

Mellin-space resummation for (boosted) top-quark pair production

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[1601.07020 & 1612.xxxxx](#)

17 Dec 2016 ‖ CLHCP 2016@PKU



Outline

I. Motivation

- Experimental
- Set-up and theoretical

2. Framework

- Two-step factorization
- Mellin-space vs Momentum-space
- RG equations
- Matching

3. Results

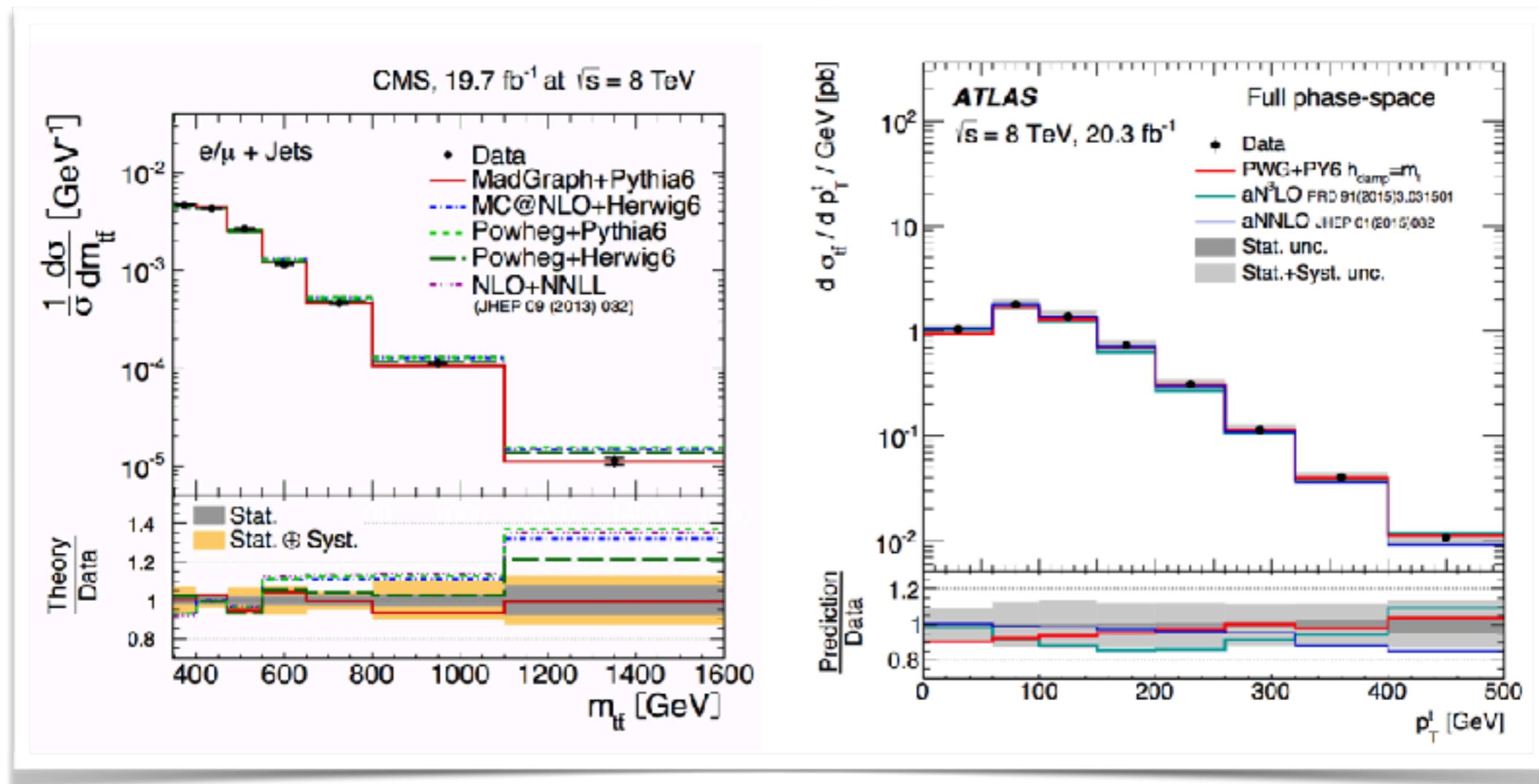
4. Conclusions and outlook

Motivation

Experimental

- LHC 8 TeV results beginning to probe the “boosted” regime

1505.04480, 1511.04716

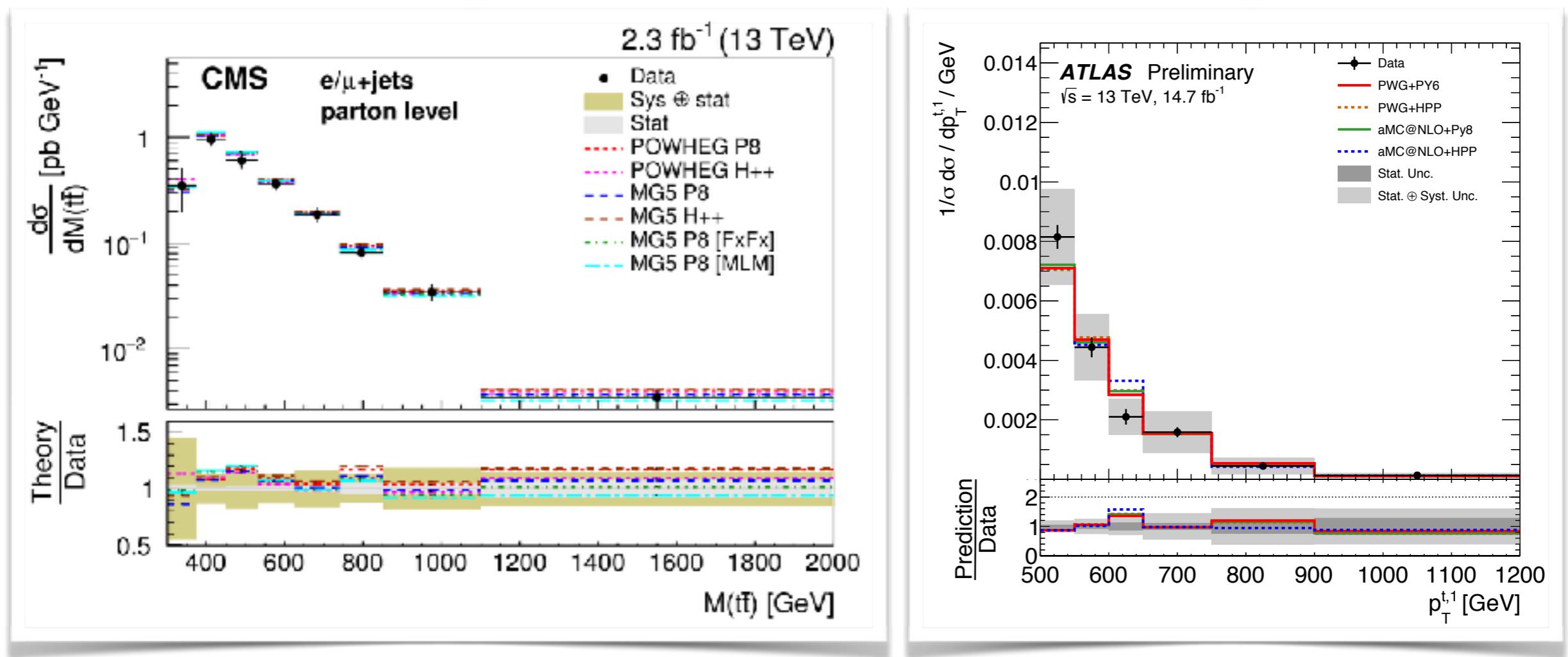


Motivation

Experimental

- Some 13 TeV results

CMS-TOP-16-008, ATLAS-CONF-2016-100

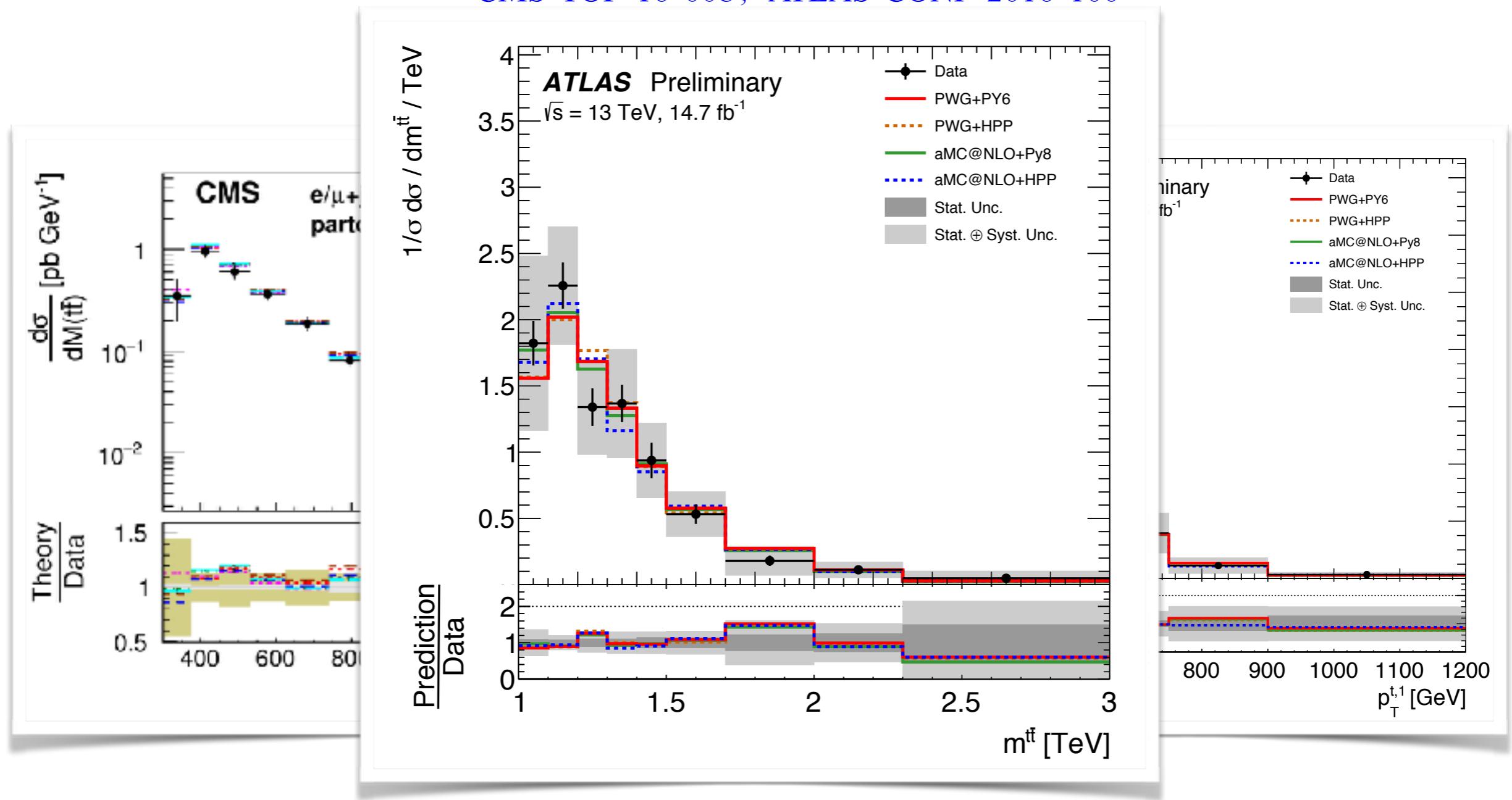


Motivation

Experimental

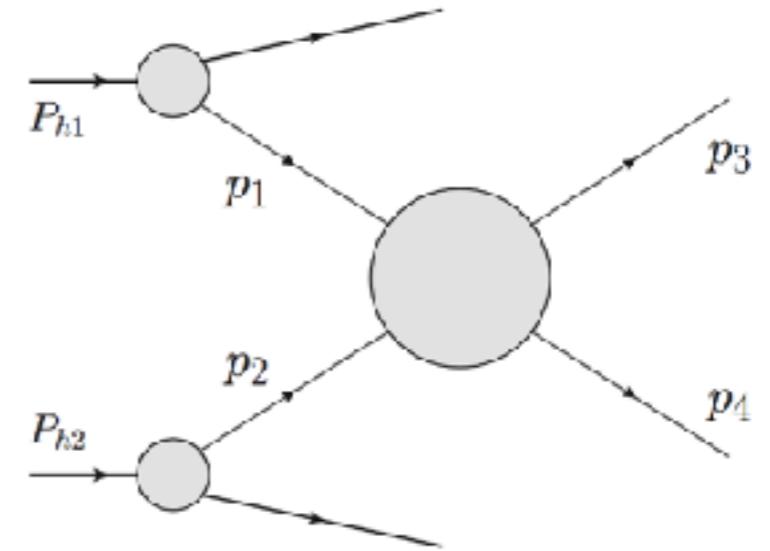
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A typical process @LHC
with partonic process:

$$i(p_1) + j(p_2) \rightarrow t(p_3) + \bar{t}(p_4) + X(p_X)$$



$$\frac{d^2\sigma(\tau)}{dM d\cos\theta} = \frac{8\pi\beta_t}{3sM} \sum_{ij} \int_\tau^1 \frac{dz}{z} \mathcal{L}_{ij}(\tau/z, \mu_f) C_{ij}(z, M, m_t, \cos\theta, \mu_f),$$

$$z \rightarrow 1$$

is partonic threshold limit(soft)

$$s = (P_1 + P_2)^2, \hat{s} = (p_1 + p_2)^2$$

$$M_{t\bar{t}}^2 = (p_3 + p_4)^2,$$

$$z = M_{t\bar{t}}^2/\hat{s}, \tau = M_{t\bar{t}}^2/s$$

Top quark is important both in SM & BSM...

Motivation

Theoretical

- Soft limit: $\hat{s}, t_1, m_t^2 \gg \hat{s}(1 - z)^2$
- Boosted soft limit: $\hat{s}, t_1 \gg m_t^2 \gg \hat{s}(1 - z)^2 \gg m_t^2(1 - z)^2$

Soft logs: $\left[\frac{\log^n(1 - z)}{1 - z} \right]_+$ Small-mass logs: $\log^n \frac{m_t}{M_{t\bar{t}}}$

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- Also a further application of SCET on top Physics

- Soft limit: $\hat{s}, t_1, m_t^2 \gg \hat{s}(1 - z)^2$
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0501229, 0601180, 1010.4509, 1208.5774, 1210.7698, 1307.2464, 1409.6959, 1409.1460

Framework

Two-step factorization

- Both soft limit and small-mass limit 1205.3662
- The two limit are commutative(one by one and cross-check)

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I. small-mass limit factorization:

$$z = E/E_{max}$$

$$\frac{d\sigma_t}{dz}(z, m_t, \mu) = \sum_a \int_z^1 \frac{dx}{x} \frac{d\tilde{\sigma}_a}{dx}(x, m_t, \mu) D_{a/t}^{(n_l+n_h)} \left(\frac{z}{x}, m_t, \mu \right)$$

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2. matching the FF to FF with only light flavor like PDF

$$D_{a/t}^{(n_l+n_h)}(z, m_t, \mu) = C_{a/t}(z, m_t, \mu) \otimes D_{t/t}^{(n_l)}(z, m_t, \mu)$$

3. small-mass limit factorization

1205.3662

$$\begin{aligned}
 C_{ij}(z, M, m_t, \cos\theta, \mu_f) &= \sum_{a,b} C_{ij}^{ab}(z, M, t_1, \mu_f) \otimes \mathbf{D}_{ab}^{(n_l)}(z, m_t, \mu_f) \\
 &\quad \otimes C_{a/t}(z, m_t, \mu_f) \otimes C_{b/\bar{t}}(z, m_t, \mu_f) \\
 &\quad \otimes C_{ff}(z, m_t, \mu_f) + \mathcal{O}\left(\frac{m_t}{M}\right)
 \end{aligned}$$

Framework

Two-step factorization(cont.)

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1205.3662

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4. soft limit factorization

$$C_{ij}^{t\bar{t}}(z, M, t_1, \mu_f) = Tr \left[\mathbf{H}_{ij}(M, t_1, \mu_f) \mathbf{S}_{ij}(\sqrt{\hat{s}}(1-z), t_1, \mu_f) \right] + \mathcal{O}(1-z)$$

$$D_{t/t}^{(n_l)}(z, m_t, \mu_f) = C_D(m_t, \mu_f) S_D(m_t(1-z), \mu_f) + \mathcal{O}(1-z)$$

The same for anti-top

Framework

Two-step factorization(cont.)

3. small-mass limit factorization

1205.3662

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The same for anti-top

collinear scale soft-collinear scale

5. final collected factorization formula
in Mellin-space:

1205.3662
1601.07020

$$\begin{aligned} \tilde{c}_{ij}(N, M, m_t, \cos\theta, \mu_f) &= Tr \left[\mathbf{H}_{ij}(M, \cos\theta, \mu_f) \tilde{\mathbf{s}}_{ij} \left(\ln \frac{M^2}{\bar{N}^2 \mu_f^2}, M, \cos\theta, \mu_f \right) \right] \\ &\times C_D^2(m_t, \mu_f) \tilde{s}_D^2 \left(\ln \frac{m_t}{\bar{N} \mu_f}, \mu_f \right) + \mathcal{O}\left(\frac{1}{N}\right) + \mathcal{O}\left(\frac{m_t}{M}\right) \end{aligned}$$

The differences are power suppressed:

$$\left[\frac{\log^m(1-z)}{1-z} \right]_+ \text{v.s.} \left[\frac{\log^n(-\log z)}{-\log z} \right]_+$$

Momentum-Space(z)

- Avoid Landau pole issues
- Soft scale chosen numerically

$$\left[\frac{\log^m \frac{M(1-z)}{\mu_s}}{1-z} \right]_+$$

Mellin-Space(N)

- Analytically get soft scale
- Easy to organize products
- Deal with Landau pole issues
- Numerically invert the transform

$$\begin{aligned}\mu_s &\sim M/\bar{N} \\ \mu_{ds} &\sim m_t/\bar{N}\end{aligned}$$

- Derive the anomalous dimension of different pieces
- Use RGEs to resum large logs

$$\begin{aligned} \tilde{c}_{ij}(N, \mu_f) &= Tr \left[\tilde{\mathbf{U}}_{ij}(\bar{N}, \mu_f, \mu_h, \mu_s) \mathbf{H}_{ij}(\mu_h) \tilde{\mathbf{U}}_{ij}^\dagger(\bar{N}, \mu_f, \mu_h, \mu_s) \tilde{\mathbf{s}}_{ij} \left(\ln \frac{M^2}{\bar{N}^2 \mu_s^2}, \mu_s \right) \right] \\ &\times U_D^2(\bar{N}, \mu_f, \mu_{dh}, \mu_{ds}) C_D^2(m_t, \mu_{dh}) \tilde{s}_D^2 \left(\ln \frac{m_t}{\bar{N} \mu_{ds}}, \mu_{ds} \right) + \mathcal{O}\left(\frac{1}{N}\right) + \mathcal{O}\left(\frac{m_t}{M}\right) \end{aligned}$$

- I. matching with soft-gluon resummation
2. matching with fixed order

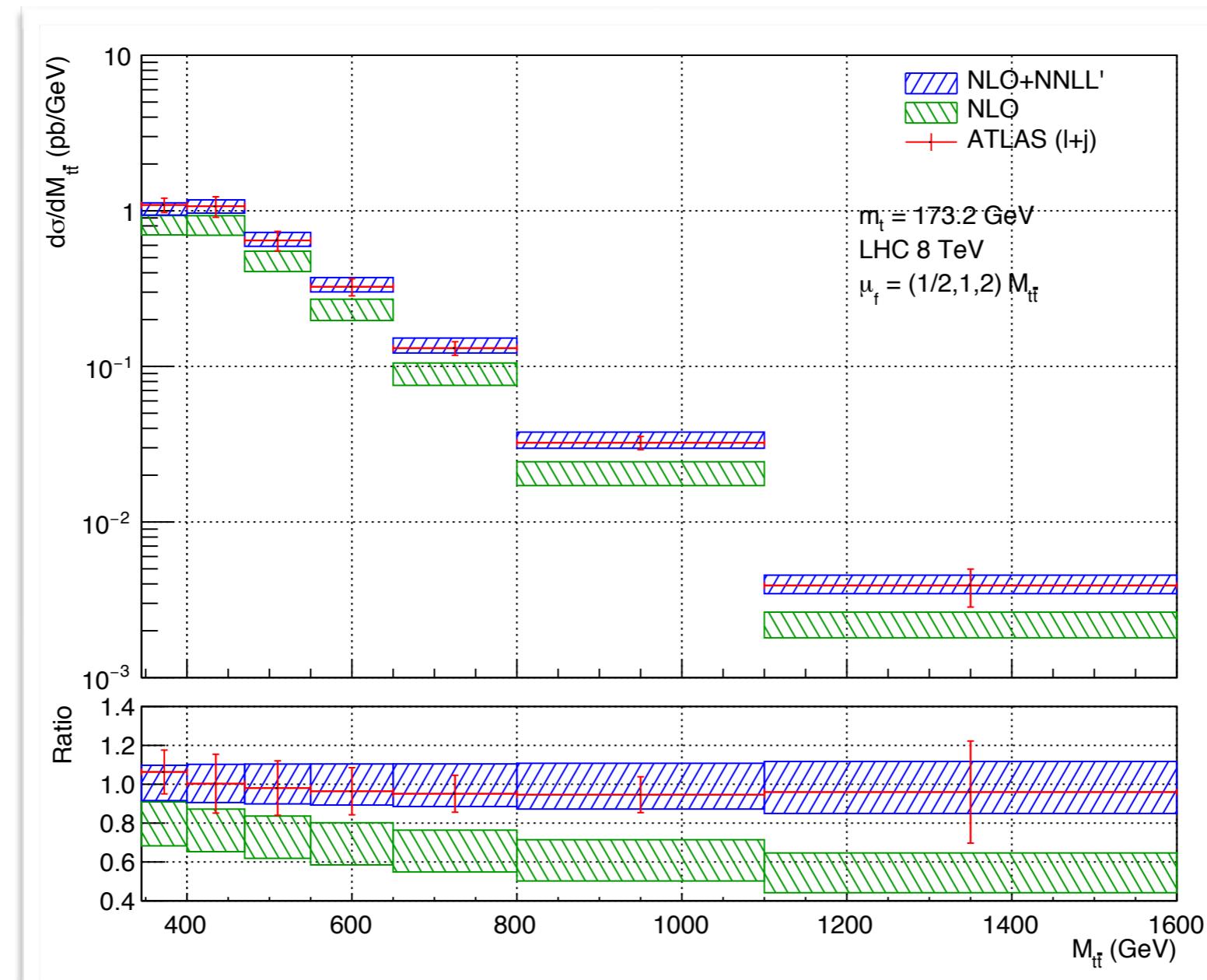
	Γ_{cusp}^i	γ_i	$\mathbf{H}, \tilde{\mathbf{s}}, c_D, \tilde{s}_D$
NLL	NLO	LO	LO
NNLL	NNLO	NLO	NLO
NNLL'	NNLO	NLO	NNLO

$$d\sigma^{\text{NLO+NNLL}'} = d\sigma^{\text{NNLL}'^b} + \left(d\sigma^{\text{NNLL}^m} - d\sigma^{\text{NNLL}^b} \Big|_{\substack{\mu_{ds}=\mu_s \\ \mu_{dh}=\mu_h}} \right) \\ + \left(d\sigma^{\text{NLO}} - d\sigma^{\text{NNLL}^m} \Big|_{\substack{\mu_s=\mu_f \\ \mu_h=\mu_f}} \right)$$

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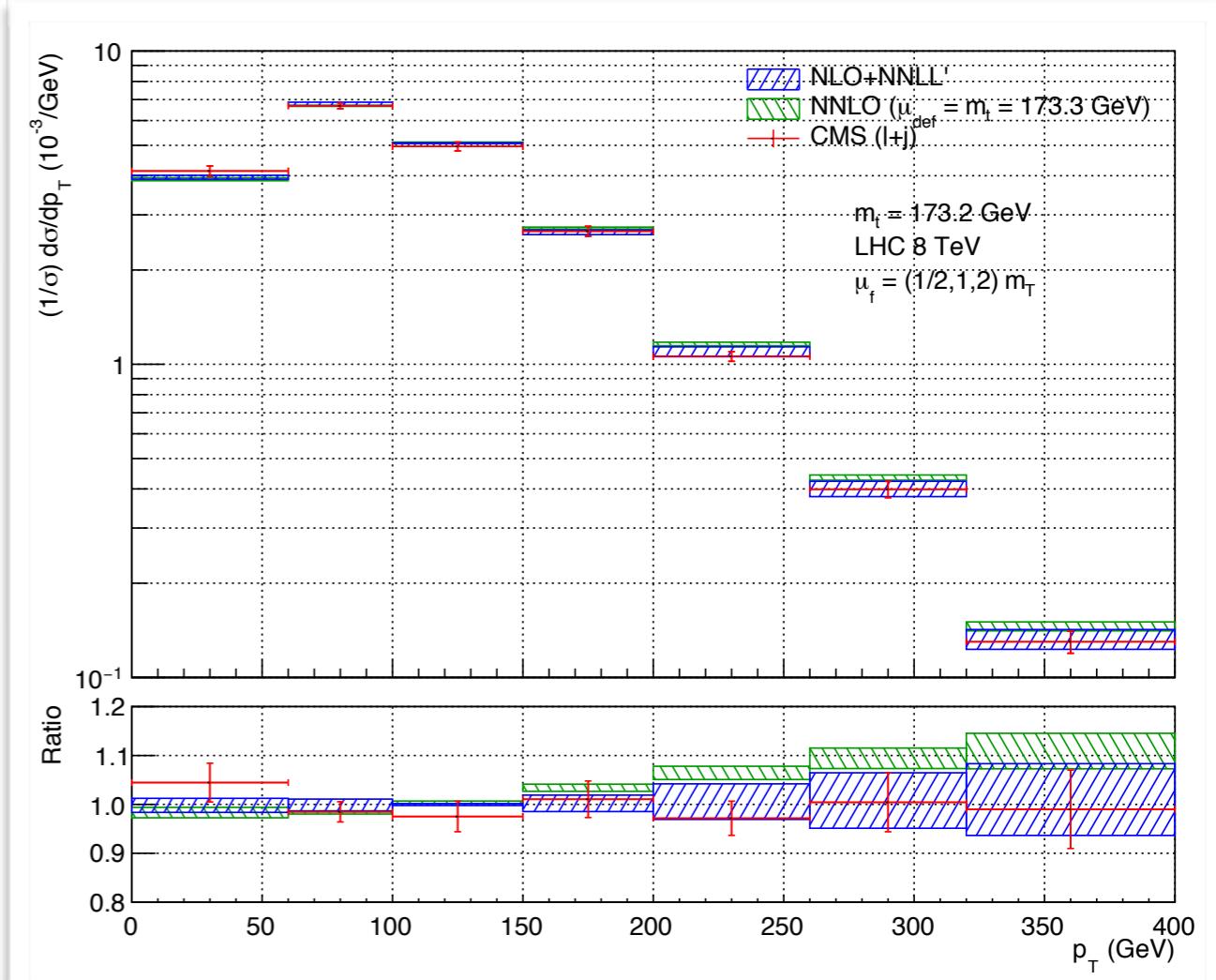
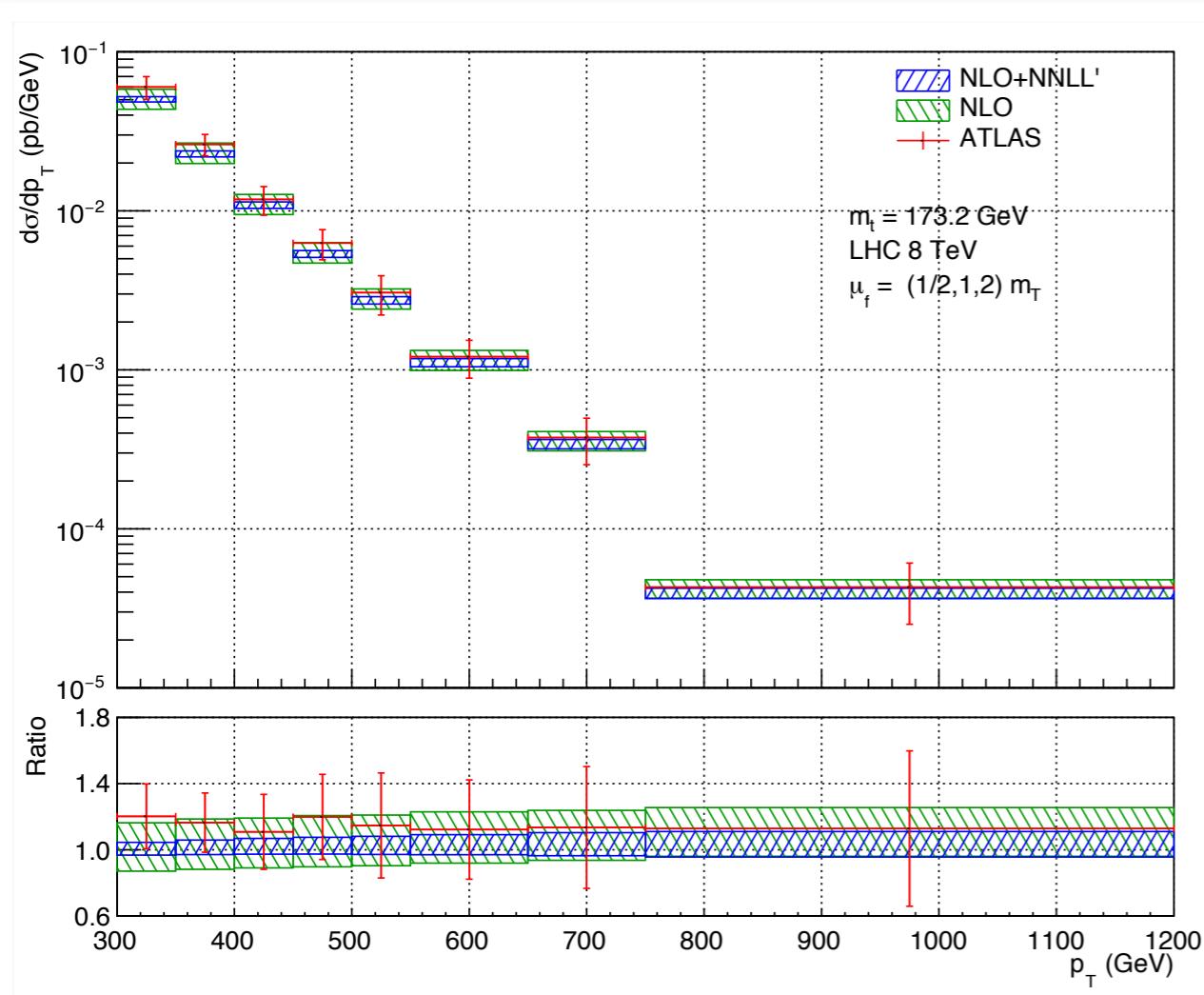
Results

M_tt distribution



Results

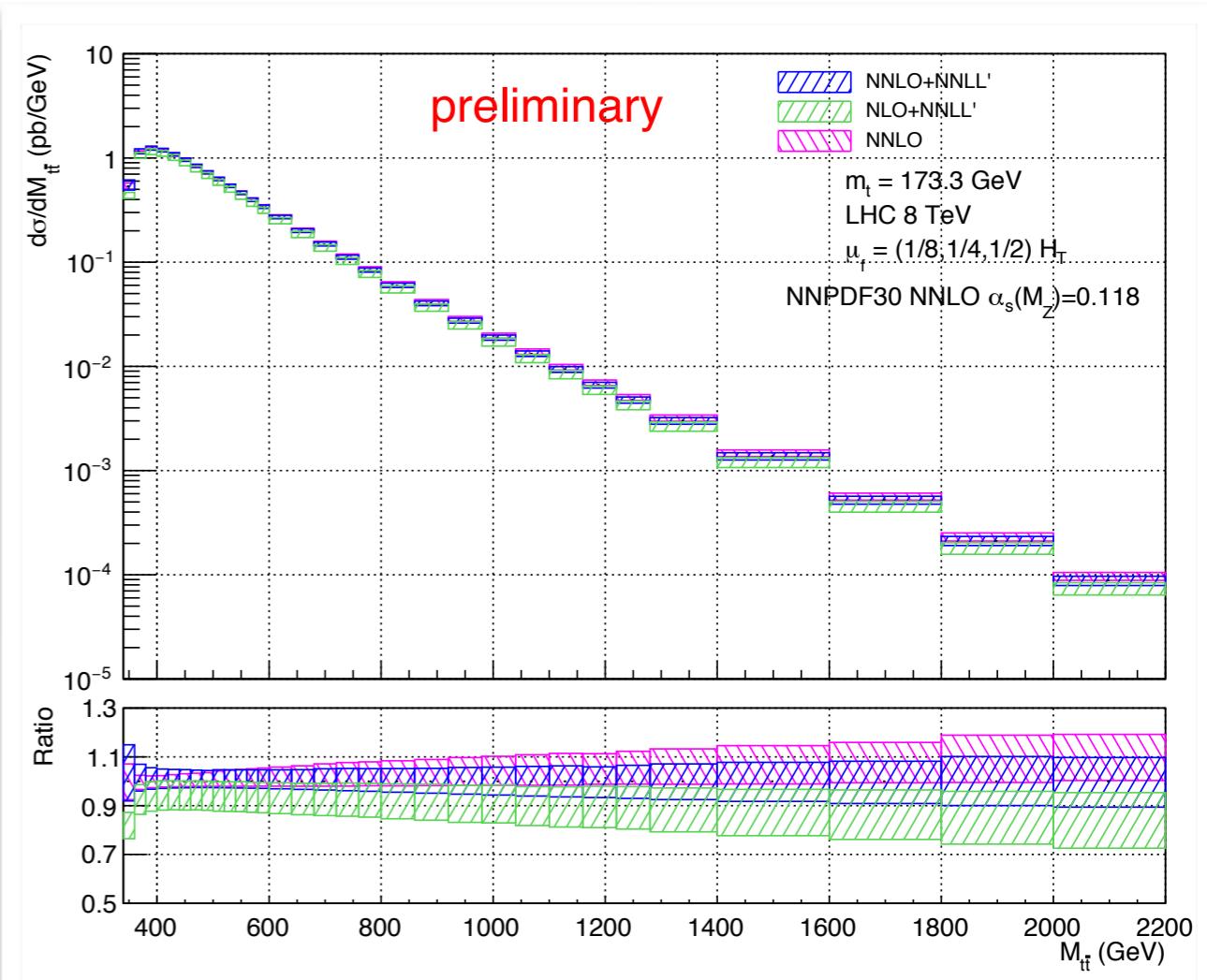
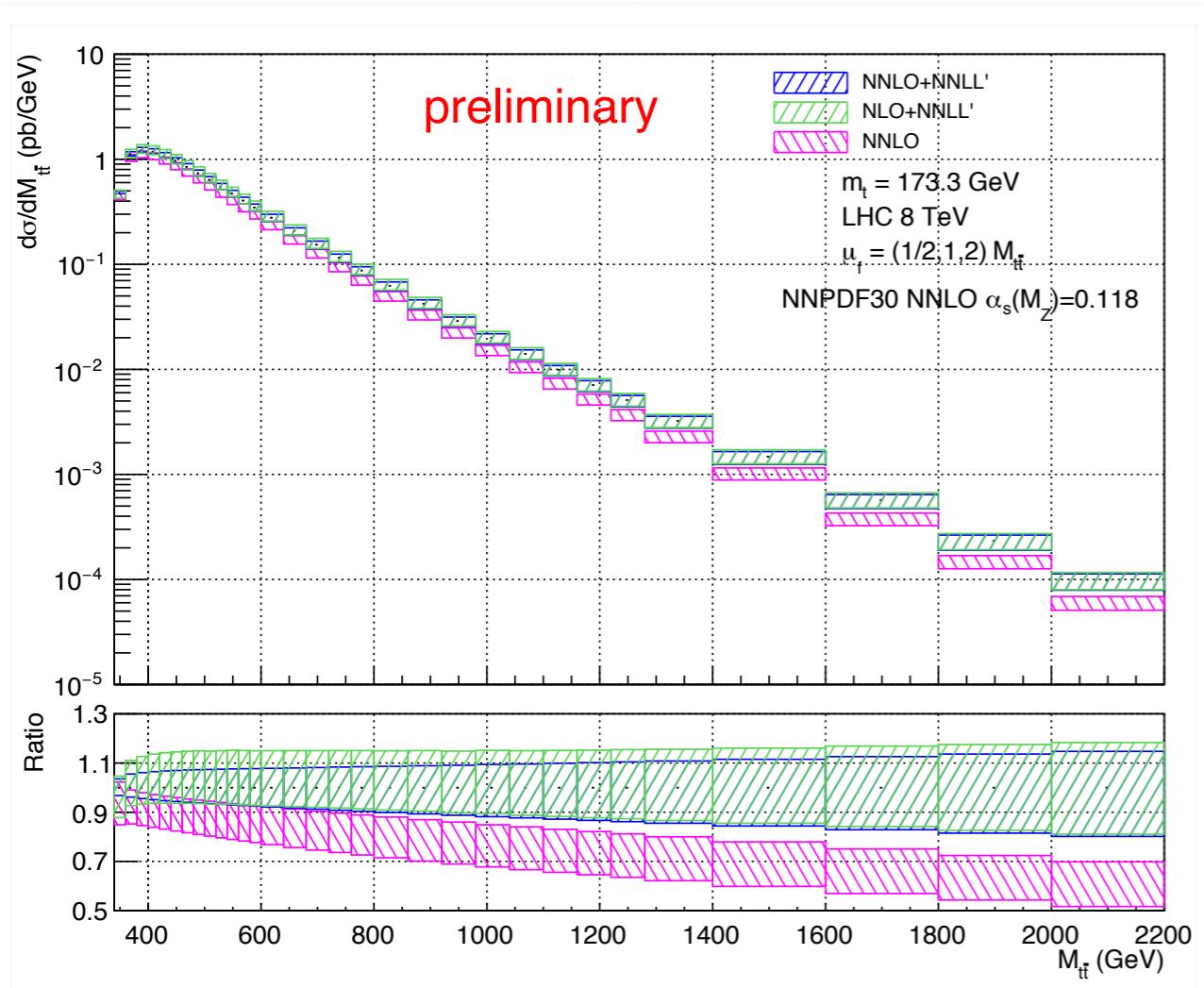
pT(top) distribution



Results

Preliminary

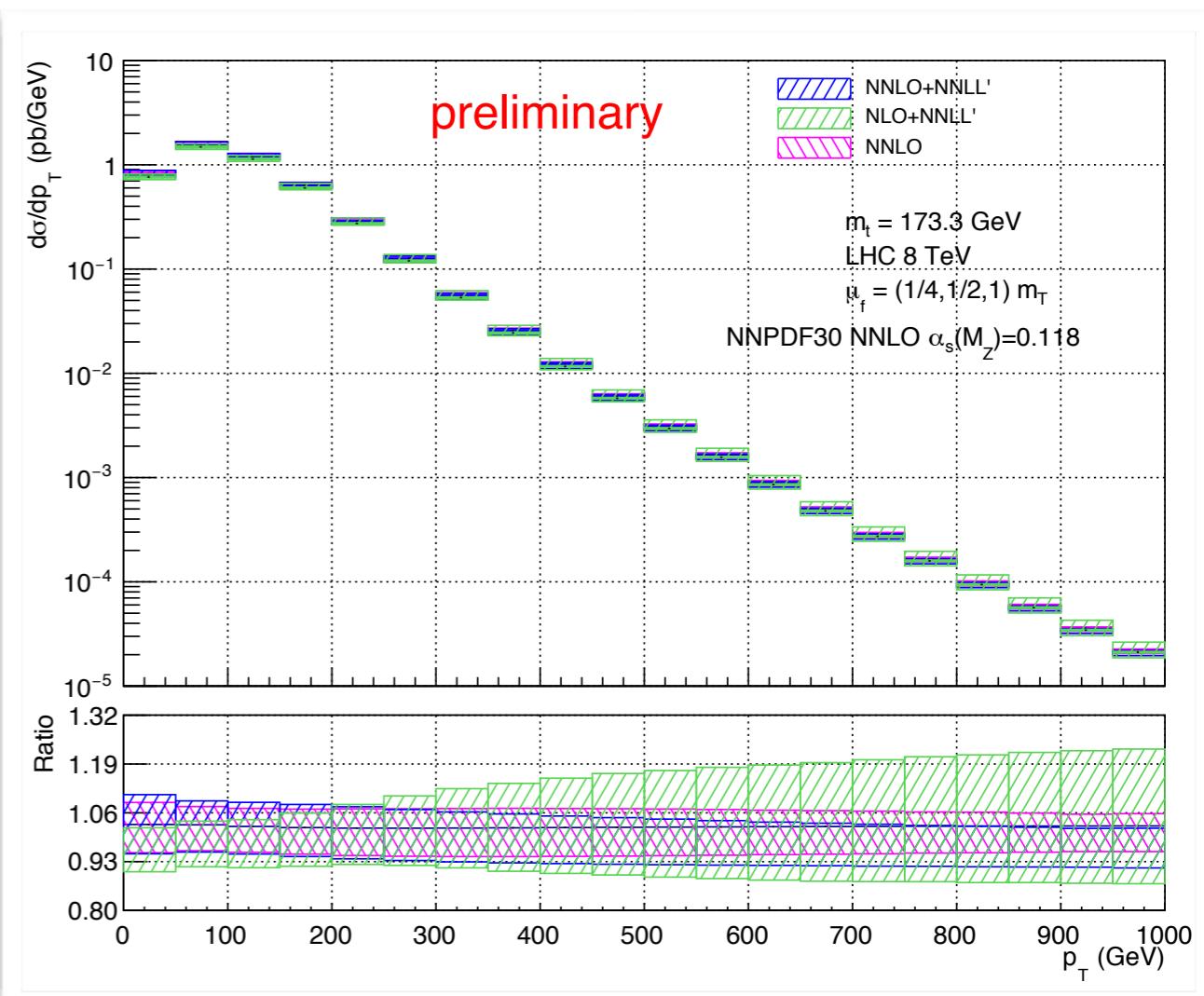
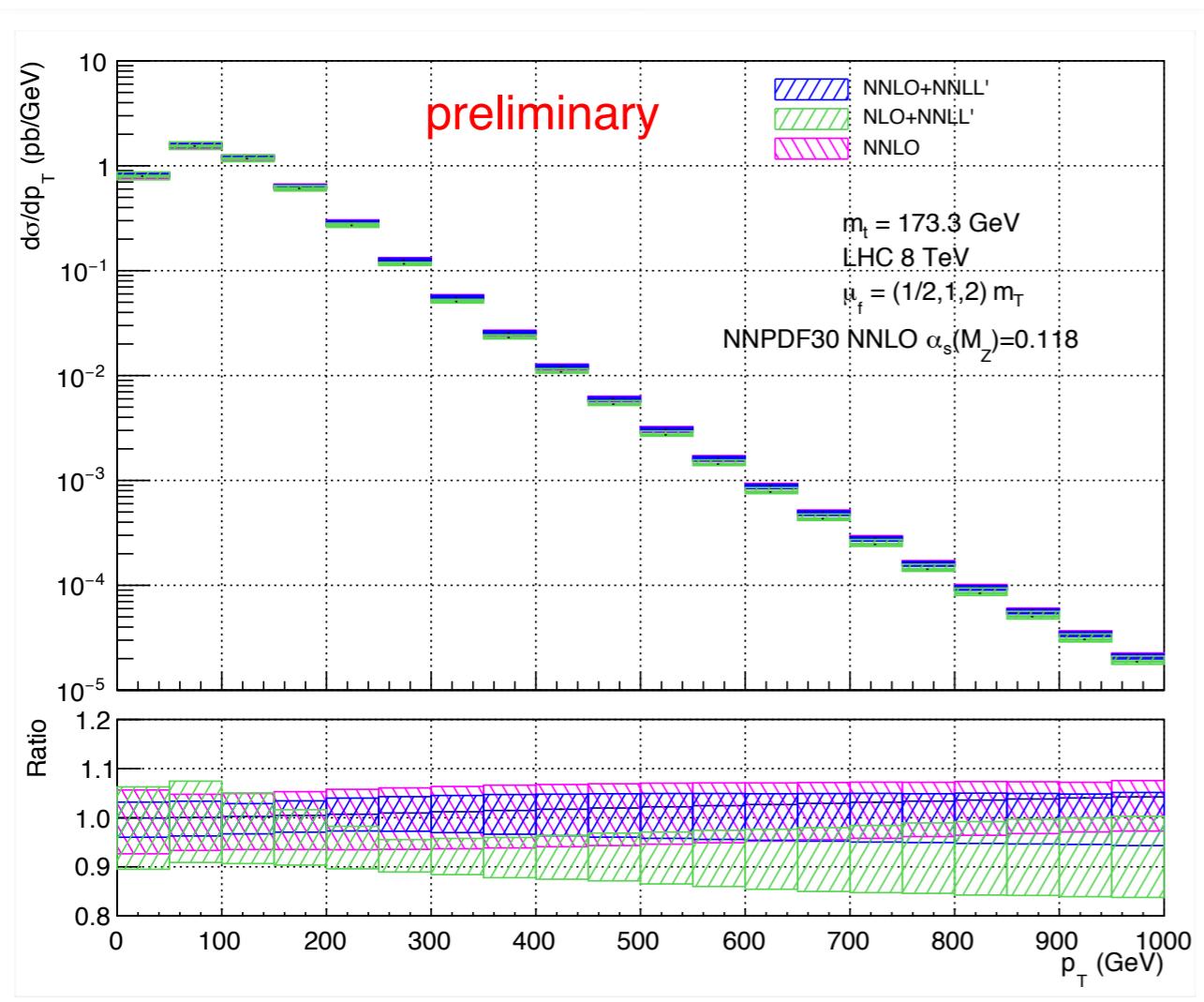
1606.03350



Results

Preliminary

1606.03350



Conclusions and future work

- Apply SCET to resum two type large logs in boosted-top pair production
- The resummation effect in boosted regime is significant
- Finish matching with NNLO results and $\text{@} 13 \text{ TeV?}$
- Rapidity distribution
- Apply in other observables, bbar system?

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Thanks!

Backup

$$\begin{aligned}
\widetilde{\mathbf{U}}(\mu_f, \mu_h, \mu_s) &= \exp \left\{ 2S_A(\mu_h, \mu_s) - a_A(\mu_h, \mu_s) \ln \frac{M^2}{\mu_h^2} + a_A(\mu_f, \mu_s) \ln \bar{N}^2 \right. \\
&\quad \left. + 2a_{\gamma^\phi}(\mu_s, \mu_f) + 2a_{\gamma^{\phi_q}}(\mu_s, \mu_f) \right\} \mathbf{u}(M, \cos \theta, \mu_h, \mu_s), \\
\widetilde{U}_D(\mu_f, \mu_{dh}, \mu_{ds}) &= \exp \left\{ -2S_{\Gamma^q}(\mu_{dh}, \mu_{ds}) + a_{\Gamma^q}(\mu_{dh}, \mu_{ds}) \ln \frac{m_t^2}{\mu_{dh}^2} - a_{\Gamma^q}(\mu_f, \mu_{ds}) \ln \bar{N}^2 \right. \\
&\quad \left. - 2a_{\gamma^S}(\mu_{dh}, \mu_{ds}) - 2a_{\gamma^{\phi_q}}(\mu_{dh}, \mu_f) \right\} \\
\mathbf{u}(M, \cos \theta, \mu_h, \mu_s) &= \mathcal{P} \exp \int_{\alpha_s(\mu_h)}^{\alpha_s(\mu_s)} \frac{d\alpha}{\beta(\alpha)} \gamma^h(M, \cos \theta, \alpha)
\end{aligned}$$

Backup

$$\begin{aligned}\tilde{\mathbf{U}}(\mu_f, \mu_h, \mu_s) &= \exp \left\{ \frac{4\pi}{\alpha_s(\mu_h)} g_1(\lambda, \lambda_f) + g_2(\lambda, \lambda_f) + \frac{\alpha_s(\mu_h)}{4\pi} g_3(\lambda, \lambda_f) \right\} \\ &\times \mathbf{u}^m(M, \cos \theta, \mu_h, \mu_s)\end{aligned}$$

$$\begin{aligned}U_D(\mu_f, \mu_{dh}, \mu_{ds}) &= \exp \left\{ \frac{4\pi}{\alpha_s(\mu_h)} g_1^D(\lambda_{dh}, \lambda_{ds}, \lambda_f) + g_2^D(\lambda_{dh}, \lambda_{ds}, \lambda_f) \right. \\ &\quad \left. + \frac{\alpha_s(\mu_h)}{4\pi} g_3^D(\lambda_{dh}, \lambda_{ds}, \lambda_f) \right\}\end{aligned}$$

	Γ_{cusp}^i	γ_i	$\mathbf{H}, \tilde{\mathbf{s}}, c_D, \tilde{s}_D$
NLL	NLO	LO	LO
NNLL	NNLO	NLO	NLO
NNLL'	NNLO	NLO	NNLO

$$\begin{aligned}\lambda &= \frac{\alpha_s(\mu_h)}{2\pi} \beta_0 \ln \frac{\mu_h}{\mu_s}, \quad \lambda_f = \frac{\alpha_s(\mu_h)}{2\pi} \beta_0 \ln \frac{\mu_h}{\mu_f} \\ \lambda_{dh} &= \frac{\alpha_s(\mu_h)}{2\pi} \beta_0 \ln \frac{\mu_h}{\mu_{dh}}, \quad \lambda_{ds} = \frac{\alpha_s(\mu_h)}{2\pi} \beta_0 \ln \frac{\mu_h}{\mu_{ds}}\end{aligned}$$