



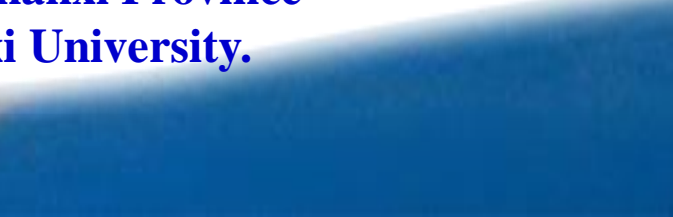
# **A method for extracting kinetic freeze-out temperature of interacting system**

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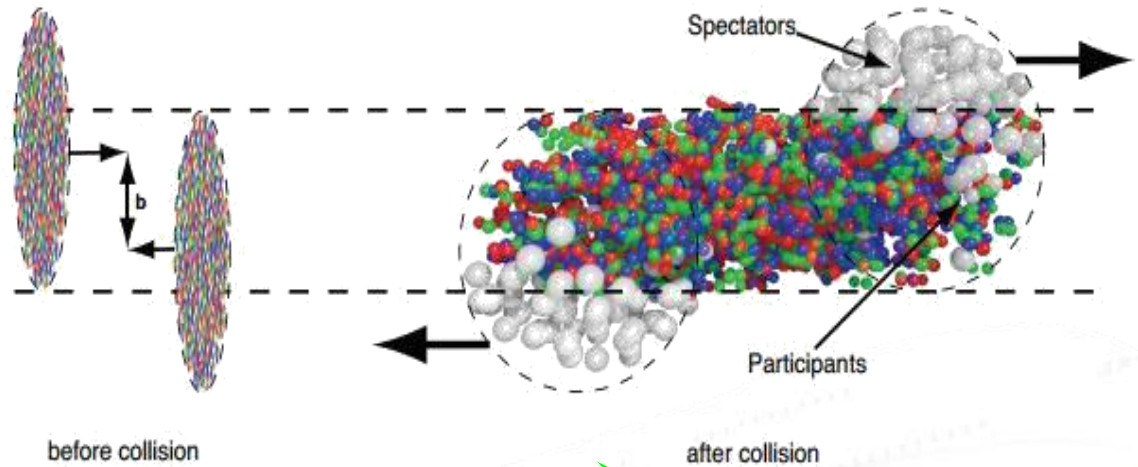
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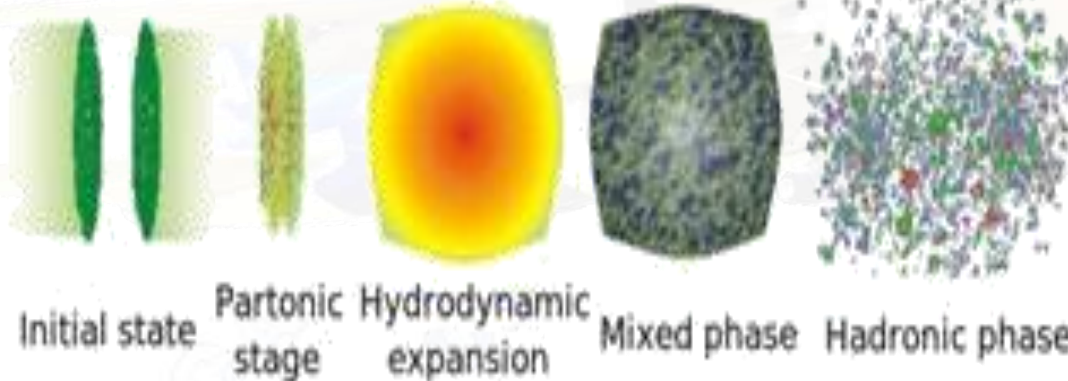


# 1 Research background

The process of specific evolution



Participant-Spectator Model

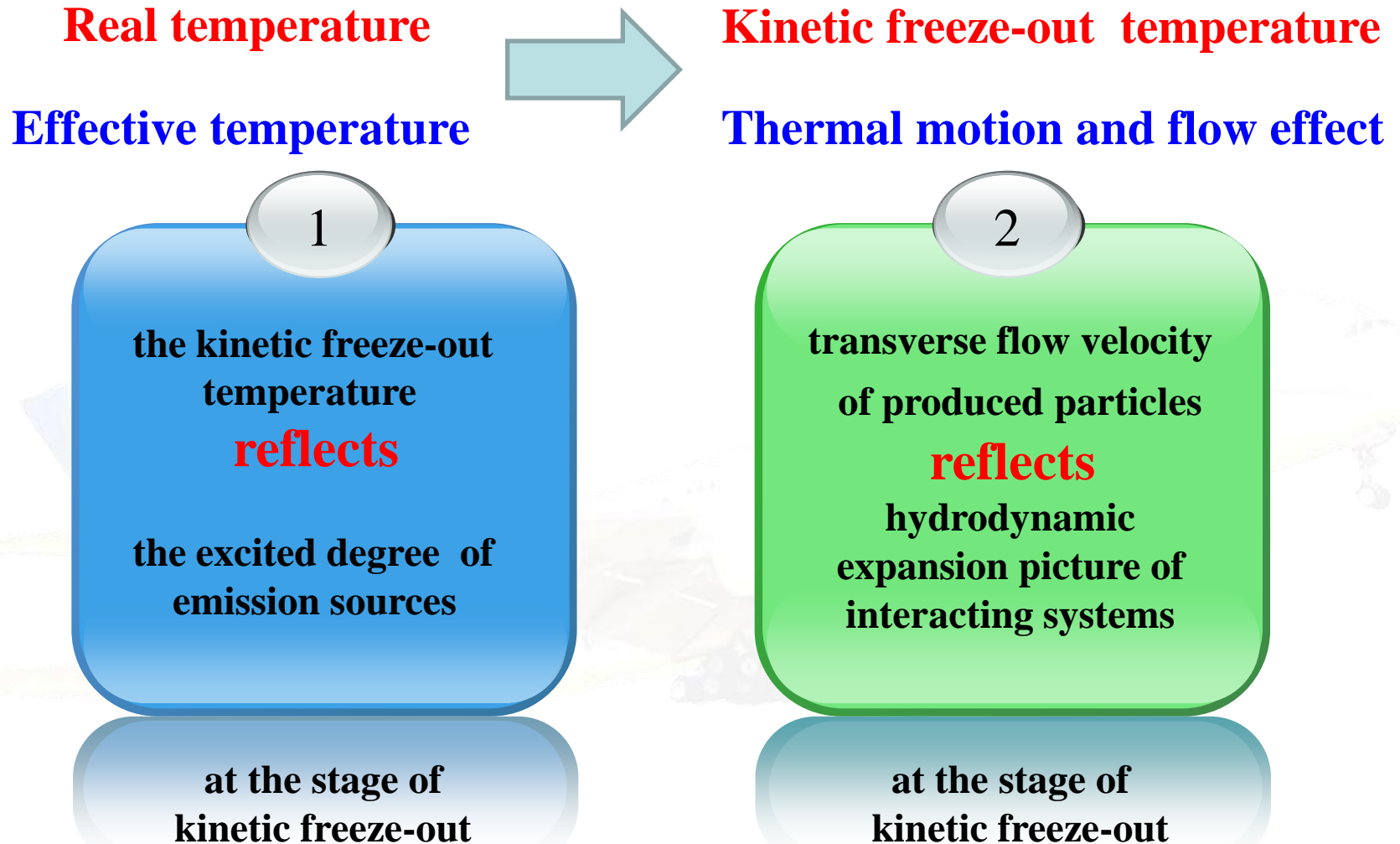


# 1 Research background

- ◆ The interacting system at the kinetic freeze-out (the last stage of collisions) stays at a thermodynamic equilibrium state or local equilibrium state, when the particle emission process is influenced not only by the thermal motion but also the flow effect.
- ◆ By analyzing the transverse momentum spectra of final-state particles, one can obtain the kinetic freeze-out temperature of interacting systems (emission sources) and the transverse flow velocity of produced particles.



# 1 Research background



# 2 Theoretical Model

## 2. Theoretical Model -multisource thermal model

2.1

The model assumes that many emission sources are formed in high energy collisions.

2.2

These sources are classified into a few groups

due to the existence of different interacting mechanisms in the collisions and different event samples measured in experiments.

2.3

We assume that these sources in the same group stay at a local equilibrium state.

They have the same excitation degree and temperature, and they can be described by different distribution laws.



# 2 Theoretical Model

**2.4**

The emission process of all the sources in different groups result in the final-state distribution.

**2.5**

The final-state distribution can be described by a multi-component distribution law.

**2.6**

Different effective temperatures can be obtained by using different distributions to describe the transverse momentum spectra of final-state particles.

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# 3 Research Methods

**A new method for  
extracting**

**Kinetic freeze-out temperature**

**Transverse flow velocity**

**Distribution  
Function**

**Tsallis-  
standard  
transverse  
momentum  
distribution**

**Tsallis  
transverse  
momentum  
distribution**

**The multi-  
component  
standard  
distribution**

**The multi-  
component  
Erlang  
distribution**





# 3 Research Methods

## **Tsallis-standard** transverse momentum distribution:

$$f_{T-S}(p_T) = \frac{1}{N} \frac{dN}{dp_T} = C_{T-S} p_T \sqrt{p_T^2 + m_0^2} \int_{y_{\min}}^{y_{\max}} \cosh y \times \left\{ \left[ 1 + \frac{q_{T-S} - 1}{T_{T-S}} \sqrt{p_T^2 + m_0^2} \cosh y \right]^{\frac{1}{q_{T-S}-1}} + S \right\}^{-1} dy.$$

F.-H. Liu, Y.-Q. Gao, H.-R. Wei,  
Adv. High Energy Phys. 2014,  
293873 (2014).

## **Tsallis** transverse momentum distribution:

$$f_T(p_T) = \frac{1}{N} \frac{dN}{dp_T} = C_{T0} p_T \sqrt{p_T^2 + m_0^2} \int_{y_{\min}}^{y_{\max}} \cosh y \times \left[ 1 + \frac{q_T - 1}{T_T} \sqrt{p_T^2 + m_0^2} \cosh y \right]^{-\frac{q_T}{q_T-1}} dy.$$

J. Cleymans, D. Worku,  
Eur. Phys. J. A 48, 160 (2012).

H. Zheng, L.-L. Zhu,  
Adv. High Energy Phys.  
2015, 180491 (2015).



# 3 Research Methods

**Standard transverse momentum distribution:**

$$f_i(p_T) = \frac{1}{N} \frac{dN}{dp_T} = C_{i0} p_T \sqrt{p_T^2 + m_0^2} \int_{y_{\min}}^{y_{\max}} \cosh y \left[ \exp \left( \frac{\sqrt{p_T^2 + m_0^2} \cosh y}{T_i} \right) + S \right]^{-1} dy,$$

**Multi-component standard distribution:**

$$f_S(p_T) = \frac{1}{N} \frac{dN}{dp_T} = \sum_{i=1}^l k_i f_i(p_T)$$

**The mean effective temperature:**

$$T_S = \sum_{i=1}^l k_i T_i$$



# 3 Research Methods

**The multi-component Erlang distribution:**

$$f_{ij}(p_{tij}) = \frac{1}{\langle p_{tij} \rangle} \exp \left[ -\frac{p_{tij}}{\langle p_{tij} \rangle} \right]$$

Erlang  
distribution

Particles generated from one emission source are assumed to obey an exponential function of transverse momentum distribution

$$f_j(p_T) = \frac{p_T^{m_j-1}}{(m_j-1)! \langle p_{tij} \rangle^{m_j}} \exp \left[ -\frac{p_T}{\langle p_{tij} \rangle} \right]$$

All the sources in the j-th group result in the folding of exponential functions

the multi-  
component  
Erlang  
distribution

$$f_E(p_T) = \sum_{j=1}^l k_j f_j(p_T)$$

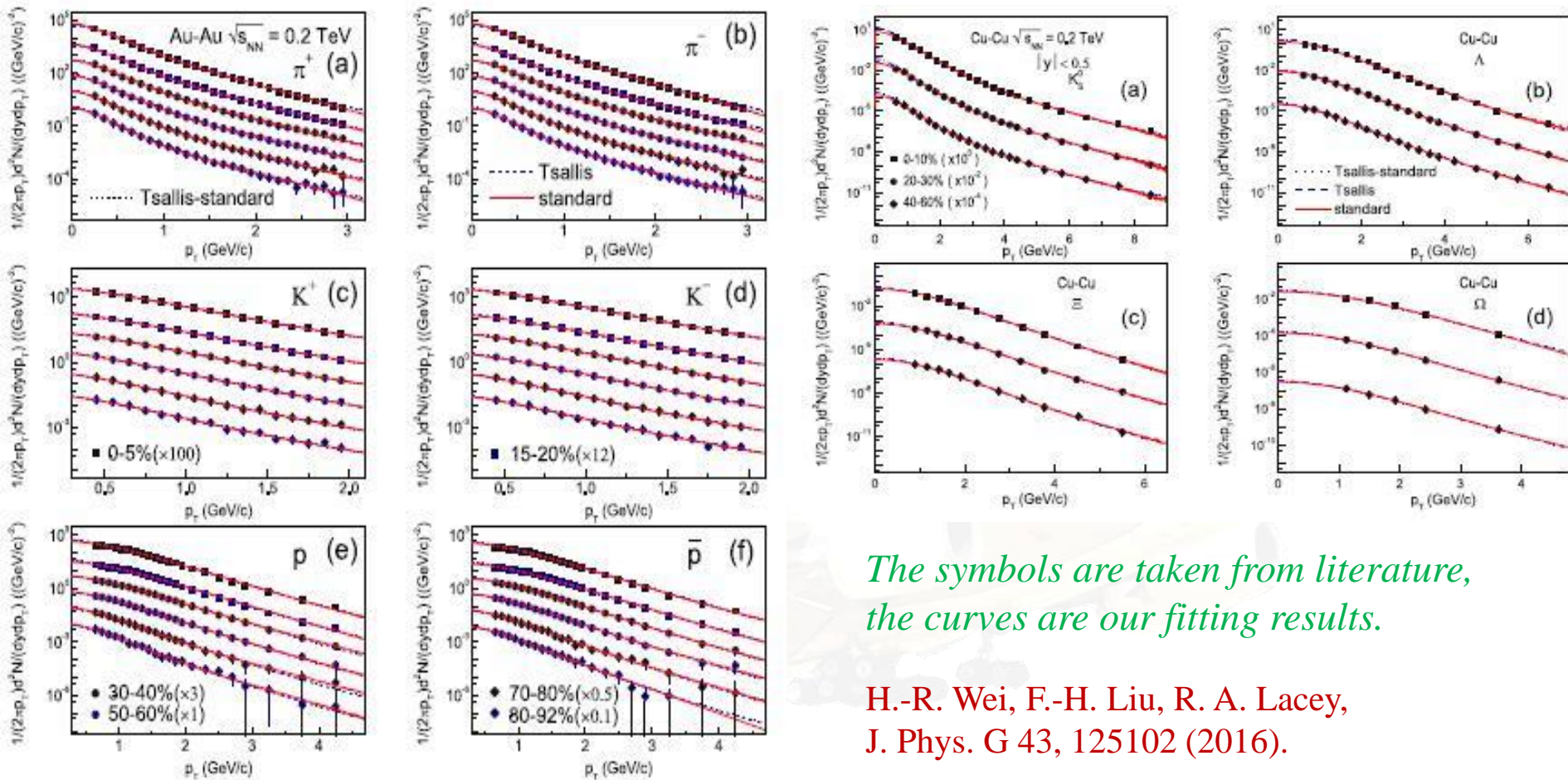
The contribution of all groups of sources

$$\langle p_T \rangle = \sum_{j=1}^l k_j m_j \langle p_{tij} \rangle$$

The mean transverse momentum



# 4 Results

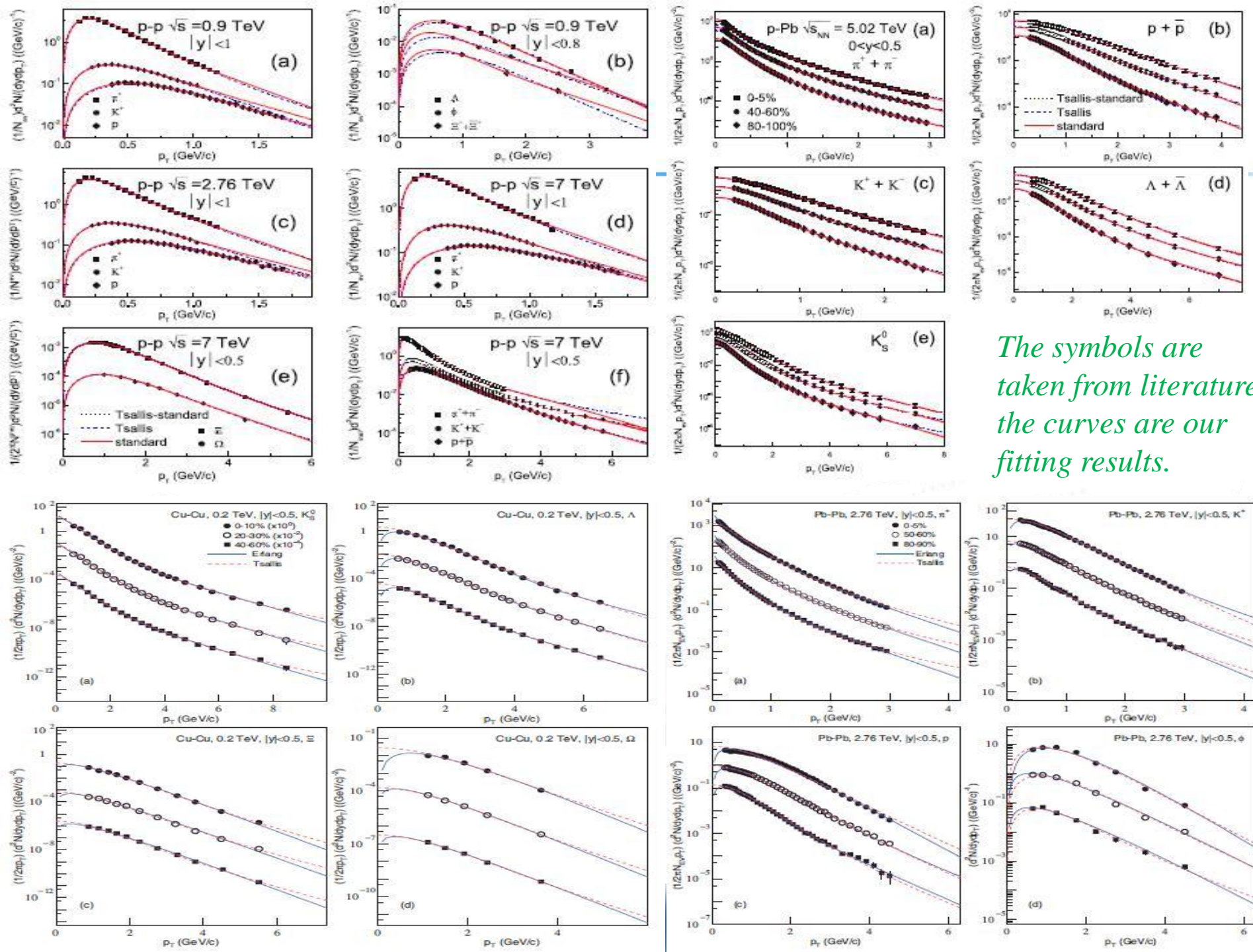


*The symbols are taken from literature,  
the curves are our fitting results.*

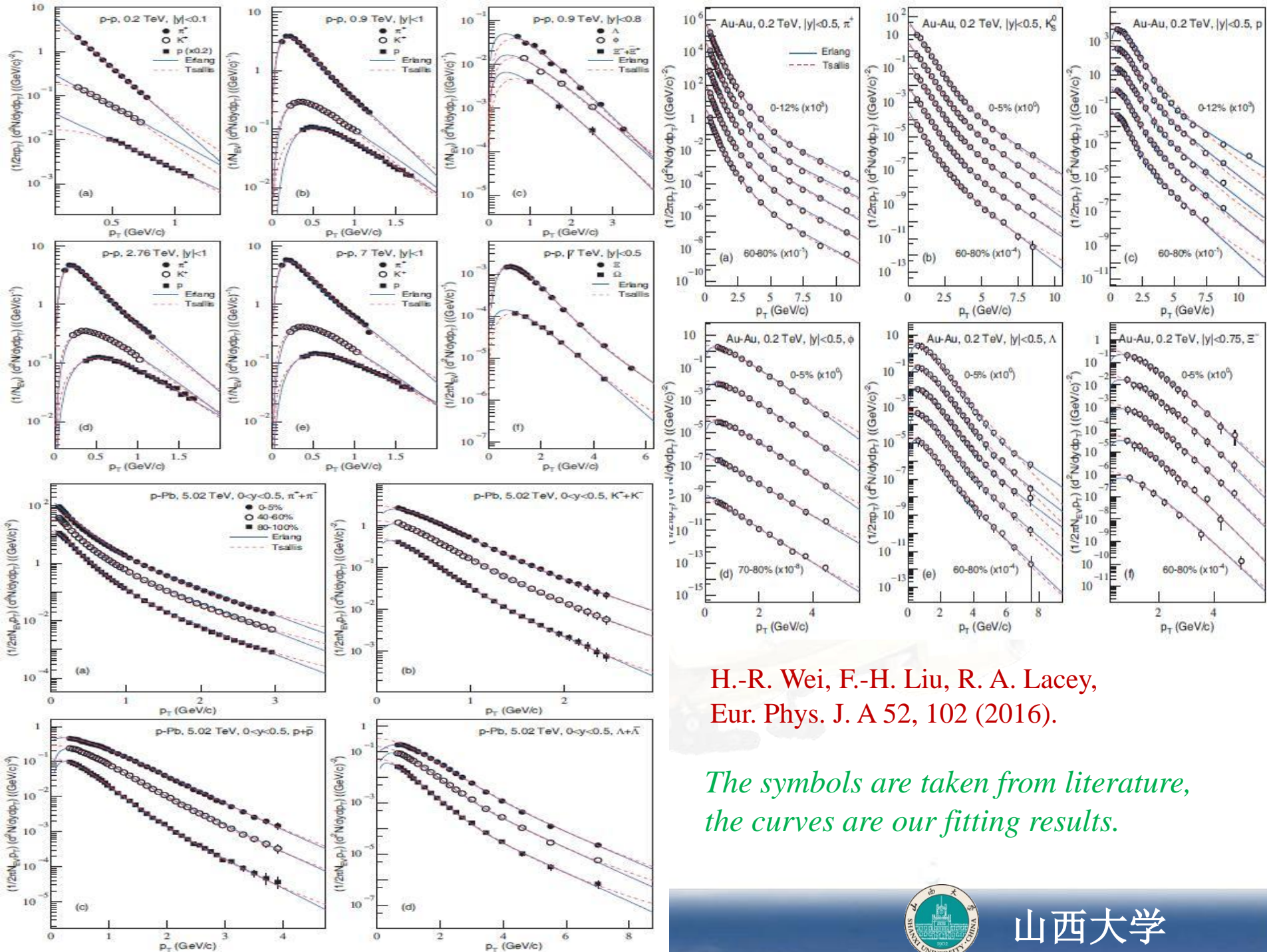
H.-R. Wei, F.-H. Liu, R. A. Lacey,  
J. Phys. G 43, 125102 (2016).











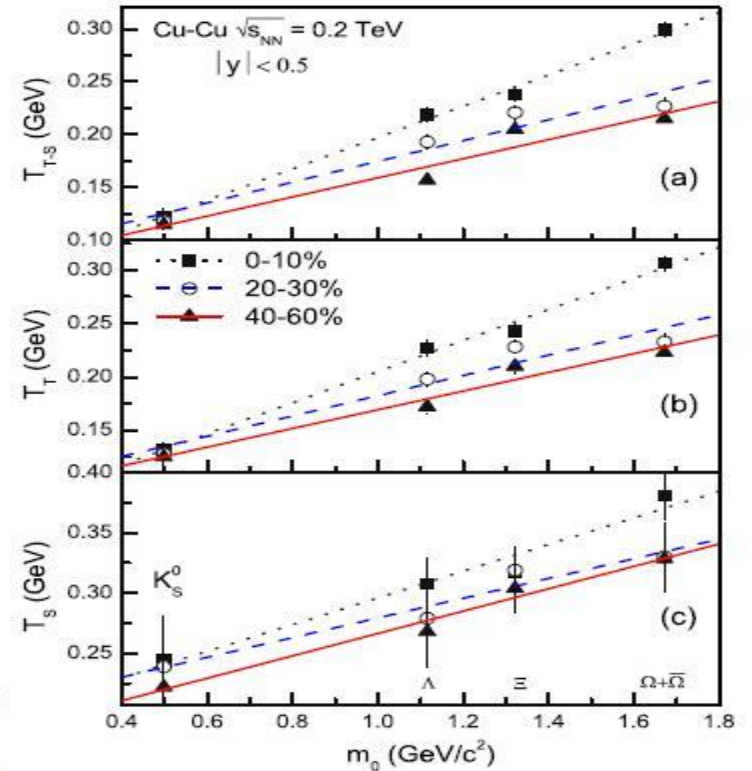
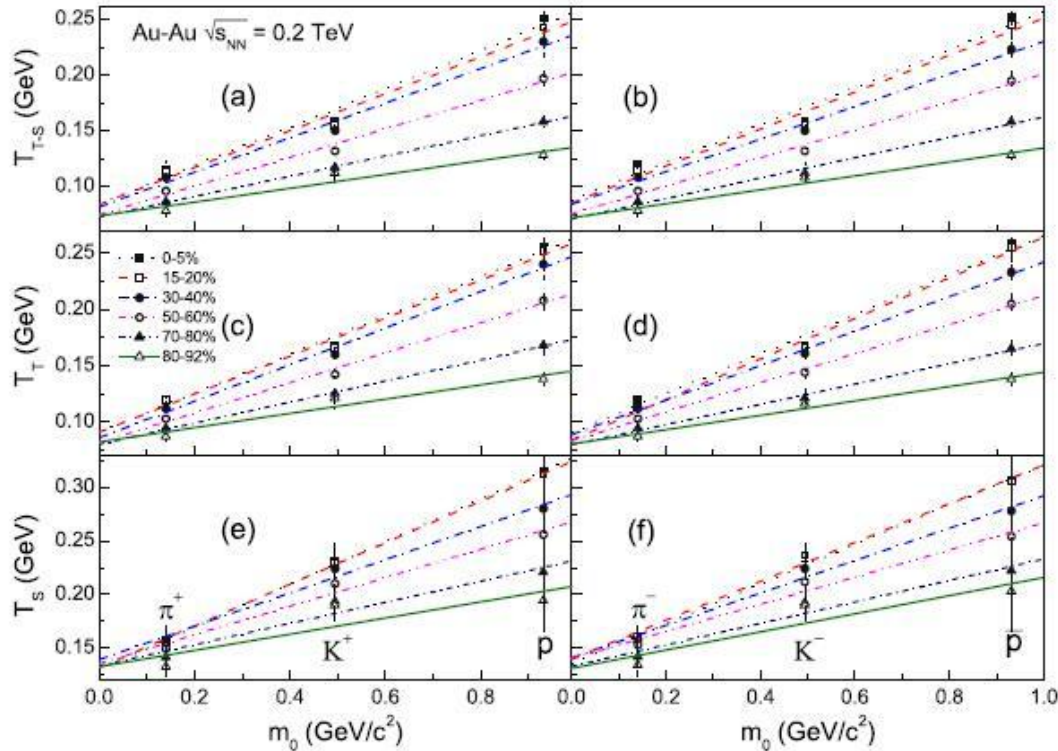
H.-R. Wei, F.-H. Liu, R. A. Lacey,  
Eur. Phys. J. A 52, 102 (2016).

*The symbols are taken from literature,  
the curves are our fitting results.*



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# 4 Results

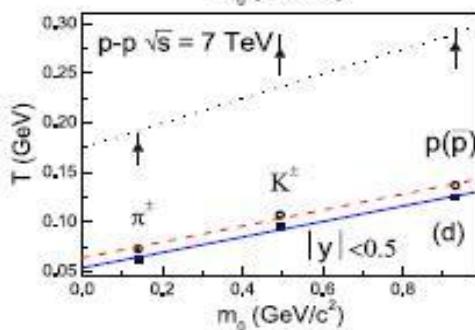
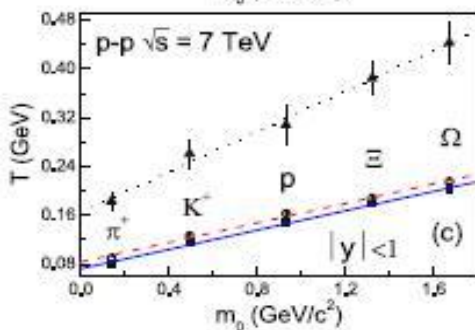
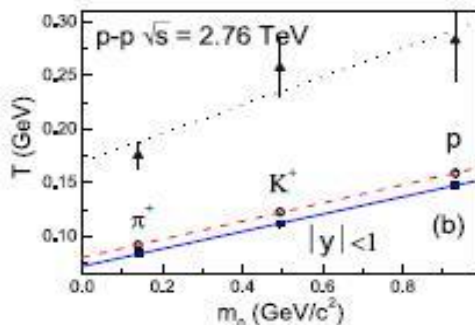
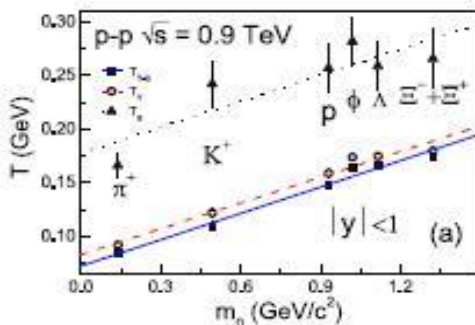
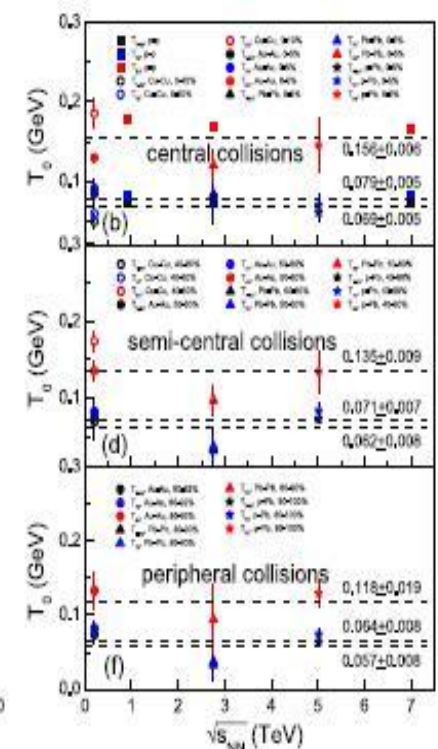
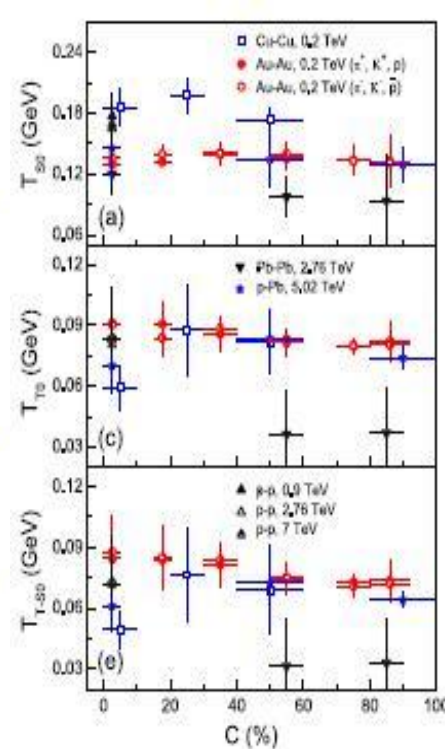
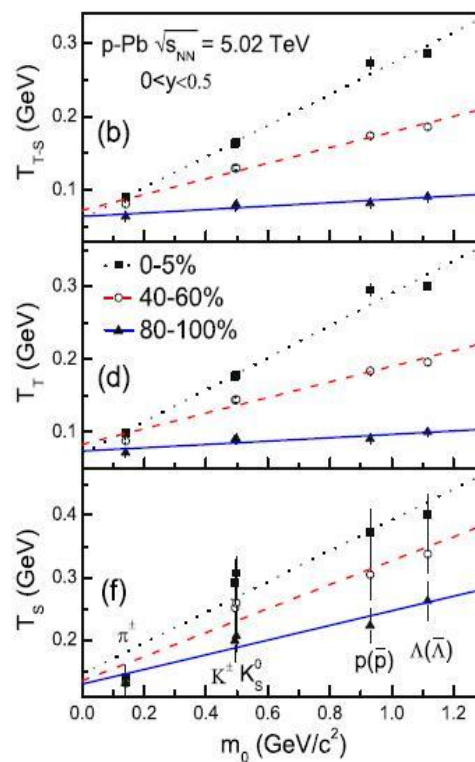
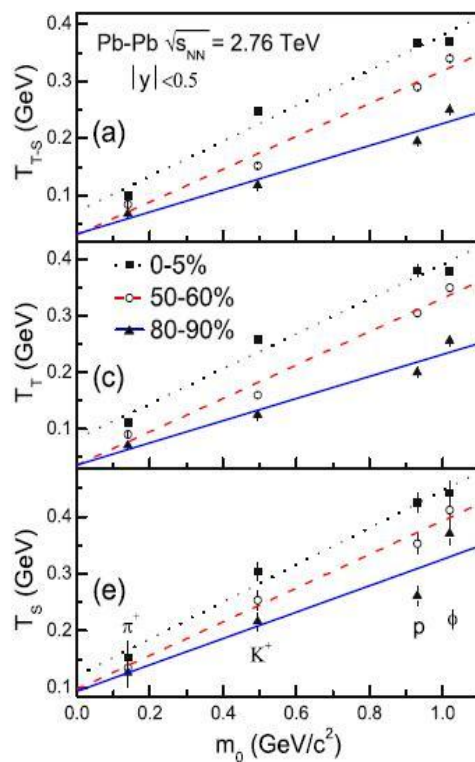


$$T = T_0 + am_0$$

- ◆ When  $m_0 = 0$ ,  $T_0$  is regarded as the real source temperature, or the kinetic freeze-out temperature of interacting system.
- ◆  $T_{T-S0}$ ,  $T_{T0}$ , and  $T_{S0}$  are extracted from Tsallis-standard, Tsallis, and two- or three component standard distributions.







$$T_{T-S0} \leq T_{T0} < T_{S0}$$

The effective temperatures  $T_{T-S}$ ,  $T_T$ , and  $T_S$  increase with the increase of centrality or particles mass



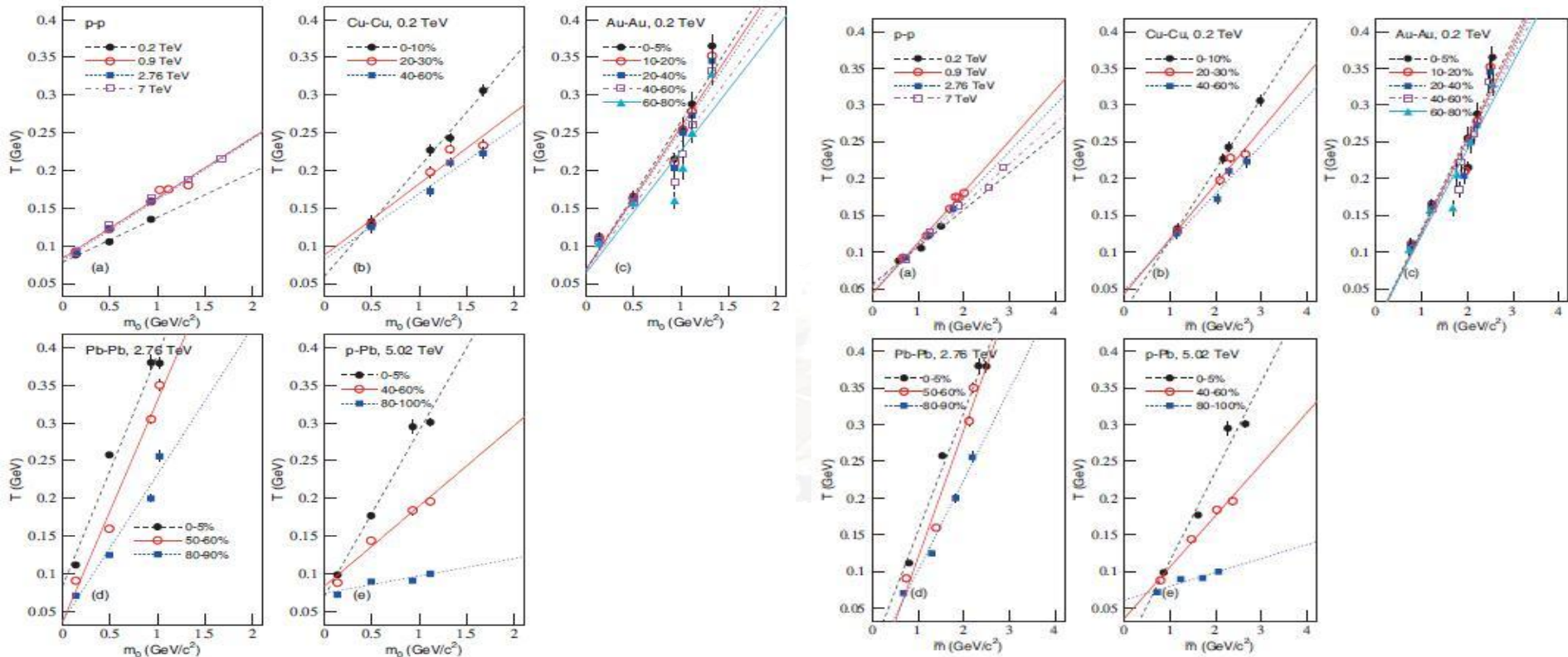
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# 4 Results

The quantity extracted directly from the distribution mentioned above is the effective temperature which includes thermal motion and flow effect of particles.

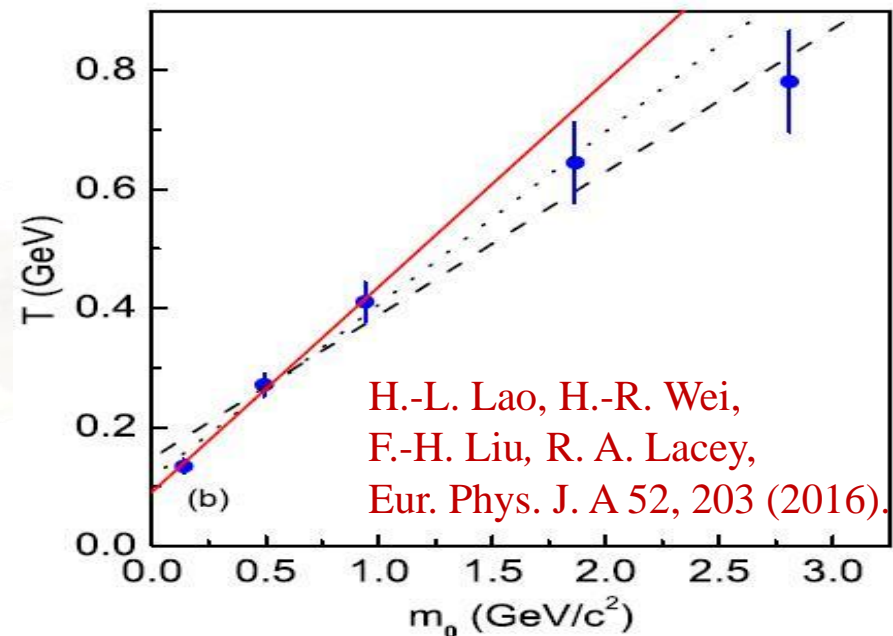
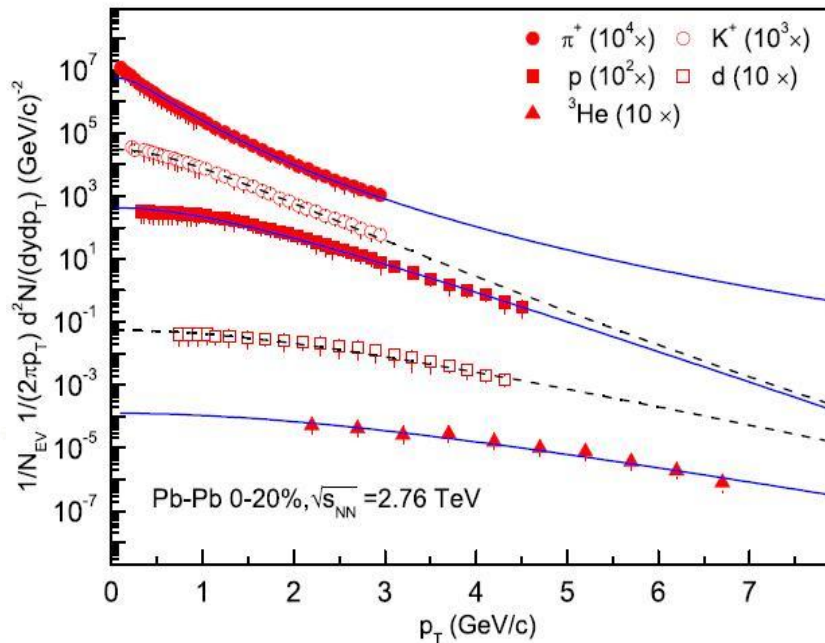
$$T = T_0 + km_0$$

$$T = C'_T + k'\bar{m}$$

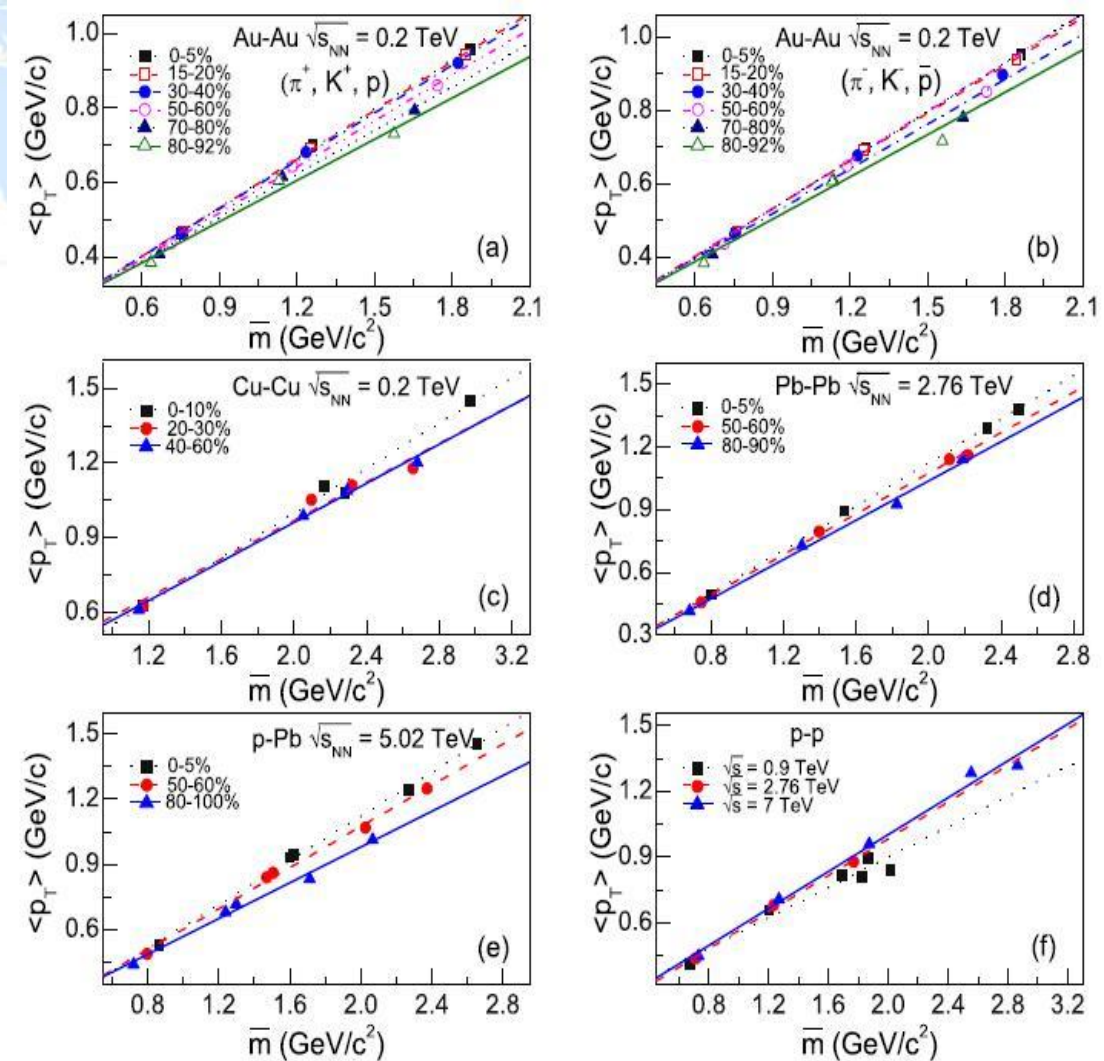


# 4 Results

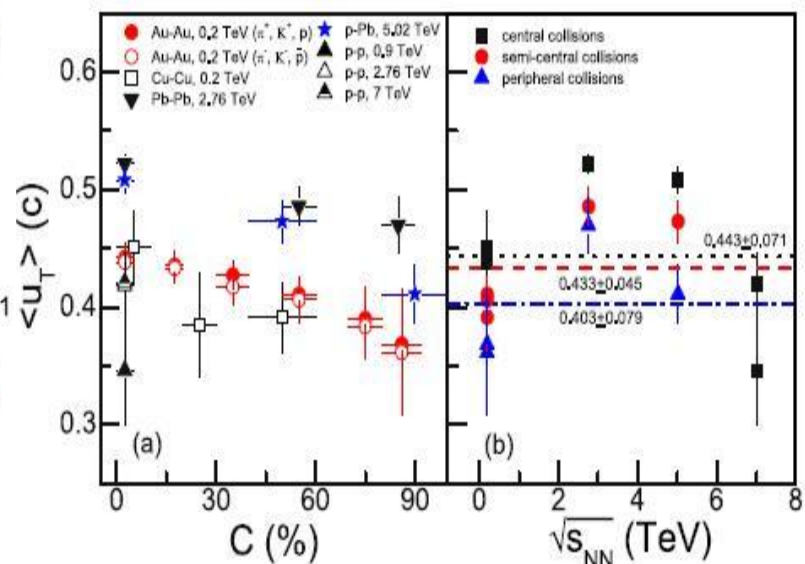
- (1) The kinetic freeze-out temperature extracted by us is 0–20% in Pb-Pb collisions at 2.76 TeV for including  $\pi^+$ ,  $K^+$ ,  $p$ ,  $d$ , and  ${}^3\text{He}$ .
- (2) The particle's mass effects of kinetic freeze-out temperature for  $d$  and  ${}^3\text{He}$  are obvious.
- (3) We have observed an evidence of mass-dependent differential kinetic freeze-out scenario or multiple kinetic freeze-out scenario.







This can provide a set of alternative methods to extract the kinetic freeze-out temperature, transverse flow velocity, and flow velocity.

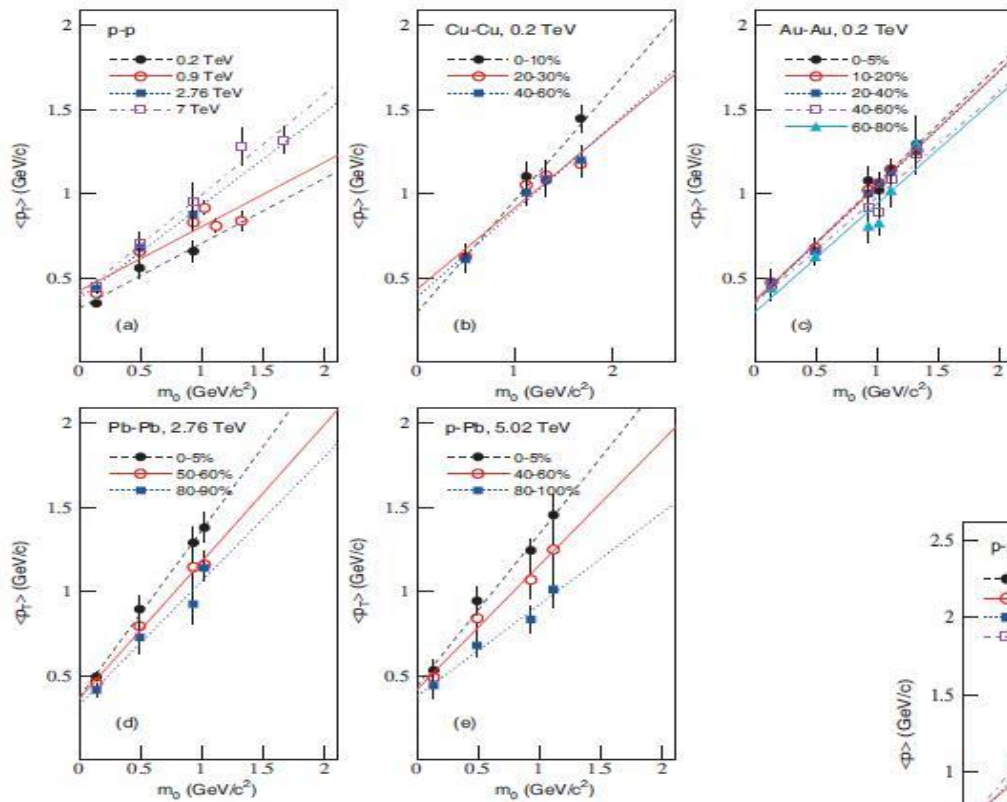


$$\langle p_T \rangle = \langle p_T \rangle_0 + \langle u_T \rangle \bar{m}$$

$$\langle p \rangle = \langle p \rangle_0 + \langle u \rangle \bar{m}$$

The slope in  $\langle p_T \rangle - \bar{m}$  (or  $\langle p \rangle - \bar{m}$ ) relation is regarded as the mean transverse flow velocity  $\langle u_T \rangle$  (or mean flow velocity  $\langle u \rangle$ ).  $\langle p_T \rangle_0$  and  $\langle p \rangle_0$  denote the mean transverse momentum and mean momentum of massless particle.

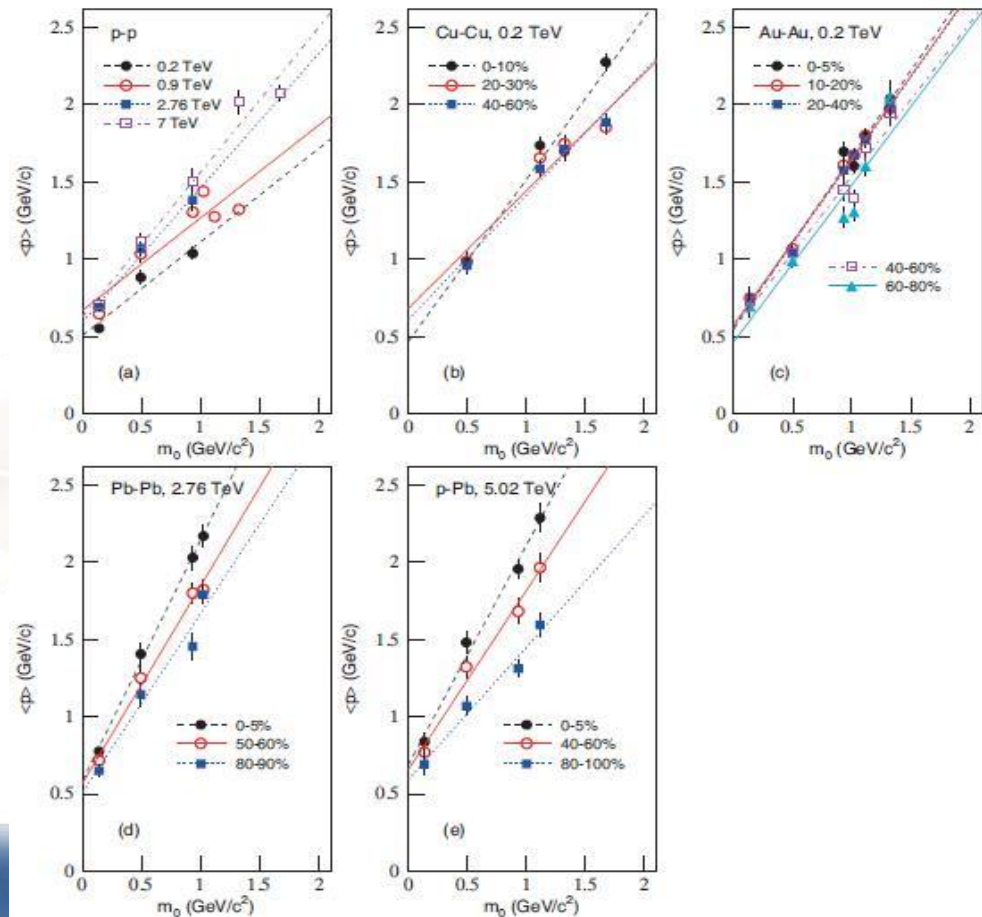




$$\langle p_T \rangle = C_0 + k_T m_0$$

$$\langle p \rangle = C_p + k_p m_0$$

- (1) The intercepts have the tendency of converging to one point.
- (2) The intercepts are nearly equal to each other or do not change obviously with center-of-mass energy or centrality.
- (3) The slopes have the tendencies of increasing with center-of-mass energy and centrality.

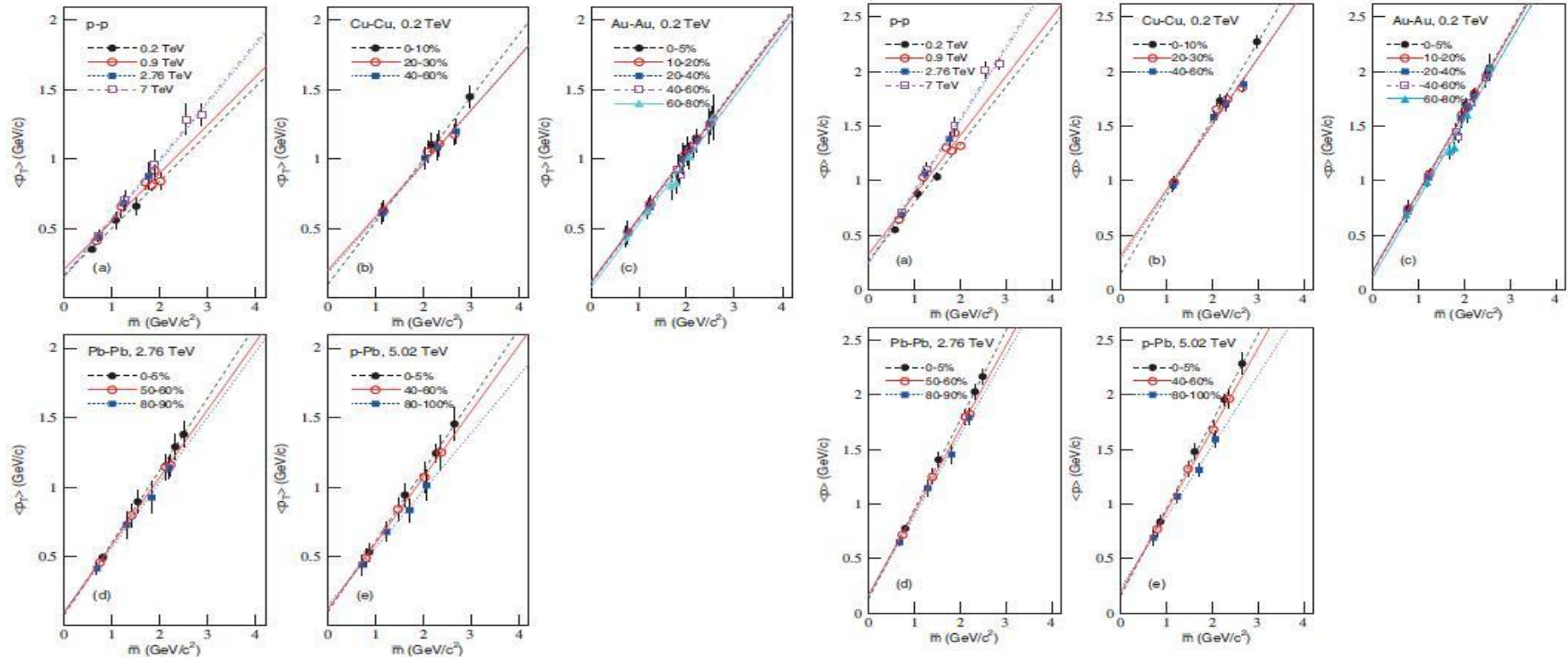


# 4 Results

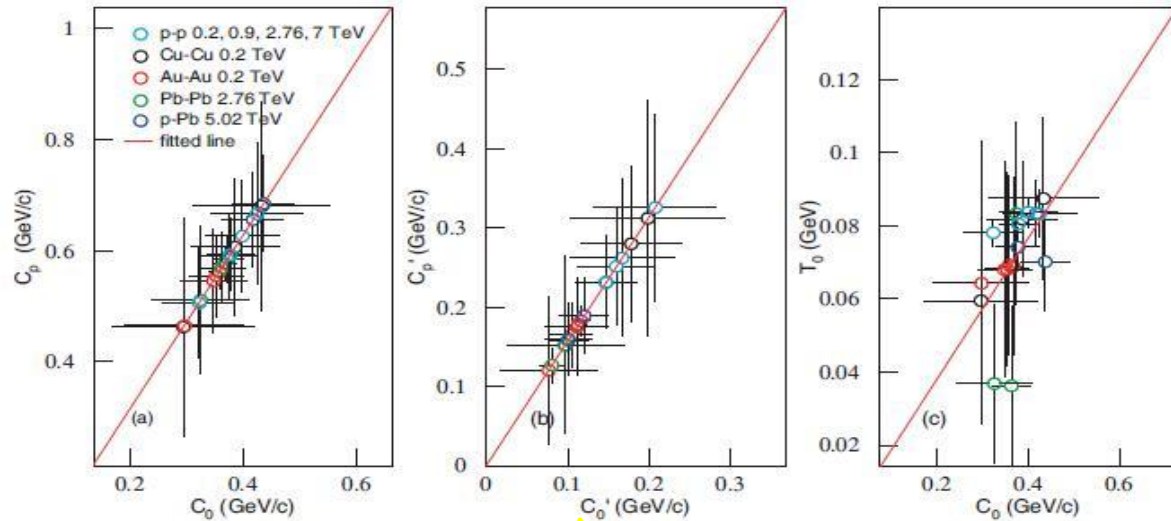
The slopes have the dimension of velocity and are considered to be related to mean transverse flow velocity and mean flow velocity.

$$\langle p_T \rangle = C'_0 + \langle u_T \rangle \bar{m}$$

$$\langle p \rangle = C'_p + \langle u \rangle \bar{m}$$







$$\begin{aligned}\langle u_T \rangle &\rightarrow (0.339 - 0.522c) \\ \langle u \rangle &\rightarrow (0.532 - 0.820c) \\ \langle k_T \rangle &\rightarrow (0.382 - 0.944c) \\ \langle k_p \rangle &\rightarrow (0.546 - 1.424c)\end{aligned}$$

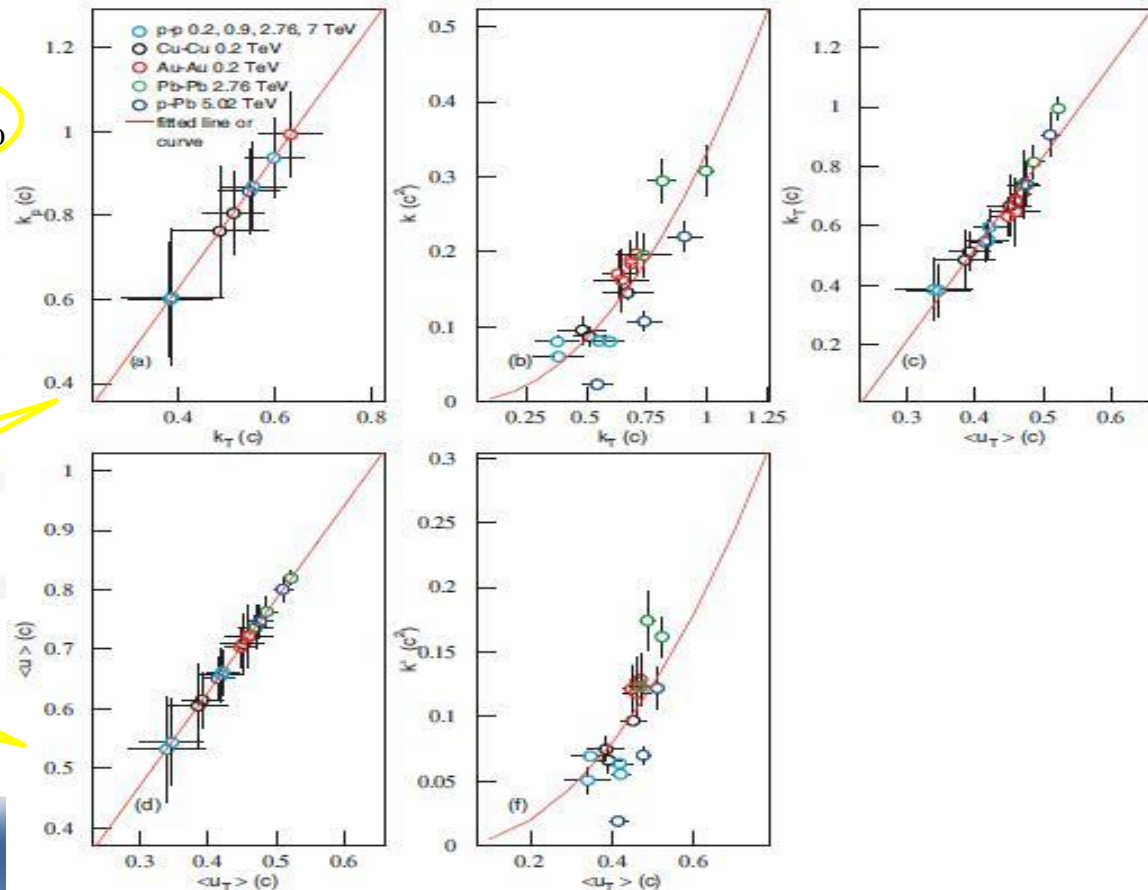
$$C_p = (\pi/2)C_0$$

$$C'_p = (\pi/2)C'_0$$

the assumption of isotropic emission in the source rest frame

$$k_p = (\pi/2)k_T$$

$$\langle u \rangle = (\pi/2) \langle u_T \rangle$$





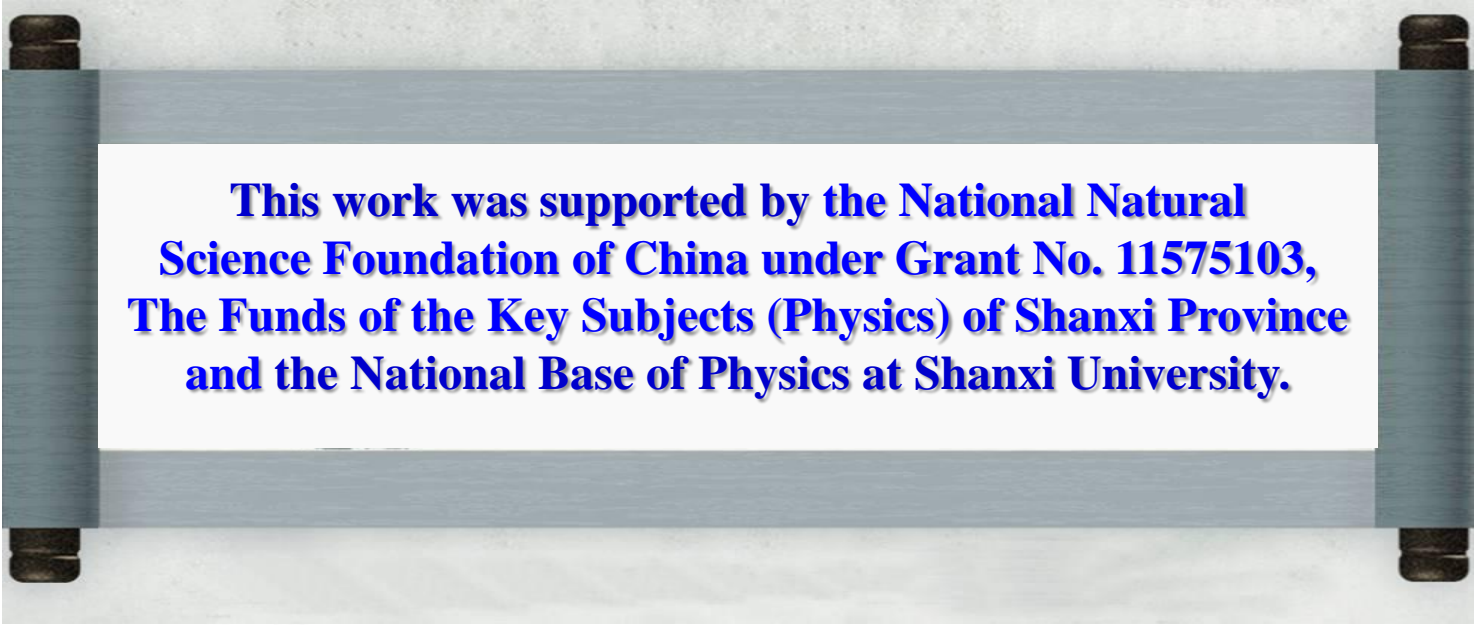
# 5 Conclusions

- 1、 We used the Tsallis-standard, Tsallis, multi-component standard and multi-component Erlang distributions to describe the transverse momentum spectra of final-state particles and extracted the effective temperature.
- 2、 We obtained the kinetic freeze-out temperature from the intercept by plotting the effective temperature versus the particle's rest mass.
- 3、 We obtained the transverse flow velocity from the slope by plotting the mean transverse momentum versus the mean moving mass.





# **Thank you for your attention!**



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Science Foundation of China under Grant No. 11575103,  
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and the National Base of Physics at Shanxi University.**