Compactifications 101

5D spacetime $\mathbb{R}^3 \times S^1$
coordinates $\{ x^\mu, y \}$, $y = y + 2\pi n \ell R$, $n \in \mathbb{Z}$

real scalar $\phi(x^\mu, y) = \sum_{k \in \mathbb{Z}} \phi_k(x^\mu) e^{iky/R}$

$\bar{\phi}(x^\mu, y) = \phi(x^\mu, y) \Rightarrow \phi^*_k(x^\mu) = \phi_{-k}(x^\mu)$

$S = -\int d^4x \, dy \, \partial \mu \phi \partial^\mu \phi = -\int d^4x \, dy \, \left( \partial \mu \phi \partial^\mu \phi + \partial y \phi \partial^y \phi \right)$

$= -\int d^4x \, dy \, \sum_{k, \ell} \left( \partial \mu \phi_k \partial^\mu \phi_{-\ell} - k \ell \phi_k \phi_{-\ell} e^{iky/R} \right)$

$= -\int d^4x \, (2\pi R) \sum_k \left( \partial \mu \phi_k \partial^\mu \phi_k + \frac{k^2}{R^2} \phi_k(x^\mu) \phi_k(x^\mu) \right)$

$k=0$ massless scalar in 4D
$k \neq 0$ KK-tower of massive scalar fields

In string compactifications we usually neglect the KK-tower, i.e. we restrict to energies below the KK-scale $E \ll M_{KK} \sim \frac{1}{R}$
Fields with spacetime index

5D vector field \( A_M(x^\mu, y) = \left( \sum_k A_{\mu k}(x^\mu) e^{iky/R} \right) \)
\( \hat{A}_{\mu k} = A_{\mu k}^* \)

\[ S = - \int \! d^4x \, dy \left( F^{MN} F_{MN} \right) = - \int \! d^4x \, dy \left( F_{\mu \nu} F^{\mu \nu} + F_{\mu y} F^{\mu y} \right) \]

\[ F_{\mu y} = \partial_y A_\mu - \partial_\mu A_y = \sum_k \frac{ik}{R} A_{\mu k} e^{iky/R} - \sum_k \partial_\mu A_{y k} e^{iky/R} \]

\[ F_{\mu y} F^{\mu y} = F_{\mu y} F_{\nu y} = \sum_k \left( - \frac{kl}{R^2} A_{\mu k} A_{\nu k} + \partial_\mu A_{y k} \partial_\nu A_{y k} \right) \]

\[ - \frac{ik}{R} A_{\mu k} \partial_\nu A_{y k} - \frac{ik}{R} A_{\nu k} \partial_\mu A_{y k} \right) \]

\[ S = - \int \! d^4x \, (2\pi R) \sum_k \left( F_{\mu k} F_{\nu k} + \frac{k^2}{R^2} A_{\mu k} A_{\nu k} \right) \]

\[ + \partial_\mu A_{y k} \partial_\nu A_{y k} - \frac{ik}{R} A_{\mu k} \partial_\nu A_{y k} \left( \partial_\nu A_{y k} + \frac{ik}{R} \hat{A}_{\nu k} \right) \]

\[ = - \int \! d^4x \, (2\pi R) \sum_k \left( F_{\mu k} F_{\nu k} + \partial_\mu A_{y k} \partial_\nu A_{y k} + \text{massive KK vectors} \right) \]

\[ \text{massless scalar} \]
$A_{y/0}$ is a real scalar that arises since the vector $A_m$ wraps a 1-cycle, i.e. the $S^1$

A higher dimensional analogue:

fields arising in type II B string theory

\begin{align*}
B_{2 \ \mu \nu} & \text{ can wrap } 2\text{-cycles} \\
C_{2 \ \mu \nu} & \text{ can wrap } 2\text{-cycles} \\
C_{4 \ \mu \nu \rho \sigma} & \text{ can wrap } 4\text{-cycles}
\end{align*}

\{ \text{ gives scalars in 4D} \}

Additional scalar can come from the metric, for $\mathbb{R}^{31} \times S^1$

\[
 g_{\mu \nu} = \begin{pmatrix}
 g_{\mu \nu} & g_{\mu y} \\
 g_{y \mu} & g_{yy}
\end{pmatrix}
\]

\[
\begin{matrix}
4D \text{ vector} & \leftrightarrow & 4d \text{ scalar}
\end{matrix}
\]
String theory is very difficult and we cannot usually solve it in non-trivial backgrounds.

The low energy limits of the super string theories are 10D supergravities (strings $\Rightarrow$ point particles)

$$E \ll M_{\text{string}} = (\alpha')^{-\frac{1}{2}}$$

We will restrict here to type IIB string theory/supergravity which is the most studied in the context of flux compactifications and string cosmology.

The 10D supergravity preserves 32 supercharges in flat space and the spectrum is

\[ \begin{align*}
\mathcal{G}^{MN} & \quad \text{NS NS sector} \\
B^{MN} & \\
\phi & \\
C_0 & \quad \text{RR sector} \\
C_2^{MN} & \\
C_4^{MNP} &
\end{align*} \]
There are also fermions coming from the NSR sector. Since we have supersymmetry we do not need to keep track of the fermions. Their action is fixed by the bosonic action combined with the SUSY transformations.

We want to compactify from 10D to 4D and preserve at most $N=1$ SUSY, i.e., 4 supercharges in 4D. The reason for this is that for 4D $N \geq 2$ the left- and right-handed fermions are related via supersymmetry and therefore have the same quantum numbers.

In the standard model of particle physics the left-handed electron and the neutrino sit in an SU(2) doublet $(e_L, \nu_L)$, the right-handed electron is an SU(2) singlet $e_R \Rightarrow N \leq 1$.

$N=0$ would mean that all SUSY is broken at the compactification scale $\Lambda \sim \frac{1}{R} \sim \frac{1}{(vol_6)^{1/2}}$. This is possible but technically harder.
We therefore restrict to 4D $\mathcal{N}=4$ models for which we have the best mathematical tools and that are the best studied models.

In 10D the 32 supersymmetry transformation parameters are two 10D Majorana-Weyl spinors $\epsilon^i$, $i=1,2$.

The 10D SUSY transformation for the gravitino $\psi_M$ (the spin $\frac{3}{2}$ superpartner of the graviton) is

$$\delta \psi_M = V_M \epsilon^i = (\partial_M - i \frac{1}{4} \omega_{M}^{AB} \Gamma_{AB}) \epsilon^i = 0$$

In flat space, the two 16 component spinors $\epsilon^i$ satisfy this equation and we preserve 32 supercharges.

For a CY$_3$ manifold, the non-trivial spin connection $\omega_{M}^{AB}$ allows only for 4 non-zero components in each $\epsilon^i$. Hence a CY$_3$ manifold preserves 8 supercharges which is $\mathcal{N}=2$ in 4D.

A compactification cannot lead directly to $\mathcal{N}=1$ in 4D since for each $\epsilon^i$ we can either get one or zero 4D spinor with 4 components. $\epsilon^1$ and $\epsilon^2$ have the same chirality and therefore we can only get $\mathcal{N}=2$ (or $\mathcal{N}=0$) or $\mathcal{N}=1$. 
To get $\mathcal{N}=1$ in 4D we need to do an additional orientifold projection:

We mod out by $\Omega_p (-1)^F I$

$\Omega_p$ string world sheet parity $\Omega_p: \sigma \to \bar{\sigma}$

$F_i$ left-moving worldsheet fermion number

$I$ spacetime involution, e.g. $I: x^\mu \to x^\mu$

$y^i \to -y^i$, $i=1, \ldots, 6$

gives 03-plane extending along $x^\mu$, sitting at $y^i=0$

An orientifold projection break $\frac{1}{2}$ of the supersymmetry flat space example $\mathcal{E}^2 = \Gamma_{0123} \mathcal{E}^2 = -\Gamma_{456789} \mathcal{E}^2$

$\Rightarrow \mathcal{N}=1$ for $\mathcal{C}_\frac{1}{3}$ compactifications with orientifold projection

The involution $I$ can have fixed points of codim 6 and codim 2, i.e. give rise to 03- and 07-planes ($05/09$ also possible)
The orientifold projection also truncates the spectrum. Since $(\Omega \rho (-1)^F I)^2 = 1$ all fields are either mapped to plus themselves or minus themselves (in the latter case they are projected out).

**Example:** $B_{mn}$ and $C_{2mn}$ are mapped to minus themselves by $\Omega \rho (-1)^F I$. So they only give rise to 4D fields, if their "legs" extend along 2-cycles that are odd under $I$.

**Flat space:** $I: x^m \mapsto x^m$, $y^i \mapsto -y^i$  
$B_{ur} \mapsto B_{ur} \}$ projected out  
$B_{ij} \mapsto B_{ij}$  
$B_{ui} \mapsto -B_{ui}$ survives  
(but no 1-cycles in CY$_3$)

$\phi, C_0$ are invariant under $\Omega \rho (-1)^F I$. They give rise to a 4D complex scalar $\tau = C_0 \exp \tau$.  
$\tau$ is called the axio-dilaton
The internal part of the metric gives rise to scalars that for a CY₃ can be conveniently packaged into a 2-form $\Omega^{a\bar{b}} = \Omega^{2,3}$ called the "Kähler form" and a 3-form $\Omega_{abc}$ (not to be confused with the world-sheet parity operator $\Omega_p$). $\Omega_{abc}$ is called the holomorphic 3-form since it has 3 holomorphic indices.

The real scalar fields $\Omega^{a\bar{b}}$ control the size of 2-cycles inside the CY₃:

The complex scalar fields inside $\Omega_{abc}$ determine the shape of the CY₃ manifold:

[2D analogue: $\Theta \mapsto \Theta^*$]
We can now reduce the 10D IIB SUGRA action

\[
S_{\text{bosonic}}^{\text{IIB}} = \frac{1}{2k_0^2} \int d^4x \, d^6y \, \sqrt{-g} \left\{ R - \frac{\mathcal{C}_M T^M}{\text{Im}(\zeta)} - \frac{G^3 \cdot \overline{G^3}}{12 \text{Im}(\zeta)} - \frac{F_5^2}{4 \cdot 5!} \right\} \\
+ \frac{1}{8 \cdot 4k_0^2} \int \frac{C_4 \cdot G_3 \cdot \overline{G_3}}{\text{Im}(\zeta)} + S_{\text{loc}}
\]

\[
G_{\mu\nu} = \begin{pmatrix} e^{2A(y)} & g_{\mu\nu} & 0 \\ 0 & e^{-2A(y)} & g_{ij} \\ 0 & g_{ij} & 0 \end{pmatrix}
\]

g_{\mu\nu} \text{ maximally symmetric: Minkowski, ds or AdS}

\[
F_{\mu\nu} = dC_\rho \quad H_3 = dB_2
\]

\[
\tau = G_0 + i e^{-\Phi}
\]

\[
G_3 = F_3 - \tau H_3
\]

\[
\tilde{F}_5 = F_5 - \frac{1}{2} C_2 \wedge H_3 + \frac{1}{2} B_2 \wedge F_3
\]

S_{\text{loc}} \text{ contribution from O-planes and/or D-branes}

G_3 \cdot \overline{G_3} \text{ and } F_5^2 \text{ denote contractions with the metric}
Now we can reduce this to 4D \( \rightarrow \) takes some time \( \rightarrow \) skip details

Simplifying assumptions:

1) Neglect KK-modes: \( E \ll M_{\text{KK}} \)

2) Neglect vectors (don't arise unless we have \( H^3_+(CY_3) \neq 0 \) no even 3-cycles)

3) Assume \( B_{MN} \& C_{2MN} \) don't contribute

\( \Rightarrow \) \( H^2_-(CY_3) = 0 \) no odd 2-cycles

The resulting 4D theory is constrained by \( \mathcal{N}=1 \) supersymmetry.

4D \( \mathcal{N}=1 \) SUGRA

In 4D \( \mathcal{N}=1 \) SUGRA we have only complex scalars

\[ \tau = C_0 \text{ + i } e^{-\phi} \] \( \text{axio-dilaton} \)

\[ \Pi^A = \int \Omega_{abc} \] \( \Sigma^A_3 \) complex structure "moduli"

\[ A = 1,2,\cdots, h^3 \] (projective coordinates, only \( h^3-1 \) independent scalars)

\[ T^B = \int C_4 + i J^B \] \( \Sigma^B_4 \) Kähler moduli

\[ B = 1,2,\cdots, h^4_+ = h^2_+ = h^4 \] number of even 4-cycles
The 4D $\mathcal{N}=1$ SUGRA is described in terms of 2 functions (since we don't have vectors):

- the holomorphic superpotential $W(\phi^i) \in \mathbb{C}$
- the real Kähler potential $K(\phi^i, \bar{\phi}^i) \in \mathbb{R}$

The bosonic action for $M_{pl} = 1$ is

$$\mathcal{L}_{\text{bosonic}} = -\int d^4x \sqrt{-g} \left[ -\frac{1}{2} R + K_{IJ} \partial^I \phi^i \partial^J \bar{\phi}^j + V(\phi^i, \bar{\phi}^i) \right]$$

$$V(\phi^i, \bar{\phi}^i) = e^K (K^{IJ} D_I W D_J \bar{W} - 3|W|^2) \in \mathbb{R}$$

$$K_{IJ} = \partial \phi^i \partial \bar{\phi}^j K(\phi^i, \bar{\phi}^j), \quad K^{IJ} \text{ inverse matrix}$$

$$K^{IJ} K_{JL} = \delta^I_L$$

$$D_I W = \partial^I W + W \partial^I K$$

In the (flux less) case we have

$$K = -2 \ln (\text{vol}_c) -2 \ln (-i(\mathcal{C} - \bar{\mathcal{C}})) - \ln \left( -i \int_{\mathcal{C}_3} \omega \bar{\omega} \right)$$

$$W = 0 \Rightarrow V = 0$$

$$\text{vol}_c = K_{B_1 B_2 B_3} T^{B_1} T^{B_2} T^{B_3}$$

$$T^B = \int_{\sum_{B_2}^B}$$

$$\text{Im} (T^B) = \frac{\partial \text{vol}_c}{\partial T^B}$$
The moduli problem

We do not see any massless scalars. If there were massless scalars then corrections might lead to minima outside the regime of validity of our theory:

small volume $\text{Re}(T^B) \ll 1 \Rightarrow \alpha'$ corrections

large coupling $\text{Im}(\tau) = e^{-\phi} \ll 1 \Rightarrow$ string loop corrections

infinity $\text{vol}_6 \rightarrow 10$ D theory (decompactification limit)

very light scalars can also lead to 5th forces and problems in the early universe cosmology.

How do we make the scalars heavy?

(see Giddings, Kachru, Polchinski
hep-th/005097)

3-form fluxes $F_3, H_3$ through 3-cycles can stabilize the axio-dilaton at small coupling and all complex structure moduli:

$$ W = \int_{C_{Y_3}} G_3(\tau) \wedge Q_3(\Pi^A) $$

Gukov, Vafa, Witten
hep-th/9906070
Note: 1) CY₃ manifolds have no 1- and 5-cycles so we cannot turn on topologically non-trivial F₄ or F₅.

2) The volume moduli are associated with even 0, 2, 4, 6-cycles but we have no fluxes with an even number of legs.

3) Tadpole: The orientifold O3-planes (as well as D3-brane) and the F₃ + H₃ flux

\[ dF₅ = ddC₄ + H₃ \wedge F₃ + S_{D3/103}^{loc} \]

\[ \int_{CY₃} dF₅ = 0 = \int_{CY₃} H₃ \wedge F₃ + Q_{D3/103}^{loc} \]

There is an upper limit on the number of fluxes we can turn on.
Let us restrict to the case of a single Kähler modulus $T$

$$\Rightarrow \ vol_6 = -i (T-T)^{\frac{3}{2}}$$

$$\kappa = -3 \ln (-i (T-T)) - \ln (-i (T-T)) - \ln (-i \tilde{\omega} \tilde{\omega}) \quad \text{for } CY_3$$

$$W = \int_{CY_3} G_3(\tau) \omega(\pi \eta)$$

The Kähler sector enjoys a no-scale property:

$$D_T W = \partial_T W + W \partial_T k = -\frac{3W}{T-T}$$

$$k_{T T} = \partial_T \partial_T k = -\frac{3}{(T-T)^2} = \frac{3}{4 \text{Im}(T)} > 0$$

$$\kappa_{T T} D_T W \overline{D_T W} = -\frac{(T-T)^2}{3} (\frac{-3W}{T-T}) \left( + \frac{3W}{T-T} \right) = 3 |W|^2$$

$$\Rightarrow \quad V = e^k \left( \kappa_{T T} D_T W \overline{D_T W} + \kappa \overline{\partial} D_T W \overline{D_T W} + k A_i \overline{A_i} D_{A_1} W \overline{D_{A_1} W} \right. \left. - 3 |W|^2 \right)$$

$$= e^k \left( \kappa \overline{\partial} D_T W \overline{D_T W} + k A_i \overline{A_i} D_{A_1} W \overline{D_{A_1} W} \right) \geq 0$$

$$\Rightarrow \quad e^k \propto \frac{1}{i (T-T)^3} \Rightarrow \quad D_T W = D_{A_1} W = 0 \quad \text{otherwise}$$

$\text{Im}(T)$ will run to infinity, i.e. to 10D flat-space.
We can find Minkowski solution \((V_{\text{min}} = 0)\) by solving \(D_T W = D_A W = 0\) but \(T\) remains a flat direction.

**SUSY is generically broken since**

\[ F_T = D_T W = - \frac{3 W_{\text{min}}}{T - \tilde{T}} \]

i.e. if \( W_{\text{min}} = \int G_3 (\tau_{\text{min}}) \cdot \Omega_3 (\tilde{\Pi}^A_{\text{min}}) \neq 0 \) then

**SUSY is broken.**

The first dS vacua (i.e. \(V_{\text{min}} > 0\)) in string theory

Kachru, Kallosh, Linde, Trivedi hep-th/0301240

String theory does not allow for exactly directions like a continuous shift symmetry

\[ \Rightarrow \text{Re} (T) \text{ is lifted by corrections} \]

1) Euclidean instantons can wrap 4 internal directions in the \(CY^3\) (these branes are instantons = localized in time)

\[ \Rightarrow W \rightarrow W + A e^{2\pi i T} \]

breaks to a discrete shift symmetry

"\(A\" is a function of the complex structure moduli \(A = A(\tilde{\Pi}^A)\)"
2) A stack of D7-branes can extend along the 4 non-compact directions and wrap an internal 4-cycle. The D7-branes give rise to a non-abelian gauge theory with gauge group SU(N).

If there is no "matter" (i.e. if \( h^{IV}(\Sigma^4) = 0 \)), then the theory undergoes gluino condensation \( \langle \gamma_2 \rangle \neq 0 \). This also leads to a superpotential contribution

\[
W \rightarrow W + A e^{2\pi i T_N}
\]

3) The Kahler potential \( K = -3 \ln (|T - \bar{T}|) \) receives a corrections (see below).

The superpotential corrections of the form

\[
\Delta W = Ae^{iaT} \quad \text{with} \quad a = \frac{s2\pi}{2\pi N}, \quad \text{D7's are exponentially small for large volume (which we need to suppress corrections)}
\]

\[
\Delta W = Ae^{-a \text{Im}(T) + ia \text{Re}(T)}
\]

So \( \Delta W \) is smaller than the flux contributions:

First stabilize all complex structure moduli and the axio-dilaton

\[
\Rightarrow \quad W = \sum_{G_3} G_3(\Sigma_{\text{min}}) \quad \Sigma_{\text{min}} = W_0 = \text{const.}
\]

An \( \{ T_{\text{min}} \} = \text{const.} \).
Now we can study the Kahler modulus stabilization \( K = -3 \ln (-i(T - \bar{T})) \)
\( W = W_0 + A e^{2aT} \)

There exist a supersymmetric minimum that satisfies \( \mathcal{D}_T W = 0 \). We find \( \text{Re}(T) = 0 \) and the following potential

\[
V(\text{Im}(T))
\]

\[
\begin{array}{c}
0 \\
\downarrow \\
\text{Im}(T)
\end{array}
\]

Problems:
1) Supersymmetry is not broken
2) The value of the potential at the minimum \( V_{\text{min}} < 0 \). Observation requires \( V_{\text{min}} > 0 \)

Solution: Add an "uplifting term"; an anti-D3-brane

This breaks supersymmetry spontaneously and adds a positive term to the potential

\[
V \rightarrow V + \frac{D}{\text{Im}(T)^3} \quad \text{(or } V + \frac{D}{\text{Im}(T)^2} \text{ with warping)}
\]

For example, for \( W_0 = 10^{-4}, A = 1 \)
\( a = 1, D = 3 \times 10^{-9} \)

one gets

\[
V(\text{Im}(T))
\]

\[
\begin{array}{c}
V_{\text{today}}^2 \\
\downarrow \\
0
\end{array}
\]
Comments:
- The SUSY breaking scale $\tilde{V}_{\text{min}} = V_{\text{today}} > 0$
- $W_0$ is usually order 1 but cancellations can lead to $|W_0| \ll 1$
- $A = 1$ is reasonable but hard to calculate in practice
- $\alpha = 1$ a stack of $N \approx 10, 2\pi$ D7-branes
- $D = 3 \times 10^{-9}$ is related to the tension of the anti-D3-brane
  If the anti-D3-brane is in a region of strong warping $e^{-A} \ll 1$, then $D$ is naturally exponentially small

The Large Volume Scenario (LVS)

Balasubramanian, Berglund, Conlon, Quevedo, hep-th/0502058

- A different way of stabilizing the Kähler moduli sector
- Leads to exponentially large volume
- Uses $\alpha'$ corrections from string theory
- Requires at least two Kähler moduli and a CY$_3$ manifold of "swiss cheese type" with negative Euler number $X(CY_3) < 0 \Rightarrow h^{2,1} > h^{1,1}$
\[ V_{\text{k"{a}hler}} = -2 \log(\text{vol}_6) \Rightarrow -2 \log \left[ \text{vol}_6 + \frac{\xi}{2} \left( \frac{\alpha'(z-\bar{z})}{2} \right)^{3/2} \right] \]

\[ \xi = - \frac{\xi(3) \chi(CY_3)}{2(2\pi)^3} \]

need \( \xi > 0 \)

The superpotential is not renormalized in perturbation theory but will again receive the same non-perturbative corrections.

"Swiss cheese volume"

Restrict to two moduli \( T_{\text{large}}, T_{\text{small}} \)

\[ \text{vol}_6 = \text{Im}(T_{\text{large}})^{3/2} \text{Im}(T_{\text{small}})^{3/2} \]

\[ \text{vol}_6 > 0 \Rightarrow \text{Im}(T_{\text{large}}) > \text{Im}(T_{\text{small}}) \]

\[ W = W_0 + A e^{i\alpha T_{\text{small}}} + \hat{A} e^{i\hat{\alpha} T_{\text{large}}} \]

\[ \text{subleading} \Rightarrow \text{neglect} \]

Upon minimizing the scalar potential one finds

\[ \text{vol}_6 \leq e^{i\alpha \text{Im}(T_{\text{small}})} |W_0| \gg 1 \]

\[ \text{Im}(T_{\text{small}}) \times \frac{\xi}{2} \gg 1 \]

\[ |W_0| \text{ does not need to be particularly small} \]
The scalar potential has a non-supersymmetric AdS minimum, i.e. $V_{\text{min}} < 0$.

One can again obtain $V_{\text{min}} > 0$ by adding an anti-D3-brane as uplift.
The scales in string compactifications

String scale \( M_s = \frac{1}{f_{\alpha''}} \)

The internal volume \( \text{vol}_6 = \sqrt{\alpha'} \Rightarrow M_{\text{KK}} \sim M_s \sqrt{f_{\alpha''}} \)

4D Planck scale \( M_{\text{pl}} \sim \frac{1}{g_s} \left( \frac{M_s}{M_{\text{KK}}} \right)^3 M_s \sim \frac{f_{\alpha''}}{g_s} M_s \), \( g_s = e^g \)

We want to neglect string loops \( \Rightarrow e^g \ll 1 \)
(extra) \( \alpha' \) corrections \( \Rightarrow \text{vol} \gg 1 \)

This leads to
\[
\begin{align*}
\text{effective 10D SUGRA} & \downarrow \\
M_{\text{KK}} & \ll M_{\text{flux}} \ll M_{\text{KK}} \ll M_s \ll M_{\text{pl}} \sim \frac{f_{\alpha''}}{g_s} M_s = 10^{19} \text{GeV} \\
\text{for LVS} & \uparrow \\
& \text{neglecting KK-tower} \\
& \text{for KKLT} & \text{large volume Small coupling}
\end{align*}
\]

For large field inflation with \( V_{\text{inf}} \sim 10^{16} \text{GeV} \) we can neglect only fields with masses above the Hubble scale \( H \sim \sqrt{\frac{V_{\text{inf}} M_{\text{pl}}}{f_{\alpha''}} \sim 10^{14} \text{GeV}} \)

So ideally we want \( M_{\text{KK}} \gg H \sim 10^{14} \text{GeV} \)

This is technically challenging but not impossible.