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Effects of Magnetic Coupling on Radiation from Accretion Disc around a Kerr Black Hole

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ABSTRACT
The effects of magnetic coupling (MC) process on the inner edge of the disc are discussed in detail. It is shown that the inner edge can deviate from the innermost stable circular orbit (ISCO) due to the magnetic transfer of energy and angular momentum between a Kerr black hole (BH) and its surrounding accretion disc. It turns out that the inner edge could move inward and outward for the BH spin a_* being greater and less than 0.3594, respectively. The MC effects on disc radiation are discussed based on the displaced inner edge. A very steep emissivity can be provided by the MC process, which is consistent with the observation of MCG-6-30-15. In addition, the BH spins of GRO J1655-40 and GRS 1915+105 are detected by X-ray continuum fitting based on this model.

Key words: accretion, accretion disc— black hole physics — magnetic field

1 INTRODUCTION

As is well shown, disc accretion is a very efficient energy source in astrophysics, which is widely used to explain the high-energy radiation of AGNs, X-ray binaries and so on. In the model of standard accretion disc (SAD), the motion of the accreting matter is assumed to be Keplerian with a small radial velocity, and the inner edge of the disc lies at the innermost stable circular orbit (ISCO) of radius r_{ISCO}, within which the matter plunges very fast onto the black hole (BH). In this case, there would be no significant torques exerted at the inner edge, and the "no torque boundary condition" is a very good approximation (Bardeen 1970, Shakura & Sunyaev 1973, Novikov & Thorne 1973, Page & Thorne 1974).

Not long ago, some authors discussed the possibility that magnetic fields may induce a nonzero torque at ISCO, which involves the magnetic connection of the matter in the plunging region inside ISCO with the matter in the disc outside ISCO (Krolik 1999; Gammie 1999; Agol & Krolik 2000). On the other hand, some authors argued that the torque at ISCO of a thin disk is very weak, since it depends on the vertical thickness of the disk (Paczynski 2000; Li 2002a, hereafter L02; Afshordi & Paczynski 2003).

Recently, much attention has been paid to the magnetic coupling (MC) between a BH and its surrounding accretion disc (Blandford 1999, Li 2002a, hereafter L02, Wang et al. 2002), which can be regarded as one of the variants of the Blandford-Znajek (BZ) process (Blandford & Znajek 1977). By virtue of the magnetic field connecting the rotating BH and the disc, energy and angular momentum are transferred between the BH and the disc, just like the energy transportation between a dynamo and a motor (Macdonald & Thorne 1982, Thorne, Price & MacDonald 1986). In other words, the BH exerts a torque at the disc in the MC process, resulting in the transfer of energy and angular momentum. The transfer direction is determined by the difference between the rotation of the BH and that of the disc, i.e., if the BH rotates faster than the disc, energy and angular momentum are extracted from the BH and transferred to the disc, otherwise the direction is reversed.

Recently, Li (2004, hereafter L04) studied the MC between a BH and its surrounding disc. It is argued that the inner edge of the disc moves out to a radius where the angular velocity of the disc is equal to that of the BH for $a_* < 0.3594$. However, the inner edge remains at ISCO for the BH spin with $a_* > 0.3594$. This result seems somewhat inconsistent. Why is the inner edge of the disc affected by the MC for $a_* < 0.3594$, but not for $a_* > 0.3594$?

In this paper, we intend to discuss the MC effects on the inner edge of the disc. In our model the inner edge of the disc is not assumed to be ISCO in advance. The MC effects on the disc radiation lie in the following aspects. The profile of the radiation flux and the interior viscous torque at the disc can be changed due to the magnetic torque exerted at the disc. It is shown that if the BH rotates faster than the disc, the disc radiation would be more concentrated at the inner disc in the MC process. This feature can be used to explain the very steep emissivity index in the inner disc, which is consistent with the *XMM-Newton* observation of the nearby bright Seyfert 1 galaxy MCG-6-30-15 (Wilms et al. 2001; Li 2002b; Wang et al. 2003 hereafter W03).

The disc radiation is derived by considering the magnetic transfer of the energy and angular momentum between a rotating BH and a relativistic thin disc (Page & Thorne 1974; L02), and a criterion of determining the inner edge of the disc is proposed based on a reasonable constraint, i.e., a reasonable disc radiation should not be negative. It is shown that the value $a_* = 0.3594$ can be regarded as a critical BH spin, which corresponds to the angular velocity of ISCO equal to that of the BH. As the BH spin a_* is less than 0.3594, i.e., the energy and angular momentum are transferred from the inner disc to the BH, and the balance of energy and angular momentum on the disc would be disrupted in the MC process. Thus the inner disc becomes unstable, as the disc cannot suffer a negative interior viscous torque. It turns out that the MC effects on the disc radiation are so strong that the inner edge of the disc has to move outward and inward from ISCO for the BH spin a_* less and greater than 0.3594, respectively.

In order to facilitate the discussion of the MC effects on the inner edge of the disc we make the following assumptions:

1.The large scale magnetic field remains constant at the BH horizon, while it varies as a power-law with the disc radius as given in W03 rather than concentrated at ISCO.

2. The disc is thin, perfectly conducting and Keplerian, lying in the equatorial plane of a Kerr BH;

3. Considering that the "no torque boundary condition" remains controversial, we assume that no torque is exerted at the inner edge of a thin disc in the MC with a central BH.

This paper is organized as follows. In §2 the properties of the inner edge of SAD and the radial profiles of specific energy and angular momentum are discussed in detail. In §3 a criterion for the inner edge of a disc is proposed by considering the MC effects. In §4 some of the characteristics and applications of the MC process are presented, including fitting the steep emissivity index of MCG-6-30-15 and the BH spins of GRO J1655-40 and GRS 1915+105. Finally, in §5, we summarize our main results. Throughout this paper Boyer-Lindquist coordinates (t, r, θ, φ) and the geometric units $G = c = 1$ are used.

2 INNER EDGE OF SAD WITH NO TORQUE BOUNDARY CONDITION

As is well known, the full description of a Kerr BH needs only two parameters, i.e., mass M and spin $a_* \equiv a/M \equiv J/M^2$ $(-1 < a_* < 1)$.

Based on Page & Thorne (1974) and L02 we derive the expressions for the radiation flux, interior viscous torque and total luminosity of a relativistic thin disc as follows.

$$F_{DA} = \frac{1}{4\pi r}\frac{-d\Omega_D/dr}{(E^\dagger - \Omega_D L^\dagger)^2}\Big[\int_{r_{in}}^{r}(E^\dagger - \Omega_D L^\dagger)\dot{M}_D\frac{dL^\dagger}{dr}dr + g_{in}(E^\dagger_{in} - \Omega_{in}L^\dagger_{in})\Big] \quad (1)$$

$$g_{DA} = \frac{E^\dagger - \Omega_D L^\dagger}{-d\Omega_D/dr}4\pi r \cdot F_{DA} \quad (2)$$

$$\mathcal{L}_{DA} = 2\int_{r_{in}}^{\infty} E^\dagger F_{DA}\cdot 2\pi r\cdot dr = \dot{M}_D(1 - E^\dagger_{in}) + g_{in}\cdot\Omega_{in} \quad (3)$$

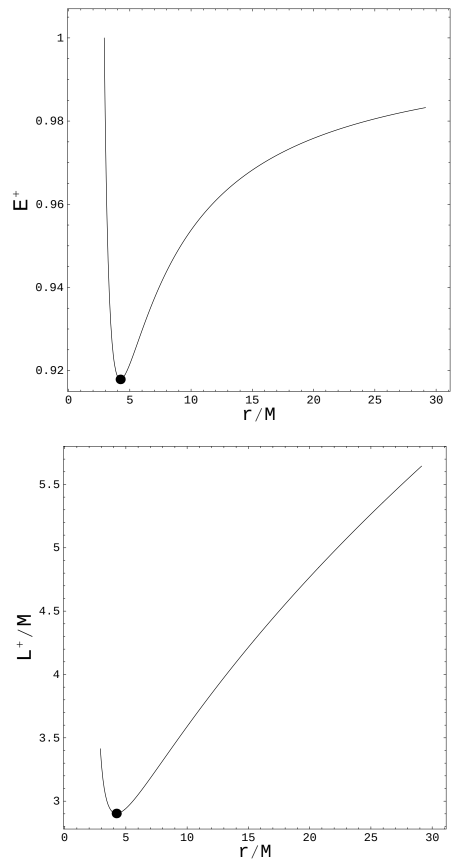

Figure 1. Specific energy and angular momentum of the accreting matter of SAD versus the disc radius with a_*=0.5. The black points indicate the position radius of ISCO.

where $\dot{M}_D$ is the accretion rate. The quantities F_{DA} and g_{DA} are the radiation flux and interior viscous torque of the disc, respectively. It is emphasized that these quantities can never be negative in any physical cases. $E^\dagger$, $L^\dagger$ and Ω_D are respectively the specific energy, specific angular momentum and angular velocity of the test particles moving along geodesic circular orbits in the equatorial plane of a Kerr BH (Bardeen, Press & Teukolsky 1972, hereafter B72).

The quantity $\mathcal{L}_{DA}$ is the total luminosity, and g_{in} is the exterior torque exerted at the inner boundary with angular velocity $\Omega_{in} = \Omega_D(r_{in})$. Generally, the "no torque boundary condition" is assumed in the theory of SAD, i.e., $g_{in} = 0$ (Page & Thorne 1974). From equation (1)-(3) we find that the radiation flux and interior viscous torque could be significantly different in the case of $g_{in} \neq 0$, by which an extra energy is provided.

The specific energy and angular momentum vary non-monotonically with the disc radius, attaining their minima at ISCO indicated by a black dot as shown in Figure 1. The radial profile of $E^\dagger$ and $L^\dagger$ is very different from the monotonous profile of these quantities obtained in Newtonian mechanics. It is the radial feature of $E^\dagger$ and $L^\dagger$ in general relativity that results in two constraints to the inner edge of SAD: (1) it is always located at or outside ISCO without extra energy and angular momentum transferred to the ac-

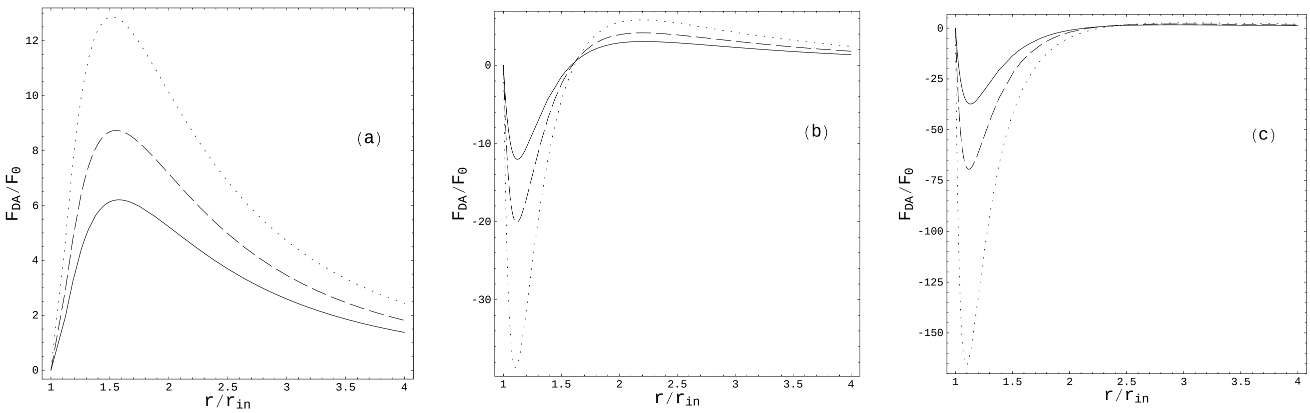

Figure 2. Radiation flux from SAD with different inner edges versus the disc radius for different BH spins, where $a_* = 0.8$, 0.6 and 0.3 correspond to dotted, dashed and solid lines, respectively. The parameter $\lambda = 1.0$, 0.90 and 0.85 correspond to panels (a), (b) and (c), respectively.

creting matter, and (2) it could move inward within ISCO, provided that some extra energy and angular momentum are transferred to the accreting matter. In §3 we shall elaborate this point in detail, and propose a criterion for the inner edge of a thin disc by considering the MC effects.

The lower limit r_{in} to the integral in equations (1) and (3) is the radius of the inner edge of the disc, which is usually assumed to lie at ISCO in SAD. Abramowitz et al. (1978) argued that if the disc is geometrically thick, its inner edge locates between ISCO and the innermost bound circular orbit.

A parameter λ is introduced to label the position of the inner edge, which is defined as

$$r_{in} \equiv \lambda^2 \, r_{ISCO}, \qquad for \quad \lambda \geqslant \sqrt{r_H/r_{ISCO}}, \tag{4}$$

where r_H is the radius of the BH horizon (B72).

Combining equations (1) with (4), we have the curves of F_{DA}/F_0 versus r/r_{in} for $g_{in} = 0$ as shown in Figure 2, where the quantity F_0 is defined as $F_0 = 10^{-4} \cdot \dot{M}_D/r_{in}^2$.

As shown in Figure 2a, the radiation flux becomes zero at r_{ISCO} with $\lambda = 1$. However, an unphysical disc region with negative radiation flux appears for λ less than unity as shown in Figures 2b and 2c, implying that the radius r_{in} cannot be smaller than r_{ISCO}. This unphysical case can be understood based on the radial profile of $E^\dagger$, $L^\dagger$ given in Figure 1.

In SAD the accreting matter outside r_{ISCO} losses its energy and angular momentum due to the differential rotation, and thus enters into the smaller circular orbit. Since the radius r_{ISCO} corresponds to the minimum specific energy and specific angular momentum as shown in Figure 1, the accreting matter has to absorb rather than release energy and angular momentum to get into the smaller orbit. The real picture inside r_{ISCO} is that the accreting matter is plunged into the BH with a huge radial velocity due to lack of enough angular momentum.

Thus we conclude that SAD with an inner edge within r_{ISCO} is not stable, while any radius larger than r_{ISCO} is possible. It is worth noting that r_{ISCO}, the radius of ISCO, is not the unique choice for the inner edge of a standard thin disc, only being the smallest radius among all of the possible ones. In addition, as argued in the next section, the inner edge of an accretion disc can deviate from ISCO in the MC process due to the magnetic transfer of energy and angular momentum between a rotating BH and its surrounding accretion disc.

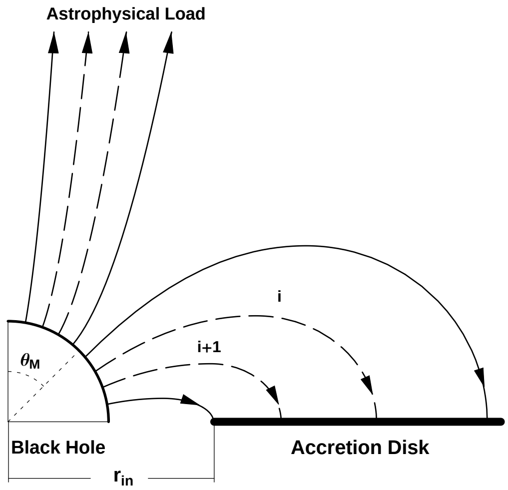

Figure 3. The poloidal magnetic field configuration in our model.

3 MC EFFECTS ON INNER EDGE OF DISC

The magnetic field configuration of our model is shown in Figure 3. The parameter n is the power-law index indicating the variation of the poloidal magnetic field with the disc radius, $B_D^P \propto r^{-n}$. The mapping relation between between the angular coordinate on the BH horizon and the radial coordinate on the disc can be derived from the conservation of the magnetic flux (Wang et al. 2002, W03) and it reads

$$\cos\theta = \int_{r_{in}}^{r} \mathrm{G}(a_*; r', n) \, dr' + C_0, \tag{5}$$

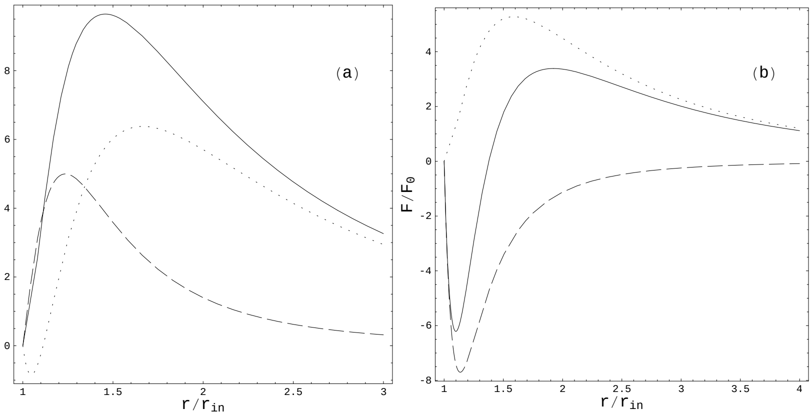

Figure 4. The radiation fluxes versus r/r_{in} with F_{total}/F_0, F_{DA}/F_0 and F_{MC}/F_0 plotted in solid, dotted and dashed lines, respectively. The parameters $n = 6$ and $\kappa_m = 1$ are taken with $\lambda = 0.974$, $a_* = 0.45$ and $\lambda = 1.000$, $a_* = 0.10$, in panels (a) and (b), respectively.

$$\mathrm{G}\,(a_*; r, n) = \frac{(r/r_{in})^{1-n}\sqrt{1+a_*^2M^2r^{-2}+2a_*^2M^3r^{-3}}}{2\sqrt{\left(1+a_*^2M^2r_{in}^{-2}+2a_*^2M^3r_{in}^{-3}\right)\left(1-2Mr^{-1}+a_*^2M^2r^{-2}\right)}} \tag{6}$$

where C_0 is an integral constant (we adopt $C_0 = \cos 0.45\pi$ in this paper). According to the conservation of the magnetic flux, B_D^P and the poloidal magnetic field on the horizon B_H are related by (Wang et al. 2002, W03)

$$\begin{aligned} d\psi &= -B_H \cdot 2\pi(r_H^2 + a^2)\sin\theta \cdot d\theta \\ &= B_D^P \cdot 2\pi\sqrt{\frac{r^4+r^2a^2+2a^2Mr}{r^2+a^2-2Mr}} \cdot dr, \end{aligned} \tag{7}$$

where $\psi = \psi(r, \theta)$ is the magnetic flux through a surface bounded by a circle with $r = constant$ and $r = constant$.

Based on Macdonald & Thorne (1982) and W03 we express the MC torque and power as follows,

$$\begin{cases} dT_{MC} = 2\left(\frac{d\psi}{2\pi}\right)^2 \frac{(\Omega_H - \Omega_D)}{dZ_H}, \\ dP_{MC} = 2\left(\frac{d\psi}{2\pi}\right)^2 \frac{\Omega_D \cdot (\Omega_H - \Omega_D)}{dZ_H}. \end{cases} \tag{8}$$

where $\Omega_H = a_*/(2\ r_H)$ is the angular velocity of the BH and

$$dZ_H = \left(\frac{R_H}{2\pi}\right)\frac{r_H^2 + a^2\cos^2\theta}{(r_H^2 + a^2)\sin\theta}\left(\frac{d\theta}{dr}\right) dr \tag{9}$$

In equations (8) and (9) Z_H is the resistance of the BH horizon with the surface resistivity given by $R_H = 4\pi/c \approx 377\ ohm$.

Incorporating the MC effects with the conservation of energy and angular momentum, we have the following relations:

$$\begin{cases} \frac{d}{dr}(\dot{M}_D L^\dagger - g_{MC}) = 4\pi r(F_{total}L^\dagger - H), \\ \frac{d}{dr}(\dot{M}_D E^\dagger - g_{MC} \cdot \Omega_D) = 4\pi r(F_{total}E^\dagger - H\Omega_D) \end{cases} \tag{10}$$

where $H \equiv (1/4\pi r) \cdot dT_{MC}/dr$ is the angular momentum flux due to the MC effects, and g_{MC} is the interior viscous torque of the disc in the MC process.

Thus we can derive the radiation flux and the total luminosity of a thin disc by resolving equation (10).

$$\begin{aligned} F_{total} = \frac{1}{4\pi r}\frac{-d\Omega_D/dr}{(E^\dagger - \Omega_D L^\dagger)^2}\int_{r_{in}}^{r}(E^\dagger - \Omega_D L^\dagger)\times \\ (\dot{M}_D \cdot dL^\dagger/dr + 4\pi r H)dr \equiv F_{DA} + F_{MC} \end{aligned} \tag{11}$$

$$g_{MC} = \frac{E^\dagger - \Omega_D L^\dagger}{-d\Omega_D/dr} 4\pi r \cdot F_{total} \tag{12}$$

$$\mathcal{L}_{total} = \dot{M}_D(1 - E_{in}^\dagger) + 4\pi\int_{r_{in}}^{\infty} H\Omega_D r \cdot dr \tag{13}$$

As the magnetic field on the BH is supported by the surrounding disc, there is some relation between B_H and $\dot{M}_D$. As a matter of fact the relation might be rather complicated, and would be very different in different situations. One possibility has been suggested by Moderski, Sikora & Lasota (1997) and is based upon the balance between the pressure of the magnetic field on the horizon and the ram pressure of the innermost parts of an accretion flow, i.e.,

$$B_H^2\big/(8\pi) = P_{ram} \sim \rho c^2 \sim \dot{M}_D\big/\left(4\pi r_H^2\right), \tag{14}$$

Considering that equation (14) is not a certain relation between B_H and $\dot{M}_D$, we rewrite it as follows,

$$B_H \equiv \sqrt{2\kappa_m \dot{M}_D/r_H^2}, \tag{15}$$

where κ_m is a parameter indicating the relative importance of the MC process with respect to the disc accretion. The MC process dominates over disc accretion for $\kappa_m \gg 1$, and it is dominated by the latter for $\kappa_m \ll 1$. We emphasize that equation (15) is independent of (14), one can always fix κ_m with any given relation between B_H and $\dot{M}_D$.

As shown in Figure 4a, the total radiation flux from the disc with the inner edge within r_{ISCO} ($\lambda = 0.974$) can be positive, though more energy and angular momentum are needed to keep the Keplerian orbits of the accreting matter within r_{ISCO}. The reason is that the energy and angular momentum are transferred from the BH to the disc for $a_* > 0.3594$ (Blandford 1999), and the positive contribution due to the MC effects exceeds the negative one due to the disc accretion. The BH spin $a_* = 0.3594$ corresponds to Ω_H equal to Ω_D at ISCO.

On the other hand, as shown in Figure 4b, the total radiation flux from the inner disc could be negative for $a_* <$

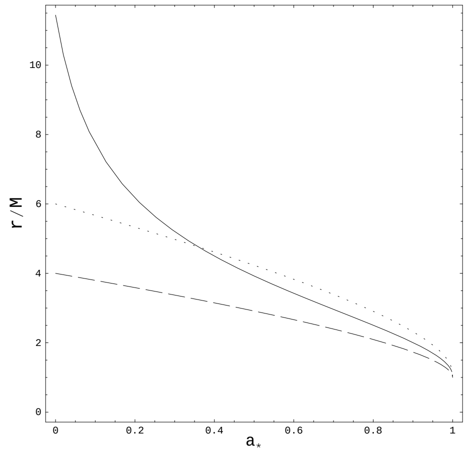

Figure 5. Curves of the radius of the inner edge (solid line), the radius of ISCO (dotted line) and the radius of the innermost bound orbit (dashed line) versus the BH spin with $n = 6$ and $\kappa_m = 1$.

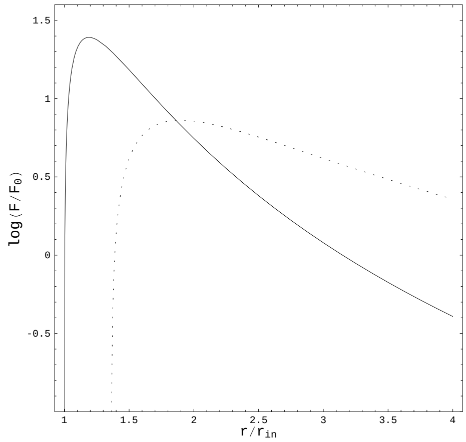

Figure 6. The curves of F_{DA}/F_0(dotted line) and F_{MC}/F_0 (solid line) versus r/r_{in} for a_*= 0.75, $n = 6$ and $\kappa_m = 1.0$.

0.3594, giving rise to an unphysical radiant region, although the inner edge is located at ISCO with $\lambda = 1.000$. This result arises from the magnetic transfer direction of the energy and angular momentum from the inner disc to the BH for $a_* < 0.3594$.

From the above discussion we find that the MC process is a very efficient mechanism in transferring energy and angular momentum between the BH and its surrounding disc. The inner region of the disc would be eventually disrupted for $a_* < 0.3594$, because too much energy and angular momentum are transferred to the BH via the MC process. Thus we infer that the inner edge will move outward beyond ISCO. On the other hand, the inner edge can be extended within ISCO for $a_* > 0.3594$, because the excess energy and angular momentum can be transferred magnetically from the rotating BH to the inner disc via the MC process.

From equation (12), we infer that g_{MC} has the same sign as the total radiation flux. Combining the MC effects with the fact that an accretion disc cannot sustain a negative viscous torque, we suggest that a reasonable inner edge should be constrained by a positive radiation flux. Just like the case of SAD, one can always find some reasonable radii for the inner edge of the disc, as long as they are far away enough from the BH, to keep the radiation flux positive throughout the whole disc. And what we have to do is to find the smallest radius as the inner edge. Following Wang (1995), we consider that the inner edge of the disc is the position where the transportation of energy and angular momentum on the disc just begins to exceed the adjustable range of the interior viscous torque. However, this criterion can hardly be formulated accurately. Alternatively, we suggest that the criterion for the inner edge of a thin disc with a smoothly distributed exterior torque can be written as

$$\left(\frac{dF_{total}}{dr}\right)_{r_{in}^+} = 0, \quad \left(\frac{d^2F_{total}}{dr^2}\right)_{r_{in}^+} > 0 \tag{16}$$

Incorporating equations (11) and (16), we have the radius of the inner edge of the disc, r_{in}/M, varying with the BH spin a_* as shown in Figure 5. Inspecting Figure 5, we find that the deviation of r_{in} from r_{ISCO} depends on the BH spin as follows: (1) $r_{in} > r_{ISCO}$ for $a_* < 0.3594$, (2) $r_{in} < r_{ISCO}$ for $a_* > 0.3594$ and (3) $r_{in} = r_{ISCO}$ at $a_* = 0.3594$.

Based on the criterion (16), we can evaluate the influence of the parameter κ_m on the deviation of r_{in} from r_{ISCO}: (1) the turning point of the deviation $a_* = 0.3594$ is independent of the value of κ_m; (2) the greater κ_m corresponds to the greater MC effects, and thus to the greater deviation, and (3) we have $r_{in} = r_{ISCO}$ for $\kappa_m = 0$, which corresponds to SAD without MC effects according to equation (15).

Inspecting equations (11) and (12), we find that the MC torque always vanishes at the inner edge, implying that the MC of the BH with a thin disc does not disrupt the no torque boundary condition. However, there is an exception that the closed field lines concentrate at the inner edge of the disc, where the MC torque related to g_{MC} at r_{in} can be written in the form

$$\begin{cases} dT_{MC}/dr \equiv 4\pi rH = (g_{MC})_{in} \cdot \delta(r - r_{in}), \\ (g_{MC})_{in} = \tau_0 \cdot sign(\Omega_H - \Omega_{in}). \end{cases} \tag{17}$$

This situation has been discussed in L04, where the MC torque does not vanish at the inner edge of the disc. For $a_* < 0.3594$, the criterion for the inner edge can be equivalently written as

$$\Omega_H(a_*) = \Omega_D(r_{in}, a_*). \tag{18}$$

Equation (18) implies that the inner edge is located at the radius where the disc matter has the same angular velocity as the BH, which is consistent with the conclusion in L04.

4 MC EFFECTS ON DISC RADIATION

The MC process has a significant effect on the disc radiation. Compared with SAD, the radiation flux is much more con-

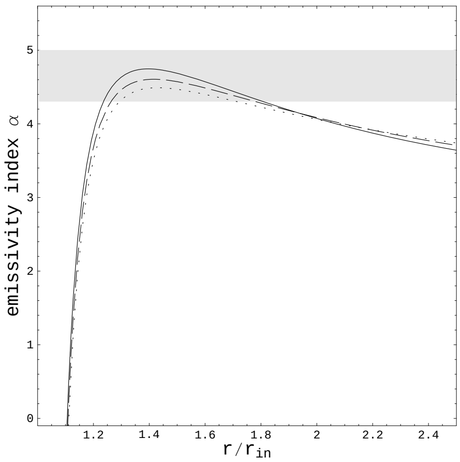

Figure 7. The curves of $\alpha \equiv -d \ln F_{total}/d \ln r$ versus r/r_{in} for $a_* = 0.998$ and $\kappa_m = 10$. The dotted, dashed and solid lines correspond to n =6, 7 and 8, respectively. The emissivity index of the Seyfert 1 galaxy MCG-6-30-15 inferred from the observation of XMM-Newton is shown by the shaded region.

centrated at the inner disc. By using equations (1) and (11) we have the curves of F_{DA}/F_0 and F_{MC}/F_0 versus r/r_{in} as shown in Figure 6.

From equation (11) we obtain a very steep emissivity index up to 4.3∼5.0 as shown in Figure 7, where the emissivity index is defined as $\alpha \equiv -d \ln F_{total}/d \ln r$. This result is consistent with the *XMM-Newton* observation of the nearby bright Seyfert 1 galaxy MCG-6-30-15 (Wilms et al. 2001; Branduardi-Raymont et al. 2001).

Zhang et al. (1997, hereafter Z97; Shafee et al. 2006) firstly invented an approach to measure the BH spin by determining the radius of ISCO in fitting the spectrum of the X-ray continuum from a thin relativistic disc. Considering the deviation of the inner edge from ISCO due to the MC effects, we modify equation (3) in Z97 as follows,

$$r_{in} = \eta D \left[\frac{F_{earth}}{2\sigma g(\theta, a_*) \cdot Q(a_*)}\right]^{1/2} \left[\frac{f_{col} f_{GR}(\theta, a_*)}{T_{col}}\right]^2 \quad (19)$$

where $g(\theta, a_*)$ and $f_{GR}(\theta, a_*)$ have the same meaning given in Z97. We ignore the effect of magnetic field on the geodesic of photons, so they are only determined by the metric around the BH. In this paper, we use the values of $g(\theta, a_*)$ and $f_{GR}(\theta, a_*)$ from Table 1 in Z97. $\eta \equiv r_{in}/r_{peak}$. And the other quantities are defined as follows:

$$\begin{array}{l} F_{earth} = g(\theta, a_*) \mathcal{L}_{total}/2\pi D^2, \\ T_{col} = f_{GR}(\theta, a_*) f_{col} \cdot T_{peak} \end{array} \quad (20)$$

Compared with Z97, the MC effects are taken into account in our model. In addition, we introduce a factor $Q(a_*)$ to modify the approximate relation between the bolometric luminosity of the disc and the peak emission region (Makishima et al. 1986) as follows,

$$\mathcal{L}_{total} \equiv 4\pi\sigma r_{peak}^2 T_{peak}^4 \cdot Q(a_*) \quad (21)$$

The MC effects on the data fitting behave at least two aspects: (1) the inner edge deviates from ISCO, and (2) the radius r_{peak} of the peak value of the radiation flux is much closer to the inner edge. Incorporating equations (19)—(21) with the criterion (16), we have the BH spins of GRO J1655-40 and GRS 1915+105 as shown in Table 1 (McClintock et al. 2006).

In Z97, the factor $Q(a_*)$ is set to unity, and it is argued that the approximation is accurate to within 10% for a wide range of the model parameter space. In this paper, we check the relation in equation (21) carefully and find that the approximation made in Z97 works well only for the non-relativistic discs. Once the relativistic effects are taken into account, the correction from $Q(a_*)$ would deviate unity significantly by a factor of several orders. On the other hand, the correction above in our paper is also partially from the MC effects. We emphasize that the results in our paper contains fully relativistic effects, which is different from the ones in Z97. And one can also find that the derived spins of the black holes in Z97 would be much smaller if the correction $Q(a_*)$ is considered seriously.

According to the observation, jets appear both in GRO J1655-40 and GRS 1915+105. As shown in Figure 3, two kinds of magnetic field configurations are contained in our model. The closed field lines connecting the BH and the disc correspond to the MC process. The open field lines connecting the BH to the remote astrophysical load correspond to BZ process, which is widely used to interpret the jet-production in X-ray binaries, AGNs and GRBs (Blandford & Znajek 1977, Blandford 1999). Based on the argument given in W03 we find that the BZ process can coexist with the MC process for the parameters in Table 1. Thus the jet-production in GRO J1655-40 and GRS 1915+105 can be interpreted naturally based on this model.

5 CONCLUSION AND DISCUSSION

In this paper the MC effects on the radiation from a relativistic thin disc are discussed, and some issues related to this toy model are addressed as follows.

(1) As argued by Uzdensky (2005), the MC of a rotating BH with its surrounding disc can be regarded as a stable magnetic connection, which is dramatically different from the magnetic connection of a rotating neutron star with a disc. Since a BH does not have a conducting surface, the magnetic field lines frozen into a rotating conducting disk can slip on the horizon. Compared with the BH, a neutron stars is a highly-conducting star. Therefore, each field line connecting the star to the disk is subject to a continuous twisting, and no steady magnetic connection exists in the case of a rotating neutron star with a disc (Ghosh, Lamb & Pethick 1977, Ghosh & Lamb 1979a, b; Eksi, Herquist & Narayan 2005).

(2) The MC discussed in this paper involves a large-scale magnetic field connecting a rotation BH with a thin disc. As argued in L02, this type of MC is very different from those involving a small-scale magnetic field connecting the matter inside ISCO with the matter outside ISCO (Krolik 1999; Gammie 1999; Agol & Krolik 2000), and the "no torque boundary condition" is assumed based on the consideration of this MC with a thin disc.

The contribution of this paper lies in the MC effects on disc radiation based on the deviation of the inner edge from ISCO, and the main results are summarized as follows.

Table 1. Measuring BH spins of GRO J1655-40 and GRS 1915+105 based on X-ray continuum with the MC effects (n=6.0 and κ_m=1.0).

Source	Observed parameters				BH spin
	$F_{earth}(erg \cdot cm^{-2} \cdot s^{-1})$	$T_{col}(k/Kev)$	θ(deg)	M($M_{\odot}$)	
GRO J1655-40	3.3×10^{-8}	1.36	70	$6.0 \sim 6.6$	$0.932 \sim 0.960$
GRS 1915+105	4.4×10^{-8}	2.27	70	$10 \sim 18$	$0.927 \sim 0.996$

It is shown that the inner edge of SAD should be located at or outside ISCO, which is regarded as the inner boundary of SAD. ISCO is the smallest one among all the possible stable circular orbits in SAD, although it is not compelling to taken ISCO as the inner edge of the disc.

Considering the magnetic transfer of energy and angular momentum between a rotating BH and its surrounding accretion disc, we propose a criterion for the position of the inner edge based on a reasonable constraint to the radiation flux from the disc. It turns out that the deviation of the inner edge from ISCO depends on the direction of the magnetic transfer, which is eventually determined by the BH spin: the radius r_{in} could be greater and less than r_{ISCO} for a_* being less and greater than 0.3594, respectively.

In addition, the MC effects on the boundary condition of the disc are discussed. It is argued that the "no torque boundary condition" is not affected at the presence of the MC process except that the magnetic field is concentrated completely at the inner edge. Only in this extreme case the MC effects can be treated effectively as a "non-zero boundary torque".

It has been argued that no stable geodesic circular orbits exist within ISCO (B72). The conclusion is derived based on geodesic of a test particle in the Kerr metric. It is worth noting that the orbits of the accreting matter cannot be regarded as geodesics because of the interior viscous torque and the presence of a strong magnetic field related to the MC process, and the result that r_{in} is less than r_{ISCO} for a disc around a fast-rotating BH seems not in conflict with the conclusion given in B72.

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