Gluon fragmentation functions in the Nambu-Jona-Lasinio model

Hsiang-nan Li (李湘楠)
Academia Sinica, Taipei
Presented at HPCPV, SJTU
Nov. 04, 2016
In collaboration with D.J. Yang

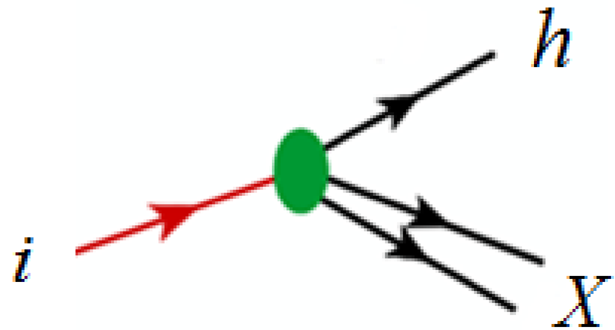
Outlines

- Introduction
- Gluon fragmentation functions
- Comparison with data
- Summary

Introduction

Fragmentation functions

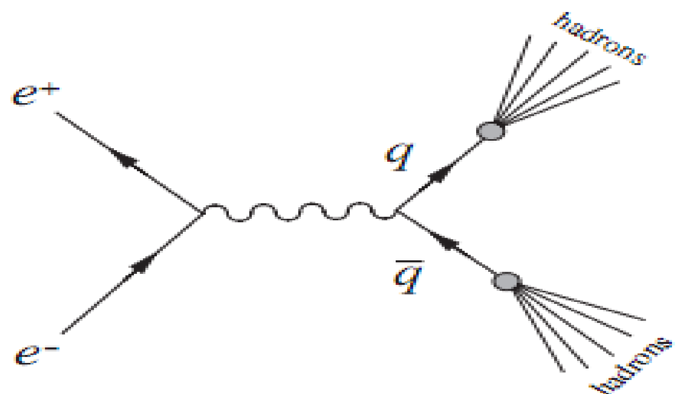
- Fragmentation functions $D_i^h(z)$ mean the probability to find the hadron h from a parton i=u, d, s,…,g with the energy fraction z.



- one needs fragmentation functions for the analysis of e^+e^- *annihilation* into hadrons, *Semi inclusive deep inelastic scattering*, and so on.

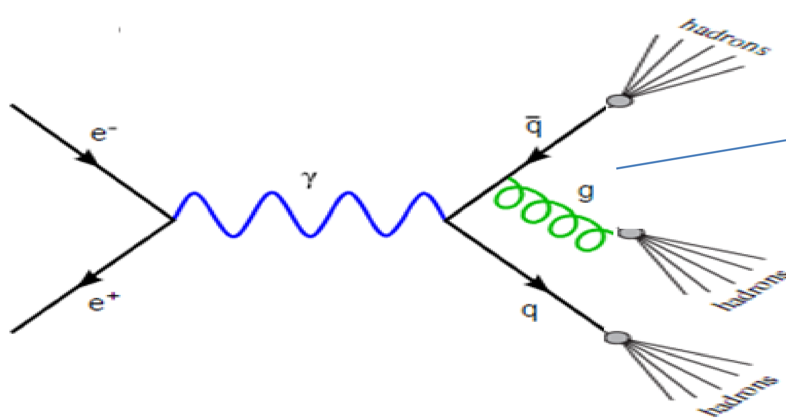
$e^{+}e^{-}$ *annihilation*: $e^{+} + e^{-} \rightarrow h + x$

- LO: $\sigma^{e^{+}e^{-}\rightarrow hx} = \sum_q \sigma^{e^{+}e^{-}\rightarrow q\bar{q}} \otimes (D_q^h + D_{\bar{q}}^h)$ (LO)



quark FF

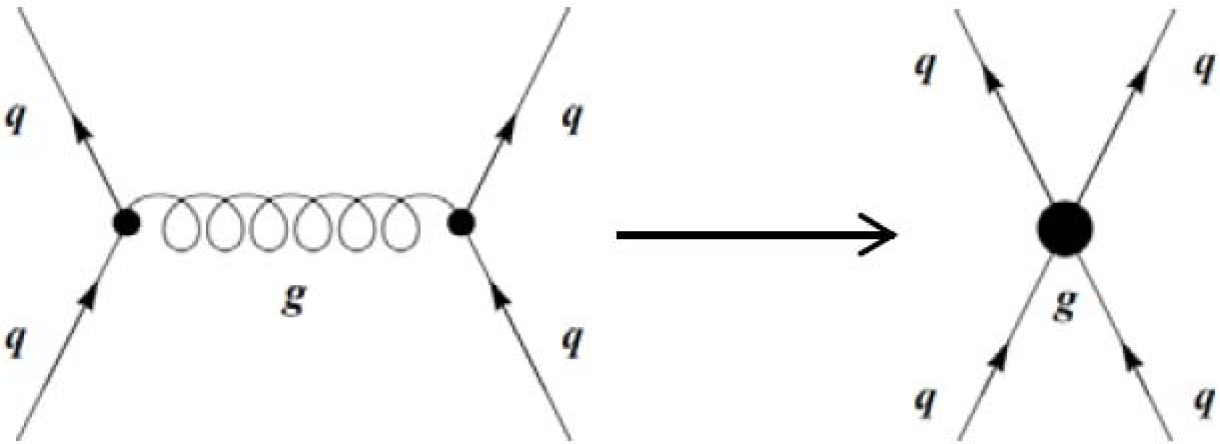
- NLO: $\sigma^{e^{+}e^{-}\rightarrow hx} = \sum_q \sigma^{e^{+}e^{-}\rightarrow gq\bar{q}} \otimes (D_q^h + D_{\bar{q}}^h + D_g^h)$ (NLO)



gluon FF

NJL model

- Gluons have been integrated out at very low energy in NJL model

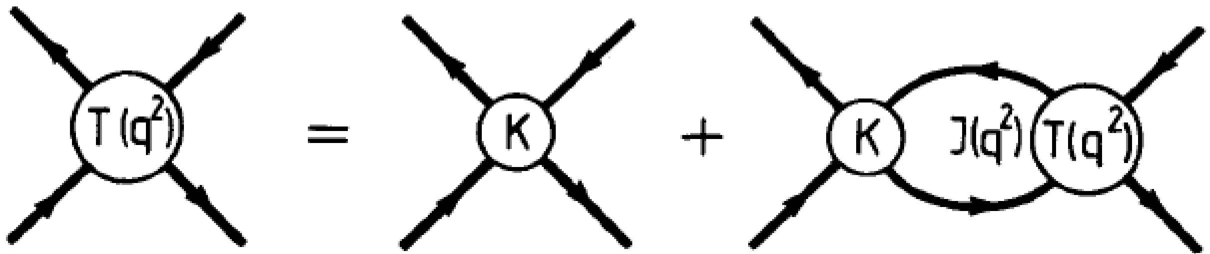


- How could it be possible to get gluon FF?
- Without gluon FF, QCD evolution cannot be complete

NG bosons

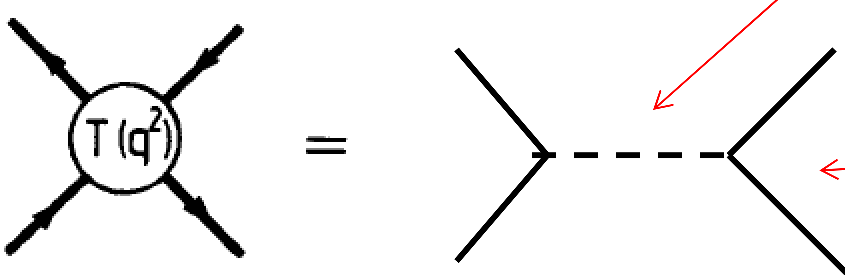
- Spontaneous chiral symmetry breaking generates NG bosons

$$G(\overline{\psi}\gamma_5\psi)^2 \Rightarrow$$



$$T_P(q^2) = K_P + K_P\, J_P(q^2)\, K_P + \ldots$$
$$= [1 - K_P\, J_P(q^2)]^{-1}\, K_P.$$

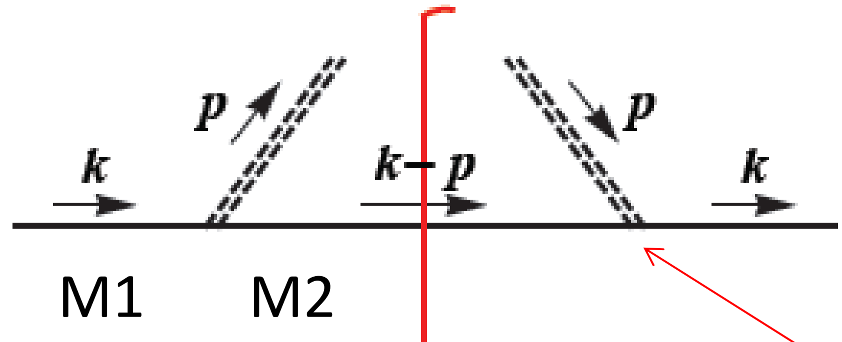
$$\langle \overline{\psi}\psi \rangle \neq 0 \Rightarrow \quad K_P\, J_P(q^2 = 0) = 1$$ ← NG boson



pseudo-scalar quark current

Elementary fragmentation functions

- Mesons of couple to pseudo-scalar quark current in NJL model
- Elementary FF



$$d_q^m(z) = -\frac{C_q^m}{2} g_{mqQ}^2 \frac{z}{2} \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\left[S_1(k)\gamma^+ S_1(k)\gamma_5(\not{k} - \not{p} + M_2)\gamma_5\right]$$
$$\times \delta(k_- - p_-/z) 2\pi\delta((k-p)^2 - M_2^2)$$
$$= \frac{C_q^m}{2} g_{mqQ}^2 \frac{z}{2} \int \frac{d^2p_\perp}{(2\pi)^3} \frac{p_\perp^2 + ((z-1)M_1 - M_2)^2)}{(p_\perp^2 + z(z-1)M_1^2 + zM_2^2 + (1-z)m_m^2)^2}$$

meson mass

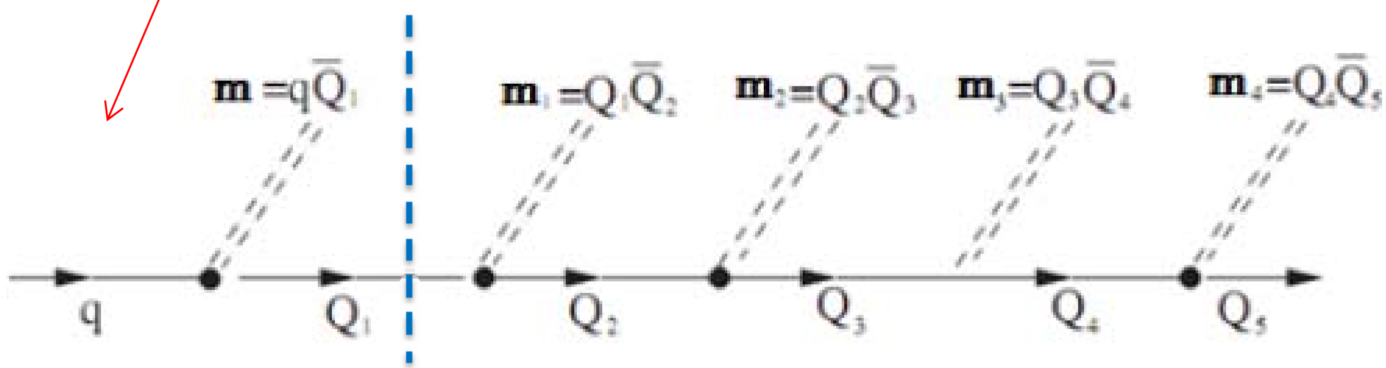
Quark fragmentation functions

- Solve evolution equation

$$D_q^m(z) = \hat{d}_q^m(z) + \sum_Q \int_z^1 \frac{dy}{y} \hat{d}_q^Q(y) D_Q^m\left(\frac{z}{y}\right)$$

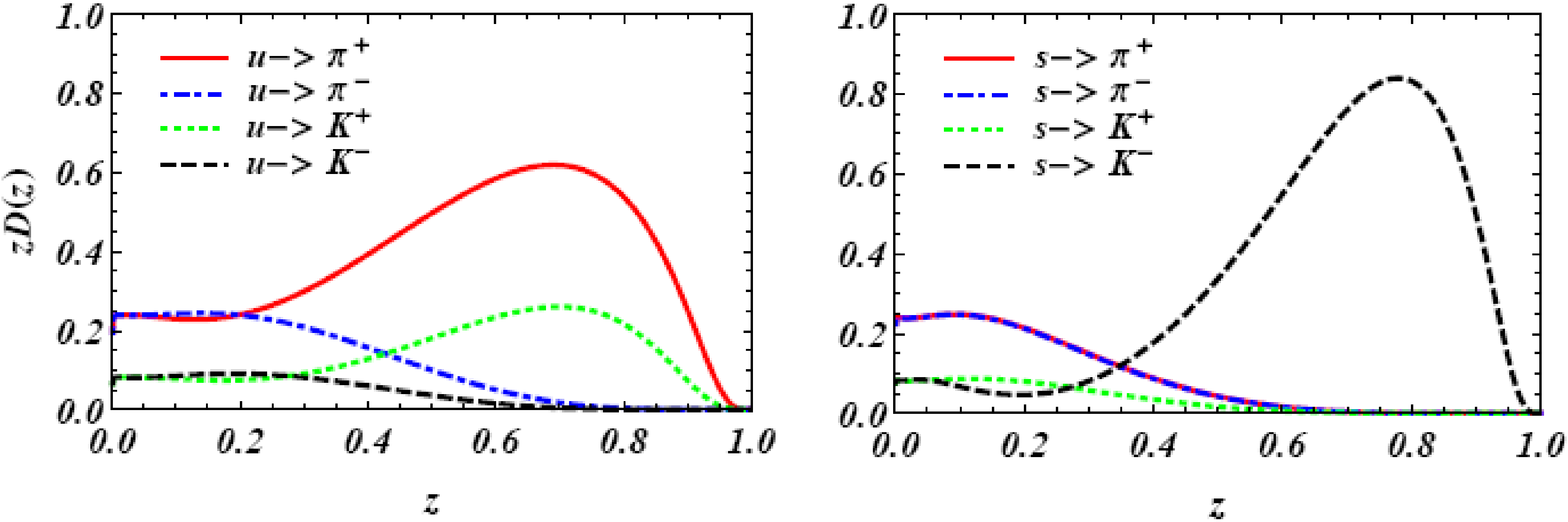
$$\hat{d}_q^Q(y) = \hat{d}_q^m(1-y)\big|_{m=q\bar{Q}}$$

- Iteration gives



- Use Monte Carlo simulation

Quark FFs at model scale



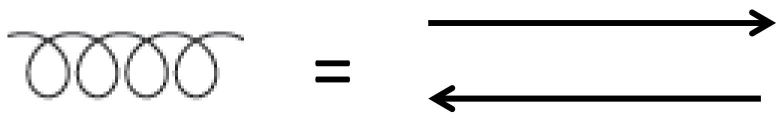
Favor FFs: $D_q^h(z)$ *with* $d_q^h(z) \neq 0$, example: $D_u^{\pi^+}(z)$

Unfavor FFs: $D_q^h(z)$ *with* $d_q^h(z) = 0$, example: $D_u^{\pi^-}(z)$

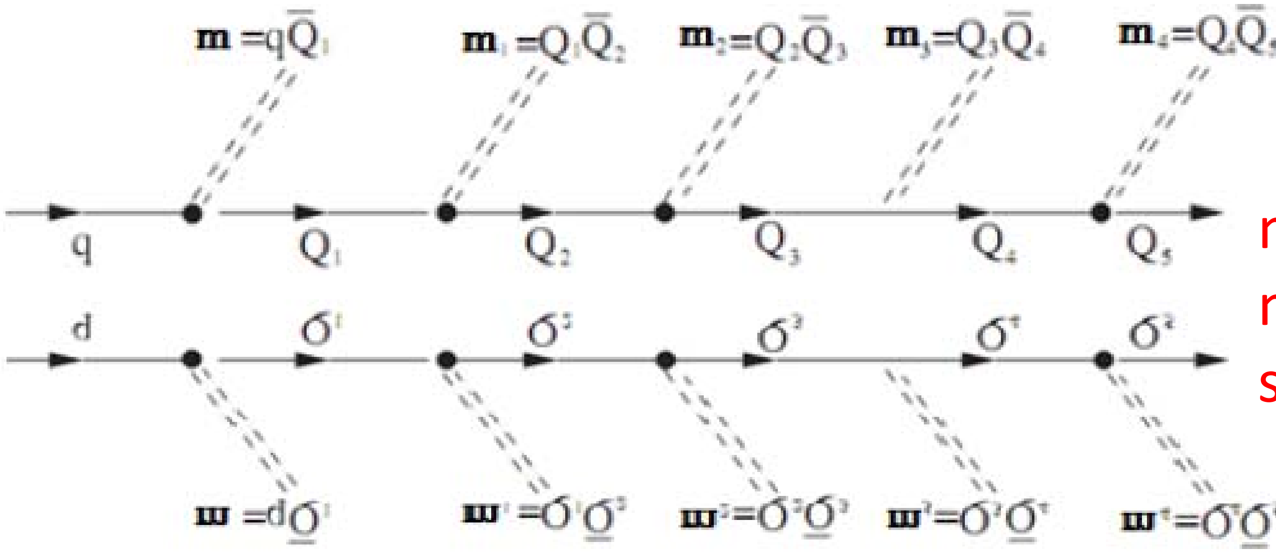
Gluon fragmentation functions

Basic idea

- Treat gluon as color dipole, formed by fictitious quark and anti-quark pair (color octet)



- Fictitious quark then radiates according to NJL



mirror symmetry maintains flavor singlet

Scheme 1

- Quark shares gluon momentum according to splitting function

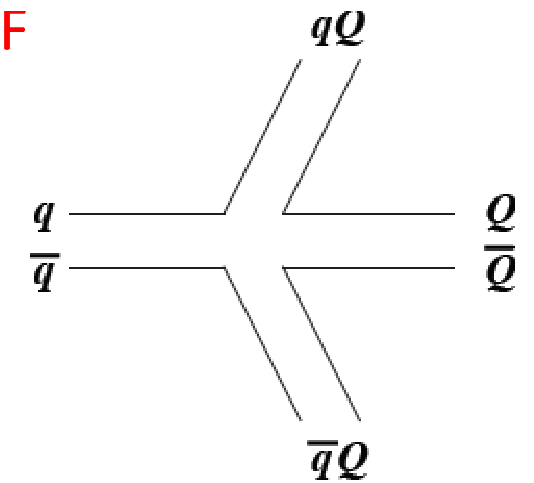
$$P_{g\to q\bar{q}}(x) = \frac{1}{2}(1 - 2x + 2x^2),$$

- Taken care of by quark FF afterwards

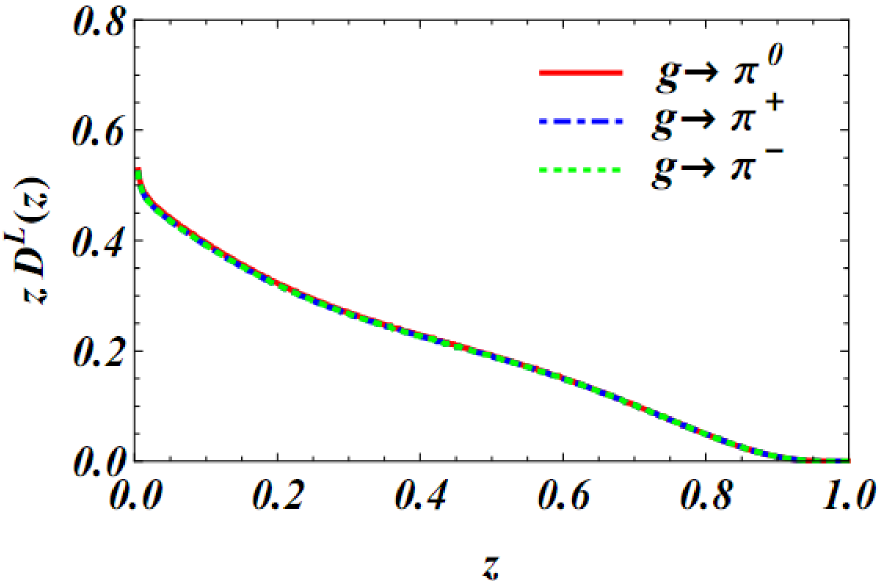
$$D_g^{Lm}(z) = \sum_q \frac{1}{3}\int_0^1 P_{g\to q\bar{q}}(x)\left[D_q^m(\frac{z}{x})\frac{1}{x} + D_{\bar{q}}^m(\frac{z}{1-x})\frac{1}{1-x}\right]dx$$

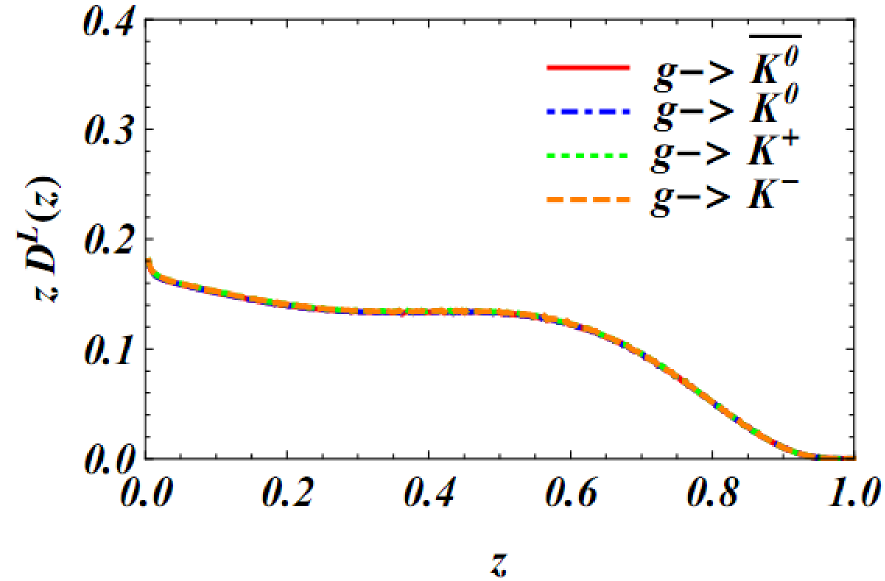
quark FF

- Corresponding Feynman diagram



Results at model scale



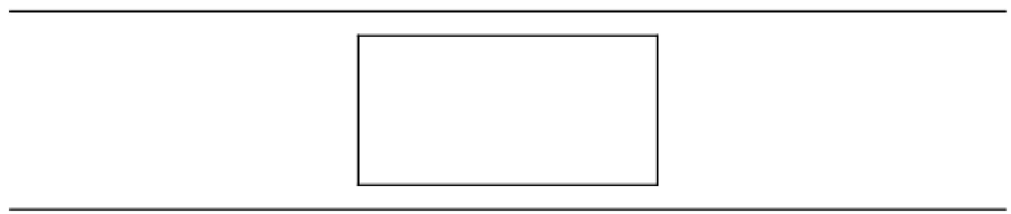


$$zD_g^{L\pi^0}(z) = zD_g^{L\pi^+}(z) = zD_g^{L\pi^-}(z)$$

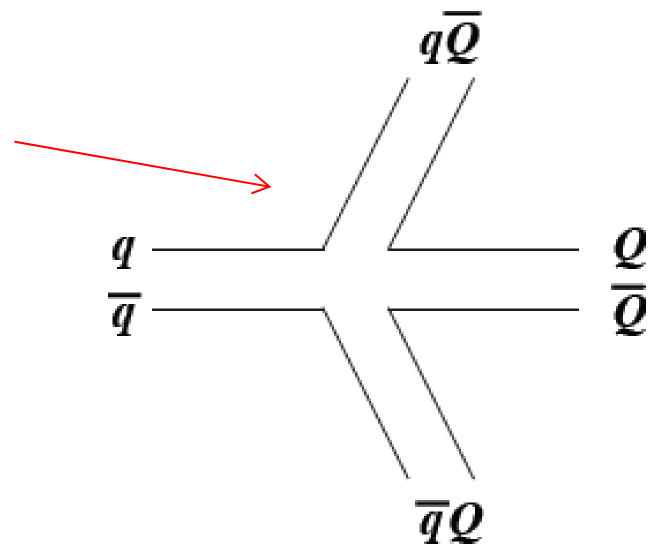
$$zD_g^{LK^+}(z) = zD_g^{LK^-}(z) = zD_g^{LK^0}(z) = zD_g^{L\overline{K^0}}(z)$$

Annihilation

- Quark mixing via annihilation color flow



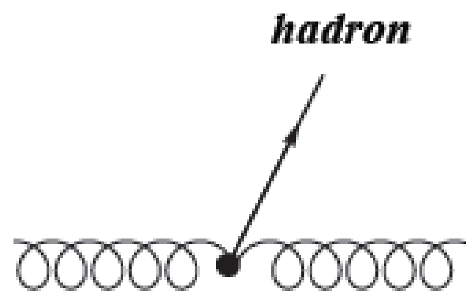
- Missing in scheme 1
- If included, quark flavor conservation can be relaxed



Scheme 2

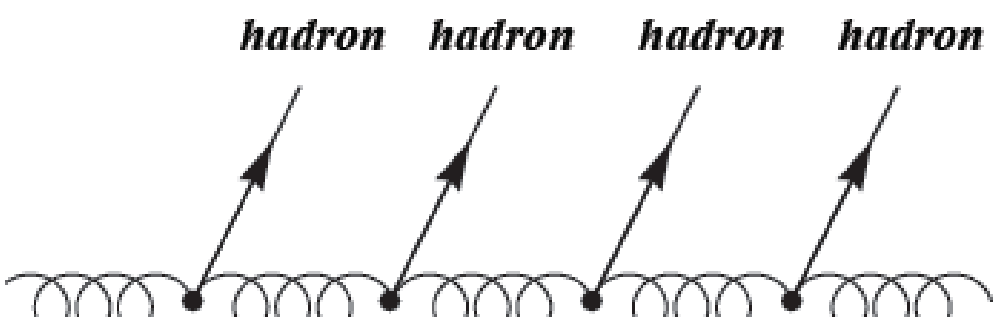
- Define gluon elementary FF first

$$d_g^m(z) = \sum_q \frac{1}{3} \int_0^1 P_{g\to q\bar{q}}(x) \left[d_q^m(\frac{z}{x})\frac{1}{x} + d_{\bar{q}}^m(\frac{z}{1-x})\frac{1}{1-x} \right] dx$$



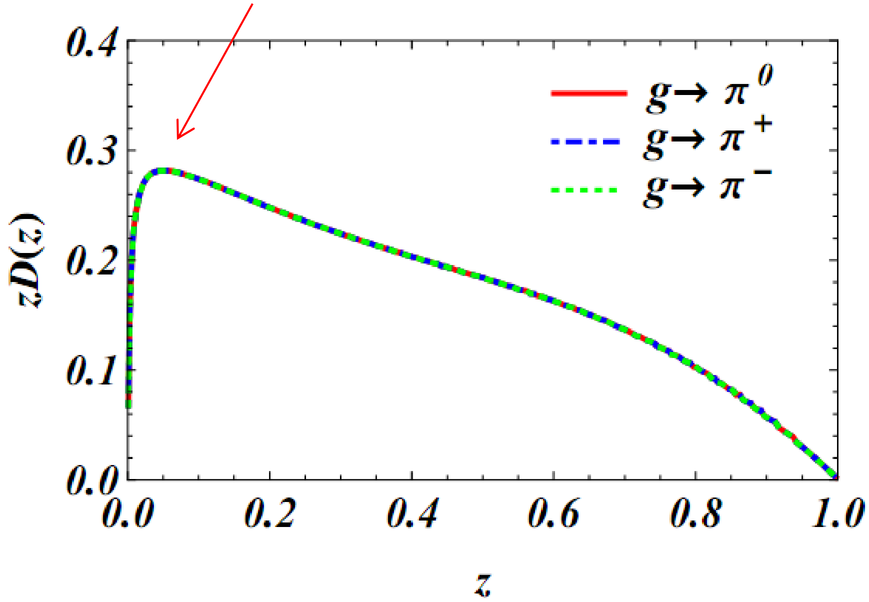
- Annihilation included
- Solve evolution equation

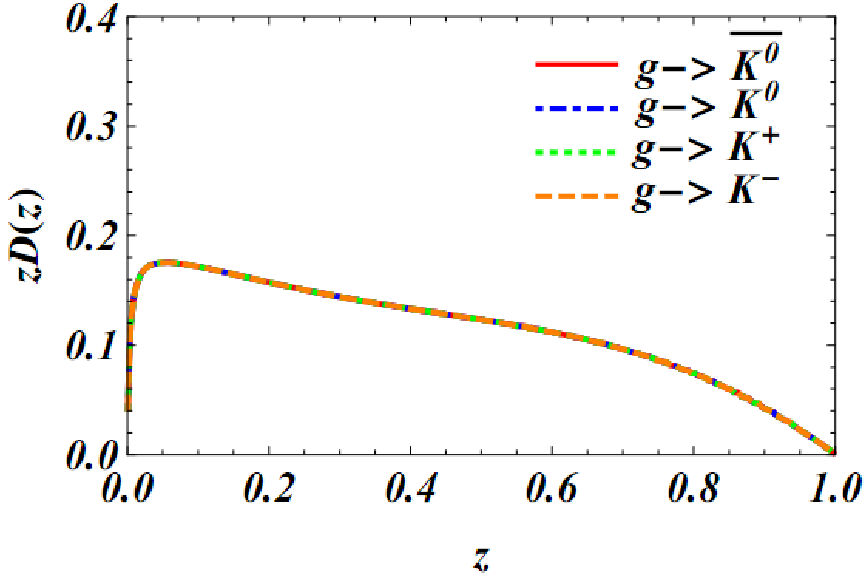
$$D_g^m(z) = \hat{d}_g^m(z) + \sum_{m'} \int_z^1 \frac{dy}{y} \hat{d}_g^{m'}(1-y) D_g^m(\frac{z}{y})$$



Results at model scale

peak shifts to larger z due to flavor blindness



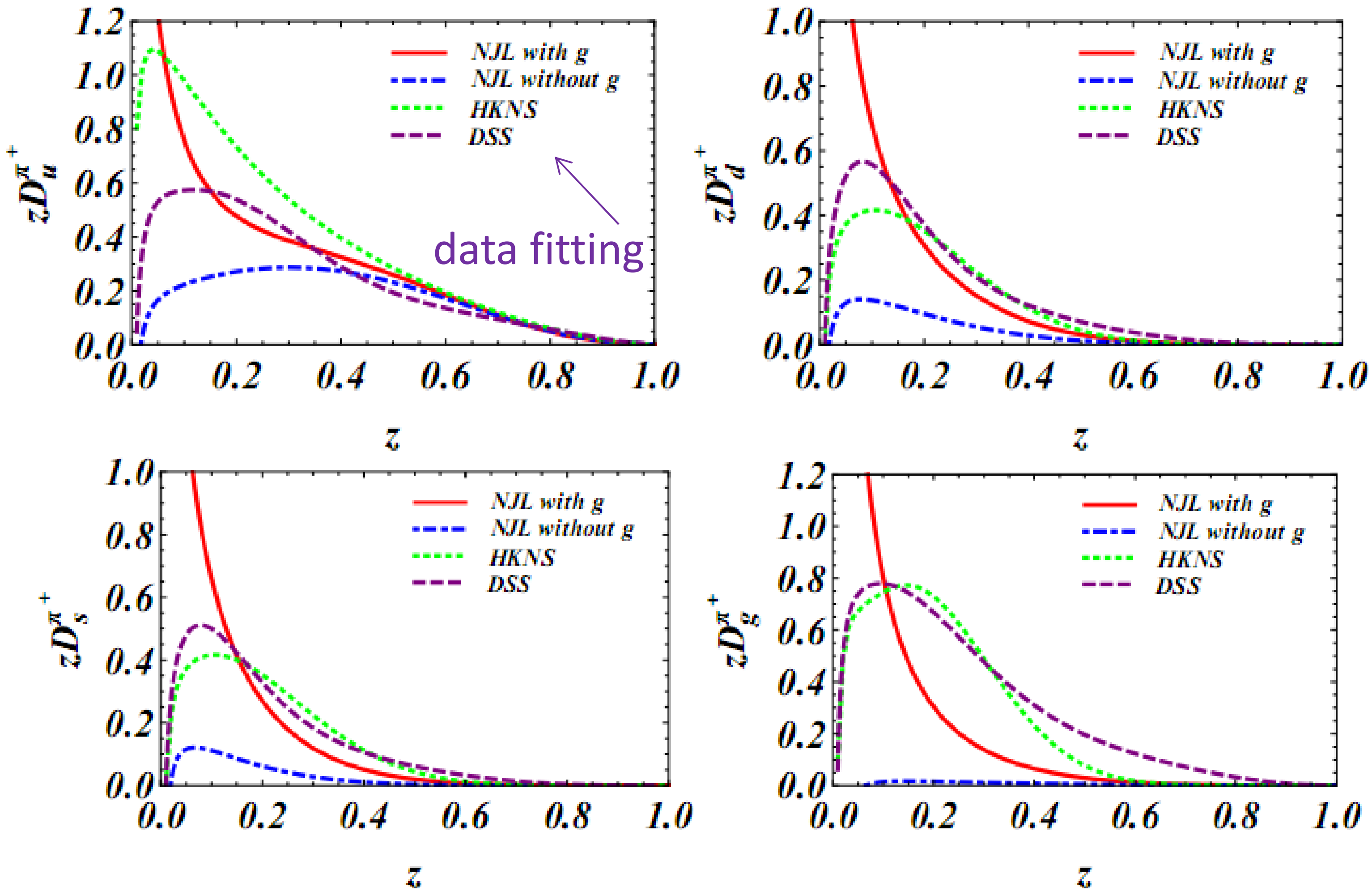


$$zD_g^{\pi^0}(z) = zD_g^{\pi^+}(z) = zD_g^{\pi^-}(z)$$

$$zD_g^{K^+}(z) = zD_g^{K^-}(z) = zD_g^{K^0}(z) = zD_g^{\overline{K^0}}(z)$$

Comparison with data

Comparison at $Q^2 = 4GeV^2$



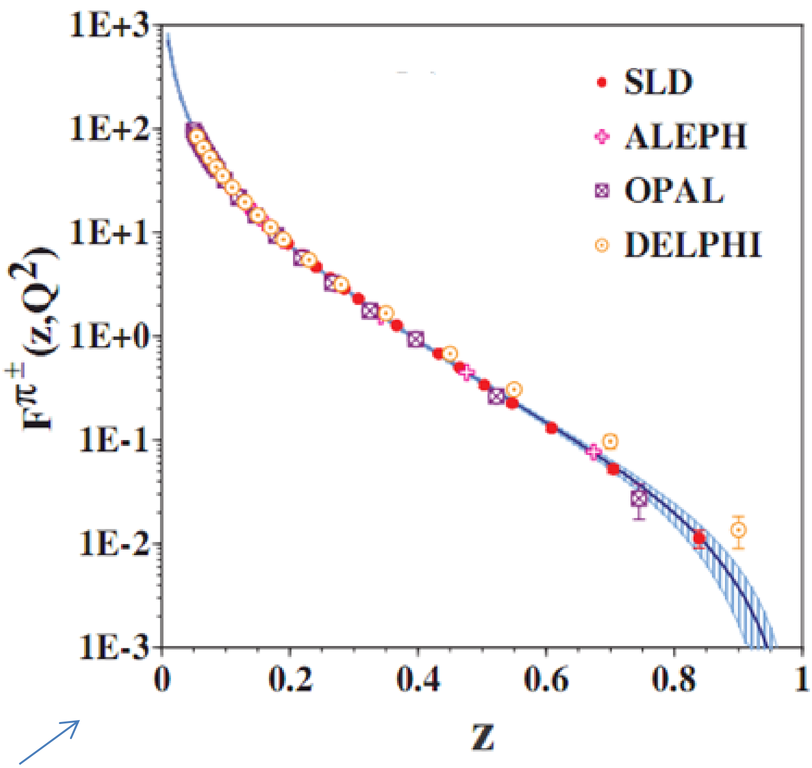
HKNS: M. Hirai, S. Kumano, T. H. Nagai, and K. Sudoh, Phys. Rev. D 75, 094009 (2007).

DSS: D. de Florian, R. Sassot, and M. Stratmann, Phys. Rev. D 75, 114010 (2007).

Comparison with data at $Q^2 = M_Z^2$

$$F^h(z,Q^2) = \frac{1}{\sigma_{tot}} \frac{d\sigma(e^+e^- \to hX)}{dz} = \sum_i C_i(z,\alpha_s) \otimes D_i^h(z,Q^2)$$

improve consistency



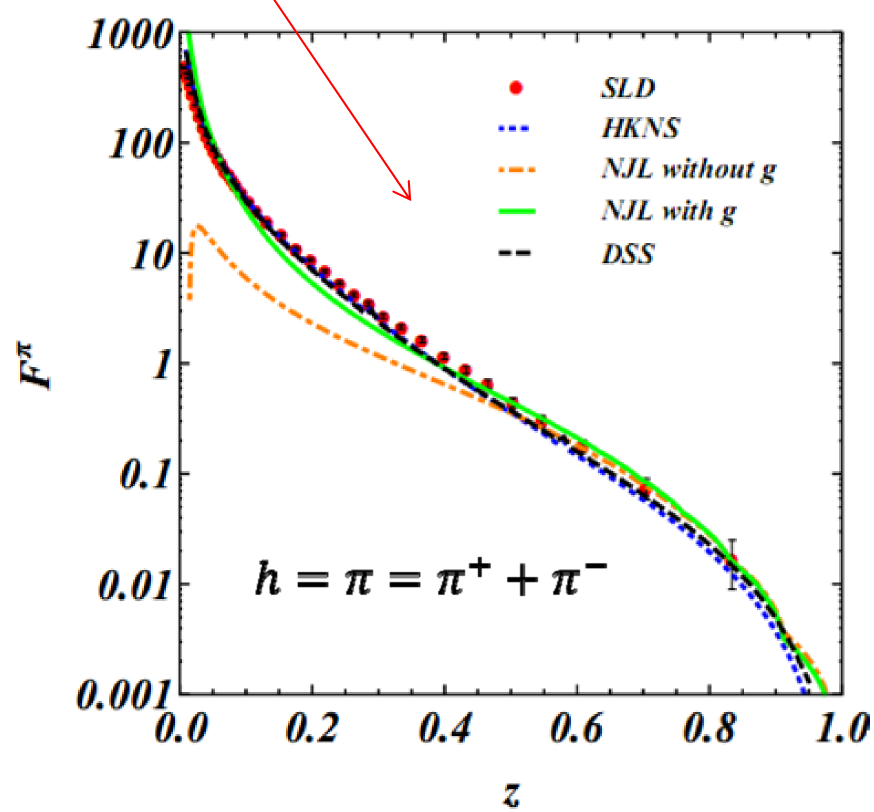
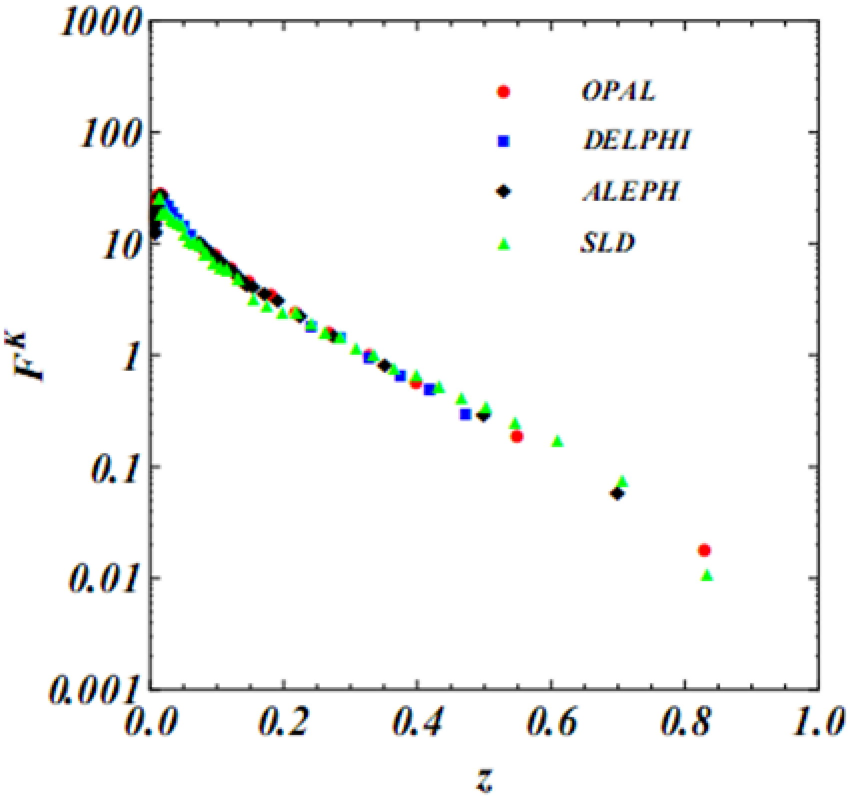


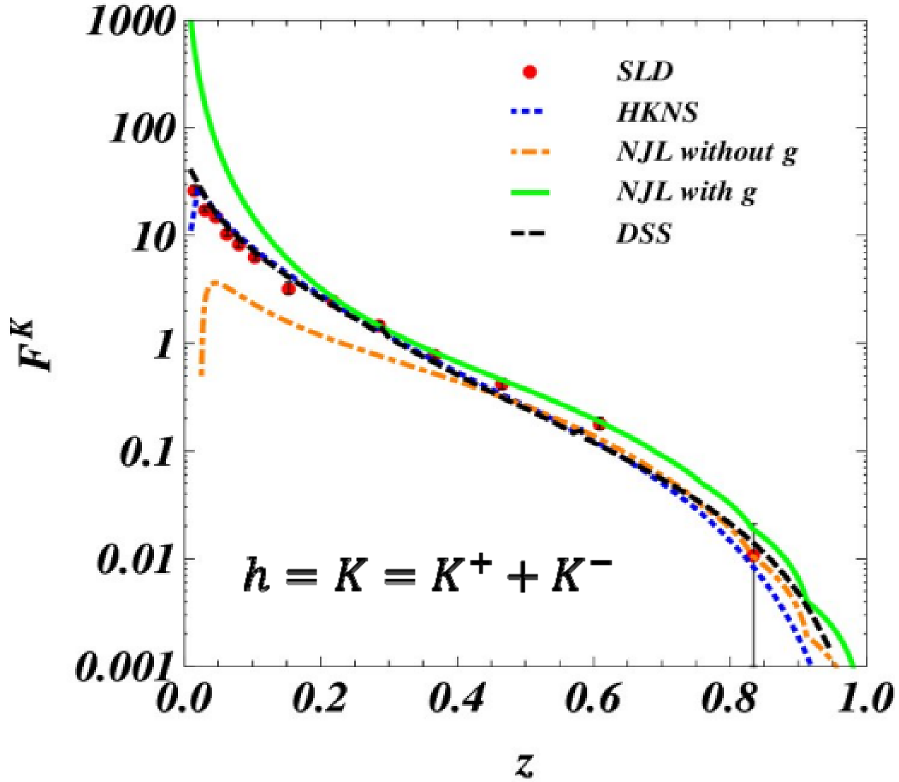
figure from HKNS paper

K. Abe et al. (SLD Collaboration), Phys. Rev. D69, 072003 (2004)

Comparison with data at $Q^2 = M_Z^2$

$$F^h(z,Q^2) = \frac{1}{\sigma_{tot}} \frac{d\sigma(e^+e^- \to hX)}{dz} = \sum_i C_i(z,\alpha_s) \otimes D_i^h(z,Q^2)$$





Summary

- NJL model can produce reasonable quark FFs, but cannot provide gluon FFs
- Without gluon FFs, QCD evolution incomplete
- Treat gluon as fictitious quark pair governed by NJL model to produce gluon FFs, insensitive to model parameters, like quark mass and model scale
- Gluon FFs improve consistency with data
- Ready to get heavy quark FFs in NJL model

  Guo Xiao-Yu, Chen Xiao-Lin, Deng Wei-Zhen (2013) Ebert, Feldmann, Friedrich, Reinhardt (1995)

Back-up slides

QCD evolution

- Without gluon FFs at model scale, QCD evolution cannot be complete

$$\frac{\partial}{\partial t} D^h_{q_i^-}(z,t) = \sum_j \int_z^1 \frac{dy}{y} P_{q_j q_i}(y)\, D^h_{q_j^-}\left(\frac{z}{y}, t\right)$$

$$\frac{\partial}{\partial t} D^h_{q_i^+}(z,t) = \int_z^1 \frac{dy}{y}\left[\sum_j P_{q_j q_i}(y)\, D^h_{q_j^+}\left(\frac{z}{y}, t\right) + 2P_{gq}(y) D^h_g\left(\frac{z}{y}, t\right)\right]$$

set to zero, spoil normalization

$$\frac{\partial}{\partial t} D^h_g(z,t) = \int_z^1 \frac{dy}{y}\left[P_{qg}(y) \sum_j D^h_{q_j^+}\left(\frac{z}{y}, t\right) + P_{gg}(y) D^h_g\left(\frac{z}{y}, t\right)\right]$$

$$D^h_{q_i^-}(z,Q^2) = D^h_{q_i}(z,Q^2) - D^h_{\bar{q}_i}(z,Q^2)$$

$P_{ji}(y)$: splitting functions

$$D^h_{q_i^+}(z,Q^2) = D^h_{q_i}(z,Q^2) + D^h_{\bar{q}_i}(z,Q^2)$$

$t = \log Q^2$

NJL model

- Nambu and Jona-Lasinio model (1961)
- Low-energy effective theory like BCS to demonstrate chiral symmetry breaking and appearance of NG boson

$$L = i\bar{\psi}\gamma_\mu \partial^\mu \psi + \frac{G}{4}[(\bar{\psi}\psi)(\bar{\psi}\psi) - (\bar{\psi}\gamma_5\psi)(\bar{\psi}\gamma_5\psi)]$$

$$= i\bar{\psi}_L \gamma_\mu \partial^\mu \psi_L + i\bar{\psi}_R \gamma_\mu \partial^\mu \psi_R + G(\bar{\psi}_L\psi_R)(\bar{\psi}_R\psi_L)$$

$U(1)_L \times U(1)_R$ symmetry

- Chiral condensate $\langle \bar{\psi}\psi \rangle \neq 0$ as $G > G_{crit}$
- Spontaneous chiral symmetry breaking gives rise to dynamical quark mass from gap eq.

Massive NG bosons

- To get massive NG bosons, need to add bare mass term, namely, explicit chiral symmetry breaking

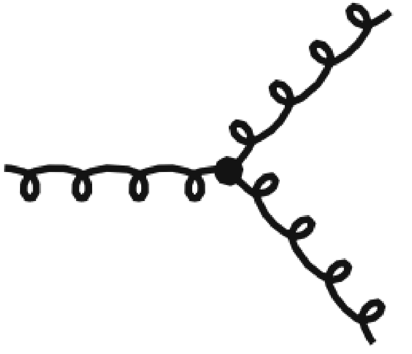
$$L_{NJL} = \bar{\psi}_q(i\partial\!\!\!/ - m_q)\psi_q + G(\bar{\psi}_q\Gamma\psi)^2$$

$$K_P\, J_P(q^2 \neq 0) = 1$$

- This NJL model has been used to calculate quark fragmentation functions of massive (physical) mesons

Multi-dipole

- The gluonic dynamics is more complicated than discussed above definitely.
- For example, $g \rightarrow g + g$



- The fictitious quark pair can split into two or more fictitious quark pairs at any stage of meson emissions.

Scheme 3

- Define elementary gluon FF

$$d_g^{Mm}(z) = \int_0^1 P_{g\to gg}(x) \left[d_g^m(\frac{z}{x})\frac{1}{x} + d_g^m(\frac{z}{1-x})\frac{1}{1-x} \right] dx,$$

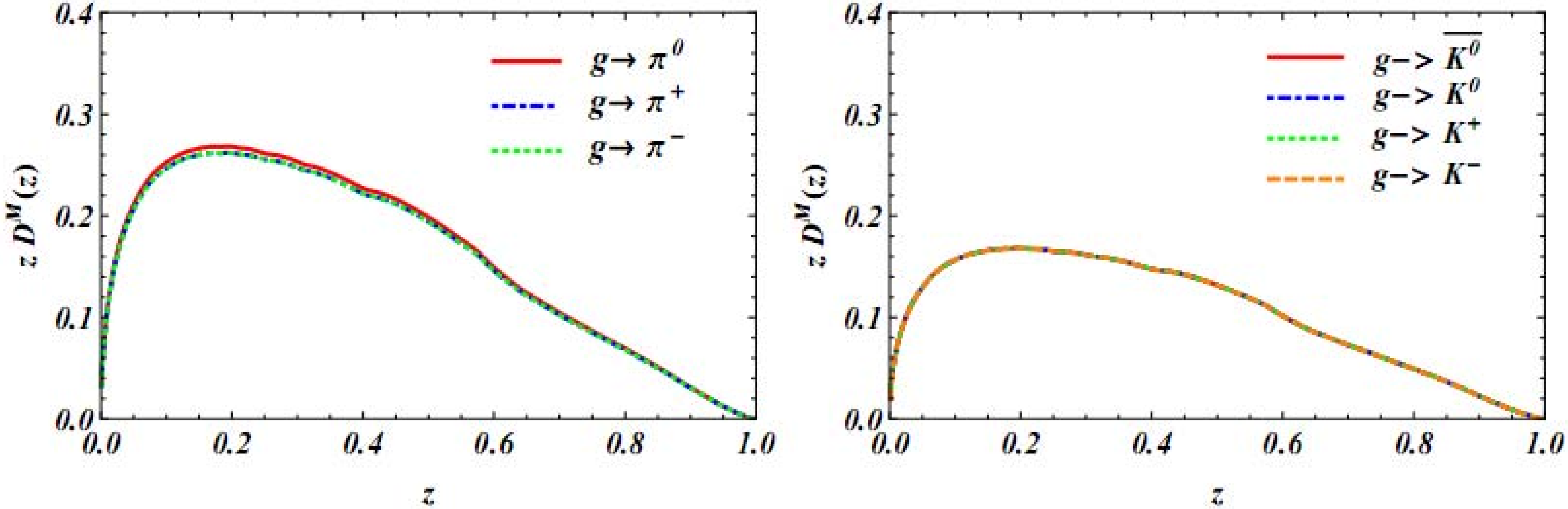
- Multi-dipole

$$P_{g\to gg}(x) = 6\left[\frac{1-x}{x} + x(1-x) + \frac{x}{1-x}\right], 0 < x < 1$$

- Solve evolution equation

$$D_g^{Mm}(z) = \hat{d}_g^{Mm}(z) + \sum_{m'} \int_z^1 \frac{dy}{y} \hat{d}_g^{Mm'}(1-y) D_g^{Mm}\left(\frac{z}{y}\right)$$

Results at model scale

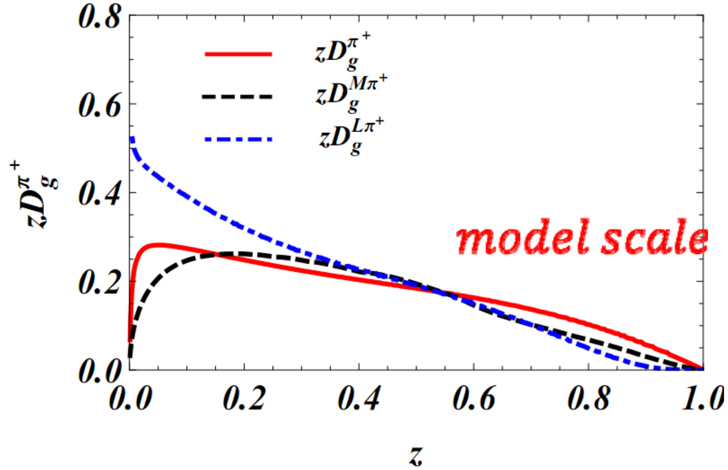


gluon branching effect is not significant
results insensitive to choices of splitting function

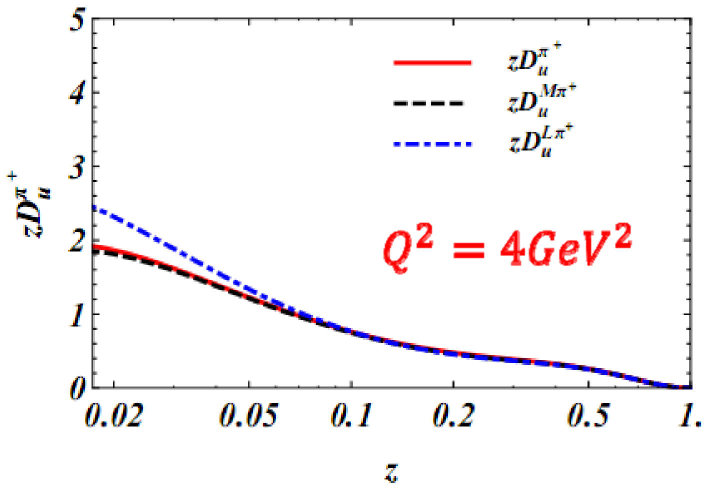
$$zD_g^{M\pi^0}(z) = zD_g^{M\pi^+}(z) = zD_g^{M\pi^-}(z)$$

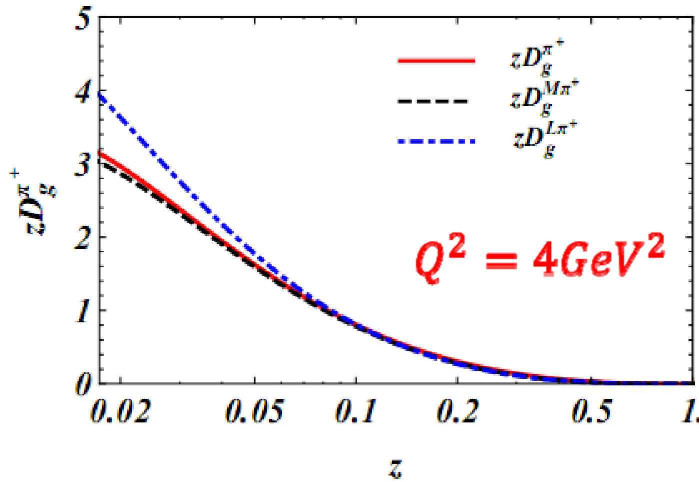
$$zD_g^{MK^+}(z) = zD_g^{MK^-}(z) = zD_g^{MK^0}(z) = zD_g^{M\overline{K^0}}(z)$$

Comparison of three schemes



Employ QCDNUM-17-00/07 (released on February 26, 2016) code to evolve Q^2 in NLO. (M. Botje, Comput. Phys. Commun. 182, 490 (2011))

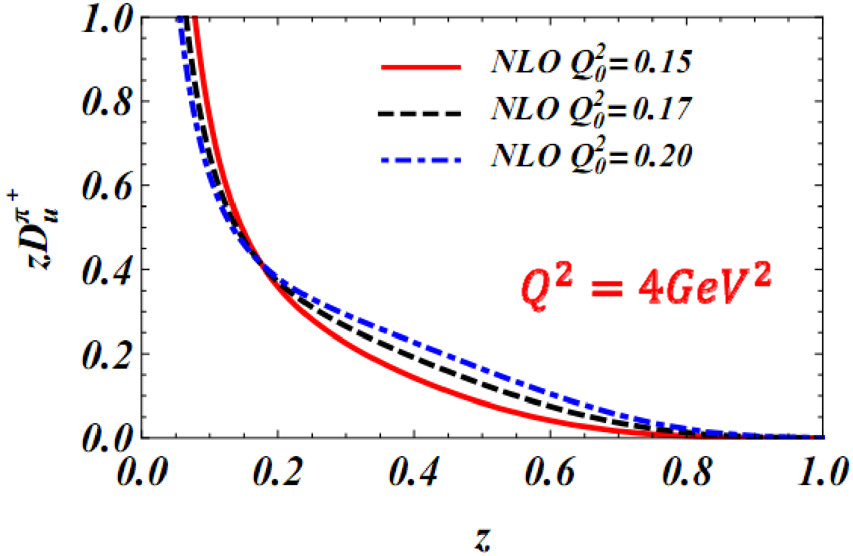


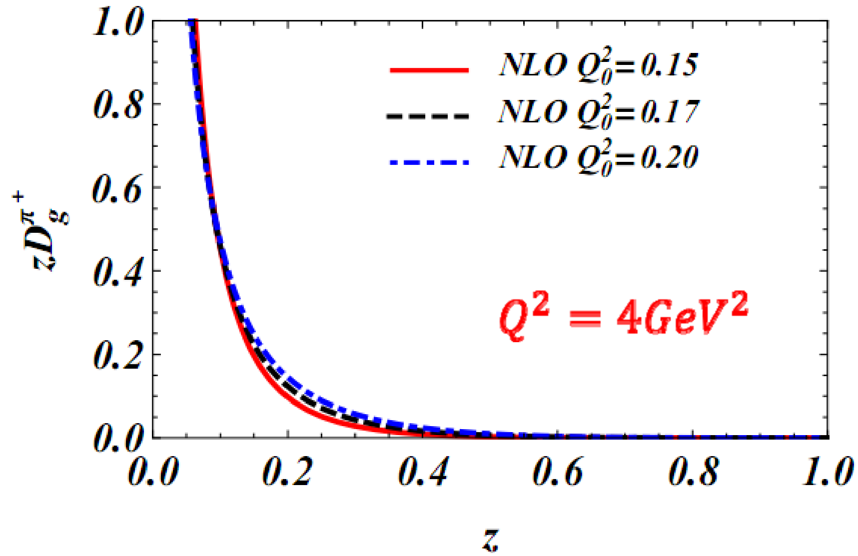


difference is pushed to small z after evolution

Sensitivity of model scale Q_0^2

- Quark FFs versus Gluon FFs





quark FFs are more sensitive to model scale than gluon FFs.
gluon FFs contribute in small z region after evolution

⇒ $e^+ + e^- \rightarrow h + x$ differential cross section at high z strongly depends on model scale due to quark FFs, but at small z it weakly depends on model scale.

Comparison at Q=Mz