

Next-to-Next-to-Leading Order N -Jettiness Soft Function for One Massive Coloured Particle Production at Hadron Colliders

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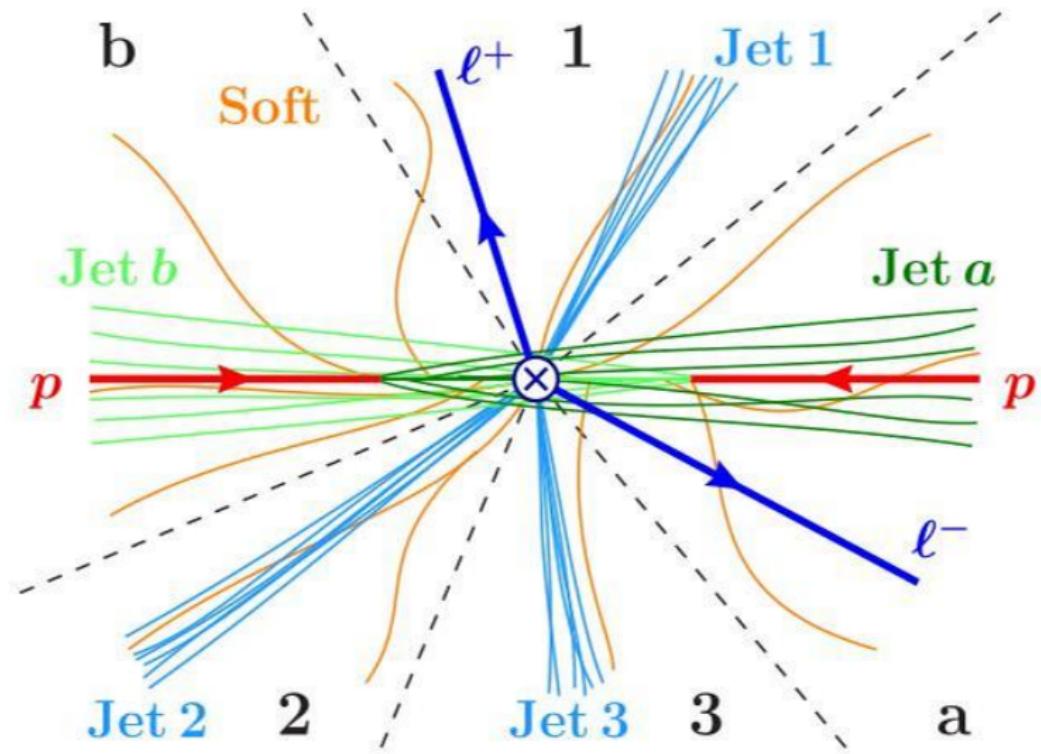
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Happy 70th Birthday to My PhD Supervisor



Processes at the LHC



N-jettiness

The N -jettiness event shape variable is defined as [Iain Stewart et al, 2010]

$$\mathcal{T}_N = \sum_k \min_i \{n_i \cdot q_k\}$$

Here n_i ($i = a, b, 1, \dots, N$) are light-like reference vectors representing the moving directions of massless external particles. When q_k is soft or collinear with any external partons $\mathcal{T}_N \rightarrow 0$.

In this limit, the cross section is factorized as [Iain Stewart et al, 2009, 2010]

$$\frac{d\sigma}{d\mathcal{T}_N} \propto \int H \otimes B_1 \otimes B_2 \otimes S \otimes \left(\prod_{n=1}^N J_n \right)$$

The beam functions are known up to NNLO [Gaunt et al, 2014]. The jet function has been calculated at NNLO [Becher et al, 2006, 2010]. The soft function has been studied up to NNLO for massless parton production [Boughezal et al 2015].

N-jettiness with massive particle

Application of factorised cross section using N-jettiness variables

- Resummed distributions [[Stewart et al, 2009-2010; Kang et al, 2012-2015; Berger, 2010; Jouttenus, 2013; Alioli, 2015, ...](#)].

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What happens for massive coloured particle production such as top quark?

- Because there are no collinear singularities along the moving direction of the heavy particle, the definition of \mathcal{T}_N can control all the infrared (both soft and collinear) singularities.
- And the effect of including heavy external particle amounts to modifying only the soft function.

Kinematics

We consider the process

$$P_1 + P_2 \rightarrow Q + X$$

where P_1 and P_2 denote incoming hadrons, Q represents the massive colored particle, and X includes any inclusive hadronic final state.

For later convenience we introduce two light-like vectors

$$n^\mu = (1, 0, 0, 1), \quad \bar{n}^\mu = (1, 0, 0, -1)$$

The momenta can be written as

$$p_1^\mu = \frac{m}{2} n^\mu, \quad p_2^\mu = \frac{m}{2} \bar{n}^\mu, \quad p_3^\mu = \frac{m}{2} (n^\mu + \bar{n}^\mu),$$

where m is the mass of particle Q . The 0-jettiness event shape variable in this process is defined as

$$\tau \equiv \mathcal{T}_0 = \sum_k \min\{n \cdot q_k, \bar{n} \cdot q_k\}$$

Soft function

The soft function is defined by the vacuum matrix element

$$S(\tau, \mu) = \sum_{X_s} \left\langle 0 \left| \bar{\mathbf{T}} Y_n^\dagger Y_{\bar{n}} Y_v \right| X_s \right\rangle \left\langle X_s \left| \mathbf{T} Y_n Y_{\bar{n}}^\dagger Y_v^\dagger \right| 0 \right\rangle$$
$$\underbrace{\delta \left(\tau - \sum_k \min \left(n \cdot \hat{P}_k, \bar{n} \cdot \hat{P}_k \right) \right)}_{\text{measurement function}}$$

where the soft Wilson lines are defined as

$$Y_n(x) = \mathbf{P} \exp \left(ig_s \int_{-\infty}^0 ds n \cdot A_s^a(x + sn) t^a \right)$$

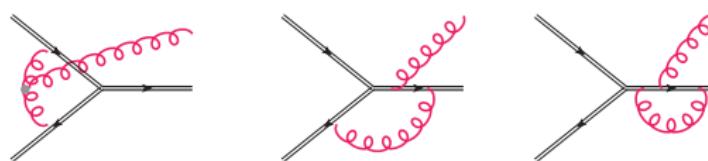
$$Y_{\bar{n}}^\dagger(x) = \bar{\mathbf{P}} \exp \left(-ig_s \int_{-\infty}^0 ds n \cdot A_s^a(x + sn) t^a \right)$$

$$Y_v^\dagger(x) = \mathbf{P} \exp \left(ig_s \int_{\infty}^0 ds n \cdot A_s^a(x + sn) t^a \right)$$

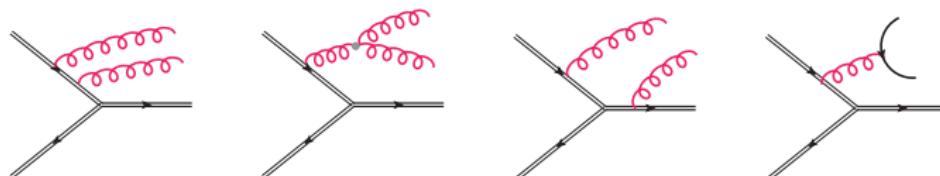
NNLO soft function

The LO and NLO soft function is easy to calculate. The NNLO contribution consists of two parts

$$S^{(2)}(\tau) = S_{\text{VR}}^{(2)}(\tau) + S_{\text{DR}}^{(2)}(\tau)$$



Virtual real corrections



Double real corrections

NNLO soft function-virtual real

The virtual-real contribution to the soft function is given by

$$S_{\text{VR}}^{(2)}(\tau) = \frac{e^{2\gamma_E \epsilon} \mu^{4\epsilon}}{\pi^{2-\epsilon}} 2\text{Re} \left[\int d^D q \delta(q^2) J_a^{\mu(0)\dagger} d_{\mu\nu}(q) J_a^{\nu(1)}(q) F(n, \bar{n}, q) \right] ,$$

where the unrenormalized one-loop soft current can be written as
[Bierenbaum et al,2011]

$$J_a^{\mu(1)}(q) = if_{abc} \sum_{i \neq j=1}^3 \mathbf{T}_i^b \mathbf{T}_j^c \left(\frac{p_i^\mu}{p_i \cdot q} - \frac{p_j^\mu}{p_j \cdot q} \right) g_{ij}(\epsilon, q, p_i, p_j) .$$

The factor $F(n, \bar{n}, q)$ is a measurement function, embodying the constraint from the 0-jettiness variable. In the center-of-mass frame, it is defined as

$$F(n, \bar{n}, q) = \delta(q^+ - \tau) \Theta(q^- - q^+) + \delta(q^- - \tau) \Theta(q^+ - q^-) ,$$

where we use the notations, $q^+ = q \cdot n$ and $q^- = q \cdot \bar{n}$.

The result of virtual real contribution is

$$\begin{aligned}s_{\text{VR}}^{(2)} = & -\frac{8C_A C_F}{\epsilon^3} + \frac{8C_A^2}{\epsilon^2} + \frac{4C_A}{3\epsilon} \left((\pi^2 - 6 - 24 \ln 2) C_A + 3\pi^2 C_F \right) \\& + \frac{4C_A}{3} \left(C_A (\pi^2 - 33\zeta_3 + 12(\ln^2 2 + 2 \ln 2)) + 16\zeta_3 C_F \right) \\& - \epsilon \frac{C_A}{15} \left[2C_A \left(\pi^4 + 30\pi^2 (3 - 4 \ln 2 + \ln^2 2) \right. \right. \\& \left. \left. - 10(24 + 3 \ln^4 2 + 72 \text{Li}_4(1/2) - 124\zeta_3 + 63\zeta_3 \ln 2) \right) + \pi^4 C_F \right] \\& + \mathcal{O}(\epsilon^2)\end{aligned}$$

NNLO soft function-double real

The full integrand for double-real emission can be found in literature [Catani et al, 2001; Czakon, 2001] . The contribution from double gluon radiation is

$$S_{\text{gg}}^{(2)}(\tau) = \frac{2e^{2\gamma_E\epsilon}}{\pi^{2-2\epsilon}} \int d^d q_1 d^d q_2 \delta(q_1^2)\delta(q_2^2) \\ \times J_{a_1 a_2}^{\mu_1 \nu_1 (0)\dagger}(q_1, q_2) d_{\mu_1 \mu_2}(q_1) d_{\nu_1 \nu_2}(q_2) J_{a_1 a_2}^{\mu_2 \nu_2 (0)}(q_1, q_2) F(n, \bar{n}, q_1, q_2)$$

where $F(n, \bar{n}, q_1, q_2)$ is the measurement function

$$F(n, \bar{n}, q_1, q_2) = \delta(q_1^+ + q_2^+ - \tau) \Theta(q_1^- - q_1^+) \Theta(q_2^- - q_2^+) \\ + \delta(q_1^+ + q_2^- - \tau) \Theta(q_1^- - q_1^+) \Theta(q_2^+ - q_2^-) \\ + \delta(q_1^- + q_2^+ - \tau) \Theta(q_1^+ - q_1^-) \Theta(q_2^- - q_2^+) \\ + \delta(q_1^- + q_2^- - \tau) \Theta(q_1^+ - q_1^-) \Theta(q_2^+ - q_2^-)$$

The whole phase space is partitioned into four pieces.

It is convenient to perform the phase space integration in the light-cone coordinates.

$$\int d^d q = \frac{1}{2} \int d^{d-2} q_T dq^+ dq^- ,$$

Then we insert two identities

$$1 = \int d\tau_1 \delta(\tau_1 - q_1^\pm), \quad 1 = \int d\tau_2 \delta(\tau_2 - q_2^\pm)$$

to extract the contributions from the two hemispheres. Finally, the integrals we need to calculate boil down to four-fold integrals over a unit hypercube. For example,

$$I = \int_0^1 dx \int_0^1 dy \int_0^1 dz \int_0^1 dt \frac{x^{1+2\epsilon} y^{-1+2\epsilon} (1-z)^{-2\epsilon} z^{-1-2\epsilon} (1-t)^{-\frac{1}{2}-\epsilon} t^{-\frac{1}{2}-\epsilon}}{(x^2 + z - zx^2)(1 - 2xy + x^2y^2 + 4txy)} .$$

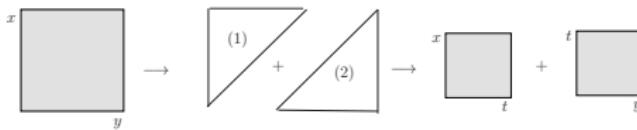
We have adopted two different methods to deal with phase space integration so that they can provide a cross-check.

- Mellin-Barnes representation

$$\frac{1}{(X+Y)^\lambda} = \frac{1}{\Gamma(\lambda)} \frac{1}{2\pi i} \int_{-i\infty}^{i\infty} dz \frac{Y^z}{X^{\lambda+z}} \Gamma(\lambda+z) \Gamma(-z)$$

- Sector decomposition method: the basic idea is to factorise the overlapping singularities

$$\int_0^1 dx \int_0^1 dy x^{-1-a\epsilon} y^{-b\epsilon} \left(x + (1-x)y \right)^{-1}$$



$$\begin{aligned} & \int_0^1 dx x^{-1-(a+b)\epsilon} \int_0^1 dt t^{-b\epsilon} \left(1 + (1-x)t \right)^{-1} \\ & + \int_0^1 dy y^{-1-(a+b)\epsilon} \int_0^1 dt t^{-1-a\epsilon} \left(1 + (1-y)t \right)^{-1} \end{aligned}$$

The final result of the double-real contribution is

$$\begin{aligned}s_{\text{DR}}^{(2)} = & \frac{8C_A C_F - 32C_F^2}{\epsilon^3} \\& + \frac{1}{\epsilon^2}(46.667C_A C_F - 8C_A^2 - 2.667n_f C_F) \\& - \frac{1}{\epsilon}(-67.226C_A C_F + 2.667n_f C_A - 5.6423C_A^2 - 4.444n_f C_F + 263.189C_F^2) \\& + (-316.07C_A C_F - 2.957n_f C_A + 54.485C_A^2 + 4.853n_f C_F + 641.097C_F^2) \\& + \epsilon(-531.488C_A C_F - 2.905n_f C_A + 92.248C_A^2 + 10.171n_f C_F + 874.517C_F^2) \\& + \mathcal{O}(\epsilon^2).\end{aligned}$$

Cross-Check with RG equations

Because of the independence of the cross section on the renormalisation scale μ , in Laplace space, RG equation for soft function is

$$\frac{d \ln \tilde{s}}{d \ln \mu} = \gamma_s = -\frac{d \ln H}{d \ln \mu} - \frac{d \ln \tilde{B}_1}{d \ln \mu} - \frac{d \ln \tilde{B}_2}{d \ln \mu}$$

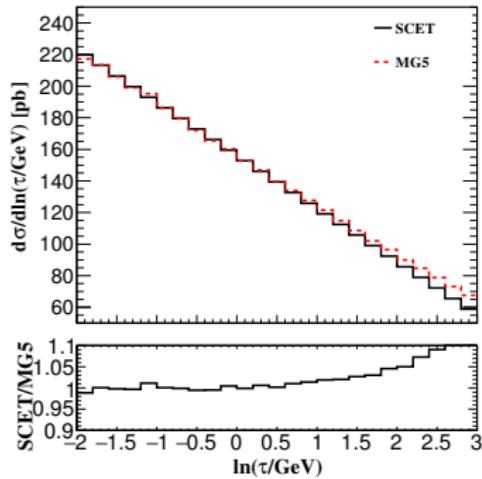
The expression for renormalisation factor Z_s is

$$\ln Z_s = \frac{\alpha_s}{4\pi} \left(\frac{\gamma_s^{(0)'} }{4\epsilon^2} + \frac{\gamma_s^{(0)} }{2\epsilon} \right) + \left(\frac{\alpha_s}{4\pi} \right)^2 \left(-\frac{3\beta_0 \gamma_s^{(0)'} }{16\epsilon^2} + \frac{\gamma_s^{(1)'} - 4\beta_0 \gamma_s^{(0)} }{16\epsilon^2} + \frac{\gamma_s^{(1)} }{4\epsilon} \right)$$

| | ϵ^{-4} | ϵ^{-3} | ϵ^{-2} | ϵ^{-1} |
|------------------|-----------------|---------------------|--------------------|-----------------|
| SCET prediction | -24.8889 | 96.8889 | 158.568 | 354.032 |
| real calculation | -24.8889 | 96.8888 | 158.577 | 353.820 |
| difference | 0 | -1×10^{-4} | 9×10^{-3} | -0.212 |

Comparison of the coefficients of ϵ^{-i} , $i = 1, 2, 3, 4$ in double real contribution in two different methods.

Application



The NLO τ distributions from
SCET and
MadGraph5_aMC@NLO

$$d\sigma = d\sigma(\tau < \tau^{\text{cut}}) + d\sigma(\tau \geq \tau^{\text{cut}})$$

- Can be used in NNLO calculation for coloured particle production using phase space slicing method
- Is one boundary condition for moving coloured particle production
- Can be extended to the case of single top production or top pair production at the LHC

Thank you !

