.48^{+0.21}_{-0.25}$	-54^{+28}_{-23}	$4.09^{+0.17}_{-0.27}$	104^{+28}_{-23}	$5.00^{+0.10}_{-0.12}$	-165^{+2}_{-3}	$1.22^{+0.66}_{-0.47}$	164^{+25}_{-27}
	3.03 ± 0.09	-85 ± 4	$0.43^{+0.13}_{-0.09}$	30^{+29}_{-34}	$0.76^{+0.07}_{-0.10}$	18 ± 19	14.24	0.0002	

$$\frac{p_c^3}{8\pi m_D^2}|\tilde{\mathcal{M}}|^2$$

SOLUTIONS IN SCHEME A

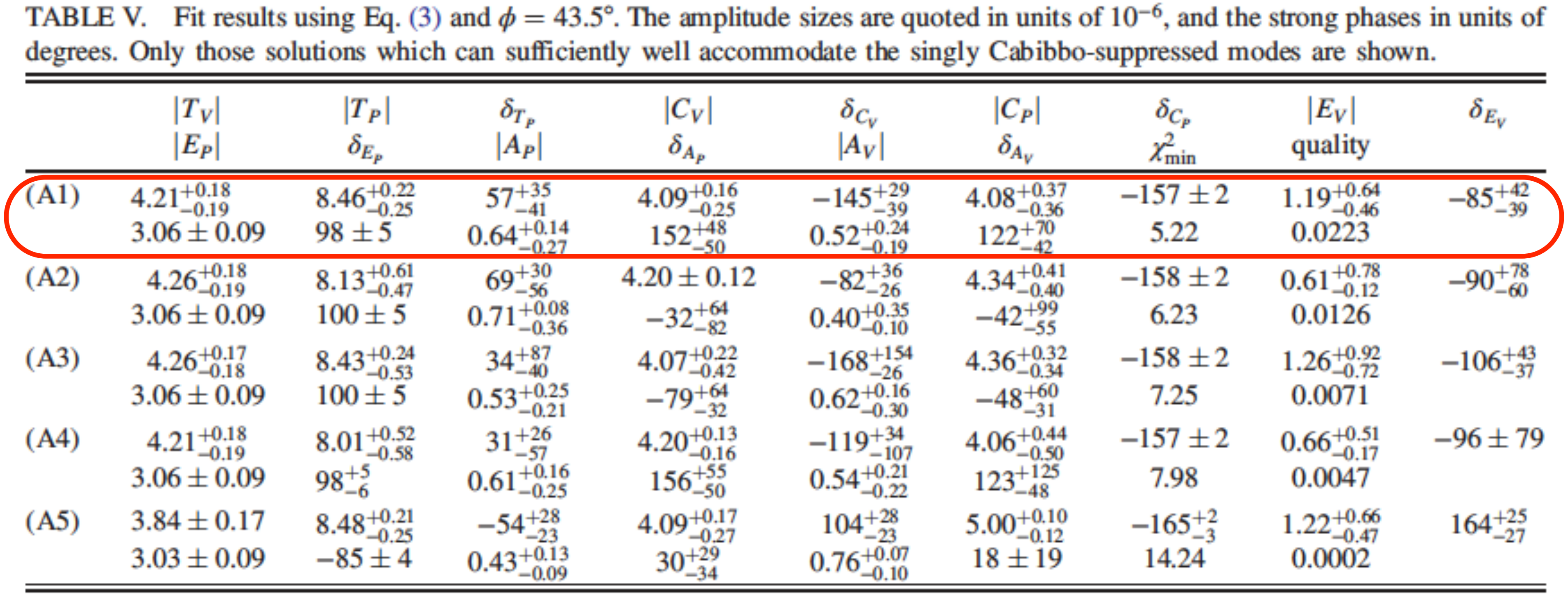


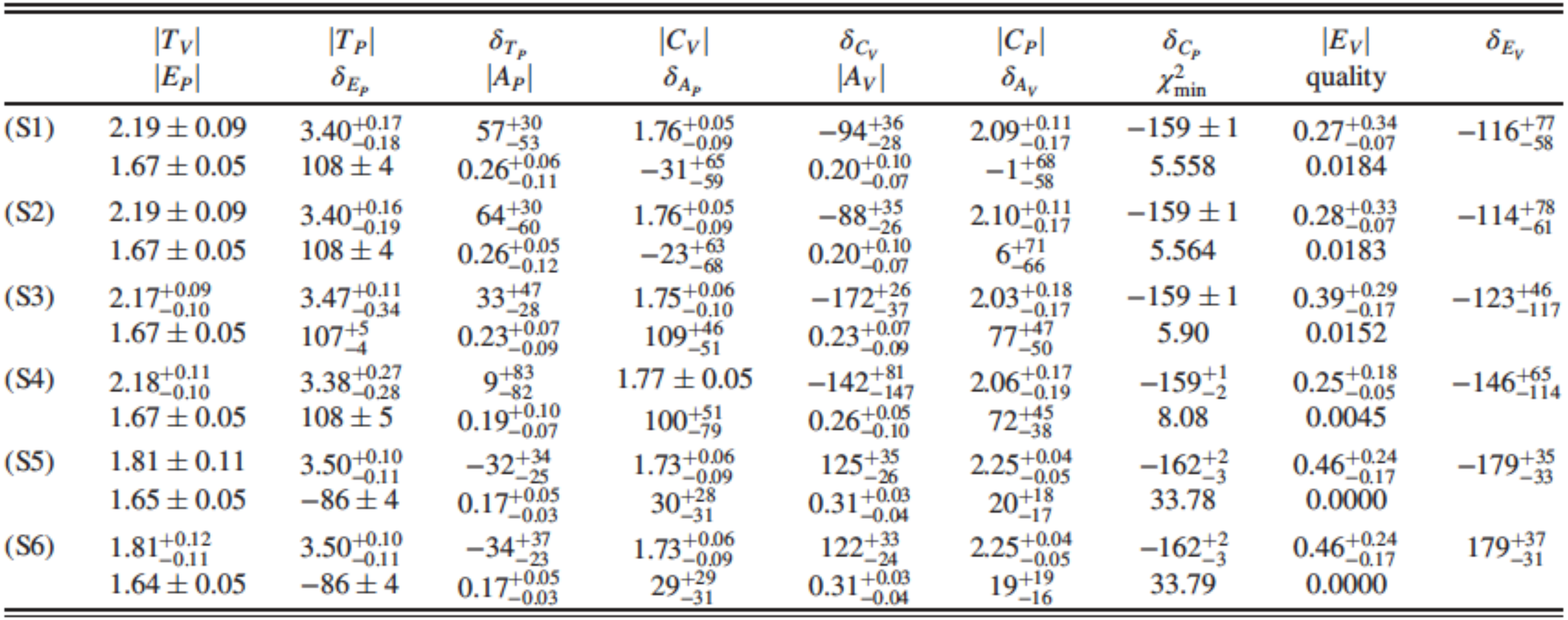
TABLE V. Fit results using Eq. (3) and $\phi = 43.5°$. The amplitude sizes are quoted in units of 10^{-6}, and the strong phases in units of degrees. Only those solutions which can sufficiently well accommodate the singly Cabibbo-suppressed modes are shown.

	$\vert T_V\vert$	$\vert T_P\vert$	δ_{T_P}	$\vert C_V\vert$	δ_{C_V}	$\vert C_P\vert$	δ_{C_P}	$\vert E_V\vert$	δ_{E_V}
	$\vert E_P\vert$	δ_{E_P}	$\vert A_P\vert$	δ_{A_P}	$\vert A_V\vert$	δ_{A_V}	χ^2_{min}	quality	
(A1)	$4.21^{+0.18}_{-0.19}$	$8.46^{+0.22}_{-0.25}$	57^{+35}_{-41}	$4.09^{+0.16}_{-0.25}$	-145^{+29}_{-39}	$4.08^{+0.37}_{-0.36}$	-157 ± 2	$1.19^{+0.64}_{-0.46}$	-85^{+42}_{-39}
	3.06 ± 0.09	98 ± 5	$0.64^{+0.14}_{-0.27}$	152^{+48}_{-50}	$0.52^{+0.24}_{-0.19}$	122^{+70}_{-42}	5.22	0.0223	
(A2)	$4.26^{+0.18}_{-0.19}$	$8.13^{+0.61}_{-0.47}$	69^{+30}_{-56}	4.20 ± 0.12	-82^{+36}_{-26}	$4.34^{+0.41}_{-0.40}$	-158 ± 2	$0.61^{+0.78}_{-0.12}$	-90^{+78}_{-60}
	3.06 ± 0.09	100 ± 5	$0.71^{+0.08}_{-0.36}$	-32^{+64}_{-82}	$0.40^{+0.35}_{-0.10}$	-42^{+99}_{-55}	6.23	0.0126	
(A3)	$4.26^{+0.17}_{-0.18}$	$8.43^{+0.24}_{-0.53}$	34^{+87}_{-40}	$4.07^{+0.22}_{-0.42}$	-168^{+154}_{-26}	$4.36^{+0.32}_{-0.34}$	-158 ± 2	$1.26^{+0.92}_{-0.72}$	-106^{+43}_{-37}
	3.06 ± 0.09	100 ± 5	$0.53^{+0.25}_{-0.21}$	-79^{+64}_{-32}	$0.62^{+0.16}_{-0.30}$	-48^{+60}_{-31}	7.25	0.0071	
(A4)	$4.21^{+0.18}_{-0.19}$	$8.01^{+0.52}_{-0.58}$	31^{+26}_{-57}	$4.20^{+0.13}_{-0.16}$	-119^{+34}_{-107}	$4.06^{+0.44}_{-0.50}$	-157 ± 2	$0.66^{+0.51}_{-0.17}$	-96 ± 79
	3.06 ± 0.09	98^{+5}_{-6}	$0.61^{+0.16}_{-0.25}$	156^{+55}_{-50}	$0.54^{+0.21}_{-0.22}$	123^{+125}_{-48}	7.98	0.0047	
(A5)	3.84 ± 0.17	$8.48^{+0.21}_{-0.25}$	-54^{+28}_{-23}	$4.09^{+0.17}_{-0.27}$	104^{+28}_{-23}	$5.00^{+0.10}_{-0.12}$	-165^{+2}_{-3}	$1.22^{+0.66}_{-0.47}$	164^{+25}_{-27}
	3.03 ± 0.09	-85 ± 4	$0.43^{+0.13}_{-0.09}$	30^{+29}_{-34}	$0.76^{+0.07}_{-0.10}$	18 ± 19	14.24	0.0002	

$$\frac{p_c^3}{8\pi m_D^2}|\tilde{\mathcal{M}}|^2$$

SOLUTIONS IN SCHEME S

TABLE VI. Same as Table V except that Eq. (4) is employed for the fit. The amplitude sizes are quoted in units of $10^{-6}(\epsilon \cdot p_D)$.



	$\lvert T_V\rvert$	$\lvert T_P\rvert$	δ_{T_P}	$\lvert C_V\rvert$	δ_{C_V}	$\lvert C_P\rvert$	δ_{C_P}	$\lvert E_V\rvert$	δ_{E_V}
	$\lvert E_P\rvert$	δ_{E_P}	$\lvert A_P\rvert$	δ_{A_P}	$\lvert A_V\rvert$	δ_{A_V}	$\chi^2_{\rm min}$	quality	
(S1)	2.19 ± 0.09	$3.40^{+0.17}_{-0.18}$	57^{+30}_{-53}	$1.76^{+0.05}_{-0.09}$	-94^{+36}_{-28}	$2.09^{+0.11}_{-0.17}$	-159 ± 1	$0.27^{+0.34}_{-0.07}$	-116^{+77}_{-58}
	1.67 ± 0.05	108 ± 4	$0.26^{+0.06}_{-0.11}$	-31^{+65}_{-59}	$0.20^{+0.10}_{-0.07}$	-1^{+68}_{-58}	5.558	0.0184	
(S2)	2.19 ± 0.09	$3.40^{+0.16}_{-0.19}$	64^{+30}_{-60}	$1.76^{+0.05}_{-0.09}$	-88^{+35}_{-26}	$2.10^{+0.11}_{-0.17}$	-159 ± 1	$0.28^{+0.33}_{-0.07}$	-114^{+78}_{-61}
	1.67 ± 0.05	108 ± 4	$0.26^{+0.05}_{-0.12}$	-23^{+63}_{-68}	$0.20^{+0.10}_{-0.07}$	6^{+71}_{-66}	5.564	0.0183	
(S3)	$2.17^{+0.09}_{-0.10}$	$3.47^{+0.11}_{-0.34}$	33^{+47}_{-28}	$1.75^{+0.06}_{-0.10}$	-172^{+26}_{-37}	$2.03^{+0.18}_{-0.17}$	-159 ± 1	$0.39^{+0.29}_{-0.17}$	-123^{+46}_{-117}
	1.67 ± 0.05	107^{+5}_{-4}	$0.23^{+0.07}_{-0.09}$	109^{+46}_{-51}	$0.23^{+0.07}_{-0.09}$	77^{+47}_{-50}	5.90	0.0152	
(S4)	$2.18^{+0.11}_{-0.10}$	$3.38^{+0.27}_{-0.28}$	9^{+83}_{-82}	1.77 ± 0.05	-142^{+81}_{-147}	$2.06^{+0.17}_{-0.19}$	-159^{+1}_{-2}	$0.25^{+0.18}_{-0.05}$	-146^{+65}_{-114}
	1.67 ± 0.05	108 ± 5	$0.19^{+0.10}_{-0.07}$	100^{+51}_{-79}	$0.26^{+0.05}_{-0.10}$	72^{+45}_{-38}	8.08	0.0045	
(S5)	1.81 ± 0.11	$3.50^{+0.10}_{-0.11}$	-32^{+34}_{-25}	$1.73^{+0.06}_{-0.09}$	125^{+35}_{-26}	$2.25^{+0.04}_{-0.05}$	-162^{+2}_{-3}	$0.46^{+0.24}_{-0.17}$	-179^{+35}_{-33}
	1.65 ± 0.05	-86 ± 4	$0.17^{+0.05}_{-0.03}$	30^{+28}_{-31}	$0.31^{+0.03}_{-0.04}$	20^{+18}_{-17}	33.78	0.0000	
(S6)	$1.81^{+0.12}_{-0.11}$	$3.50^{+0.10}_{-0.11}$	-34^{+37}_{-23}	$1.73^{+0.06}_{-0.09}$	122^{+33}_{-24}	$2.25^{+0.04}_{-0.05}$	-162^{+2}_{-3}	$0.46^{+0.24}_{-0.17}$	179^{+37}_{-31}
	1.64 ± 0.05	-86 ± 4	$0.17^{+0.05}_{-0.03}$	29^{+29}_{-31}	$0.31^{+0.03}_{-0.04}$	19^{+19}_{-16}	33.79	0.0000	

$$\frac{p_c}{8\pi m_D^2} \sum_{\rm pol.} |\mathcal{M}|^2$$

SOLUTIONS IN SCHEME S

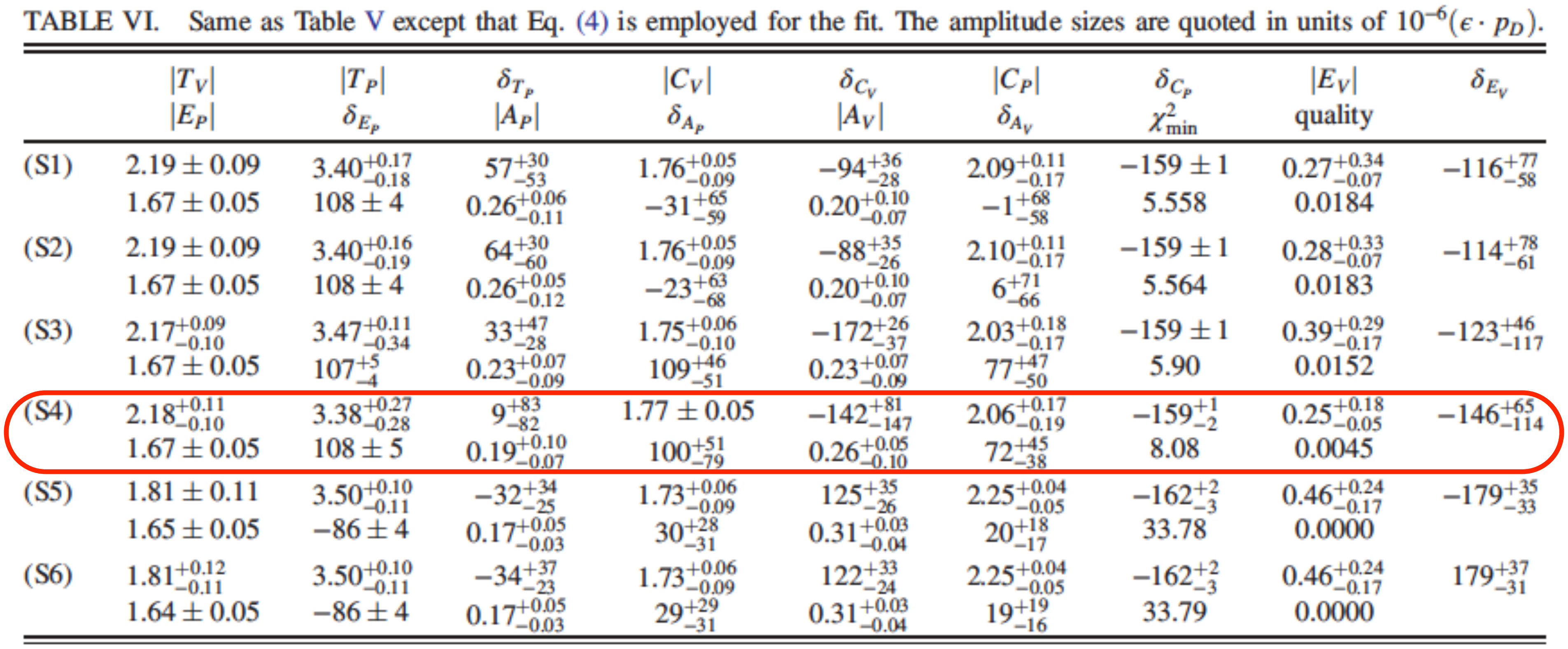


TABLE VI. Same as Table V except that Eq. (4) is employed for the fit. The amplitude sizes are quoted in units of $10^{-6}(\epsilon \cdot p_D)$.

	$\lvert T_V\rvert$	$\lvert T_P\rvert$	δ_{T_P}	$\lvert C_V\rvert$	δ_{C_V}	$\lvert C_P\rvert$	δ_{C_P}	$\lvert E_V\rvert$	δ_{E_V}
	$\lvert E_P\rvert$	δ_{E_P}	$\lvert A_P\rvert$	δ_{A_P}	$\lvert A_V\rvert$	δ_{A_V}	$\chi^2_{\rm min}$	quality	
(S1)	2.19 ± 0.09	$3.40^{+0.17}_{-0.18}$	57^{+30}_{-53}	$1.76^{+0.05}_{-0.09}$	-94^{+36}_{-28}	$2.09^{+0.11}_{-0.17}$	-159 ± 1	$0.27^{+0.34}_{-0.07}$	-116^{+77}_{-58}
	1.67 ± 0.05	108 ± 4	$0.26^{+0.06}_{-0.11}$	-31^{+65}_{-59}	$0.20^{+0.10}_{-0.07}$	-1^{+68}_{-58}	5.558	0.0184	
(S2)	2.19 ± 0.09	$3.40^{+0.16}_{-0.19}$	64^{+30}_{-60}	$1.76^{+0.05}_{-0.09}$	-88^{+35}_{-26}	$2.10^{+0.11}_{-0.17}$	-159 ± 1	$0.28^{+0.33}_{-0.07}$	-114^{+78}_{-61}
	1.67 ± 0.05	108 ± 4	$0.26^{+0.05}_{-0.12}$	-23^{+63}_{-68}	$0.20^{+0.10}_{-0.07}$	6^{+71}_{-66}	5.564	0.0183	
(S3)	$2.17^{+0.09}_{-0.10}$	$3.47^{+0.11}_{-0.34}$	33^{+47}_{-28}	$1.75^{+0.06}_{-0.10}$	-172^{+26}_{-37}	$2.03^{+0.18}_{-0.17}$	-159 ± 1	$0.39^{+0.29}_{-0.17}$	-123^{+46}_{-117}
	1.67 ± 0.05	107^{+5}_{-4}	$0.23^{+0.07}_{-0.09}$	109^{+46}_{-51}	$0.23^{+0.07}_{-0.09}$	77^{+47}_{-50}	5.90	0.0152	
(S4)	$2.18^{+0.11}_{-0.10}$	$3.38^{+0.27}_{-0.28}$	9^{+83}_{-82}	1.77 ± 0.05	-142^{+81}_{-147}	$2.06^{+0.17}_{-0.19}$	-159^{+1}_{-2}	$0.25^{+0.18}_{-0.05}$	-146^{+65}_{-114}
	1.67 ± 0.05	108 ± 5	$0.19^{+0.10}_{-0.07}$	100^{+51}_{-79}	$0.26^{+0.05}_{-0.10}$	72^{+45}_{-38}	8.08	0.0045	
(S5)	1.81 ± 0.11	$3.50^{+0.10}_{-0.11}$	-32^{+34}_{-25}	$1.73^{+0.06}_{-0.09}$	125^{+35}_{-26}	$2.25^{+0.04}_{-0.05}$	-162^{+2}_{-3}	$0.46^{+0.24}_{-0.17}$	-179^{+35}_{-33}
	1.65 ± 0.05	-86 ± 4	$0.17^{+0.05}_{-0.03}$	30^{+28}_{-31}	$0.31^{+0.03}_{-0.04}$	20^{+18}_{-17}	33.78	0.0000	
(S6)	$1.81^{+0.12}_{-0.11}$	$3.50^{+0.10}_{-0.11}$	-34^{+37}_{-23}	$1.73^{+0.06}_{-0.09}$	122^{+33}_{-24}	$2.25^{+0.04}_{-0.05}$	-162^{+2}_{-3}	$0.46^{+0.24}_{-0.17}$	179^{+37}_{-31}
	1.64 ± 0.05	-86 ± 4	$0.17^{+0.05}_{-0.03}$	29^{+29}_{-31}	$0.31^{+0.03}_{-0.04}$	19^{+19}_{-16}	33.79	0.0000	

$$\frac{p_c}{8\pi m_D^2} \sum_{\text{pol.}} |\mathcal{M}|^2$$

GENERAL OBSERVATIONS

- Among all the theory parameters, the uncertainties associated with $|E_P|$, δE_P, $|C_P|$ and δC_P are much smaller than the others. Moreover, their best-fit values are quite stable across different solutions.
- The flavor amplitudes generally respect the following hierarchy pattern: $|T_P| > |T_V| \sim |C_{P,V}| > |E_P| > |E_V| \sim |A_{P,V}|$.

  ➠ large $|T_P|$ driven by large rates of $D^0 \to K^- \rho^+$ and $\underline{K}^0 \rho^+$

Cheng and CWC 2010

$$T = 3.08 \pm 0.06, \qquad C = (2.46^{+0.06}_{-0.07})e^{-i(152\pm1)^\circ},$$

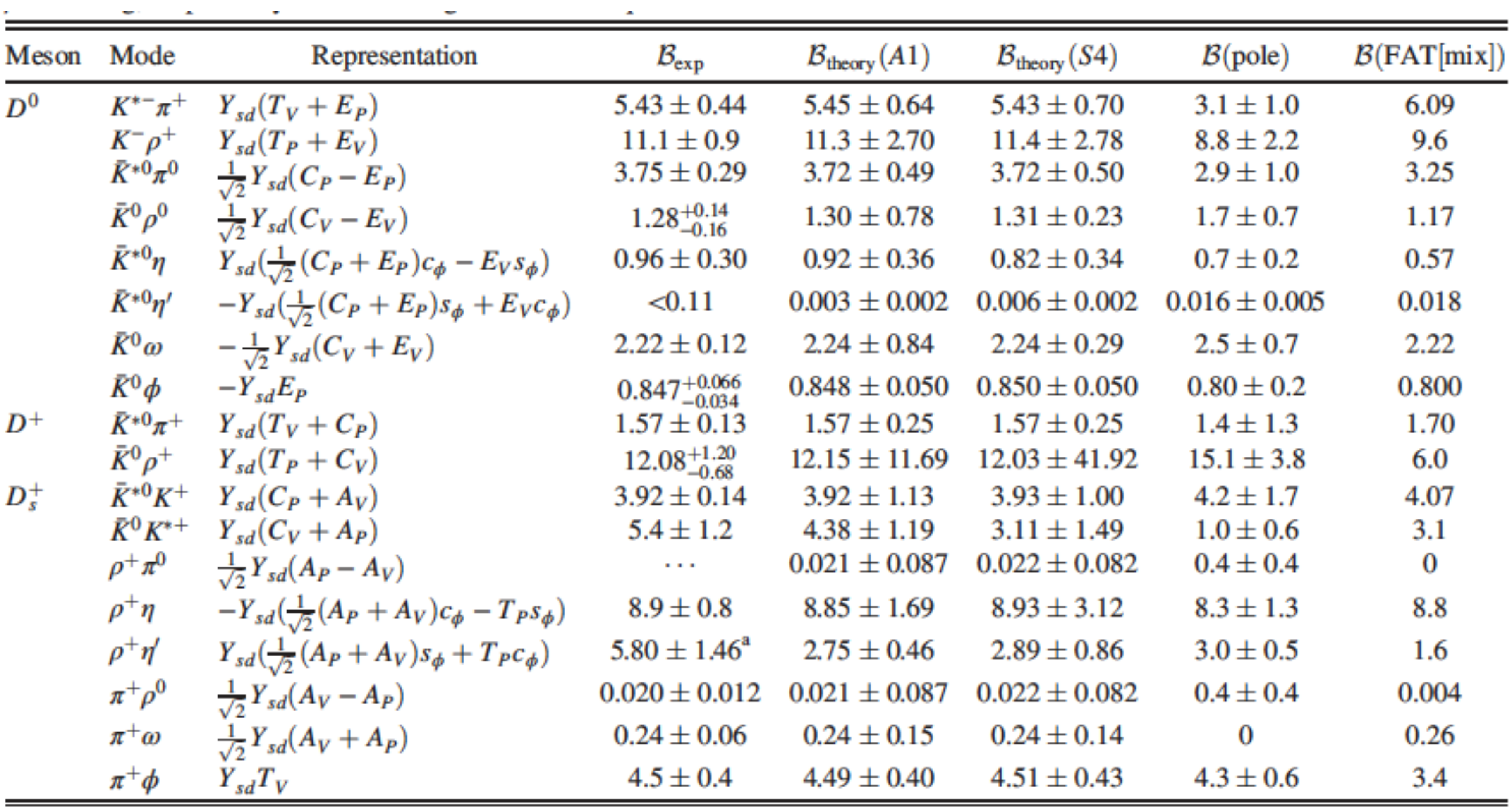
$$E = (1.66 \pm 0.06)e^{i(120\pm2)^\circ}, \qquad A = (0.34^{+0.17}_{-0.18})e^{i(70^{+10}_{-27})^\circ}$$

- The relation $E_V \approx -E_P$ advocated by some analysis is disfavored by the data.

Rosner 1999

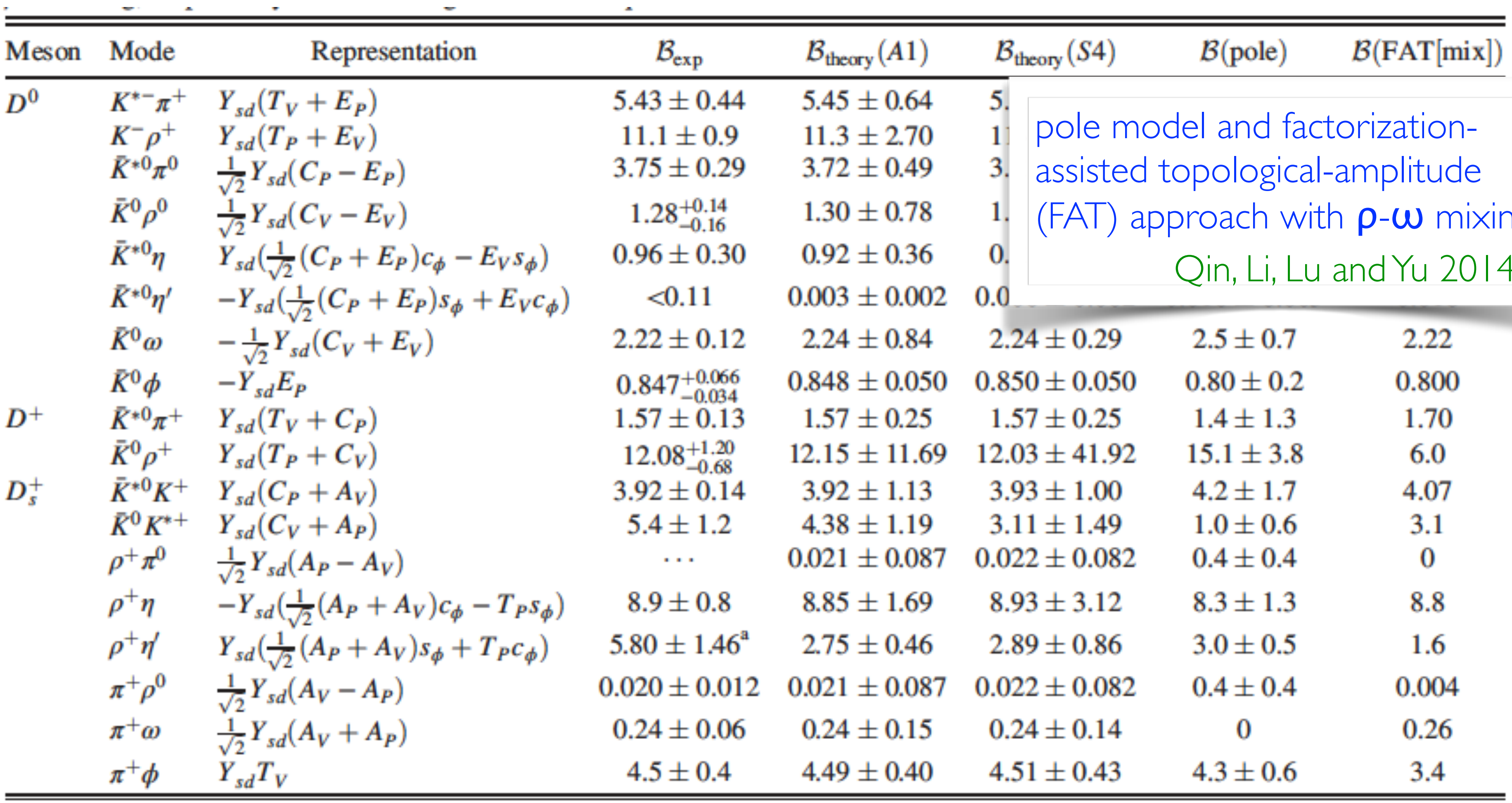
- Though with large uncertainties, A_P and A_V are only about one order of magnitude smaller than T and C amplitudes.

PREDICTIONS — CF MODES



Meson	Mode	Representation	$\mathcal{B}_{\rm exp}$	$\mathcal{B}_{\rm theory}(A1)$	$\mathcal{B}_{\rm theory}(S4)$	$\mathcal{B}$(pole)	$\mathcal{B}$(FAT[mix])
D^0	$K^{*-}\pi^+$	$Y_{sd}(T_V+E_P)$	5.43 ± 0.44	5.45 ± 0.64	5.43 ± 0.70	3.1 ± 1.0	6.09
	$K^-\rho^+$	$Y_{sd}(T_P+E_V)$	11.1 ± 0.9	11.3 ± 2.70	11.4 ± 2.78	8.8 ± 2.2	9.6
	$\bar{K}^{*0}\pi^0$	$\frac{1}{\sqrt{2}}Y_{sd}(C_P-E_P)$	3.75 ± 0.29	3.72 ± 0.49	3.72 ± 0.50	2.9 ± 1.0	3.25
	$\bar{K}^0\rho^0$	$\frac{1}{\sqrt{2}}Y_{sd}(C_V-E_V)$	$1.28^{+0.14}_{-0.16}$	1.30 ± 0.78	1.31 ± 0.23	1.7 ± 0.7	1.17
	$\bar{K}^{*0}\eta$	$Y_{sd}(\frac{1}{\sqrt{2}}(C_P+E_P)c_\phi-E_Vs_\phi)$	0.96 ± 0.30	0.92 ± 0.36	0.82 ± 0.34	0.7 ± 0.2	0.57
	$\bar{K}^{*0}\eta'$	$-Y_{sd}(\frac{1}{\sqrt{2}}(C_P+E_P)s_\phi+E_Vc_\phi)$	<0.11	0.003 ± 0.002	0.006 ± 0.002	0.016 ± 0.005	0.018
	$\bar{K}^0\omega$	$-\frac{1}{\sqrt{2}}Y_{sd}(C_V+E_V)$	2.22 ± 0.12	2.24 ± 0.84	2.24 ± 0.29	2.5 ± 0.7	2.22
	$\bar{K}^0\phi$	$-Y_{sd}E_P$	$0.847^{+0.066}_{-0.034}$	0.848 ± 0.050	0.850 ± 0.050	0.80 ± 0.2	0.800
D^+	$\bar{K}^{*0}\pi^+$	$Y_{sd}(T_V+C_P)$	1.57 ± 0.13	1.57 ± 0.25	1.57 ± 0.25	1.4 ± 1.3	1.70
	$\bar{K}^0\rho^+$	$Y_{sd}(T_P+C_V)$	$12.08^{+1.20}_{-0.68}$	12.15 ± 11.69	12.03 ± 41.92	15.1 ± 3.8	6.0
D_s^+	$\bar{K}^{*0}K^+$	$Y_{sd}(C_P+A_V)$	3.92 ± 0.14	3.92 ± 1.13	3.93 ± 1.00	4.2 ± 1.7	4.07
	$\bar{K}^0K^{*+}$	$Y_{sd}(C_V+A_P)$	5.4 ± 1.2	4.38 ± 1.19	3.11 ± 1.49	1.0 ± 0.6	3.1
	$\rho^+\pi^0$	$\frac{1}{\sqrt{2}}Y_{sd}(A_P-A_V)$	$\cdots$	0.021 ± 0.087	0.022 ± 0.082	0.4 ± 0.4	0
	$\rho^+\eta$	$-Y_{sd}(\frac{1}{\sqrt{2}}(A_P+A_V)c_\phi-T_Ps_\phi)$	8.9 ± 0.8	8.85 ± 1.69	8.93 ± 3.12	8.3 ± 1.3	8.8
	$\rho^+\eta'$	$Y_{sd}(\frac{1}{\sqrt{2}}(A_P+A_V)s_\phi+T_Pc_\phi)$	5.80 ± 1.46[a]	2.75 ± 0.46	2.89 ± 0.86	3.0 ± 0.5	1.6
	$\pi^+\rho^0$	$\frac{1}{\sqrt{2}}Y_{sd}(A_V-A_P)$	0.020 ± 0.012	0.021 ± 0.087	0.022 ± 0.082	0.4 ± 0.4	0.004
	$\pi^+\omega$	$\frac{1}{\sqrt{2}}Y_{sd}(A_V+A_P)$	0.24 ± 0.06	0.24 ± 0.15	0.24 ± 0.14	0	0.26
	$\pi^+\phi$	$Y_{sd}T_V$	4.5 ± 0.4	4.49 ± 0.40	4.51 ± 0.43	4.3 ± 0.6	3.4

PREDICTIONS — CF MODES

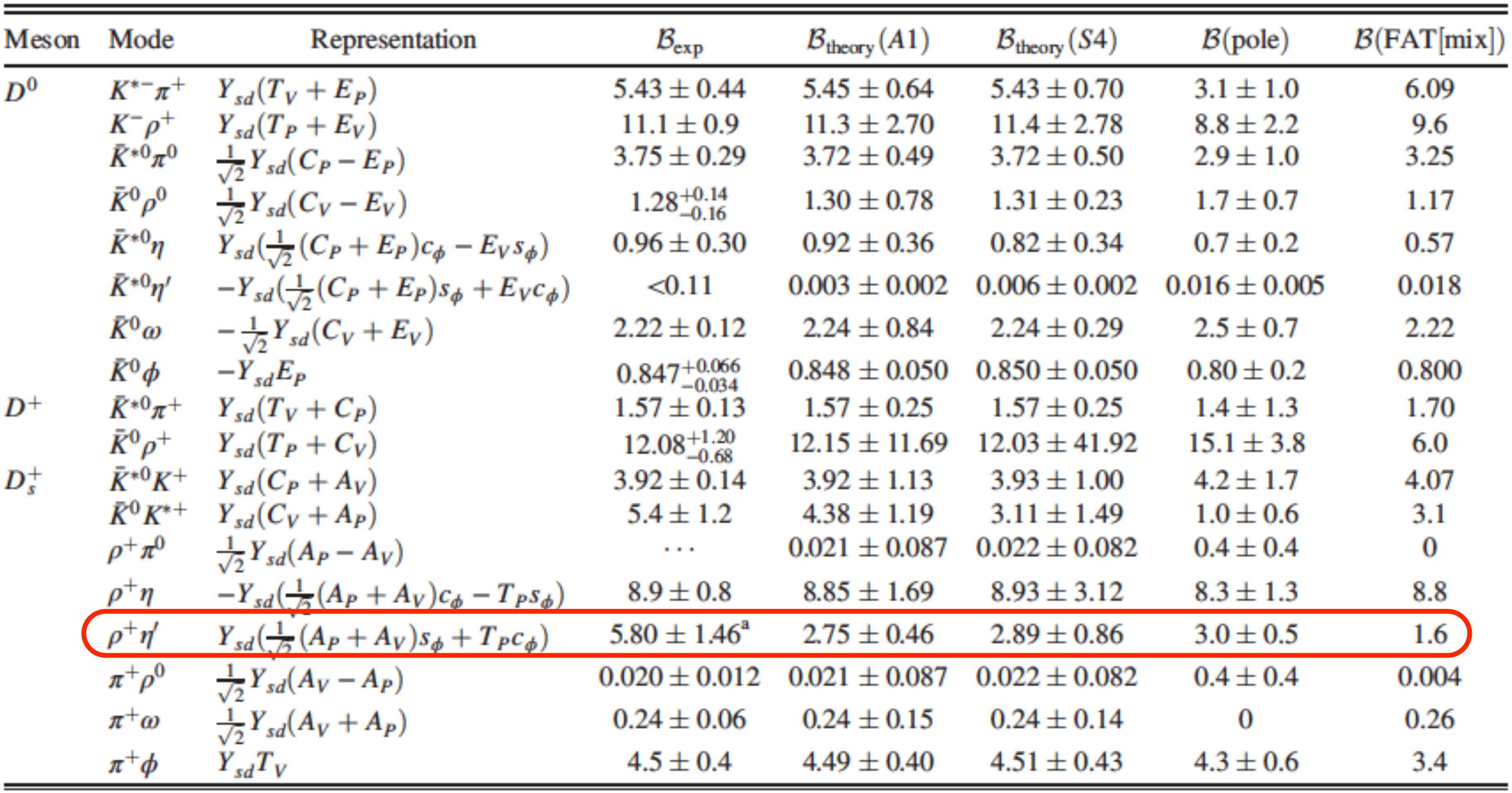


Meson	Mode	Representation	$\mathcal{B}_{\rm exp}$	$\mathcal{B}_{\rm theory}(A1)$	$\mathcal{B}_{\rm theory}(S4)$	$\mathcal{B}$(pole)	$\mathcal{B}$(FAT[mix])
D^0	$K^{*-}\pi^+$	$Y_{sd}(T_V+E_P)$	5.43 ± 0.44	5.45 ± 0.64	5.		
	$K^-\rho^+$	$Y_{sd}(T_P+E_V)$	11.1 ± 0.9	11.3 ± 2.70	1		
	$\bar{K}^{*0}\pi^0$	$\frac{1}{\sqrt{2}}Y_{sd}(C_P-E_P)$	3.75 ± 0.29	3.72 ± 0.49	3.		
	$\bar{K}^0\rho^0$	$\frac{1}{\sqrt{2}}Y_{sd}(C_V-E_V)$	$1.28^{+0.14}_{-0.16}$	1.30 ± 0.78	1.		
	$\bar{K}^{*0}\eta$	$Y_{sd}(\frac{1}{\sqrt{2}}(C_P+E_P)c_\phi-E_Vs_\phi)$	0.96 ± 0.30	0.92 ± 0.36	0.		
	$\bar{K}^{*0}\eta'$	$-Y_{sd}(\frac{1}{\sqrt{2}}(C_P+E_P)s_\phi+E_Vc_\phi)$	<0.11	0.003 ± 0.002	0.0		
	$\bar{K}^0\omega$	$-\frac{1}{\sqrt{2}}Y_{sd}(C_V+E_V)$	2.22 ± 0.12	2.24 ± 0.84	2.24 ± 0.29	2.5 ± 0.7	2.22
	$\bar{K}^0\phi$	$-Y_{sd}E_P$	$0.847^{+0.066}_{-0.034}$	0.848 ± 0.050	0.850 ± 0.050	0.80 ± 0.2	0.800
D^+	$\bar{K}^{*0}\pi^+$	$Y_{sd}(T_V+C_P)$	1.57 ± 0.13	1.57 ± 0.25	1.57 ± 0.25	1.4 ± 1.3	1.70
	$\bar{K}^0\rho^+$	$Y_{sd}(T_P+C_V)$	$12.08^{+1.20}_{-0.68}$	12.15 ± 11.69	12.03 ± 41.92	15.1 ± 3.8	6.0
D_s^+	$\bar{K}^{*0}K^+$	$Y_{sd}(C_P+A_V)$	3.92 ± 0.14	3.92 ± 1.13	3.93 ± 1.00	4.2 ± 1.7	4.07
	$\bar{K}^0K^{*+}$	$Y_{sd}(C_V+A_P)$	5.4 ± 1.2	4.38 ± 1.19	3.11 ± 1.49	1.0 ± 0.6	3.1
	$\rho^+\pi^0$	$\frac{1}{\sqrt{2}}Y_{sd}(A_P-A_V)$	$\cdots$	0.021 ± 0.087	0.022 ± 0.082	0.4 ± 0.4	0
	$\rho^+\eta$	$-Y_{sd}(\frac{1}{\sqrt{2}}(A_P+A_V)c_\phi-T_Ps_\phi)$	8.9 ± 0.8	8.85 ± 1.69	8.93 ± 3.12	8.3 ± 1.3	8.8
	$\rho^+\eta'$	$Y_{sd}(\frac{1}{\sqrt{2}}(A_P+A_V)s_\phi+T_Pc_\phi)$	$5.80\pm1.46^{\rm a}$	2.75 ± 0.46	2.89 ± 0.86	3.0 ± 0.5	1.6
	$\pi^+\rho^0$	$\frac{1}{\sqrt{2}}Y_{sd}(A_V-A_P)$	0.020 ± 0.012	0.021 ± 0.087	0.022 ± 0.082	0.4 ± 0.4	0.004
	$\pi^+\omega$	$\frac{1}{\sqrt{2}}Y_{sd}(A_V+A_P)$	0.24 ± 0.06	0.24 ± 0.15	0.24 ± 0.14	0	0.26
	$\pi^+\phi$	$Y_{sd}T_V$	4.5 ± 0.4	4.49 ± 0.40	4.51 ± 0.43	4.3 ± 0.6	3.4

pole model and factorization-assisted topological-amplitude (FAT) approach with **ρ-ω** mixing

Qin, Li, Lu and Yu 2014

PREDICTIONS — CF MODES



Meson	Mode	Representation	$\mathcal{B}_{\text{exp}}$	$\mathcal{B}_{\text{theory}}(A1)$	$\mathcal{B}_{\text{theory}}(S4)$	$\mathcal{B}$(pole)	$\mathcal{B}$(FAT[mix])
D^0	$K^{*-}\pi^+$	$Y_{sd}(T_V + E_P)$	5.43 ± 0.44	5.45 ± 0.64	5.43 ± 0.70	3.1 ± 1.0	6.09
	$K^-\rho^+$	$Y_{sd}(T_P + E_V)$	11.1 ± 0.9	11.3 ± 2.70	11.4 ± 2.78	8.8 ± 2.2	9.6
	$\bar{K}^{*0}\pi^0$	$\frac{1}{\sqrt{2}}Y_{sd}(C_P - E_P)$	3.75 ± 0.29	3.72 ± 0.49	3.72 ± 0.50	2.9 ± 1.0	3.25
	$\bar{K}^0\rho^0$	$\frac{1}{\sqrt{2}}Y_{sd}(C_V - E_V)$	$1.28^{+0.14}_{-0.16}$	1.30 ± 0.78	1.31 ± 0.23	1.7 ± 0.7	1.17
	$\bar{K}^{*0}\eta$	$Y_{sd}(\frac{1}{\sqrt{2}}(C_P + E_P)c_\phi - E_V s_\phi)$	0.96 ± 0.30	0.92 ± 0.36	0.82 ± 0.34	0.7 ± 0.2	0.57
	$\bar{K}^{*0}\eta'$	$-Y_{sd}(\frac{1}{\sqrt{2}}(C_P + E_P)s_\phi + E_V c_\phi)$	<0.11	0.003 ± 0.002	0.006 ± 0.002	0.016 ± 0.005	0.018
	$\bar{K}^0\omega$	$-\frac{1}{\sqrt{2}}Y_{sd}(C_V + E_V)$	2.22 ± 0.12	2.24 ± 0.84	2.24 ± 0.29	2.5 ± 0.7	2.22
	$\bar{K}^0\phi$	$-Y_{sd}E_P$	$0.847^{+0.066}_{-0.034}$	0.848 ± 0.050	0.850 ± 0.050	0.80 ± 0.2	0.800
D^+	$\bar{K}^{*0}\pi^+$	$Y_{sd}(T_V + C_P)$	1.57 ± 0.13	1.57 ± 0.25	1.57 ± 0.25	1.4 ± 1.3	1.70
	$\bar{K}^0\rho^+$	$Y_{sd}(T_P + C_V)$	$12.08^{+1.20}_{-0.68}$	12.15 ± 11.69	12.03 ± 41.92	15.1 ± 3.8	6.0
D_s^+	$\bar{K}^{*0}K^+$	$Y_{sd}(C_P + A_V)$	3.92 ± 0.14	3.92 ± 1.13	3.93 ± 1.00	4.2 ± 1.7	4.07
	$\bar{K}^0K^{*+}$	$Y_{sd}(C_V + A_P)$	5.4 ± 1.2	4.38 ± 1.19	3.11 ± 1.49	1.0 ± 0.6	3.1
	$\rho^+\pi^0$	$\frac{1}{\sqrt{2}}Y_{sd}(A_P - A_V)$	$\cdots$	0.021 ± 0.087	0.022 ± 0.082	0.4 ± 0.4	0
	$\rho^+\eta$	$-Y_{sd}(\frac{1}{\sqrt{2}}(A_P + A_V)c_\phi - T_P s_\phi)$	8.9 ± 0.8	8.85 ± 1.69	8.93 ± 3.12	8.3 ± 1.3	8.8
	$\rho^+\eta'$	$Y_{sd}(\frac{1}{\sqrt{2}}(A_P + A_V)s_\phi + T_P c_\phi)$	5.80 ± 1.46^{a}	2.75 ± 0.46	2.89 ± 0.86	3.0 ± 0.5	1.6
	$\pi^+\rho^0$	$\frac{1}{\sqrt{2}}Y_{sd}(A_V - A_P)$	0.020 ± 0.012	0.021 ± 0.087	0.022 ± 0.082	0.4 ± 0.4	0.004
	$\pi^+\omega$	$\frac{1}{\sqrt{2}}Y_{sd}(A_V + A_P)$	0.24 ± 0.06	0.24 ± 0.15	0.24 ± 0.14	0	0.26
	$\pi^+\phi$	$Y_{sd}T_V$	4.5 ± 0.4	4.49 ± 0.40	4.51 ± 0.43	4.3 ± 0.6	3.4

PREDICTIONS — CF MODES

- While the predicted BR($D_s^+ \to \rho^+\eta$) is close to CLEO's (8.9±0.8)%, the predicted BR($D_s^+ \to \rho^+\eta'$) is substantially below the recent BES-III's (5.80±1.46)%.
- All existing model calculations yield around 3%.
- If BR($D_s^+ \to \rho^+\eta'$) still remains to be of order 6% in the future experiments, this may hint at a sizeable flavor singlet contribution unique to the η_0 production.
- This issue should be clarified both experimentally and theoretically.

Buccella, Lusignoli, Miele, Pugliese and Santorelli 1995
Cheng and CWC 2010
Bhattacharya and Rosner 2010
Qin, Li, Lu and Yu 2014

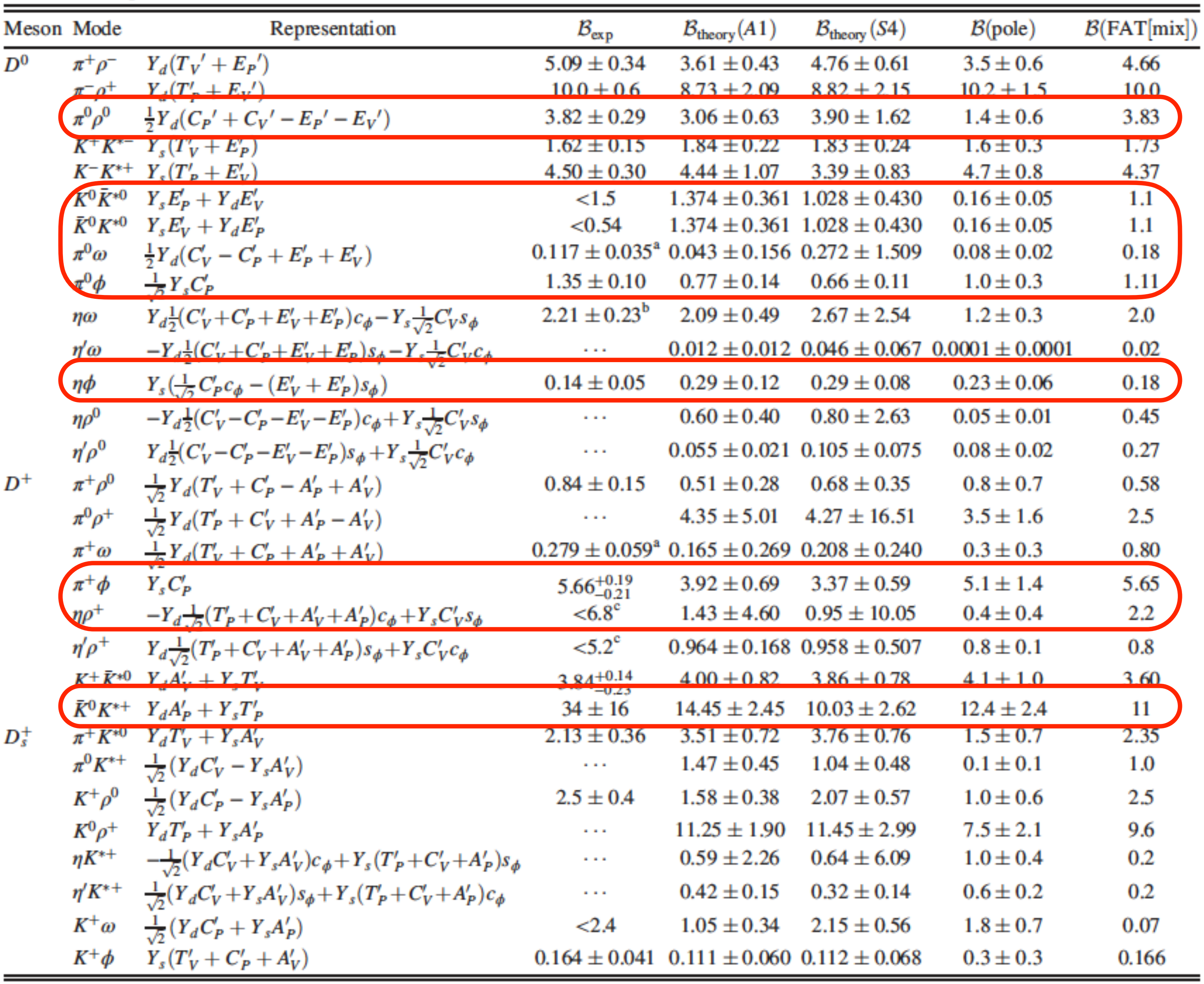
Distinguishing Schemes

- It is noted that $|C_P|$ and $|C_V|$ are comparable in solutions (A), but have a small hierarchy in solutions (S).
- As a way to tell which scheme is preferred, one can resort to the $D_s^+ \to \underline{K}^{*0}K^+$ decay, dominated by C_P, and the $\underline{K}^0K^{*+}$ decay, dominated by C_V.

D_s^+	$\bar{K}^{*0}K^+$	$Y_{sd}(C_P + A_V)$	3.92 ± 0.14	3.92 ± 1.13	3.93 ± 1.00
	$\bar{K}^0K^{*+}$	$Y_{sd}(C_V + A_P)$	5.4 ± 1.2	4.38 ± 1.19	3.11 ± 1.49

- Current data slightly favor (A1) over (S4).
- Since the $\underline{K}^{*0}K^+$ decay has been measured several times with similar results before and the $\underline{K}^0K^{*+}$ decay was last measured in 1989, it is obvious that the latter should be updated.

PREDICTIONS — SCS MODES

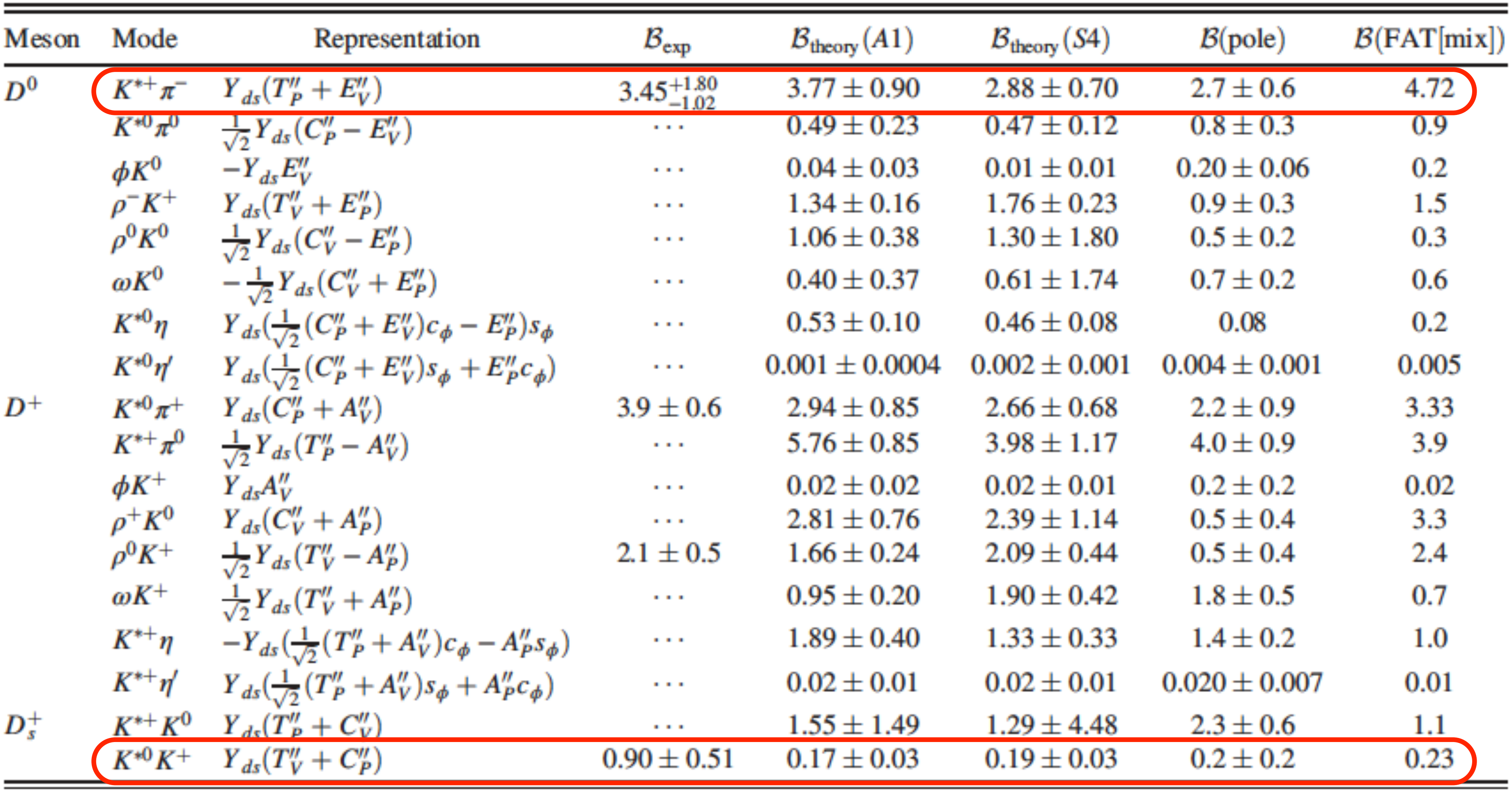


Meson	Mode	Representation	$\mathcal{B}_{\rm exp}$	$\mathcal{B}_{\rm theory}(A1)$	$\mathcal{B}_{\rm theory}(S4)$	$\mathcal{B}$(pole)	$\mathcal{B}$(FAT[mix])
D^0	$\pi^+\rho^-$	$Y_d(T_V{}' + E_P{}')$	5.09 ± 0.34	3.61 ± 0.43	4.76 ± 0.61	3.5 ± 0.6	4.66
	$\pi^-\rho^+$	$Y_d(T'_P + E_V{}')$	10.0 ± 0.6	8.73 ± 2.09	8.82 ± 2.15	10.2 ± 1.5	10.0
	$\pi^0\rho^0$	$\frac{1}{2}Y_d(C_P{}' + C_V{}' - E_P{}' - E_V{}')$	3.82 ± 0.29	3.06 ± 0.63	3.90 ± 1.62	1.4 ± 0.6	3.83
	K^+K^{*-}	$Y_s(T'_V + E'_P)$	1.62 ± 0.15	1.84 ± 0.22	1.83 ± 0.24	1.6 ± 0.3	1.73
	K^-K^{*+}	$Y_s(T'_P + E'_V)$	4.50 ± 0.30	4.44 ± 1.07	3.39 ± 0.83	4.7 ± 0.8	4.37
	$K^0\bar{K}^{*0}$	$Y_sE'_P + Y_dE'_V$	<1.5	1.374 ± 0.361	1.028 ± 0.430	0.16 ± 0.05	1.1
	$\bar{K}^0K^{*0}$	$Y_sE'_V + Y_dE'_P$	<0.54	1.374 ± 0.361	1.028 ± 0.430	0.16 ± 0.05	1.1
	$\pi^0\omega$	$\frac{1}{2}Y_d(C'_V - C'_P + E'_P + E'_V)$	$0.117 \pm 0.035^{\rm a}$	0.043 ± 0.156	0.272 ± 1.509	0.08 ± 0.02	0.18
	$\pi^0\phi$	$\frac{1}{\sqrt{2}}Y_sC'_P$	1.35 ± 0.10	0.77 ± 0.14	0.66 ± 0.11	1.0 ± 0.3	1.11
	$\eta\omega$	$Y_d\frac{1}{2}(C'_V + C'_P + E'_V + E'_P)c_\phi - Y_s\frac{1}{\sqrt{2}}C'_Vs_\phi$	$2.21 \pm 0.23^{\rm b}$	2.09 ± 0.49	2.67 ± 2.54	1.2 ± 0.3	2.0
	$\eta'\omega$	$-Y_d\frac{1}{2}(C'_V + C'_P + E'_V + E'_P)s_\phi - Y_s\frac{1}{\sqrt{2}}C'_Vc_\phi$	$\cdots$	0.012 ± 0.012	0.046 ± 0.067	0.0001 ± 0.0001	0.02
	$\eta\phi$	$Y_s(\frac{1}{\sqrt{2}}C'_Pc_\phi - (E'_V + E'_P)s_\phi)$	0.14 ± 0.05	0.29 ± 0.12	0.29 ± 0.08	0.23 ± 0.06	0.18
	$\eta\rho^0$	$-Y_d\frac{1}{2}(C'_V - C'_P - E'_V - E'_P)c_\phi + Y_s\frac{1}{\sqrt{2}}C'_Vs_\phi$	$\cdots$	0.60 ± 0.40	0.80 ± 2.63	0.05 ± 0.01	0.45
	$\eta'\rho^0$	$Y_d\frac{1}{2}(C'_V - C'_P - E'_V - E'_P)s_\phi + Y_s\frac{1}{\sqrt{2}}C'_Vc_\phi$	$\cdots$	0.055 ± 0.021	0.105 ± 0.075	0.08 ± 0.02	0.27
D^+	$\pi^+\rho^0$	$\frac{1}{\sqrt{2}}Y_d(T'_V + C'_P - A'_P + A'_V)$	0.84 ± 0.15	0.51 ± 0.28	0.68 ± 0.35	0.8 ± 0.7	0.58
	$\pi^0\rho^+$	$\frac{1}{\sqrt{2}}Y_d(T'_P + C'_V + A'_P - A'_V)$	$\cdots$	4.35 ± 5.01	4.27 ± 16.51	3.5 ± 1.6	2.5
	$\pi^+\omega$	$\frac{1}{\sqrt{2}}Y_d(T'_V + C'_P + A'_P + A'_V)$	$0.279 \pm 0.059^{\rm a}$	0.165 ± 0.269	0.208 ± 0.240	0.3 ± 0.3	0.80
	$\pi^+\phi$	$Y_sC'_P$	$5.66^{+0.19}_{-0.21}$	3.92 ± 0.69	3.37 ± 0.59	5.1 ± 1.4	5.65
	$\eta\rho^+$	$-Y_d\frac{1}{\sqrt{2}}(T'_P + C'_V + A'_V + A'_P)c_\phi + Y_sC'_Vs_\phi$	$<6.8^{\rm c}$	1.43 ± 4.60	0.95 ± 10.05	0.4 ± 0.4	2.2
	$\eta'\rho^+$	$Y_d\frac{1}{\sqrt{2}}(T'_P + C'_V + A'_V + A'_P)s_\phi + Y_sC'_Vc_\phi$	$<5.2^{\rm c}$	0.964 ± 0.168	0.958 ± 0.507	0.8 ± 0.1	0.8
	$K^+\bar{K}^{*0}$	$Y_dA'_V + Y_sT'_V$	$3.84^{+0.14}_{-0.23}$	4.00 ± 0.82	3.86 ± 0.78	4.1 ± 1.0	3.60
	$\bar{K}^0K^{*+}$	$Y_dA'_P + Y_sT'_P$	34 ± 16	14.45 ± 2.45	10.03 ± 2.62	12.4 ± 2.4	11
D_s^+	π^+K^{*0}	$Y_dT'_V + Y_sA'_V$	2.13 ± 0.36	3.51 ± 0.72	3.76 ± 0.76	1.5 ± 0.7	2.35
	π^0K^{*+}	$\frac{1}{\sqrt{2}}(Y_dC'_V - Y_sA'_V)$	$\cdots$	1.47 ± 0.45	1.04 ± 0.48	0.1 ± 0.1	1.0
	$K^+\rho^0$	$\frac{1}{\sqrt{2}}(Y_dC'_P - Y_sA'_P)$	2.5 ± 0.4	1.58 ± 0.38	2.07 ± 0.57	1.0 ± 0.6	2.5
	$K^0\rho^+$	$Y_dT'_P + Y_sA'_P$	$\cdots$	11.25 ± 1.90	11.45 ± 2.99	7.5 ± 2.1	9.6
	ηK^{*+}	$-\frac{1}{\sqrt{2}}(Y_dC'_V + Y_sA'_V)c_\phi + Y_s(T'_P + C'_V + A'_P)s_\phi$	$\cdots$	0.59 ± 2.26	0.64 ± 6.09	1.0 ± 0.4	0.2
	$\eta'K^{*+}$	$\frac{1}{\sqrt{2}}(Y_dC'_V + Y_sA'_V)s_\phi + Y_s(T'_P + C'_V + A'_P)c_\phi$	$\cdots$	0.42 ± 0.15	0.32 ± 0.14	0.6 ± 0.2	0.2
	$K^+\omega$	$\frac{1}{\sqrt{2}}(Y_dC'_P + Y_sA'_P)$	<2.4	1.05 ± 0.34	2.15 ± 0.56	1.8 ± 0.7	0.07
	$K^+\phi$	$Y_s(T'_V + C'_P + A'_V)$	0.164 ± 0.041	0.111 ± 0.060	0.112 ± 0.068	0.3 ± 0.3	0.166

PREDICTIONS — SCS MODES

- Measurements of SCS decay modes are useful in distinguishing different solutions:
  - Among solutions (A), (A1) is more preferred.
  - Among solutions (S), (S4) is more preferred.
- We tried a fit to only SCS modes. Not only did we obtain more solutions, we also could not get small χ^2 results. ➠ these data present inconsistency within the framework
- In contrast, all the solutions can explain DCS decay data sufficiently well.

PREDICTIONS — DCS MODES



Meson	Mode	Representation	$\mathcal{B}_{\text{exp}}$	$\mathcal{B}_{\text{theory}}(A1)$	$\mathcal{B}_{\text{theory}}(S4)$	$\mathcal{B}(\text{pole})$	$\mathcal{B}(\text{FAT[mix]})$
D^0	$K^{*+}\pi^-$	$Y_{ds}(T''_P + E''_V)$	$3.45^{+1.80}_{-1.02}$	3.77 ± 0.90	2.88 ± 0.70	2.7 ± 0.6	4.72
	$K^{*0}\pi^0$	$\frac{1}{\sqrt{2}}Y_{ds}(C''_P - E''_V)$	$\cdots$	0.49 ± 0.23	0.47 ± 0.12	0.8 ± 0.3	0.9
	ϕK^0	$-Y_{ds}E''_V$	$\cdots$	0.04 ± 0.03	0.01 ± 0.01	0.20 ± 0.06	0.2
	$\rho^- K^+$	$Y_{ds}(T''_V + E''_P)$	$\cdots$	1.34 ± 0.16	1.76 ± 0.23	0.9 ± 0.3	1.5
	$\rho^0 K^0$	$\frac{1}{\sqrt{2}}Y_{ds}(C''_V - E''_P)$	$\cdots$	1.06 ± 0.38	1.30 ± 1.80	0.5 ± 0.2	0.3
	ωK^0	$-\frac{1}{\sqrt{2}}Y_{ds}(C''_V + E''_P)$	$\cdots$	0.40 ± 0.37	0.61 ± 1.74	0.7 ± 0.2	0.6
	$K^{*0}\eta$	$Y_{ds}(\frac{1}{\sqrt{2}}(C''_P + E''_V)c_\phi - E''_P)s_\phi$	$\cdots$	0.53 ± 0.10	0.46 ± 0.08	0.08	0.2
	$K^{*0}\eta'$	$Y_{ds}(\frac{1}{\sqrt{2}}(C''_P + E''_V)s_\phi + E''_P c_\phi)$	$\cdots$	0.001 ± 0.0004	0.002 ± 0.001	0.004 ± 0.001	0.005
D^+	$K^{*0}\pi^+$	$Y_{ds}(C''_P + A''_V)$	3.9 ± 0.6	2.94 ± 0.85	2.66 ± 0.68	2.2 ± 0.9	3.33
	$K^{*+}\pi^0$	$\frac{1}{\sqrt{2}}Y_{ds}(T''_P - A''_V)$	$\cdots$	5.76 ± 0.85	3.98 ± 1.17	4.0 ± 0.9	3.9
	ϕK^+	$Y_{ds}A''_V$	$\cdots$	0.02 ± 0.02	0.02 ± 0.01	0.2 ± 0.2	0.02
	$\rho^+ K^0$	$Y_{ds}(C''_V + A''_P)$	$\cdots$	2.81 ± 0.76	2.39 ± 1.14	0.5 ± 0.4	3.3
	$\rho^0 K^+$	$\frac{1}{\sqrt{2}}Y_{ds}(T''_V - A''_P)$	2.1 ± 0.5	1.66 ± 0.24	2.09 ± 0.44	0.5 ± 0.4	2.4
	ωK^+	$\frac{1}{\sqrt{2}}Y_{ds}(T''_V + A''_P)$	$\cdots$	0.95 ± 0.20	1.90 ± 0.42	1.8 ± 0.5	0.7
	$K^{*+}\eta$	$-Y_{ds}(\frac{1}{\sqrt{2}}(T''_P + A''_V)c_\phi - A''_P s_\phi)$	$\cdots$	1.89 ± 0.40	1.33 ± 0.33	1.4 ± 0.2	1.0
	$K^{*+}\eta'$	$Y_{ds}(\frac{1}{\sqrt{2}}(T''_P + A''_V)s_\phi + A''_P c_\phi)$	$\cdots$	0.02 ± 0.01	0.02 ± 0.01	0.020 ± 0.007	0.01
D_s^+	$K^{*+}K^0$	$Y_{ds}(T''_P + C''_V)$	$\cdots$	1.55 ± 1.49	1.29 ± 4.48	2.3 ± 0.6	1.1
	$K^{*0}K^+$	$Y_{ds}(T''_V + C''_P)$	0.90 ± 0.51	0.17 ± 0.03	0.19 ± 0.03	0.2 ± 0.2	0.23

PREDICTIONS — DCS MODES

- For observed modes, our predictions are consistent with data within 1σ, except for $D_s^+ \to K^{*0}K^+$ whose measured value is significantly larger than theory predictions, though its error bar is also large.
- The $D^+ \to K^{*0}\pi^+$ and ρ^0K^+ modes involve respectively A_V and A_P, without which their predicted BFs are smaller than the measured values.

  ➠ clear indication the necessity of $A_{P,V}$

SU(3) BREAKING

- If we assume for factorizable amplitudes (T and C) that the effective Wilson coefficients $a_{1,2}$ are the same, their sizes will differ mode by mode due to differences in the final-state meson masses, decay constants, and form factors.

$$\frac{T'_{P,\bar{K}^0 K^{*+}}}{T_{P,\bar{K}^0 \rho^+}} = \frac{f_{K^*}}{f_\rho} \frac{F_1^{DK}(m^2_{K^{*+}})}{F_1^{DK}(m^2_{\rho^+})} \simeq 1.09$$

$$\frac{C'_{P,\pi^+\phi}}{C_{P,\bar{K}^{*0}\pi^+}} = \frac{f_\phi}{f_{K*}} \frac{F_1^{D\pi}(m^2_\phi)}{F_1^{D\pi}(m^2_{K^{*0}})} \simeq 1.07$$

- Although some of the modes have better agreement with data after the above-mentioned symmetry breaking is included, some others deviate from measurements even more regardless of which solution we take.

SUMMARY

- Using $SU(3)_F$ symmetry as a working assumption along with latest data, we have updated a global χ^2 fit to CF decay BFs.
- Thanks to recent measurement of $BR(D_s^+ \to \pi^+\rho^0)$, we have determined for the first time $A_{P,V}$.
  ➠ a determination of $BR(D_s^+ \to \pi^0\rho_+)$ useful in confirming the information and reducing uncertainties associated with $A_{P,V}$
- Though serious $SU(3)_F$ violation is seen, we have used SCS data and our predictions to find favored solutions.
- We have tried by including $SU(3)_F$ breaking in T and C to see if there is a better agreement with data. However, the conclusion is mixed, and the exact $SU(3)_F$ approach is still sufficiently adequate to provide an overall explanation for the current data.

Thank You!