Axions

1. What are axions?

These lectures are about axions and why they represent (possibly) the most promising way to connect string compactifications to observational physics.

What are they? I use the word 'axion', scalar field $a$ that have an exact periodicity

$$a(\theta) = a(\theta + 2\pi).$$

That is, the axion field $a$ is an angular field, and is periodic: the state of the Hilbert space we with angular vev $\theta$ is identical to that with angular vev $\theta + 2\pi$. (cf axion monodromy, different usage).

As a scalar field has canonical mass dimension 1, we can also write this as

$$\langle a_{\theta} \rangle \text{ and } \langle a_{\theta} + 2\pi fa \rangle$$

are the same.
The moduli space of the axion field

can then be viewed as a circle

with circumference $2\pi f_a$.

The field range of the axion is then $2\pi f_a$.

$f_a$ is also called the axion decay constant

(this language is historical; also definitions may vary
by factor of $2\pi$ etc)

The exact symmetry under $a \rightarrow a + 2\pi f_a$
also resubmits the form of the potential

\[ V(a) \]

\[ 0 \quad 2\pi f_a \quad 4\pi f_a \]

not allowed

\[ 0 \quad 2\pi f_a \quad 4\pi f_a \]

allowed
As we shall see, this has important consequences for axion masses:

Axions are naturally extremely (exponentially) light. Unlike e.g. the Higgs field, perturbative quantum loops do not tend to renormalize the mass in a quadratically divergent fashion.
Axions

II. Why care about axions?

Why am I giving these lectures?

If string theory is to be a theory of physics, it must connect to the world of observations and experiment.

Axions provide one of the best ways to connect Planck-scale physics to observation, and to probe the deep UV through low-energy physics.

Several reasons:

* Axions are generic consequences of string compactifications: almost all compactifications contain axions in the low-energy spectrum.

* Searches for axions can probe couplings suppressed by scales $M > 10^{12}$ GeV: far higher than those accessible to colliders such as the LHC.

* The technology is orthogonal to those at the LHC: if the LHC does not find anything, there is no 20-year delay.
Axions are light - there is no energetic obstruction to producing axions.
III. Axions in Field Theory

Axions originate in the strong CP problem: why does the neutron have no electric dipole moment?

The gauge part of the QCD Lagrangian is

$$\frac{-1}{4 g^2} \text{Tr}(F_{\mu \nu} F^{\mu \nu}) + \frac{\theta}{8 \pi} \exp(\frac{-i g}{\theta} F_{\mu \nu} F^{\mu \nu})$$

The strong CP problem can be restated as:

Why is $\frac{\theta}{2\pi} \ll 10^{-10}$?

Such a fine-tuning of the $\theta$ angle of the strong force seems rather awkward to explain.

However, if $\theta$ is promoted to a dynamical field, so we have

$$\frac{-1}{4 g^2} \text{Tr}(F_{\mu \nu} F^{\mu \nu}) + \frac{\theta}{8 \pi} \exp(\frac{-i g}{\theta} F_{\mu \nu} F^{\mu \nu})$$

Then non-perturbative QCD instanton effects generate a potential for $\theta$, minimized at $\langle \theta \rangle = 0$.
Potential is approximately

\[ V(x) = \lambda_{QCD} c \cos \left( \frac{a}{f_a} x \right) \]

\[ m_a^2 = V'' = \frac{\lambda_{QCD}}{f_a^2} \]

\[ M_a = 10^{-2} \text{eV} \left( \frac{10^{10} \text{eV}}{f_a} \right) \]

Small mass is because mass only arises from non-perturbative effects.

The QCD axion is the original example of the axion.

A natural generalisation is to axion-like particles

These have a topological coupling to electromagnetism but not to the strong force.

\[ \frac{1}{2} \partial_a \phi \partial^a \chi + \frac{g}{f_M} \partial_M F^M_{\mu \nu} \]

\( \partial \phi \) gives a 28 year decay.
IV. Axions in String Theory

If string theory is true, what is the right low-energy Lagrangian of the world?

This depends on the compactification, but under general circumstances, axions are present.

(Although, the coupling to electromagnetism or QCD are much more model-dependent.)

Heterotic: World-sheet action is

\[ S_{\text{world}} = \frac{1}{2\pi \alpha'} \int \sqrt{g} + i B_2. \]

\[ \text{In path integral:} \]

\[ \int D\phi \ e^{-S[\phi]}. \]

\[ S_{B_2} \text{ appears as a phase \to physics \ is identical} \]

for \( S_{B_2} = b_2 \) and \( S_{B_2} = b_2 + 2\pi i \).

Dimensional reduction of \( B_2 \rightarrow \sum c_i b_2 \wedge \omega^i \) \( \text{(where } \omega^i \text{ are basis of 2-forms)} \)

\( \Rightarrow \) each non-trivial 2-cycle gives rise to an axion in the 4d effective field theory.
It also follows that such axions are irrelevant in perturbation theory.

String perturbation theory involves an expansion about the trivial embedding of the worldsheet.

As \( \int \sigma^2 \) is topological, once all embedding

contradicts to zero it vanishes.

Only for non-flat embedding weighted by \( e^{-t} \)

are sensitive to \( \int \sigma^2 \).

**Type IIA/IB**

Both theories contain D-branes, as well as N-R form \( \{ C_0, C_1, C_2, C_3, \ldots \} \).

The N-R form only couple directly to D-branes.

The D-brane action is

\[
\frac{2\pi}{(2\pi)^{-1}} \int d^d x \sqrt{g} e^{-\phi} + i C_n.
\]

Take a D7-brane wrapped on a 4-cycle. Then

the dimensionally reduced action is (confining W3 theory)

\[
\frac{9}{\text{Vol}(S^4)} \int d^4 x \sqrt{g} F_{\mu\nu} F^{\mu\nu} + C_n F_{\mu\nu}.
\]

\( C_n \rightarrow \Sigma C_n. \)
As for the heterotic string, reduction of
RCL form fields (in this case RCL form fields C_i)
along non-trivial extra-dimensional cycles produces
axions.

\[ \int C_i \to \text{axions, one for each independent cycle.} \]

String perturbation theory is insensitive to the exact
value of this.

It is only non-perturbatively (with the inclusion of
\text{D-instantons,}}
\text{gauge group})
that dependence on the
absolute value of \( \int C_i \) can be obtained.

\underline{Generally:} (most of the time)

Axions are associated with non-contractible
cycles in the extra-dimensional geometry.
Such cycles are common (e.g., CY compactifications) and can lead to \( O(100) \)
axion in low-energy theory.

Axion potentials are generated by effects that
are non-perturbative in \( g_s, \alpha' \), exponential \( \sim \frac{\text{Vol}}{g_s} \)

\( \therefore \) produce very small masses
also open string axion.

These are analogous to field theoretic axions.

If you have a U(1) charged field (where U(1) may be anomalous), and charge may be non-linearly realized,

then the residual symmetry contains an axion.

In this case the field range relates to the ren of the charged field $f_a - <\phi>$. 
V. Consequences for Low Energy Theory

Many string compactifications fit into structure of $N=1$ supersymmetric 4d effective field theory.

Described by Kähler potential and superpotential

$$L = \int d^4 x d^2 \sigma d^2 \theta \ K(\sigma, \theta) + \int d^4 x d^2 \theta \ W(\Phi)$$

Axion appear as imaginary parts of chiral multiplets

$$\tilde{\phi} = -2 + \circ \ 1 \ a = a + 1$$

Perturbative symmetry $\ln (T) \Rightarrow \ln (\bar{T}) + \epsilon$ implies

in perturbation theory

$$\left< \psi \bar{T} \psi \right> = \left< \psi (T + \bar{T}) \psi \right>$$

$$W(T) = W_0 \ (\text{constant})$$

The metric on moduli space is given by $k_{\bar{T} T}$

and the field range can be found by $\frac{1}{2\pi} \int_{\mathcal{M}} \sqrt{g}$. 

$\sigma = 0$
Example

\[ k = -3 \frac{M_p^2 \ln (1+1)}{4 \pi^2} \quad \text{ (no scale)} \]

\[ K_{tt} = + \frac{3 M_p^2}{(1+1)^2} = \frac{3 M_p^2}{4 \pi^2} \]

1. Metric for axion is

\[ \int d^4 x \frac{3 M_p^2}{4 \pi^2} \partial_\mu \partial_\mu a = \int d^4 x \left( \frac{3 M_p^2}{2 \pi^2} \right) \frac{1}{2} \partial_\mu \partial_\mu a. \]

1. Axion field range is

\[ \int_0^1 \frac{\sqrt{3} M_p}{\sqrt{2} \pi} = \sqrt{\frac{3}{2}} \frac{M_p}{\pi}. \]

1. In geometric regime (TTTT), field range is sub-planckian:

\[ \sqrt{\frac{3}{2}} \frac{M_p}{\pi} \ll M_p. \]

It is a big open question whether controlled models exist with true Planckian fields parametrically.
And if there is a limit, what is it?

Q. In a vacuum solution of string theory, what is the largest allowed axion field range?

As devoted in other lectures, this question is very important for models of inflation in string theory.

Trans-Planckian field excursions during inflation correspond to observable levels of tensor modes

\[ \frac{(\Delta \phi)^2}{M_p^2} \]

As their potential is naturally flat, axions are good inflation candidates in string theory.

For this I-modulus example, field range grows as \( \sim \frac{1}{T^2} \).

What happens to field range at small volumes?

- For cases with N=2 strings, where we have full calculations controlled via mirror symmetry, field range (up) at \( \frac{1}{M_p} \).

\[ \text{String duality symmetries cut off field ranges at small radii.} \]

- For three options:
  - Max field range \( \sim M_p \)
  - Max field range finite, but \( \gg M_p \)
  - Max field range unbounded.
VI. Axion Dark Matter

Axions exist are also a good dark matter candidate.

This is true both for the QCD axion and also for more general axions, provided they are massive and sufficiently long-lived.

The basic physics is similar in both cases — the misalignment mechanism — but easier to develope for more general axions.

How does it work?

1. We assume in the early universe $m_a < \lesssim H < f_a$ during inflation.

\[
-H\cdots
\]

During this period axion with a Hubble scale.

2. Dynamical equation for scalar field in de Sitter space is

\[
\ddot{\phi} + 3H \dot{\phi} + m_a^2 \phi = 0
\]

For $H > m_a^2$, Hubble friction dominates and field does not evolve.
While $H \approx m$, scalar field does not move, but is misaligned from 10 minimum.

As universe cools after inflation, $H$ reduces.

When $H \approx m$, field starts oscillating about minimum.

Initially, energy in axion field: $m^2(\Delta \phi)^2$.

Overall energy: $V = 3H^2m^2 - 3m^2M_p^2$.

Oscillating axion field now behaves as dark matter candidate (coherent oscillation of scalar field).

While universe's energy density is in form of radiation, its importance continues to grow.
Note (1) Amount of dark matter depends on initial misalignment angle.

(2) The larger $m_A$ is, the earlier it starts oscillating.

(3) For the QCD axion, the calculation are more subtle as $m_a = m_a(T)$ (as mass comes from interaction, and their strength depends on the创下 temperature which set $x_s(T)$).

The above misalignment mechanism can lead to axion dark matter.
VII. Axion as Dark Radiation.

Another important role for axions is as dark radiation candidates. This can place significant constraints on string compactifications.

String theory contains modular/penose, long-lived weakly interacting.

What is dark radiation? It is parametrized by \( N_{\text{eff}} \) and represents additional dark relativistic energy.

(annually, \( N_{\text{eff}} = 3.046 \) (three species of neutrino))

Additionally, \( \Delta N_{\text{eff}} \leq 0.5 \)

Axions can be dark radiation as they are light (and so will propagate relativistically).

How are they produced?

After inflation, we have reheating: inflation oscillates and decays.

However, as matter \( \propto \frac{1}{a^3} \) and radiation \( \propto \frac{1}{a^4} \), reheating is dominated by last scalar to decay.
Last scale to decay are those with weaker coupling.

String compactifications always contain moduli with gravitationally strong coupling:

\[ \Gamma \propto \left( \frac{1}{m^3} \right) \sim \left( \frac{100 \text{ TeV}}{m} \right)^7 \]

Moduli come to dominate the universe before decaying, late.

Visible decay of moduli (\( \phi \rightarrow gg, \phi \rightarrow \gamma \gamma \)) lead to reheating of the universe into hot big bang.

Hidden sector decays (e.g., \( \phi \rightarrow q \bar{q} \)) lead to dark radiation.

\[ \tau \phi \sim -3 \ln(1 + \tau) \]

\[ \phi = \sqrt{\frac{3}{2}} h_T \rightarrow \frac{3}{4} \phi^2 + \frac{3}{4} \phi \alpha \phi \alpha + \frac{3}{4} e^{-2\sqrt{2} \phi} \phi \alpha \phi \alpha \]

After \( \phi \) gets away, leads to

\( \phi \rightarrow q \bar{q} \) and dark cosmic axion background.

Avoiding over-production of dark radiation plus strong constraints on string \( \rightarrow \) cosmology.
VIII Constraints and Searches for Axions

I want to describe briefly some of the methods used to constrain (or search for) axions and axion-like particles.

For both cases I use SN1987A. These constraints come from the coupling to electromagnetism.

\[ aF_{\text{em}} \]

Pion-nucleon effect


\[
\begin{tikzpicture}
  \draw[thick,->] (0,0) -- (1,1);
  \draw[thick,->] (1,1) -- (2,0);
  \draw[thick,->] (2,0) -- (3,1);
\end{tikzpicture}
\]

Axions produced in the core of stars (or supernovae) provide an additional cooling channel.

Absence of such additional cooling constrains the QCD axion decay constant

\[ (F < 10^{-4} \text{ GeV}, \quad m_a < 10^{-3} \text{ eV}) \]
For sufficiently light axion or axion-like particles, $a \rightarrow \gamma$ back conversion can occur in astrophysical magnetic fields.

The coupling $\frac{e F^a}{M} = \frac{g}{E_0}$ produces a 2-particle axion-photon coupling in the presence of background $B$ fields.

Relativistic axions (ALPs) can convert both into photons.
For all $\alpha$, this contains the couple $M$ to be $M \geq 2\times 10^7$ N.