Generalized Splitting Amplitude from Effective Field Theory

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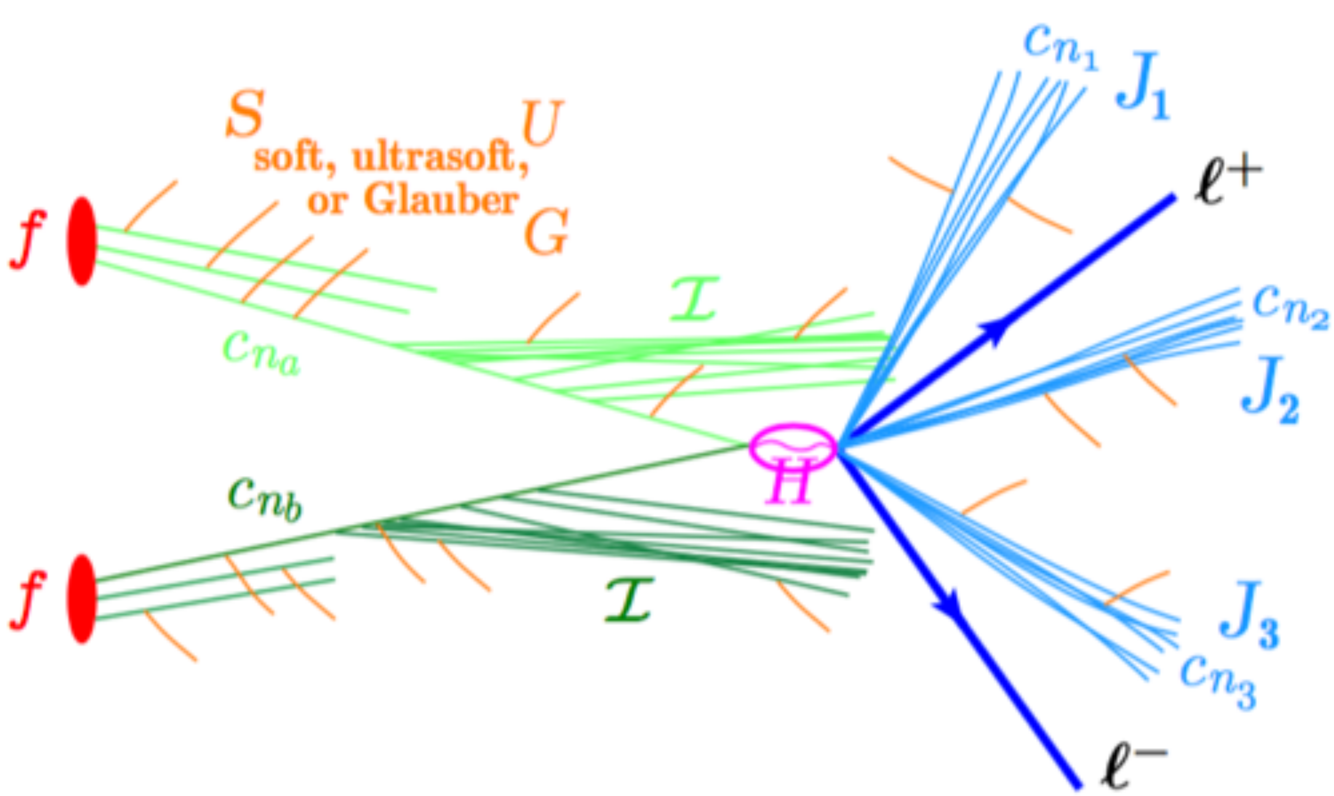
My talk

- Factorization violation in hard scattering processes :   Strict factorization breaking in space-like collinear limit
- Generalized splitting amplitude: Catani's formula; effective-theory description
- Glauber mode and the associated factorization-breaking effects

Factorization theorems at the LHC

$$\sigma(m_H) = \int dY \sum_{i,j} \int \frac{d\xi_a}{\xi_a}\frac{d\xi_b}{\xi_b}\, H_{ij}^{\rm incl}\Big(\frac{x_a}{\xi_a}, \frac{x_b}{\xi_b}, m_H, \mu\Big)\, f_i(\xi_a,\mu) f_j(\xi_b,\mu)$$



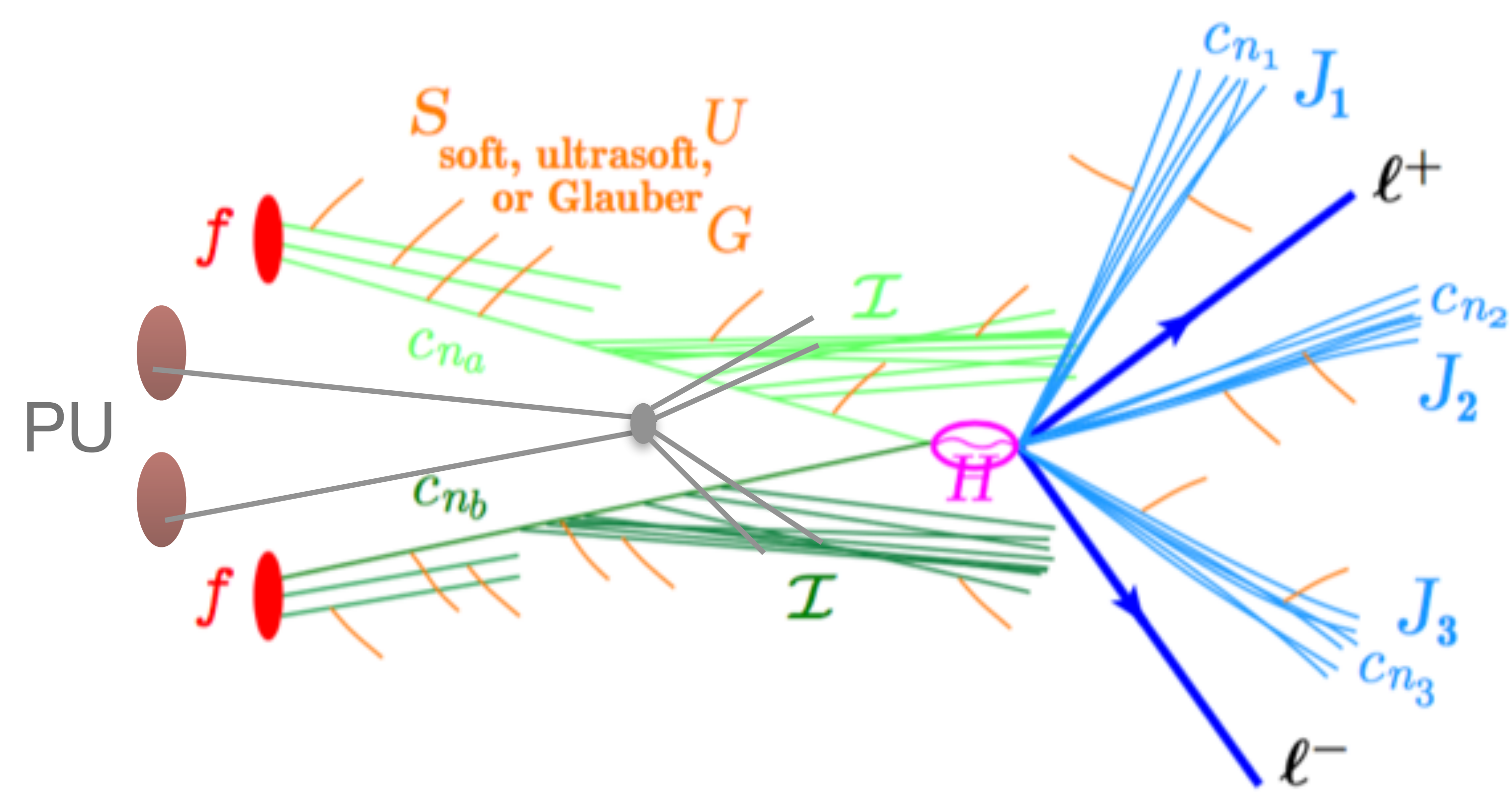
Njettiness? Beam thrust? Jet mass?

$$\hat\sigma_\kappa = \sum_{\kappa_i} {\rm tr}\; H^N_{\kappa_H} \mathcal{I}_{\kappa_a} \mathcal{I}_{\kappa_b} J_{\kappa_1} \times \cdots \times J_{\kappa_N} S^N_{\kappa_S}$$ **?**

only exists for:

- Drell-Yan like processes
- inclusive single hadron production
- double-hard scattering

What invalidate factorization



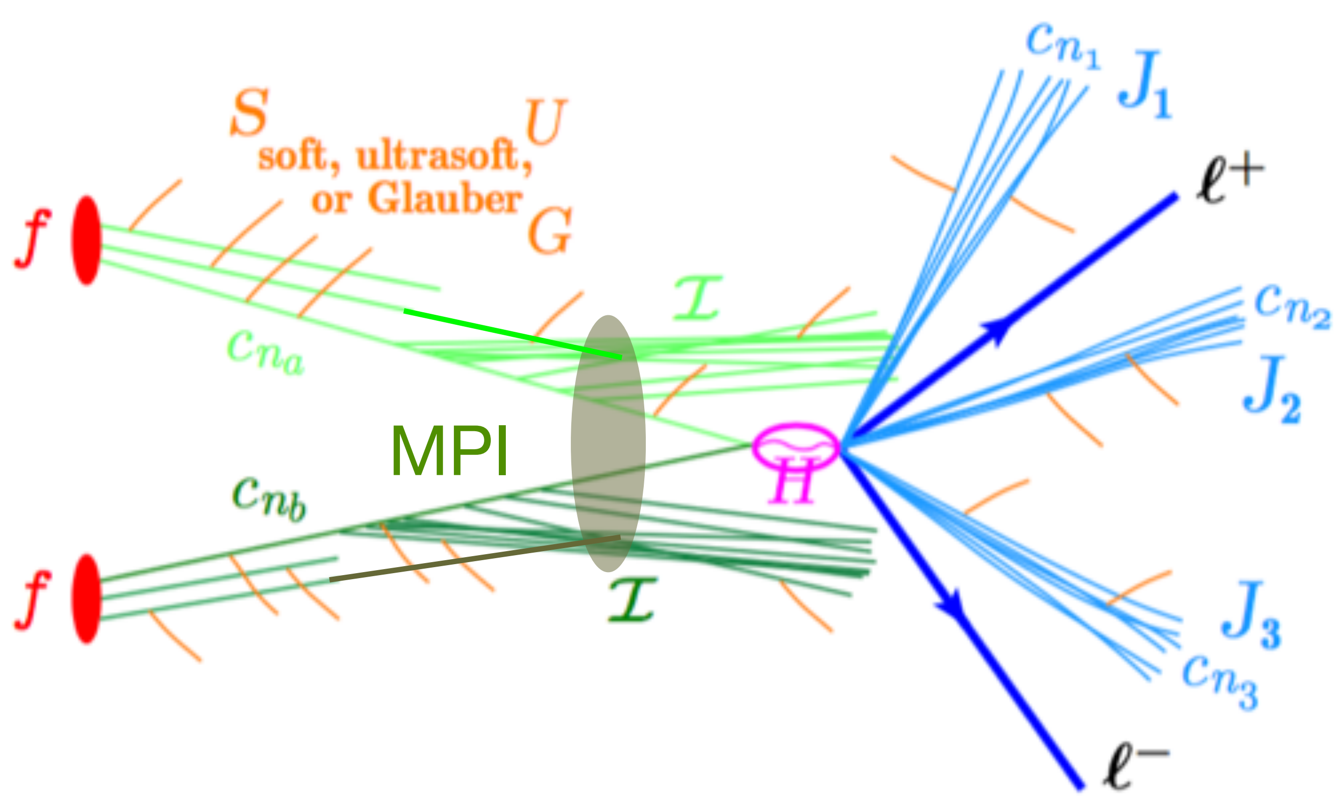
Contaminations:

- Pile-up
- MPI
- Non-global soft emissions

Solution:

Groom it away!

What invalidate factorization



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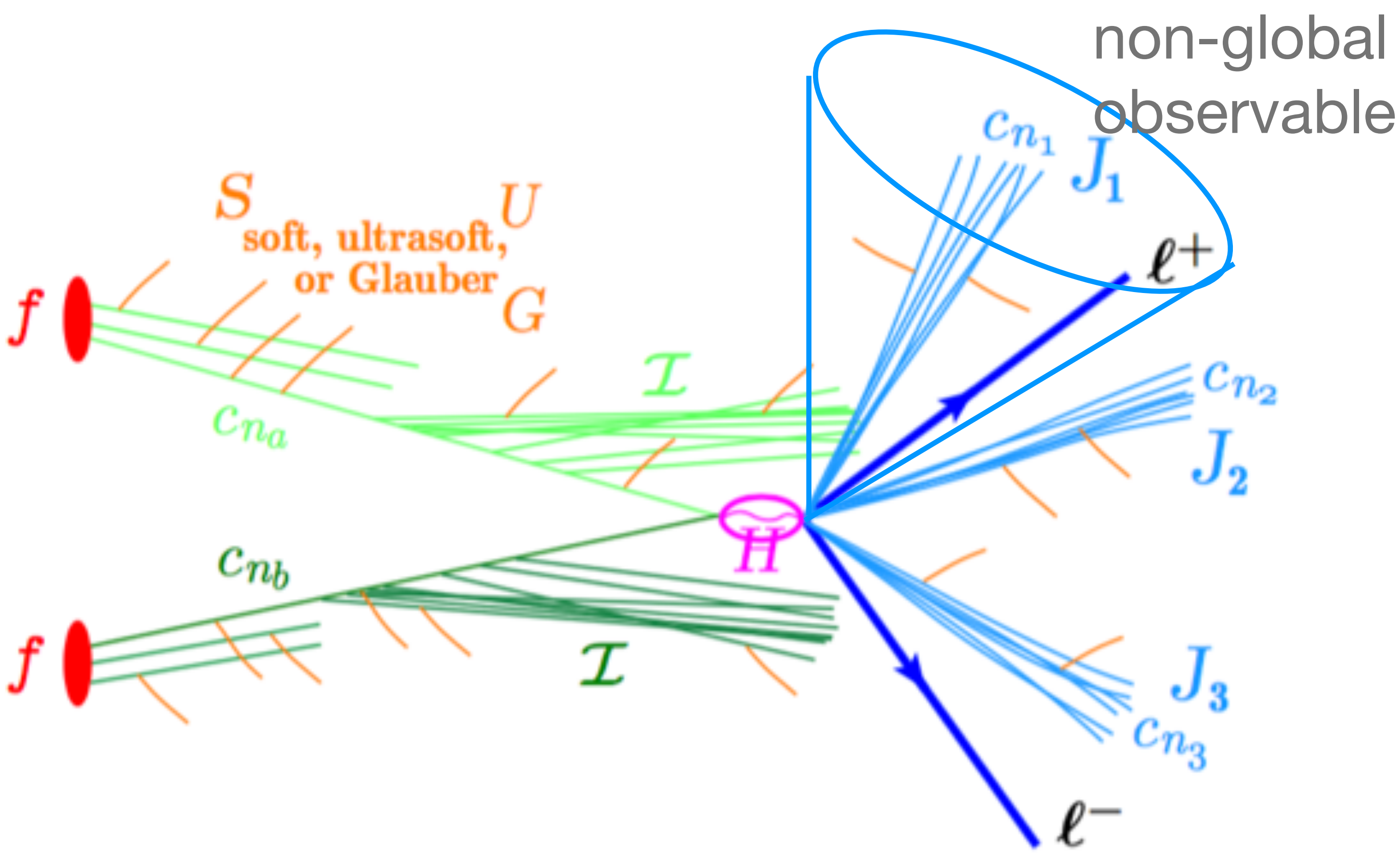

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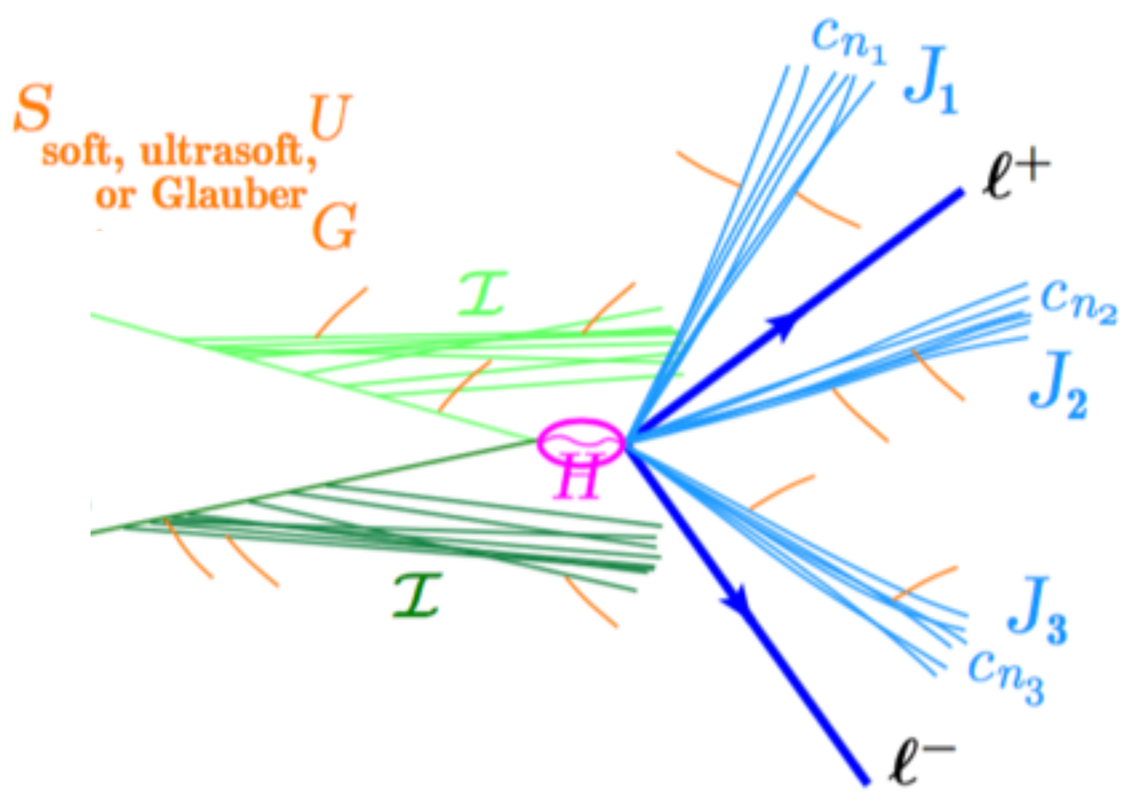
Solution:

Groom it away!

Factorization property of perturbative amplitude

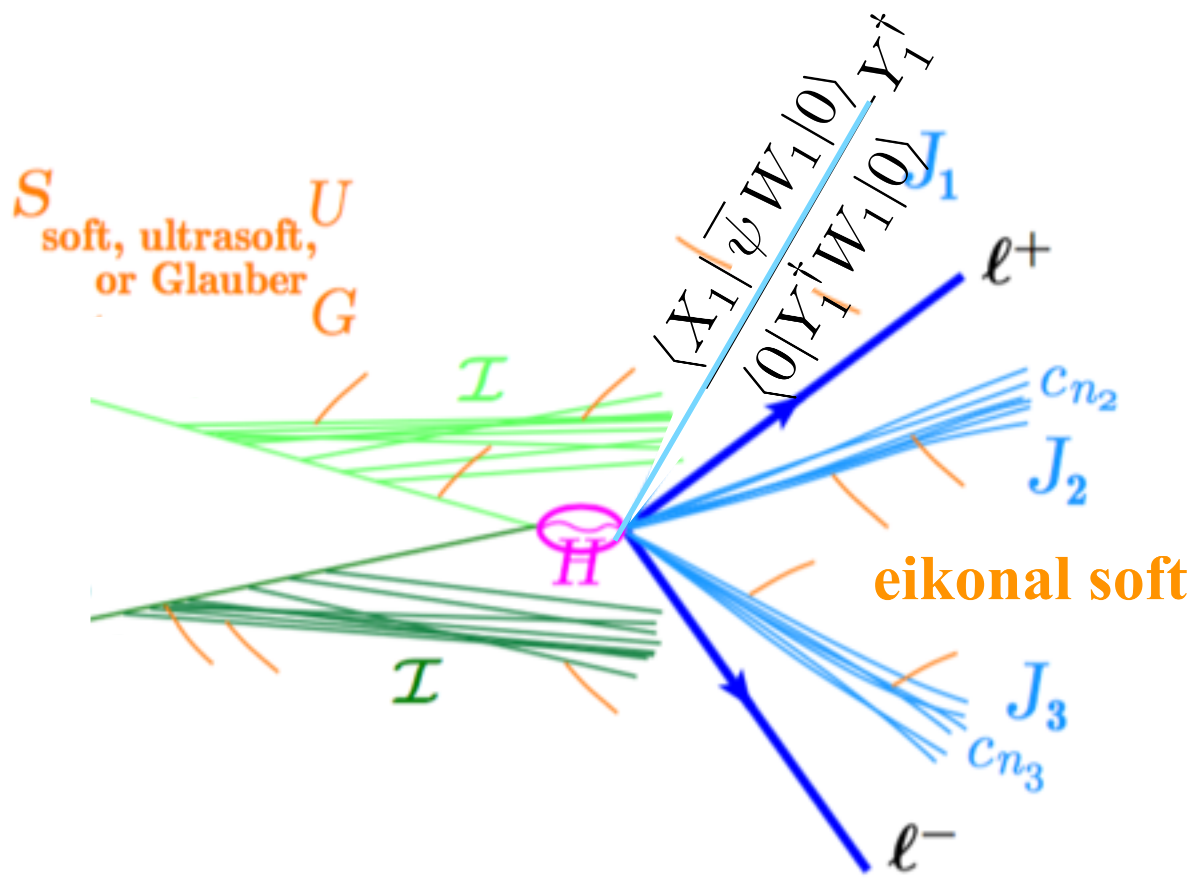
collinear universality of gauge theory amplitude challenged by quantum loop corrections



- Factorization will potentially break down due to long-distance interaction between the collinear and non-collinear partons.

Collinear factorization for perturbative amplitude

T.L. vs. S.L. collinear splitting



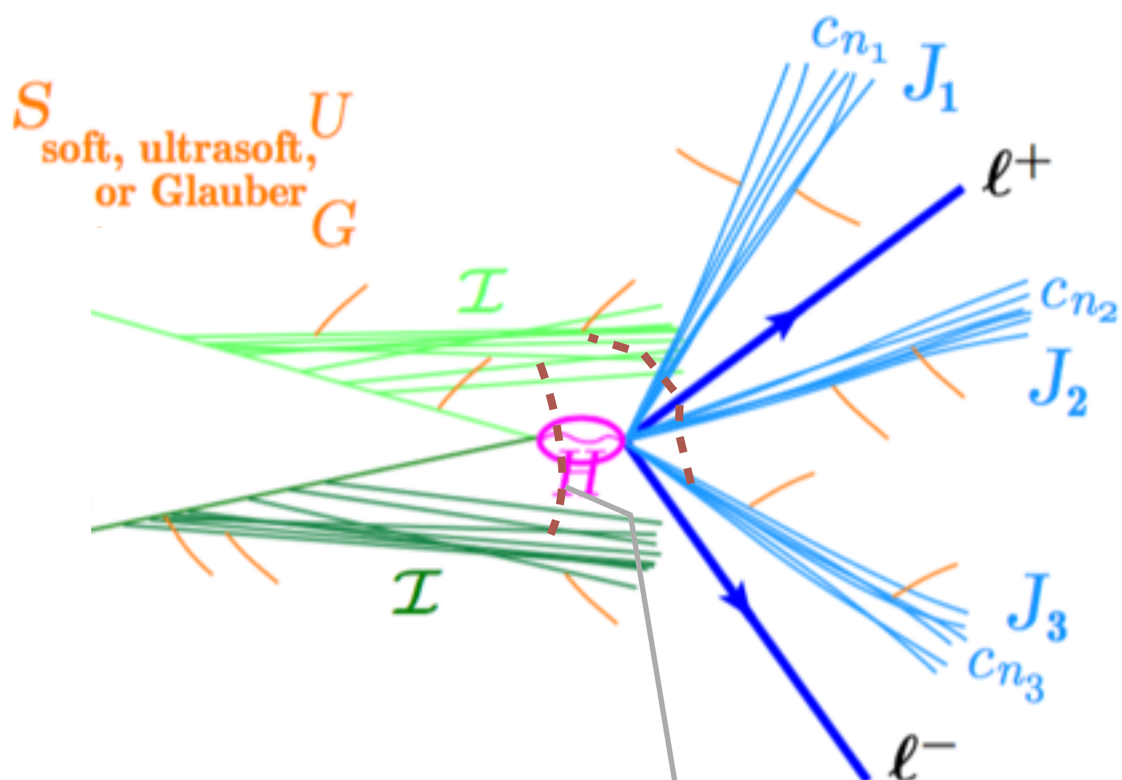
For time-like splitting, factorization is restored by **color coherence** in eikonal limit.

soft radiation can only probe the total charge of collinear partons

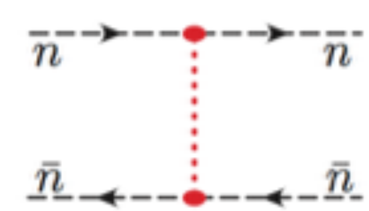
$$\mathcal{M}(\{P_i\}) \cong \frac{\langle P_i|\,\overline{\psi}W_{t_i}\,|0\rangle}{\langle 0|\,Y_{n_i}^\dagger W_{t_i}\,|0\rangle}\cdots\frac{\langle 0|\,W_{t_i}^\dagger\psi\,|P_j\rangle}{\langle 0|\,W_{t_i}^\dagger Y_{n_j}\,|0\rangle}\,\langle 0|\,Y_{n_i}^\dagger\cdots Y_{n_j}\,|0\rangle$$

proven and validated
by explicit calculation

T.L. vs. S.L. collinear splitting



In space-like regime, color coherence breaks down. Eikonalization not manifestly true.



$$V_{\mathrm{G}}(q_\perp) = -\frac{8\pi\alpha_s(\mu)}{\vec{q}_\perp^{\,2}} = \frac{8\pi\alpha_s(\mu)}{t}$$

S.L. splitting amplitude not well studied, except for its IR poles.

Glauber/Coulomb potential: static in light-cone time

solve QED: $e^{i\phi_G}$

Splitting amplitude

strict collinear factorization:

$$|\mathcal{M}^{(0)}(p_1 \cdots p_{n+1})\rangle \cong \boldsymbol{Sp}^{(0)}(p_1, p_2, P)\, |\overline{\mathcal{M}}^{(0)}(P, p_3 \cdots p_n)\rangle$$

generalized factorization
formula by Catani:

$$|\mathcal{M}(p_1, p_2, \cdots p_n)\rangle \cong \boldsymbol{Sp}(p_1, p_2, P; p_3, \cdots p_n)\ |\overline{\mathcal{M}}(P, p_3, \cdots p_n)\rangle$$

all order structure of IR singularity of gauge theory amplitude:

$$|\mathcal{M}\rangle = (\mathbf{1} - \boldsymbol{I}_M)^{-1}\, |\mathcal{M}^{\text{fin.}}\rangle \equiv \boldsymbol{V}_M(\epsilon)\, |\mathcal{M}^{\text{fin.}}\rangle$$

$$\boldsymbol{V}_M(\epsilon) \equiv \exp\Big\{ - \boldsymbol{I}_{M,\text{cor.}}(\epsilon) \Big\}$$

$$\boxed{\boldsymbol{Sp} = \boldsymbol{V}^{-1}(\epsilon)\, \boldsymbol{Sp}^{\text{fin.}}\, \overline{\boldsymbol{V}}(\epsilon)}$$

IR poles extracted from the collinear limit of the n-point formula, compared with the (n-1)-point formula

Catani's formula

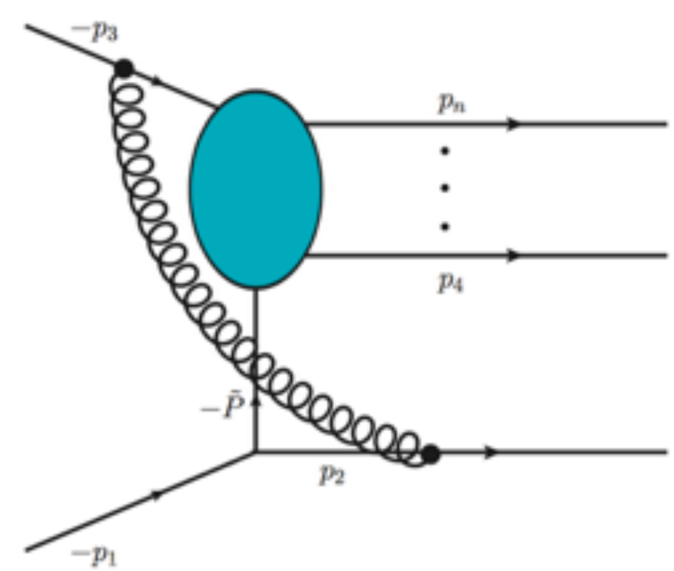
one-loop splitting amplitude

$$\boldsymbol{I}_C^{(1)}(\epsilon) = \frac{\alpha_s}{2\pi}\frac{1}{2}\left\{\left(\frac{1}{\epsilon^2}C_{\widetilde{P}} + \frac{1}{\epsilon}\gamma_{\widetilde{P}}\right) - \sum_{i=1}^{m}\left(\frac{1}{\epsilon^2}C_i + \frac{1}{\epsilon}\gamma_i - \frac{2}{\epsilon}C_i \ln|z_i|\right)\right.$$

$$\left. -\frac{1}{\epsilon}\sum_{\substack{i,l\in C\\ i\neq l}} \boldsymbol{T}_i\cdot\boldsymbol{T}_l \ln\left(\frac{-s_{il}-i0}{|z_i||z_l|\mu^2}\right)\right\} + \boxed{\boldsymbol{\Delta}_C^{(1)}(\epsilon)}$$

absorptive part; purely imaginary

radiative part; real; factorized



$$\boldsymbol{\Delta}_C^{(1)}(\epsilon) = \frac{\alpha_s}{2\pi}\frac{i\pi}{\epsilon}\sum_{\substack{i\in C\\ j\in NC}} \boldsymbol{T}_i\cdot\boxed{\boldsymbol{T}_j}\,\Theta(-z_i)\mathrm{sign}(s_{ij})$$

dependence on non-collinear partons

two-loop splitting amplitude

$$\boldsymbol{Sp}^{(2)} = \left\{ \frac{1}{2} \left(\boldsymbol{I}_{\rm cor}^{(1)} - \overline{\boldsymbol{I}}_{\rm cor}^{(1)} \right)^2 - \boxed{\frac{1}{2} \left[\boldsymbol{I}_{\rm cor}^{(1)}, \overline{\boldsymbol{I}}_{\rm cor}^{(1)} \right]} + \left(\boldsymbol{I}_{\rm cor}^{(2)} - \overline{\boldsymbol{I}}_{\rm cor}^{(2)} \right) \right\} \cdot \boldsymbol{Sp}^{(0)}$$
$$+ \left\{ \boldsymbol{I}_{\rm cor}^{(1)} - \overline{\boldsymbol{I}}_{\rm cor}^{(1)} \right\} \cdot \boldsymbol{Sp}^{(1),{\rm fin.}} + \left[\overline{\boldsymbol{I}}_{\rm cor}^{(1)}, \boldsymbol{Sp}^{(1),{\rm fin.}} \right] + \boldsymbol{Sp}^{(2),{\rm fin.}}$$

two-loop non-abelian web; subleading pole

exponentiation of one-loop result

two -loop

$\Delta_C^{(2)}(\epsilon)$

$$\left(\frac{\alpha_{\rm S}(\mu^2)}{2\pi} \right)^2 \left(-\frac{1}{2\,\epsilon^2} \right) \pi\, f_{abc} \sum_{i \in C} \sum_{\substack{j,k \in NC \\ j \neq k}} T_i^a\, T_j^b\, T_k^c\, \Theta(-z_i)\, {\rm sign}(s_{ij})\, \Theta(-s_{jk})\, \ln \left(-\frac{s_{j\widetilde{P}}\, s_{k\widetilde{P}}}{s_{jk}\, \mu^2} - i0 \right)$$

leading pole; breaks non-abelian exponentiation

Squared amplitude

$$\mathbf{P} \equiv [\,\boldsymbol{Sp}\,]^{\dagger}\;\boldsymbol{Sp}$$

$$\widetilde{\boldsymbol{\Delta}}_P^{(2)}(\epsilon) = \widetilde{\boldsymbol{\Delta}}_C^{(2)}(\epsilon) + \text{ h.c.}$$

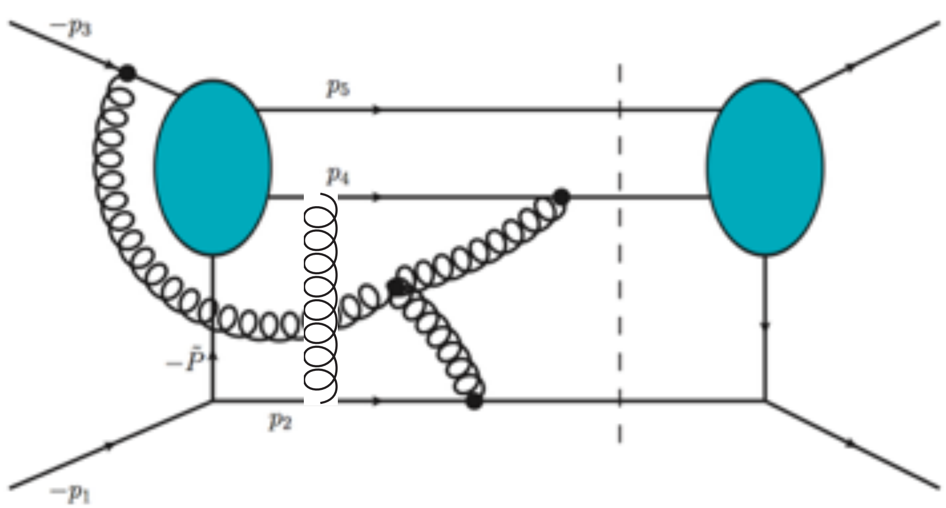
its expectation value on a QCD tree amplitude is zero

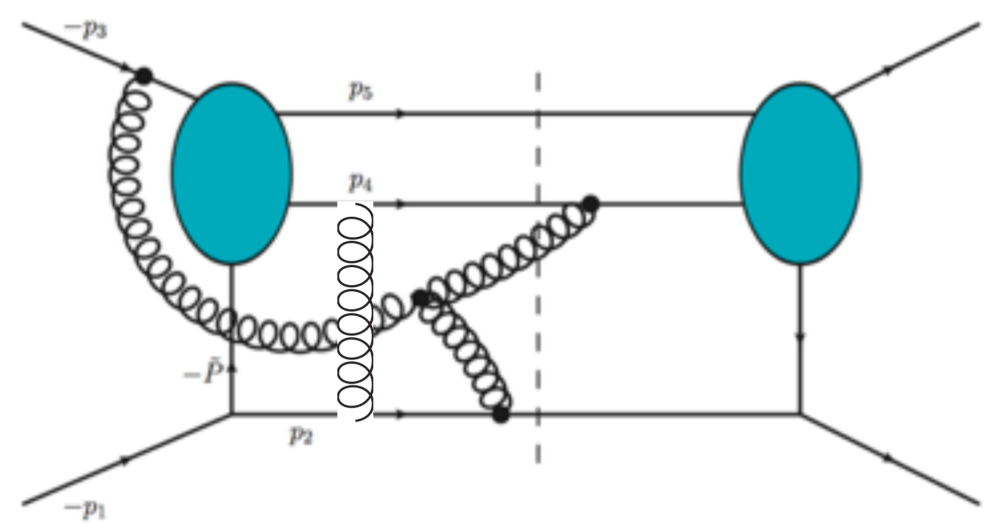
$$\langle\overline{\mathcal{M}}^{(0)}|Sp^{(0)\dagger}\,\Delta_P^{(2)}\,Sp^{(0)}|\overline{\mathcal{M}}^{(0)}\rangle = 0$$

(pure QCD.)

Forshaw, Seymour, Si´odmok

Super-leading-logs in gap-between-jets cross section
comes from 3-loop factorization-breaking terms in Sp





SCET approach

appropriate for the study the infrared universality of amplitudes

- Allows seperating the relavant degrees of freedom associated with different infrared regions.

Advantages

- check the generalized factorization property of gauge theory amplitude (both IR poles and finite terms.)

- powerful in determining the existence of factorization-breaking effects for certain physical observables

IR sensitive regions (power-counting analysis)

power-counting behavior under a particular scaling:

$$I = \int d^4\ell \, \frac{\text{Numerator}}{[(p_a+\ell)^2+i\varepsilon][(p_b-\ell)^2+i\varepsilon][\ell^2+i\varepsilon]}$$

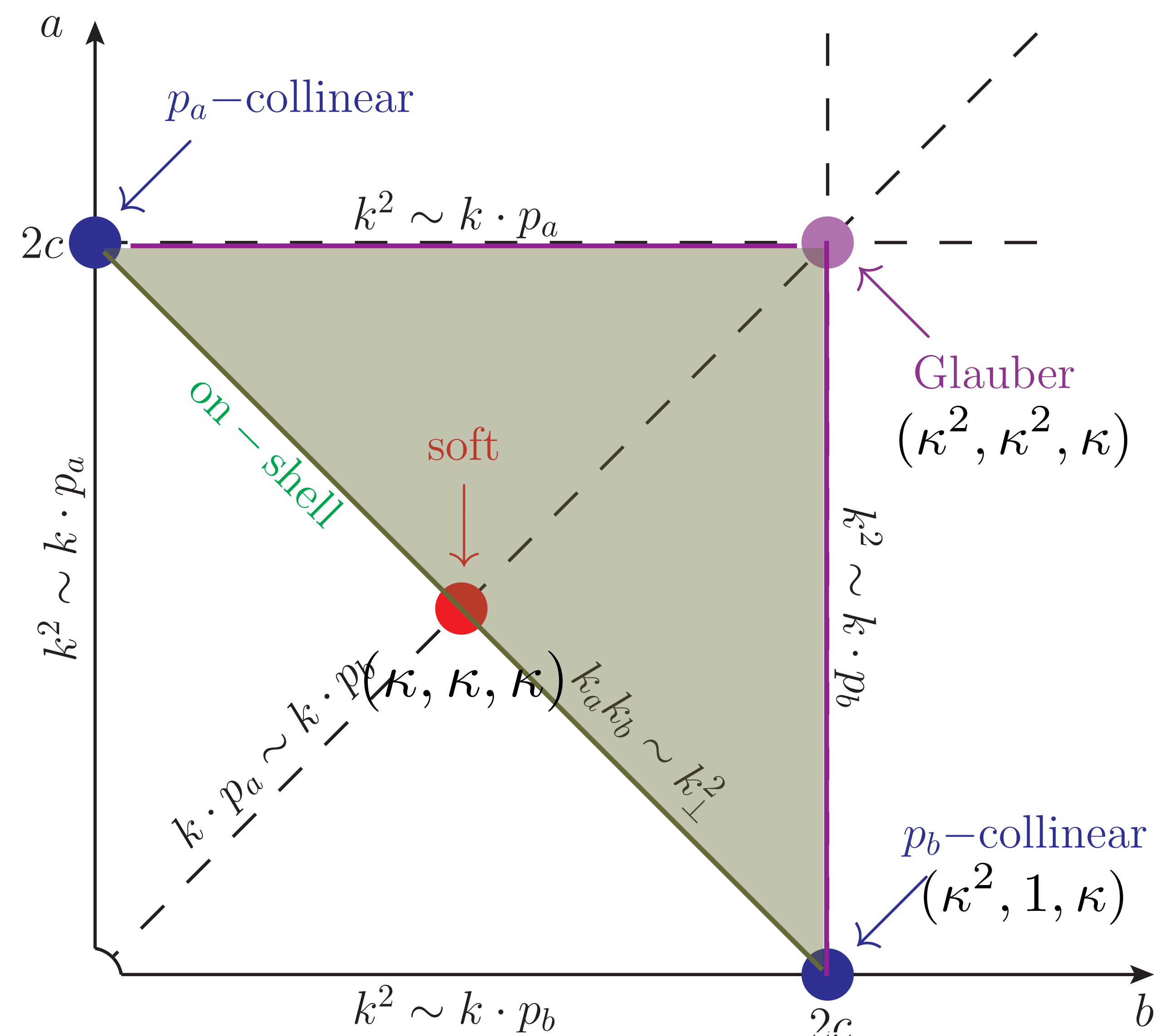
zoom into a particular infrared regime:

$$\ell^\mu \to \kappa^a(\ell\cdot n_b)\,\frac{n_a^\mu}{2} + \kappa^b(\ell\cdot n_a)\,\frac{n_b^\mu}{2} + \kappa\ell_\perp^\mu$$

$$I \to \int \frac{\bar{d}^4\ell \; \kappa^{a+b+2}}{[\kappa^{a+b}(n_a\cdot\ell)(n_b\cdot\ell) - \kappa^2\ell_\perp^2 + i\varepsilon]}$$

$\sim \kappa^0$ power-counting divergent

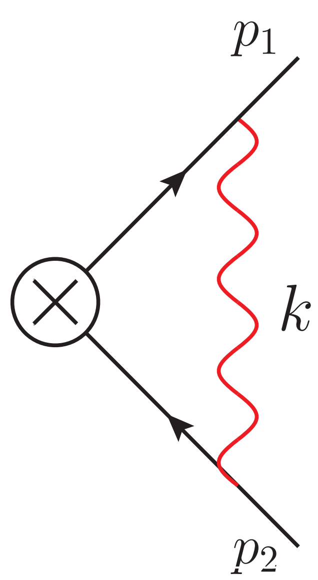
$\sim \mathcal{O}(\kappa)$ power-counting finite

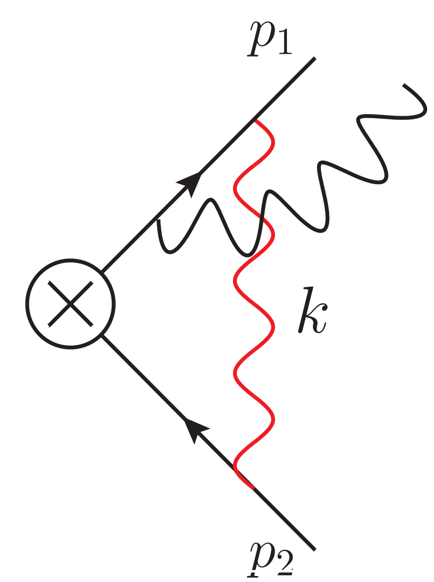


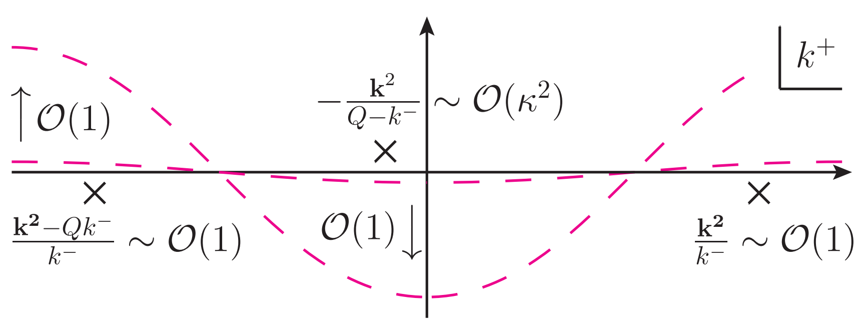
- independent IR modes: soft, collinear, glauber

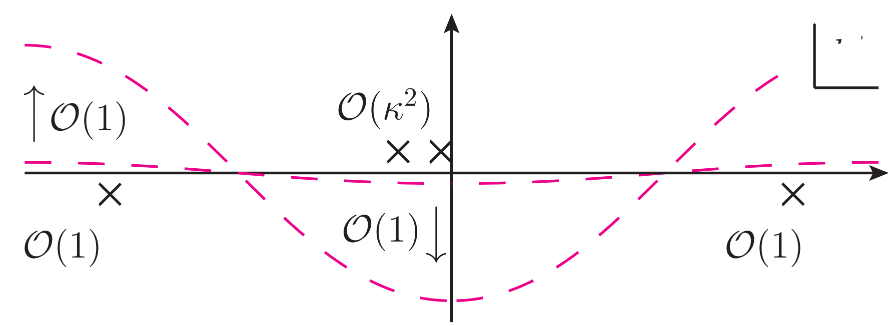
When is glauber relavant? (pinch analysis)

- No pinched singularity in glauber region

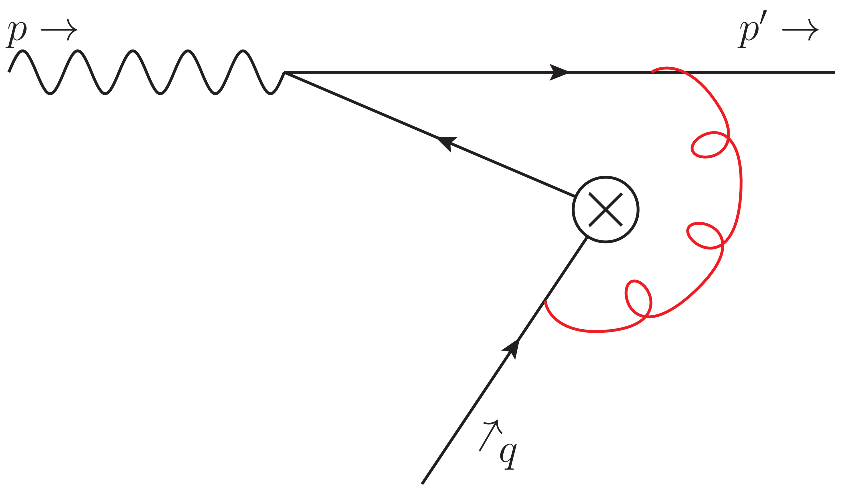


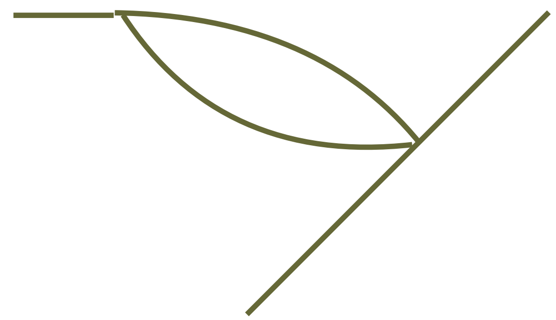


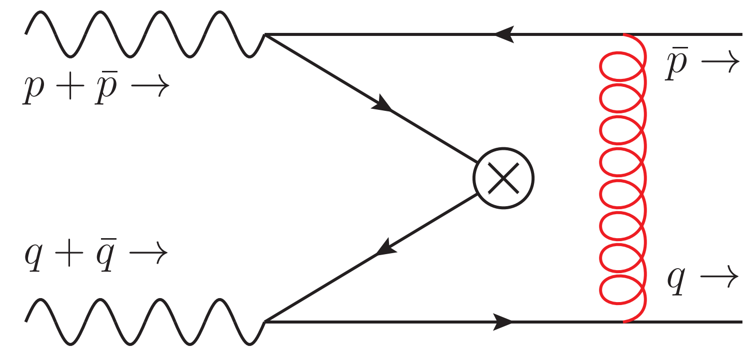


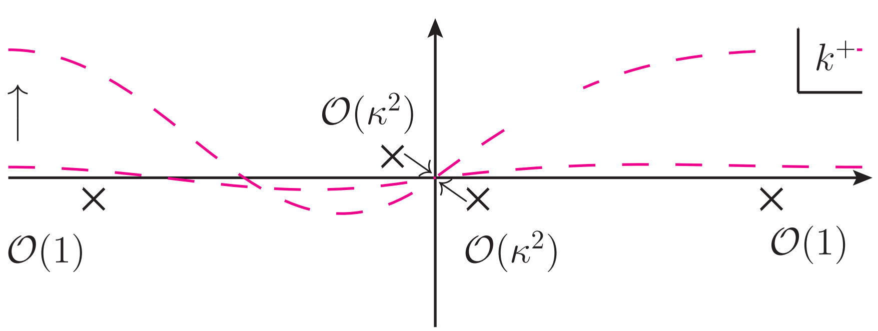


- pinched singularity in glauber regime

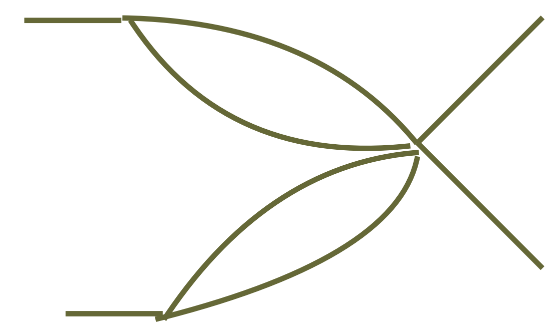








space-time picture of pinch surfaces



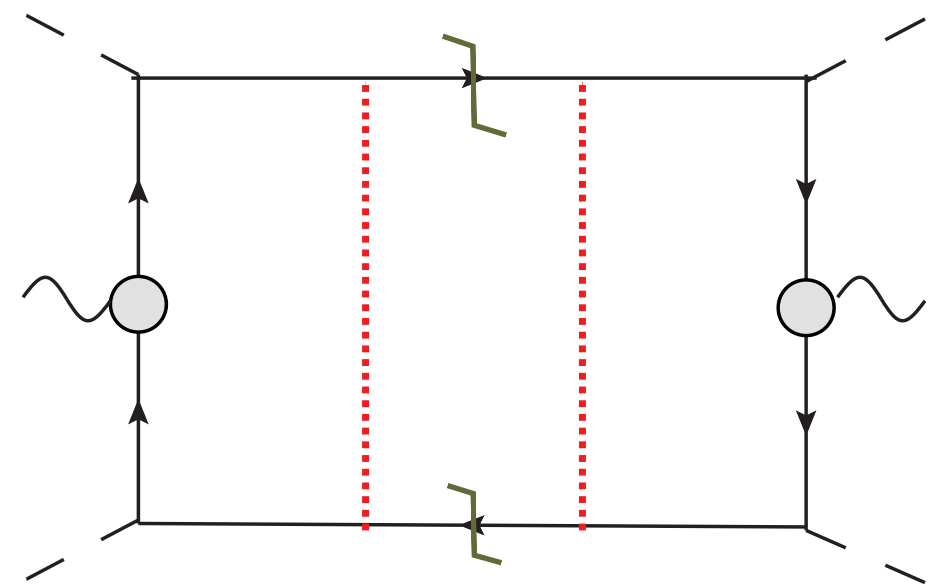
Glauber mode challenges the validation of eikonalization. It is responsible for the breaking of collinear factorization for initial state splitting

Progress with glaubers

- Collins-Soper-Sterman: glauber cancels at cross-section level for inclusive Drell-Yan or its qT spectrum

- Gaunt : when does CSS argument breaks down; sensitivity to MPI

- Ian & Ira : A new effective lagragian is proposed to incorporate glauber modes into the SCET framework



pT v.s. ET

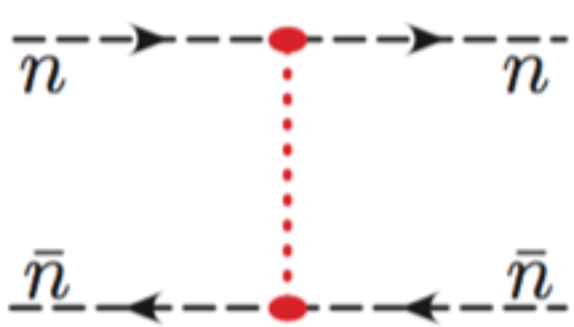
SCET lagrangian with Glaubers

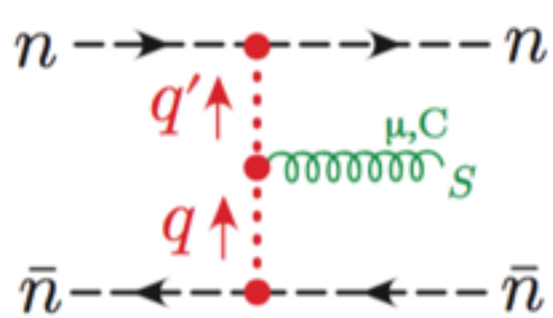
$$\mathcal{L}_{\mathrm{SCET_{II}}}^{\mathrm{hardscatter}} = \sum_K C_K^{\mathrm{II}} \otimes O_K^{\mathrm{II}}(\{\xi_{n_i}, A_{n_i}\}, \psi_S, A_S)$$

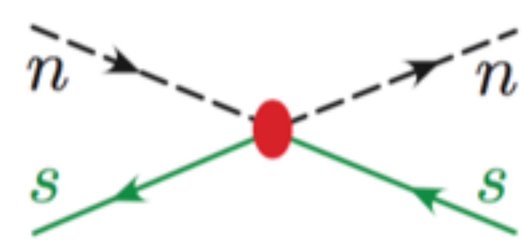
gauge invariant glauber potential

$$\mathcal{L}_{\mathrm{SCET_{II}}}^{(0)} = \Big[\mathcal{L}_{\mathrm{S}}^{(0)}(\psi_{\mathrm{S}}, A_{\mathrm{S}}) + \sum_{n_i} \mathcal{L}_{n_i}^{(0)}(\xi_{n_i}, A_{n_i})\Big] + \mathcal{L}_G^{\mathrm{II}(0)}(\{\xi_{n_i}, A_{n_i}\}, \psi_{\mathrm{S}}, A_{\mathrm{S}})$$

$$\begin{aligned}\mathcal{L}_G^{\mathrm{II}(0)} &= e^{-ix\cdot\mathcal{P}} \sum_{n,\bar{n}} \sum_{i,j=q,g} O_{ns\bar{n}}^{ij} + e^{-ix\cdot\mathcal{P}} \sum_{n} \sum_{i,j=q,g} O_{ns}^{ij} \\ &\equiv e^{-ix\cdot\mathcal{P}} \sum_{n,\bar{n}} \sum_{i,j=q,g} \mathcal{O}_n^{iB} \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_s^{BC} \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_{\bar{n}}^{jC} + e^{-ix\cdot\mathcal{P}} \sum_{n} \sum_{i,j=q,g} \mathcal{O}_n^{iB} \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_s^{j_n B} .\end{aligned}$$







Formulation of Sp in SCET

hard scattering mediated by local operator $\mathcal{O}_4 = \frac{1}{\Lambda^3}\bar{\psi}\bar{\psi}\psi\psi$.

$$q(P)\bar{q}(p_2) \to \bar{q}(p_3)q(p_4) \qquad \overline{\mathcal{M}}(P, p_2, p_3, p_4)$$

$$p_g \parallel p_1:\ q(p_1)\bar{q}(p_2) \to \bar{q}(p_3)q(p_4)g(p_g) \qquad \mathcal{M}(p_1, p_g, p_2, p_3, p_4)$$

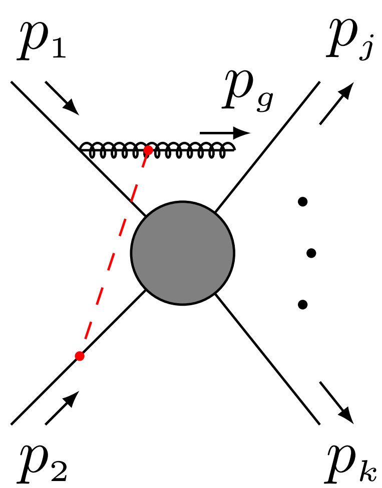
$$\boldsymbol{Sp}(p_1, p_g; P; p_2, p_3, p_4) = \frac{\langle p_1; p_2|\,(\overline{\chi}_{n_1}\bar{S}_{n_1})(\bar{S}^\dagger_{n_2}\chi_{n_2})(\overline{\chi}_{n_3}S_{n_3})(S^\dagger_{n_4}\chi_{n_4})\,|p_g; p_3; p_4\rangle}{\langle P; p_2|\,(\overline{\chi}_{n_1}\bar{S}_{n_1})(\bar{S}^\dagger_{n_2}\chi_{n_2})(\overline{\chi}_{n_3}S_{n_3})(S^\dagger_{n_4}\chi_{n_4})\,|p_3; p_4\rangle}$$

traditional SCET ignoring glauber:

$$\boldsymbol{Sp}^{\text{fac.}}(p_1, p_g; P) \equiv \frac{\langle p_1|\,\overline{\chi}_{n_1}\,|p_g\rangle}{\langle P|\,\overline{\chi}_{n_1}\,|0\rangle}$$

all factorization breaking terms comes from glauber !

One loop



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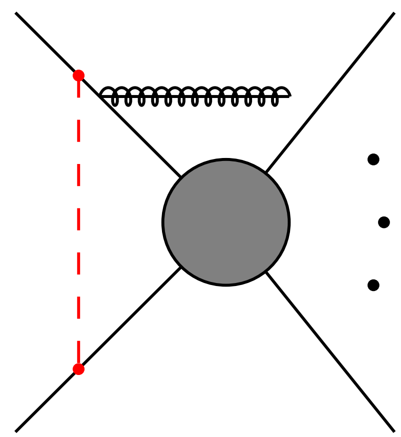

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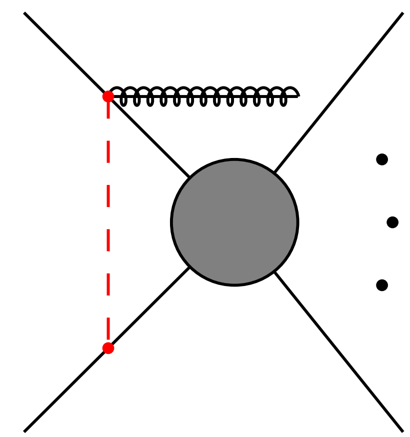

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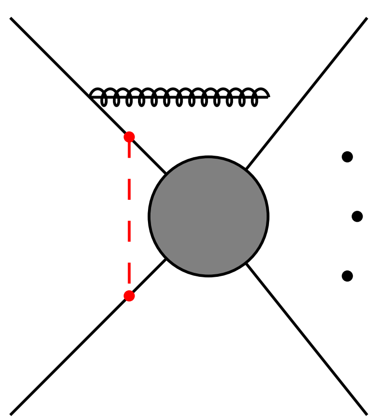
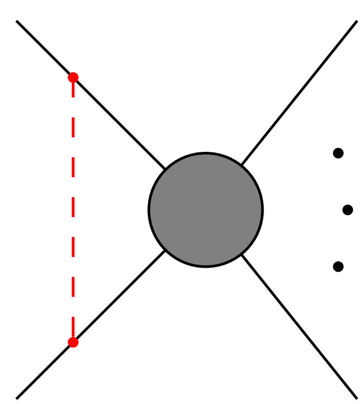


$-\ Sp^{(0)} \times$



non-vanishing only if glauber exchange takes place **after** the splitting

$$= -\frac{\alpha_s}{2\pi}(i\pi)\left(\frac{1}{\epsilon}+\ln\frac{\tilde{\mu}^2}{-s_{1g}}-\ln\frac{-z}{1-z}+\mathcal{O}(\epsilon)\right)(\boldsymbol{T}_g\cdot\boldsymbol{T}_2)\,\boldsymbol{Sp}_+^{(0)}(q^-,g^\pm)\,\overline{\mathcal{M}}^{(0)}P\rangle$$

summing over all diagrams:

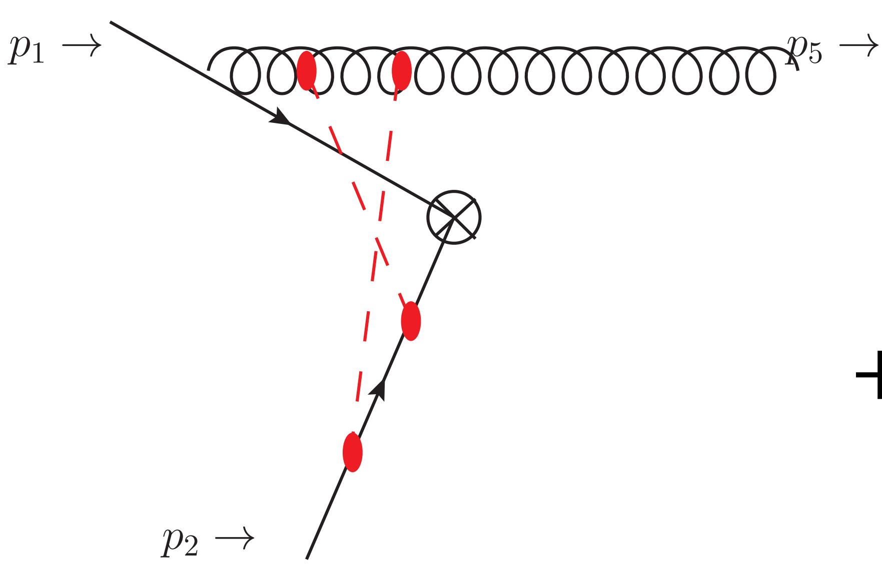
$$\boldsymbol{Sp}^{(1),non-fac.} = \frac{\alpha_s}{2\pi}\left(\frac{\tilde{\mu}^2}{-s_{1g}}\right)^{\epsilon}\frac{-i}{\epsilon}f_I(\epsilon;z)\left(-\boldsymbol{T}_g\cdot\boldsymbol{T}_2+\boldsymbol{T}_g\cdot\boldsymbol{T}_3+\boldsymbol{T}_g\cdot\boldsymbol{T}_4\right)\boldsymbol{Sp}^{(0)}\overline{\mathcal{M}}^{(0)}$$

$$= \underbrace{\frac{\alpha_s}{2\pi}\left(\frac{\tilde{\mu}^2}{-s_{1g}}\right)^{\epsilon}\frac{-i}{\epsilon}\sum_{\substack{i\in C\\ j\in NC}}\boldsymbol{T}_i\cdot\boldsymbol{T}_j\,\mathrm{sign}(s_{ij})\,f_I(\epsilon;z_i)}_{}\;\boldsymbol{Sp}^{(0)}\overline{\mathcal{M}}^{(0)}$$

agrees with $\Delta^{(1)}(\epsilon)$

Two loop (two colored collinear sectors)

— double glauber exchange accounts for the exponentiation of $\Delta^{(1)}(\epsilon)$, cancels at squared amplitude level



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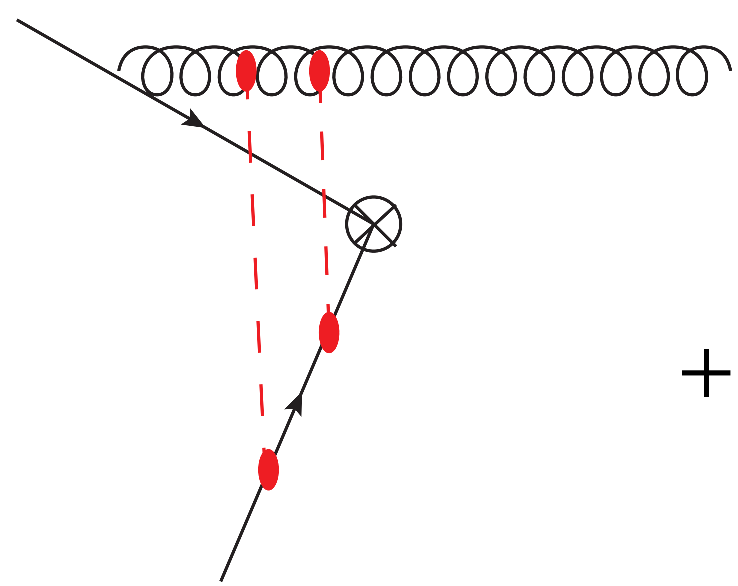

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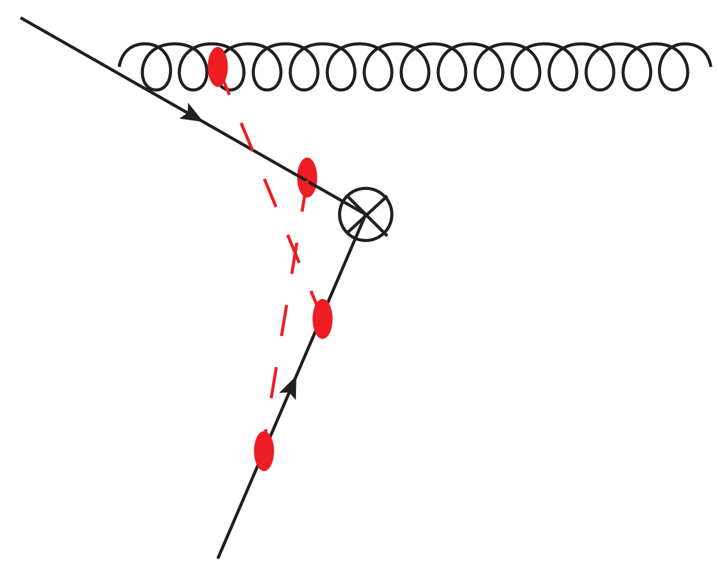

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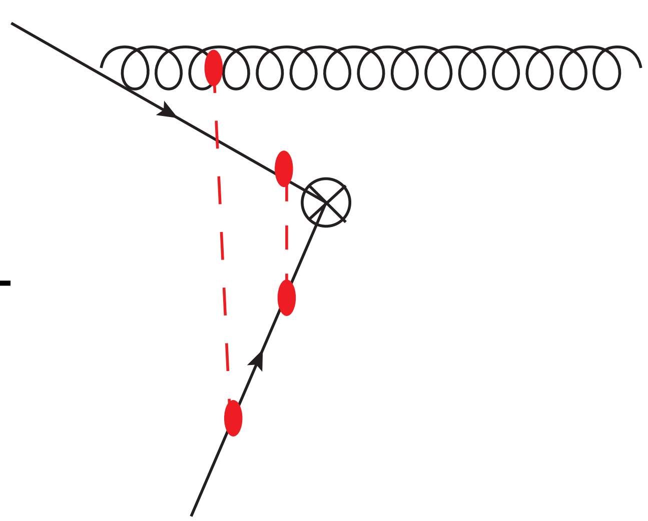

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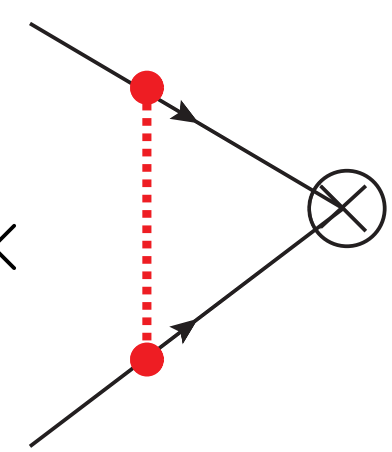
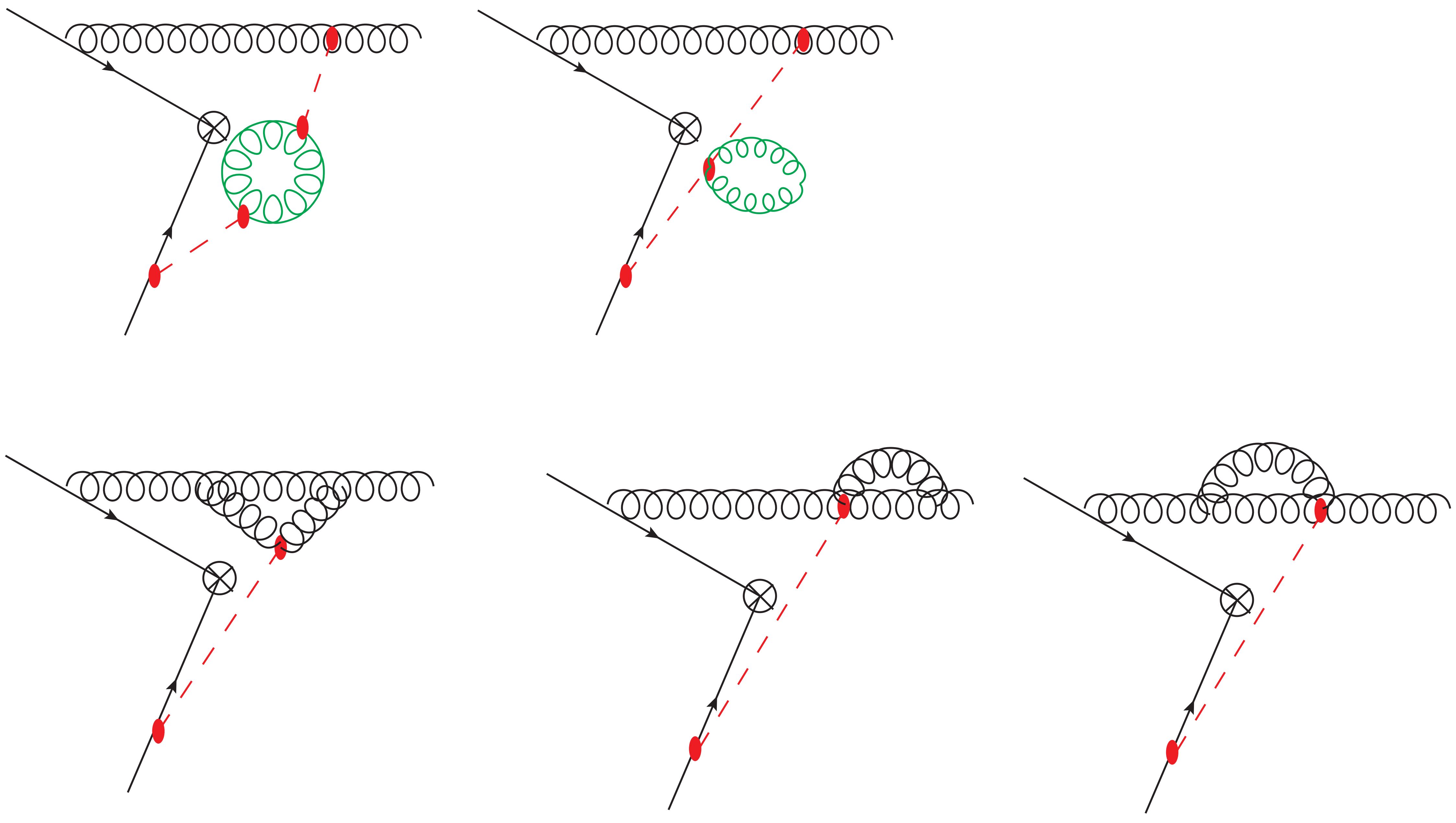
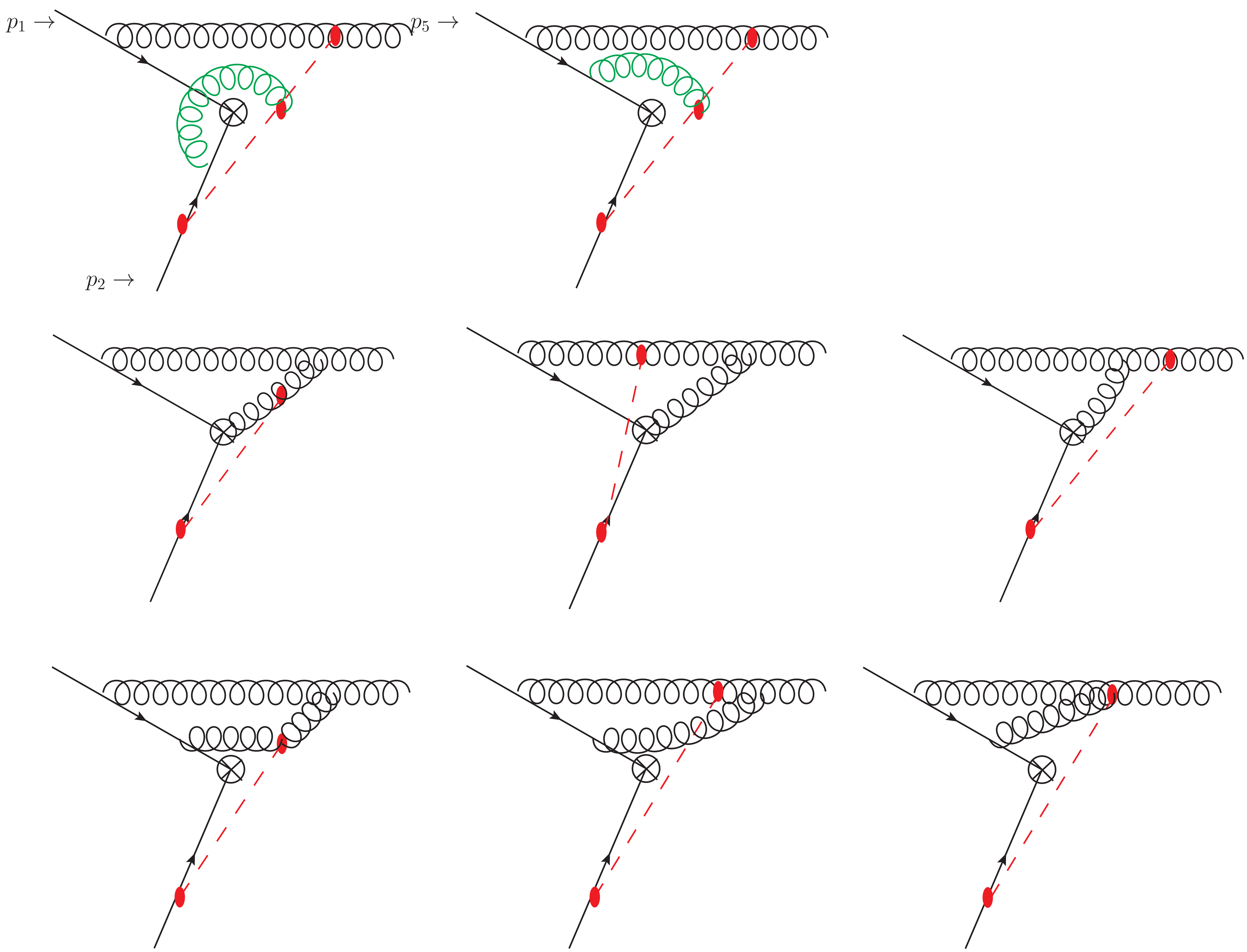
$Sp^{(1),non-fac.} \times$



— glauber-collinear and glauber-soft mixing diagrams: purely imaginary, cancels with h.c.





$p_1 \rightarrow$
$p_5 \rightarrow$
$p_2 \rightarrow$

Two-loop generalized splitting amplitude for Drell-Yan like process

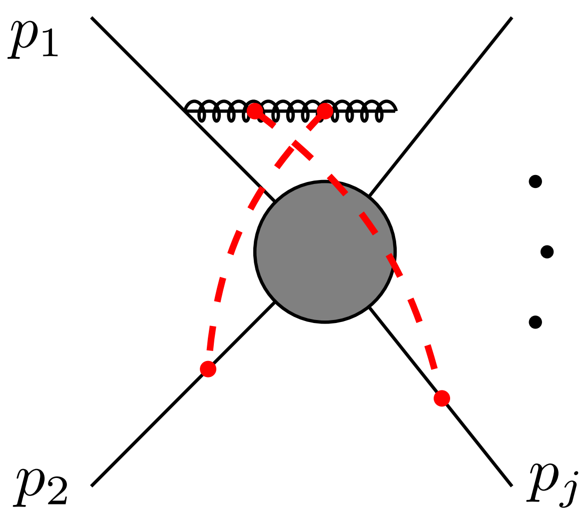
Glauber effects cancel at amplitude squared
level ! (both IR singular and finite terms)

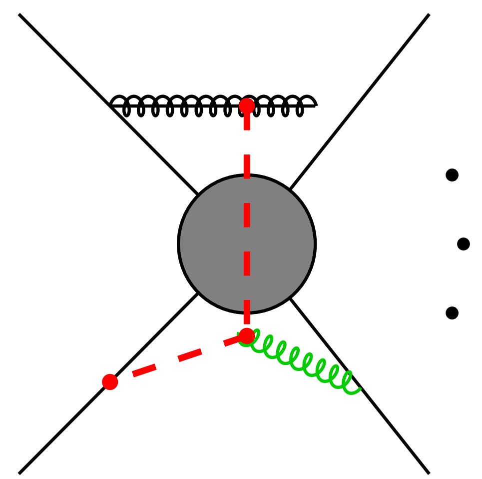
One needs at least three colored collinear sectors

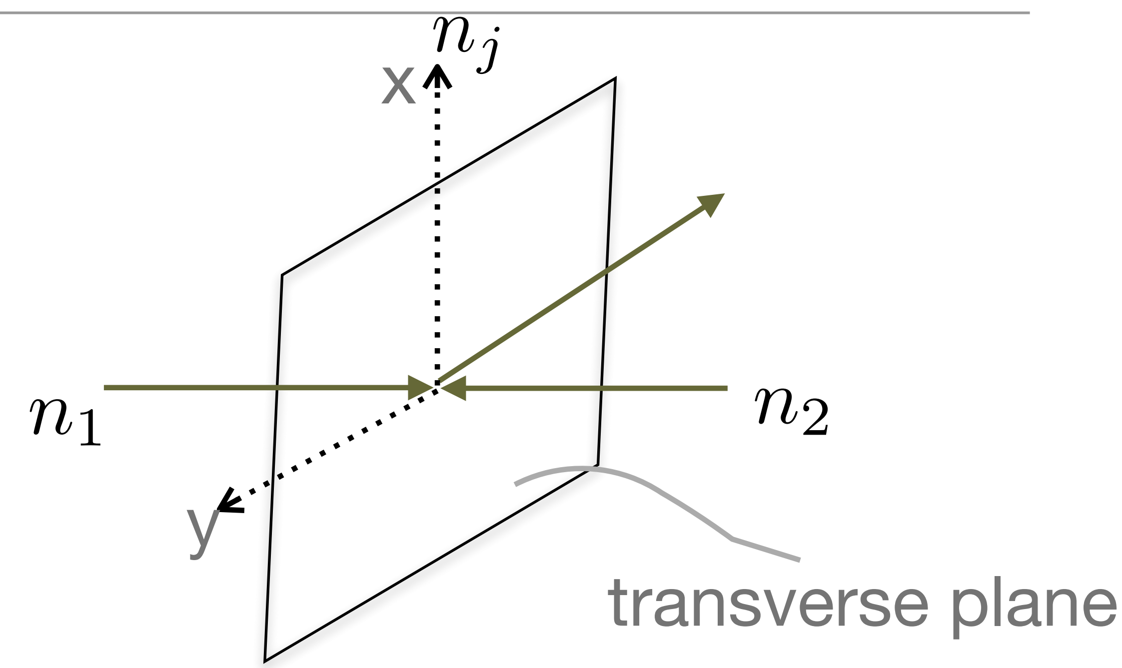
(2 incoming, 1 outgoing).

Difficulty with multi-leg at high-loop order

- Multiple colored collinear sectors







$$\propto \int đ^{d-2}\ell_\perp đ^{d-2}k_\perp \frac{(p_{5,\perp}+\ell_\perp)^\mu}{|p_{5,\perp}+\ell_\perp|^2}\frac{1}{|\ell_\perp|^2}\frac{1}{|\ell_\perp-k_\perp|^2}\frac{2\ell^x-k^x}{i|k^y|}\log\frac{k^x+i|k^y|}{k^x-i|k^y|}$$

After light-cone decomposition and power expansion, the form of SCET glauber potential loses Lorentz covariance.

- Rapidity regulator that preserves Lorentz covariance?

Outlook

- determine whether glauber matters for an observable
- violation of pdf factorization?
- factorization breaking v.s. sensitivity to MPI
- systematic resummation of log enhancement due to factorization-breaking effects

谢 谢！